Towards a consistent nonperturbative quark-gluon vertex from QCD symmetries

QCD-TNT 4 – Unraveling the organization of the QCD tapestry

Ilha Bela, São Paulo, August 31 to September 4, 2015

Bruno El-Bennich

Laboratório de Física Teórica e Computacional
Universidade Cruzeiro do Sul, São Paulo

Instituto de Física Teórica
Universidade Estadual Paulista, São Paulo

Different parts of this work done in collaboration with

Ishtiaq Ahmed, Eduardo Rojas and Orlando Oliveira.

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPESP
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Whence cometh this alarum and the noise?

William Shakespeare, Henry IV, Part I

Part I
Whence

The Lagrangian of QCD

- Lagrangian of QCD
  - *G* = gluon fields
  - *Ψ* = quark fields

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$
$$= \bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a\,,$$

- The key to complexity in QCD … gluon field strength tensor

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^a_\mu G^b_\nu$$

- Generates gluon self-interactions, whose consequences are quite extraordinary

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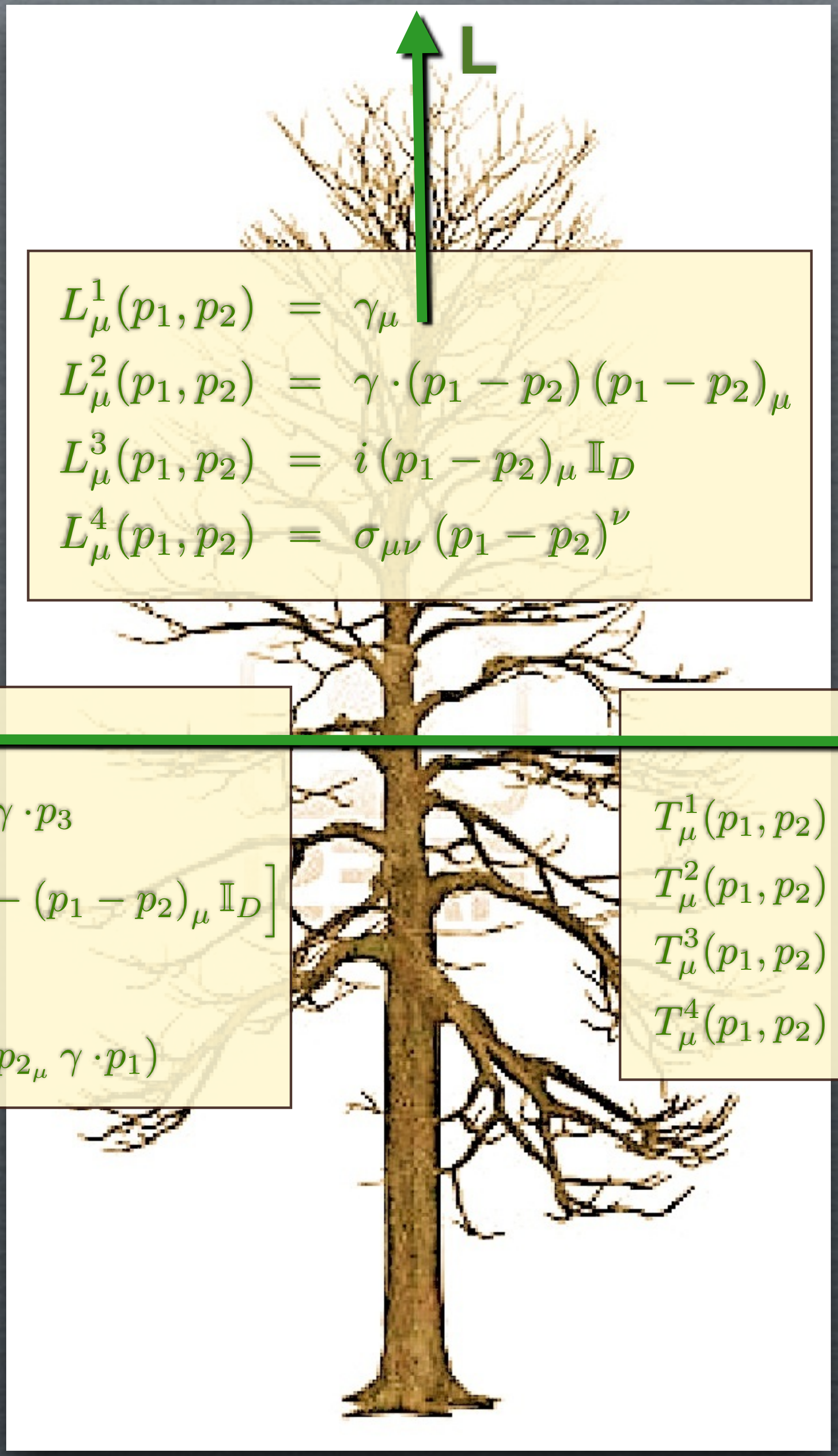
This complexity also affects the bare quark-gluon vertex in a nonperturbative manner!

Nonperturbative quark-gluon vertex



$$
\begin{aligned}
L^1_\mu(p_1,p_2) &= \gamma_\mu \\
L^2_\mu(p_1,p_2) &= \gamma\cdot(p_1-p_2)\,(p_1-p_2)_\mu \\
L^3_\mu(p_1,p_2) &= i\,(p_1-p_2)_\mu\,\mathbb{I}_D \\
L^4_\mu(p_1,p_2) &= \sigma_{\mu\nu}\,(p_1-p_2)^\nu
\end{aligned}
$$

$$
\begin{aligned}
T^1_\mu(p_1,p_2) &= i\,\left[p_{1\mu}\,(p_2\cdot p_3) - p_{2\mu}\,(p_1\cdot p_3)\right]\mathbb{I}_D \\
T^2_\mu(p_1,p_2) &= i\,T^1_\mu\,\gamma\cdot(p_1-\ p_2) \\
T^3_\mu(p_1,p_2) &= p_3^2\,\gamma_\mu - p_{3\mu}\,\gamma\cdot p_3 \\
T^4_\mu(p_1,p_2) &= -i\,T^1_\mu(p_1,p_2)\,\sigma_{\alpha\beta}\,p_2^\alpha p_1^\beta
\end{aligned}
$$

$$
\begin{aligned}
T^5_\mu(p_1,p_2) &= \sigma_{\mu\nu}\,p_3^\nu \\
T^6_\mu(p_1,p_2) &= -\gamma_\mu\,(p_1^2-p_2^2) - (p_1-p_2)_\mu\,\gamma\cdot p_3 \\
T^7_\mu(p_1,p_2) &= \frac{i}{2}\,(p_2^2-p_1^2)\left[\gamma_\mu\,\gamma\cdot(p_1-p_2) - (p_1-p_2)_\mu\,\mathbb{I}_D\right] \\
&+ (p_1-p_2)_\mu\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta \\
T^8_\mu(p_1,p_2) &= i\,\gamma_\mu\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta - (p_{1_\mu}\gamma\cdot p_2 + p_{2_\mu}\,\gamma\cdot p_1)
\end{aligned}
$$

QCD's Dyson-Schwinger Equations

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations:

$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) := i\gamma\cdot p\,A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^\Lambda \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)
\end{aligned}
$$

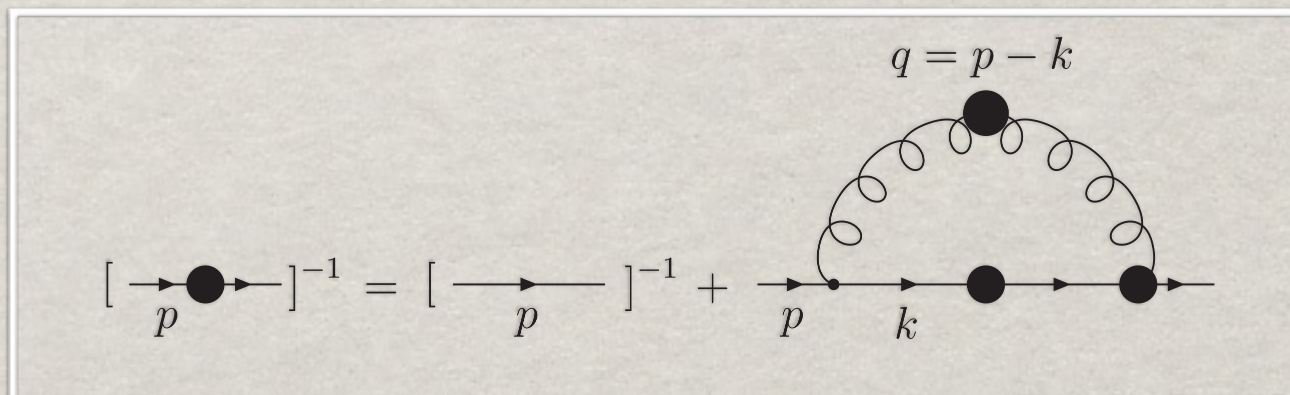
with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

each satisfies it's own DSE

$$
\begin{aligned}
D_{\mu\nu} &: \quad \text{dressed-gluon propagator} \\
\Gamma^a_\nu(q,p) &: \quad \text{dressed quark-gluon vertex} \\
Z_2 &: \quad \text{quark wave function renormalization constant} \\
Z_1 &: \quad \text{quark-gluon vertex renormalization constant}
\end{aligned}
$$

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

where ζ is the renormalization point.



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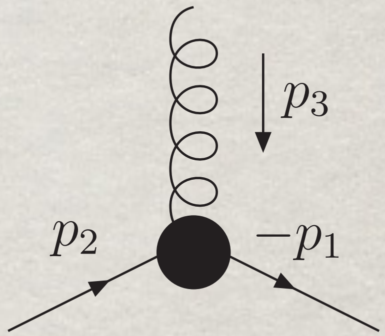
$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\rm bm}) + \Sigma(p) := i\gamma \cdot$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma$$

$$\Gamma^a_\nu(q,p)$$

with the *running* mass function $M(p^2)$).



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$D_{\mu\nu}$	:	dressed-gluon propagator
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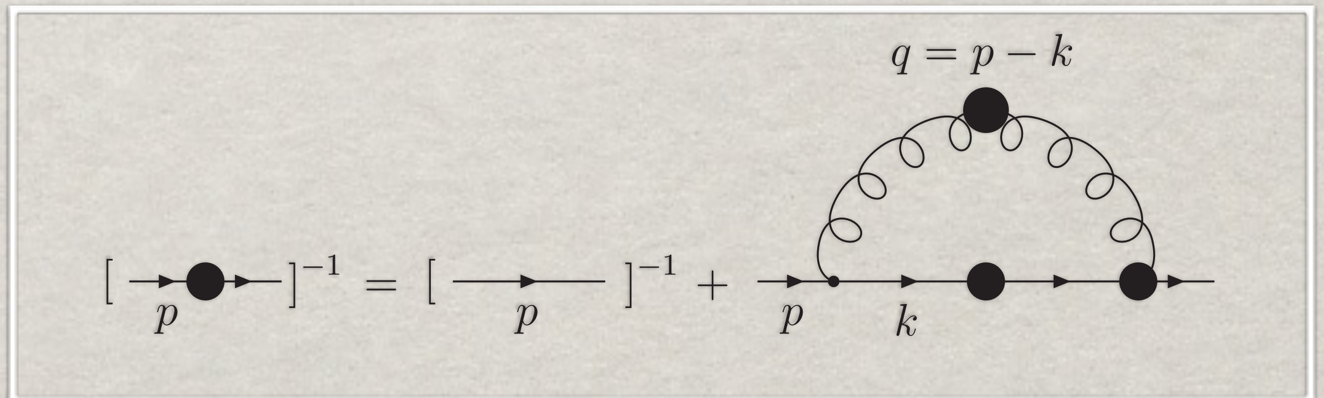

Bound State Equations for Pseudoscalars

For RL truncation one may use the Bethe-Salpeter equation (BSE):

$$\Gamma_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu - g^2 \int_q^\Lambda D^{\alpha\beta}(k-q)\frac{\lambda^a}{2}\gamma_\alpha\; S_f(q_+)\,\Gamma_{5\mu}(q;P)\,S_g(q_-)\frac{\lambda^a}{2}\gamma_\beta$$

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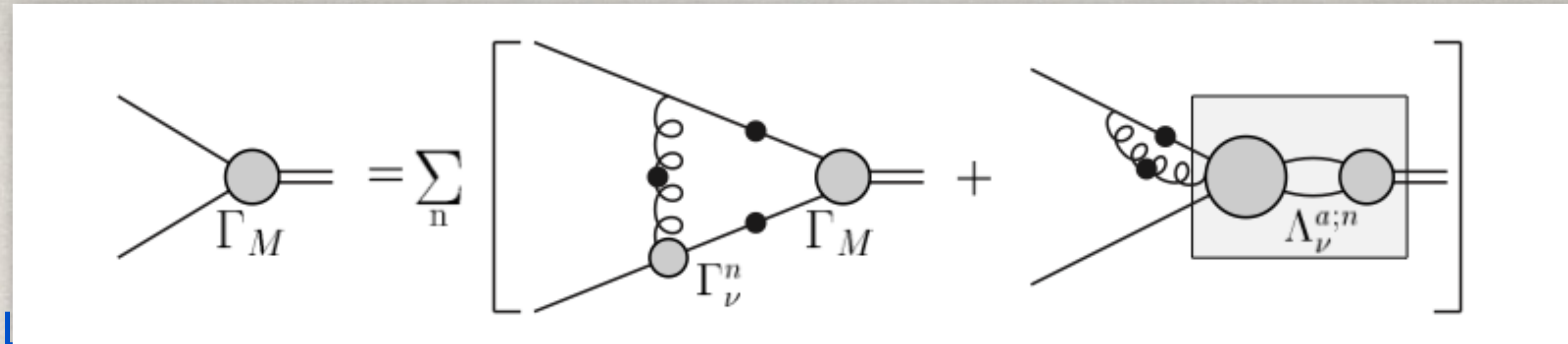
However, when using a quark-gluon vertex beyond RL, the exact BSE valid for any symmetry-preserving ansatz of this vertex must be employed:

$$\begin{aligned}\Gamma^{fg}_{5\mu}(k;P) \;&=\; Z_2\gamma_5\gamma_\mu - g^2 \int_q^\Lambda D^{\alpha\beta}(k-q)\frac{\lambda^a}{2}\gamma_\alpha\, S_f(q_+)\,\Gamma^{fg}_{5\mu}(q;P)\,S_g(q_-)\frac{\lambda^a}{2}\Gamma^g_\beta(q_-,k_-)\\ &+\; g^2\int_q^\Lambda D^{\alpha\beta}(k-q)\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P)\end{aligned}$$

L. Chang & C.D. Roberts (2009)



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Valid for RL truncation because for $\Gamma_\mu(k,p) = \gamma_\mu \quad \Longrightarrow \quad \Lambda^{fg}_{5\mu\beta}(k,q;P) = 0$

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L. Chang & C.D. Roberts (2009)

$\Lambda^{fg}_{5\mu\beta}(k,q;P)$ is a 4-point Schwinger function; completely defined by the dressed quark propagator.

Motivations and methods

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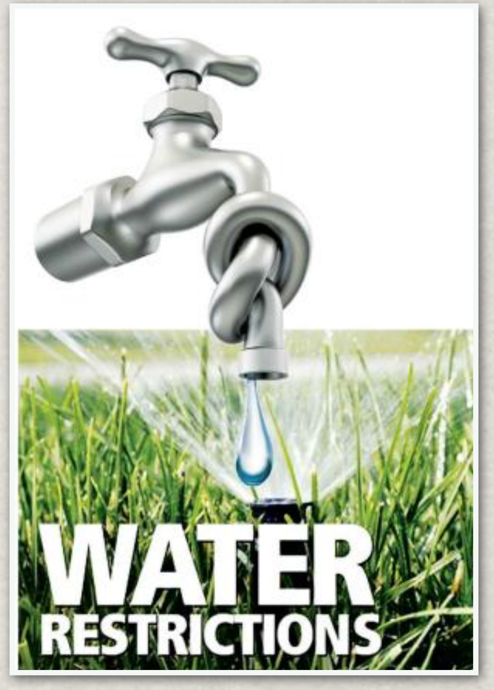
- Combining constraints from lattice-QCD gluon, ghost and quark propagators to study the DSE kernels and DCSB.
- To produce sufficient dynamical chiral symmetry breaking (DCSB) the transverse vertex components are mandatory.
- Using constraints from Slavnov-Taylor identities in the longitudinal AND transverse vertex tensor structure.

Nonperturbative quark-gluon vertex: *restrictions*



- Must satisfy Slavnov-Taylor identities.
- $\Gamma_\mu(k,p)$ must be free of kinematic singularities for $k^2 \to p^2$.
- Must transform as bare vertex γ_μ under C, P and T transformations.
- Correct weak-coupling limit: must reduce to the perturbative limit when coupling is small.
- Vertex ansatz should lead to gauge independent physical observables, i.e. fermion mass and condensate, meson masses and decay constants.

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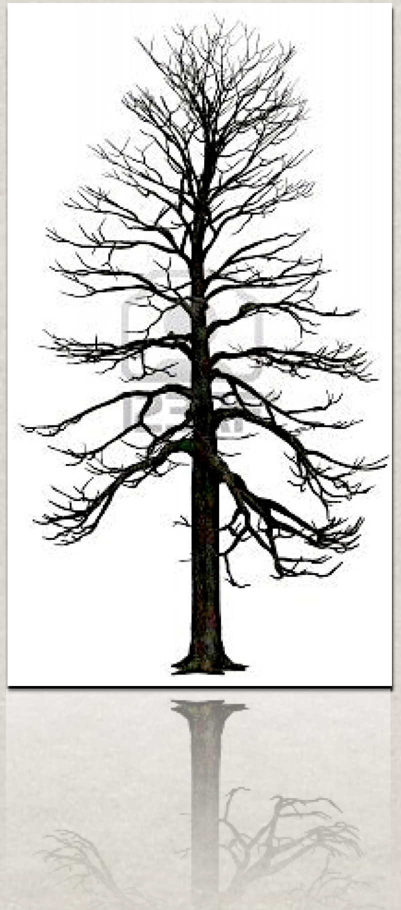
Nonperturbative quark-gluon vertex: tensor structure

The fermion-gauge-boson vertex can be decomposed into longitudinal and transverse components: $\Gamma_\mu(p_1,p_2,p_3) = \Gamma^{\rm L}_\mu(p_1,p_2,p_3) + \Gamma^{\rm T}_\mu(p_1,p_2,p_3)$.



$$\begin{aligned} \Gamma^{\rm L}_\mu(p_1,p_2,p_3) &= \sum_{i=1}^{4} \lambda_i(p_1,p_2,p_3)\, L^i_\mu(p_1,p_2) \\ \Gamma^{\rm T}_\mu(p_1,p_2,p_3) &= \sum_{i=1}^{8} \tau_i(p_1,p_2,p_3)\, T^i_\mu(p_1,p_2) \end{aligned}$$

$$\Gamma_\mu(p_1,p_2,p_3)\big|_{p_1^2=p_2^2=p_3^2=\mu^2} = \gamma_\mu$$

$$p_3 \cdot \Gamma^{\rm T}(p_1,p_2,p_3) = 0$$

Nonperturbative quark-gluon vertex: tensor structure

Which independent tensor structures to specify the longitudinal and transverse vertex? Following Ball and Chiu (1980), one can write:

$$
\begin{aligned}
L^1_\mu(p_1,p_2) &= \gamma_\mu \\
L^2_\mu(p_1,p_2) &= \gamma\cdot(p_1-p_2)\,(p_1-p_2)_\mu \\
L^3_\mu(p_1,p_2) &= i\,(p_1-p_2)_\mu\,\mathbb{I}_D \\
L^4_\mu(p_1,p_2) &= \sigma_{\mu\nu}\,(p_1-p_2)^\nu \\
\\
T^1_\mu(p_1,p_2) &= i\,\left[p_{1\mu}\,(p_2\cdot p_3) - p_{2\mu}\,(p_1\cdot p_3)\right]\mathbb{I}_D \\
T^2_\mu(p_1,p_2) &= i\,T^1_\mu\gamma\cdot(p_1-p_2)\ , \\
T^3_\mu(p_1,p_2) &= p_3^2\,\gamma_\mu - p_{3\mu}\ \gamma\cdot p_3 := p_3^2\gamma^{\mathrm{T}}_\mu \\
T^4_\mu(p_1,p_2) &= -i\,T^1_\mu(p_1,p_2)\,\sigma_{\alpha\beta}\,p_2^\alpha p_1^\beta \\
T^5_\mu(p_1,p_2) &= \sigma_{\mu\nu}\,p_3^\nu\ , \\
T^6_\mu(p_1,p_2) &= -\gamma_\mu\,(p_1^2-p_2^2) - (p_1-p_2)_\mu\ \gamma\cdot p_3 \\
T^7_\mu(p_1,p_2) &= \frac{i}{2}\,(p_2^2-p_1^2)\left[\gamma_\mu\gamma\cdot(p_1-p_2) - (p_1-p_2)_\mu\,\mathbb{I}_D\right] \\
&+ (p_1-p_2)_\mu\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta \\
T^8_\mu(p_1,p_2) &= i\,\gamma_\mu\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta - (p_{1_\mu}\,\gamma\cdot p_2 + p_{2_\mu}\,\gamma\cdot p_1)
\end{aligned}
$$

Nonperturbative quark-gluon vertex: *Rainbow-Ladder*

Since the Dyson-Schwinger Equation for QCD imply an infinite tower of non-linear integral equations, a symmetry preserving truncation scheme must be employed. The leading term in such a scheme is the Rainbow-Ladder (RL) truncation (Abelian approach).

$$K^{rs}_{tu}(q,k;P) = -4\pi\alpha(Q^2)D^{\rm free}_{\rho\sigma}(Q)\,\left[\frac{\lambda^a}{2}\gamma_\rho\right]_{ts}\left[\frac{\lambda^a}{2}\gamma_\sigma\right]_{ru}$$

RL truncation satisfies vector and flavour non-singlet axial-vector Ward-Takahashi identities but has bad gauge dependence $\Rightarrow$ Landau gauge!

Nonperturbative quark-gluon vertex: symmetries

Which steps to remedy this gauge dependence?

Clearly must be beyond the RL — bare vertex violates gauge variance:

$$iq^\mu \gamma_\mu \;\neq\; i\gamma\cdot k\, A(k^2) + B(k^2) - i\gamma\cdot p\, A(p^2) - B(p^2)$$

Best "prepared" with Landau gauge to minimize dependence.

First step is the proposal for the longitudinal vertex by Ball and Chiu:

$$\Gamma^{\mathrm{L}}_\mu(p_1,p_2,p_3) \;=\; \sum_{i=1}^{4} \lambda_i(p_1,p_2,p_3)\, L^i_\mu(p_1,p_2)$$

$$\lambda_1(p_1,p_2,p_3) \;=\; \tfrac{1}{2}\left[A(p_1^2)+A(p_2^2)\right] \qquad \lambda_2(p_1,p_2,p_3) \;=\; \tfrac{1}{2(p_2^2-p_1^2)}\left[A(p_2^2)-A(p_1^2)\right]$$

$$\lambda_3(p_1,p_2,p_3) \;=\; \frac{1}{p_1^2-p_2^2}\left[B(p_1^2)-B(p_2^2)\right] \qquad \lambda_4(p_1,p_2,p_3) \;=\; 0$$

Widely employed in phenomenology and satisfies WGTI; however transverse part remains undetermined — what about gauge covariance?

Nonperturbative quark-gluon vertex: *ansätze*

A step further is the Curtis-Pennington ansatz:

$$\Gamma_\mu^{\rm CP}(p_1,p_2,p_3) = \Gamma_\mu^{\rm BC}(p_1,p_2,p_3) + \Gamma_{\mu T}^{\rm CP}(p_1,p_2,p_3)$$

$$\begin{aligned}
\Gamma_{\mu T}^{\rm CP}(p_1,p_2,p_3) &= \tau_6(p_1^2,p_2^2)\, T_\mu^6(p_1,p_2) \\
\tau_6(p_1^2,p_2^2) &= \frac{1}{2}\frac{A(p_1^2)-A(p_2^2)}{d(p_1^2,p_2^2)} \\
d(p_1^2,p_2^2) &= \frac{(p_1^2-p_2^2)^2 + \left[\mathcal{M}^2(p_1^2)+\mathcal{M}^2(p_2^2)\right]^2}{p_1^2+p_2^2}
\end{aligned}$$

Gauge dependence reduced about 50% compared to RL truncation in neighborhood of the Landau gauge (fix point) for critical coupling in QED.

A. Bashir, R. Bermudez, L. Chang, C.D. Roberts (2009)

Nonperturbative quark-gluon vertex: *ansätze*

Beyond rainbow-ladder approximations

Systematic truncation scheme; extends the rainbow-ladder approximation and ensures axial-vector Ward-Takahashi identity (employs model gluon propagator but proof is independent).

$$\Gamma^g_\mu(k,p) = \gamma_\mu + \frac{1}{6}\int \frac{d^4 l}{(2\pi)^4} g^2 D_{\rho\sigma}(p-l)\gamma_\rho S(l+k-p)\gamma_\mu S(l)\gamma_\sigma$$

A. Bender, L. von Smekal & C. D. Roberts, Phys. Lett. B380, 7 (1996)

Dominant non-Abelian contributions to the dressed quark-gluon vertex stemming from the gluon self-interaction have been taken into account. The corresponding Bethe-Salpeter kernel satisfies the axial-vector Ward-Takahashi identity $\Rightarrow$ allows to investigate the influence of the gluon self-interaction in (light) mesons.

C. S. Fischer & R. Williams, Phys. Rev. Lett. 103, 122001 (2009)

Nonperturbative quark-gluon vertex: ansätze

Bashir, Bermúdez, Chang & Roberts (2012)

Guided by perturbation theory; draws on comparison with structural dependence of the Ball-Chiu vertex on A(p²) and B(p²); perturbative limit of the transverse vertex conforms with its one loop expansion in the asymptotic limit of $k^2 \gg p^2$.

$$
\begin{aligned}
\tau_1(k^2,p^2) &= \frac{a_1\,\Delta_B(k^2,p^2)}{(k^2+p^2)} \\
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\tau_4(k^2,p^2) &= \frac{a_4\,\Delta_B(k^2,p^2)}{[k^2+M^2(k^2)[p^2+M^2(p^2)]} \\
\tau_5(k^2,p^2) &= a_5\,\Delta_B(k^2,p^2) \\
\tau_6(k^2,p^2) &= \frac{a_6(k^2+p^2)\,\Delta_A(k^2,p^2)}{[(k^2-p^2)^2+(M^2(k^2)+M^2(p^2))^2]} \\
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$$k = p_1$$

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Qin, Chang, Liu, Roberts & Schmidt (2013)

Make use of longitudinal + transverse WGTI to constrain the Abelian gauge-boson-fermion vertex. Agreement of τ_3 τ_5 and τ_8 with functional form derived by Bashir et al.

Based on Abelian longitudinal and transverse vertex of Qin et al. in the background field method and all-order relation connecting this background-field vertex with the quark-gluon vertex.

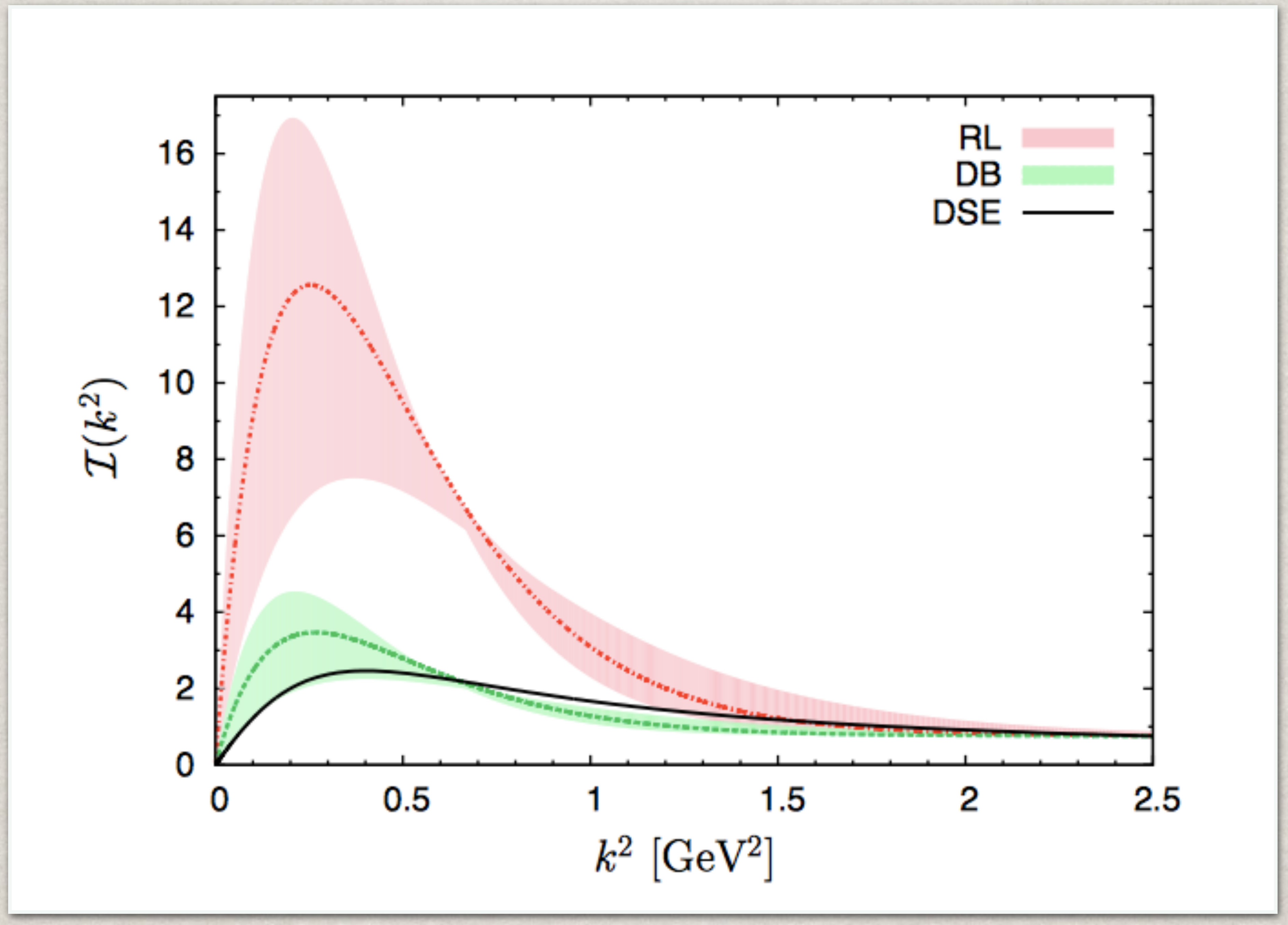
A.C. Aguilar, D. Binosi, D. Ibañez & J. Papavassiliou, Phys.Rev. D90, 6 (2014)

And thence from Athens turn away our eyes to seek new friends and stranger companies?

William Shakespeare, *A Midsummer Night's Dream*

Part II
Thence

Comparison of top-down and bottom-up interactions



Binosi, Chang, Papavassiliou & Roberts, Phys. Lett. B742 (2015)

Abelian Ward-Takahashi identities: divergence and curl

Ward-Takahashi identity:

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

Transverse Ward-Takahashi identities:

$$\begin{aligned} q_\mu\Gamma_\nu(k,p) - q_\nu\Gamma_\mu(k,p) &= S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu}S^{-1}(k) \\ &+ 2im\Gamma_{\mu\nu}(k,p) + t_\lambda\varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) + A^V_{\mu\nu}(k,p) \end{aligned}$$

$$\begin{aligned} q_\mu\Gamma^A_\nu(k,p) - q_\nu\Gamma^A_\mu(k,p) &= S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k) \\ &+ t_\lambda\varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) + V^A_{\mu\nu}(k,p) \end{aligned}$$

What is the origin of these transverse identities?

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$$\psi(x) \longrightarrow \psi'(x) = \psi(x) + ig\alpha(x)\psi(x),\ \ \bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x),$$

$$\big\downarrow$$

$$q_\mu \Gamma^\mu_V(p_1, p_2) = S_F^{-1}(p_1) - S_F^{-1}(p_2),$$

$$\delta_T \psi(x) = \frac{1}{4} g\alpha(x)\epsilon^{\mu\nu}\sigma_{\mu\nu}\psi(x), \quad \delta_T \bar{\psi}(x) = \frac{1}{4} g\alpha(x)\epsilon^{\mu\nu}\bar{\psi}(x)\sigma_{\mu\nu},$$

$$\big\downarrow$$

Infinitesimal Lorentz transformation

H.-x. He Phys.Rev. D80 (2009)

$$\begin{aligned} & iq^\mu \Gamma^\nu_V(p_1, p_2) - iq^\nu \Gamma^\mu_V(p_1, p_2) \\ = \ & S_F^{-1}(p_1)\sigma^{\mu\nu} + \sigma^{\mu\nu} S_F^{-1}(p_2) + 2m\Gamma^{\mu\nu}_T(p_1, p_2) \\ & + (p_{1\lambda} + p_{2\lambda})\varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_1, p_2) - \int \frac{d^4k}{(2\pi)^4} 2k_\lambda \varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_1, p_2; k), \end{aligned}$$

Non-Abelian Ward-Takahashi identities: divergence and curl

Slavnov-Taylor identity:

$$q_\mu \, i\Gamma^a_\mu(k,p) \quad = \quad F(q^2)\left[S^{-1}(k)H^a(k,p) - \bar{H}^a(k,p)S^{-1}(p)\right]$$

Transverse Slavnov-Taylor identities:

$$\begin{aligned} q_\mu \Gamma^a_\nu(k,p) - q_\nu \Gamma^a_\mu(k,p) &= F(q^2)\left[S^{-1}(p)\sigma_{\mu\nu}H^a(k,p) + \bar{H}^a(k,p)\sigma_{\mu\nu}S^{-1}(k)\right] \\ &+ 2im\,\Gamma^a_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^{aA}_\rho(k,p) + A^{aV}_{\mu\nu}(k,p) \end{aligned}$$

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Slavnov-Taylor Identity

One can relate the longitudinal form factors λ_i to the quark propagator's scalar and vector pieces, $B(p^2)$ and $A(p^2)$ via an STI:

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Ghost dressing function

Quark-ghost scattering kernel

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Ghost dressing function

Quark-ghost scattering kernel

Decomposition of H(k,p) and its conjugate in terms of Lorentz covariants:

$$H(p_1,p_2,p_3) \quad = \quad X_0\,\mathbb{I}_D + i\,X_1\,\gamma\cdot p_1 + i\,X_2\,\gamma\cdot p_2 + i\,X_3\,\sigma_{\alpha\beta}\,p_1^\alpha p_2^\beta$$

$$\overline{H}(p_2,p_1,p_3) \quad = \quad \overline{X}_0\,\mathbb{I}_D - i\,\overline{X}_2\,\gamma\cdot p_1 - i\,\overline{X}_1\,\gamma\cdot p_2 + i\,\overline{X}_3\,\sigma_{\alpha\beta}\,p_1^\alpha p_2^\beta$$

$$X_i(p,k,q) = \bar{X}_i(k,p,q) \qquad\qquad X_i \equiv X_i(p_1,p_2,p_3)$$

Davydychev, Osland & Saks (2001)

Longitudinal form factors of the quark-gluon vertex from the STI

Since the transverse part of the quark-gluon vertex does not contribute to the "longitudinal" STI, the longitudinal form factors λ_i may be written in terms of $A(p^2)$, $B(p^2)$, $F(p^2)$ and the quark-ghost scattering functions X_i .

$$
\begin{aligned}
\lambda_1^{\mathrm{QCD}} &= \frac{F(q^2)}{2}\Big\{A(k^2)\left[X_0 + (k^2 - k\cdot p)\,X_3\right] + A(p^2)\left[\overline{X}_0 + (p^2 - k\cdot p)\,\overline{X}_3\right] \\
&+ B(k^2)\left[X_1 + X_2\right] + B(p^2)\left[\overline{X}_1 + \overline{X}_2\right]\Big\}\,, \\
\lambda_2^{\mathrm{QCD}} &= \frac{F(q^2)}{2(p^2 - k^2)}\Big\{A(k^2)\left[(k^2 + k\cdot p)\,X_3 - X_0\right] + A(p^2)\left[\overline{X}_0 - (p^2 + k\cdot p)\,\overline{X}_3\right] \\
&+ B(k^2)\left[X_2 - X_1\right] + B(p^2)\left[\overline{X}_1 - \overline{X}_2\right]\Big\}\,, \\
\lambda_3^{\mathrm{QCD}} &= \frac{F(q^2)}{k^2 - p^2}\Big\{A(k^2)\left[k^2\,X_1 + k\cdot p\,X_2\right] - A(p^2)\left[p^2\,\overline{X}_1 + k\cdot p\,\overline{X}_2\right] \\
&+ B(k^2)\,X_0 - B(p^2)\,\overline{X}_0\Big\}\,, \\
\lambda_4^{\mathrm{QCD}} &= \frac{-F(q^2)}{2}\Big\{A(k^2)\,X_2 - A(p^2)\,\overline{X}_2 + B(k^2)\,X_3 - B(p^2)\,\overline{X}_3\Big\}
\end{aligned}
$$

Aguilar & Papavassiliou (2011)

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$$X_1(q^2), X_2(q^2), X_3(q^2) \to 0$$

$$\begin{aligned}
\lambda_1^{\rm QCD} &= \frac{X_0(q^2)\,F(q^2)}{2}\left[A(k^2)+A(p^2)\right] \\
\lambda_2^{\rm QCD} &= \frac{X_0(q^2)\,F(q^2)}{2\,(k^2-p^2)}\left[A(k^2)-A(p^2)\right] \\
\lambda_3^{\rm QCD} &= \frac{X_0(q^2)\,F(q^2)}{k^2-p^2}\left[B(k^2)-B(p^2)\right] \\
\lambda_4^{\rm QCD} &= 0
\end{aligned}$$

Aguilar & Papavassiliou (2011)

Decoupling the transverse STIs

Consider the transverse STI which involves the axialvector vertex:

$$
\begin{aligned}
q_\mu \Gamma_\nu^{aA}(k,p) - q_\nu \Gamma_\mu^{aA}(k,p) &= F(q^2)\left[S^{-1}(p)\sigma_{\mu\nu}^5 H^a(k,p) - \bar{H}^a(k,p)\sigma_{\mu\nu}^5 S^{-1}(k)\right] \\
&+ t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho^a(k,p) + V_{\mu\nu}^{aA}(k,p)
\end{aligned}
$$

Contract it with the two tensors:

$$
\begin{aligned}
T_{\mu\nu}^1 &= \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_{\rm D} \qquad\qquad t = k + p \\
T_{\mu\nu}^2 &= \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta
\end{aligned}
$$

and use the relations:

$$
\begin{aligned}
T_{\mu\nu}^1\, t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) &= t^2\, q\cdot\Gamma(k,p) - q\cdot t\, t\cdot\Gamma(k,p) \\
T_{\mu\nu}^2\, t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) &= \gamma\cdot t\, q\cdot\Gamma(k,p) - q\cdot t\gamma\cdot\Gamma(k,p)
\end{aligned}
$$

H.-x. He (2009) ; S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

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$$\begin{aligned} q_\mu \Gamma^{aA}_\nu(k,p) - q_\nu \Gamma^{aA}_\mu(k,p) &= F(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu} H^a(k,p) - \bar{H}^a(k,p)\sigma^5_{\mu\nu} S^{-1}(k)\right] \\ &+ t_\lambda \varepsilon_{\lambda\mu\nu\rho} \Gamma^a_\rho(k,p) + V^{aA}_{\mu\nu}(k,p) \end{aligned}$$

Contract it with the two tensors:

$$\begin{aligned} T^1_{\mu\nu} &= \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_{\mathrm{D}} \qquad t = k + p \\ T^2_{\mu\nu} &= \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta \end{aligned}$$

and use the relations:

$$\begin{aligned} T^1_{\mu\nu}\, t_\lambda \varepsilon_{\lambda\mu\nu\rho} \Gamma_\rho(k,p) &= t^2\, q\cdot\Gamma(k,p) - q\cdot t\, t\cdot\Gamma(k,p) \\ T^2_{\mu\nu}\, t_\lambda \varepsilon_{\lambda\mu\nu\rho} \Gamma_\rho(k,p) &= \gamma\cdot t\, q\cdot\Gamma(k,p) - q\cdot t \gamma\cdot\Gamma(k,p) \end{aligned}$$

$$\begin{aligned} q\cdot t\, t\cdot\Gamma^a(k,p) &= T^1_{\mu\nu} F(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu} H^a(k,p) - \bar{H}^a(k,p)\sigma^5_{\mu\nu} S^{-1}(k)\right] \\ &+ t^2\, q\cdot\Gamma^a(k,p) + T^1_{\mu\nu} V^{aA}_{\mu\nu}(k,p), \\ q\cdot t\,\gamma\cdot\Gamma^a(k,p) &= T^2_{\mu\nu} F(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu} H^a(k,p) - \bar{H}^a(k,p)\sigma^5_{\mu\nu} S^{-1}(k)\right] \\ &+ \gamma\cdot t\, q\cdot\Gamma^a(k,p) + T^2_{\mu\nu} V^{aA}_{\mu\nu}(k,p). \end{aligned}$$

H.-x. He (2009) ; S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

These two identities involve ONLY the vector vertex!

$$
\begin{aligned}
q\cdot t\, t\cdot \Gamma^a(k,p) &= T^1_{\mu\nu}\, F(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu}H^a(k,p) - \bar{H}^a(k,p)\sigma^5_{\mu\nu}S^{-1}(k)\right] \\
&+ t^2\, q\cdot \Gamma^a(k,p) + T^1_{\mu\nu}V^{aA}_{\mu\nu}(k,p), \\
q\cdot t\, \gamma\cdot \Gamma^a(k,p) &= T^2_{\mu\nu}\, F(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu}H^a(k,p) - \bar{H}^a(k,p)\sigma^5_{\mu\nu}S^{-1}(k)\right] \\
&+ \gamma\cdot t\, q\cdot \Gamma^a(k,p) + T^2_{\mu\nu}V^{aA}_{\mu\nu}(k,p).
\end{aligned}
$$

The unfamiliar complicated components in these identities can simply be decomposed:

$$
\begin{aligned}
iT^1_{\mu\nu}V^{aA}_{\mu\nu}(k,p) &= \mathbf{I}_{\rm D}\, Y_1(k,p) + \gamma\cdot q\, Y_2(k,p) + \gamma\cdot t\, Y_3(k,p) + [\gamma\cdot q, \gamma\cdot t]Y_4(k,p) \\
iT^2_{\mu\nu}V^{aA}_{\mu\nu}(k,p) &= \mathbf{I}_{\rm D}\, Y_5(k,p) + \gamma\cdot q\, Y_6(k,p) + \gamma\cdot t\, Y_7(k,p) + [\gamma\cdot q, \gamma\cdot t]Y_8(k,p)
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$$

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&+ t^2\, q\cdot\Gamma^a(k,p) + T^1_{\mu\nu}V^{aA}_{\mu\nu}(k,p), \\
q\cdot t\, \gamma\cdot \Gamma^a(k,p) &= T^2_{\mu\nu}\, F(q^2)\left[S^{-1}(p)\sigma^5_{\mu\nu}H^a(k,p) - \bar{H}^a(k,p)\sigma^5_{\mu\nu}S^{-1}(k)\right] \\
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\end{aligned}
$$

S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

Not known for now until some *Ansatz/Model* for them is found:

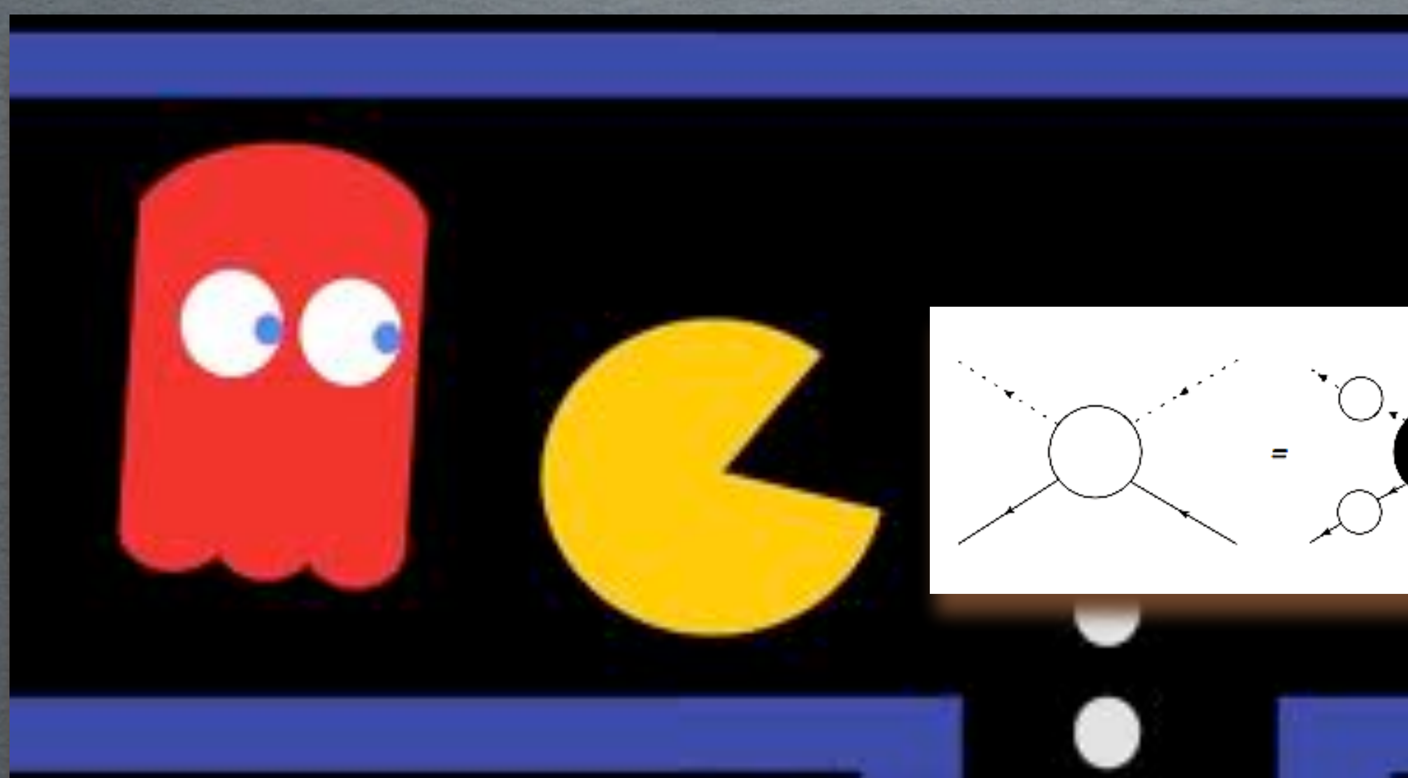
$$
V^{aA}_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4}\, 2k_\lambda\, \varepsilon_{\lambda\mu\nu\rho}\, \Gamma^a_\rho(k,p)
$$

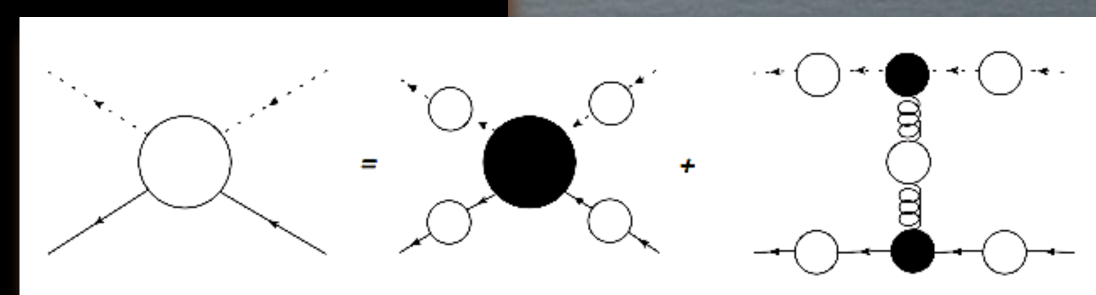
Transverse form factors of the quark-gluon vertex from transverse STIs

$$
\begin{aligned}
\tau_3^{\rm QCD} &= F(q^2)X_0(k,p)\frac{A(k^2)-A(p^2)}{2\,(k^2-p^2)} + \frac{2Y_2\left(k^2-p^2\right)-(Y_3-Y_5)(k+p)^2}{8\,(k^2-p^2)\,(k^2p^2-(k\cdot p)^2)}, \\
\tau_5^{\rm QCD} &= -F(q^2)X_0(k,p)\frac{B(k^2)-B(p^2)}{k^2-p^2} - \frac{Y_1}{8\,(k^2p^2-(k\cdot p)^2)} - \frac{2Y_4+Y_6}{2(k^2-p^2)}, \\
\tau_8^{\rm QCD} &= -F(q^2)X_0(k,p)\frac{A(k^2)-A(p^2)}{k^2-p^2} - \frac{2Y_8}{k^2-p^2} - \frac{(k-p)^2Y_2+(k^2-p^2)Y_3}{4(k^2-p^2)\,(k^2p^2-(k\cdot p)^2)}
\end{aligned}
$$

$$
\begin{aligned}
\tau_1^{\rm QCD} &= -\frac{Y_1}{2\,(k^2-p^2)\,(k^2p^2-(k\cdot p)^2)} \\
\tau_2^{QCD} &= -\frac{Y_5-3Y_3}{4\,(k^2-p^2)\,(k^2p^2-(k\cdot p)^2)}, \\
\tau_4^{QCD} &= \frac{Y_1-(6Y_4+Y_6)\left(k^2-p^2\right)-Y_7\,(k+p)^2}{2\,(k^2-p^2)^2\,(k^2p^2-(k\cdot p)^2)}, \\
\tau_6^{QCD} &= \frac{2Y_2\,(k-p)^2-(Y_3-Y_5)(k^2-p^2)}{8\,(k^2-p^2)\,(k^2p^2-(k\cdot p)^2)}, \\
\tau_7^{QCD} &= \frac{Y_1\,(k-p)^2-4Y_7(k^2p^2-(k\cdot p)^2)}{4\,(k^2-p^2)\,(k^2p^2-(k\cdot p)^2)}
\end{aligned}
$$

Quark-ghost and ghost-gluon scattering





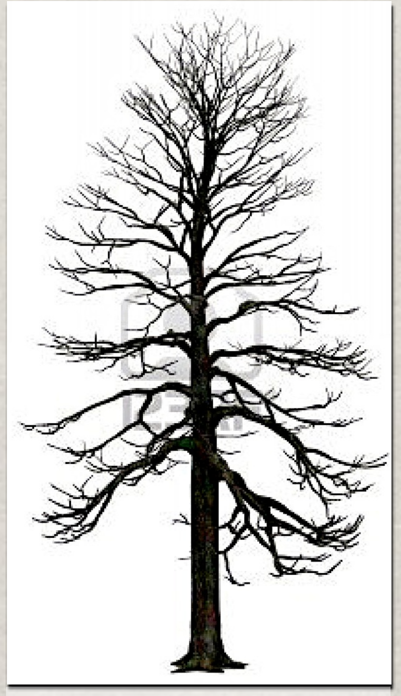
Quark-ghost scattering

The functions X_i are known in pQCD at one-loop order, see Davydychev, Osland & Saks (2001).

Nonperturbative approach to quark-ghost scattering kernel by Aguilar & Papavassiliou (2011) with the approximation $X_{1,2,3} = 0$. This yields a "ghost-improved" Ball-Chiu vertex.

$$\tilde{\Gamma}_\mu^{\mathrm{BC}} = X_0(p_3^2) F(p_3^2)\, \Gamma_\mu^{\mathrm{BC}} \longleftarrow X_0 = 1 + \mathcal{O}(g^2) \text{ and } X_i = \mathcal{O}(g^2),\ i = 1,2,3$$

$$\begin{aligned}
\lambda_1(p_1,p_2,p_3) &= \frac{X_0(p_3^3)\,F(p_3^2)}{2}\,[A(p_1^2) + A(p_2^2)] \\
\lambda_2(p_1,p_2,p_3) &= \frac{X_0(p_3^3)\,F(p_3^2)}{2\,(p_2^2 - p_1^2)}\,[A(p_2^2) - A(p_1^2)] \\
\lambda_3(p_1,p_2,p_3) &= \frac{X_0(p_3^3)\,F(p_3^2)}{p_1^2 - p_2^2}\,[B(p_1^2) - B(p_2^2)] \\
\lambda_4(p_1,p_2,p_3) &= 0
\end{aligned}$$



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$$\begin{aligned}
\lambda_1(p_1,p_2,p_3) &= \frac{X_0(p_3^3)\,F(p_3^2)}{2}\,[A(p_1^2) + A(p_2^2)] \\
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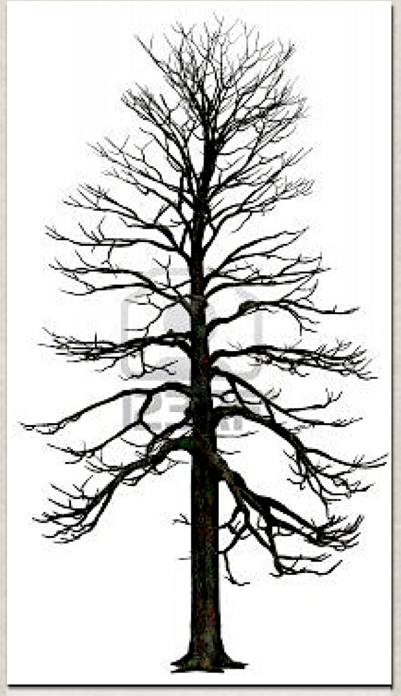
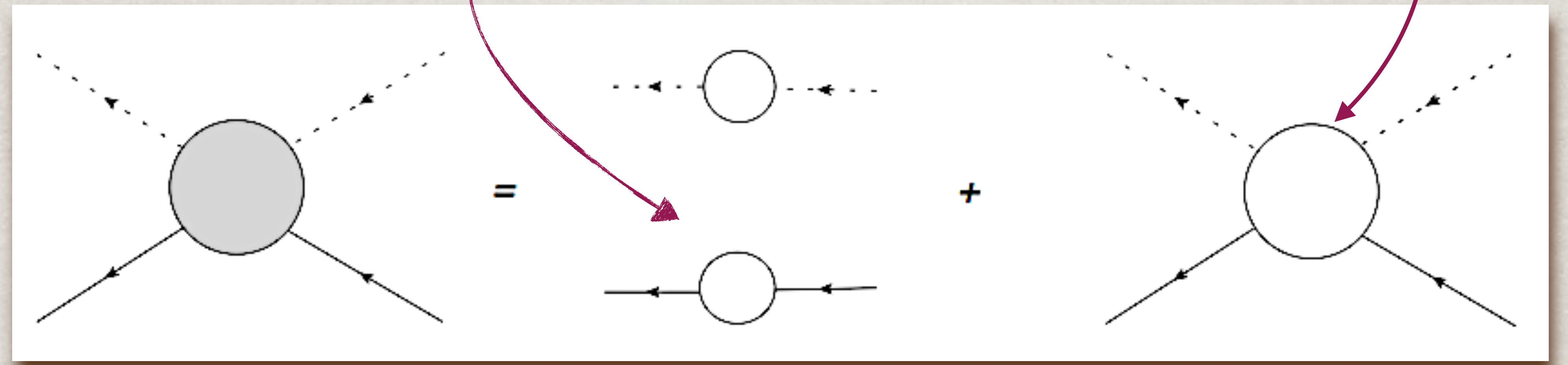
\lambda_3(p_1,p_2,p_3) &= \frac{X_0(p_3^3)\,F(p_3^2)}{p_1^2 - p_2^2}\,[B(p_1^2) - B(p_2^2)] \\
\lambda_4(p_1,p_2,p_3) &= 0
\end{aligned}$$



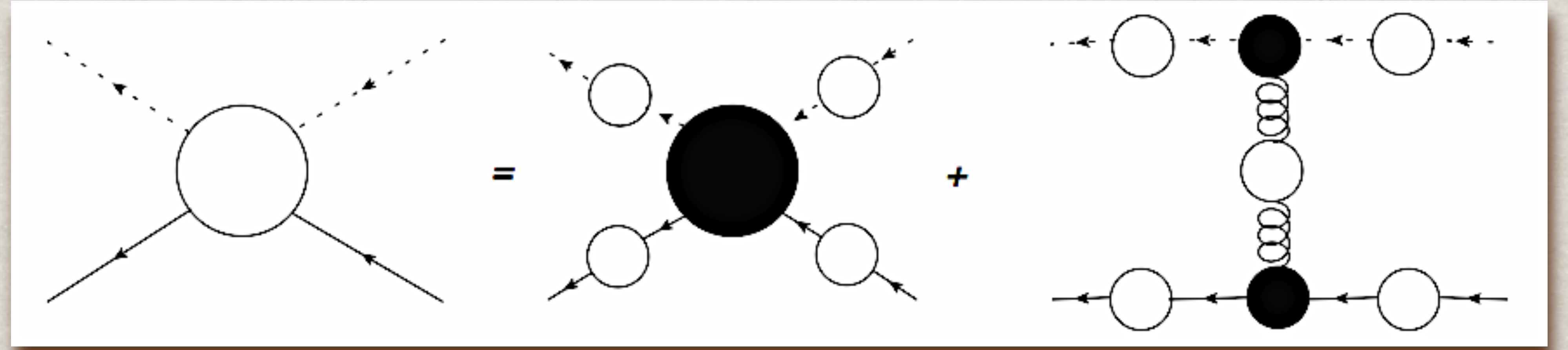
One still needs a nonperturbative model for $X_0(p_1,p_2,p_3)$!

Quark-ghost scattering

The quark-quark-ghost-ghost scattering amplitude is given by connected and a disconnected diagrams.



Connected diagrams in terms of dressed vertices:



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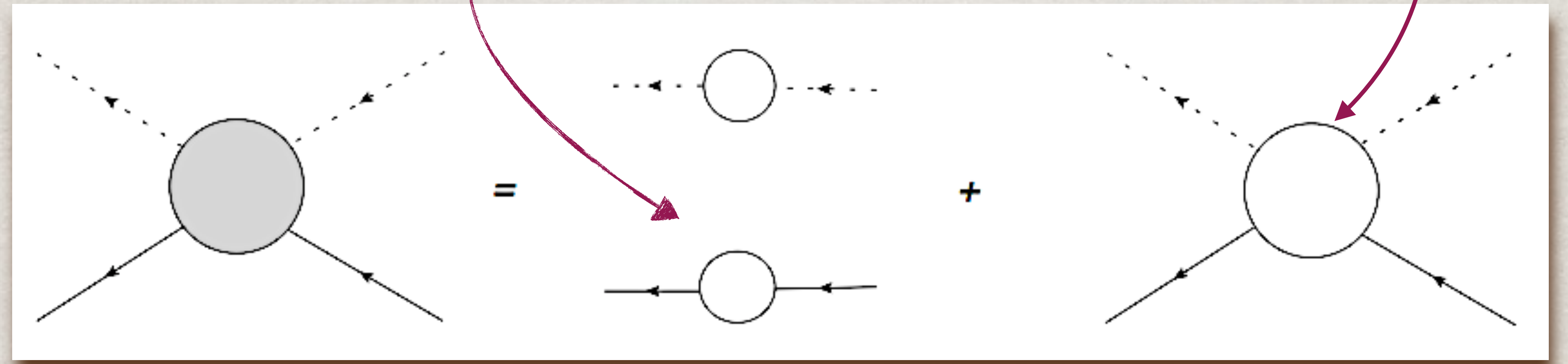

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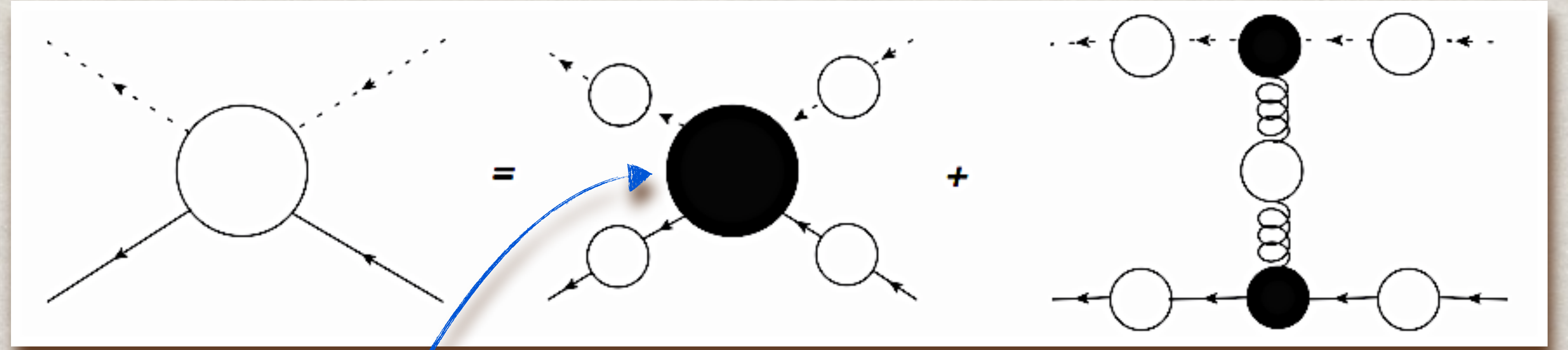


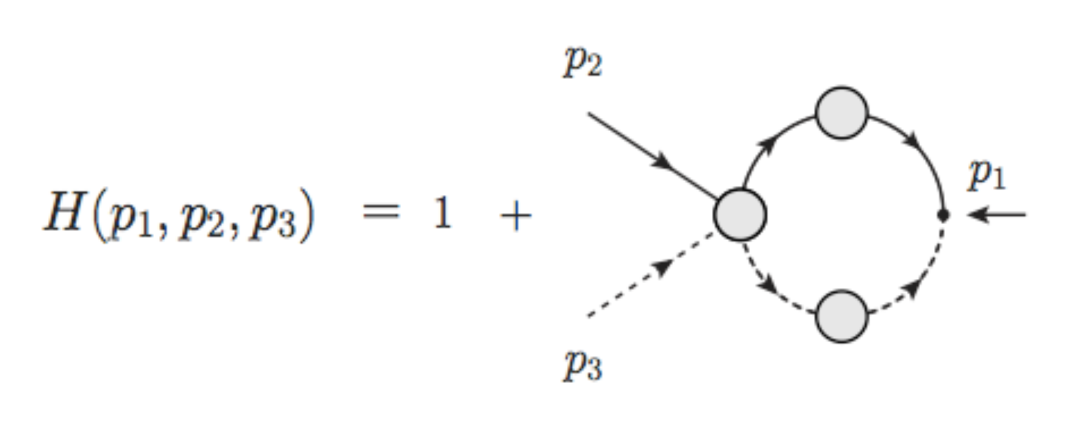
No tree-level contributions! Perturbative expansion starts with $\mathcal{O}(g^4)$ two-gluon exchange!

Quark-ghost-gluon-ghost scattering

Aguilar & Papavassiliou (2011)

Quark-ghost-gluon-ghost scattering

To obtain $H(p_1,p_2,p_3)$, an outgoing quark-ghost pair of the scattering amplitude must be contracted forming an non-standard QCD vertex.



Aguilar & Papavassiliou (2011)

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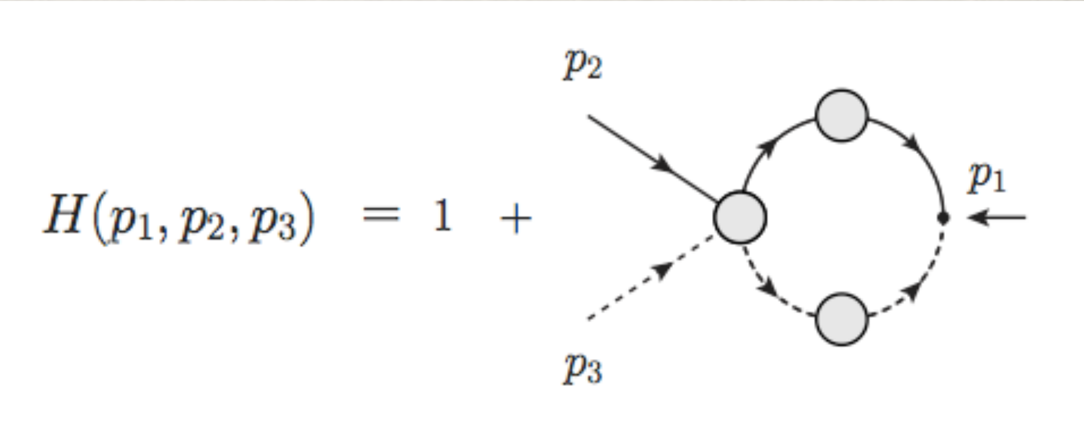
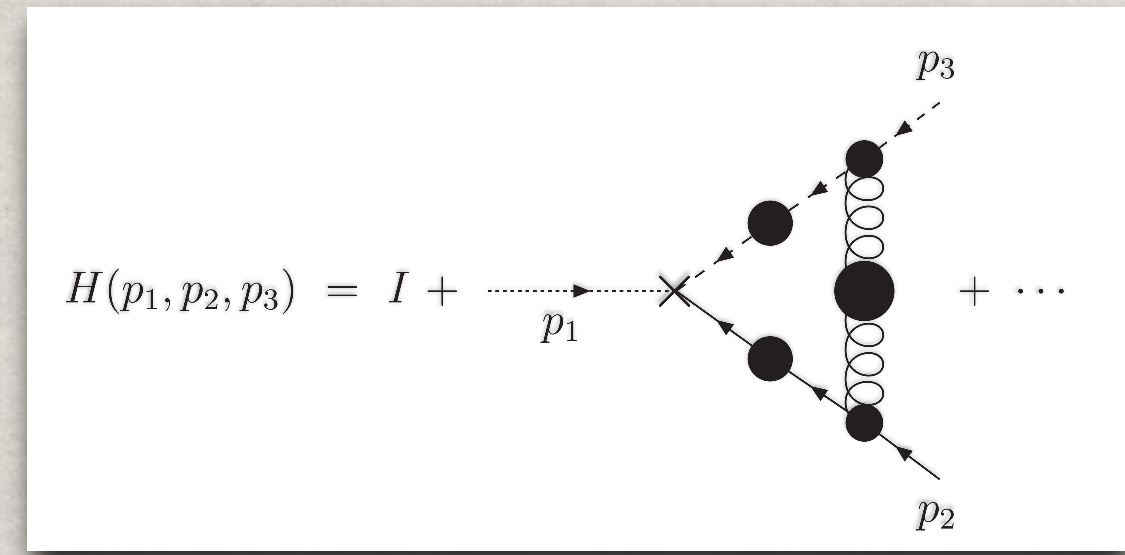




A tractable calculation of the functions $X_i(p_1,p_2,p_3)$ can be based on the dressed one-loop approximation.

$$H^{[1]}(p_1, p_2, p_3) = 1 + i\,\frac{g^2 C_F}{2}\int_k^{\Lambda} D(p_3 - k)G_\nu(p_3 - k)\Delta^{\mu\nu}(k)S(k + p_2)\Gamma_\mu(p_2, -p_2 - k, k)$$

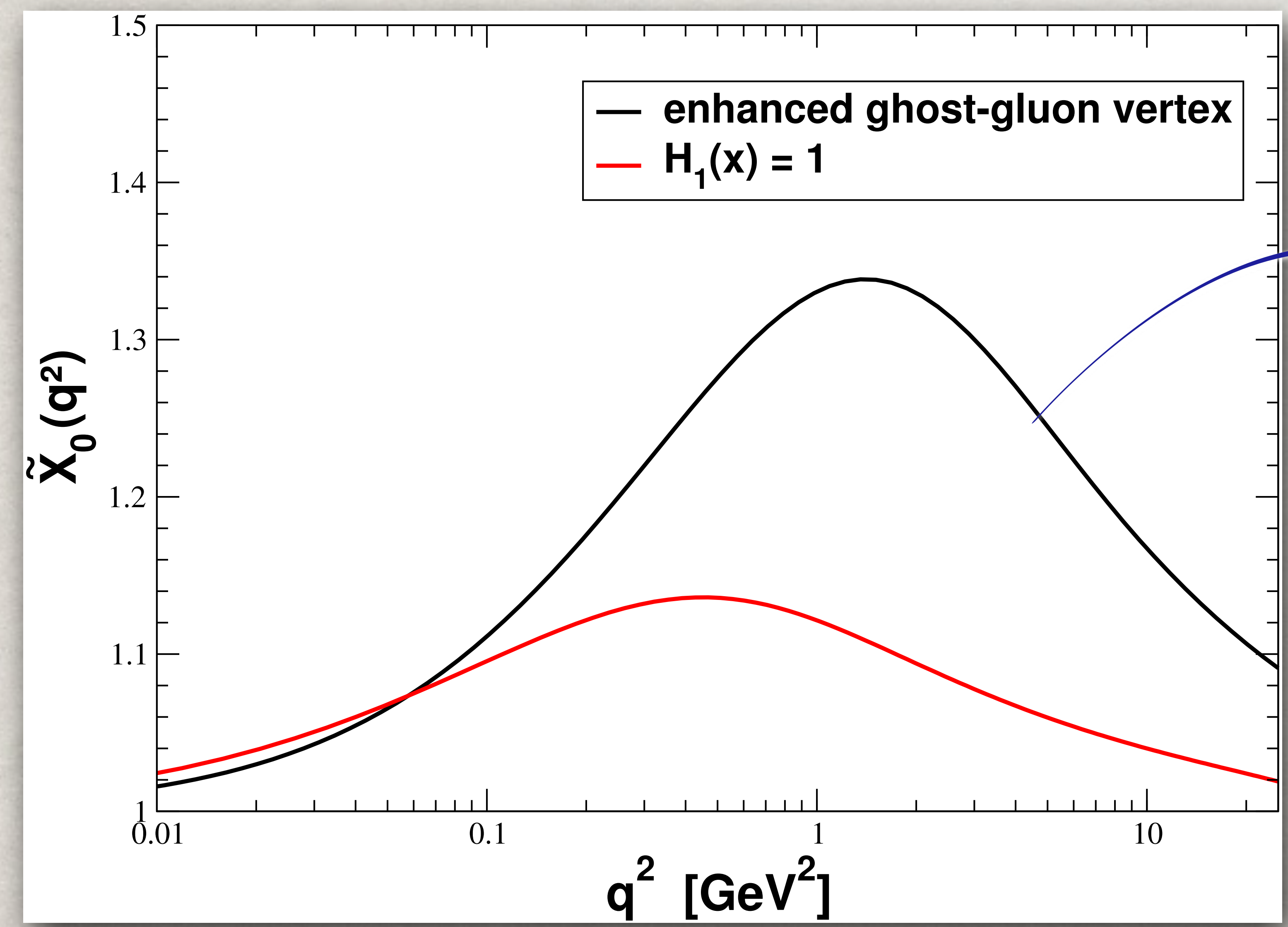
Aguilar & Papavassiliou (2011)

Quark-ghost-gluon-ghost scattering

$$
\begin{aligned}
X_0 &= \tfrac{1}{4}\,\mathrm{Tr}\,\{\mathrm{H}\} \\
X_1 &= \frac{p_2^2\,\mathrm{Tr}\,\{\gamma\cdot p_1\, H\} - (p_1\cdot p_2)\,\mathrm{Tr}\,\{\gamma\cdot p_2\, H\}}{4\,[p_1^2 p_2^2 - (p_1\cdot p_2)^2]} \\
X_2 &= \frac{p_1^2\,\mathrm{Tr}\,\{\gamma\cdot p_2\, H\} - (p_1\cdot p_2)\,\mathrm{Tr}\,\{\gamma\cdot p_1\, H\}}{4\,[p_1^2 p_2^2 - (p_1\cdot p_2)^2]} \\
X_3 &= \frac{\mathrm{Tr}\,\{\tilde{\sigma}_{\alpha\rho}\, p_1^{\alpha} p_2^{\rho}\, H\}}{4\,[(p_1\cdot p_2)^2 - p_1^2 p_2^2]}
\end{aligned}
$$

Aguilar & Papavassiliou (2011)

Result for X_0



Ghost-gluon vertex lattice-QCD parametrization

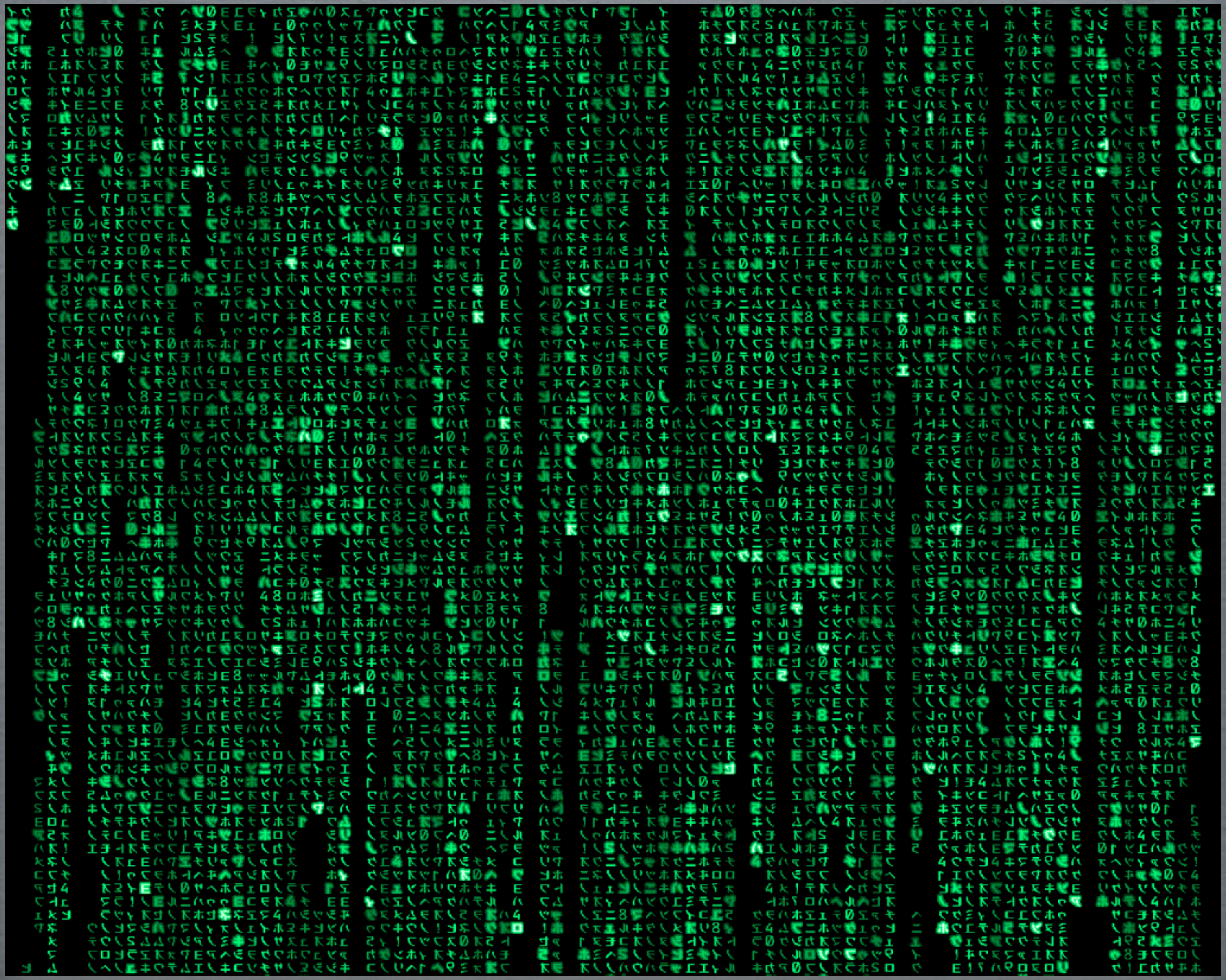
$$H_1(x) = c\left(1 + \frac{a^2x^2}{x^4+b^4}\right) + (1-c)\,\frac{w^4}{w^4+x^4}$$

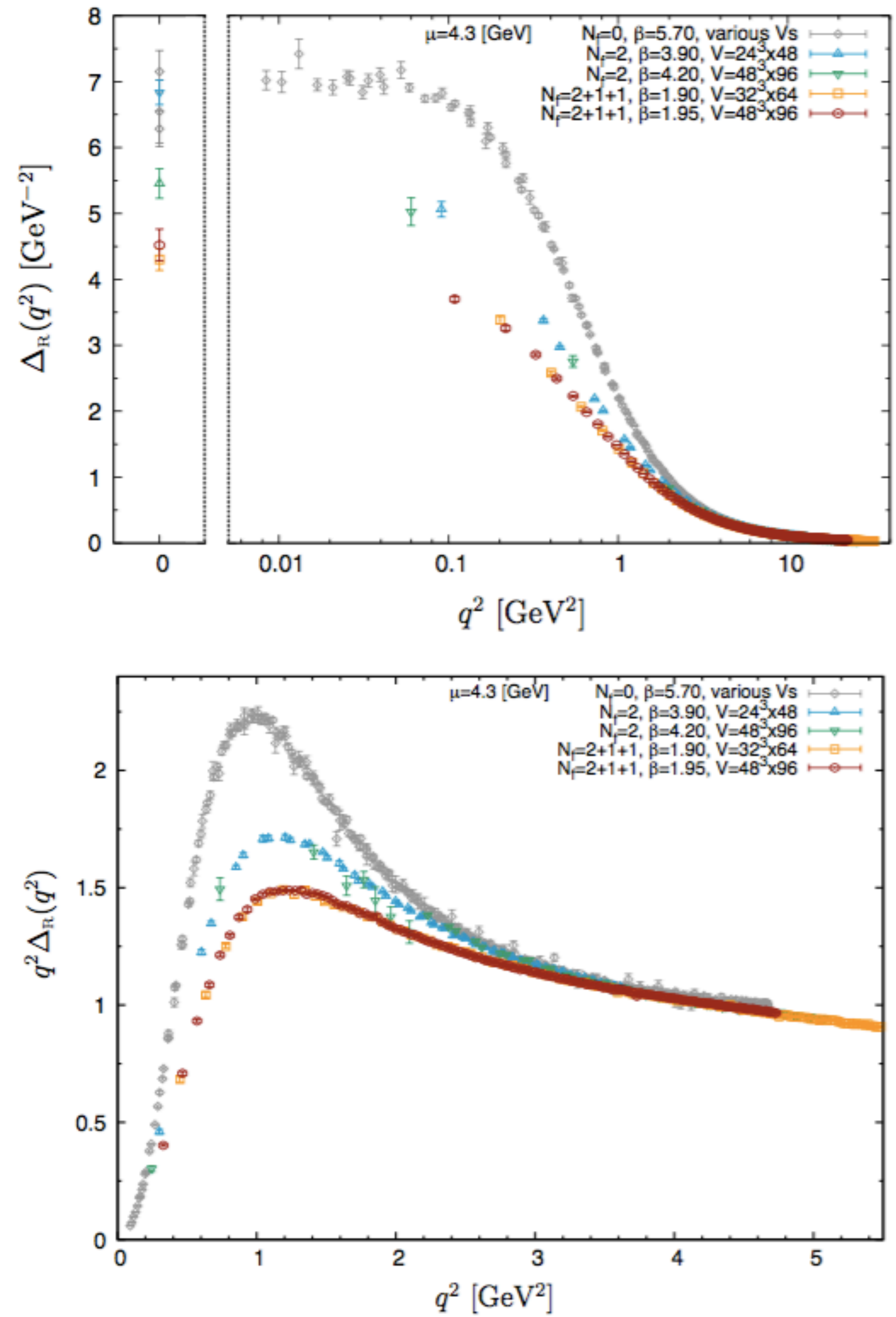
Dudal, Oliveira, Rodríguez-Quintero (2012)

Form factor contributions from X_1 , X_2 and X_3 are suppressed relative to X_0.

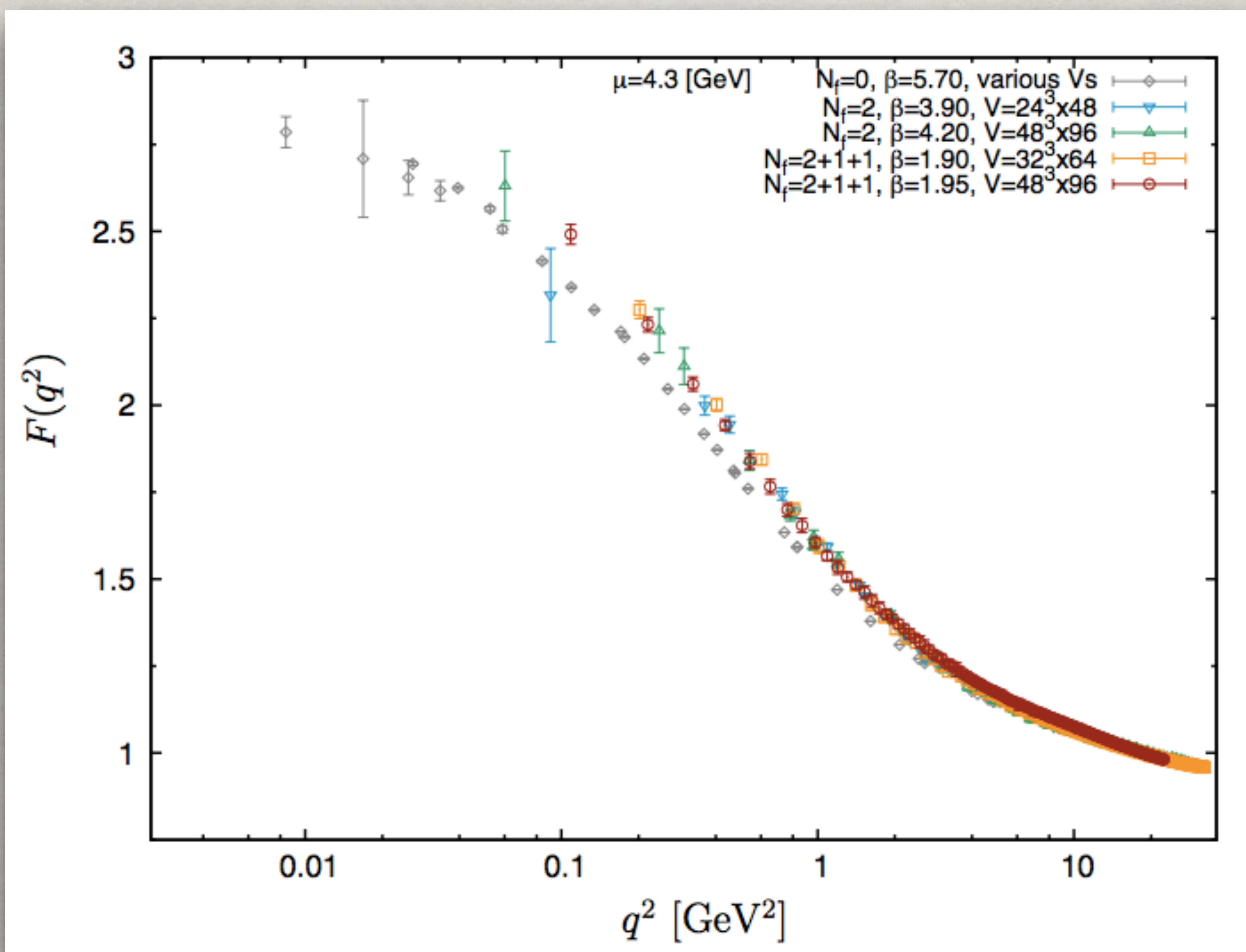
E. Rojas, J.P.B.C. de Melo, B. El-Bennich, O. Oliveira, T. Frederico (2013)

Propagators from lattice-regularized QCD





Ayala, Bashir, Binosi, Cristoforetti & Rodríguez-Quintero (2012)



Ayala, Bashir, Binosi, Cristoforetti & Rodríguez-Quintero (2012)

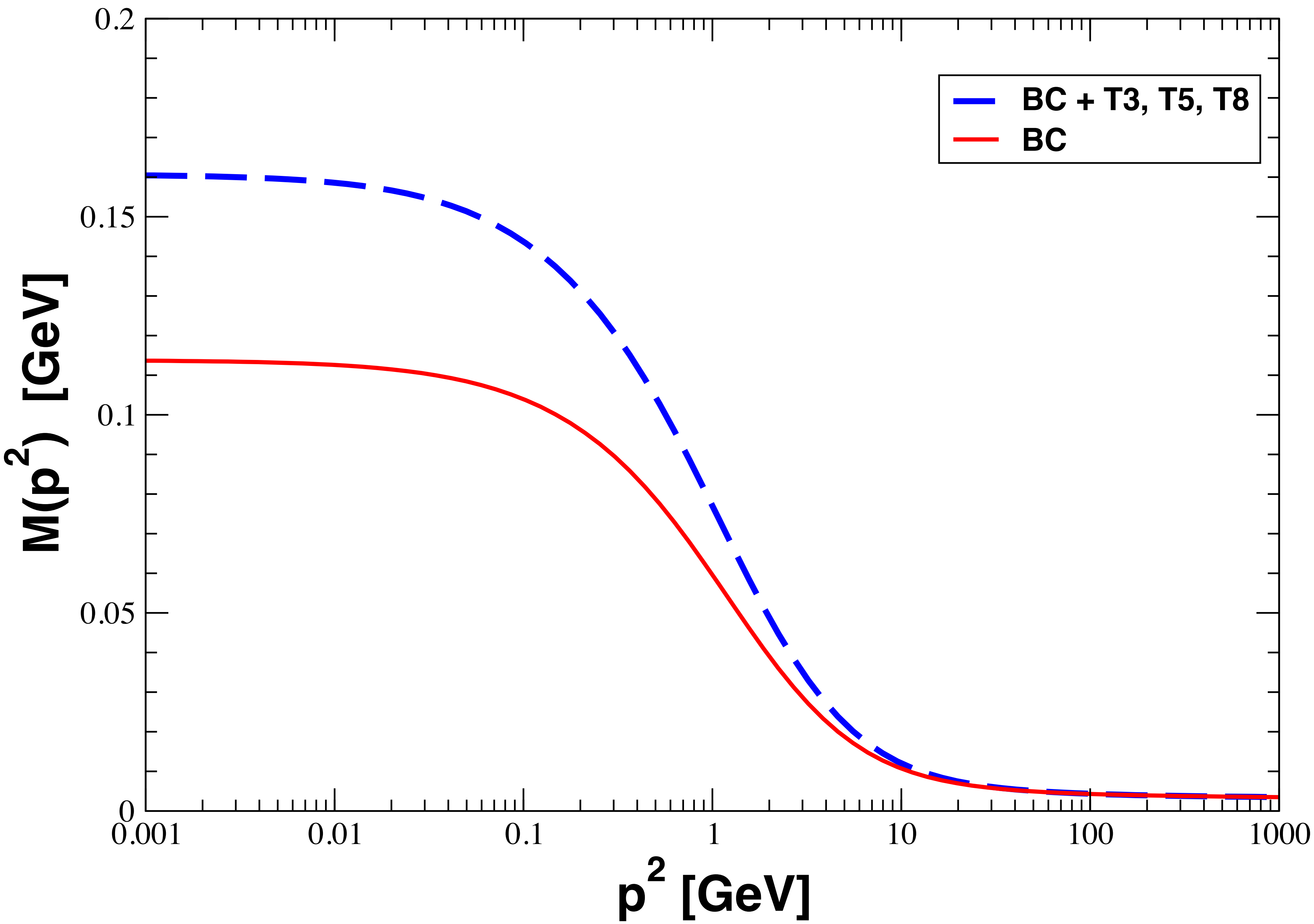
Use with STI derived quark-gluon vertex in the limit:

Use with STI derived quark-gluon vertex in the limit:

$$X_1(q^2), X_2(q^2), X_3(q^2) \to 0 \qquad \text{and} \qquad Y_1(q^2) \ \dots \ Y_8(q^2) \to 0$$

$$\begin{aligned}
\lambda_1^{\rm QCD} &= \frac{X_0(q^2)\,F(q^2)}{2}\left[A(k^2)+A(p^2)\right] \\
\lambda_2^{\rm QCD} &= \frac{X_0(q^2)\,F(q^2)}{2\,(k^2-p^2)}\left[A(k^2)-A(p^2)\right] \\
\lambda_3^{\rm QCD} &= \frac{X_0(q^2)\,F(q^2)}{k^2-p^2}\left[B(k^2)-B(p^2)\right] \\
\lambda_4^{\rm QCD} &= 0
\end{aligned}$$

$$\begin{aligned}
\tau_3^{\rm QCD} &= F(q^2)X_0(k,p)\frac{A(k^2)-A(p^2)}{2\,(k^2-p^2)} \\
\tau_5^{\rm QCD} &= -F(q^2)X_0(k,p)\frac{B(k^2)-B(p^2)}{k^2-p^2} \\
\tau_8^{\rm QCD} &= -F(q^2)X_0(k,p)\frac{A(k^2)-A(p^2)}{k^2-p^2}
\end{aligned}$$


BC + T3, T5, T8
BC
0.2
0.15
0.1
0.05
0
0.001
0.01
0.1
1
10
100
1000
M(p²) [GeV]
p² [GeV]

Concluding remarks

- We did not solve the inhomogeneous Bethe-salpeter equation for the quark-gluon vertex.
- Instead, derived a quark-gluon vertex from symmetries.
- No free parameters, only ingredients are the $N_f = 4$ gluon and ghost propagators from lattice QCD.
- Current status of DCSB not satisfying when only known terms are kept.
- Next step: use Y_i form factors from nonlocal tensors.
- If the same vertex form is employed with a phenomenological interaction, (Qin, Chang & Roberts) then DCSB is large $\Rightarrow$ $M(0) \simeq 370$ MeV.