Gluons and ghosts at (non-)vanishing temperatures

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From Green functions to 'observables'

Basic building blocks of functional equations: n-point functions $\Gamma_{i_1 \ldots i_n}$

Effective action: generating functional of 1PI Green functions

$$\Gamma[\Phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_1 \ldots \Phi_n \Gamma_{i_1 \ldots i_n}$$

$\longleftrightarrow$

The set of **all** Green functions describes the theory completely.

$$\Gamma_{ij} = \frac{\delta^2 \Gamma[\Phi]}{\delta\Phi_i \delta\Phi_j},$$
$$\Gamma_{ijk} = \frac{\delta^3 \Gamma[\Phi]}{\delta\Phi_i \delta\Phi_j \delta\Phi_k}, \quad \ldots$$

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Green functions $\to$ 'observables'?

Examples:

- Bound state equations $\to$ masses and properties of hadrons
- Analytic properties of Green functions $\to$ confinement
- (Pseudo-)Order parameters $\to$ Phases and transitions

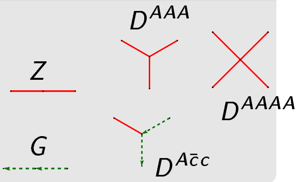
Landau gauge Yang-Mills theory

Gluonic sector of quantum chromodynamics: Yang-Mills theory

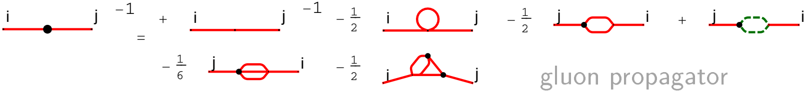
$$\mathcal{L} = \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + i\, g\, [\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$$

Landau gauge

- simplest one for functional equations
- $\partial_\mu \boldsymbol{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \boldsymbol{A}_\mu)^2, \quad \xi \to 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\boldsymbol{c}}\,(-\Box + g\,\boldsymbol{A}\,\times)\,\boldsymbol{c}$



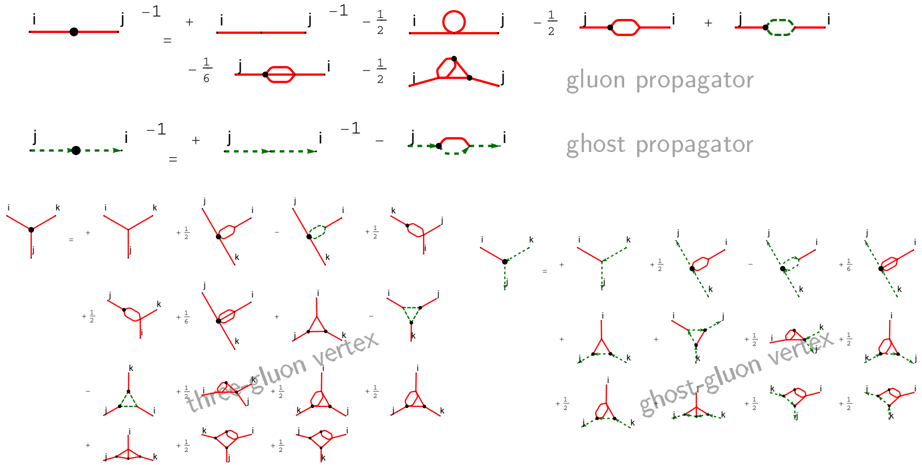
The tower of DSEs



gluon propagator

ghost propagator

The tower of DSEs



Infinitely many equations. In QCD, every n-point function depends on $(n+1)$- and possibly $(n+2)$-point functions.

Truncating the equations

Truncation

- Drop quantities (unimportant?)
- Model quantities (good models available?)
- Use fits

Ideally: Find a truncation that has no parameters and yields quantitative results.

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Ideally: Find a truncation that has no parameters and yields quantitative results.

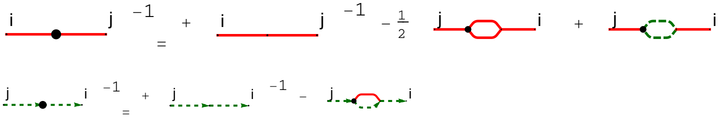
Guides

- Perturbation theory
- Symmetries
- Lattice
- Analytic results

Practical obstacle: Manage the system of equations. $\rightarrow$ Automatization tools [Alkofer, MQH, Schwenzer '08; Braun, MQH '11; MQH, Mitter '11; `http://tinyurl.com/dofun2`; `http://tinyurl.com/crasydse`]

Truncation of Yang-Mills system

Coupled system of propagators with models for three-point functions:



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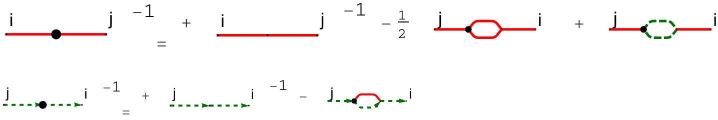
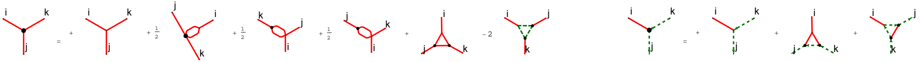


Include three-point functions dynamically:



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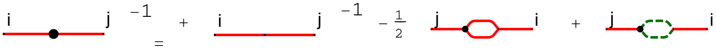

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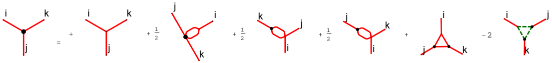

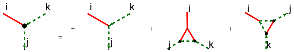

Open questions:

- Four-gluon vertex (in this truncation scheme no dependence on higher n-point functions)
- Two-loop diagrams

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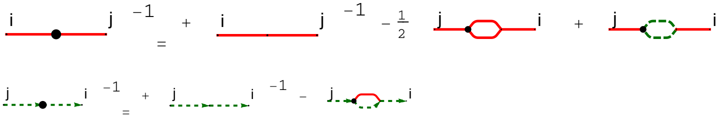

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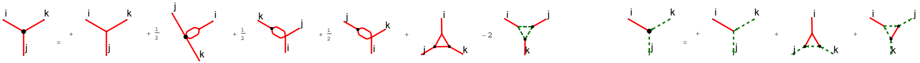
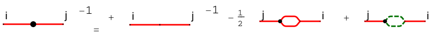


Open questions:

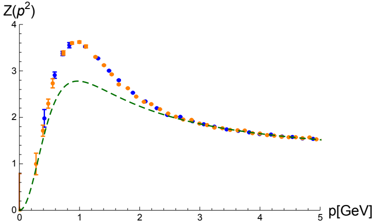
- Four-gluon vertex (in this truncation scheme no dependence on higher n-point functions)
- Two-loop diagrams
- Technical questions: spurious divergences in gluon propagator, RG resummation

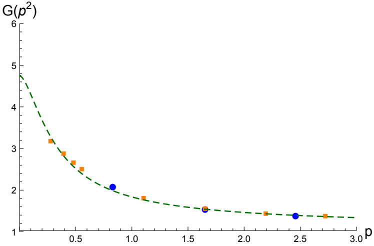
Propagators I



Long-time 'standard' truncation

- No four-gluon vertex
- Ghost-gluon vertex: bare
- Three-gluon vertex: model





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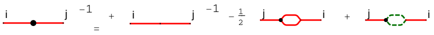
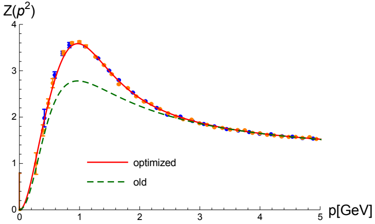


Long-time 'standard' truncation

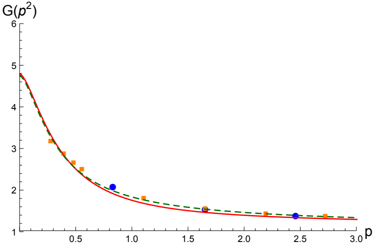
- No four-gluon vertex
- Ghost-gluon vertex: bare $\rightarrow$ dressed (dynamic)
- Three-gluon vertex: model $\rightarrow$ optimized model

Missing strength in mid-momentum regime:

- neglected diagrams?
- vertices?



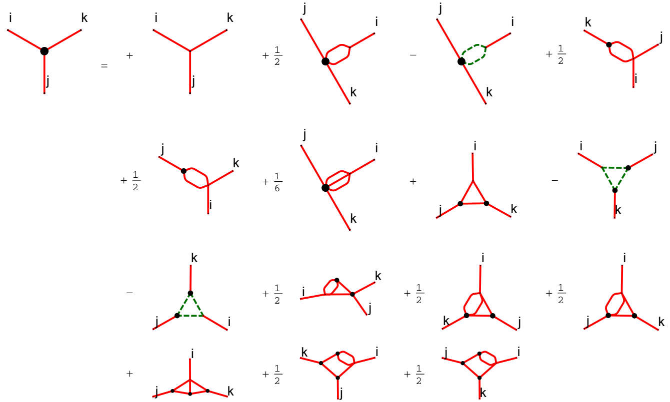
[MQH, von Smekal '12; lattice; Sternbeck '06]



$\rightarrow$ Role of three-gluon vertex?

$\rightarrow$ Use as input in other calculations.

Three-gluon vertex DSE



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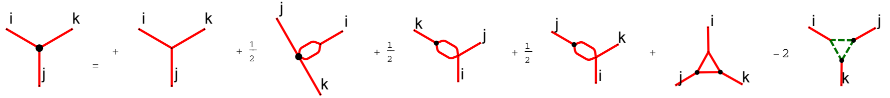


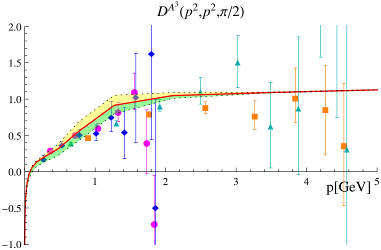
- Keep only diagrams with primitively divergent Green functions.
- Tree-level tensor only.

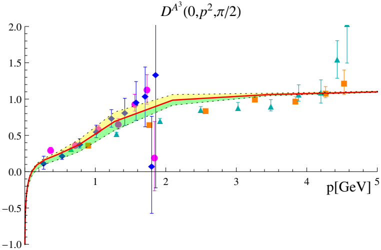
Four-gluon vertex model:

$$D^{A^4}(p,q,r,s) = (a\tanh(b/\bar{p}^2)+1)\,D^{A^4}_{RG}(p,q,r,s)$$

$\rightarrow$ Test model dependence by varying a and b.

The three-gluon vertex





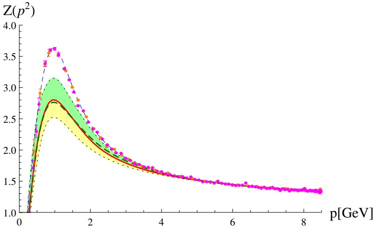
[Blum, MQH, Mitter, von Smekal '14; lattice: Cucchieri, Maas, Mendes '08]

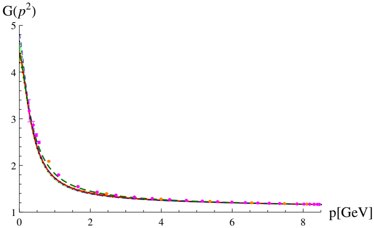
$\rightarrow$ Truncation reliable. Neglected terms, including two-loop, suppressed.

- Zero crossing (in this tensor)
  See also [Peláez, Tissier, Wschebor '13; Aguilar, Binosi, Ibáñez, Papavassiliou '13; Eichmann, Williams, Alkofer, Vujinovic '14],
  lattice seen in $d = 2, 3$ [Maas '07; Cucchieri, Maas, Mendes '08].
- Results for other dressings [Eichmann, Williams, Alkofer, Vujinovic '14]: very small.

Propagators II: Limits of one-loop truncation

Feed three-gluon vertex into gluon propagator DSE:





[Blum, MQH, Mitter, von Smekal '13]

→ Gap in midmomentum regime must be due to missing two-loop diagrams!

NB: Employed projection of three-gluon vertex is the same as in gluon loop of gluon propagator DSE! → Error from neglected tensors small.

Regularization and UV behavior

Ways of regularization

- Lattice
- Pauli-Villars
- Analytic reg., proper time, ...; useful for analytic calculations
- Dimensional regularization → numerical difficult, esp. for power law divergences [Phillips, Afnan, Henry-Edwards '99]
- UV cutoff:

$$\int_0^\infty dq \to \int_0^\Lambda dq$$

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Several methods for subtraction of spurious divergences used.
Prerequisite for a quantitatively accurate description: a good understanding of how to subtract them.

Subtraction of divergences of gluon propagator

1. Logarithmic divergences handled by subtraction at p_0.
2. Quadratic divergences also subtracted, coefficient C_{sub}.

$$Z(p^2)^{-1} := \underbrace{Z_\Lambda(p^2)^{-1}}_{\text{calculated right-hand side (log. divergences handled)}} - C_{\text{sub}} \left(\frac{1}{p^2} - \frac{1}{p_0^2} \right)$$

How to determine C_{sub}?

Calculation of C_{sub}

Can be calculated analytically!

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Use projector $P^{\zeta}_{\mu\nu}(p) = g_{\mu\nu} - \zeta p_\mu p_\nu / p^2$ for gluon propagator DSE.

Consider ghost loop: 

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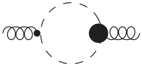
Consider ghost loop: 

Approximation: $G\left((p+q)^2\right) \to G\left(q^2\right) \Rightarrow$ perform angle integrals

$$I_{gh}(x) = \frac{N_c\, g^2}{192\pi^2} \int_x^{\Lambda^2} dy \Big(\underbrace{x\,(\zeta - 2)}_{\text{log. div.} \checkmark} - \underbrace{(\zeta - 4) y}_{\text{quad. div.}} \Big) \frac{G(y)^2}{xy} + \ldots$$

$x = p^2, y = q^2, z = (p+q)^2$

Calculation of C_{sub}

$$I_{gh}^{spur}(x) \propto \frac{1}{x} \int_{x_1}^{\Lambda^2} dy\, G_{UV}^2(y)$$

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If $G_{UV}(y)$ runs logarithmically:

$$\to \frac{\Lambda_{\mathrm{QCD}}^2}{x}(-1)^{2\delta}\Gamma(1+2\delta, -\ln(\Lambda^2/\Lambda_{\mathrm{QCD}}^2))$$

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What about the finite part?

- Perturbatively no mass term should be generated.
- Independent of UV parametrization.
- Dim. reg. calculation yields the same finite part.

Form of spurious divergences

Up to now approximated analytic calculation.

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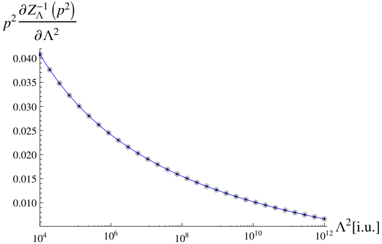
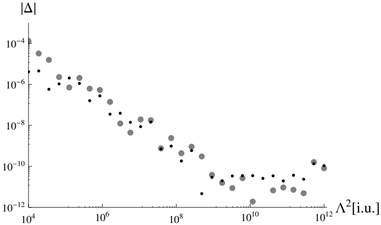
Compare derivatives of analytic result with *full* numeric calculation:





[MQH, von Smekal '14]

- Full agreement
  $\rightarrow$ Approximations well justified, anal. expressions can be used.
- Independent of external momentum
  $\rightarrow$ Spurious divergences of purely perturbative origin.
  $\Rightarrow$ Subtraction should not interfere with non-perturbative part.

Take home messages from d=4

- Subtraction of spurious divergences:
  Should not interfere with non-perturbative part
- Three-point functions: important
- Two-loop diagrams: likely important

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$\boxed{\text{Is this sufficient?}}$

Yang-Mills theory in 3 dimensions: Propagator results

$$d = 3$$

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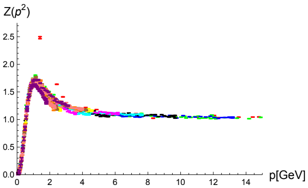
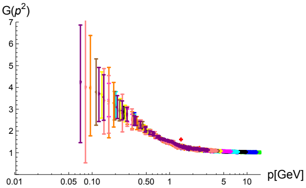
Historically interesting because cheaper on the lattice → easier to reach the IR, e.g., [Cucchieri '99; Cucchieri, Mendes, Taurines '03; Maas '08, '14; Cucchieri, Dudal, Mendes, Vandersickel '11; Bornyakov, Mitrjushkin, Rogalyov '11, '13]





[Maas '14]

Continuum results:

- Coupled propagator DSEs: Maas, Wambach, Grüter, Alkofer '04
- (R)GZ: Dudal, Gracey, Sorella, Vandersickel, Verschelde '08
- DSEs of PT-BFM: Aguilar, Binosi, Papavassiliou '10

Yang-Mills theory in 3 dimensions: Why again?

NB: Numerically not cheaper for functional equations.

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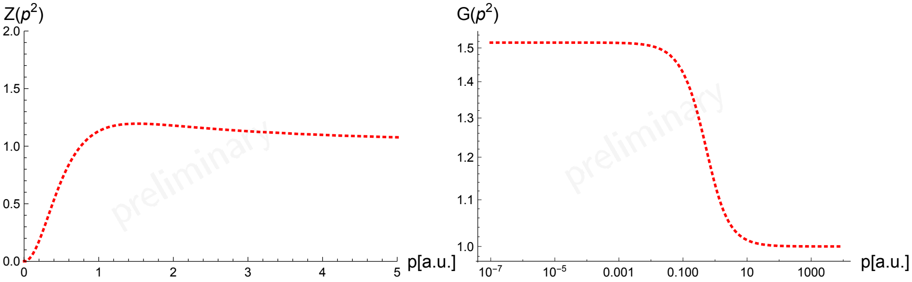
Advantages:

- UV finite: no renormalization, no anomalous running
- Spurious divergences easier to handle

$\Rightarrow$ Many complications from $d = 4$ absent!

A solution for the propagators

'Standard' truncation of propagators:
1-loop, bare ghost-gluon vertex, three-gluon vertex model



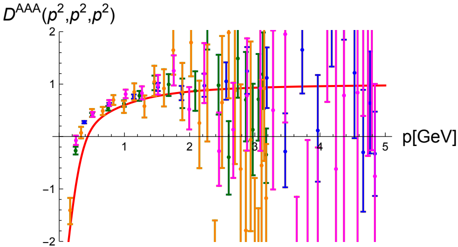
Form of spurious divergences (analytic):

$$C_{sub} = a\,\Lambda + b\,\ln\Lambda$$

Three-gluon vertex model

$$D^{A^3}(x,y,z) = \frac{\overline{p}^2}{\overline{p}^2 + L^2} - G(\overline{p}^2)^3 \frac{L^6}{(L^2+x)(L^2+y)(L^2+z)}$$

$$\overline{p}^2 = \frac{x+y+z}{2}$$

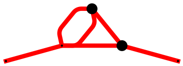


Lattice: [Cucchieri, Maas, Mendes '08]

Not possible to raise the gluon bump further by playing with the vertex models!

Two-loop diagrams

Squint:



Sunset:

4d: [Bloch '03; Mader, Alkofer '12; Meyers, Swanson '14]

Main obstacle: spurious divergences

Spurious divergences

Leading order corrections to subtraction coefficient: $g^4 \to \log(\Lambda)$

Determined by a fit:

- Very small.
- Still large effect.

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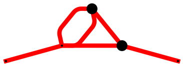

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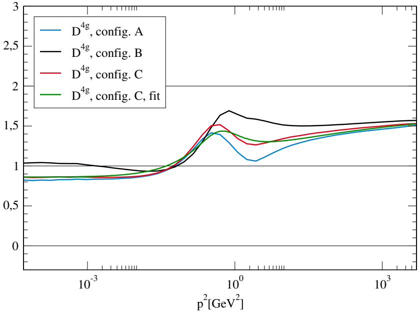
New vertex enters: Four-gluon vertex

Four-gluon vertex

4d: Solution of four-gluon DSE
(full momentum dependence)



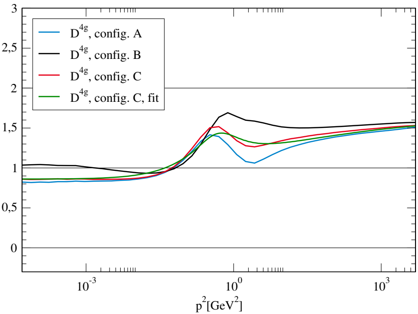
[Cyrol, MQH, von Smekal '14]

Similar results by

[Binosi, Ibáñez, Papavassiliou '14]

Four-gluon vertex

4d: Solution of four-gluon DSE (full momentum dependence)



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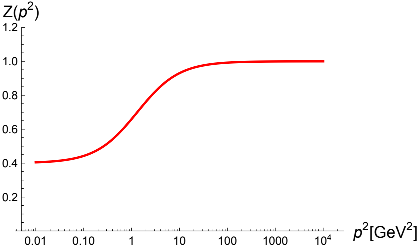
3d:

Ansatz:

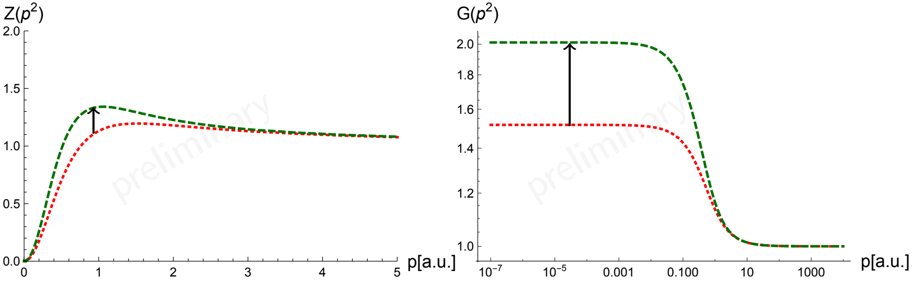
$$D^{AAAA}(x) = \frac{a\,b}{x+b} + 1$$



$a = -0.6,\ b = 1.31\ GeV^2$

A solution for the propagators with two-loop diagrams

Improved truncation of propagators:
1- and 2-loops, bare ghost-gluon vertex, three-gluon vertex model



$\rightarrow$ Two-loop diagrams essential to allow raising the gluon bump.

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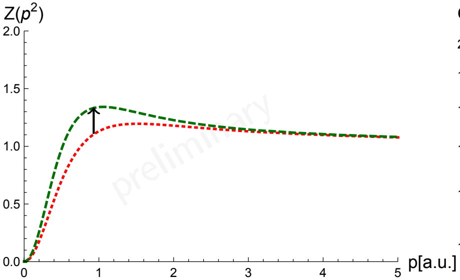
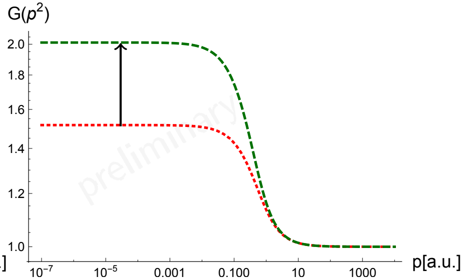
1- and 2-loops, bare ghost-gluon vertex, three-gluon vertex model

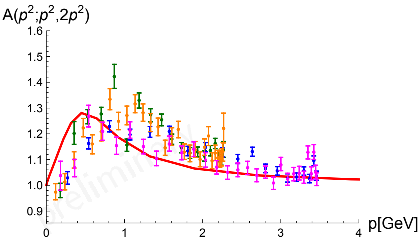


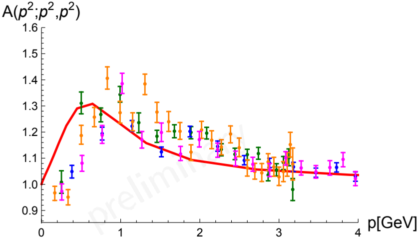


→ Two-loop diagrams essential to allow raising the gluon bump.

⇒ Next step: include vertices dynamically.

Ghost-gluon vertex

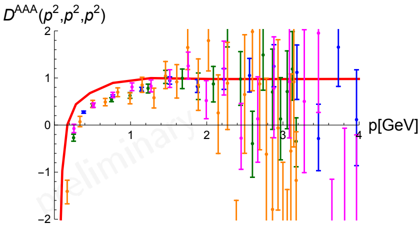


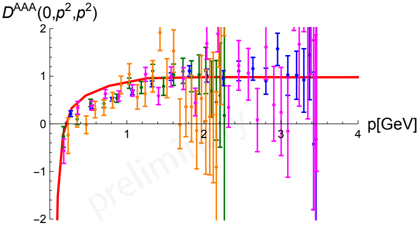


Lattice: [Cucchieri, Maas, Mendes '08]

Full momentum dependence calculated!

Three-gluon vertex





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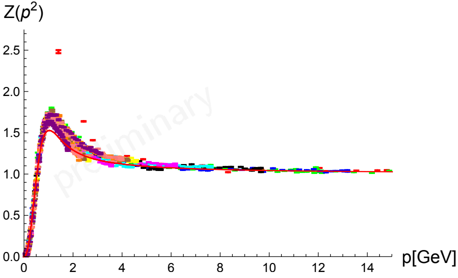
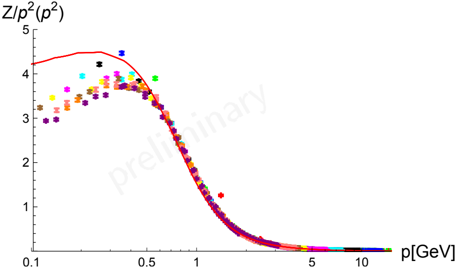
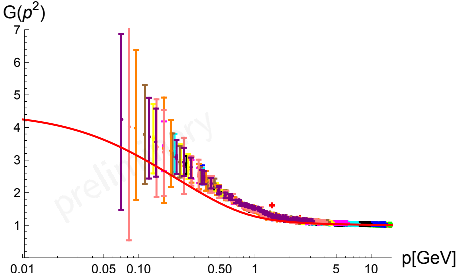
Full momentum dependence calculated!

Propagators



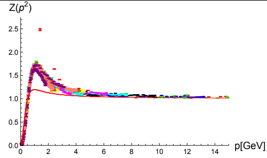
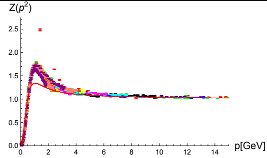
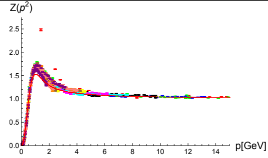




Lattice: [Maas '14]

- Quantitative improvement!
- Small gaps left.

Overview $d = 3$

2-point		3-point	4-gluon	est. max. deviation	
one-loop	two-loop				
✓	–	model	–	38%	
✓	✓	model	model	23%	
✓	✓	✓	model	8%	

Take home messages d=3

- Simpler: Useful testbed for truncations.
- Extensions of truncations: Two-loop diagrams and dynamic three-point functions improve results considerably.
- Only one remaining input: Four-gluon vertex
  Caveats: further tensors, self-consistent solution

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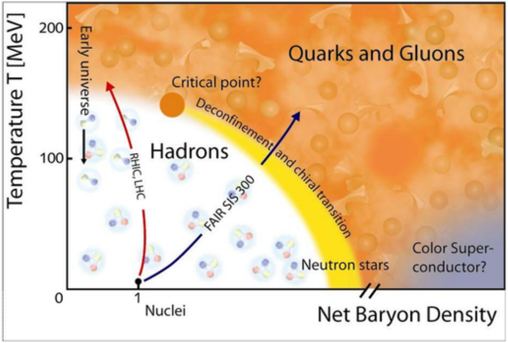
Parameter-less truncation possible!

Phases of QCD

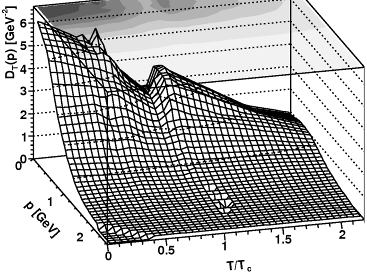


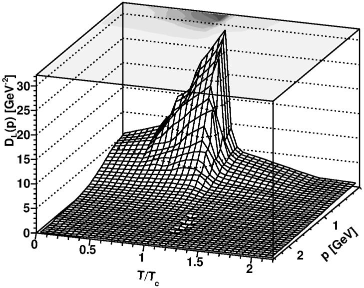
Challenges at non-zero density:

- Lattice: complex phase problem → complex Langevin, Lefschetz thimble, dual variables, Taylor expansion, reweighting, ...
- Functional framework: truncations

Gluon propagators

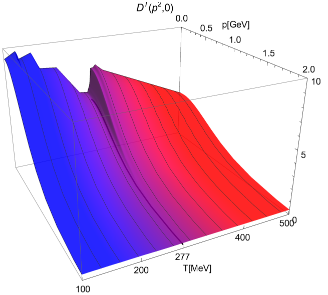
Lattice data [Fischer, Maas, Müller '10]:

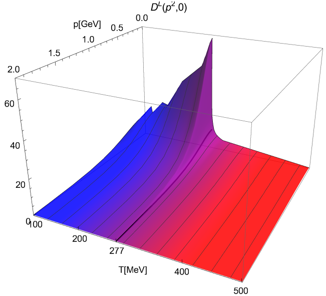




Gluon propagators

Fits, based on [Maas, Pawlowski, von Smekal, Spielmann '11]:



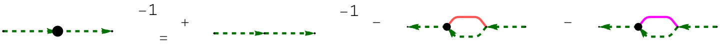


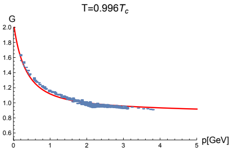
Input for DSEs to calculate quantities difficult for lattice

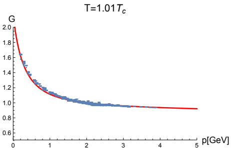
- Gluon DSE inconveniently difficult: spurious divergences, two-loop diagrams
- No truncation effects for input!
- Lattice artifacts [Cucchieri, Mendes '11]?

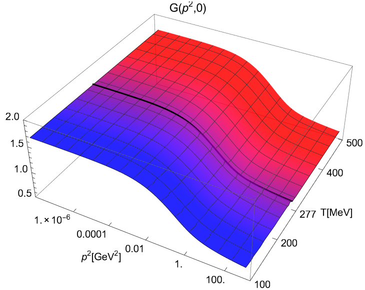
Simple test: Ghost propagator

Ghost dressing $G(p^2)$ from DSE [MQH, von Smekal '13]:









Lattice: [Fischer, Maas, Müller '10]

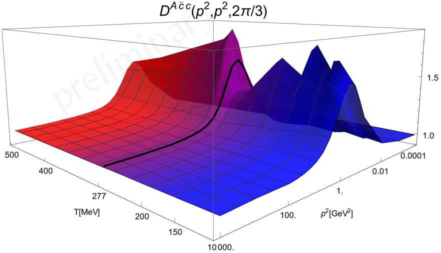
Propagators from other sources:

Lattice: [Fischer, Maas, Müller '10, Cucchieri, Mendes '11, Silva, Oliveira, Bicudo, Cardoso '13]

FRG: [Fister, Pawlowski '11]

Three-point warm-up: Ghost-gluon vertex

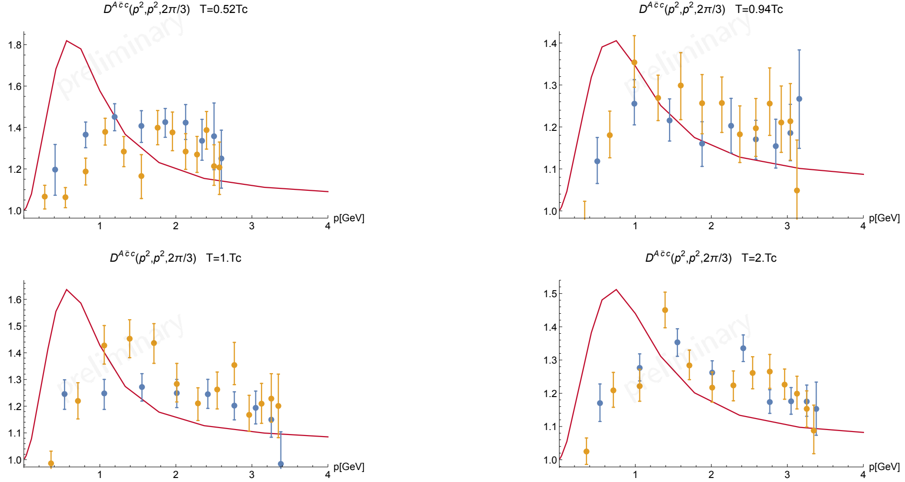
DSE calculation: self-consistent solution of truncated DSE, zeroth Matsubara frequency only



- Vertices quite expensive on lattice.
- Full momentum dependence from functional equations.

Vertex from FRG: [Fister, Pawlowski '11]

Ghost-gluon vertex: Continuum and lattice



Lattice: [Fister, Maas '14]

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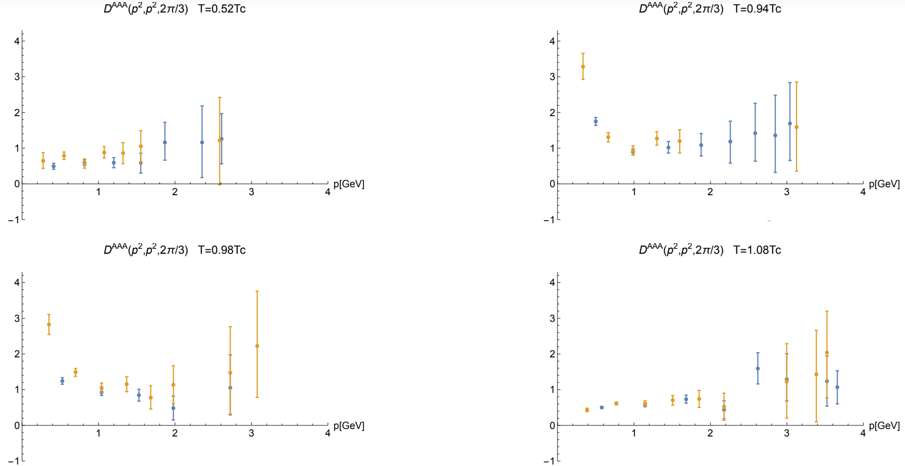

Lattice [Fister, Maas '14]: No zero crossing around T_c?

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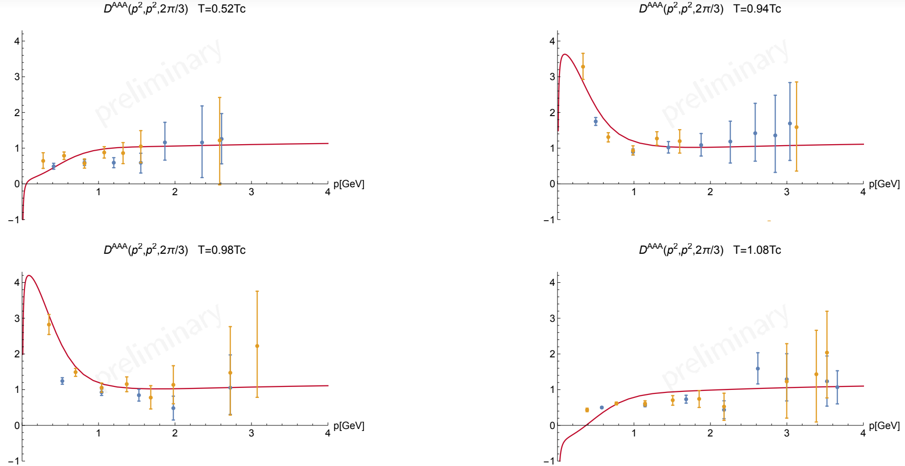
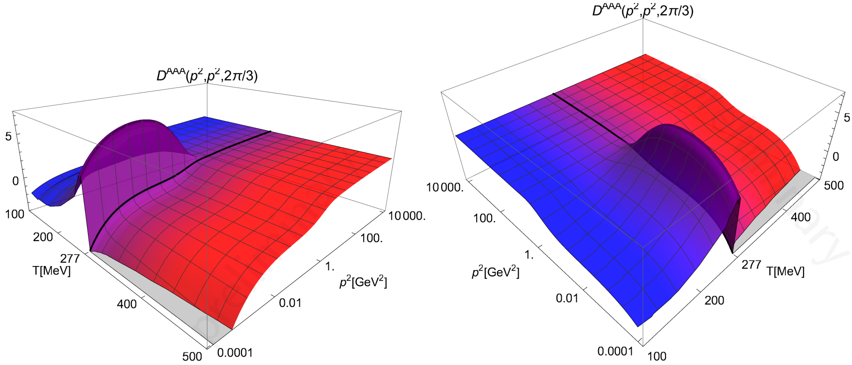


Lattice [Fister, Maas '14]: No zero crossing around T_c?

Lattice volume artifacts inherited from lattice gluon propagators in functional calculation!

Three-gluon vertex

DSE calculation: semi-perturbative approximation (first iteration only)



Summary and conclusions

- 3d Yang-Mills theory as a testbed for truncations of DSEs
- Various improvements (two-loop diagrams, dynamic three-point functions) lead to results close to lattice results.
- Missing piece to show quantitative correctness: four-gluon vertex

Implications

- An example of a self-consistent, self-contained truncation of a set of DSEs with quantitative results???
- What about 4d?

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- First results for three-point functions
- Basis for model building
- Effects of dressed vertices, e.g., in Polyakov loop potential?
- Basis for extension to QCD and $\mu > 0$

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Thank you for your attention.