European Centre for Theoretical Studies in Nuclear Physics and Related Areas

FONDAZIONE BRUNO KESSLER

INFN Istituto Nazionale di Fisica Nucleare

TIFPA Trento Institute for Fundamental Physics and Applications

From continuum QCD to hadron phenomenology

Daniele Binosi

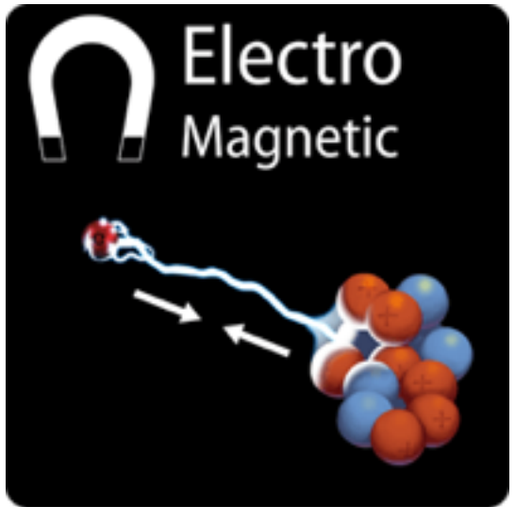
ECT* - Fondazione Bruno Kessler, Italy

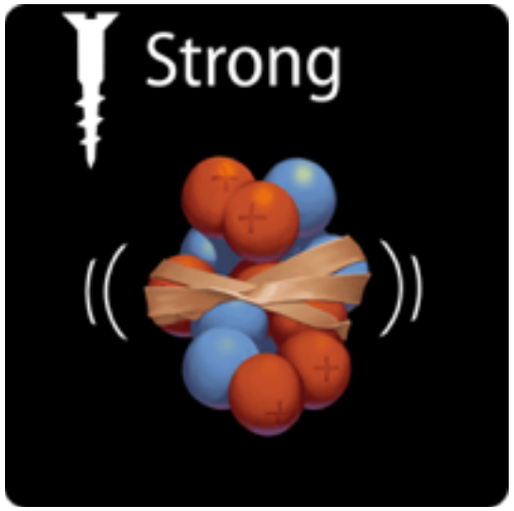
QCD-TNT 4, Ilha Belha, Brazil

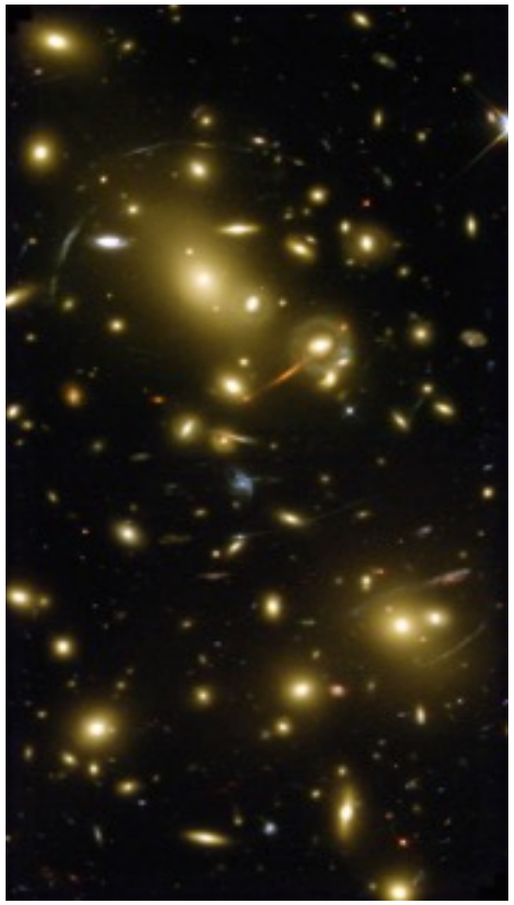
August 31 - September 4, 2015

thank **you**

Standard Model







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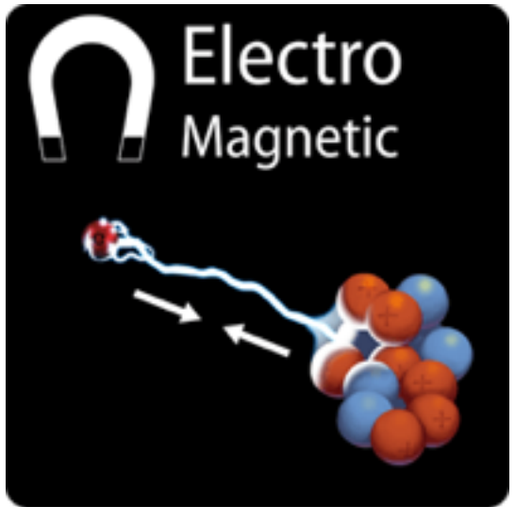

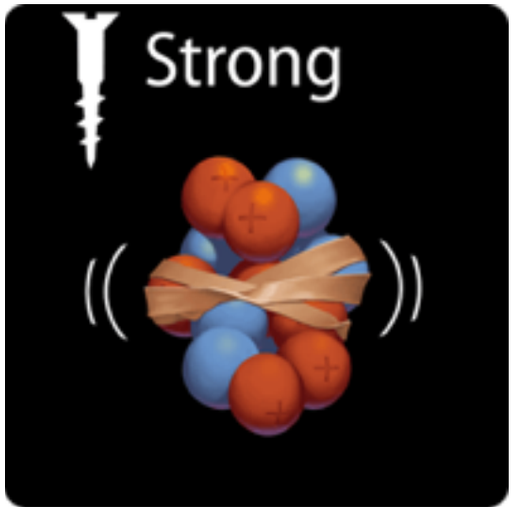

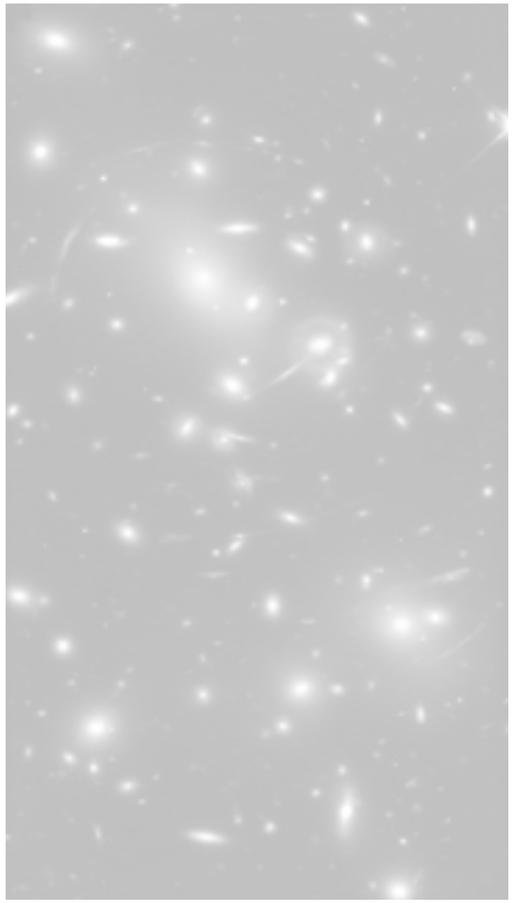

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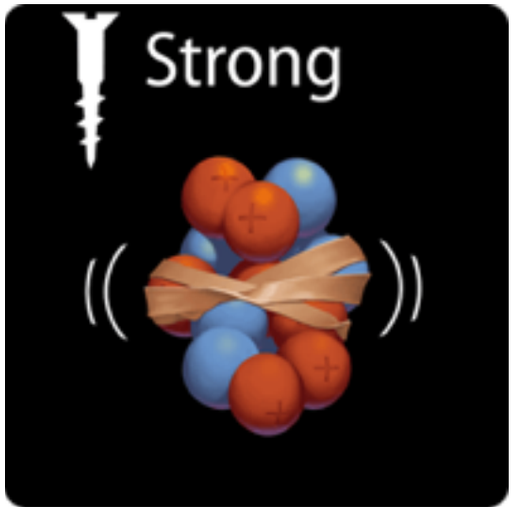
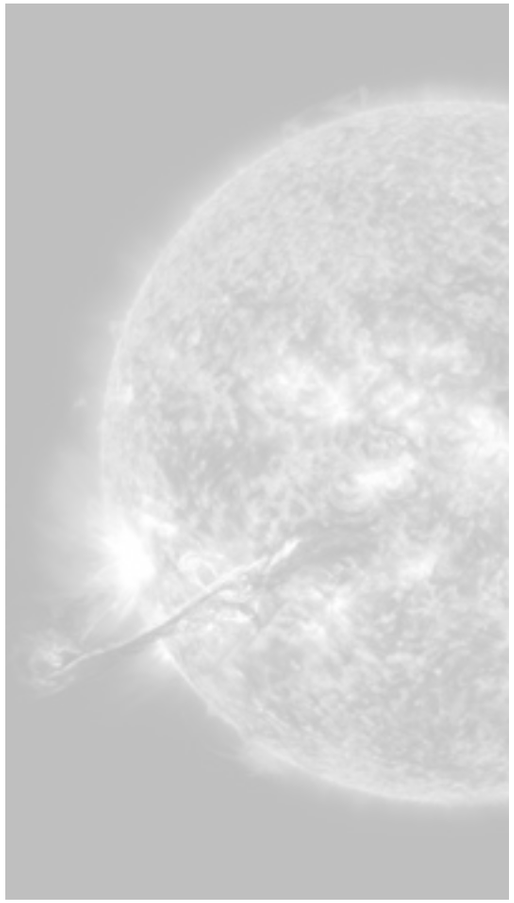
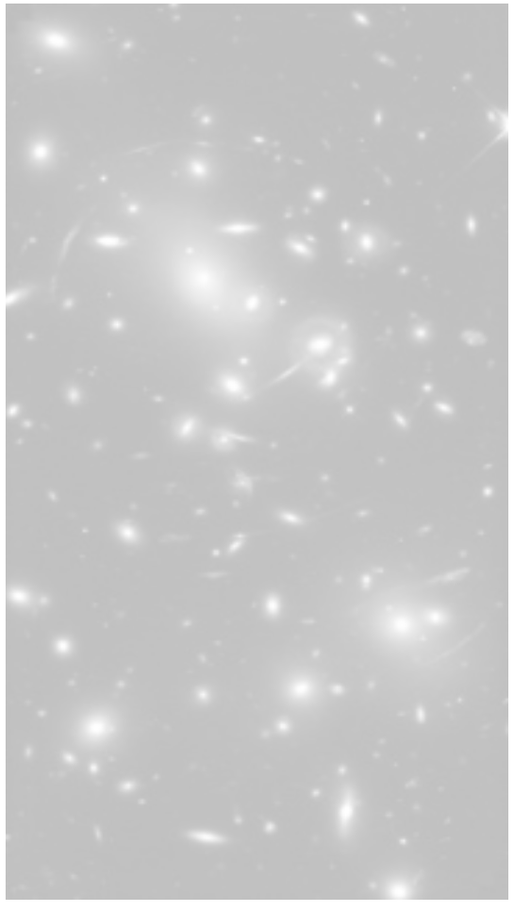
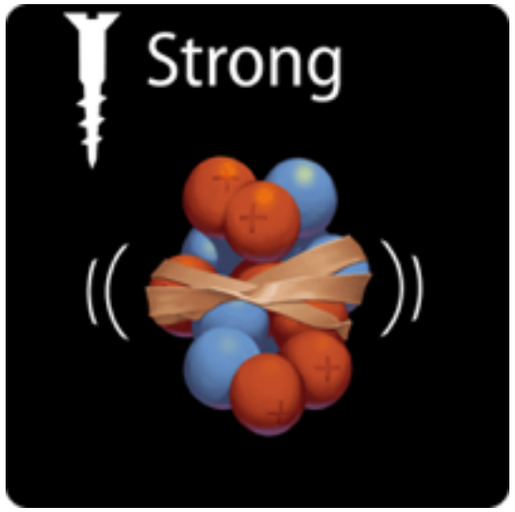
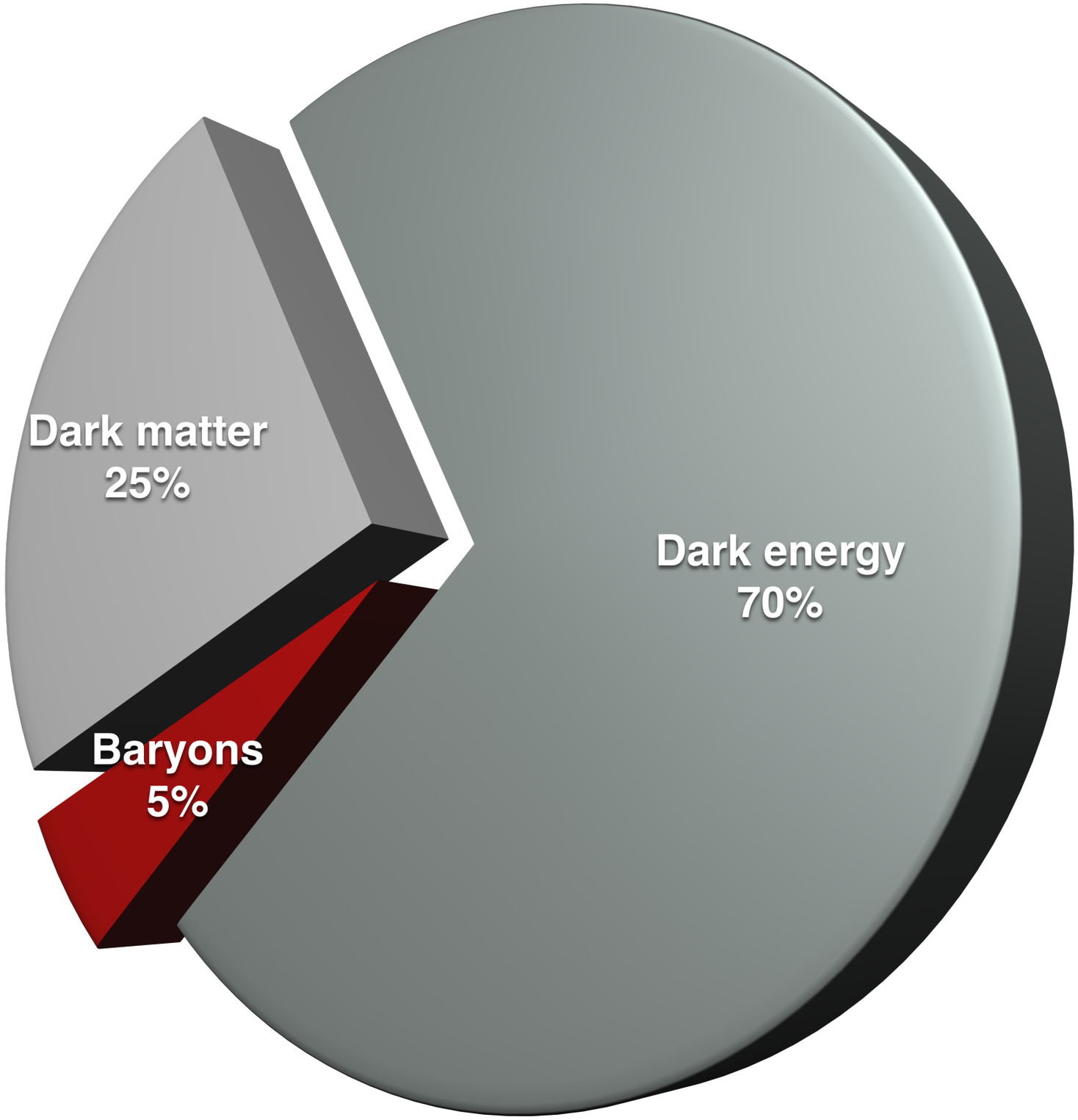






Standard Model: QCD





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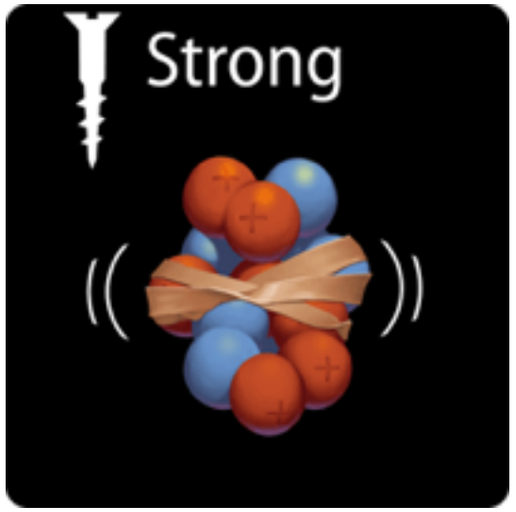


- **QCD action:**
  matter (fermions); gauge (gluons+ghosts)

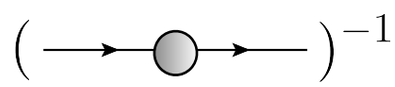
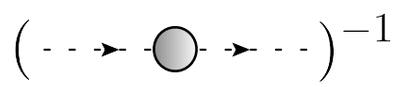
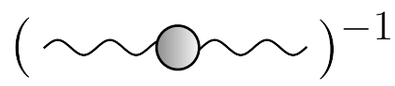
$$S_{\mathrm{QCD}} = \int \mathrm{d}^4x \, (\mathcal{L}_{\mathrm{I}} + \mathcal{L}_{\mathrm{GF+FPG}})$$

$$\mathcal{L}_{\mathrm{I}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \bar{\psi}^i_{\mathrm{f}} (i\gamma^\mu D_\mu - m)_{ij} \psi^j_{\mathrm{f}}$$

$$\mathcal{L}_{\mathrm{GF+FPG}} = s(\bar{c}^a \mathcal{F}^a - \xi/2 \bar{c}^a b^a)$$

  - **Gauge invariant part**
  - **Gauge fixing + FP ghost:**
    BRST exact, does not appear in the spectrum

- **Properties are encoded in Green's functions:**
  Schwinger-Dyson (SD) equations are their quantum eom

  - **Quark propagator:** 
  - **Ghost propagator:** 
  - **Gluon propagator:** 

- **Nonperturbative, covariant, IR/UV, light/heavy quarks; but:**
  infinite system of coupled integral equations

  - **Truncations:**
    possibly gauge invariant
  - **Ansätze/input:**
    pQCD, lattice, hadron properties

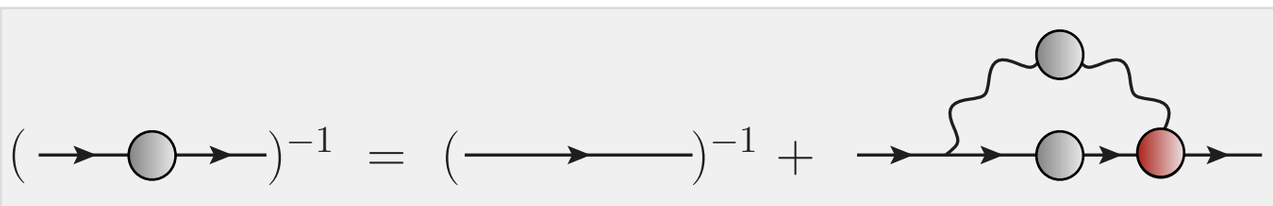
Dynamical quark mass

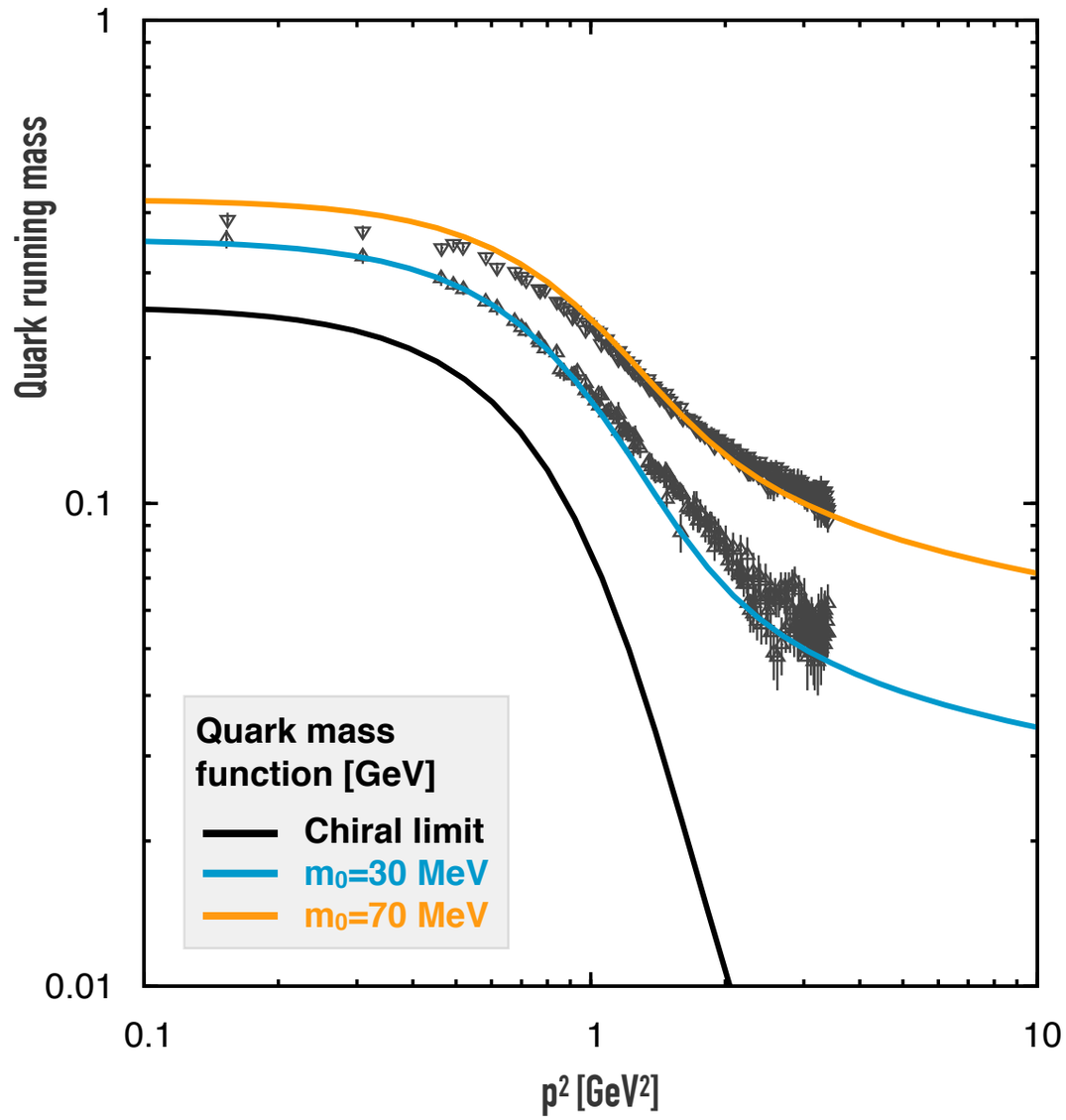
- **Dynamical chiral symmetry breaking:**
  generates “constituent-quark” masses

- **Add nothing to QCD:**
  effect achieved purely through the theory’s dynamics

- **Most important mass generating mechanism:**
  responsible for ~98% of the proton’s mass (Higgs mechanism almost irrelevant for light quarks)

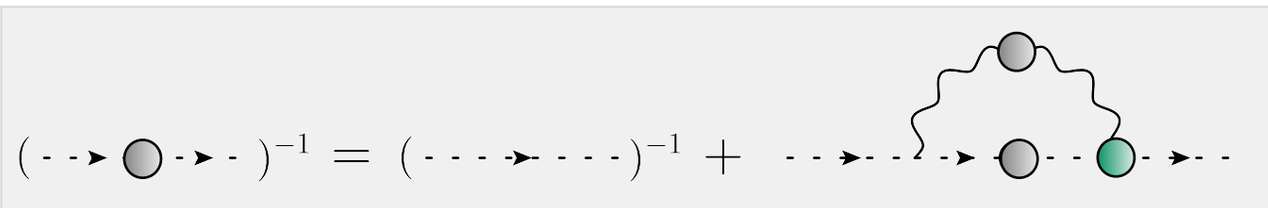
- **Realized through the quark gap equation:**

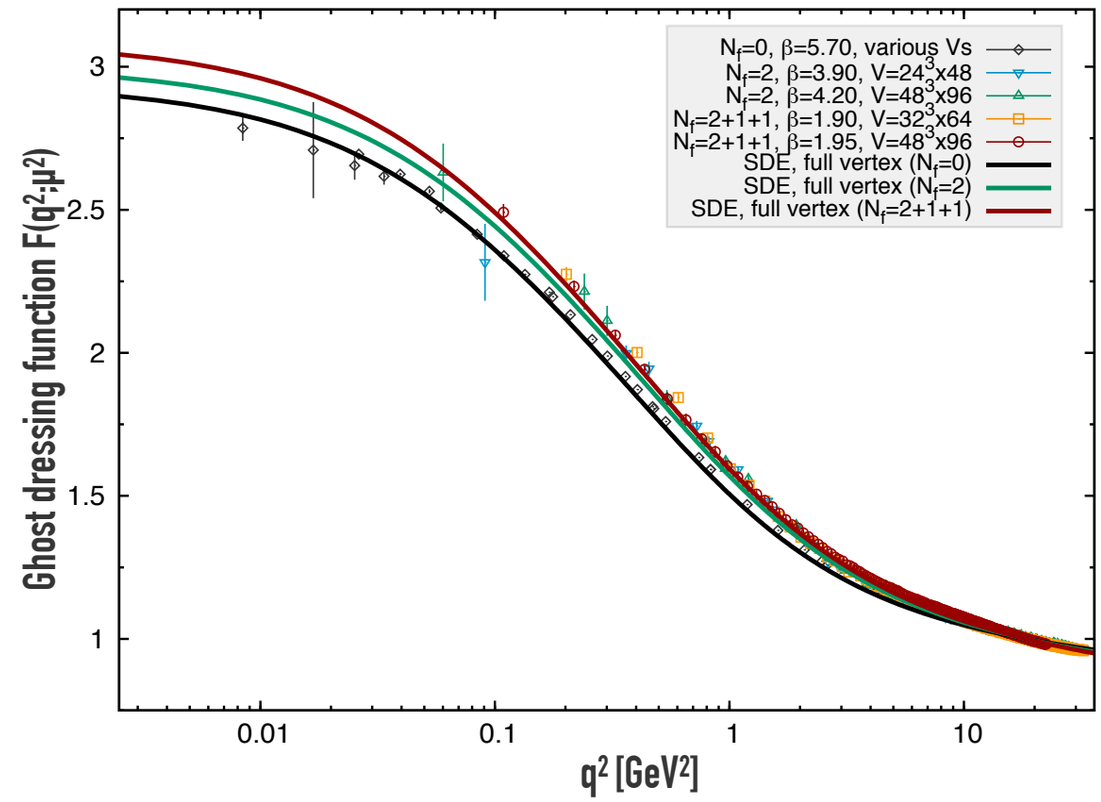




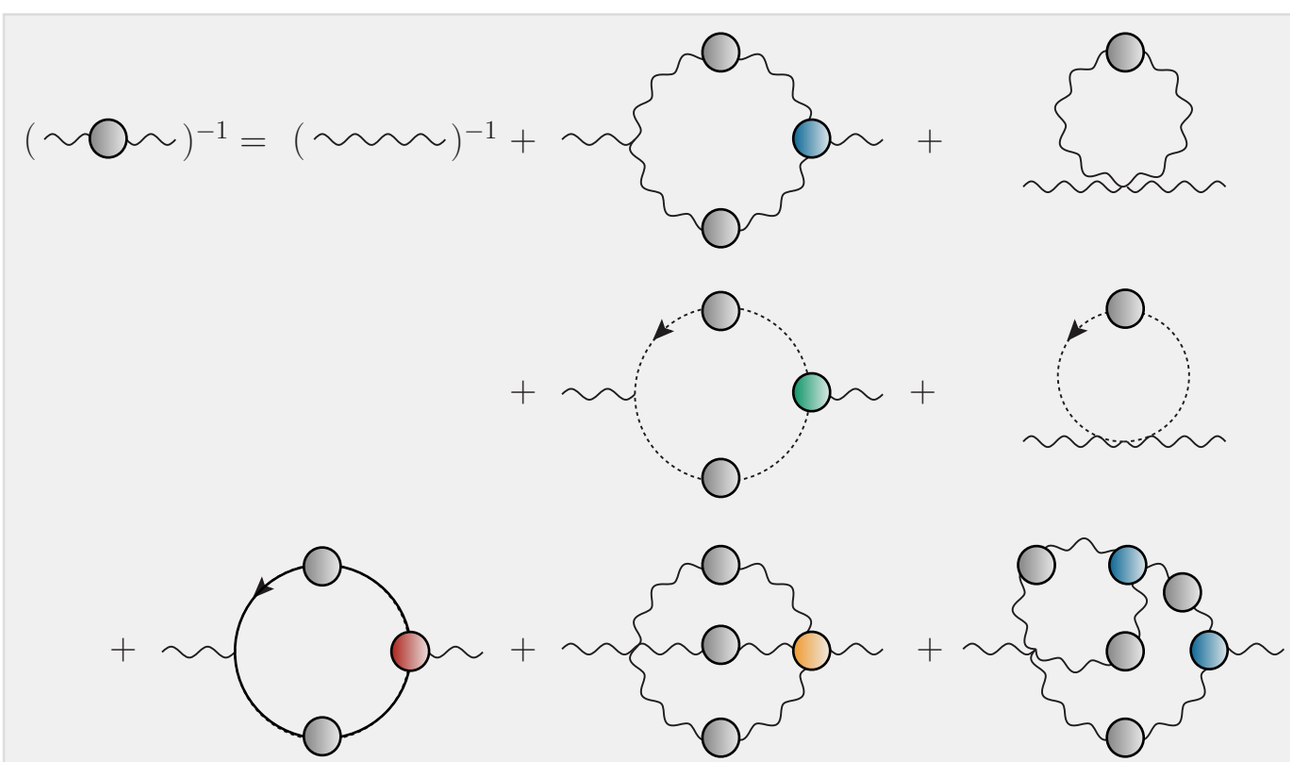
QCD gap equations

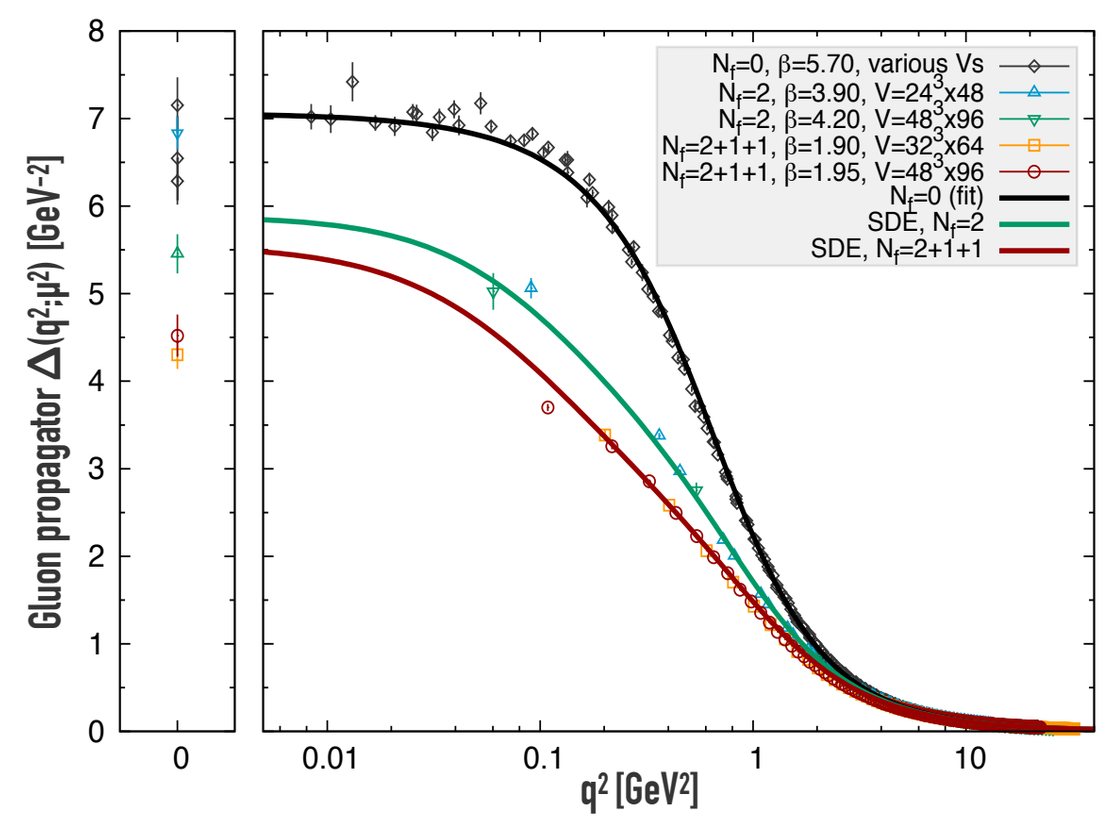
- **Ghost SD (gap) equation:**
  Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, PRD 86 (2012)





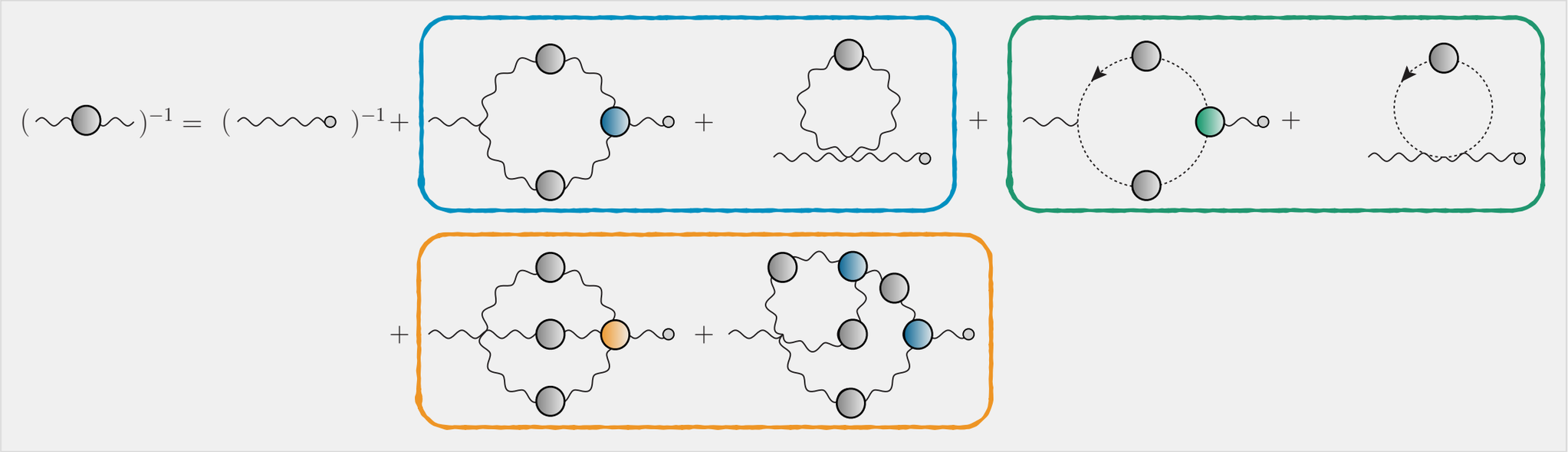
- **Gluon SD (gap) equation:**
  Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, PRD 86 (2012)
  Aguilar, DB, Papavassiliou, PRD 86 (2012): PRD 88 (2013)



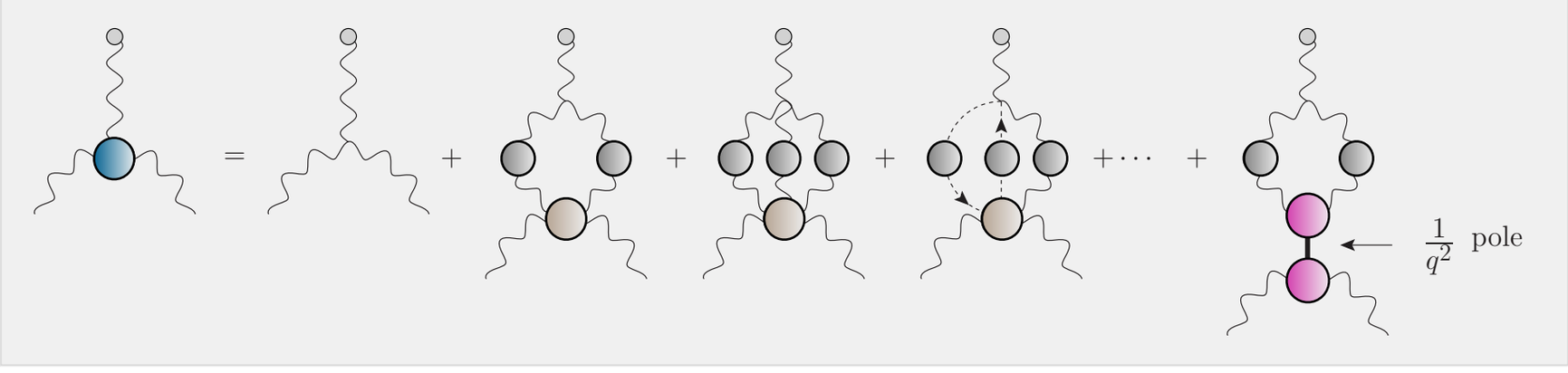


Gluons, too, get a dynamical mass!

- **Pinch-Technique Background Field Method (PT-BFM):**
  ensures the gauge invariant truncations of ghost/gluon contributions
  DB, Papavassiliou, PRD 77 (2008); JHEP 0811 (2008)

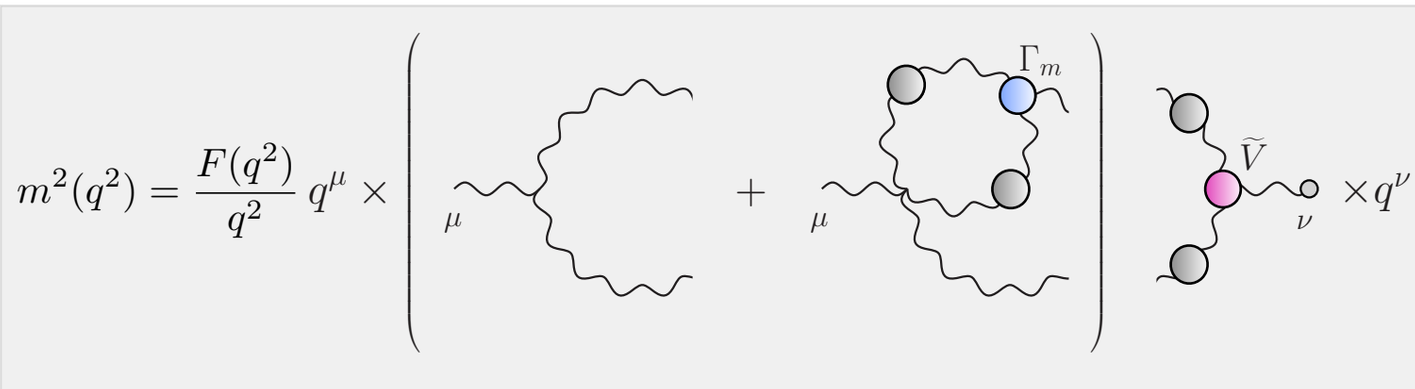


- **Related to conventional propagator by (background-quantum) identities:**
  generated by extra (antiBRST) symmetry of the PT-BFM action
  DB, Quadri, PRD 88 (2013)
- **Schwinger mechanism:**
  implemented through longitudinally coupled massless poles
  Aguilar, DB, Papavassiliou, PRD 84 (2011)



Gluons, too, get a dynamical mass!

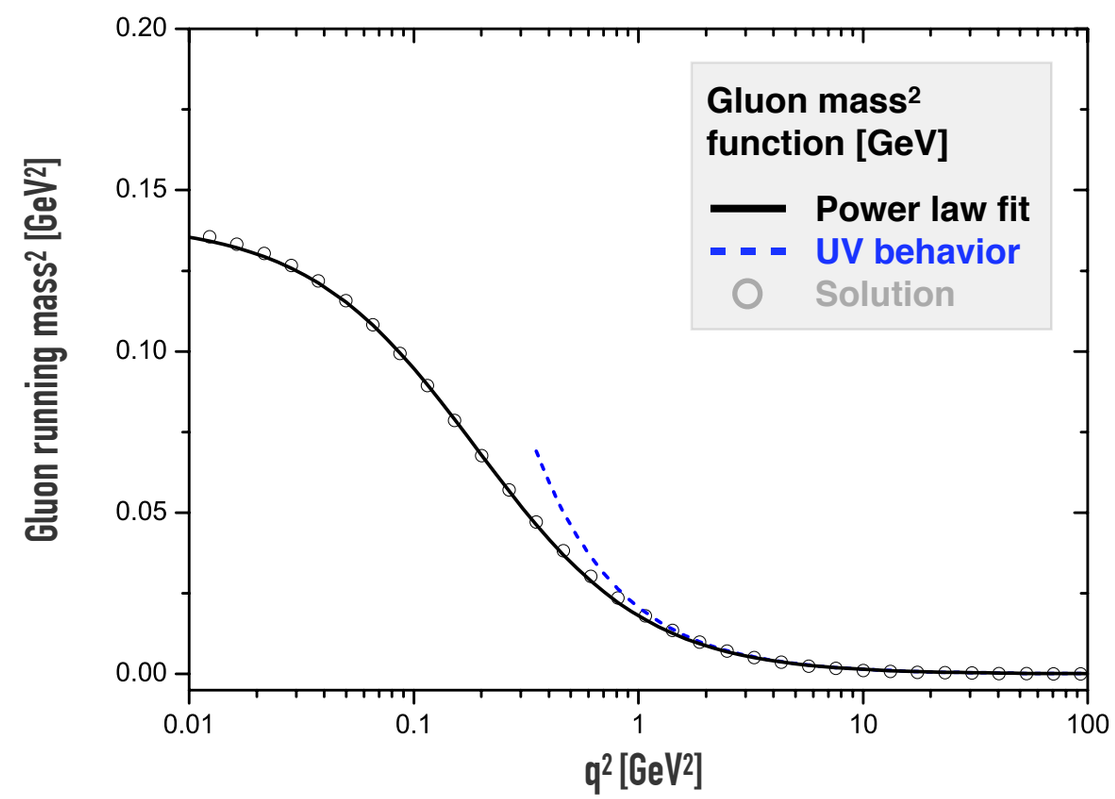
- **Gluon mass equation:**
  DB, Ibañez, Papavassiliou, PRD 84 (2011)



- **Asymptotic freedom:**
  UV behavior is controllable
- **Dynamically generated masses:**
  QCD generates its own IR cutoffs
- **If λ > 2/m ~ 1 fm:**
  quarks *and* gluons decouple
  (**confinement!?**)

- **Running gluon mass displays power law behavior:**
  invisible to perturbation theory
  Aguilar, DB, Papavassiliou, PRD 89 (2014)

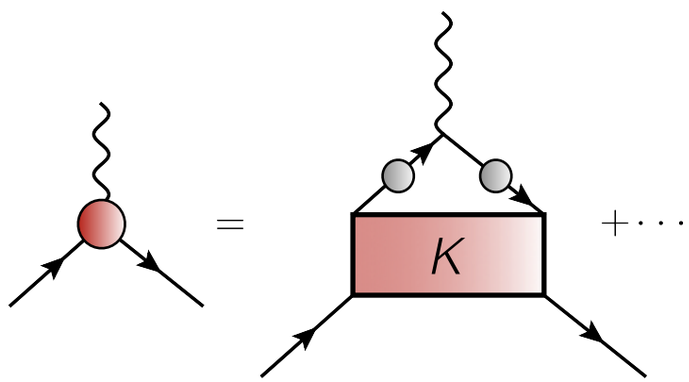
$$m^2(q^2) = \frac{m_0^2}{1 + (q^2/\mathcal{M}^2)^{1+\gamma}}$$

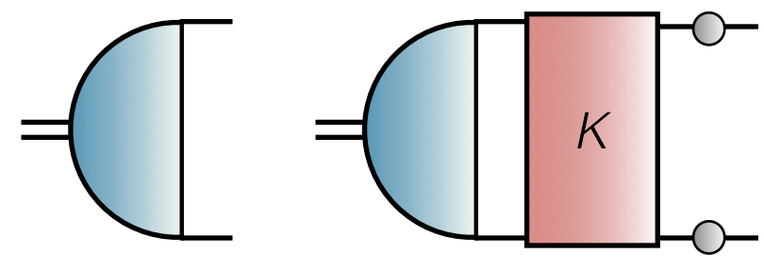


- **Gluons are "cannibals":**
  they eat each other to become massive
  - **Must impact** *any* continuum study
  - Should play a role **in gluon saturation**

Running charge: top-down vs bottom-up

- **SD and Bethe-Salpeter equations:**

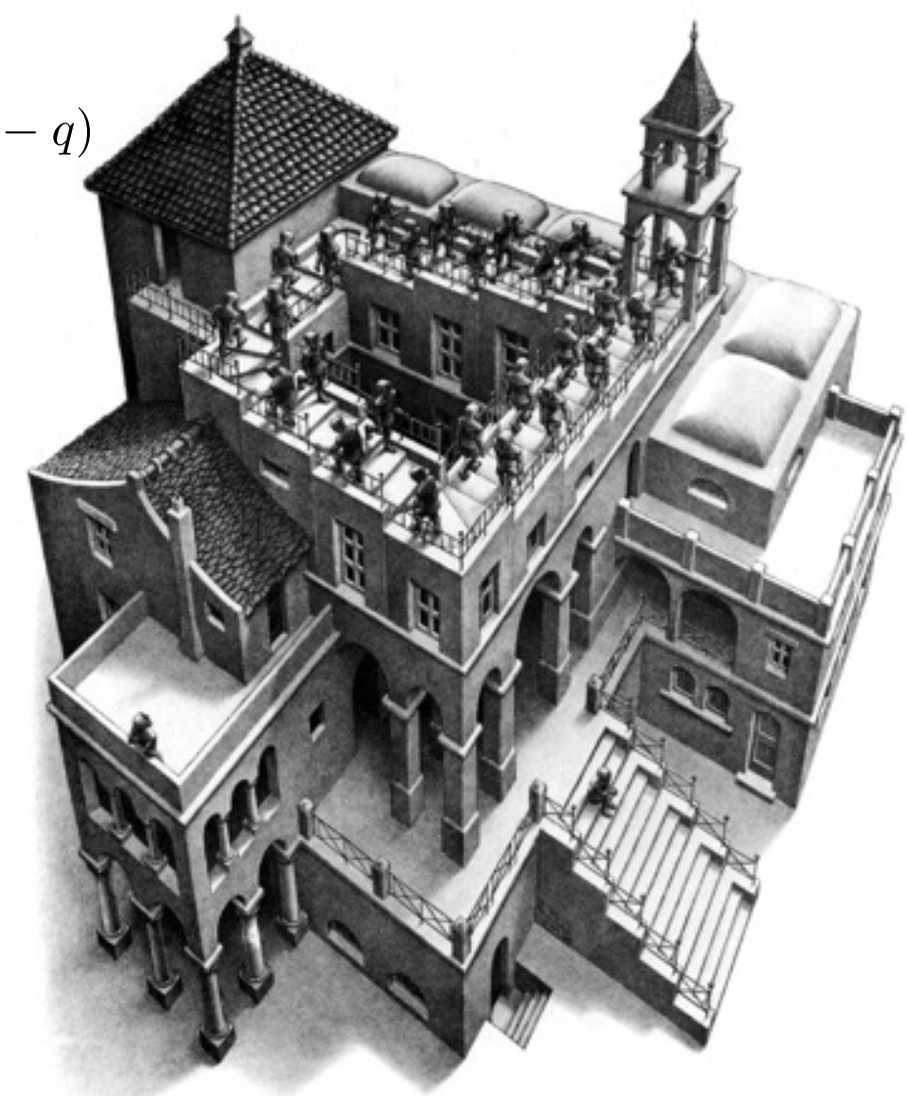




$$S^{-1}(p) = (i\gamma\cdot p + m_0) + \frac{4}{3}g^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \mathcal{D}^{\mathrm{eff}}(k-q)\gamma_\mu S(k)\Gamma_\nu(q,p)P^{\mu\nu}(k-q)$$

- **Top-down approach:**
  "ab-initio" computation of the interaction via direct analysis of the gauge sector SD equations

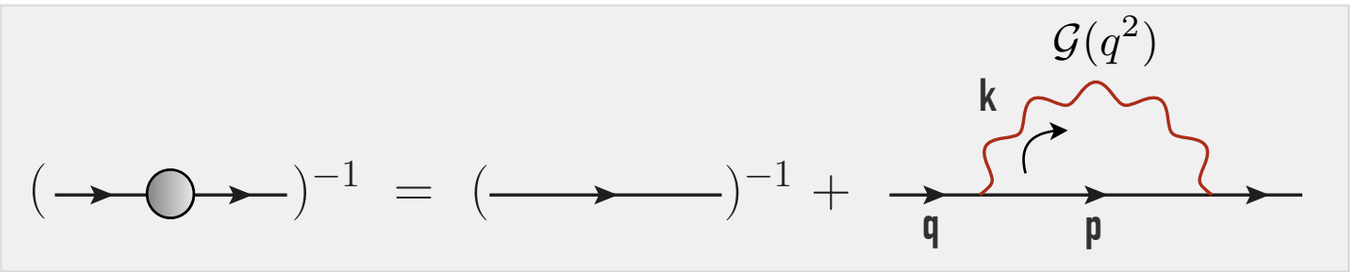
- **Bottom-up approach:**
  infer interaction by fitting data relevant for reproducing bound-state properties

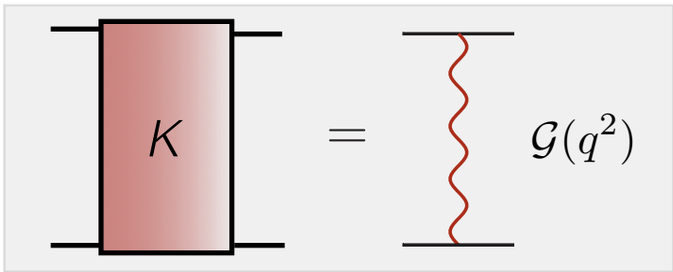


M. C. Escher, Ascending and Descending (1960)

Running charge: bottom-up approach - I

- **Use rainbow-ladder truncation:**
  tree level vertex + one gluon exchange effective kernel



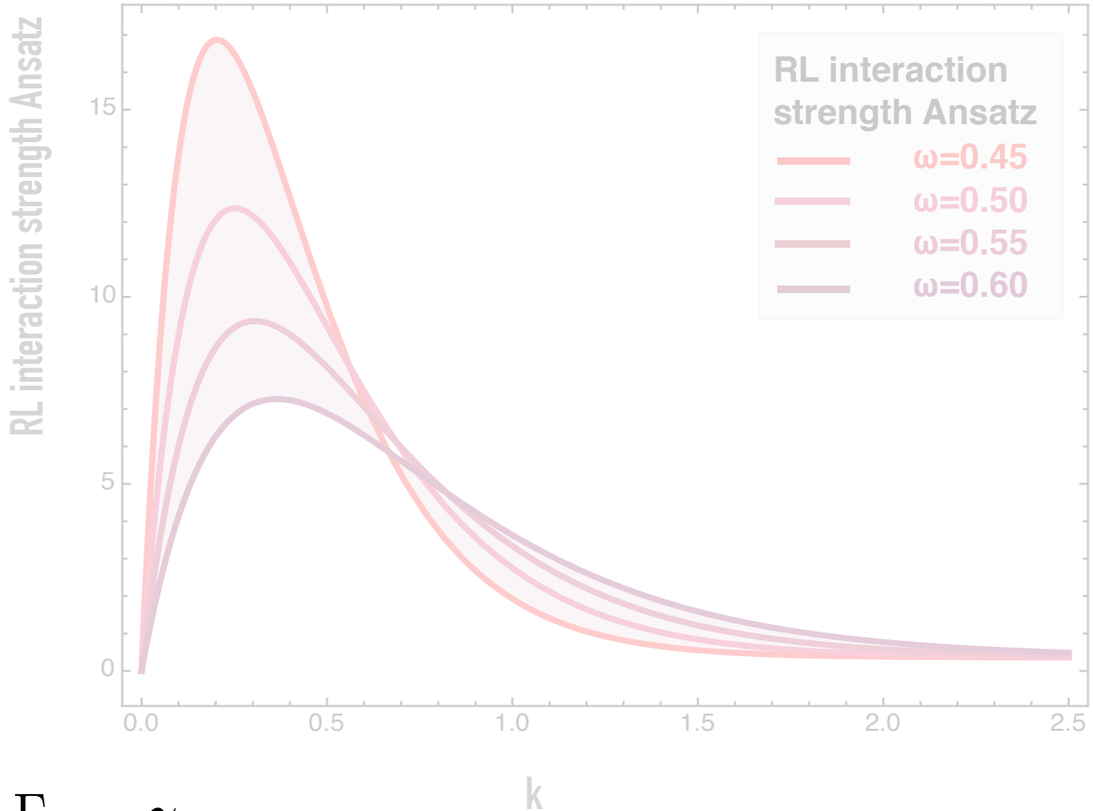


- **Interaction strength Ansatz**
  IR: Gaussian; UV: perturbative tail
  Maris, Roberts, Tandy, PRC 56 (1997); PRC 60 (1999)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{\rm IR}(k^2) + \mathcal{G}_{\rm UV}(k^2)}{4\pi}$$

$$\mathcal{G}_{\rm IR}(k^2) = \frac{8\pi^2}{\omega^5} \varsigma^3 {\rm e}^{-k^2/\omega^2}$$

$$\mathcal{G}_{\rm UV}(k^2) = \frac{96\pi^2}{25} \frac{1 - e^{-k^2/1[{\rm GeV}^2]}}{k^2 \log[{\rm e}^2 - 1 + (1 + k^2/\Lambda^2)^2]}$$



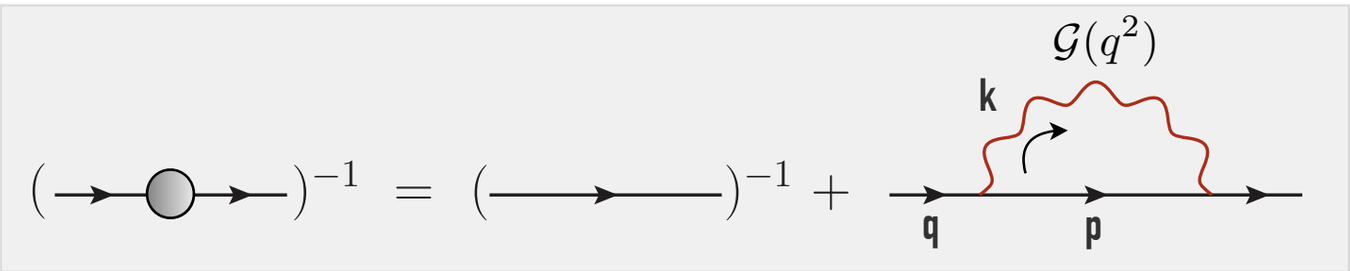
- **One parameter interaction** ($\Lambda_{\overline{\rm MS}}$=234 [MeV])
  ς fitted to obtain pion decay constant

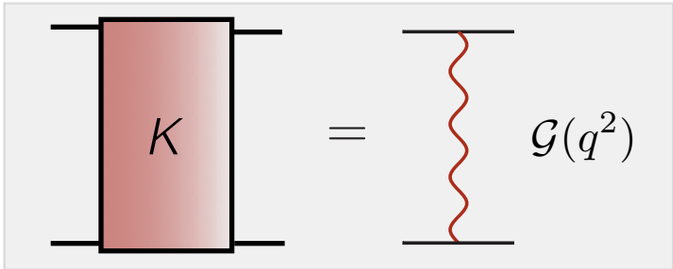
$$\varsigma_{\rm RL} = 0.87 \ [{\rm GeV}]$$

- **Depends on truncation used:** $\Gamma_\nu = \gamma_\nu$
- **Many observables invariant while** $\omega \in [0.4, 0.6]$

Running charge: bottom-up approach - I

- **Use rainbow-ladder truncation:**
  tree level vertex + one gluon exchange effective kernel





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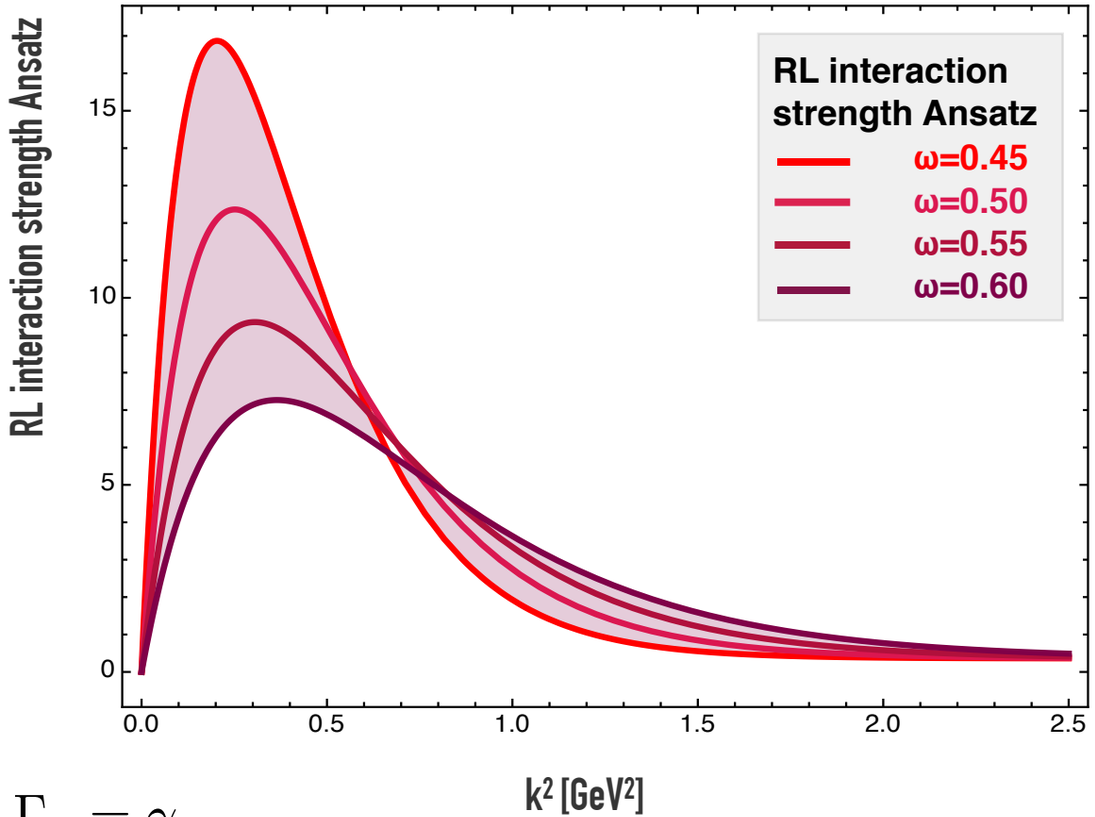

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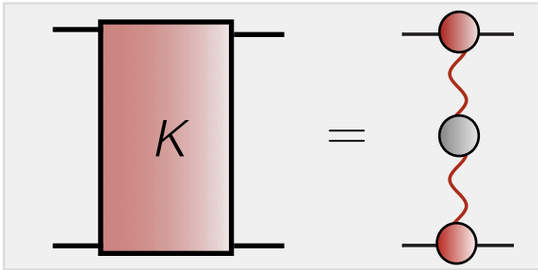
- **Depends on truncation used:** $\Gamma_\nu = \gamma_\nu$
- **Many observables invariant while** $\omega \in [0.4, 0.6]$

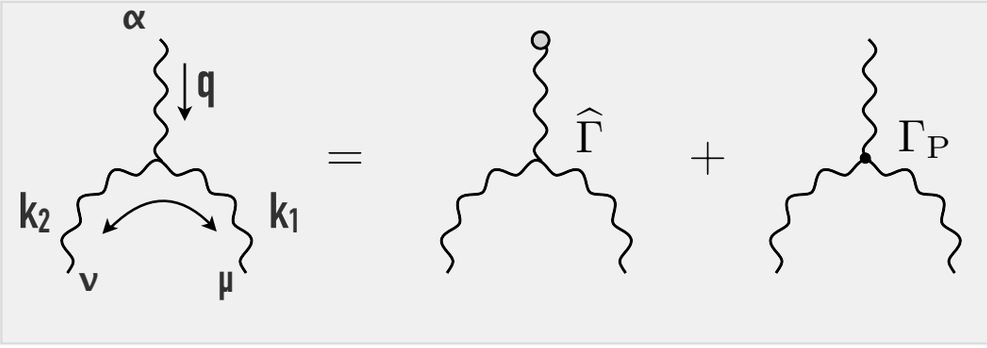
Running charge: top-down approach

- **Universal (process-independent) contribution:**
  originates entirely from the gauge sector



- **Fundamental quantities: PT-BFM propagators/vertices**
  satisfy Abelian-like Slavnov-Taylor (ST) identities

- **How to get them?**
  use the PT algorithm
  Cornwall, Papavassiliou, PRD 40 (1989)

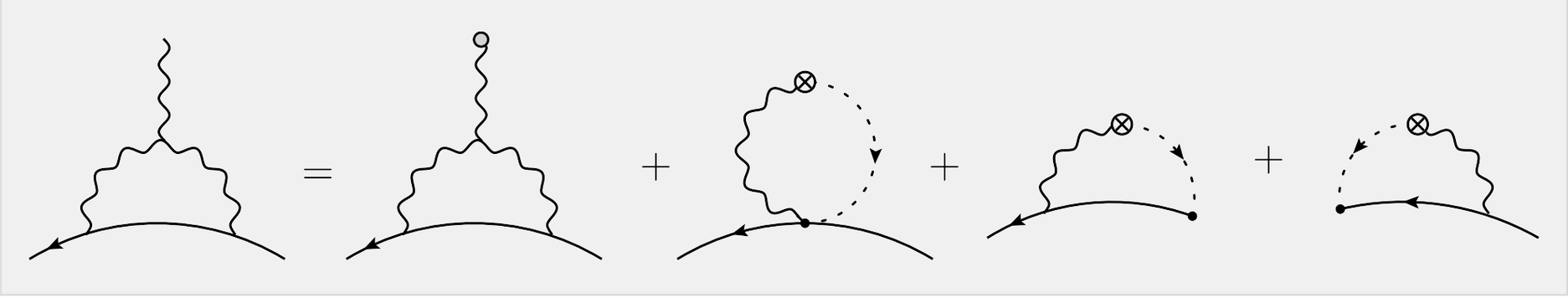


$$\widehat{\Gamma}^{\alpha\mu\nu} = (k_2 - k_1)^{\alpha} g^{\mu\nu} + 2q^{\nu} g^{\alpha\mu} - 2q^{\mu} g^{\alpha\nu}$$

$$\Gamma_{\mathrm{P}}^{\alpha\mu\nu} = k_1^{\mu} g^{\alpha\nu} - k_2^{\nu} g^{\alpha\mu}$$

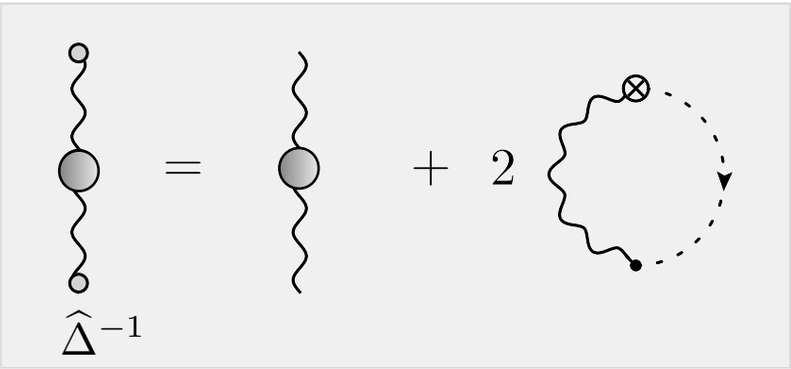
- **longitudinal momenta**
  trigger elementary Ward identities

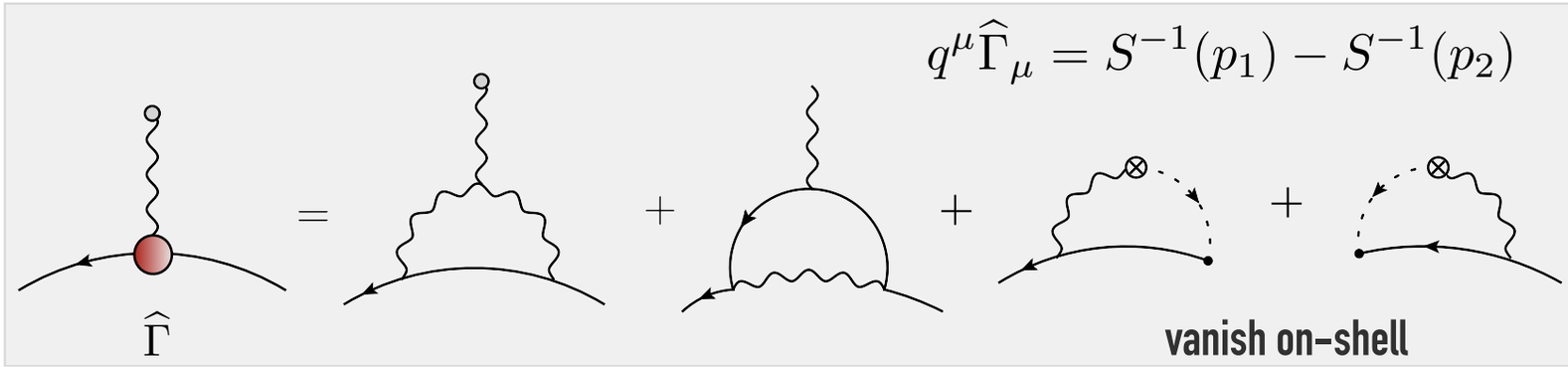
- **Apply the PT to the quark-gluon vertex**
  one loop result:



Running charge: top-down approach

- **Allot pieces to different Green's functions**
  construct $\widehat{\Delta}$ and $\widehat{\Gamma}_\mu$





- **Crucial all-order equivalence: PT=BFM**
  yields Feynman rules for systematic calculation

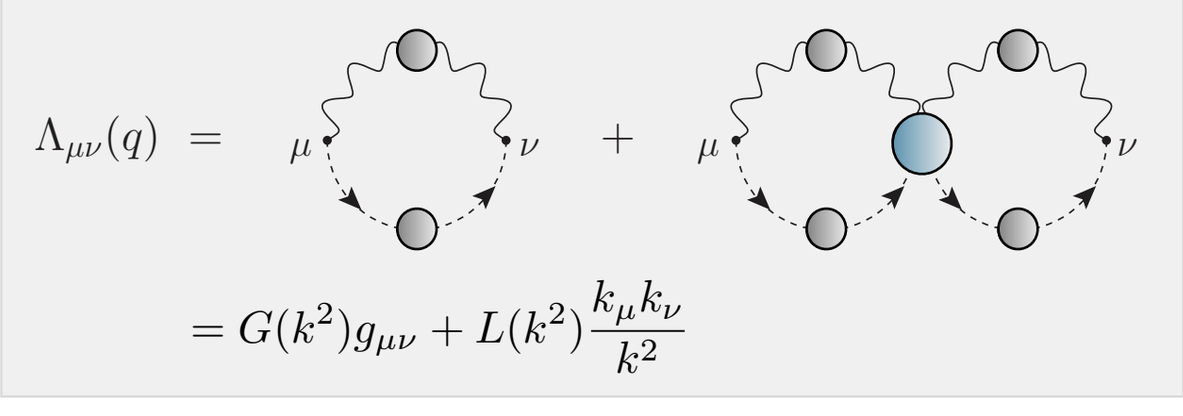
$$\widehat{\Delta} \sim \frac{1}{q^2[1 + bg^2 \log q^2/\mu^2]}; \qquad b = 11C_A/48\pi^2$$

- **Absorbs all the RG logs**
  as the photon in QED
- **Renormalizes as** Z_g^{-2}

- **An additional equivalence holds: antiBRST+BRST=BFM**
  plethora of symmetry identities, in particular BQ identities

DB, Quadri, PRD 88 (2013)

$$\Delta(q^2) = [1 + G(q^2)]^2 \widehat{\Delta}(q^2)$$



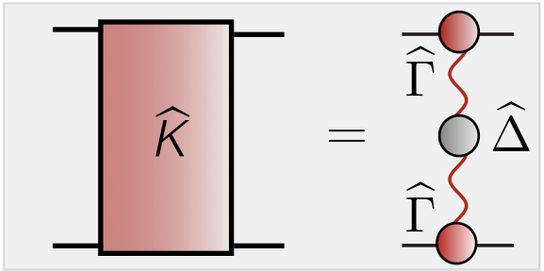
- ***G* special PT-BFM function:**
  determined by ghost-gluon dynamics
- **Combination** 1+*G* **appears in *all* BQIs**
  fundamental non-Abelian quantity
- ***G* is related (Landau gauge) to the ghost dressing:**
  use ghost gap equation to constrain *1+G, L*

$$F^{-1}(k^2;\mu^2) = 1 + G(k^2;\mu^2) + L(k^2;\mu^2)$$

Running charge: top-down approach

- **Convert vertices/propagators into PT-BFM ones**
  new RG invariant combination appears

$$\widehat{d}(k^2) = \alpha(\mu^2)\widehat{\Delta}(k^2;\mu^2)$$



- **Use symmetry identity**
  to identify the interaction strength
  Aguilar, DB, Papavassiliou, Rodriguez-Quintero, PRD 90 (2009)
  DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)

$$\mathcal{I}(k^2) = k^2\widehat{d}(k^2)$$

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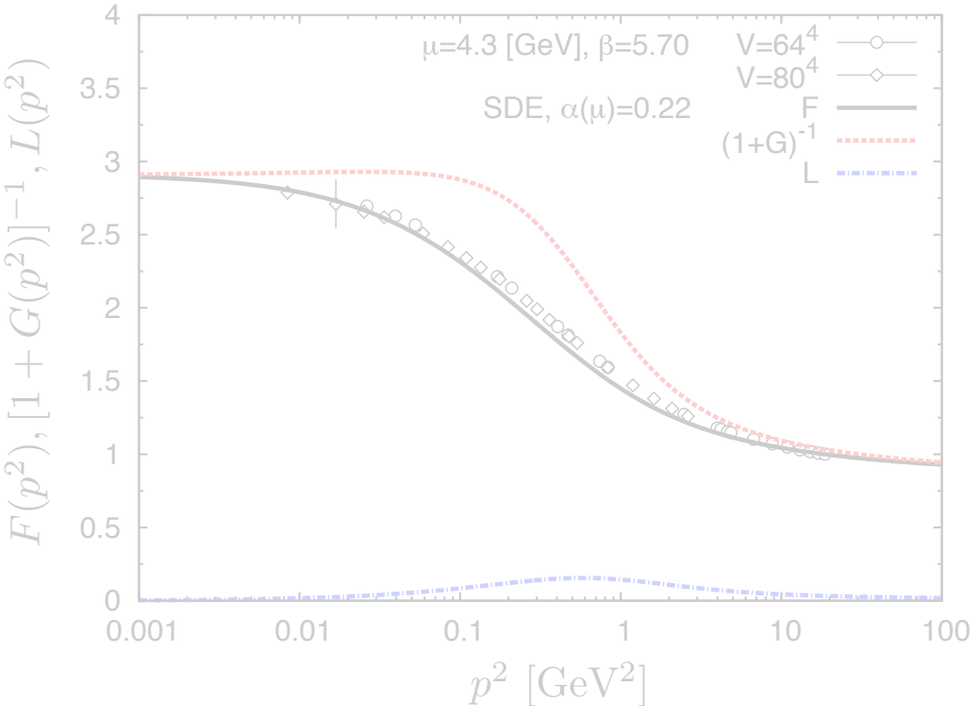

- 1+*G* **and** *L* **determined by their own SDEs**
  under simplifying assumptions:

$$1+G(p^2) = Z_c + \frac{g^2 C_{\rm A}}{d-1}\int_k \left[(d-2) + \frac{(k\cdot p)^2}{k^2p^2}\right] B_1(-k,0,k)\Delta(k)D(k+p)$$

$$L(p^2) = \frac{g^2 C_{\rm A}}{d-1}\int_k \left[1 - \frac{(k\cdot p)^2}{k^2p^2}\right] B_1(-k,0,k)\Delta(k)D(k+p)$$

- **Main source of uncertainties:**
  needs assumptions on ghost vertex behavior
- **Parametrized by** δ∈[0,1]
  lower bound (δ=0): *1/F=1+G*

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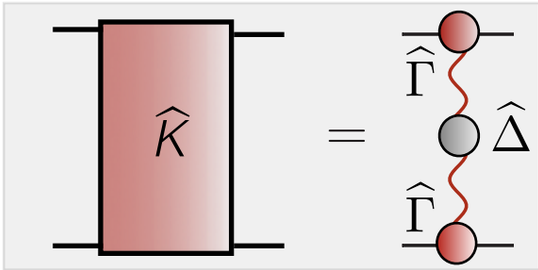

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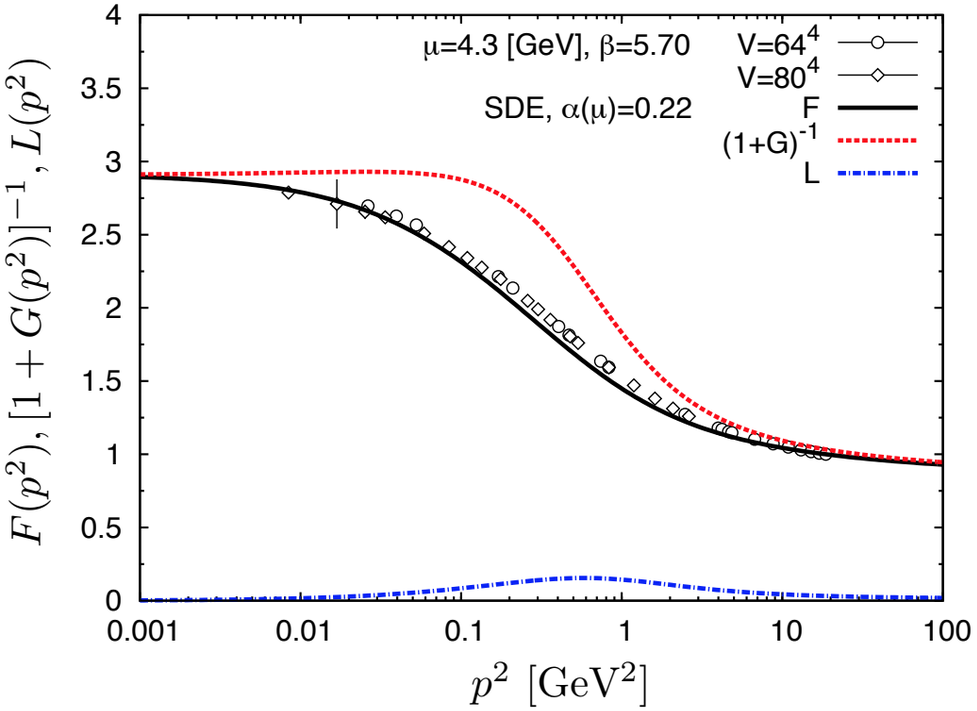

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Running charge: top-down approach

- **Input to the top-down approach:**
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- **Solve ghost gap equation self-consistently:**
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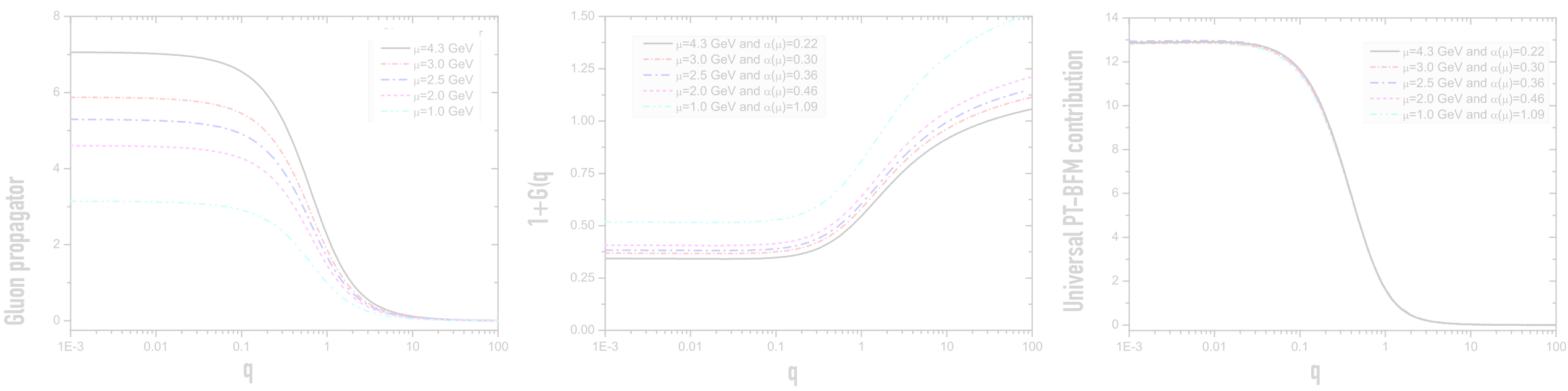
  DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)



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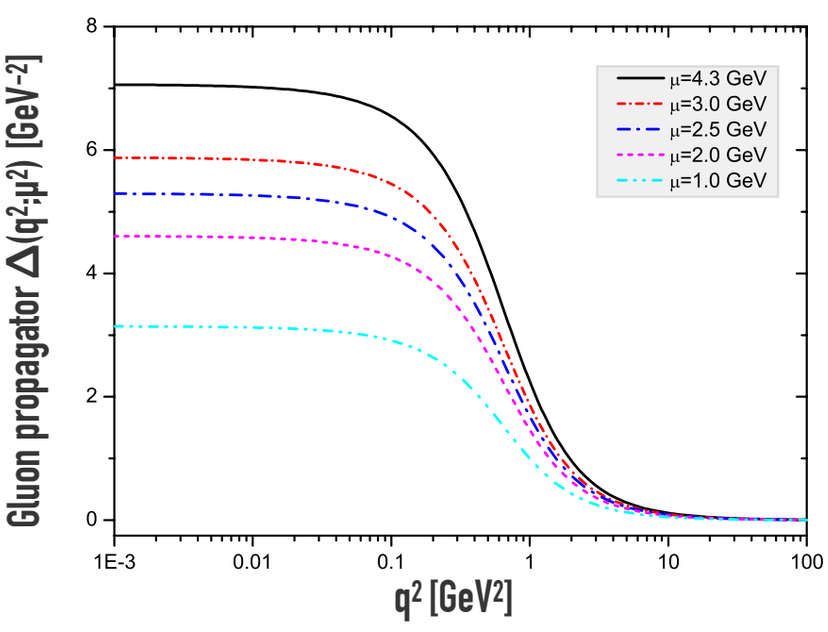
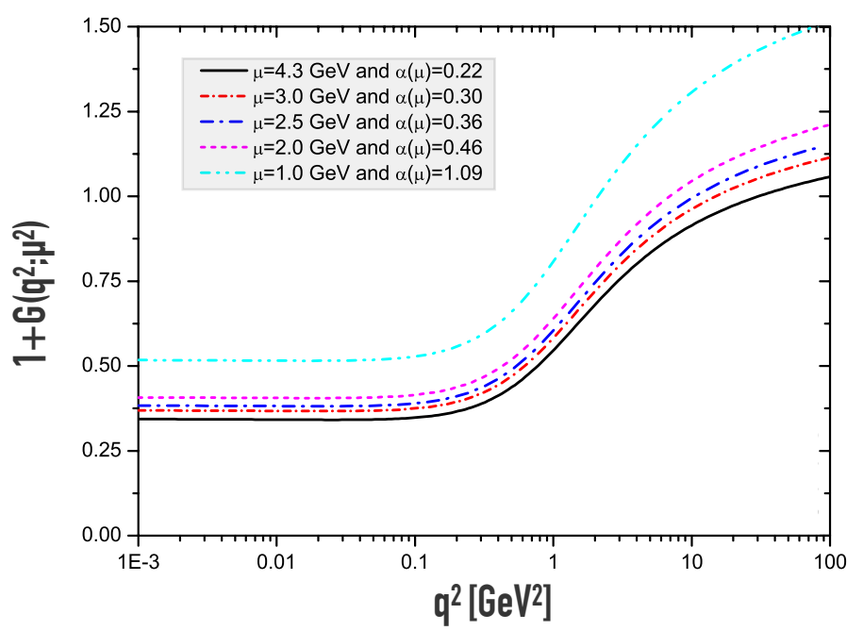
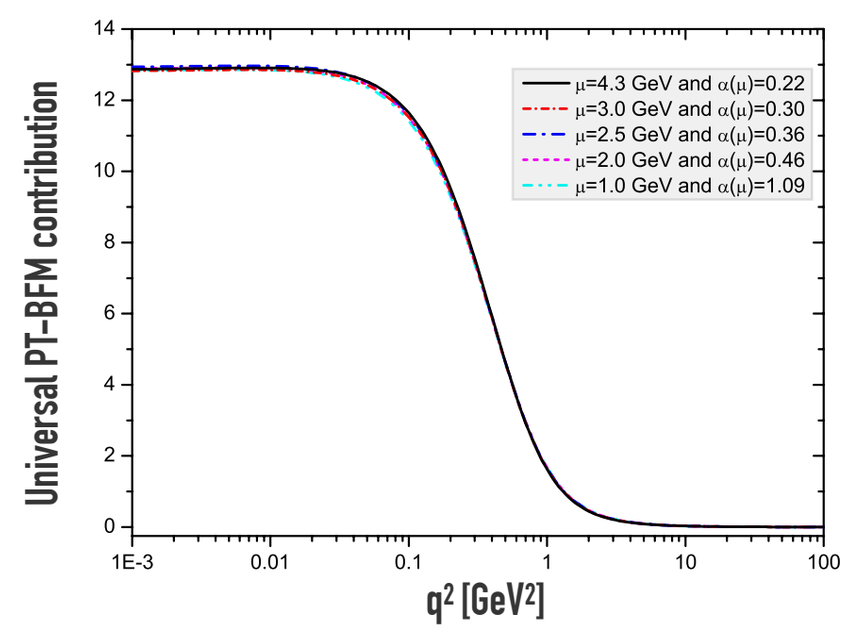






Running charge: top-down approach

- **Accurately interpolated by:**

$$\alpha_s(k^2) = 12.9\ m_g^2(k^2)$$

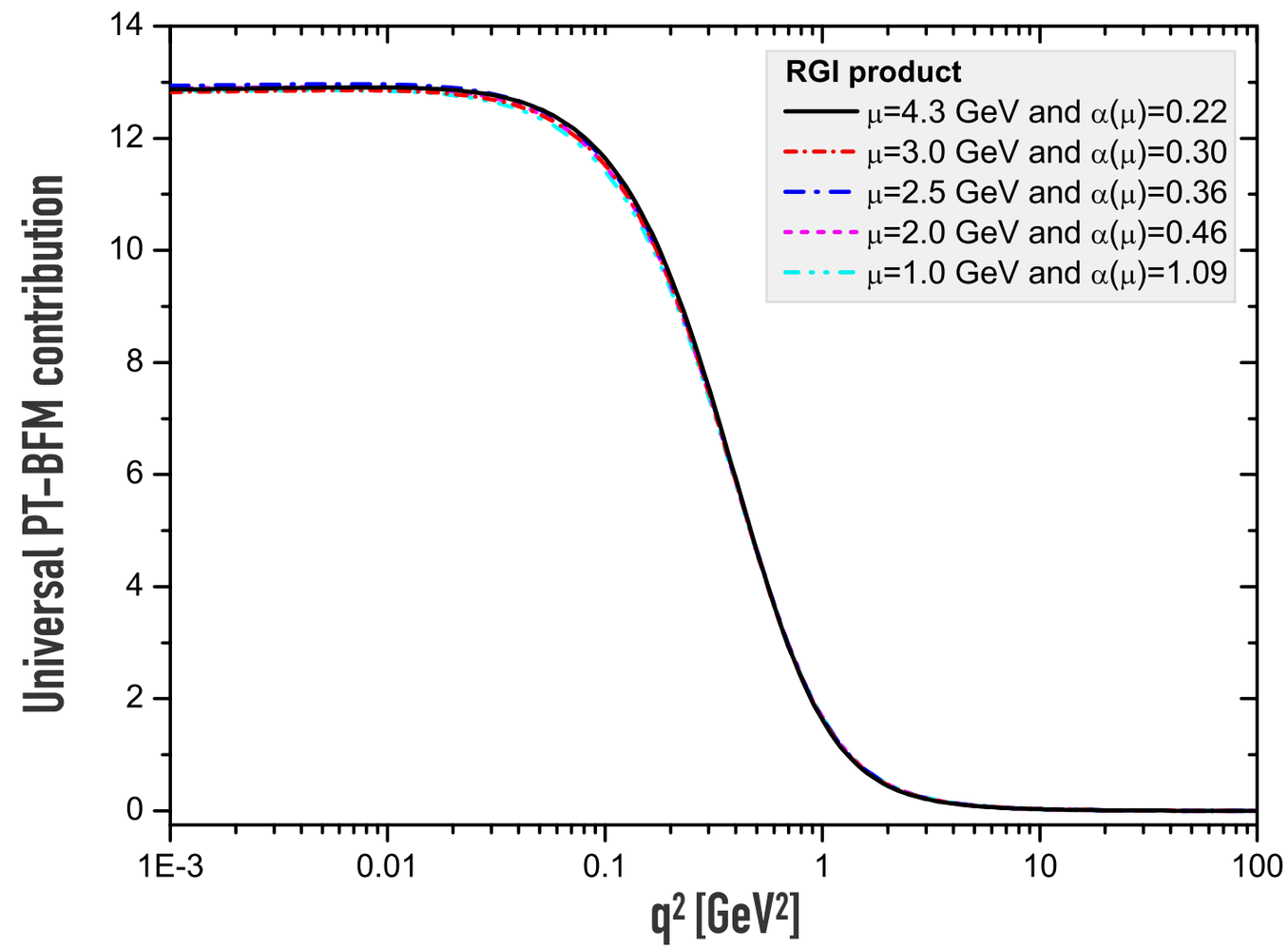
$$m_g^2(k^2) = \frac{0.22 + 0.019k^2}{1 + 1.76k^2} \qquad 1 < k < 4.3[\mathrm{GeV}]$$

- **Extrapolate in the far IR**

$$\alpha_s(0) = 2.77 \approx 0.9\pi$$

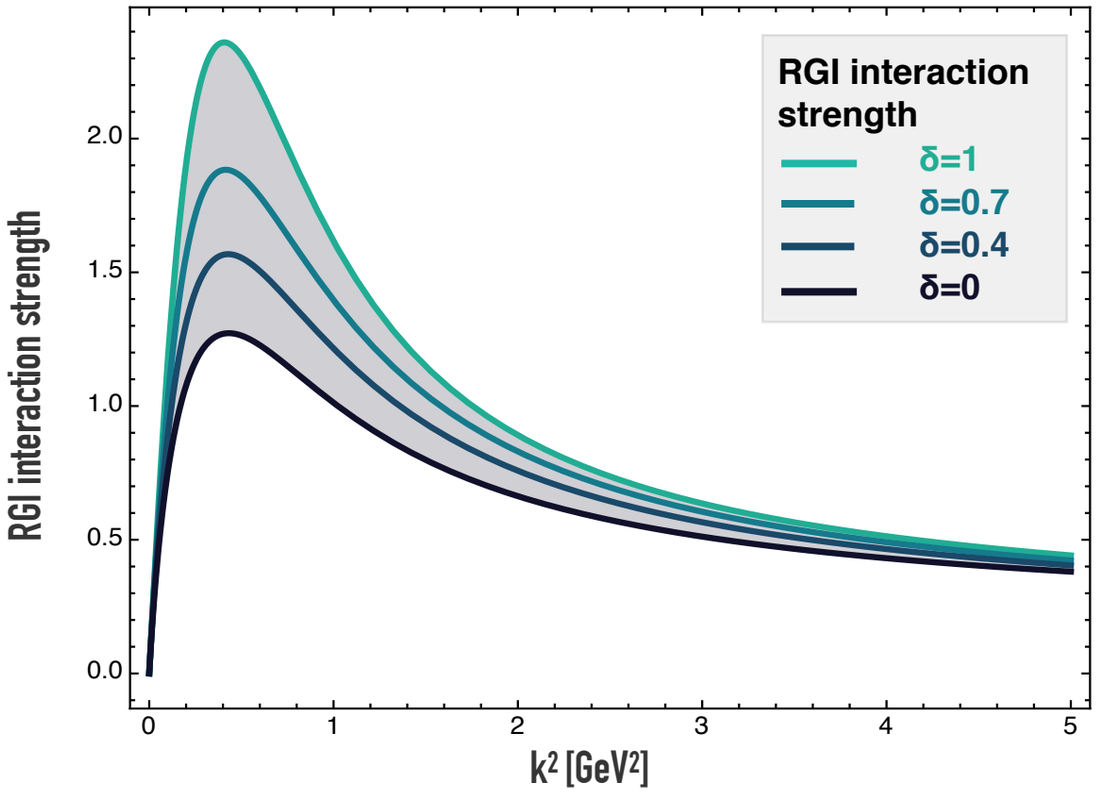
$$m_g^2(0) = (0.46)^2[\mathrm{GeV}^2]$$

- **Gluon mass scale 'natural'**
  slightly bigger than light quarks mass



- **Strength of the coupling also interesting**
  - greater than that required for DCSB in strong coupling QED (π/3) or QCD gap equation model (π/3.5)
  - consistent with π value often imagined necessary to describe strong-interaction phenomena (see, e.g., SJB talk)

Top-down vs bottom-up comparison - I



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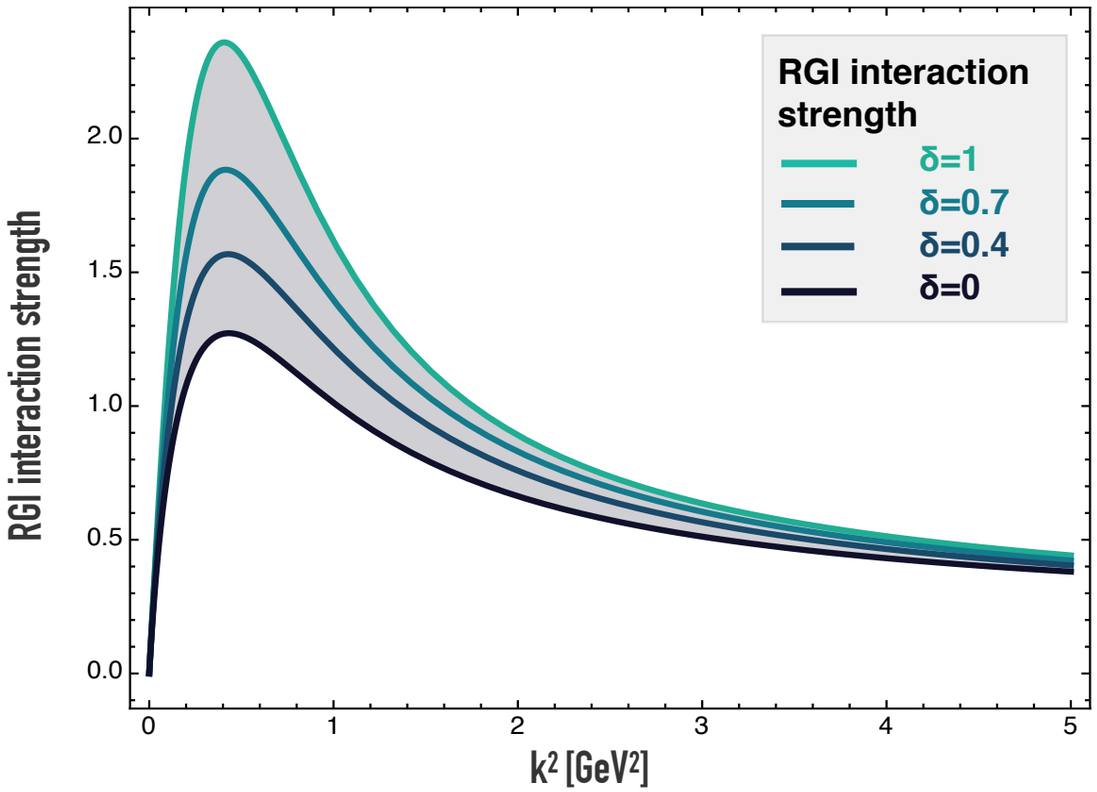
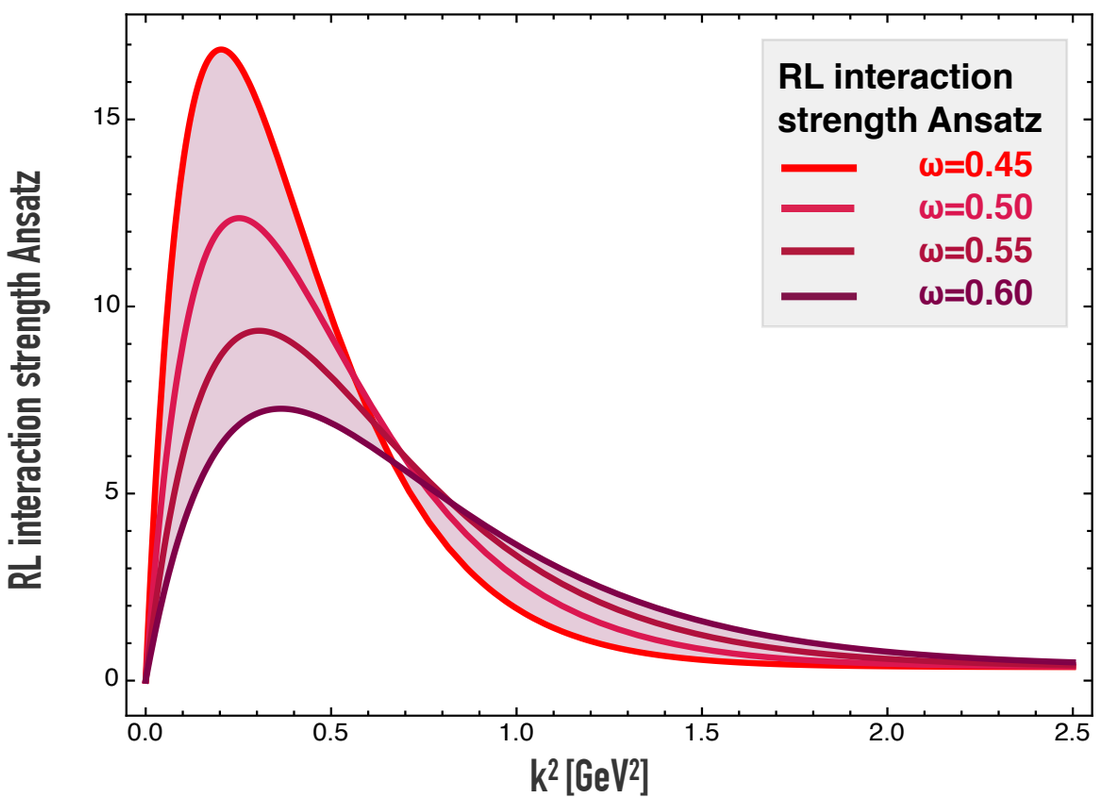
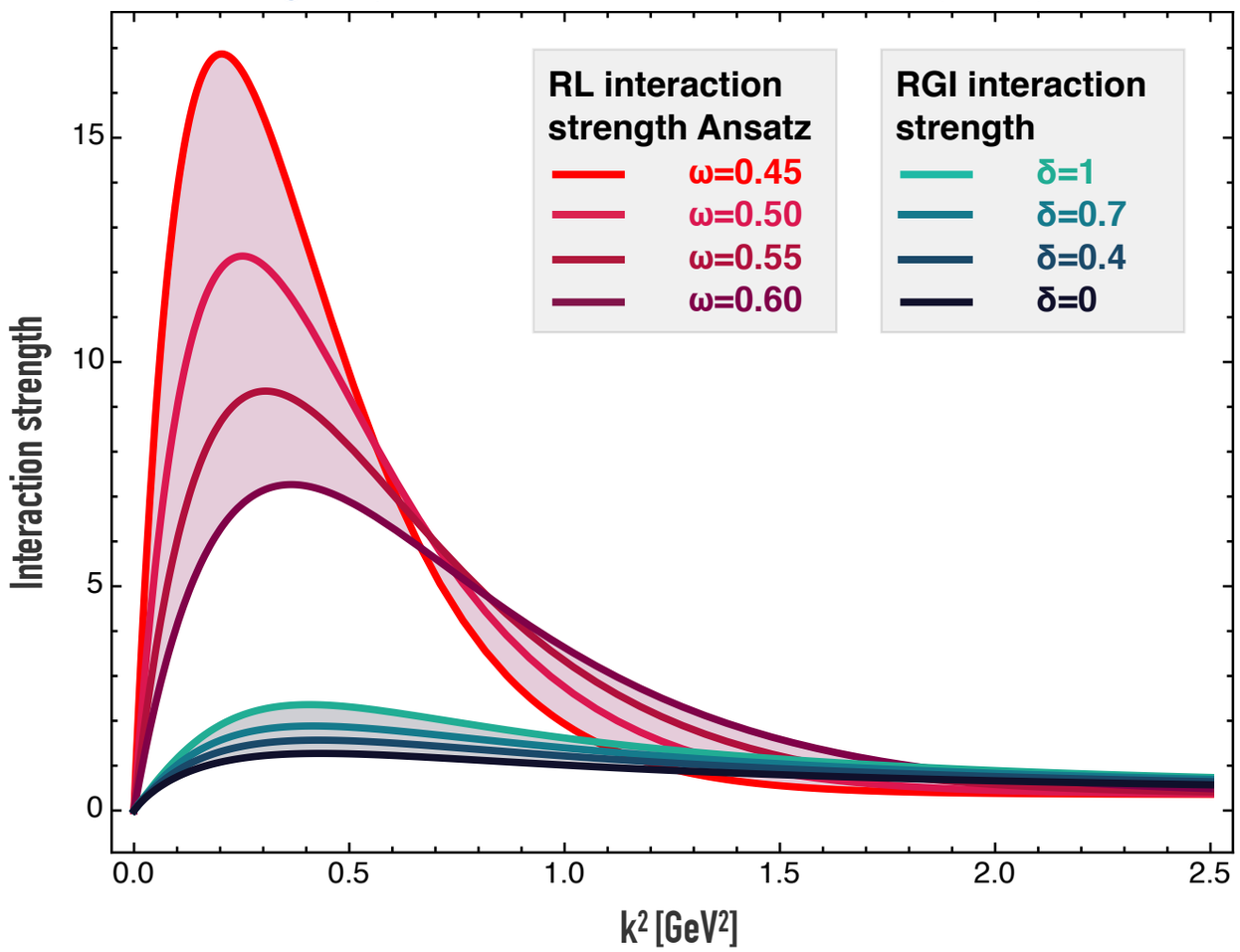




M. C. Escher,

Top-down vs bottom-up comparison - I

DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)



M. C. Escher, Belvedere (1958)

Running charge: bottom-up approach - II

- **Go beyond RL truncation:**
  implement dynamical symmetry breaking into bound-state equations

- **Same RL interaction strength Ansatz**
  IR: Gaussian; UV: perturbative tail
  Qin, Chang, Liu, Roberts and Wilson, PRC 84 (2011)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{\rm IR}(k^2) + \mathcal{G}_{\rm UV}(k^2)}{4\pi}$$

$$\mathcal{G}_{\rm IR}(k^2) = \frac{8\pi^2}{\omega^5} \varsigma^3 {\rm e}^{-k^2/\omega^2}$$

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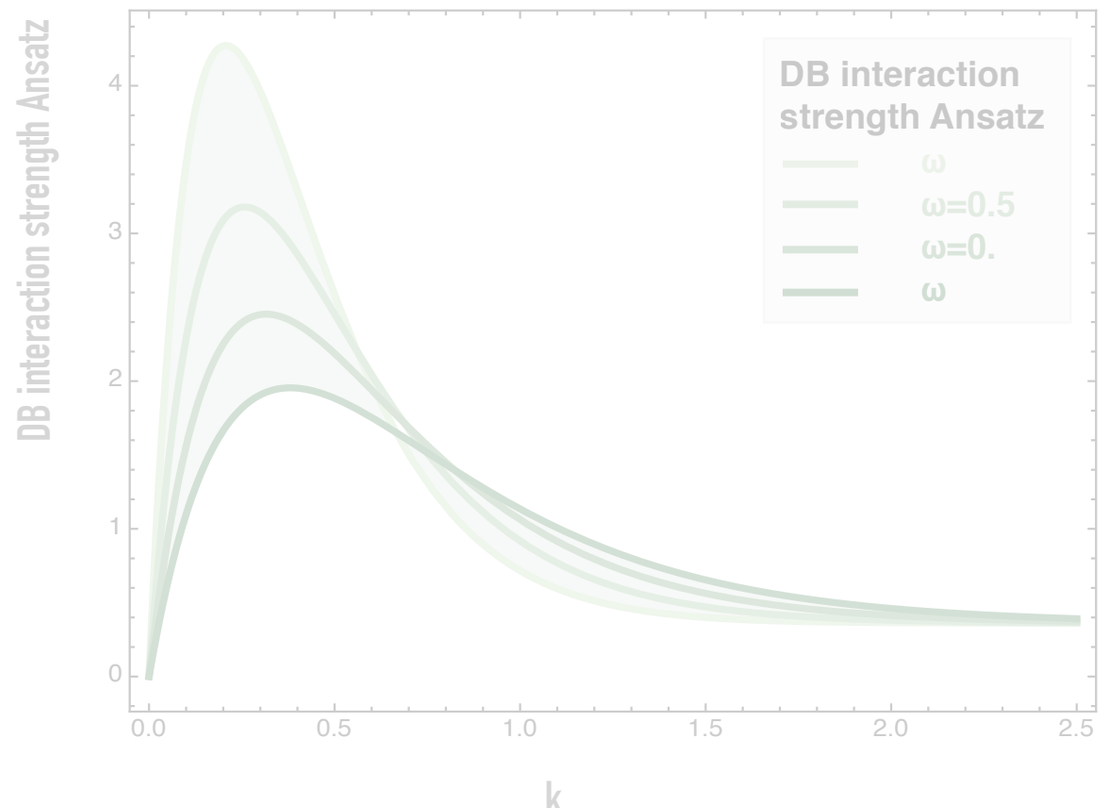

- **Quark-gluon vertex:**

$$\Gamma_\nu = \Gamma_\nu^{\rm BC} + \Gamma_\nu^{\rm ACM}$$

  - **Ball-Chiu vertex**
    completely determined by fermion propagator
    Ball, Chiu PRD 22 (1980)

  - **Anomalous chromo-magnetic vertex**
    transverse part (undetermined by STI)

- **ς fitted to**

$$\varsigma_{\rm DB} = 0.55\ [{\rm GeV}]$$

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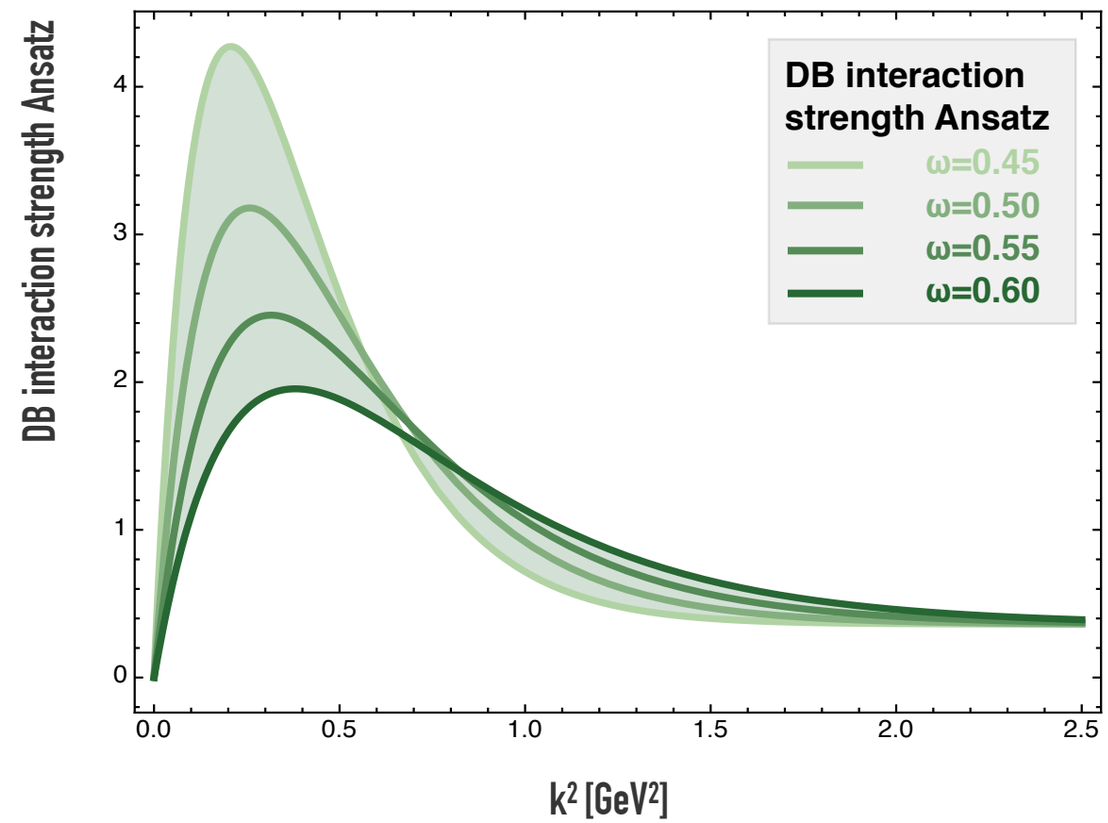

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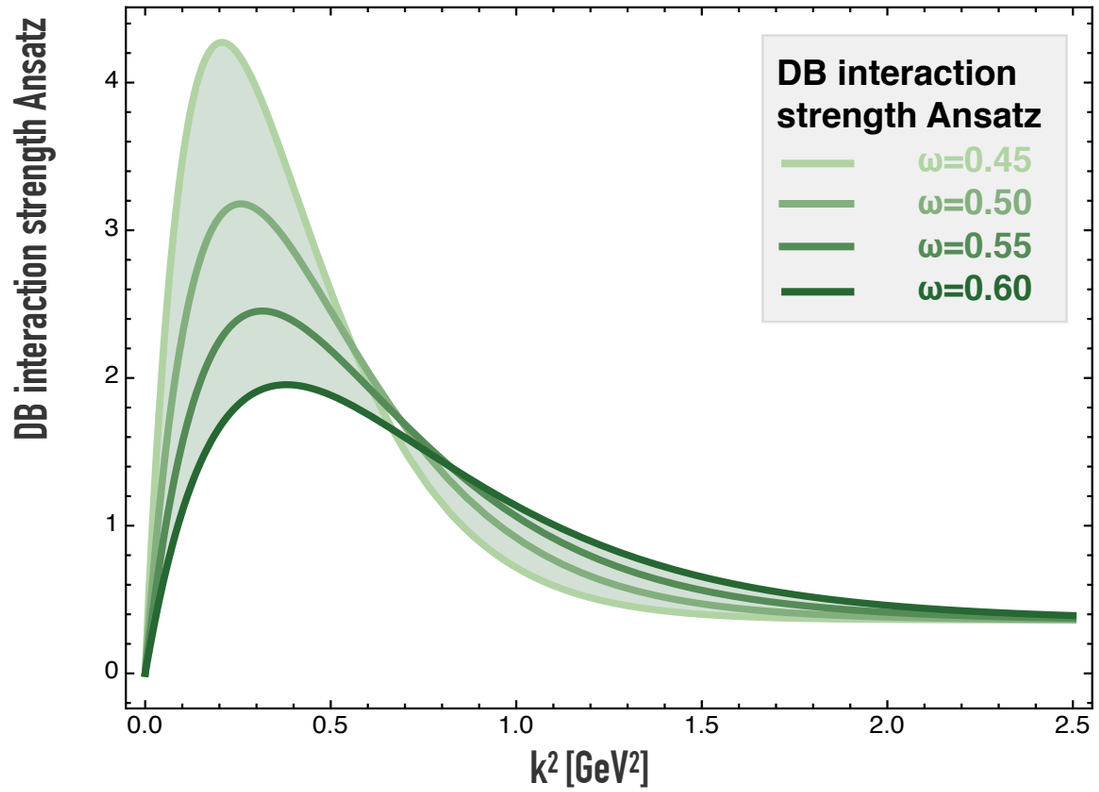
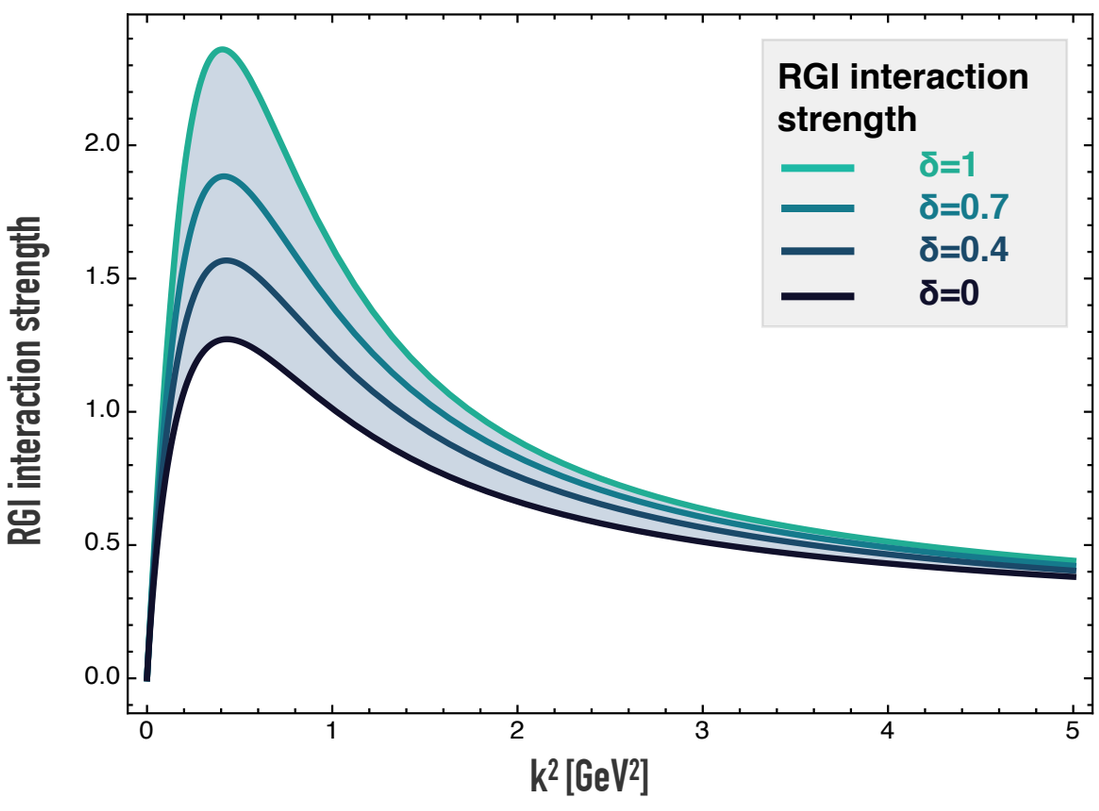
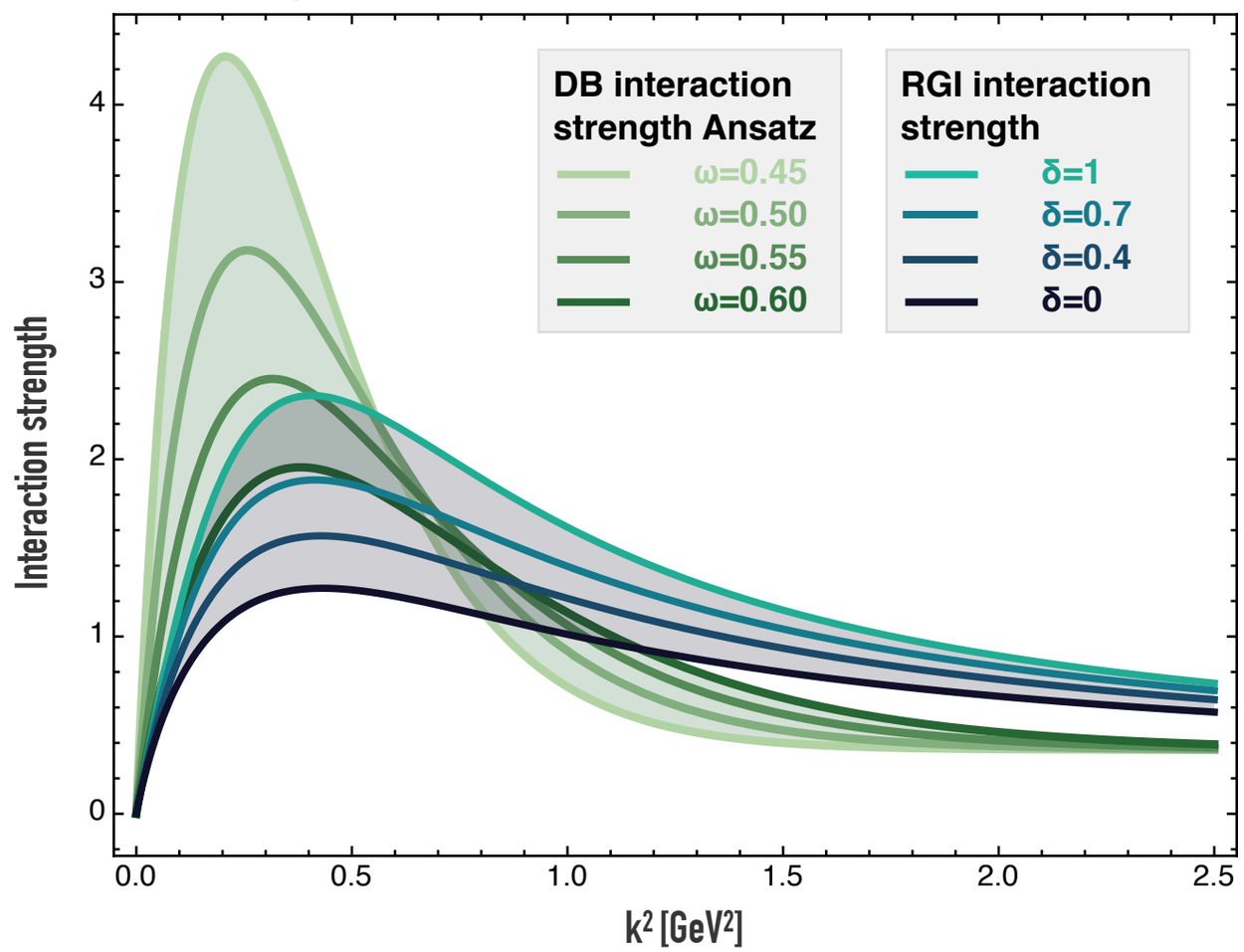




M. C. Escher, Print

Top-down vs bottom-up comparison - II

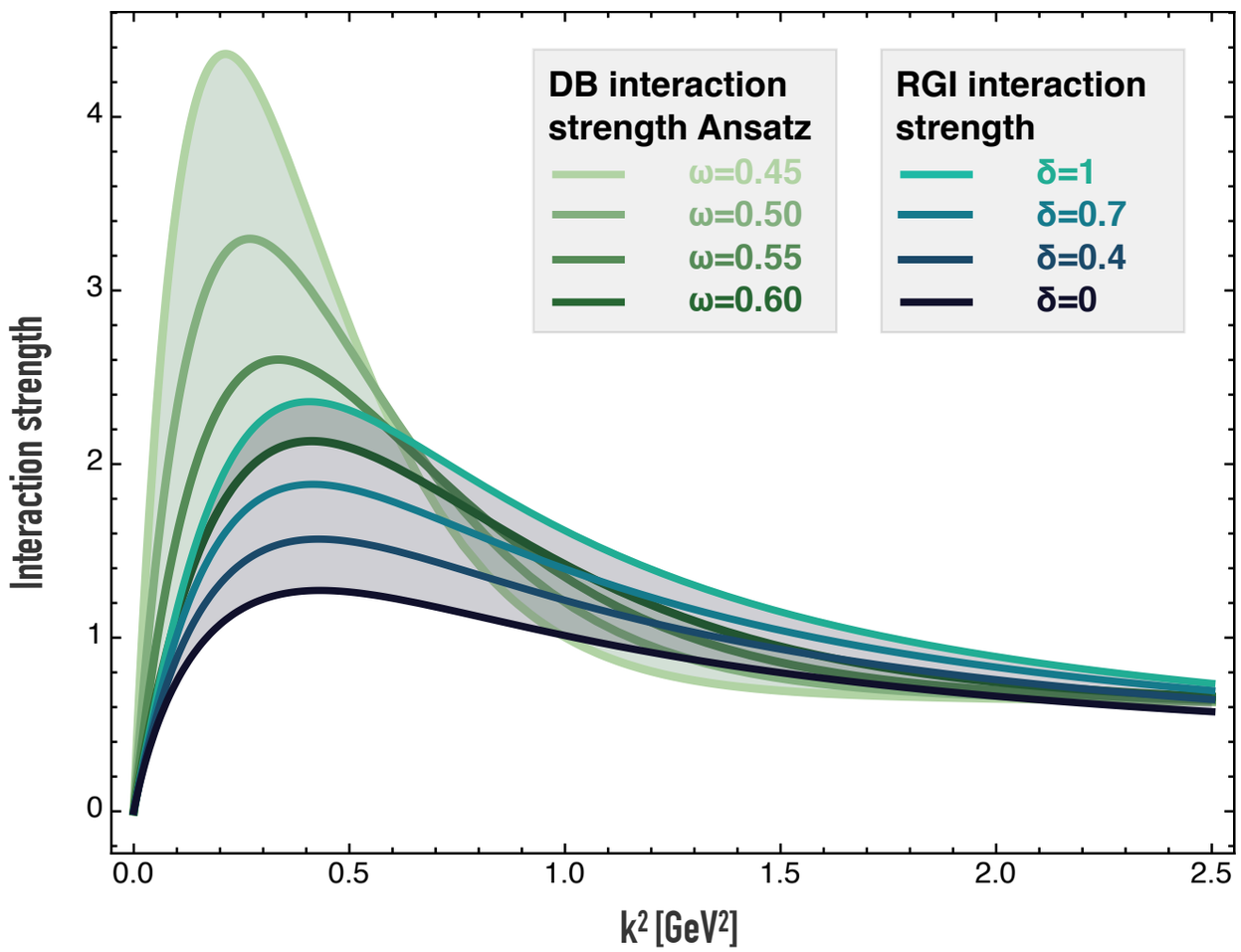
DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)



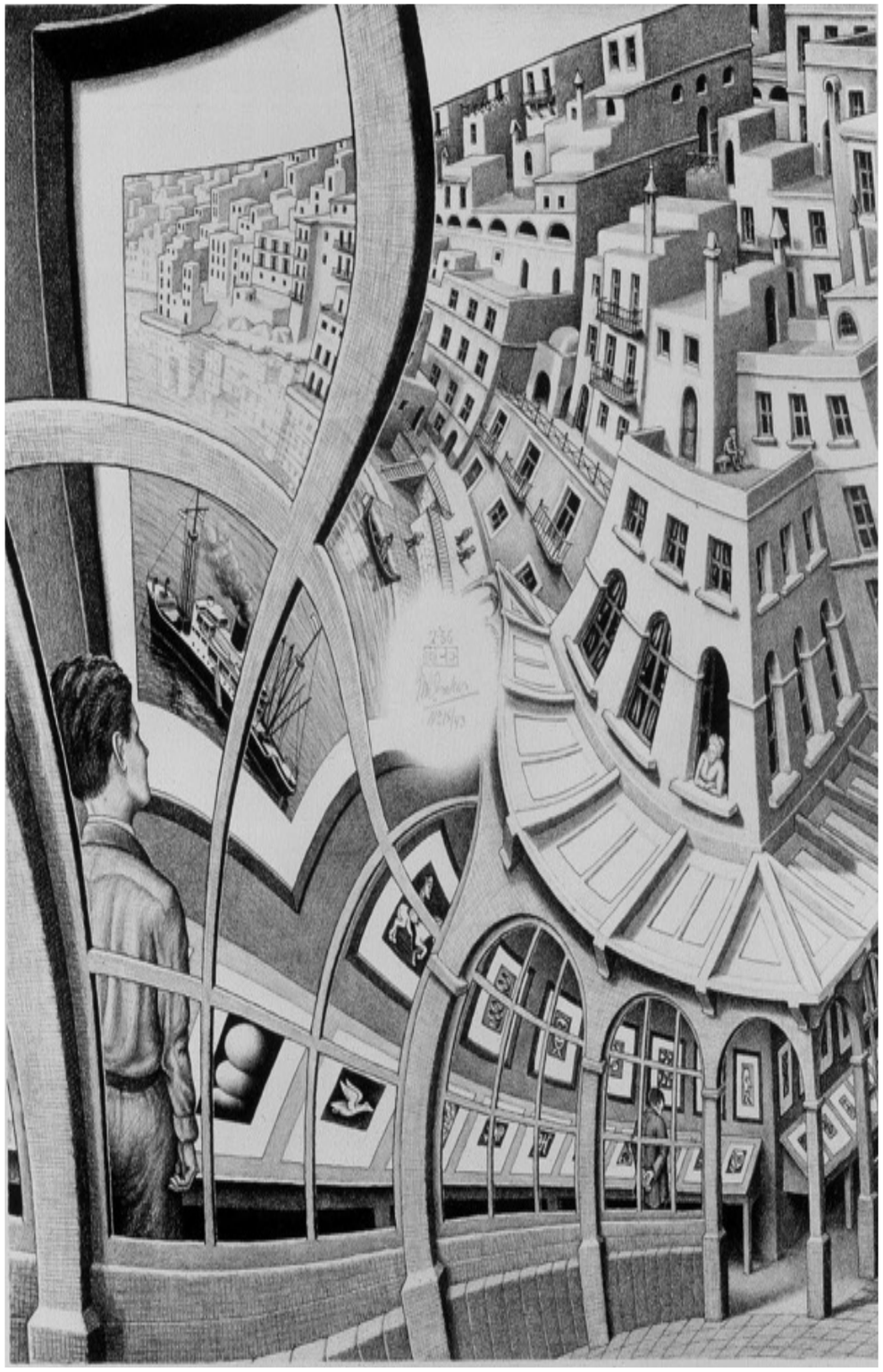
$\Lambda_{\overline{MS}}$=234 [MeV]

M. C. Escher,

Top-down vs bottom-up comparison - II



Λ_{MOM}=570 [MeV]



M. C. Escher, Print Gallery (1956)

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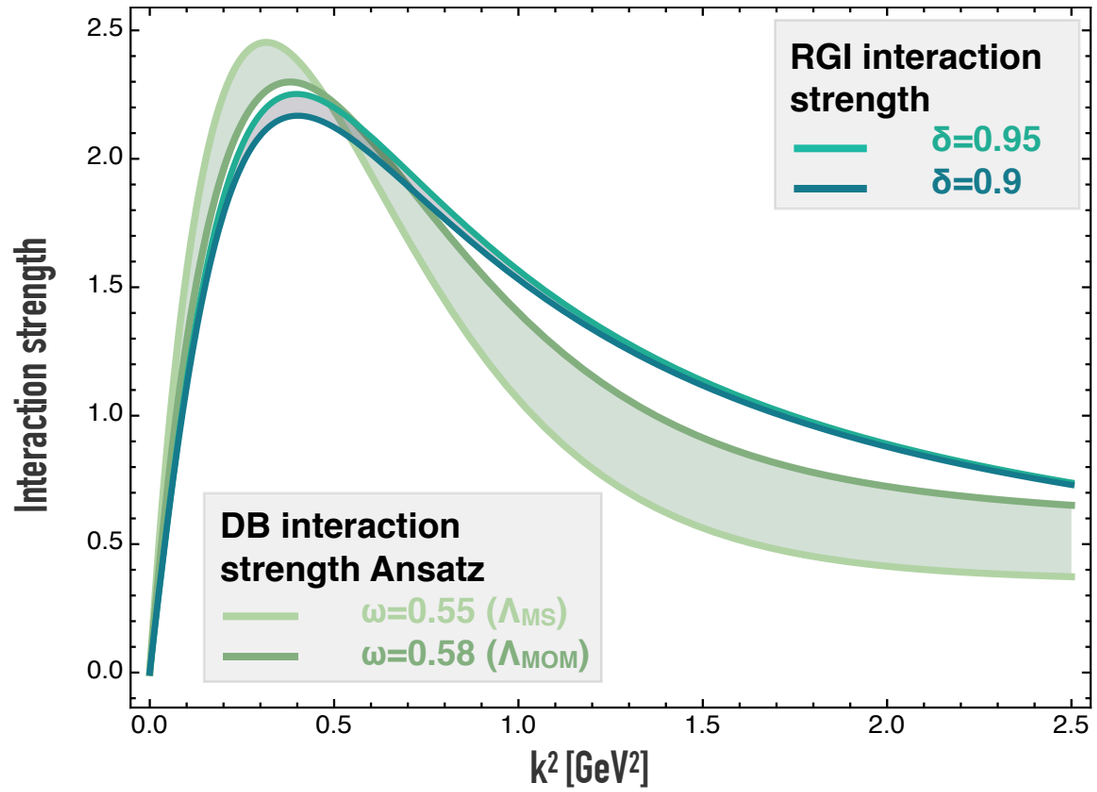
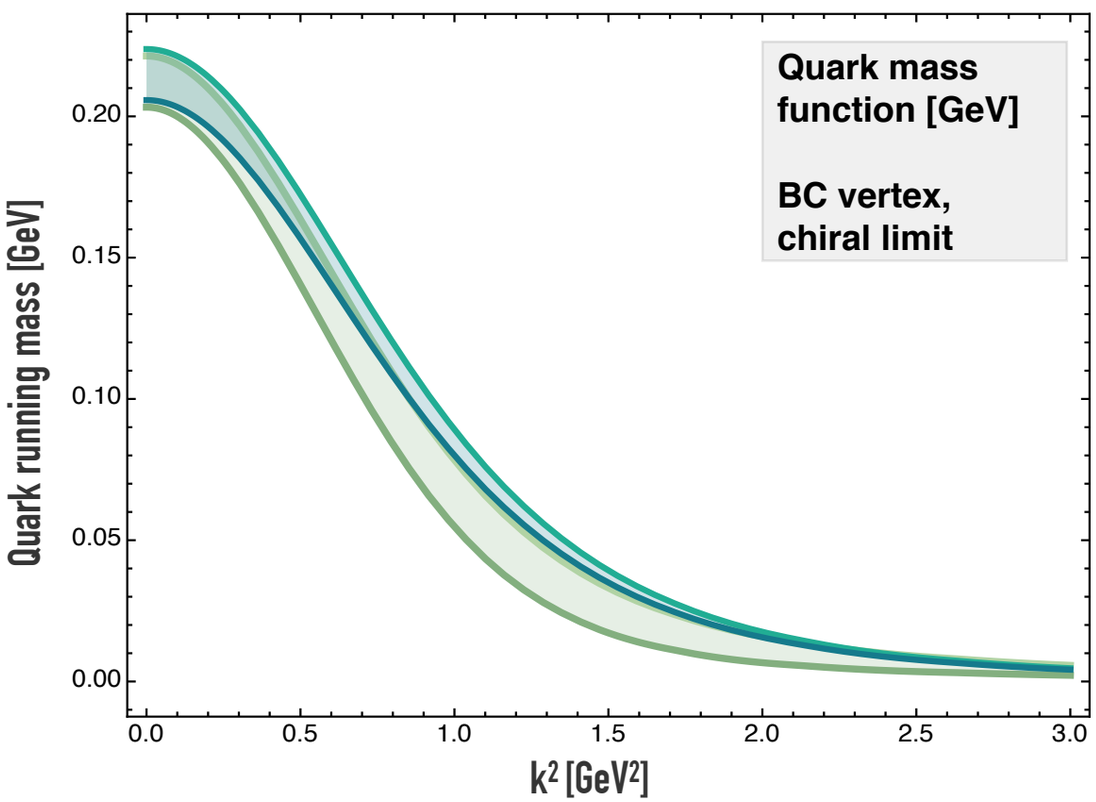
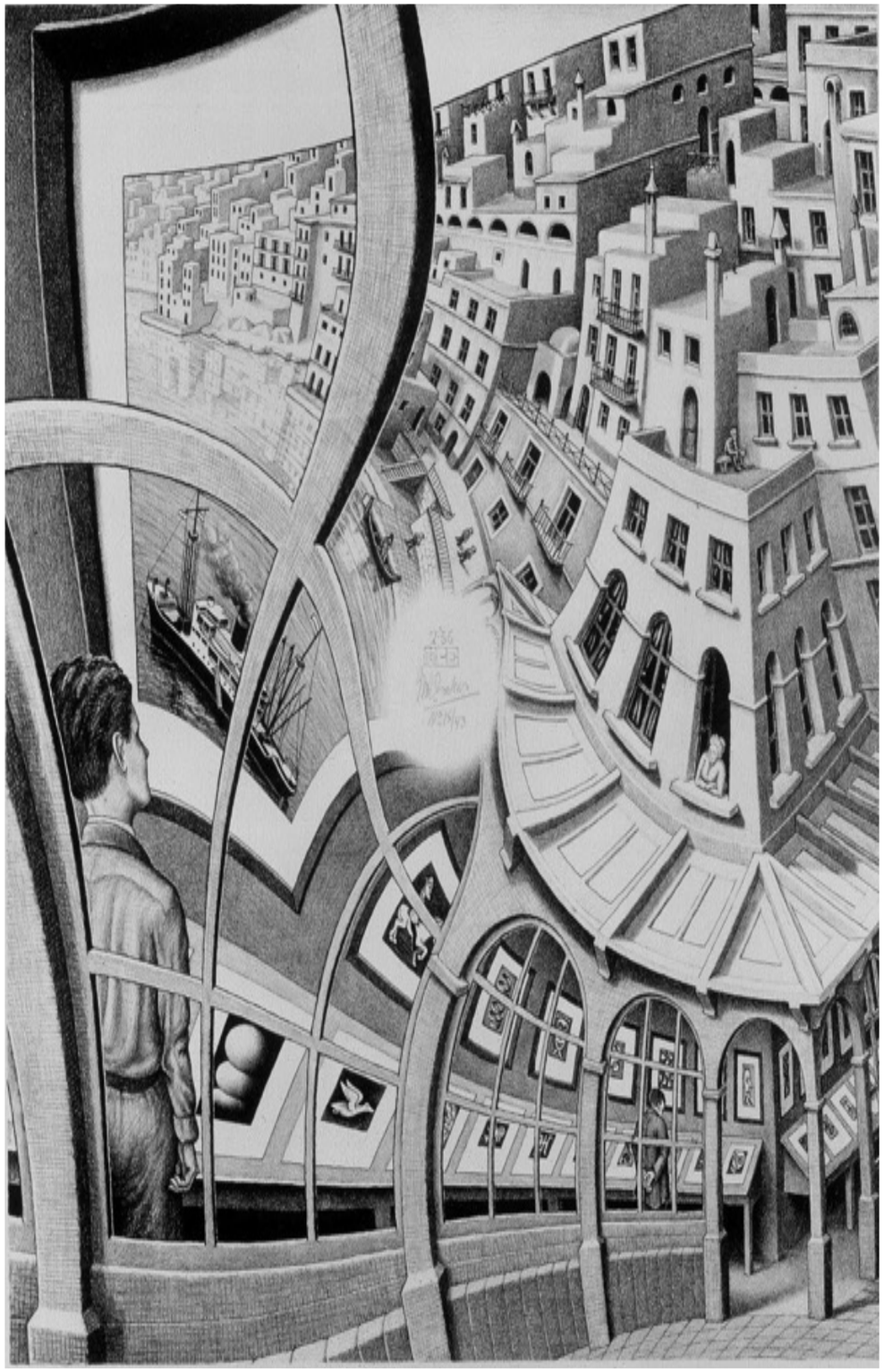




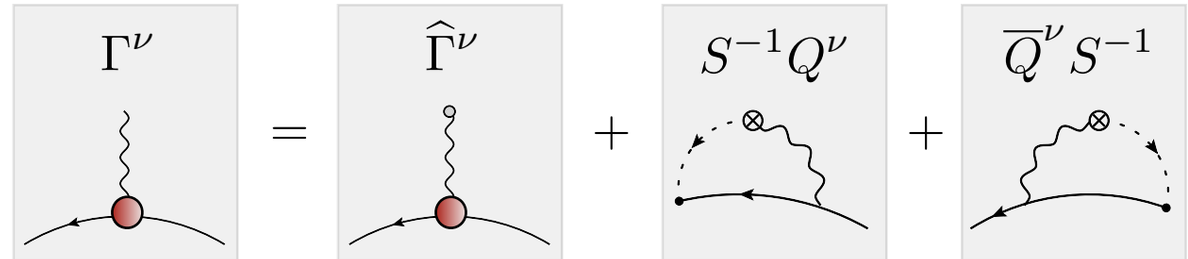


M. C. Escher, Print Gallery (1956)

Quark-gluon vertex

- **Complete quark-gluon vertex** solves all symmetry identities

  Aguilar, DB, Ibañez, Papavassiliou, PRD 90 (2014)

$[1+G]\times$ 

$$\mathcal{G}_q\Gamma_1^L = [1-L(q^2)F(q^2)]\left\{\widehat{\Gamma}_1^L + A_1\left[\frac{1}{2}(q\cdot t)K_3^L - (p_1\cdot t)K_4^L\right] - B_1K_1^L + A_2\left[-\frac{1}{2}(q\cdot t)\overline{K}_3^L + (p_2\cdot t)\overline{K}_4^L\right] - B_2\overline{K}_1^L\right\},$$

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$$\mathcal{G}_q\Gamma_2^T = \widehat{\Gamma}_2^T + A_1\left[-\frac{1}{q\cdot t}K_4^L + \frac{1}{2}K_1^T + \frac{1}{2}(p_1\cdot q)K_4^T - \frac{1}{2}K_7^T\right] - B_1K_2^T + A_2\left[-\frac{1}{q\cdot t}\overline{K}_4^L + \frac{1}{2}\overline{K}_1^T - \frac{1}{2}(p_2\cdot q)\overline{K}_4^T - \frac{1}{2}\overline{K}_7^T\right] - B_2\overline{K}_2^T + \frac{2}{q^2}L(q^2)F(q^2)\left\{A_1\left[\frac{1}{2}K_3^L + \frac{p_1\cdot q}{q\cdot t}K_4^L\right] - B_1K_2^L + A_2\left[\frac{1}{2}\overline{K}_3^L - \frac{p_2\cdot q}{q\cdot t}\overline{K}_4^L\right] - B_2\overline{K}_2^L + \frac{A_1-A_2}{2(q\cdot t)}\right\},$$

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$$\mathcal{G}_q\Gamma_4^T = \widehat{\Gamma}_4^T + A_1\left[K_2^T - \frac{2}{q\cdot t}K_3^T + \frac{1}{q\cdot t}K_8^T\right] - B_1K_4^T + A_2\left[\overline{K}_2^T + \frac{2}{q\cdot t}\overline{K}_3^T - \frac{1}{q\cdot t}\overline{K}_8^T\right] - B_2\overline{K}_4^T + \frac{4}{q^2(q\cdot t)}L(q^2)F(q^2)\left\{\frac{A_1}{2}\left[-K_1^L + (q\cdot t)K_2^L\right] - B_1K_4^L + \frac{A_2}{2}\left[\overline{K}_1^L + (q\cdot t)\overline{K}_2^L\right] - B_2\overline{K}_4^L\right\},$$

$$\mathcal{G}_q\Gamma_5^T = \widehat{\Gamma}_5^T + \frac{A_1}{2}\left[-K_1^L - q^2K_3^T - (q\cdot t)K_6^T - (p_1\cdot t)K_8^T\right] - B_1K_5^T + \frac{A_2}{2}\left[-\overline{K}_1^L - q^2\overline{K}_3^T - (q\cdot t)\overline{K}_6^T - (p_2\cdot t)\overline{K}_8^T\right] - B_2\overline{K}_5^T,$$

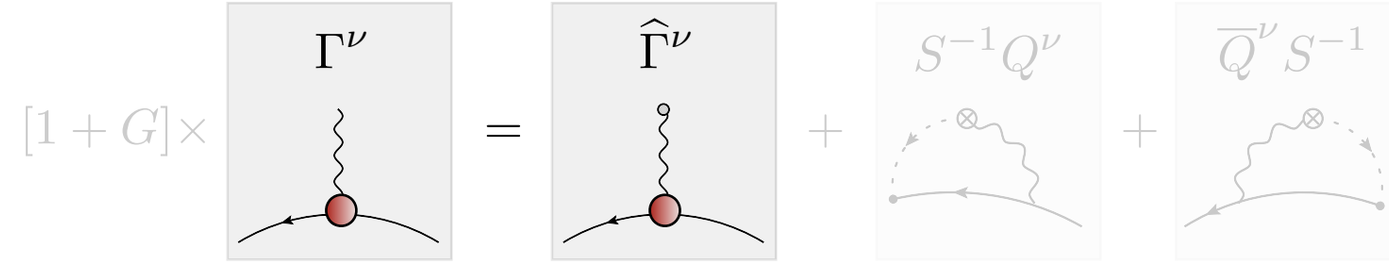
$$\mathcal{G}_q\Gamma_6^T = \widehat{\Gamma}_6^T + \frac{A_1}{2}\left[-K_3^L - \frac{q^2}{2}K_1^T + \frac{q^2}{2}(p_1\cdot t)K_4^T - K_5^T - (p_1\cdot t)K_7^T\right] - B_1K_6^T + \frac{A_2}{2}\left[\overline{K}_3^L + \frac{q^2}{2}\overline{K}_1^T - \frac{q^2}{2}(p_2\cdot t)\overline{K}_4^T + \overline{K}_5^T + (p_2\cdot t)\overline{K}_7^T\right] - B_2\overline{K}_6^T,$$

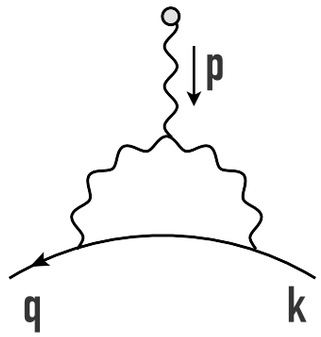
$$\mathcal{G}_q\Gamma_7^T = \widehat{\Gamma}_7^T + A_1\left[-K_2^L - \frac{q^2}{q\cdot t}K_3^T - K_6^T + \frac{p_1\cdot q}{q\cdot t}K_8^T\right] - B_1K_7^T + A_2\left[-\overline{K}_2^L + \frac{q^2}{q\cdot t}\overline{K}_3^T + \overline{K}_6^T + \frac{p_2\cdot q}{q\cdot t}\overline{K}_8^T\right] - B_2\overline{K}_7^T + \frac{2}{q\cdot t}L(q^2)F(q^2)\left\{\frac{A_1}{2}\left[-K_1^L + (q\cdot t)K_2^L\right] - B_1K_4^L + \frac{A_2}{2}\left[\overline{K}_1^L + (q\cdot t)\overline{K}_2^L\right] - B_2\overline{K}_4^L\right\},$$

$$\mathcal{G}_q\Gamma_8^T = \widehat{\Gamma}_8^T + A_1\left[K_4^L - K_5^T + \frac{1}{2}(q\cdot t)K_7^T\right] - B_1K_8^T + A_2\left[-\overline{K}_4^L - \overline{K}_5^T - \frac{1}{2}(q\cdot t)\overline{K}_7^T\right] - B_2\overline{K}_8^T.$$

Quark-gluon vertex

- **Complete quark-gluon vertex** solves all symmetry identities
  Aguilar, DB, Ibañez, Papavassiliou, PRD 90 (2014)





t=q+k

p=q-k

- **Abelian-like ST identity** determines vertex up to transverse parts

$$\widehat{\Gamma}^\nu = \Gamma^\nu_{\rm BC} + \widehat{\Gamma}^\nu_{\rm T}$$

- **Ball-Chiu vertex:** completely specified by the fully dressed quark propagator
  Ball, Chiu PRD 22 (1980)

$$\Gamma^\nu_{\rm BC} = \gamma^\nu \Sigma_A + \frac{1}{2} t_\nu \left[(\gamma\cdot t)\Delta_A - i\Delta_B\right]$$

$$\Sigma_\Phi(q^2,k^2) = \frac{1}{2}[\Phi(q^2)+\Phi(k^2)]$$

$$\Delta_\Phi(q^2,k^2) = \frac{\Phi(q^2)-\Phi(k^2)}{q^2-k^2}$$

- **Transverse part:** minimum required for meson spectrum description, including scalar meson and radial excitations
  Qin, Roberts, … in preparation

$$a^\nu_T = a^\nu + (a\cdot p)p^\nu/p^2$$

$$T^\nu_1(q,k) = \frac{i}{2} t^\nu_T;$$

$$T^\nu_2(q,k) = \frac{1}{2}(\gamma\cdot t) t^\nu_T;$$

$$T^\nu_3(q,k) = \gamma^\nu_T;$$

$$\tau_4(q,k) = a_4 \frac{4\Delta_B}{t_T\cdot t_T} \qquad T^\nu_4(q,k) = -\frac{1}{2} t^\nu_T k^\mu q^\rho \sigma_{\mu\rho};$$

$$T^\nu_5(q,k) = \sigma^{\nu\mu} p_\mu, \qquad \tau_5(q,k) = a_5 \Delta_B$$

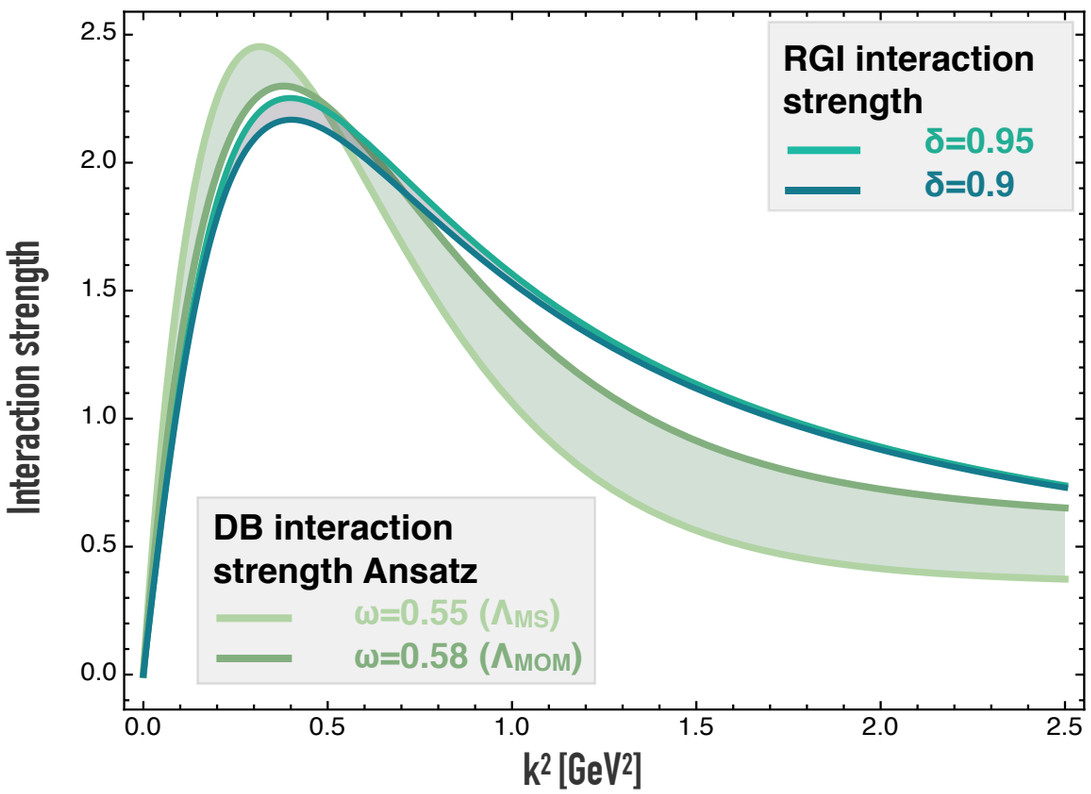
$$T^\nu_6(q,k) = -\gamma^\nu(q^2-k^2) + t^\nu \gamma\cdot p$$

$$T^\nu_7(q,k) = \frac{i}{2}(q^2-k^2)[\gamma^\nu(\gamma\cdot t) - t^\nu] + t^\nu k^\mu q^\rho \sigma_{\mu\rho}$$

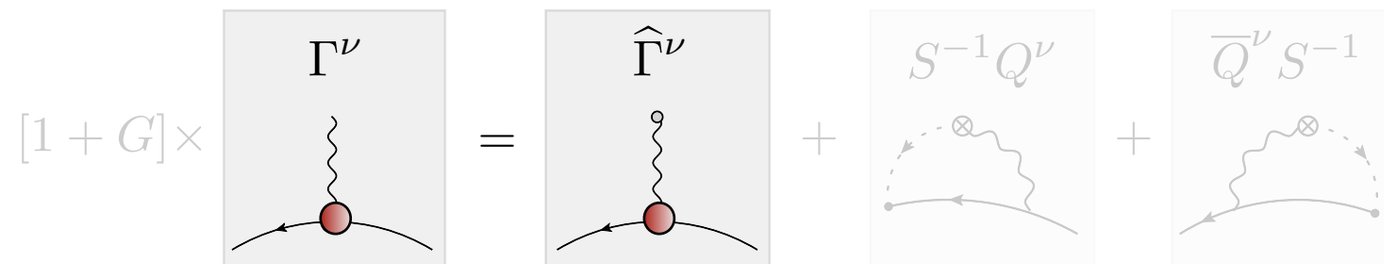
$$T^\nu_8(q,k) = q^\nu \gamma\cdot k - k^\nu(\gamma\cdot q) - i\gamma^\nu k^\mu q^\rho \sigma_{\mu\rho} \qquad \tau_8(q,k) = a_8 \Delta_A$$

Parameter space scan

- **Interaction strength**



- **Interaction vertex**

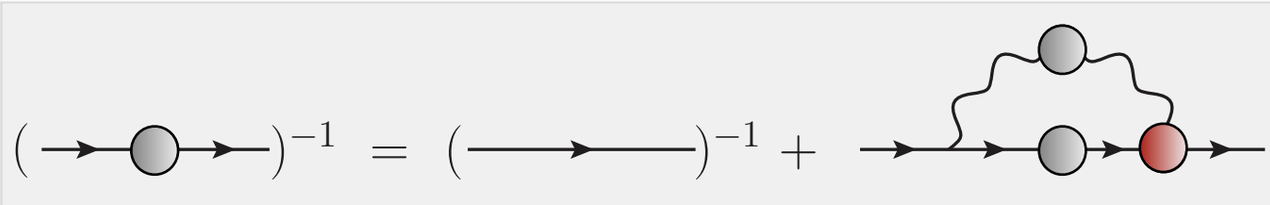


$$\widehat{\Gamma}^\nu = \Gamma^\nu_{\rm BC} + \widehat{\Gamma}^\nu_{\rm T}$$

$$\Gamma^\nu_{\rm BC} = \gamma^\nu \Sigma_A + \frac{1}{2} t_\nu \left[(\gamma\cdot t)\Delta_A - i\Delta_B\right]$$

$$\widehat{\Gamma}^\nu_T = -2a_4 t^\nu_T k^\mu q^\rho \sigma_{\mu\rho} \frac{\Delta_B}{t_T\cdot t_T} + a_5 \sigma^{\nu\mu} p_\mu \Delta_B$$
$$+ a_8 [q^\nu \gamma\cdot k - k^\nu(\gamma\cdot q) - i\gamma^\nu k^\mu q^\rho \sigma_{\mu\rho}]\Delta_A$$

- **Solve gap equation**
  in this new scheme
  DB, Papavassiliou, Roberts, … in preparation

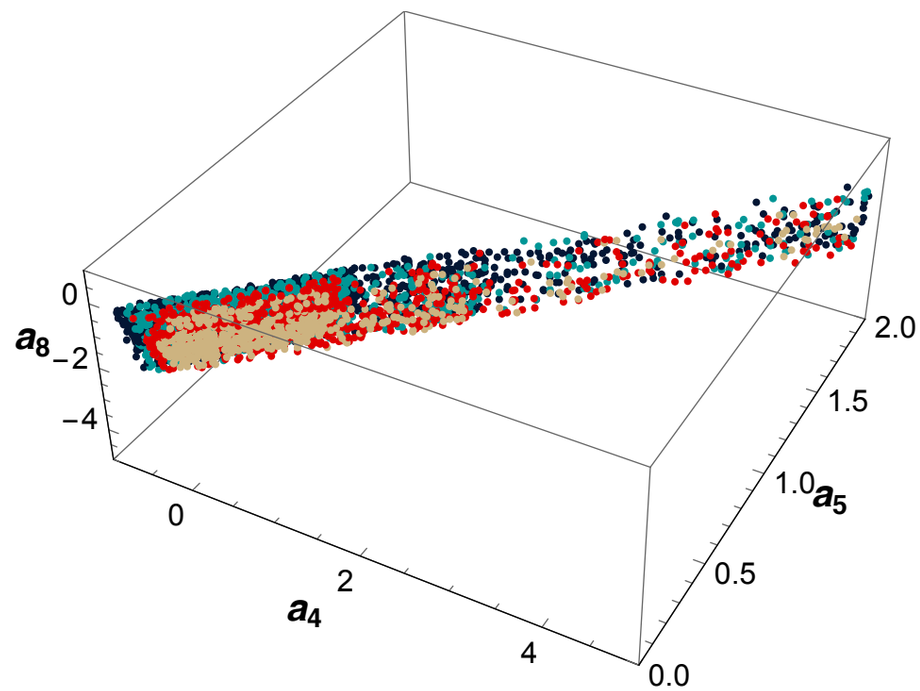


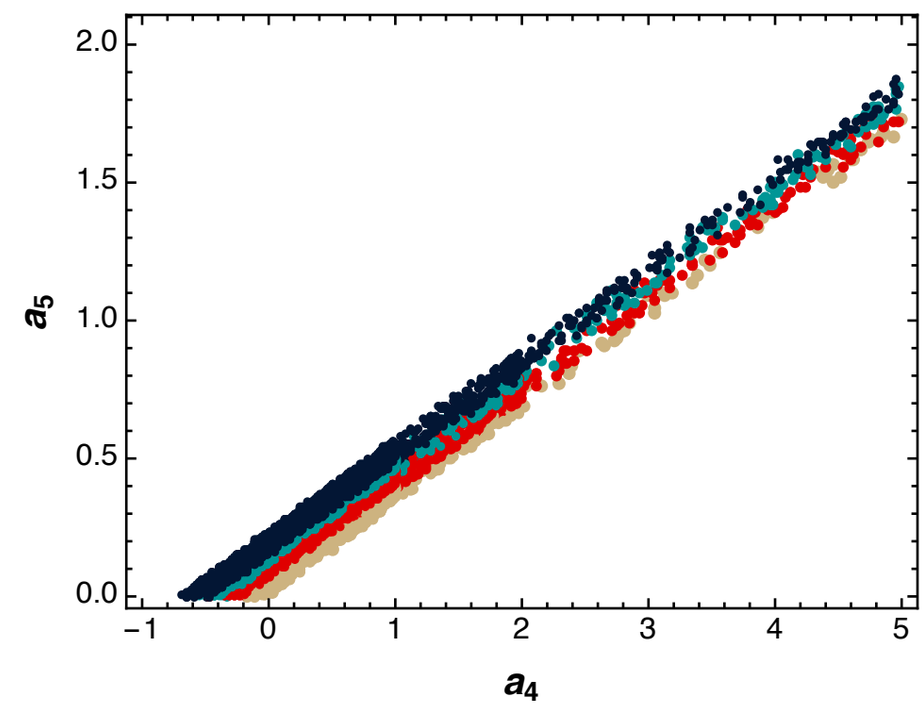
- **Scan the parameter space**
  look for *sensible* solutions varying a_4, a_5 and a_8
- **'Fitness' criteria**
  M(0)∈[250,450] MeV
  good value of pion decay constant
  non-negative and sizeable ACM distribution

Numerical results

- **Full scan with 2x10^5 runs:** mass cut only

Mass cut
- 400-450 MeV
- 350-400 MeV
- 300-350 MeV
- 250-300 MeV





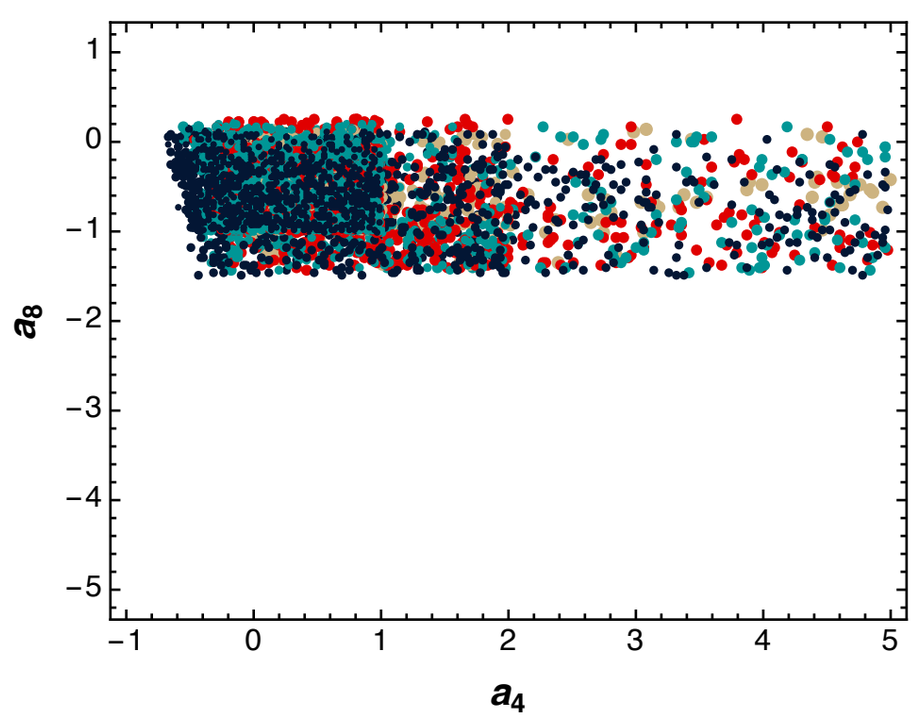
- **Parameter variation**
  $a_4 \in [-1,5]$
  $a_5 \in [0,2]$
  $a_8 \in [-5,1]$

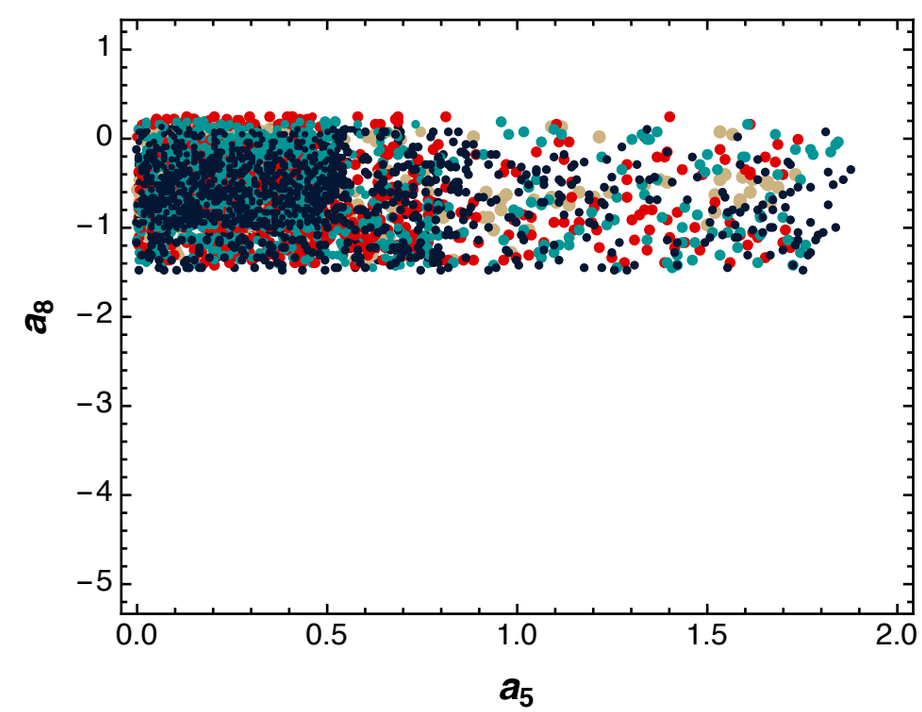
- **Small solutions' volume**
  - 2341 (1.20%)
  - 1318 (0.66%)
  - 1331 (0.67%)
  - 789 (0.40%)

- **Pion decay constant** always within 20% of physical value

- **Emergent linear relation**

$$a_4 \sim 3.03 \times a_5$$





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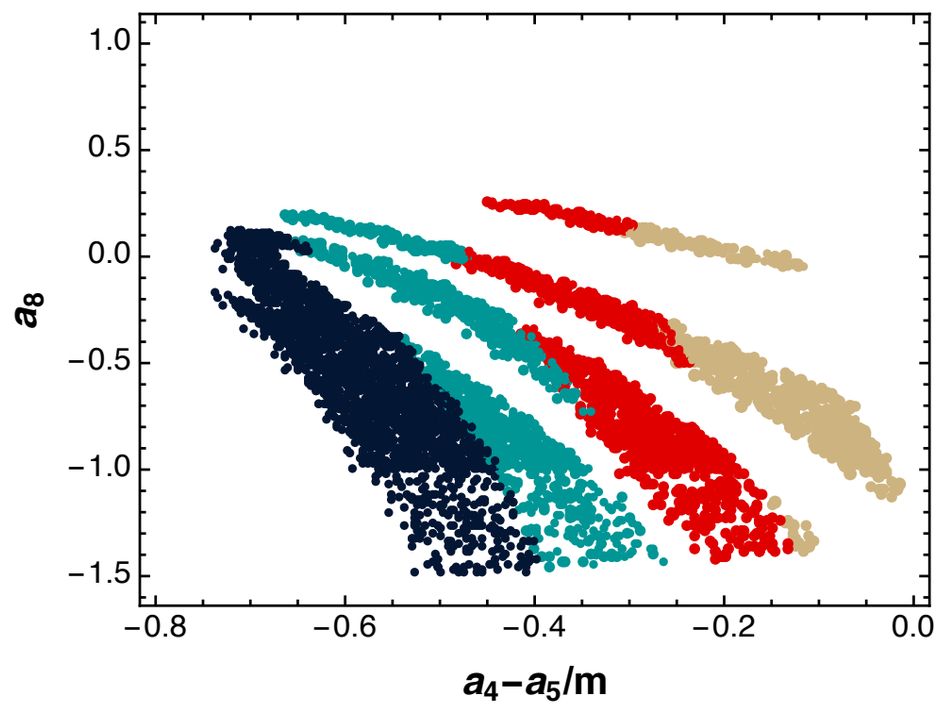
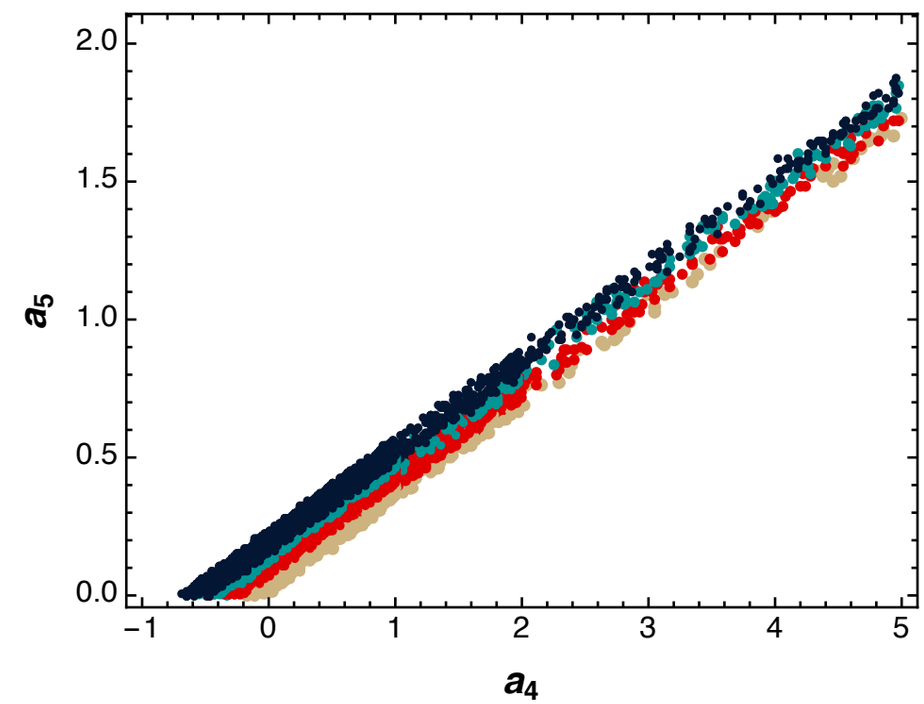
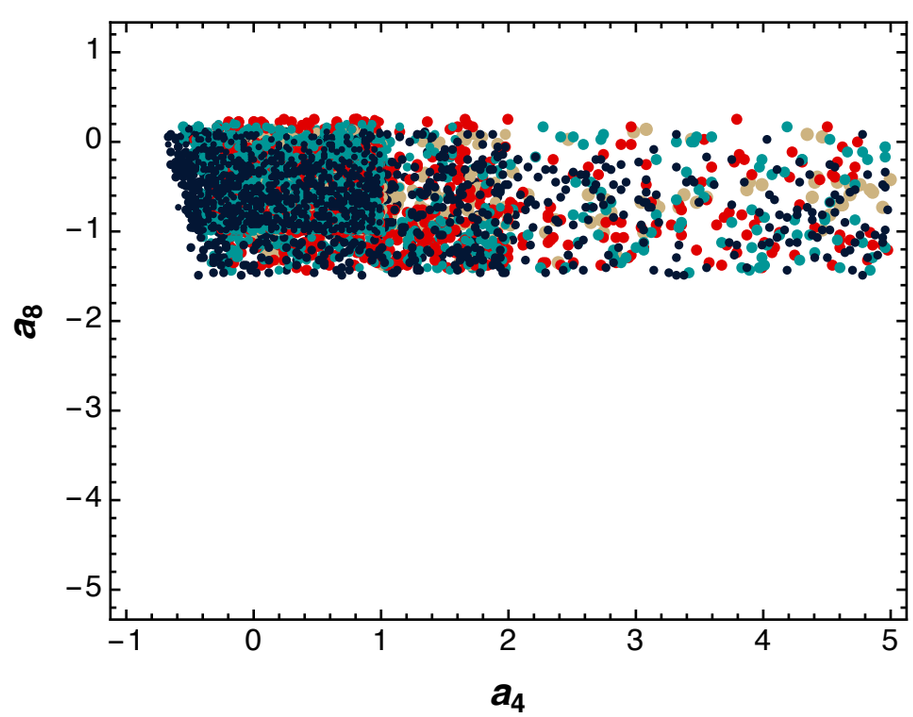
- **Pion decay constant** always within 20% of physical value

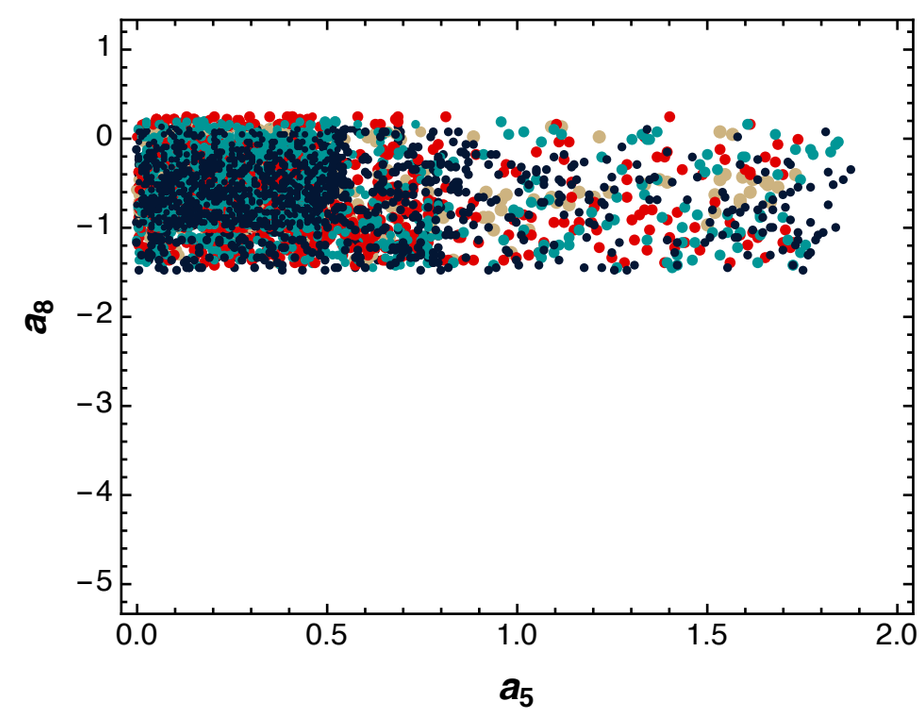
- **Emergent linear relation**
  $a_4 \sim 3.03 \times a_5$

- **Solution space is not convex**









Additional restrictions

- **Sensible ordering of the ρ and σ meson iff**

  Qin, Roberts, … in preparation

$a_4/a_5 \sim 2.3$

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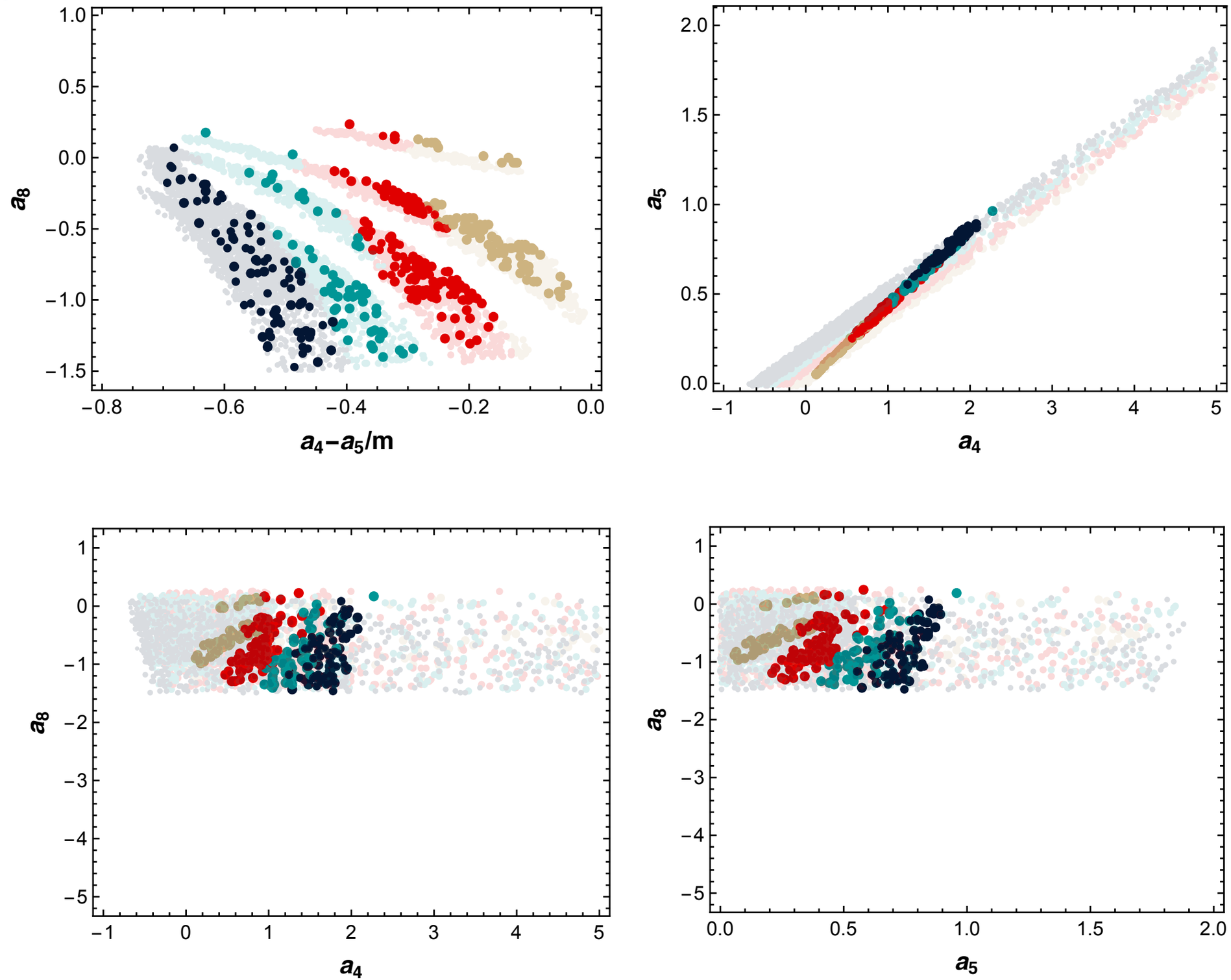

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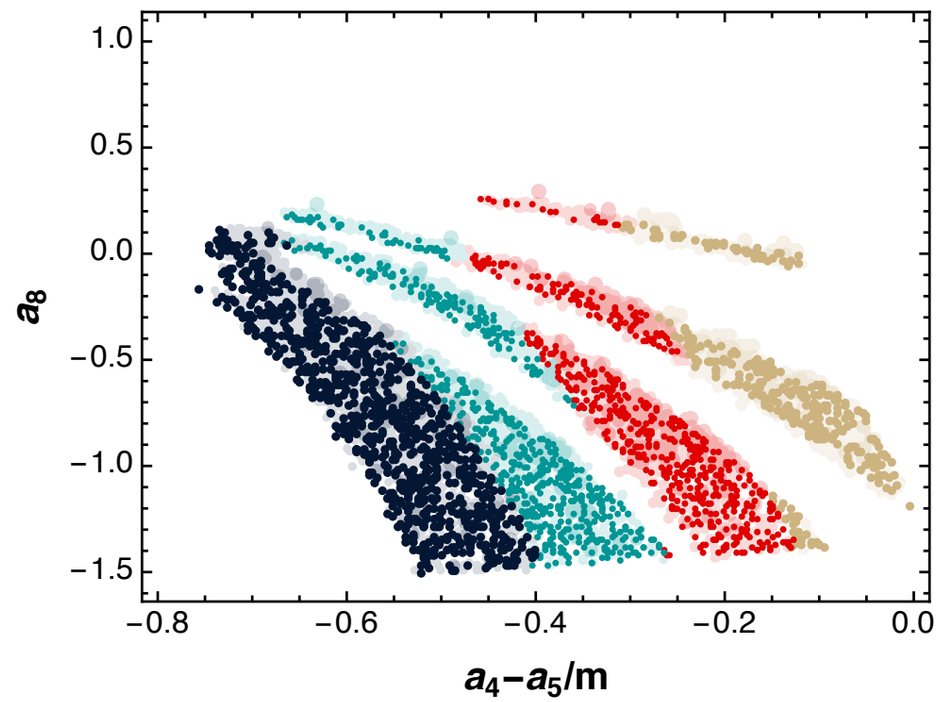
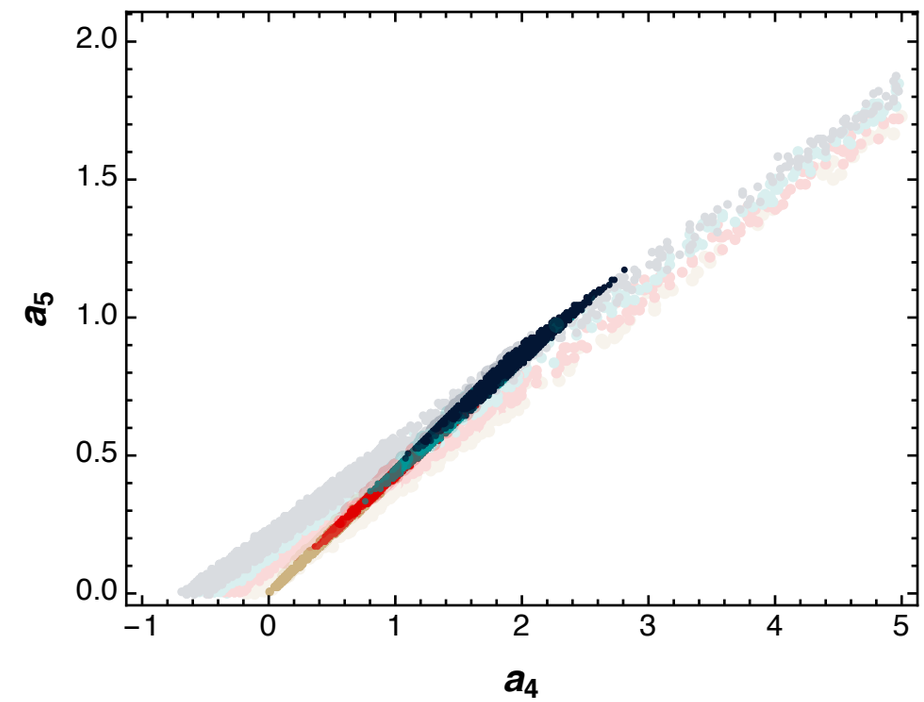
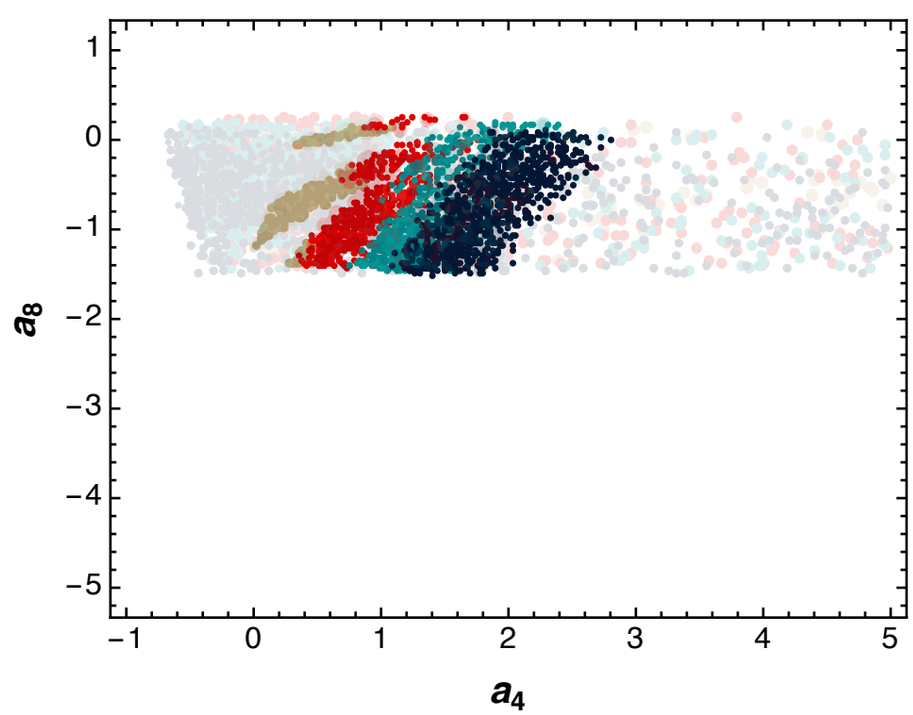
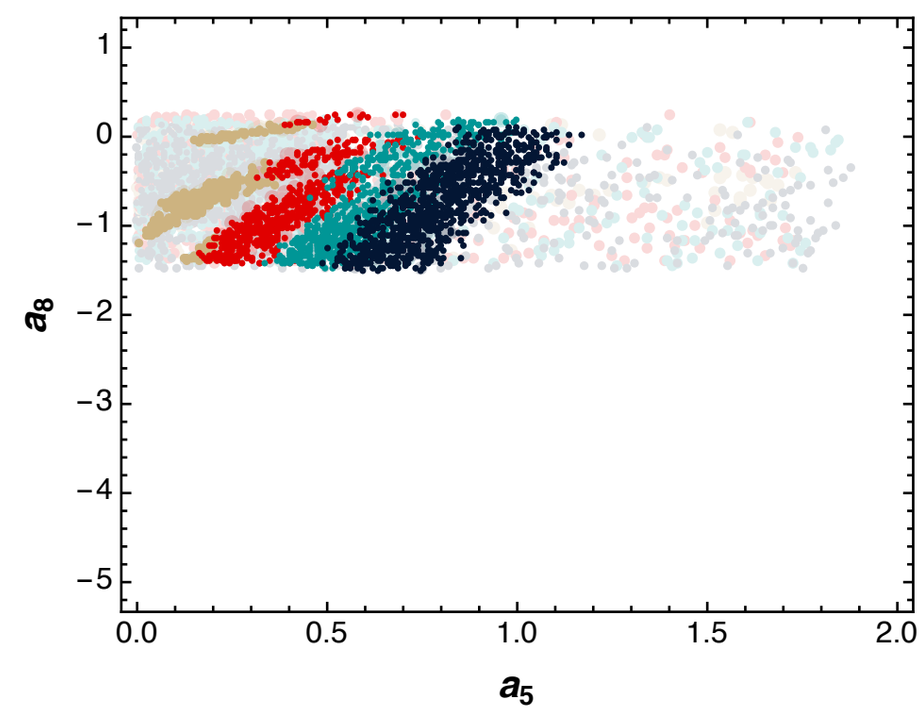
- **Full simulation 4x10⁴**

  $a_4/a_5 \in [2.2, 2.4]$









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- **Full simulation 4x10⁴**

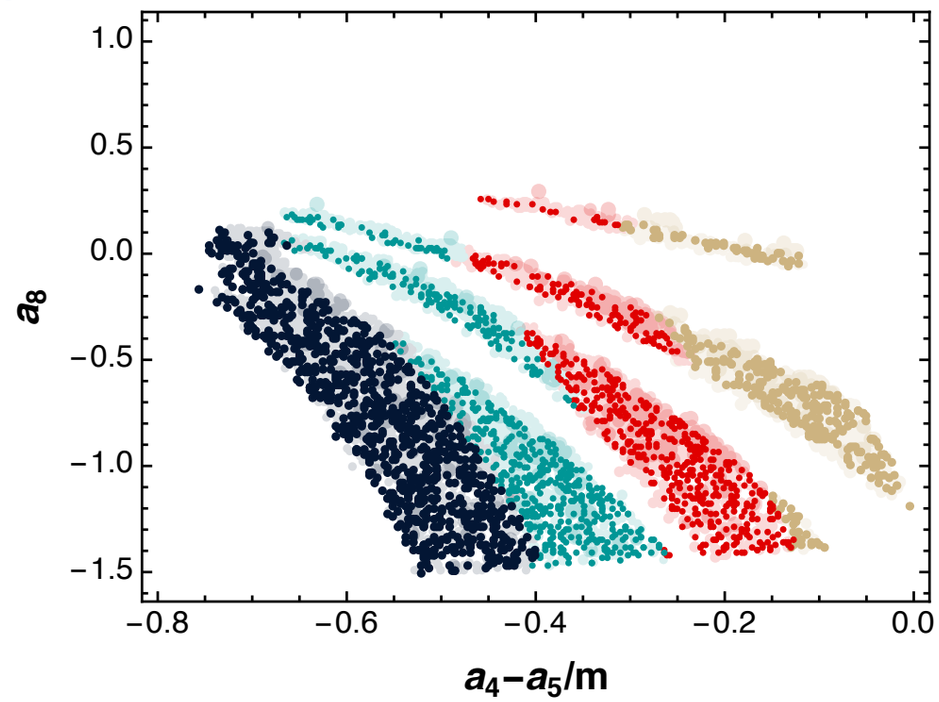
  $a_4/a_5 \in [2.2, 2.4]$

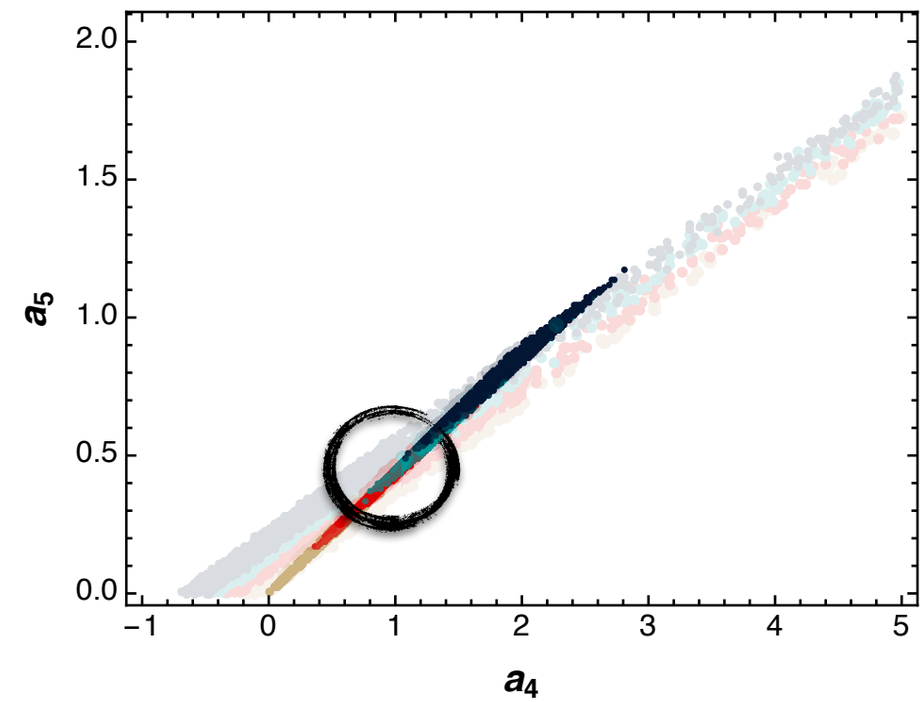
- **'Natural' values**

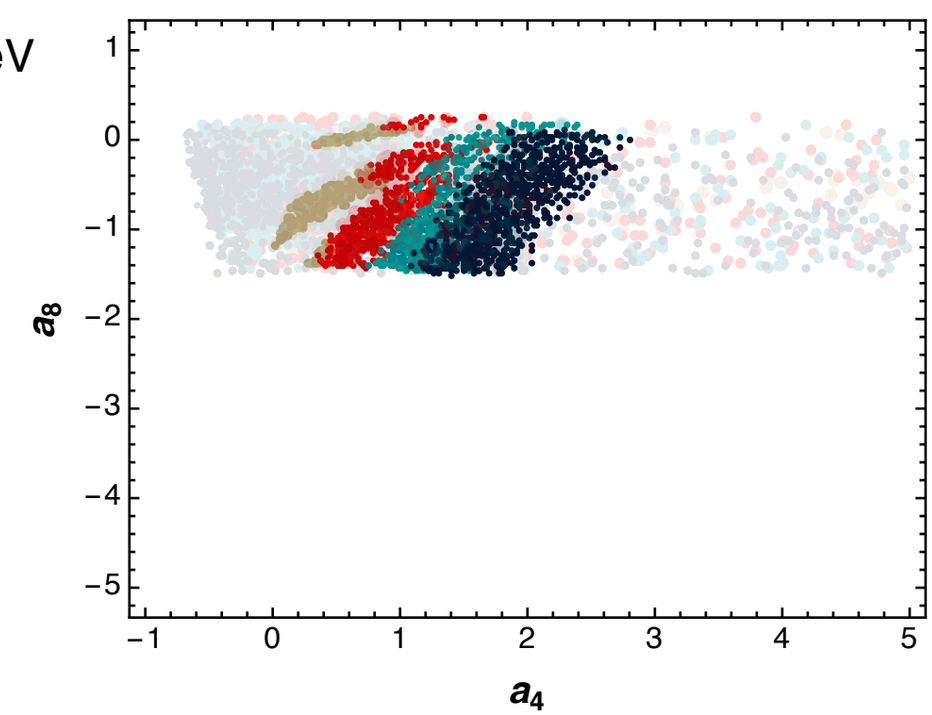
  $a_4 \sim 1.15$

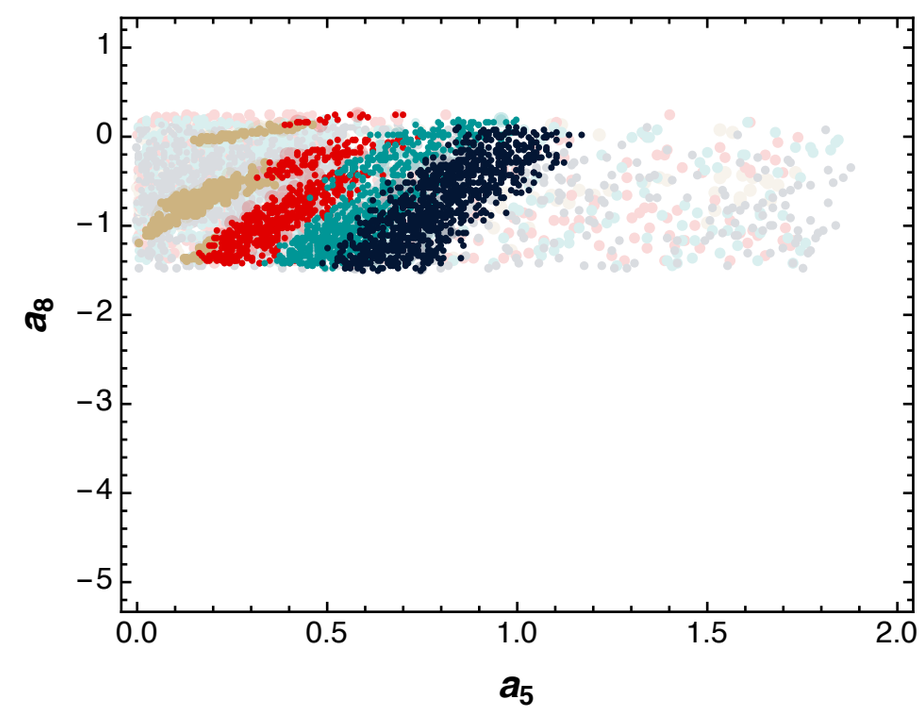
  $a_5 \sim 0.5$

  selects M(0)∈[300,400] MeV









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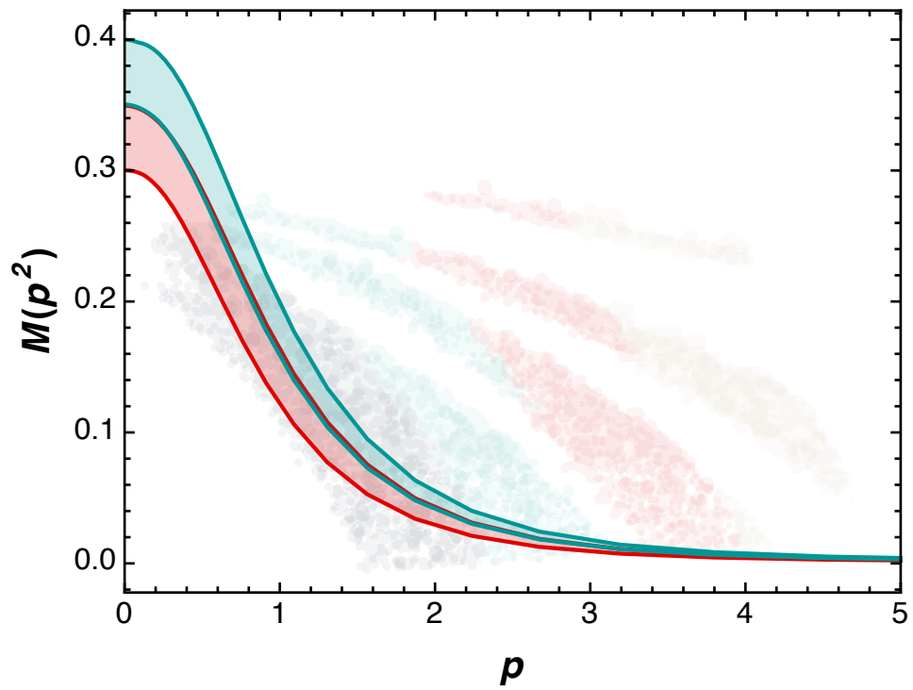
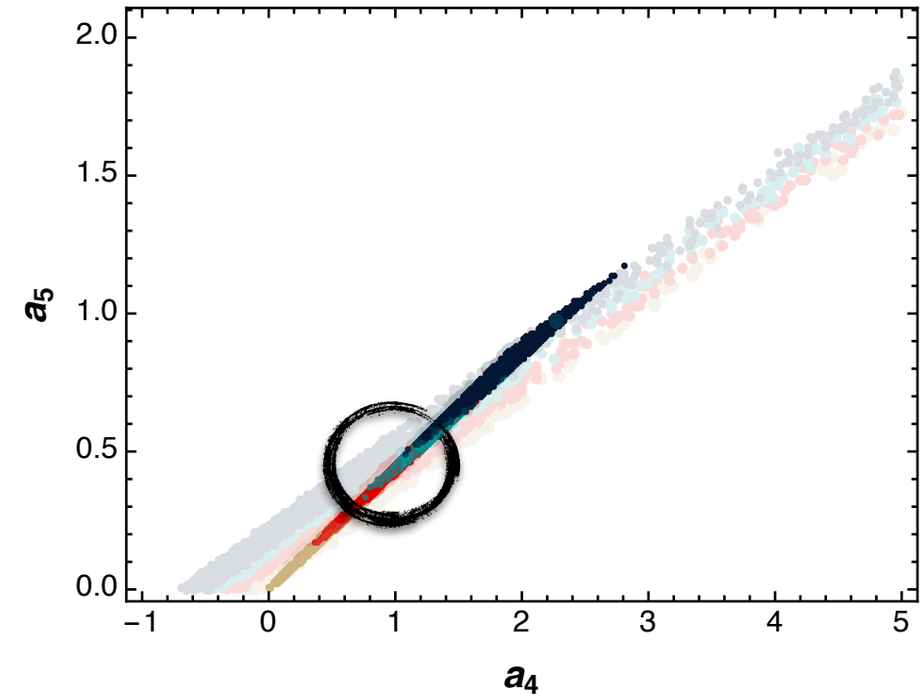
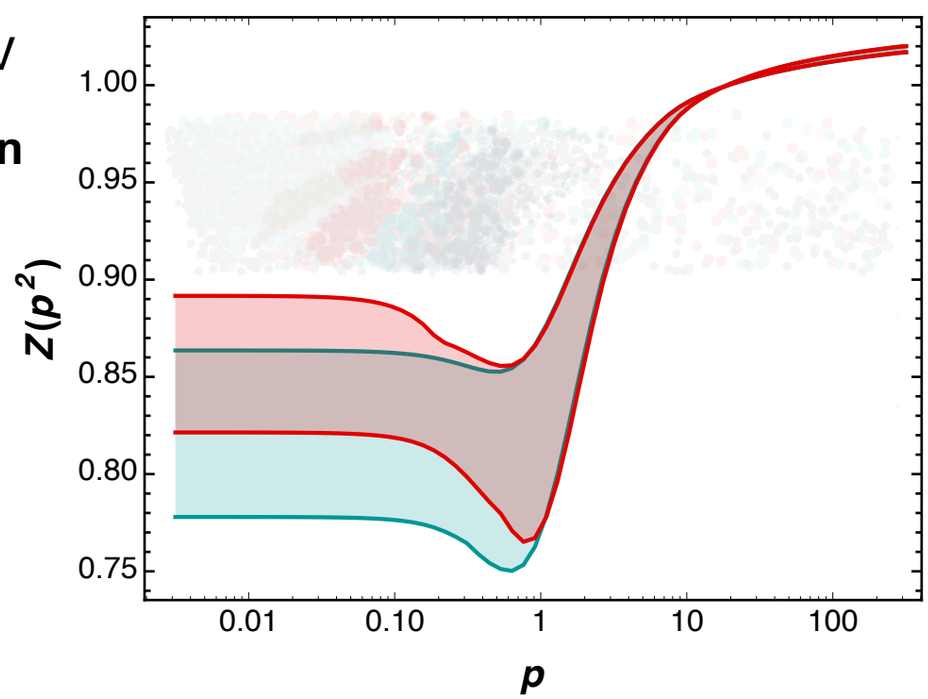
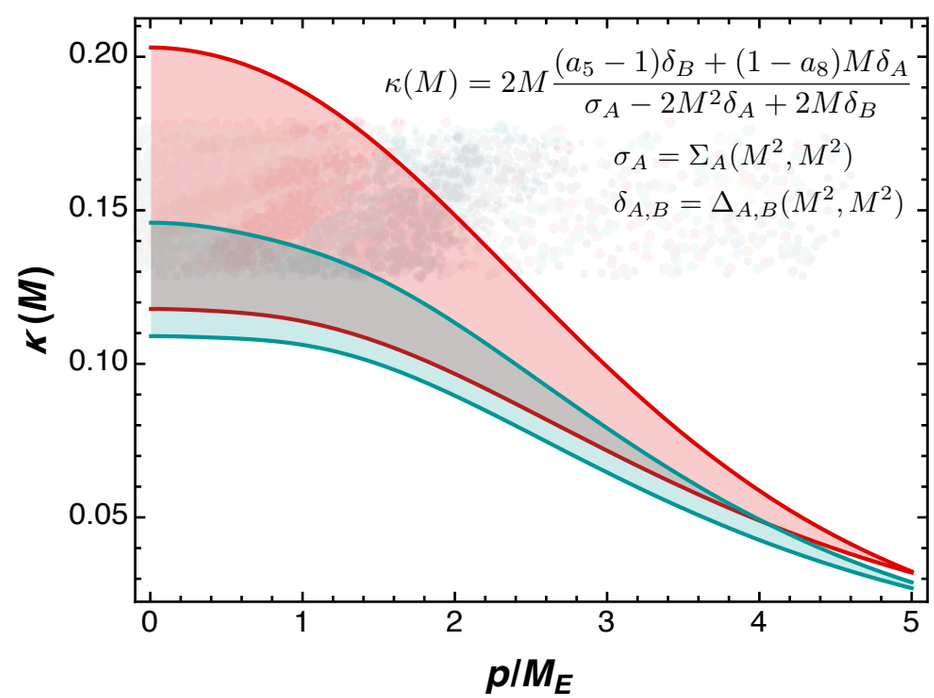
  selects M(0)∈[300,400] MeV

- **Solutions ACM distribution**
  sizeable $\kappa(0)\sim$ [0.11,0.20]









Summary

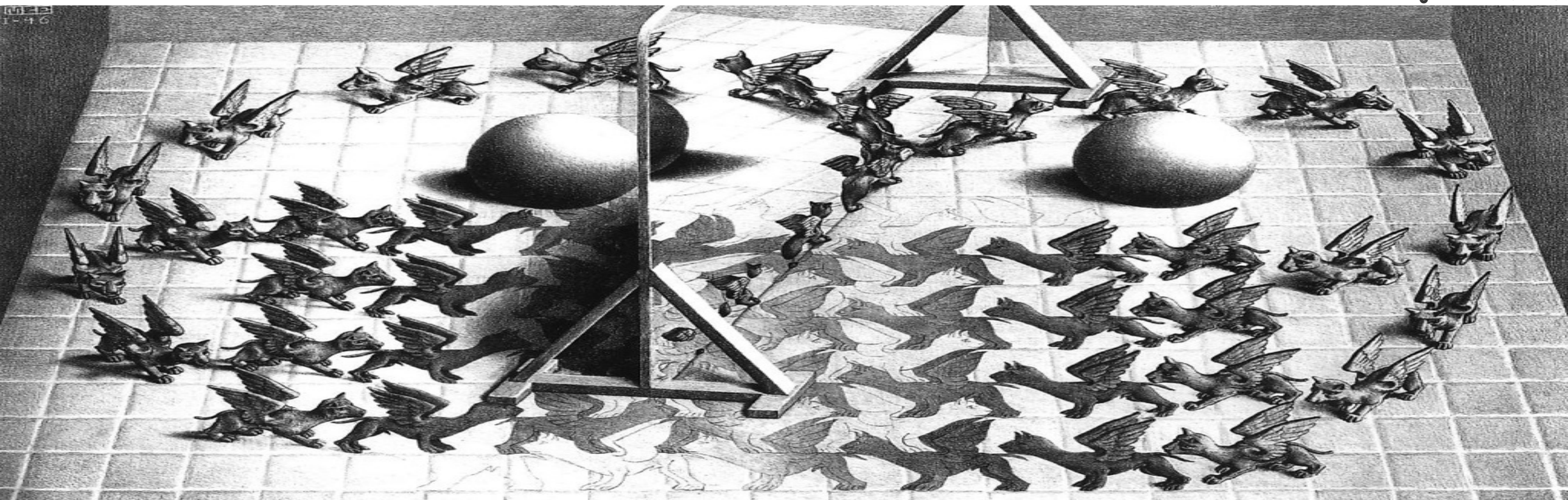
So far:

- QCD gauge sector interaction reconciled with that required to describe ground-state properties
- Preliminary analysis of the associated vertex

Next:

- Complete and publish :)

Long term:

- Refine parameter scan possibly adding all transverse terms
- Fulfil the promise and proceed to parameter-free predictions

M. C. Escher, Magic Mirror (1946)