Hadronic three-body heavy-meson decays in QCD factorization and CP violation

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Workshop “10 ANOS DO LABORATÓRIO DE FISICA TEÓRICA E COMPUTACIONAL (LFTC) - HADRON PHYSICS GROUP"

LPNHE
PARIS

Outline

1. INTRODUCTION
   - Motivations
   - Quasi-two body factorization
2. $B \to [K\pi^{\pm}]\pi^{\mp}$ decays
   - $B \to [K\pi^{+}]\pi^{-}$ amplitudes
   - Scalar and vector πK form factors
3. $D^0 \to K_S^0\pi^+\pi^-$ decays
   - Motivations
   - Quasi-two-body channel amplitudes
   - Scalar and vector $\pi\pi$ form factors
4. SOME CONCLUSION
   - Backup slides

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- Belle and BABAR comprehensive Dalitz plot analysis

$\to$ Partial wave analysis of decay amplitudes

$\to$ Hadronic physics: constraints on final state meson-meson interactions

- Weak - strong phase difference $\Rightarrow$ matter-antimatter *CP* asymmetry: e. g. direct $A_{CP}[\rho^0 K^{\pm}] = 0.30 \pm 0.14$ Belle & BABAR Collaborations
- W-boson exchange + m_b large $\to$ systematic perturbative calculation: QCD factorization (QCDF) - but final state strong interaction important source of uncertainties

$\Rightarrow$ $B \to M_1 M_2$ decays well described by QCDF based on an expansion in $\alpha_S(Q^2)$ and Λ_{QCD}/m_b

[cf. e.g. M. Beneke, Nucl. Phys. B (Proc. Suppl.) **170**, 57 (2007)]

- Such studies could allow to test Standard Model and QCD

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QCD factorization for $B \to M_1 M_2$

$\rightarrow$ Systematic approach : Factorization and perturbative expansion within QCD in terms of small parameters Λ_{QCD}/m_b and α_S:

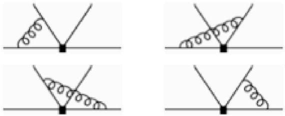
$$\langle M_1 M_2 | H_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) \langle M_1 M_2 | O_i(\mu) | B \rangle$$

V_{CKM}: quark mixing couplings, $G_F = 1.166 \times 10^{-5}$ GeV^{-2}: Fermi coupling
$C_i(\mu)$ Wilson coefficient (W exchange); renormalization scale: $\mu \sim m_b$ et $\sqrt{m_b \Lambda_{QCD}}$
O_i left-handed quark current-current operators; Large W mass + OPE

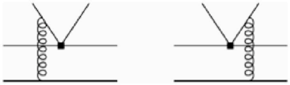
$$\boxed{\text{Factorisation : } \langle M_1 M_2 | O_i(\mu) | B \rangle = \langle M_1 | J_1 | 0 \rangle \langle M_2 | J_2 | B \rangle \, [r_n \alpha_s^n + O(\Lambda_{QCD}/m_b)]}$$

Radiative corrections:

At the weak vertex $\rightarrow$



Hard gluon exchange with the spectator quark $\rightarrow$



QCD factorization for $B \to M_1 M_2 M_3$

- No derivation for $B \to M_1 M_2 M_3$, but in B rest frame:
  $M_1, M_2, M_3 \sim$ aligned: $M_1 M_2 \sim$ same direction if $m_{M_1 M_2} \lesssim 2$ GeV
  $\Rightarrow$ quasi two-body $B \to [M_1 M_2] M_3$ with $[M_1 M_2]$ from $q\bar{q}$ pair
- $B \to [\pi^+\pi^-]K$: $B \to f_0(980)K$, $B \to \rho(770)^0 K$
  [B. El-Bennich, A. Furman, R.Kamiński, L. Leśniak, B. L., Phys. Rev. D **74**:114009, 2006]
  $B \to [K\pi^\pm]\pi^\mp$: $B \to K_0^*(1430)\pi$, $B \to K^*(892)\pi$
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  $\to$ short distance physics: perturbative QCD - leading order (LO) + next-to-leading order (NLO) vertex and penguin corrections + four complex free parameters fitted to $B \to K^*(892)\pi$ branching ratio and CP asymmetries (Belle and BABAR Collaboration)
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Effective Weak Hamiltonian

- The decay amplitude of the $B \to [K\pi^{\pm}]\,\pi^{\mp}$ processes are given by

$$\langle \pi^{\mp}[K\pi^{\pm}]|H_{eff}|B\rangle$$

where H_{eff} is a sum of local operators O_i multiplied by the product of the short range Wilson coefficients $C_i(\mu)$ and the CKM matrix elements:

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V^*_{us}(C_1(\mu)O^u_1 + C_2(\mu)O^u_2) - V_{tb}V^*_{ts}\sum_{i=3}^{10} C_i(\mu)O_i\right]$$

O_3 and O_4 in QCD factorization

- Factorization:

$$\langle\pi^-(K^-\pi^+)|C_3O_3+C_4O_4|B^-\rangle = a_4\langle\pi^-|\bar{d}\gamma^\mu(1-\gamma^5)b|B^-\rangle\langle K^-\pi^+|\bar{s}\gamma_\mu(1-\gamma^5)d|0\rangle$$

$$a_4 = C_4(\mu) + \frac{1}{N_c}C_3(\mu)\ .$$

- B to π transition:

$$\langle\pi^-(p_{\pi^-})|\bar{d}\gamma^\mu(1-\gamma_5)b|B^-(p_{B^-})\rangle$$

$$= \left[(p_{B^-}+p_{\pi^-})^\mu - \frac{M_B^2-m_\pi^2}{q^2}q^\mu\right]F_1^{B\to\pi}(q^2) + \frac{M_{B^-}^2-m_\pi^2}{q^2}q^\mu F_0^{B\to\pi}(q^2)\ ,$$

$$q = p_{B^-} - p_{\pi^-} = p_{K^-} + p_{\pi^+}\ ,$$

$$\begin{cases} F_0^{B\to\pi}(q^2) & : \quad B\pi \text{ scalar (transition) form factor} \\ F_1^{B\to\pi}(q^2) & : \quad B\pi \text{ vector (transition) form factor} \end{cases}$$

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Quark current product

- $K\pi$ transition to vacuum:

$$\langle K^-(p_{K^-})\pi^+(p_{\pi^+})|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle$$
$$= \left[(p_{K^-} - p_{\pi^+})_\mu - \frac{m_K^2 - m_\pi^2}{q^2} q_\mu\right] f_1^{K^-\pi^+}(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} q_\mu f_0^{K^-\pi^+}(q^2)\,.$$

$$\begin{cases} f_0^{K^-\pi^+}(q^2) & : \quad K\pi \text{ strange scalar form factor} \\ f_1^{K^-\pi^+}(q^2) & : \quad K\pi \text{ strange vector form factor} \end{cases}$$

- Product of the bilinear quark currents:

$$\langle \pi^-|\bar{d}\gamma^\mu(1-\gamma_5)b|B^-\rangle\langle K^-\pi^+|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle$$
$$= F_0^{B\to\pi}(q^2) f_0^{K^-\pi^+}(q^2)(M_B^2 - m_\pi^2)\frac{m_K^2 - m_\pi^2}{q^2} +$$
$$+ F_1^{B\to\pi}(q^2) f_1^{K^-\pi^+}(q^2)\left[m_{K^-\pi^-}^2 - m_{\pi^+\pi^-}^2 - (M_B^2 - m_\pi^2)\frac{m_K^2 - m_\pi^2}{q^2}\right],$$

with $\quad m_{K^-\pi^-}^2 - m_{\pi^+\pi^-}^2 - (M_B^2 - m_\pi^2)\dfrac{m_K^2 - m_\pi^2}{q^2} = 4\mathbf{p}_{\pi^+}\cdot\mathbf{p}_{\pi^-} = 4|\mathbf{p}_{\pi^+}|\,|\mathbf{p}_{\pi^-}|\cos\theta\,,$

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$$+ F_1^{B\to\pi}(q^2) f_1^{K^-\pi^+}(q^2)\left[m^2_{K^-\pi^-} - m^2_{\pi^+\pi^-} - (M_B^2 - m_\pi^2)\frac{m_K^2 - m_\pi^2}{q^2}\right],$$

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Complete weak effective Lagrangian

- Amplitude for $K\pi$ S-wave:

$$\mathcal{M}_S^- \equiv \langle \pi^- [K^-\pi^+]_S | H_{eff} | B^- \rangle = \frac{G_F}{\sqrt{2}} (M_B^2 - m_\pi^2) \frac{m_K^2 - m_\pi^2}{q^2} \times$$
$$\times F_0^{B^- \to \pi^-}(q^2)\, f_0^{K^-\pi^+}(q^2) \Big\{ \lambda_u \left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u \right) + \lambda_c \left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c \right) -$$
$$- \frac{2q^2}{(m_b - m_d)(m_s - m_d)} \left[\lambda_u \left(a_6^u(S) - \frac{a_8^u(S)}{2} + c_6^u \right) + \lambda_c \left(a_6^c(S) - \frac{a_8^c(S)}{2} + c_6^c \right) \right] \Big\},$$

- Amplitude for $K\pi$ P-wave:

$$\mathcal{M}_P^- \,\mathbf{p}_{\pi^-} \cdot \mathbf{p}_{\pi^+} \equiv \langle \pi^- [K^-\pi^+]_P | H_{eff} | B^- \rangle = 2\sqrt{2} G_F \,\mathbf{p}_{\pi^-} \cdot \mathbf{p}_{\pi^+}$$
$$\times F_1^{B^- \to \pi^-}(q^2)\, f_1^{K^-\pi^+}(q^2) \Big\{ \lambda_u \left(a_4^u(P) - \frac{a_{10}^u(P)}{2} + c_4^u \right) + \lambda_c \left(a_4^c(P) - \frac{a_{10}^c(P)}{2} + c_4^c \right) +$$
$$+ 2 \frac{\sqrt{q^2}}{m_b} \frac{f_V^\perp}{f_V} \left[\lambda_u \left(a_6^u(P) - \frac{a_8^u(P)}{2} + c_6^u \right) + \lambda_c \left(a_6^c(P) - \frac{a_8^c(P)}{2} + c_6^c \right) \right] \Big\}.$$

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$$\times F_0^{B^-\to\pi^-}(q^2)\, f_0^{K^-\pi^+}(q^2)\Big\{\lambda_u\left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u\right) + \lambda_c\left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c\right) -$$
$$- \frac{2q^2}{(m_b - m_d)(m_s - m_d)}\left[\lambda_u\left(a_6^u(S) - \frac{a_8^u(S)}{2} + c_6^u\right) + \lambda_c\left(a_6^c(S) - \frac{a_8^c(S)}{2} + c_6^c\right)\right]\Big\},$$

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$$\times F_1^{B^-\to\pi^-}(q^2)\, f_1^{K^-\pi^+}(q^2)\Big\{\lambda_u\left(a_4^u(P) - \frac{a_{10}^u(P)}{2} + c_4^u\right) + \lambda_c\left(a_4^c(P) - \frac{a_{10}^c(P)}{2} + c_4^c\right) +$$
$$+ 2\frac{\sqrt{q^2}}{m_b}\frac{f_V^\perp}{f_V}\left[\lambda_u\left(a_6^u(P) - \frac{a_8^u(P)}{2} + c_6^u\right) + \lambda_c\left(a_6^c(P) - \frac{a_8^c(P)}{2} + c_6^c\right)\right]\Big\}.$$

Complete amplitude

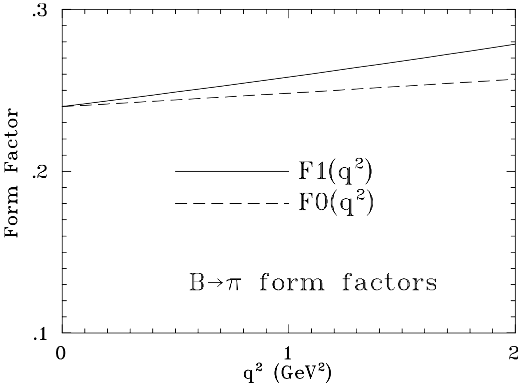
- Effective coefficients $a_i^p(S)$, $a_i^p(P)$: QCD $O(\alpha_s)$ vertex and penguins corrections.
- c_4^u, c_4^c, c_6^u, c_6^c: phenomenological penguin parameters.
- Complete amplitude : $\mathcal{M}^- = \mathcal{M}_S^- + \mathcal{M}_P^- \, \mathbf{p}_{\pi^-} \cdot \mathbf{p}_{\pi^+}$.
- B_0 amplitude: tree diagram a_1 contribution.
- B^+, $\bar{B}_0$ amplitudes: from B^-, B_0 with $V_{CKM} \to V_{CKM}^*$.

Scalar $F_0^{B\to\pi}(q^2)$ and vector $F_1^{B\to\pi}(q^2)$

Contains non-perturbative physics from hadronization of quarks currents

- Light-cone sum rules: A. Khodjamirian, T. Mannel, N. Offen, PRD**75**: 054013,2007.
- Light front: C-D. Lu, W. Wang, Z-T. Wei, Phys. Rev. D**76**:014013, 2007.
- Lattice-regularized QCD: K.C. Bowler et al., UKQCD Coll. Nucl. Phys. B**619**:507-537, 2001.
- Quarks models:
  - non-relativistic: C. Albertus et al., Phys. Rev. D**72**:094022, 2005.
  - relativistic: D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D**75**:074008, 2007.
- Double dispersion relation: D. Melikhov EPJ direct C**2**:1, 2002.

$F_0(m_{K_0^*}(1430))$ and $F_1(m_{K^*}(892))$



Schwinger-Dyson equation in QCD: M. A. Ivanov et al., Phys. Rev. D**76**:034018, 2007.

Here we use $F_0(m_{K_0^*}(1430)) = 0.266, \; F_1(m_{K^*}(892)) = 0.25.$

Strange scalar and vector $K\pi$ form factor

- Appear also in semi-leptonic decays: $\tau \to K\pi\nu_\tau, \ K \to \pi\nu_e \cdots$
- Accurate information on πK scattering available:

$\Rightarrow \ K^- p \to K^-\pi^+ n$ (11 GeV/c) — D. Aston et al., LASS Coll., NPB**296**:493,1988.

$\Rightarrow \ K^\pm p \to K^\pm\pi^+ n, \ K^\pm\pi^-\Delta^{++}$ (13 GeV/c) — P. Estabrooks et al., NPB**133**:490,1978.

- Analyticity + unitarity + $K\pi$ scattering + dispersion relation: $f_{0,1}^{K^-\pi^+}(q^2)$.
- Constraints: chiral perturbation calculations + QCD asymptotic counting rules.

Scalar form factor: two-channel ($K\pi$, $K\eta'$) Mukhelishvili-Omnès equation system

- With $t = (p_K - p_{\eta'})^2 = q^2$

$$\langle K^+|\bar{u}\gamma^\mu s|\eta'\rangle = f_+^{K^+\eta'}(t)(p_K + p_{\eta'})^\mu + f_-^{K^+\eta'}(t)(p_K - p_{\eta'})^\mu$$

$\Rightarrow$ Coupled channel involves:

$$F_1(t) \equiv \sqrt{2}\, f_0^{K^+\pi^0}(t) = f_0^{K^-\pi^+}(t),$$

$$F_2(t) = \sqrt{\frac{2}{3}}\left[\frac{m_K^2 - m_{\eta'}^2}{m_K^2 - m_\pi^2} f_+^{K^+\eta'}(t) + \frac{t}{m_K^2 - m_\pi^2} f_-^{K^+\eta'}(t)\right]$$

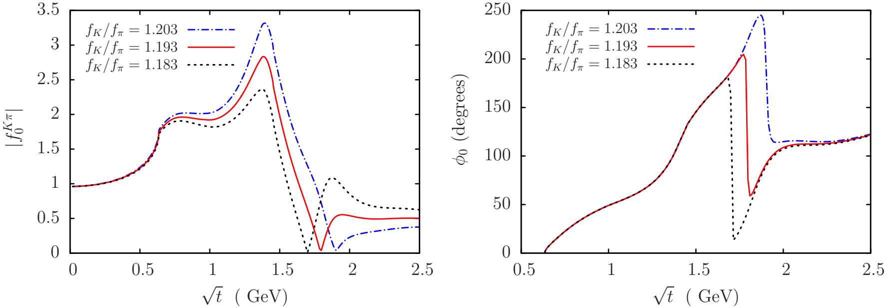
- Dispersion relations ($t_1 = (m_K + m_\pi)^2, \quad t_2 = (m_K + m'_\eta)^2$):

$$F_i(t) = \frac{1}{\pi}\int_{(m_\pi + m_K)^2}^{\infty} \frac{\mathrm{Im}\, F_i(t')\, dt'}{t' - t} \quad , \quad i = 1, 2$$

$$\mathrm{Im}\, F_1(t) = \theta(t - t_1)\frac{2q_{K\pi}(t)}{\sqrt{t}}\, T_{11}^*(t)\, F_1(t) + \theta(t - t_2)\frac{2q_{K\eta'}(t)}{\sqrt{t}}\, T_{12}^*(t)\, F_2(t),$$

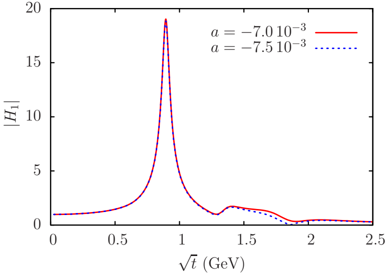
$$\mathrm{Im}\, F_2(t) = \theta(t - t_1)\frac{2q_{K\pi}(t)}{\sqrt{t}}\, T_{12}^*(t)\, F_1(t) + \theta(t - t_2)\frac{2q_{K\eta'}(t)}{\sqrt{t}}\, T_{22}^*(t)\, F_2(t)$$

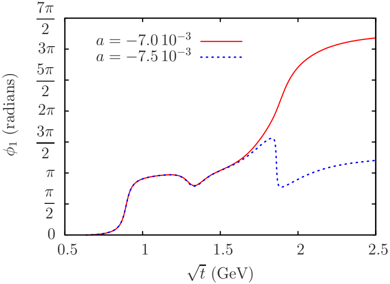
Strange scalar form factor $f_0^{K^-\pi^+} \equiv F_1(t) = |F_1(t)| \exp i\phi_1(t)$



- Variation with input at Cheng-Dashen point, $F_1(\Delta_{K\pi} \equiv m_K^2 - m_\pi^2) = f_K/f_\pi - 3.1 \times 10^{-3}$.
- Agreement with M. Jamin, J.A. Oller, A. Pich, Nucl. Phys. B**622**:341,1990 and M. Jamin, A. Pich, J.Portoles, Phys. Lett. B**640**:176-181,2006.

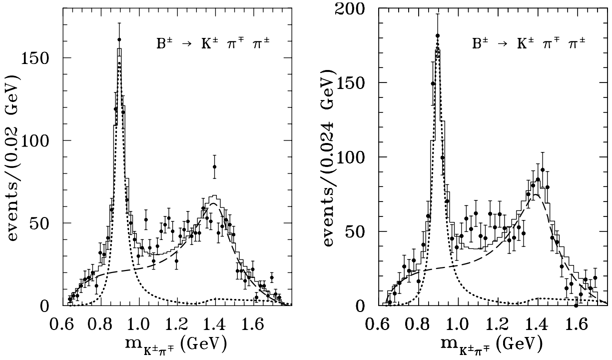
Strange vector form factor $f_1^{K^-\pi^+} \equiv H_1(t) = |H_1(t)| \exp i\phi_1(t)$





- Three-channel, $K\pi(H_1(t))$, $K^*\pi(H_2(t))$, $K\rho(H_3(t))$ Mukhelishvili-Omnès equation system.
- Variations with the flavor symmetry breaking parameter a,
  $H_2(0) = (1.41 \pm 0.09 - 65.4a)$ GeV^{-1}, $H_3(0) = (-1.34 \pm 0.07 - 654a)$ GeV^{-1}.
- Used with success in $\tau \to K\pi\nu_\tau$ and $\tau \to K\pi\pi\nu_\tau$, B. Moussallam, E.P.J.C. C**53**:401, 2008.

$m_{K\pi}$ distributions in $\bar{B}^{\pm} \to \bar{K}^{\pm}\pi^{\pm}\pi^{\mp}$



Belle data PRL**96**:251803, 2006 BABAR data PRD**78**:012004, 2008

– – – S – wave of our model
· · · · · · P – wave of our model
——— Histogram sum of S and P waves

- Dominance of $K^*(892)$, $K^*(1430)$ - $\kappa(800)$ visible

Parametrization of $B^- \to [K^-\pi^+]_S\pi^-$ amplitude

- To constrain Dalitz-plot analyses, we suggest the parametrization, with $r_{K\pi} = f_K/f_\pi$), and c_0, c_1 free complex parameters,

$$\mathcal{M}_S\left(m^2_{K\pi}, c_0, c_1, r_K\right) = F_0^{B\pi}\left(m^2_{K\pi}\right) f_0^{K\pi}\left(m^2_{K\pi}, r_K\right) \left(\frac{c_0}{m^2_{K\pi}} + c_1\right)$$

- Based on our QCDF amplitude ($q^2 = m^2_{K\pi}$)

$$\mathcal{M}_S^- \equiv \langle \pi^- \; [K^-\pi^+]_S | H_{eff} | B^- \rangle = \frac{G_F}{\sqrt{2}}(M_B^2 - m_\pi^2)\frac{m_K^2 - m_\pi^2}{q^2} \times$$
$$\times F_0^{B^- \to \pi^-}(q^2)\, f_0^{K^-\pi^+}(q^2) \left\{ \lambda_u \left(a_4^u(S) - \frac{a_{10}^u(S)}{2} + c_4^u \right) + \lambda_c \left(a_4^c(S) - \frac{a_{10}^c(S)}{2} + c_4^c \right) \right.$$
$$\left. - \frac{2q^2}{(m_b - m_d)(m_s - m_d)} \left[\lambda_u \left(a_6^u(S) - \frac{a_8^u(S)}{2} + c_6^u \right) + \lambda_c \left(a_6^c(S) - \frac{a_8^c(S)}{2} + c_6^c \right) \right] \right\}$$

$\Rightarrow$ It will take into account energy dependance from weak amplitude and decribe the contribution of the very broad $K_0^*(800)$ and wide $K_0^*(1430)$ resonances

High-statistics data from Belle and BABAR Collaborations

- Factorizaton less predictive [$m_c \sim 1.2$ GeV]

⇒ Approach with Wilson coefficients phenomenological: possible importance non-factorizable corrections

- *CP* self-conjugate decay: $K_S^0 \simeq \dfrac{(K^0 + \bar{K}^0)}{\sqrt{2}}$

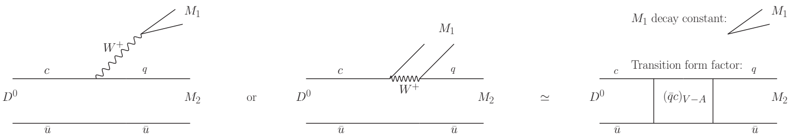
  ⇒ D^0-$\bar{D}^0$ mixing measurements by Belle and BABAR (+ LHCb) Collaborations: test of Standard Model.

- Cabibbo-Kobayashi-Maskawa (CKM) angle γ from analyses of $B^\pm \to D^0 K^\pm$, $D^0 \to K_S^0 h^+ h^- (h = \pi, K)$.

- Publication: J.-P. Dedonder, R. R Kamiński, L. Leśniak and B. Loiseau, Phys. Rev. D **89**, 094018 (2014), *Dalitz plot studies of $D^0 \to K_S^0\pi^+\pi^-$ decays in a factorization approach.*

Operator Product Expansion + large W mass $\Rightarrow$ two-body factorization approximation

Factorization: $\langle M_1 M_2|O_i(\mu)|D^0\rangle = \langle M_1|j_1|0\rangle\langle M_2|j_2|D^0\rangle$ + higher order corrections



- Works well, see e.g.: M. Beneke, M. Neubert, Nuc. Phys. B **675**, 333 (2003).
- No three-body factorization scheme $\Rightarrow$ quasi-two-body approximation:

$$\bar{K}^0 h^+ h^- \simeq [\bar{K}^0 h^\pm]_L\, h^\mp \text{ or } \bar{K}^0\, [h^+ h^-]_L \text{ with } h = \pi, K \text{ e.g.}$$

$$\{M_1 = [\pi^+ K^0]_L, M_2 = \pi^-\},\ \{M_1 = [K^+ \bar{K}^0]_L, M_2 = K^-\},\ \{M_1 = \bar{K}^0, M_2 = [h^+ h^-]_L\}$$

where the state $[m_1 m_2]_L$ in $L = S, P$ or D wave originates from a $q'\bar{q}$ state.

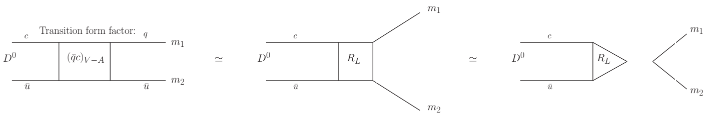
- For instance, with $V_{CKM} = V_{cs}^* V_{ud} \equiv \Lambda_1$: $\left\langle \bar{K}^0\ h^- h^+ \right|\ H_{eff}\ \left| D^0 \right\rangle \to$

$$G_F \Lambda_1 a_2 \left\langle \bar{K}^0 |(\bar{s}d)_{V-A}|0\right\rangle \left\langle [h^+ h^-]_L|(\bar{u}c)_{V-A}|D^0\right\rangle \propto i f_{K^0}\, p_{K^0} \cdot \left\langle \bar{D}^0 [h^+ h^-]_L|(\bar{u}c)_{V-A}|0\right\rangle$$

Decay constant $\uparrow$ Transition form factor $\uparrow$

- If $M_1 = [\pi^+ K^0]_L \leftrightarrow$ Form Factor $\left\langle [\pi^+ K^0]_L|(\bar{s}u)_{V-A}|0\right\rangle$: $[\pi^+ K^0]_L$ interaction

Form factor for $[m_1 m_2]_L$ in a $L(S, P$ or $D)$ wave $[s_\pm = (p_{h^\pm} + p_{K^0})^2, s_0 = (p_{h^+} + p_{h^-})^2]$



- Contribution of resonances, e.g. from $q\bar u$, in a given $[m_1 m_2]_L$ channel:

$$\left\langle [m_1 m_2]_L | (\bar q c)_{V-A} | D^0 \right\rangle \simeq \sum_{R_L} F^{D^0 R_L}(m^2_{m_3}) G_{R_L}(s) \left\langle R_L^{[m_1 m_2]} | q\bar u \right\rangle$$

$$\simeq c_{R_L} \chi \sum_{R_L} F^{D^0 R_L}(m^2_{m_3}) \langle [m_1 m_2]_L | (q\bar u)_{V-A} | 0 \rangle \simeq c_{R_L} \chi F^{D^0 \tilde R_L}(m^2_{m_3}) F_L^{m_1 m_2}(s).$$

Vertex function $G_{R_L}(s) = \chi \langle [m_1 m_2]_L | (q\bar u)_{V-A} | 0 \rangle$ and $c_{R_L} = \left\langle R_L^{[m_1 m_2]} | q\bar u \right\rangle$
$m_i (i = 1, 2, 3)$: $K^0(\bar K^0)$, $h^\pm$; $\tilde R_L$: dominant resonance; $s = (p_1 + p_2)^2$.

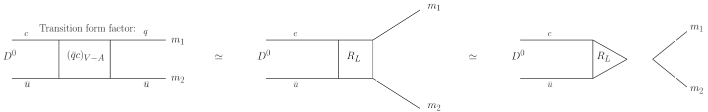
- With $L = S$, $q = u$, $m_3 = \bar K^0$, $m_1 = \pi^+$, $m_2 = \pi^-$, $\tilde R_S = f_0(980)$, $c_{f_0} = 1/\sqrt{2}$:

$$T^{[\text{Cabibbo Favored}]}_{\bar K^0\ [\pi^+\pi^-]_S}(s_0, s_-, s_+) = -\frac{G_F}{2}\, a_2\, \Lambda_1\, \chi\, (m^2_{D^0} - s_0)\, f_{K^0}\, F_0^{D^0 f_0(980)}(m^2_{K^0})\, F_0^{\pi^+\pi^-}(s_0)$$

$\rightarrow$ Form factor $F_0^{\pi^+\pi^-}(s_0)$ includes contribution of $f_0(500), f_0(980), f_0(1400)$

$\rightarrow$ If a resonance is largely dominant, like the $\rho(770)^0$ in $[\pi^+\pi^-]_P$ then $\chi \propto 1/f_\rho$

Form factor for $[m_1 m_2]_L$ in a $L(S, P$ or $D)$ wave $[s_\pm = (p_{h^\pm} + p_{K^0})^2, s_0 = (p_{h^+} + p_{h^-})^2]$



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Vertex function $G_{R_L}(s) = \chi \left\langle [m_1 m_2]_L | (q\bar{u})_{V-A} | 0 \right\rangle$ and $c_{R_L} = \left\langle R_L^{[m_1 m_2]} | q\bar{u} \right\rangle$
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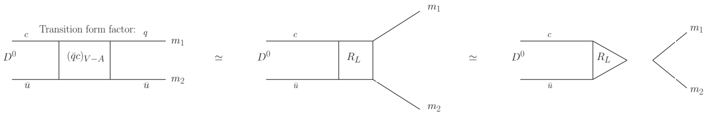

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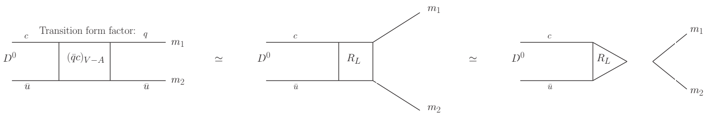

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Dominant resonances in the quasi-two-body channel amplitudes for $D^0 \to K_S^0\pi^+\pi^-$

Table: CF and DCS quasi-two-body channel amplitudes

Amplitude	Quasi two-body channel	Dominant resonances	Form Factor
$\mathcal{M}_{1\pi}$: CF(Tr+An)	$[K_S^0\,\pi^-]_S\,\pi^+$	$K_0^*(800)^-$, $K_0^*(1430)^-$	$F_0^{K\pi}(s_-)$
$\mathcal{M}_{2\pi}$: CF+DCS(Tr+An)	$K_S^0\,[\pi^+\pi^-]_S$	$f_0(500)$, $f_0(980)$, $f_0(1400)$	$F_0^{\pi\pi}(s_0)$
$\mathcal{M}_{3\pi}$: CF(Tr+An)	$[K_S^0\,\pi^-]_P\,\pi^+$	$K^*(892)^-$	$F_1^{K\pi}(s_-)$
$\mathcal{M}_{4\pi}$: CF+DCS(Tr+An)	$K_S^0\,[\pi^+\pi^-]_P$	$\rho(770)^0$	$F_1^{\pi\pi}(s_0)$
$\mathcal{M}_{5\pi}$: CF+DCS(Tr+An)	$K_S^0\,[\pi^+\pi^-]_\omega$	$\omega(782)$	Breit-Wigner
$\mathcal{M}_{6\pi}$: CF(Tr+An)	$[K_S^0\,\pi^-]_D\,\pi^+$	$K_2^*(1430)^-$	Breit-Wigner
$\mathcal{M}_{7\pi}$: CF+DCS(Tr+An)	$K_S^0\,[\pi^+\pi^-]_D$	$f_2(1270)$	Breit-Wigner
$\mathcal{M}_{8\pi}$: DCS(Tr+An)	$[K_S^0\,\pi^+]_S\,\pi^-$	$K_0^*(800)^+$, $K_0^*(1430)^+$	$F_0^{K\pi}(s_+)$
$\mathcal{M}_{9\pi}$: DCS(Tr+An)	$[K_S^0\,\pi^+]_P\,\pi^-$	$K^*(892)^+$	$F_1^{K\pi}(s_+)$
$\mathcal{M}_{10\pi}$: DCS(An)	$[K_S^0\,\pi^+]_D\,\pi^-$	$K_2^*(1430)^+$	Breit-Wigner

Here: $s_\pm = (p_{\pi^\pm} + p_{K^0})^2$, $s_0 = (p_{\pi^+} + p_{\pi^-})^2$

$\Gamma_1^{n(s)*}(s_0) \propto \langle[\pi\pi]_S\,|n\bar{n}(s\bar{s})|0\rangle$ and $\Gamma_2^{n(s)*}(s_0) \propto \langle[K\bar{K}]_S\,|n\bar{n}(s\bar{s})|0\rangle$ with $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$

- *Model*: relativistic 3-coupled channel equation

$$\Gamma_i^{n(s)*}(s) = R_i^{n(s)}(E) + \sum_{j=1}^{3} R_j^{n(s)}(E) H_{ij}(E), \quad i = 1, 2, 3,$$

$$H_{ij}(E) = \int \frac{d^3p}{(2\pi)^3} T_{ij}(E, k_i, p) \frac{1}{E - 2\sqrt{p^2 + m_j^2} + i\epsilon} \frac{k_j^2 + \kappa^2}{p^2 + \kappa^2}.$$

- $E = \sqrt{s}$; p off-shell momentum; $i, j = 1, 2, 3$: $\pi\pi$, $K\bar{K}$, effective $(2\pi)(2\pi)$ channels; CMS $k_j = \sqrt{s/4 - m_j^2}$, $m_1 = m_\pi$, $m_2 = m_K$, $m_3 = m_{(2\pi)} = 700$ MeV.
  $\to$ For the T matrix: solution A of R. Kamiński, L. Leśniak, B. Loiseau, Eur. Phys. J. **C9**, 141 (1999).
- $\to R_i^{n(s)}(E) = (\alpha_i^{n(s)} + \tau_i^{n(s)} E + \omega_i^{n(s)} E^2)/(1 + cE^4)$, $i = 1, 2, 3$, production functions.
  $\to$ Fitted parameter c controls high energy behavior.
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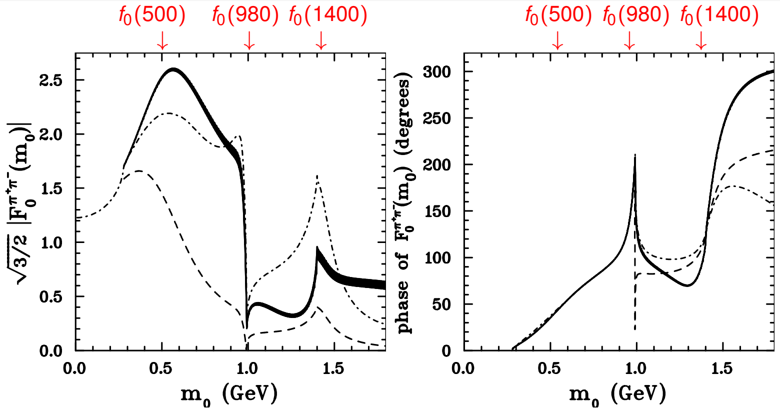
Unitary scalar-isoscalar $\pi\pi$ form factor $F_0^{\pi\pi}(\propto \Gamma_1^n)$ used in our model for $D^0 \to K^0_S\pi^+\pi^-$

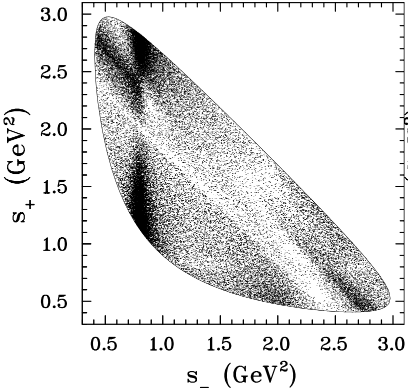


$\Rightarrow F_0^{\pi\pi}(m)$: unitarity + analyticity + $\pi\pi$ data. Dark band: variation when $\kappa(= 306. \pm 3.$ MeV) and $c(= 0.29 \pm 0.02$ GeV^{-4}) vary within their errors. Dashed line: $\kappa = 2$ GeV $c = 19.5$ GeV^{-4} [$B \to 3\pi$, J.-P. Dedonder *et al.* Acta Phys. Pol. B **42**, 2013 (2011)]. Dotted-dashed line: Moussallam calculation [Eur. Phys. J. C. **14**, 111 (2000)] using Muskhelishvili-Omnès equations

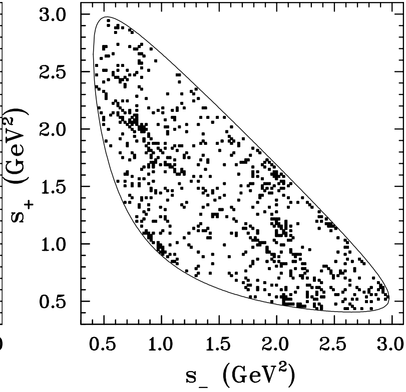
$K\pi$ and $\pi\pi$ vector form factor used in our model for $D^0 \to K^0_S\pi^+\pi^-$

- Mass and width of the $K^*(892)$ meson are free parameters entering also in the $K\pi$ vector form factor taken from the Belle Collaboration fit to the $\tau^- \to K^0_S\pi^-\nu_\tau$ decays
Contributions of $K^*(892)$ and $K^*(1410)$ resonances taken but not that of the $K^*(1680)$
- Alternatively to this experimental parameterization we use the model of the $K\pi$ vector form factor of D. R. Boito *et al.* [JHEP **1009**, 031 (2010)] in which some constraints from analyticity and elastic unitarity are incorporated
- Two types of the pion vector form factor tested:
- the experimental parameterization used by Belle Collaboration [2008] in the data analysis of $\tau^- \to \pi^-\pi^0\nu_\tau$ decays
- the unitary parametrization of Hanhart [PLB **170**, 710 (2012)] which also fit the $\tau^- \to \pi^-\pi^0\nu_\tau$ data

Fit to Belle Dalitz plot and distribution of χ^2 values larger than 4



$K^*(892)^-$ ↑. Fit to Belle Dalitz plot data



Black square: $\chi^2 > 4$

- Use of quasi-two-body factorization approximation and unitarity constraints to describe the final state meson-meson interactions in hadronic three-body heavy-meson decays
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- In $D^0 \to K^0_S\pi^+\pi^-$ decays final state strong $K^0_S\pi^{\pm}$ and $\pi^+\pi^-$ interactions in S, P and D states described through corresponding form factors including many resonances.
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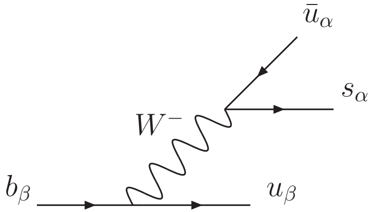
O_1 and O_2: left-hand current-current operators

- For instance (α and β color indices)

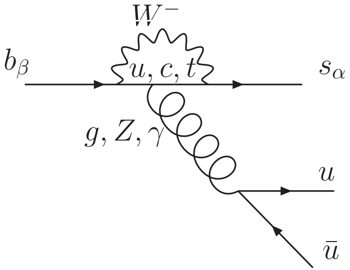
$$O_1^u = \bar{s}_\alpha\gamma^\nu(1-\gamma_5)u_\alpha \otimes \bar{u}_\beta\gamma_\nu(1-\gamma_5)b_\beta$$



$O_3 \cdots O_{10}$: QCD + electroweak penguin operators

- For instance

$$O_4 = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) b_\beta \otimes \sum_{q=u,d,s,c} \bar{q}_\beta \gamma_\mu (1-\gamma_5) q_\alpha$$

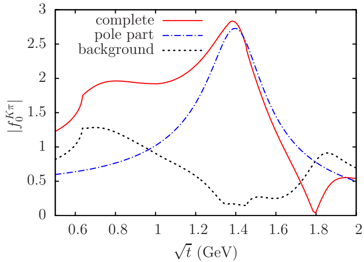


Pole in the scalar form factor $f_0^{K^+\pi^-}(t)$

- A resonance can be associated with a pole of the scattering matrix in the complex energy plane on the second Riemann sheet:

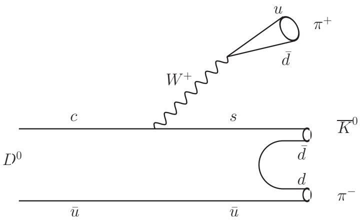
$$f_0^{pole}(t) = \frac{f_0^{K\pi}(t_0)}{\alpha(t-t_0)} = \frac{-0.341 - i1.506}{(0.839 + i1.171)(t-t_0)}; \ \sqrt{t_0} = 1.403 - i0.136 \text{ GeV}$$

PDG'06: $M_{K_0^*(1430)} - i\Gamma K_0^*(1430)/2 = 1.415(6) - i0.145(11)$ GeV



Modulus of the scalar form factor $f_0^{K\pi}(t)$ compared with its pole part $f_0^{pole}(t)$ $(f_0^{\text{background}}(t) = f_0^{K\pi}(t) - f_0^{\text{pole}}(t))$

Favored and suppressed tree amplitudes in $D^0 \to K_S^0\pi^+\pi^-$

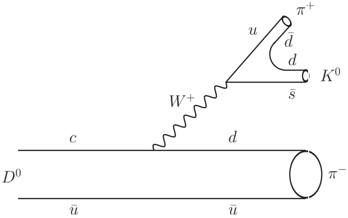


$c \to s u \bar{d}$ transition :

$$\propto \frac{G_F}{\sqrt{2}} a_1(m_c)\Lambda_1\{\equiv V_{cs}^* V_{ud}\}\ (\bar{s}c)_{V-A}\ (\bar{u}d)_{V-A}$$

$\to V_{cs} \approx V_{ud} \approx \cos\theta_C \approx 0.975,$
θ_C Cabibbo angle

$\Rightarrow$ 7 Cabibbo favored (CF) tree (Tr) amplitudes:
$[K\pi]_{S,P,D}\ \pi + K\ [\pi\pi]_{S,P,D} + \ K\ \omega\ \{\omega \to [\pi\pi]_P$ by G-parity violation}



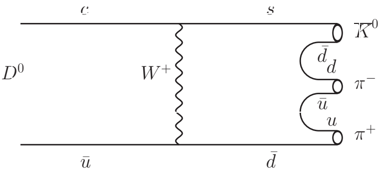
$c \to d u \bar{s}$ transition :

$$\propto \frac{G_F}{\sqrt{2}} a_1(m_c)\Lambda_2\{\equiv V_{cd}^* V_{us}\}\ (\bar{d}c)_{V-A}\ (\bar{u}s)_{V-A}$$

$\to V_{cd} \approx -\lambda,\ V_{us} \approx \lambda,\ \lambda = \sin\theta_C,$

$\Rightarrow$ 6 doubly Cabibbo suppressed (DCS) tree (Tr) amplitudes as no W coupling to $[K\pi]_D$ state

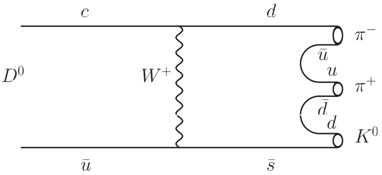
Annihilation - t-channel W-exchange amplitudes in $D^0 \to K^0_S\pi^+\pi^-$



$c \to s u \bar{d}$ $(c\bar{u} \to s\bar{d})$:

$$\propto \frac{G_F}{\sqrt{2}} a_2(m_c) \cos^2\theta_C \, (\bar{s}c)_{V-A} \, (\bar{d}u)_{V-A}$$

$\Rightarrow$ 7 Cabibbo favored annihilation (An) amplitudes



$c \to d u \bar{s}$ $(c\bar{u} \to d\bar{s})$:

$$\propto -\frac{G_F}{\sqrt{2}} a_2(m_c) \sin^2\theta_C \, (\bar{d}c)_{V-A} \, (\bar{s}u)_{V-A}$$

$(\sin\theta_C = 0.225)$

$\Rightarrow$ 7 doubly Cabibbo suppressed annihilation (An) amplitudes

Total of 27 non-zero amplitudes: 13 tree and 14 annihilation

Branching fractions (Br) for different quasi two-body channels

Table: Sum Br=132.81 % $\leftrightarrow$ interferences.
$\mathcal{M}_{1\pi}$: $K_0^*(800)^-$, $K_0^*(1430)^-$; $\mathcal{M}_{2\pi}$:$f_0(500)$, $f_0(980)$, $f_0(1400)$;
$\mathcal{M}_{3\pi}$:$K^*(892)^-$; $\mathcal{M}_{4\pi}$: $\rho(770)^0$

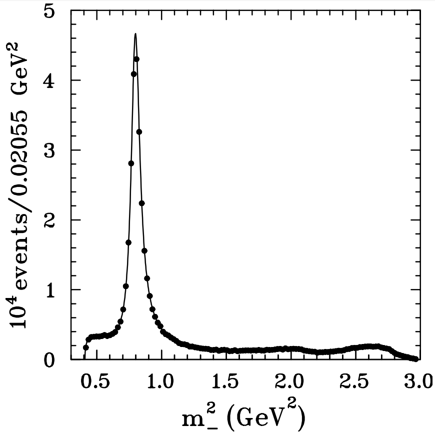
Amplitude	channel	Br	tree	ann. low
$\mathcal{M}_{1\pi}$	$[K^0_S\,\pi^-]_S\,\pi^+$	$25.03 \pm 3.61 \pm 0.18$	8.24 ± 0.10	7.88 ± 0.11
$\mathcal{M}_{2\pi}$	$K^0_S[\pi^-\pi^+]_S$	$16.92 \pm 1.27 \pm 0.02$	14.70 ± 0.17	2.92 ± 0.09
$\mathcal{M}_{3\pi}$	$[K^0_S\,\pi^-]_P\,\pi^+$	$62.72 \pm 4.45 \pm 0.15$	24.69 ± 5.65	8.74 ± 2.97
$\mathcal{M}_{4\pi}$	$K^0_S[\pi^-\pi^+]_P$	$21.96 \pm 1.55 \pm 0.06$	4.36 ± 0.06	6.74 ± 0.04
$\mathcal{M}_{5\pi}$	$K^0_S\omega$	$0.79 \pm 0.07 \pm 0.04$	0.24 ± 0.01	0.16 ± 0.02
$\mathcal{M}_{6\pi}$	$[K^0_S\,\pi^-]_D\,\pi^+$	$1.41 \pm 0.11 \pm 0.04$		
$\mathcal{M}_{7\pi}$	$K^0_S[\pi^-\pi^+]_D$	$2.15 \pm 0.19 \pm 0.10$		
$\mathcal{M}_{8\pi}$	$[K^0_S\,\pi^+]_S\,\pi^-$	$0.56 \pm 0.07 \pm 0.03$	0.07 ± 0.00	0.29 ± 0.02
$\mathcal{M}_{9\pi}$	$[K^0_S\,\pi^+]_P\,\pi^-$	$0.64 \pm 0.06 \pm 0.02$	0.77 ± 0.15	0.01 ± 0.01
$\mathcal{M}_{10\pi}$	$[K^0_S\,\pi^+]_D\,\pi^-$	$0.63 \pm 0.07 \pm 0.11$	0	0.63 ± 0.11

- Branching fractions compare well with those of Belle 's analysis

$\rightarrow$ Belle $\text{Br}_{K^0_S\sigma_1} + \text{Br}_{K^0_S f_0(980)} + \text{Br}_{K^0_S\sigma_2} + \text{Br}_{K^0_S f_0(1370)}$ = 18.6% $\sim$ 16.9% value

- Annihilation contributions can be important

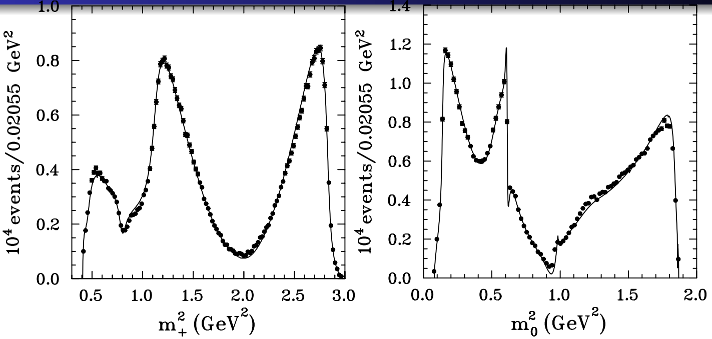
$K^0_S\pi^-$ effective mass squared distributions ($m^2_- \equiv s_-$)



$K^*(892)^-$ ↑

⇒ Our model (solid curve) compared with Belle data (points with error bars).

$K_S^0\pi^+$ and $\pi^+\pi^-$ effective mass squared distributions ($m_+^2 \equiv s_+$ and $m_0^2 \equiv s_0$)



⇒Left panel: comparison of the $K_S^0\pi^+$ effective mass squared distributions for the best fit (solid curve) with the Belle data (points with error bars). Right panel: as in left panel but for the $\pi^+\pi^-$ effective mass squared.

- The small shoulder at $m_0^2 = 1.2$ GeV^2 could correspond to the $\pi\pi \to \eta\eta$ contribution introduced in Belle'analyzis but not included in our approach.