Ghost and gluon propagators in covariant gauges

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Based on: A.C.A., D. Binosi and J. Papavassiliou PRD 91 (2015)
A.C. A., D. Binosi and J. Papavassiliou (in preparation)

QCD-TNT4 August 31- September, 04 2015 – Ilhabela, São Paulo, Brazil

Outline of the talk

- Ghost in linear covariant gauges
- Numerical results for the ghost
- Bound state formalism and the gluon mass in covariant gauges
- Numerical results
- Conclusions

Motivation

- An impressive amount of information has been accumulated over the years on the IR behavior of the Green's functions of QCD in the Landau gauge.

- Characteristic features such as the infrared finiteness of the gluon propagator and of the ghost dressing function have lead to the confirmation or reassesements of our theoretical ideas and popular nonperturbative mechanisms.

◎ **Natural questions arise:**

- ✓ Which of these features, if any, persist away from the Landau gauge ?
- ✓ Is the gluon mass generation particular to the Landau gauge ?
- ✓ Is the ghost propagator still massless, with saturating dressing function?
- ✓ How exactly is the gauge-independence of certain special quantities realized ?
- ✓ Can we begin to discern the gauge cancellation patterns familiar from perturbation theory ?

✓ With what precision are observables really gauge-independent within our present truncations schemes and computational limitations of the Schwinger-Dyson equations ?

✓ Do certain features associated with confinement persist ? Existence and location of inflection point ?

Related works:

M. A. L. Capri et al. arXiv:1506.06995 [hep-th].
M. A. L. Capri, A. D. Pereira, R. F. Sobreiro and S. P. Sorella, arXiv:1505.05467 [hep-th].
M. Q. Huber, Phys. Rev. D91, 085018 (2015).
A. Cucchieri, T. Mendes and E. M. S. Santos, Phys. Rev. Lett.103, 141602 (2009).
A. Cucchieri, T. Mendes, G. M. Nakamura and E. M. S. Santos, PoS FACESQCD, 026 (2010).
P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, arXiv:1505.05897 [hep-lat].

QCD Lagrangian

- The QCD lagrangian is given by

$$\mathcal{L}_{QCD} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{1}{2\xi}(\partial^\mu A_\mu^a)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc}(\partial^\mu \bar{c}^a) A_\mu^b c^c$$

where the gluonic field strength tensor

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

- Already at tree level the gluon propagator acquires explict dependence on ξ

$$i\Delta_{\mu\nu}^{[0]}(q) = -\frac{i}{q^2}\left[P_{\mu\nu}(q) + \xi\frac{q_\mu q_\nu}{q^2}\right];$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2},$$

- The ξ-dependence infests all Green's functions in higher orders

Perturbative results

- **Gluon propagator**

$$\Delta^{-1}_{[1]}(q^2) = q^2\left[1+\frac{\alpha_s C_A}{8\pi}\left(\frac{13}{3}-\xi\right)\log\left(\frac{q^2}{\mu^2}\right)\right]$$

- **Auxiliary function**
  (see Daniele's talk)

$$G^{[1]}(q^2) = \left[1+\frac{\alpha_s C_A}{16\pi}(3+\xi)\log\left(\frac{q^2}{\mu^2}\right)\right]$$

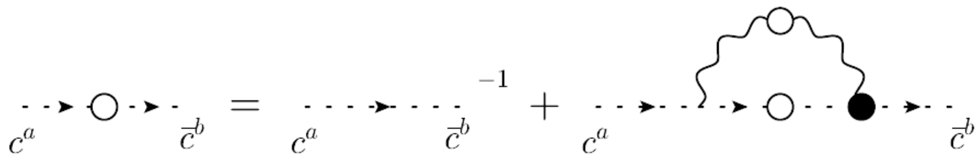
- **RGI combinations**

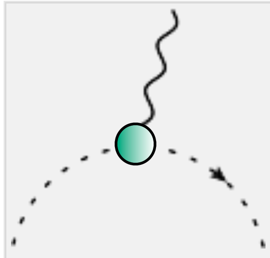
$$\alpha_s\frac{\Delta^{[1]}(q^2)}{[1+G^{[1]}(q^2)]^2} = \frac{\alpha_s}{q^2\left[1+\frac{11C_A}{12\pi}\alpha_s\log\left(\frac{q^2}{\mu^2}\right)\right]}.$$

RGI and ξ -independent !

Ghost in linear covariant gauges

◎ **The SDE for the ghost propagator**





$$\Gamma^\nu(k+q,-k,-q) = \mathcal{A}q^\nu + \mathcal{B}k^\nu$$

At three level:

$$\mathcal{A}^{(0)} = 1$$
$$\mathcal{B}^{(0)} = 0$$

$$\begin{aligned} D^{-1}(q^2) &= q^2 - i\Pi(q^2) \\ &= q^2 + ig^2 C_A \int_k (k+q)^\mu D(k+q)\Delta_{\mu\nu}(k)\Gamma^\nu(k+q,-k,-q), \end{aligned}$$

◎ **The gluon propagator acquires an explict and an implicit ξ dependence**

$$i\Delta_{\mu\nu}(q,\xi) = -i\left[\Delta(q^2,\xi)P_{\mu\nu}(q) + \xi\frac{q_\mu q_\nu}{q^4}\right];$$

$$iD(q^2,\xi) = i\frac{F(q^2,\xi)}{q^2}$$

$$D^{-1}(q^2) = q^2 + ig^2 C_A q^\mu q^\nu \int_k D(k+q)\Delta(k)P_{\mu\nu}(k)\mathcal{A}$$
$$+ i\xi g^2 C_A \int_k D(k+q)\left(1+\frac{k\cdot q}{k^2}\right)\left(\mathcal{B}+\frac{k\cdot q}{k^2}\mathcal{A}\right),$$

- **Solve the equation:**
  keep tree-level vertices ($\mathcal{A}^{(0)} = 1$, $\mathcal{B}^{(0)} = 0$) but fully-dressed propagators

- **Assuming that** $\Delta(q^2) \to \Delta_{\mathrm{L}}(q^2)$
  The main underlying assumption → $\Delta(q^2)$ saturates in the IR
  For small ξ →

Cucchieri, Mendes, Nakamura, **PoS FACESQCD (2010).**
P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, **arXiv:1505.05897 [hep-lat].**

Main features of the ghost equation

- **The Euclidean version of the ghost equation is**

$$F^{-1}(x) = 1 - \frac{\alpha_s C_A}{2\pi^2} \overbrace{\int_0^\infty \mathrm{d}y\, y\, F(y) \int_0^\pi \mathrm{d}\theta\, \frac{\sin^4\theta}{z} \Delta_{\mathrm{L}}(z)}^{\textbf{IR finite}} + \xi \frac{\alpha_s C_A}{16\pi} \left[\underbrace{\frac{1}{x^2} \int_0^x \mathrm{d}y\, y F(y)}_{\textbf{IR finite}} + \underbrace{\int_x^\infty \mathrm{d}y\, \frac{F(y)}{y}}_{\textbf{IR divergent}} \right],$$

- **Last term dominates (x→ 0)**

$$F^{-1}(x) \underset{x\to 0}{\sim} \xi \frac{\alpha_s C_A}{16\pi} \int_x^\infty \mathrm{d}y\, \frac{F(y)}{y}.$$

A. C. A., D. Binosi, J. Papavassiliou, **PRD 91 (2015).**

- **Can be easily converted to a differential equation**
  (differentiate with respect to x)

$$F'(x) \underset{x\to 0}{\sim} \xi c \frac{F^3(x)}{x}$$

$$F(x) \underset{x\to 0}{\sim} \frac{1}{\sqrt{a - 2\xi c \log(x/\mu^2)}},$$

$$c = \frac{\alpha_s C_A}{16\pi}$$

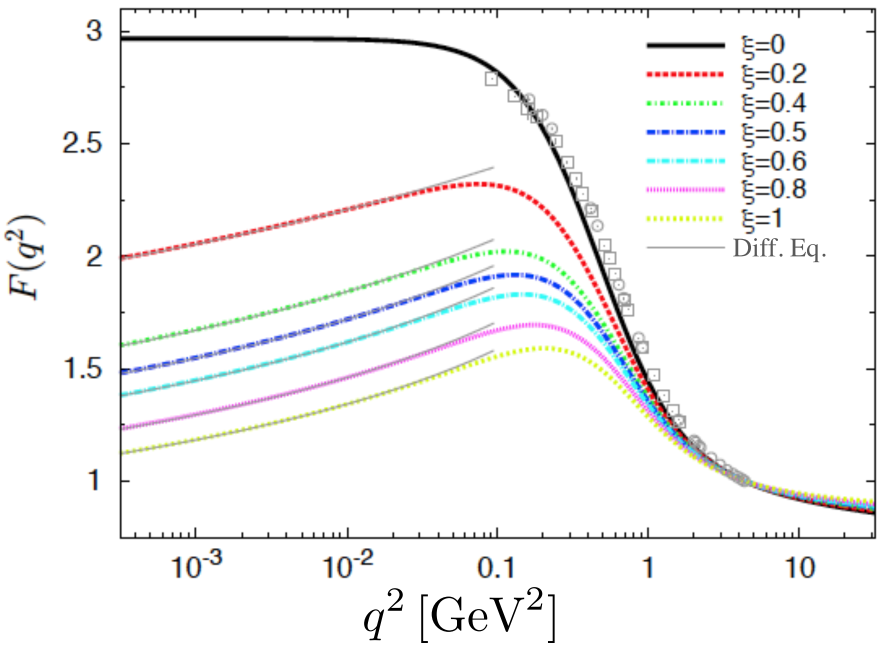
$$F(0) = 0$$

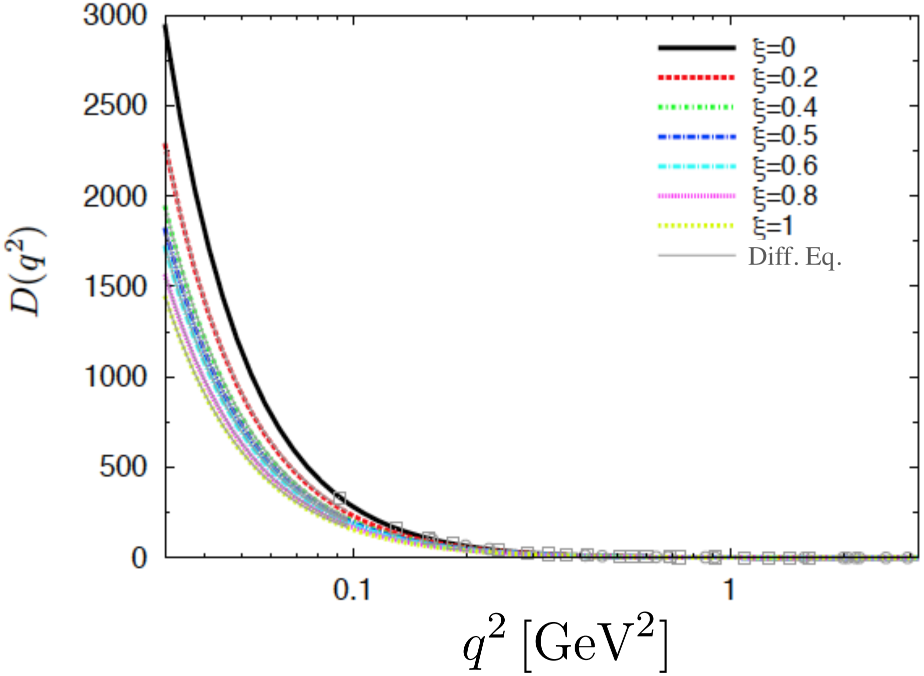
Very different from Landau gauge !

A. C. A., D. Binosi, J.Papavassliou, PRD 91 (2015).

Numerical results

◎ **Full numerical treatment confirms the qualitative result obtained differential equation**





◎ **Around ~ 300 MeV we recover**

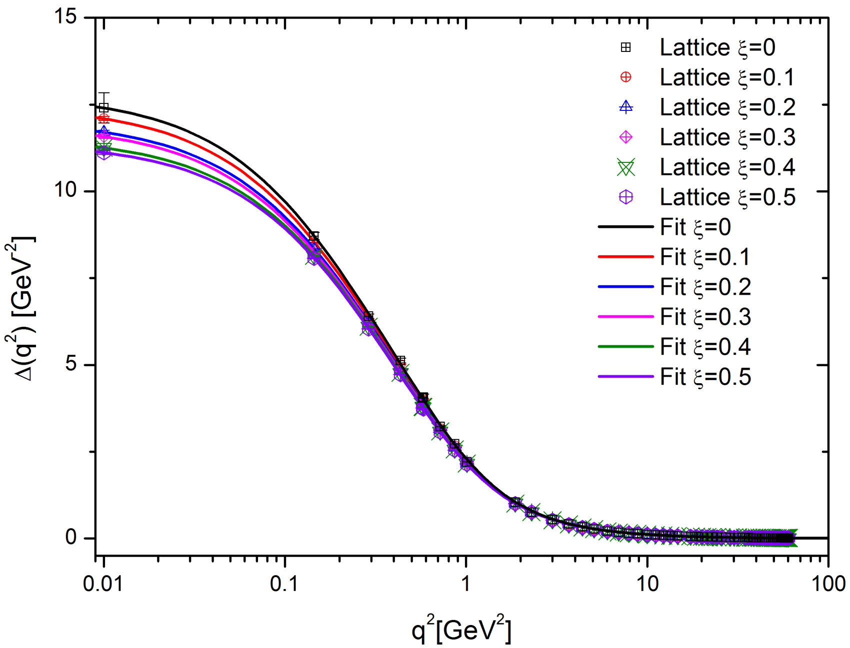
$$F(x) \underset{x\to 0}{\sim} \frac{1}{\sqrt{a - 2\xi c \log(x/\mu^2)}},$$

$$a = a(\xi) = 0.12(1+\xi); \qquad c = 0.035.$$

F(q^2) displays a characteristic maximum

Lattice data in ξ gauges

- For small ξ there is lattice simulation which indicates that the gluon propagator (in away the Landau gauge) is still finite IR



- The gluon propagator can be parametrized (Euclidean space) by

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

- Saturation is explained through **gluon mass generation**

J. M. Cornwall, Phys. Rev. D26, 1453 (1982).

$$\Delta^{-1}(0) = m^2(0)$$

P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, arXiv:1505.05897 [hep-lat].

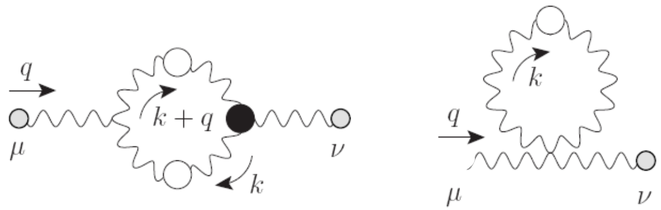
No mass without poles

PT - BFM
Ward identities

\+

Seagull
cancellation

$$\Delta^{-1}(0) = \lim_{q^2\to 0} \Big\{ \quad \Big\}$$



$$\sim \int_k k^2 \frac{\partial \Delta(k^2)}{\partial k^2} + \frac{d}{2}\int_k \Delta(k) = 0$$

Infinitesimal WI
No Poles

Seagull
condition

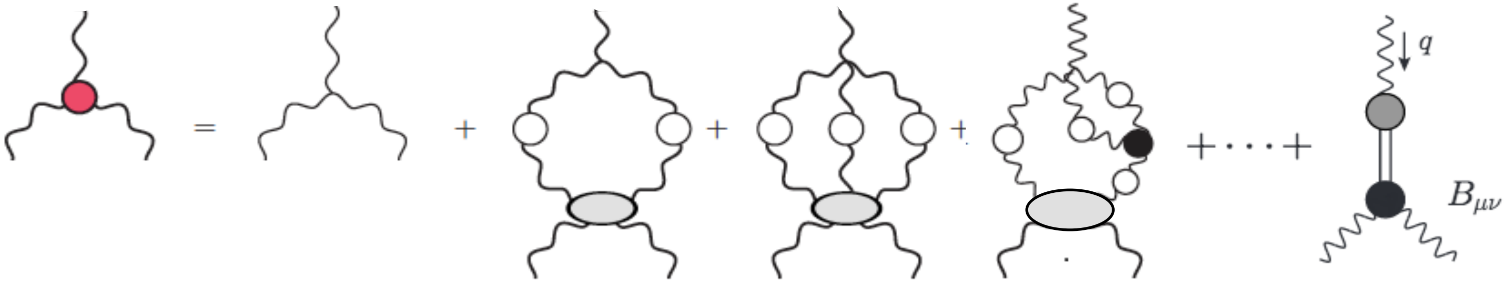
A.C. A. and J. Papavassiliou. Phys. Rev. D 81, 034003 (2010).

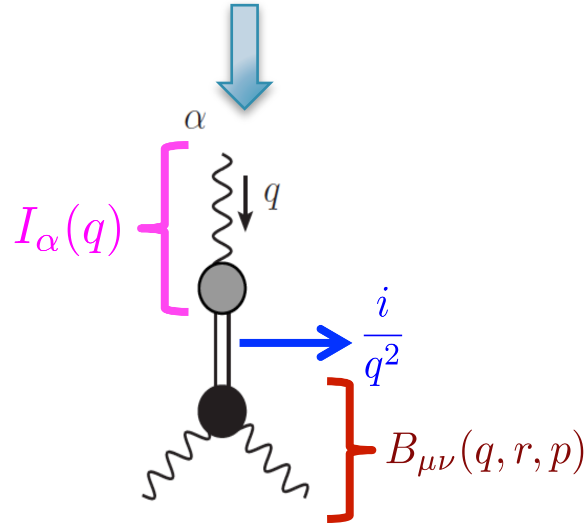
- **To evade the seagull cancellation**: *introduce massless poles*
- **Schwinger Mechanism:**

J.S. Schwinger, Phys. Rev.125, 397 (1962);
Phys.Rev.128, 2425 (1962).

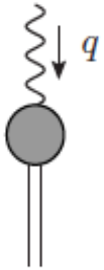
- ✓ **Massless bound state (colored) excitations;**
- ✓ **Must be generated dynamically → if not there is no gluon mass**
- ✓ **Longitudinal coupled;**
- ✓ **Decouple from on-shell amplitudes.**

The nonperturbative three-gluon vertex becomes



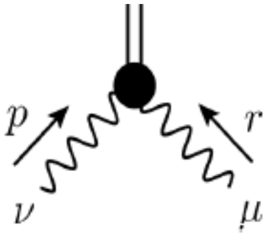


Transition amplitude

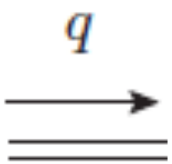


$$= I_\alpha(q) = q_\alpha(q) I(q^2)$$

Bound-state wave functions (vertex)



Massless bound-state propagator

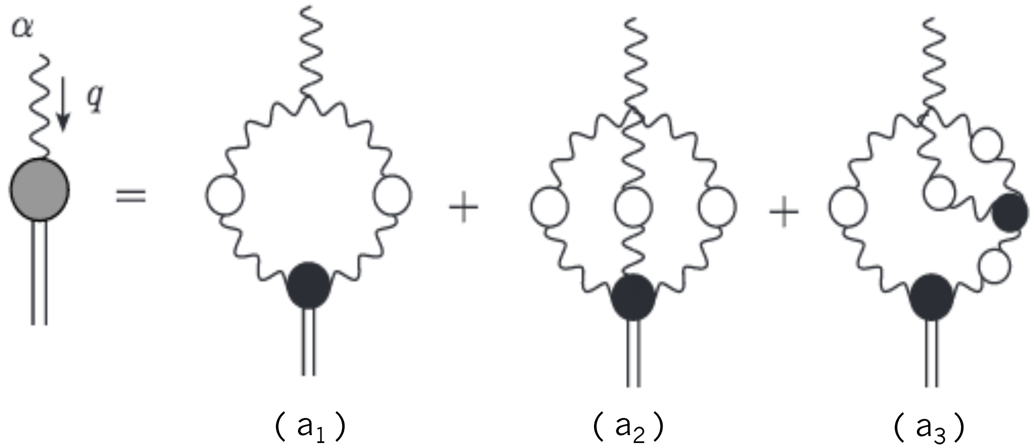


$$= i/q^2$$

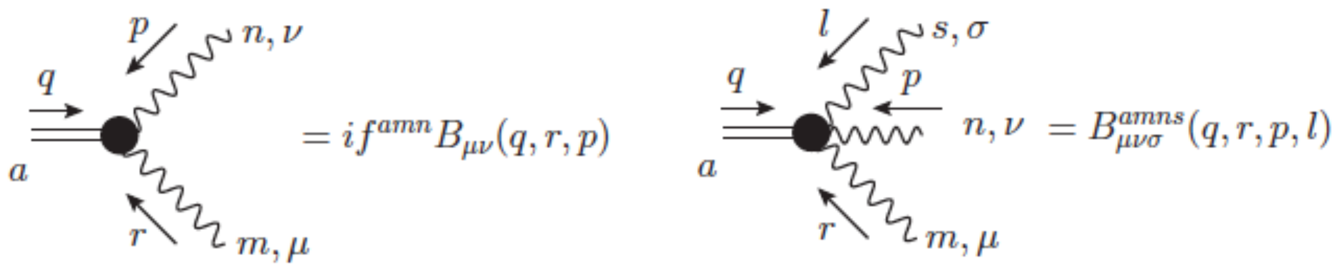
$$U_{\alpha_\mu\nu}(q,r,p) = I_\alpha(q) \left(\frac{i}{q^2}\right) B_{\mu\nu}(q,r,p)$$

D. Ibañez and J.Papavassiliou, PRD 87 034008 (2013).

Skeleton expansion of the transition amplitude

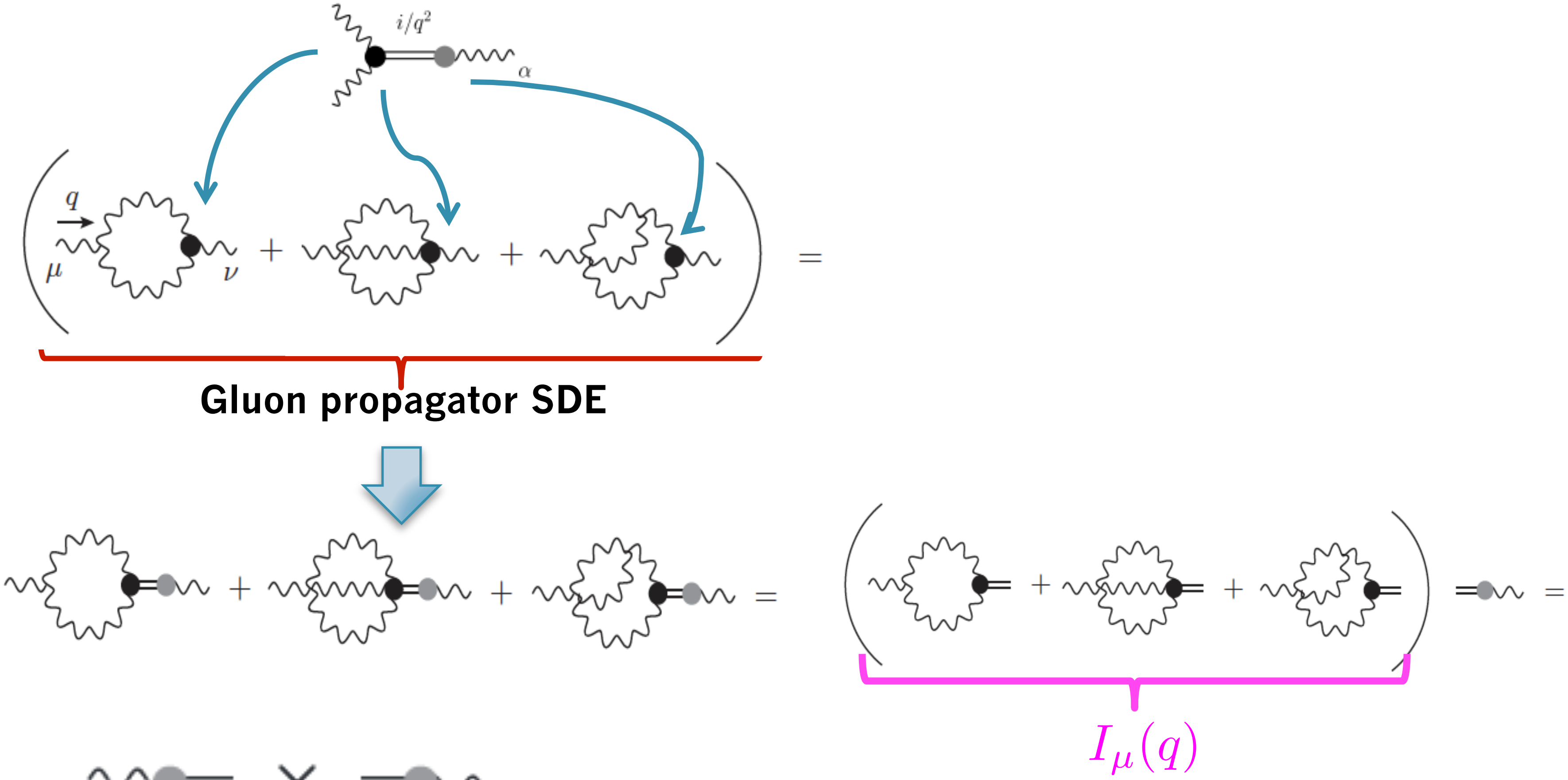


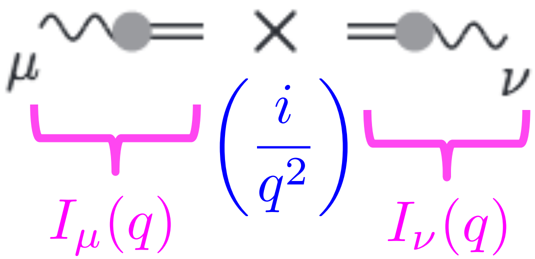
Effective vertices (Bound-state wave functions)



Relating the gluon mass with the transition amplitude

- **Insert the pole vertices U in the SDE of the gluon propagator**

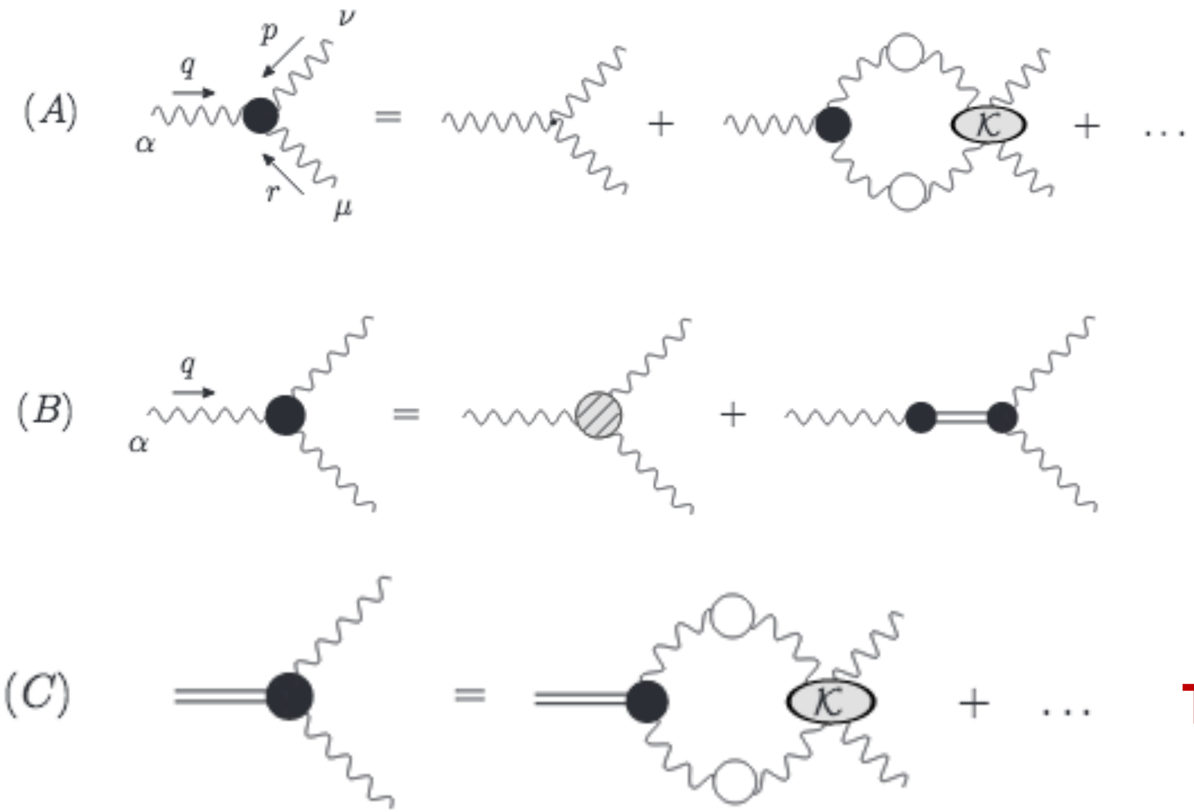




$$m^2(q^2) = g^2 I^2(q^2)$$

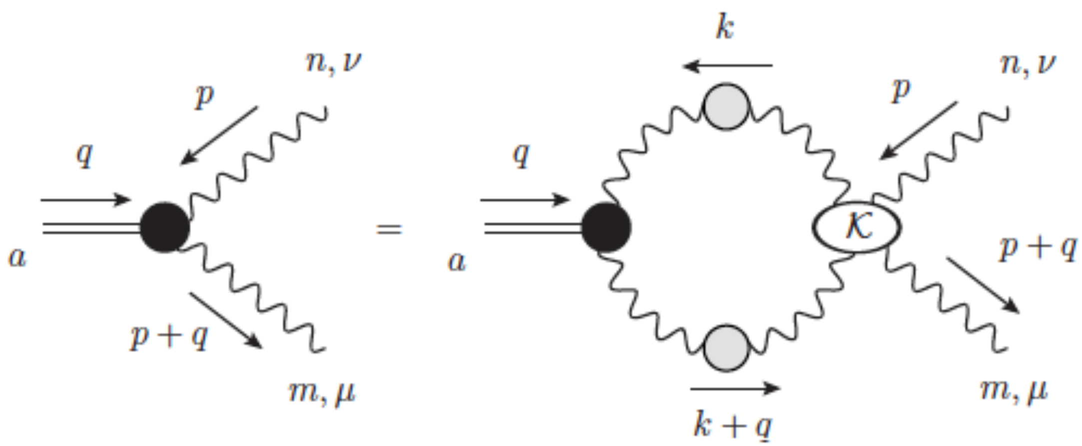
The BSE for the bound state wave function

- The separation of the vertex in regular and pole parts



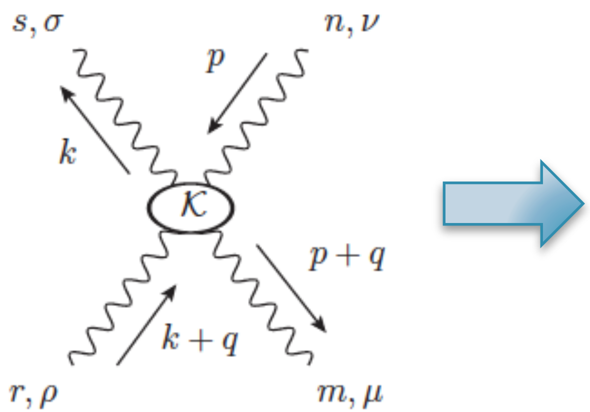
The final BSE

R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973);
E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974);
E.C.Poggio, E.Tomboulis and S.H.Tye, Phys.Rev.D 11, 2839 (1975);
R. Jackiw, In *Erice 1973, New York 1975, 225-251.



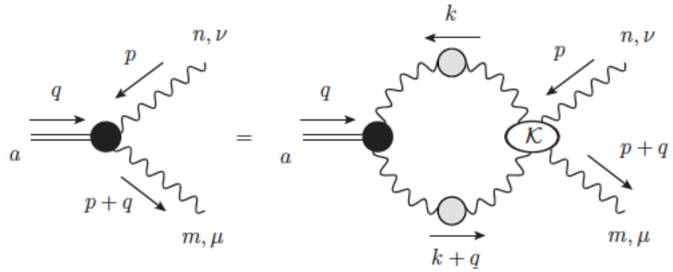
$$B^{amn}_{\mu\nu} = \int_k B^{abc}_{\alpha\beta}\Delta^{\alpha\rho}(k+q)\Delta^{\beta\sigma}(k)\mathcal{K}^{cnmb}_{\sigma\nu\mu\rho} \quad .$$

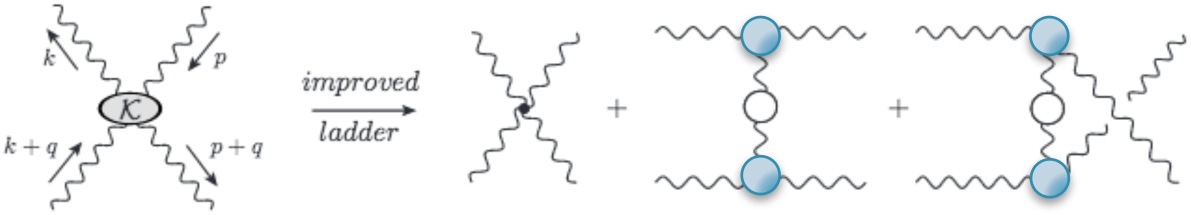
◎ **Internal gluon propagators are fully dressed.**



The four-gluon BS kernel

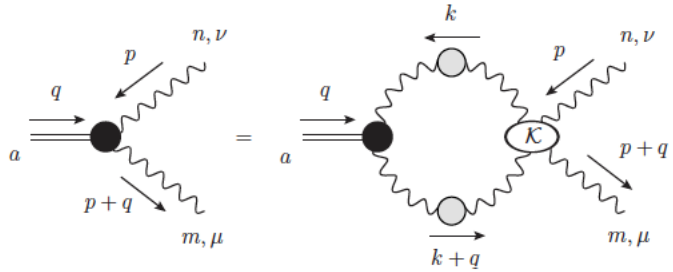
◎ Improved ladder approximation

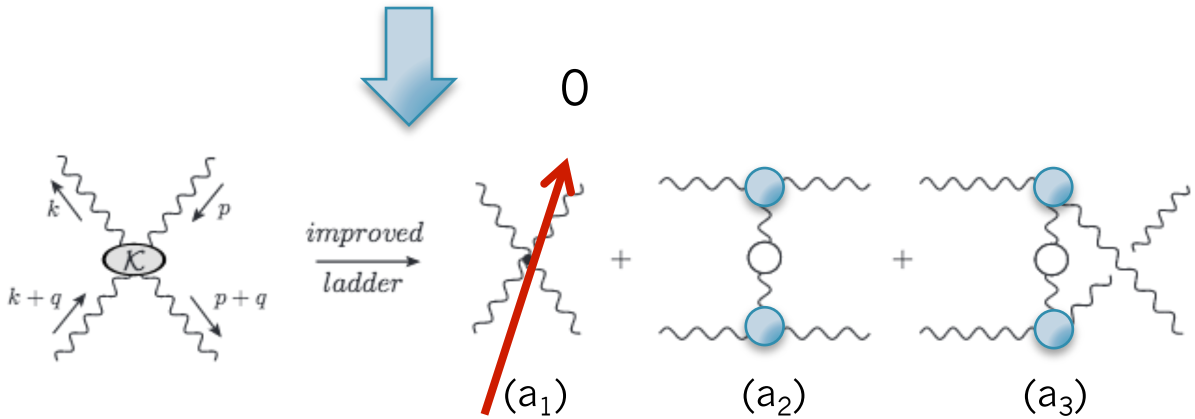




(a_1) (a_2) (a_3)

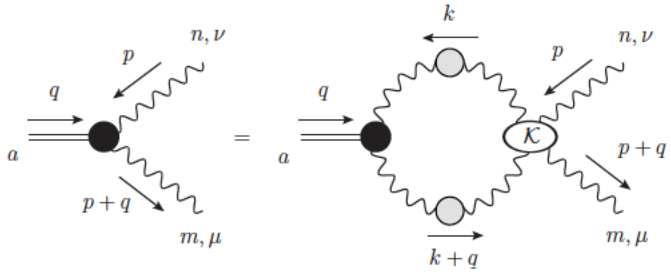
◎ Improved ladder approximation

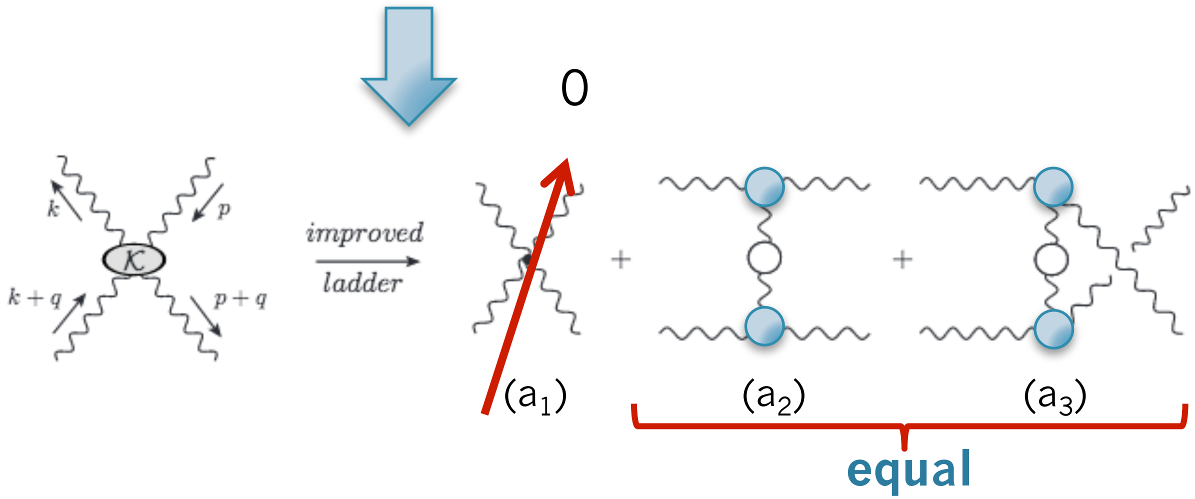




◎ a_1 vanishes because the color structure

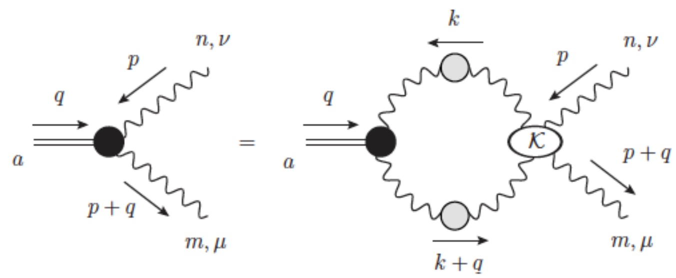
- **Improved ladder approximation**

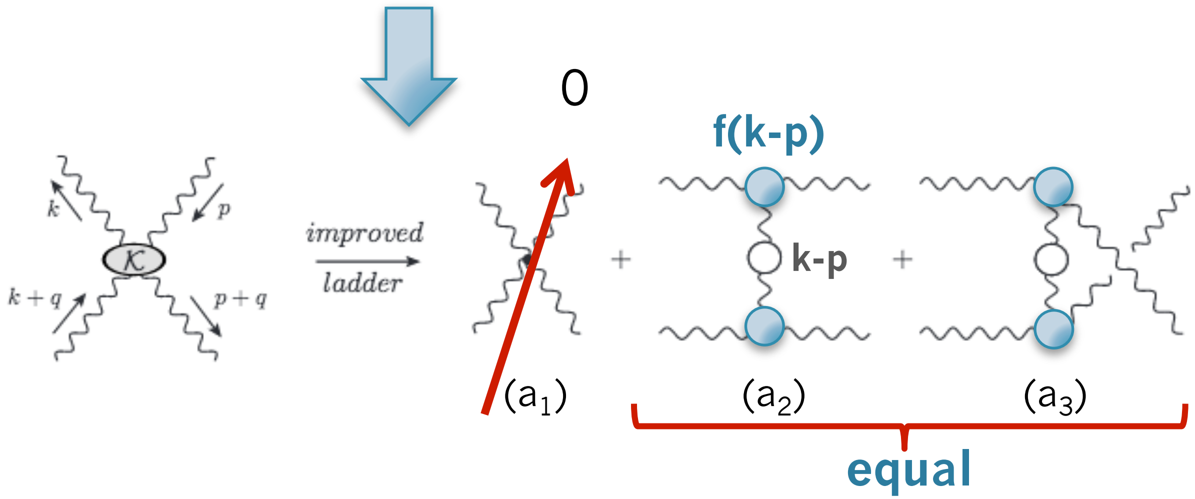




- **a_1 vanishes because the color structure**
- **a_2 and a_3 are equal (symmetry factor of ½)**

Improved ladder approximation





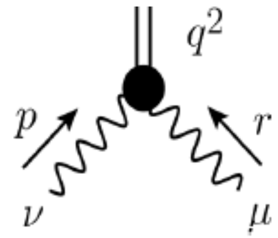
- a_1 vanishes because the color structure
- a_2 and a_3 are equal (symmetry factor of ½)

$$B_{\mu\nu}(q,p) = -2\pi i \alpha_s C_A \int_k B_{\alpha\beta}(k,p)\Delta^{\alpha\rho}(k+q)\Delta^{\beta\sigma}(k)K_{\sigma\nu\mu\rho}\,,$$

$$K_{\sigma\nu\mu\rho} = \Delta^{\gamma\lambda}(k-p)f^2(k-p)\Gamma^{(0)}_{\sigma\gamma\nu}\Gamma^{(0)}_{\mu\lambda\rho}$$

f(k-p) provides additional strength

◎ $B_{\mu\nu}$ can be decomposed as



$$B_{\mu\nu}(q,r,p) = B_1 g_{\mu\nu} + B_2 p_\mu p_\nu + B_3 r_\mu r_\nu + B_4 r_\mu p_\nu + B_5 p_\mu r_\nu$$

◎ Bose symmetry imposes

$$B_{\mu\nu}(0,-p,p) = -B_{\nu\mu}(0,p,-p)\,,$$

$$r = q - p$$

then

$$B_i(0,-p,p) = 0;\ \ i = 1,4,5$$

$$B_2(0,-p,p) = -B_3(0,-p,p)$$

- **Expansion on both sides leads to**

$$B_1'(p^2) = -\frac{2\pi i}{3}\alpha_s C_A \int_k \frac{(p\cdot k)}{p^2} B_1'(k^2) f^2(k-p)\mathcal{A}(p,k)$$

where derivative is defined as

$$B_1'(-p,p) \equiv \lim_{q\to 0}\left\{\frac{\partial B_1(q,-p-q,p)}{\partial\,(p+q)^2}\right\}$$

and

$$\mathcal{A}(p,k) = \mathcal{A}_0(p,k) + \xi\mathcal{A}_1(p,k) + \xi^2\mathcal{A}_2(p,k) + \xi^3\mathcal{A}_3(p,k)$$

Then in the Euclidean space the BSE reads

$$B_1'(x) = -\frac{\alpha_s C_A}{12\pi^2} \int_0^\infty dy y B_1'(y) \sqrt{\frac{y}{x}} \int_0^\pi d\theta \sin^4\theta \cos\theta f^2(z) \left[A_0 + \xi A_1 + \xi^2 A_2 + \xi^3 A_3\right]$$

where

$$A_0 = \left[z + 10(x+y) + \frac{1}{z}(x^2 + y^2 + 10xy)\right] \Delta^2(y)\Delta(z)$$

$$A_1 = (y-x)^2(2 + \cos^2\theta)\frac{\Delta^2(y)}{z^2}$$

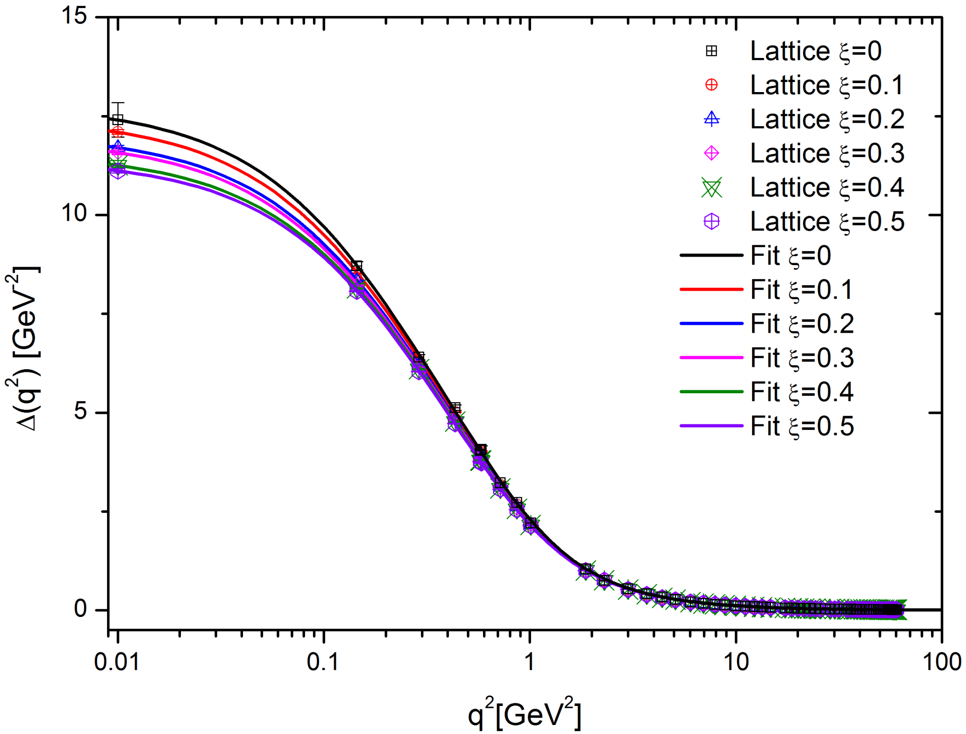
$$A_2 = \left(1 + 2\sqrt{\frac{x}{y}}\cos\theta\right)^2 (3z - y\sin^2\theta)\frac{\Delta(z)}{yz}$$

$$A_3 = \frac{x^2\sin^2\theta}{y^2 z^2}$$

$$x \equiv p^2 \quad ; \quad y \equiv k^2 \quad ; \quad z \equiv (p+k)^2$$

Gluon propagator from the lattice

- **We use the lattice results for solving the BSE**

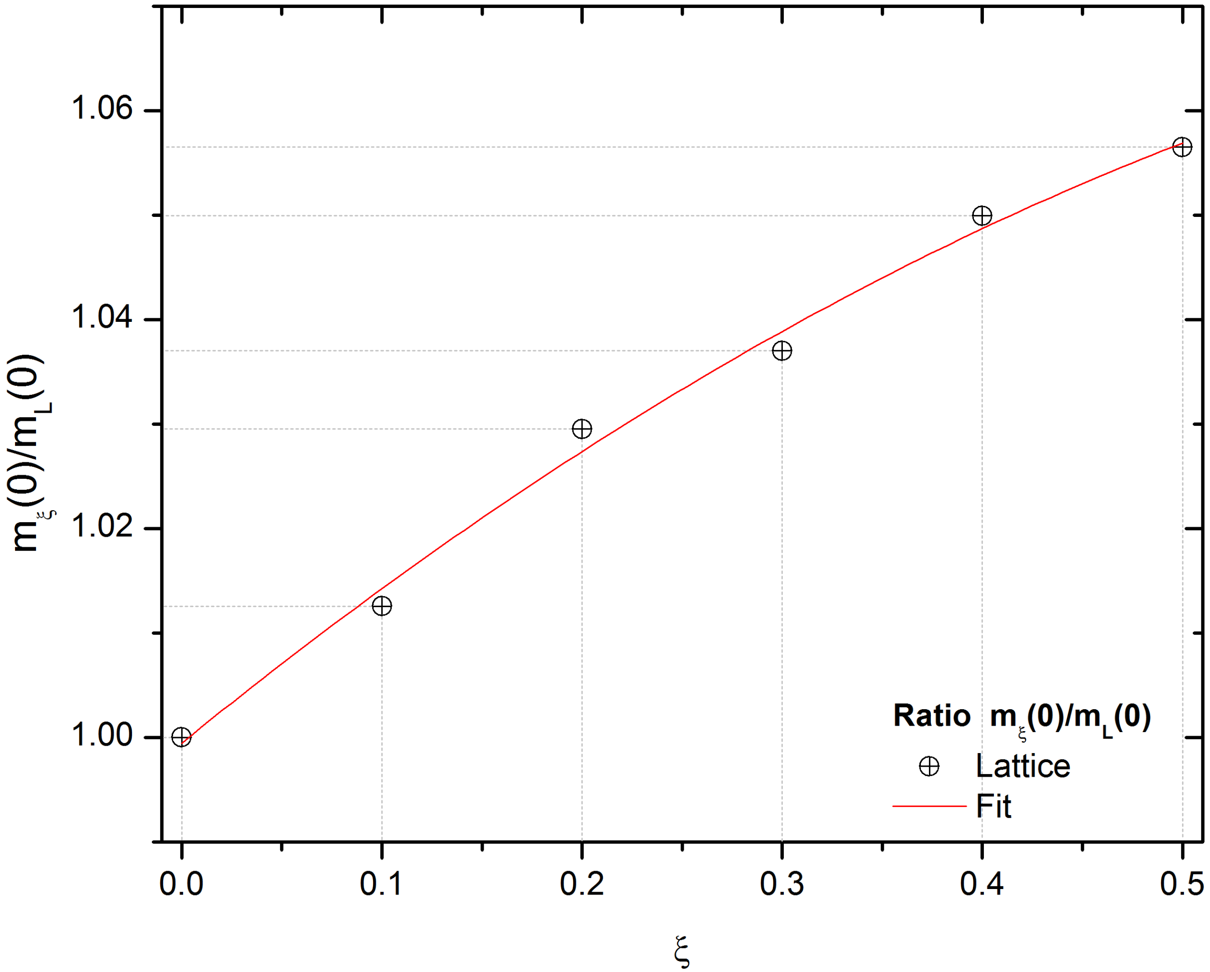


$$m^2(q^2) = \frac{m^4}{q^2 + \rho_2 m^2}$$

$$\Delta^{-1}(q^2) = m^2(q^2) + q^2 + \left[\frac{C_A}{32\pi^2}\left(\frac{13}{3} - \xi\right) g^2 \ln\left(\frac{q^2 + \rho_1 m^2(q^2)}{\mu^2}\right)\right]$$

P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, arXiv:1505.05897 [hep-lat].

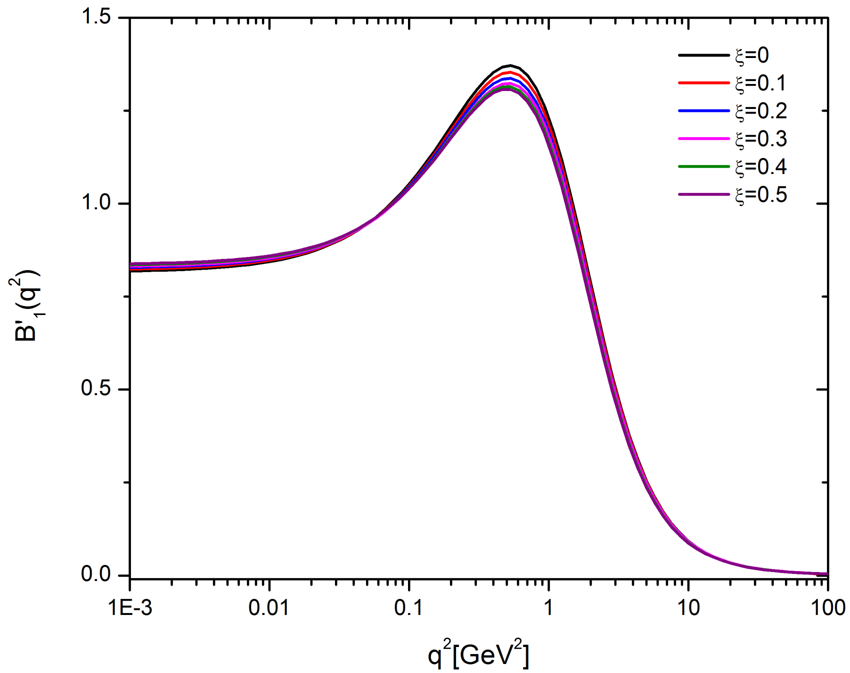
Lattice ξ dependence of the m(0)

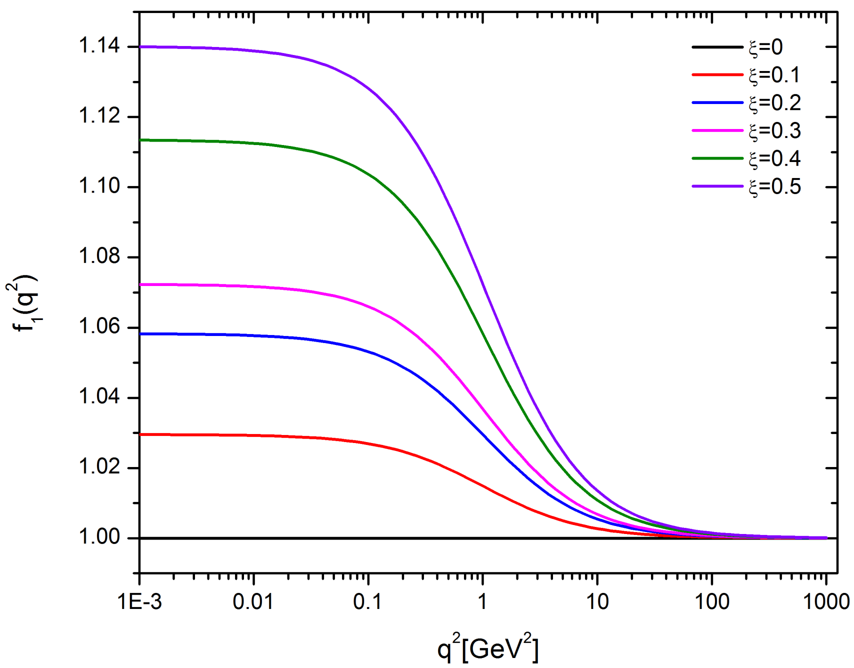


$$\frac{m_\xi(0)}{m_L(0)} = a + b\xi + c\xi^2$$

$$a = 1.0$$
$$b = 0.15$$
$$c = -0.08$$

BS function for $f_1\ (q^2)$



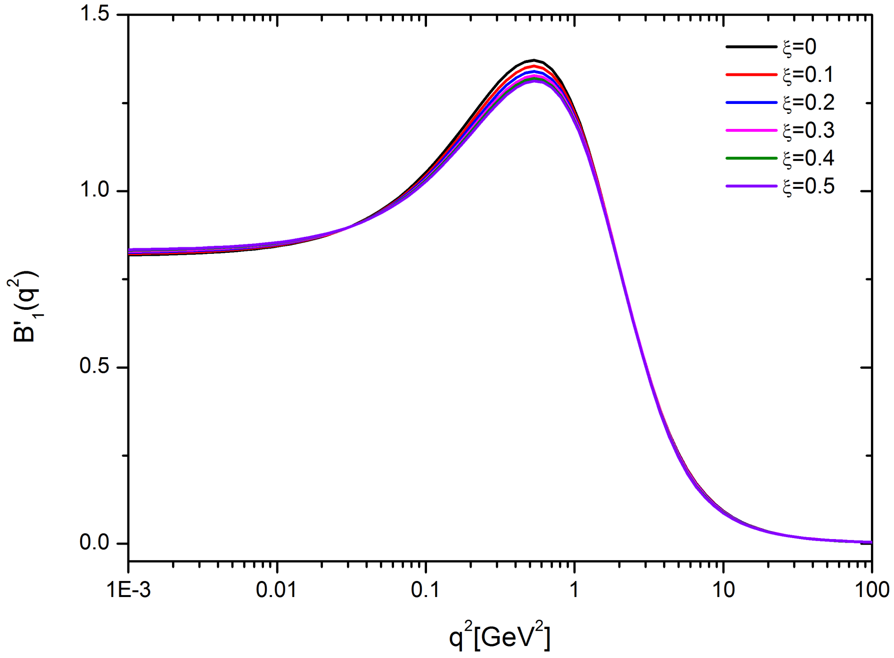


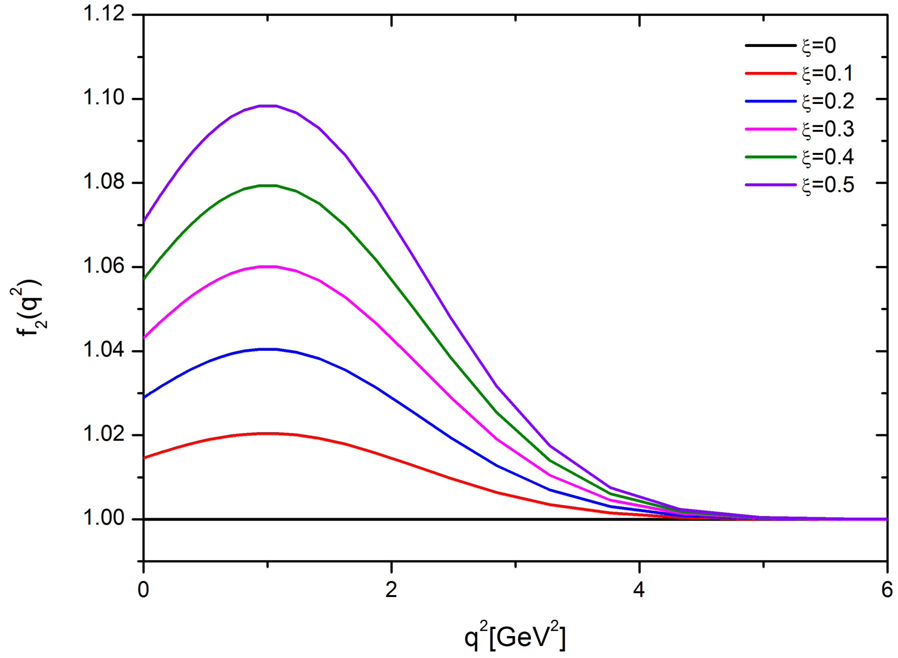
- **Ansatz for the form factor of the three gluon vertex $f(q^2) \to f_1(q^2)$ (one for each vertex)**

$$f_1^2(q^2) = 1 + \frac{0.6\xi}{1 + q^2/q_0^2}$$

$$q_0^2 = 1\,\mathrm{GeV}^2$$

BS function for $f_2\,(q^2)$





- **Ansatz for the form factor of the three gluon vertex $f(q^2) \rightarrow f_2(q^2)$ (one for each vertex)**

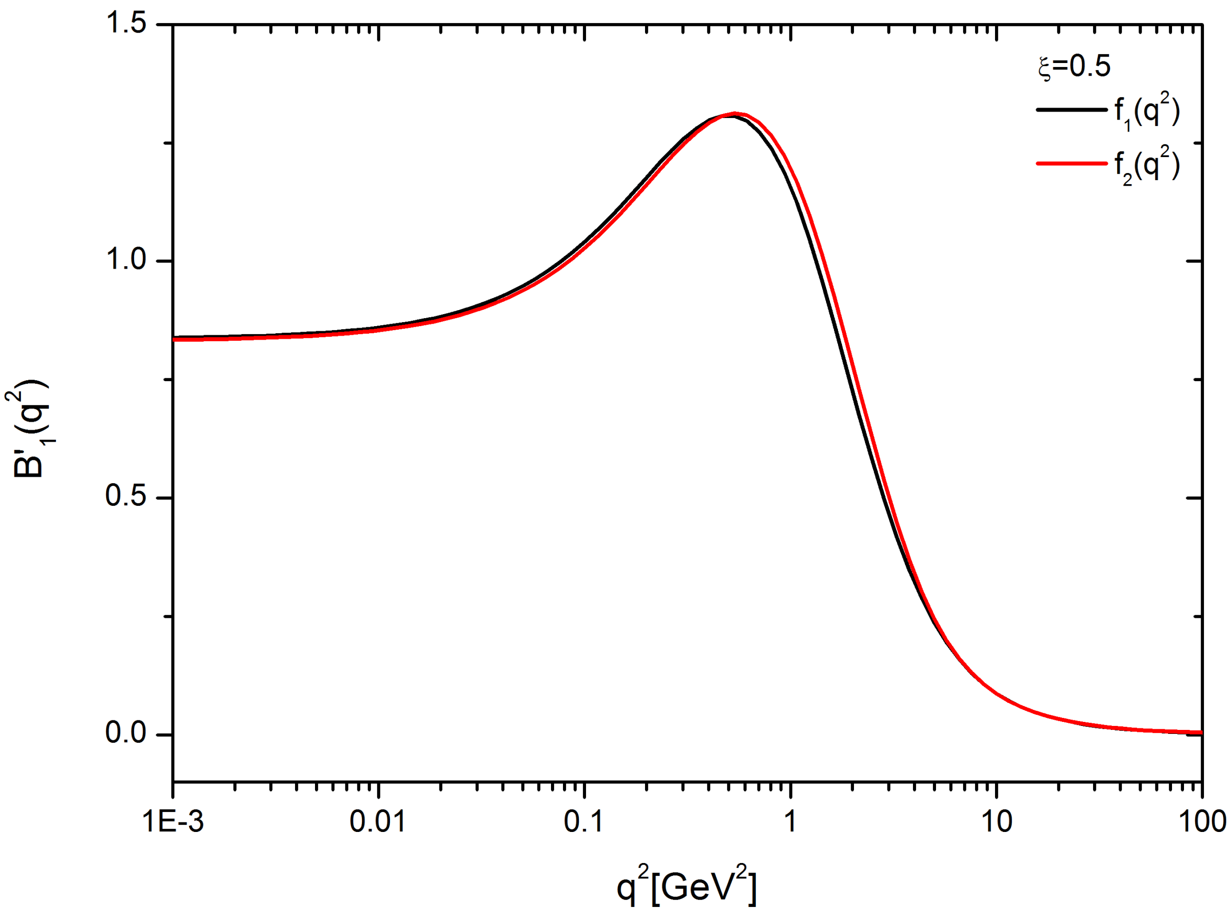
$$f_2^2(q^2) = 1 + \xi \frac{A}{\omega} e^{-\frac{2(q^2 - q_0^2)}{\omega^2}}$$

$$\omega = 2.42\,\mathrm{GeV}$$

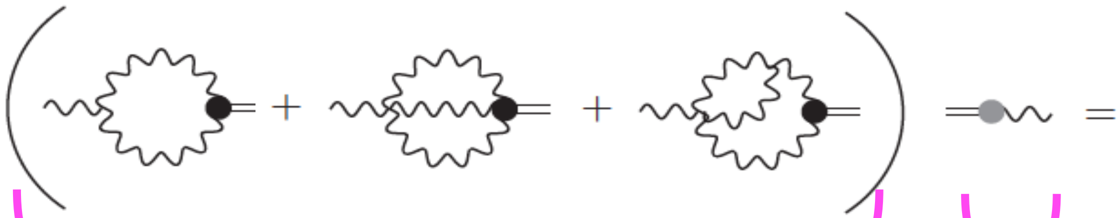
$$A = 1\,\mathrm{GeV}$$

$$q_0^2 = 1\,\mathrm{GeV}^2$$

Comparison f_1 and f_2 cases



◎ Using the relation



$$m(q^2) = gI(q^2)$$

$I_\mu(q)$ $\left(\frac{i}{q^2}\right) I_\nu(q)$

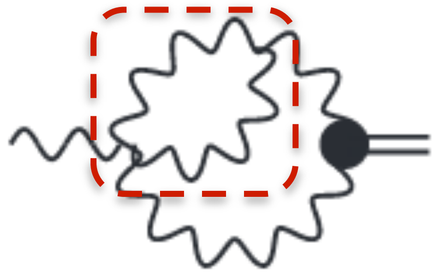
◎ We can derive the zero mass equation $m(0) = gI(0)$

$$m(0) = \underbrace{-\frac{3}{4}C_A g \int_k \Delta(k^2)\left[1 - \frac{3}{2}g^2 C_A Y(k^2)\right]\left[2k^2\Delta(k^2)B_1' + \xi\overline{B}_3\right]}_{gI(0)}$$

◎ We will approximate Y by its tree level

$$Y(k^2) = -\frac{1}{(4\pi)^2}\frac{5}{4}\log\frac{k^2}{\mu^2}.$$

$Y(k^2)$

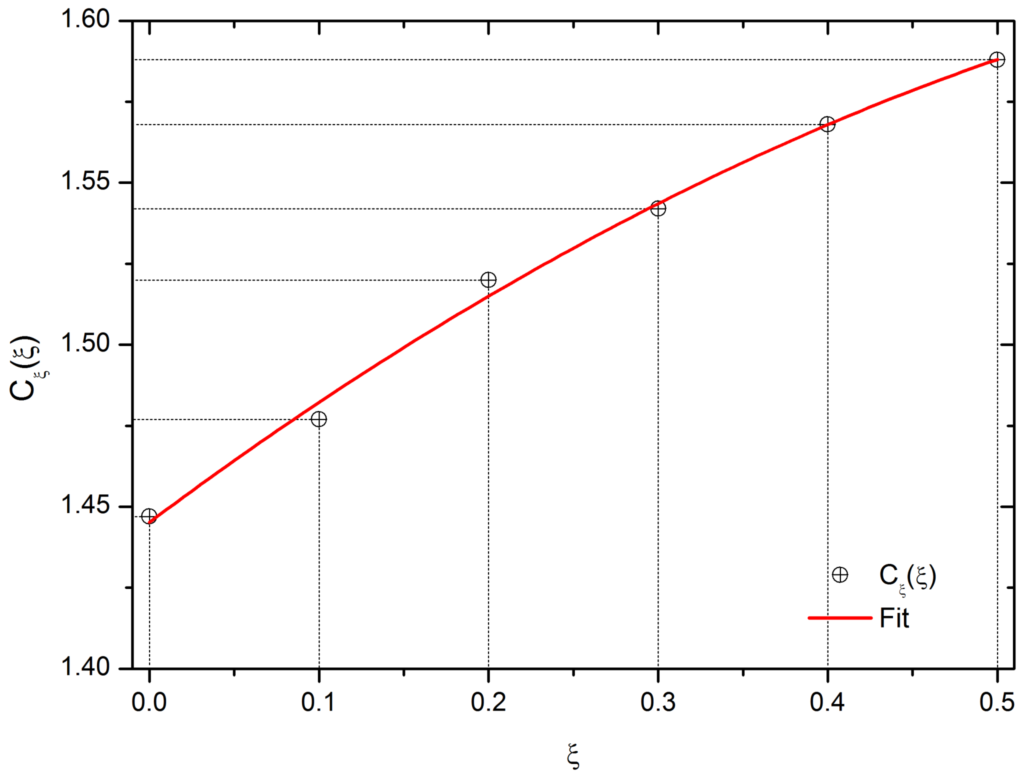


D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. 86 , 085033 (2012).

In order to mimic further corrections that may be added to the skeleton of $Y(k^2)$ we will make the replacement

$$Y(k^2) \to C_\xi Y(k^2)$$

We fix C_ξ in order to recover the value of $m(0)$



$$a_1 = 1.44$$
$$b_1 = 0.35$$
$$c_1 = -0.21$$

$$C_\xi(\xi) = a_1 + b_1\xi + c_1\xi^2$$

Conclusions

- We have studied the behavior of the ghost and gluon propagators away from the Landau gauge. Preliminary study of general patterns and tendencies under certain simplifying asumptions.

- The ghost dressing functions shows a behavior fairly different from that of the Landau gauge (vanishes at the origin for every ξ).

- Gluon mass generation persists (with minor adjustments). Smooth continuous transition from one gauge to the next.

- Future: if the tendency of the propagators is to drop, something else must increase in order to keep the gauge-independent observables fixed. Do vertices rise ?