TOWARDS A CONSISTENT NONPERTURBATIVE QUARK-GLUON VERTEX FROM QCD SYMMETRIES

QCD-TNT 4 — Unraveling the organization of the QCD tapestry Ilha Bela, São Paulo, August 31 to September 4, 2015





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Different parts of this work done in collaboration with Ishtiaq Ahmed, Eduardo Rojas and Orlando Oliveira.



Whence cometh this alarum and the noise? William Shakespeare, Henry IV, Part I

PART I WHENCE

The Lagrangian of QCD

Generates gluon self-interactions, whose consequences are quite extraordinary

The Lagrangian of QCD

Lagrangian of QCD $\mathcal{L}_{QCD} = \bar{\psi}_i (i\gamma^{\mu}(D_{\mu})_{ij} - m \,\delta_{ij}) \,\psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$ $- G = \text{gluon fields} = \bar{\psi}_i (i\gamma^{\mu}\partial_{\mu} - m)\psi_i - gG^a_{\mu}\bar{\psi}_i\gamma^{\mu}T^a_{ij}\psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a,$ $- \Psi = \text{quark fields}$ The key to complexity in QCD ... gluon field strength tensor $G^a_{\mu\nu} = \partial_{\mu}G^a_{\nu} - \partial_{\nu}G^a_{\mu} + gf^{abc}G^a_{\mu}G^b_{\nu}$

Generates gluon self-interactions, whose consequences are quite extraordinary

> This complexity also affects the bare quarkgluon vertex in a nonperturbative manner!

Nonperturbative quark-gluon vertex

$$T_{\mu}^{5}(p_{1},p_{2}) = \sigma_{\mu\nu}p_{3}^{\nu}$$

$$T_{\mu}^{4}(p_{1},p_{2}) = \sigma_{\mu\nu}(p_{1}-p_{2})\mu I_{D}$$

$$L_{\mu}^{4}(p_{1},p_{2}) = i(p_{1}-p_{2})\mu I_{D}$$

$$L_{\mu}^{4}(p_{1},p_{2}) = \sigma_{\mu\nu}(p_{1}-p_{2})^{\nu}$$

$$T_{\mu}^{5}(p_{1},p_{2}) = -\gamma_{\mu}(p_{1}^{2}-p_{2}^{2}) - (p_{1}-p_{2})_{\mu}\gamma \cdot p_{3}$$

$$T_{\mu}^{7}(p_{1},p_{2}) = \frac{i}{2}(p_{2}^{2}-p_{1}^{2})\left[\gamma_{\mu}\gamma \cdot (p_{1}-p_{2}) - (p_{1}-p_{2})_{\mu}I_{D}\right]$$

$$+ (p_{1}-p_{2})_{\mu}\sigma_{\alpha\beta}p_{1}^{\alpha}p_{2}^{\beta}$$

$$T_{\mu}^{8}(p_{1},p_{2}) = i\gamma_{\mu}\sigma_{\alpha\beta}p_{1}^{\alpha}p_{2}^{\beta} - (p_{1,\mu}\gamma \cdot p_{2}+p_{2,\mu}\gamma \cdot p_{1})$$

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QCD's Dyson-Schwinger Equations

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations:

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_{\mu} S(q) \Gamma^a_{\nu}(q,p)$$

with the running mass function $M(p^2) = B(p^2)/A(p^2)$.

each satisfies it's own DSE

 $D_{\mu\nu}$: dressed-gluon propagator

 $\Gamma^a_{\nu}(q,p)$: dressed quark-gluon vertex \checkmark

 Z_2 : quark wave function renormalization constant

 Z_1 : quark-gluon vertex renormalization constant

 $S^{-1}(p)|_{p^2=\zeta^2} = i\gamma \cdot p + m(\zeta)$ where ζ is the renormalization point.



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S

Bound State Equations for Pseudoscalars

For RL truncation one may use the Bethe-Salpeter equation (BSE):

$$\Gamma_{5\mu}(k;P) = Z_2 \gamma_5 \gamma_\mu - g^2 \int_q^{\Lambda} D^{\alpha\beta}(k-q) \frac{\lambda^a}{2} \gamma_\alpha \ S_f(q_+) \Gamma_{5\mu}(q;P) \ S_g(q_-) \frac{\lambda^a}{2} \gamma_\beta$$

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However, when using a quark-gluon vertex beyond RL, the exact BSE valid for any symmetry-preserving ansatz of this vertex must be employed:

$$\Gamma_{5\mu}^{fg}(k;P) = Z_2 \gamma_5 \gamma_\mu - g^2 \int_q^{\Lambda} D^{\alpha\beta}(k-q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q;P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_{\beta}^g(q_-,k_-)$$

$$+ g^2 \int_q^{\Lambda} D^{\alpha\beta}(k-q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k,q;P)$$
L. Chang & C.D. Roberts (2009)



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Valid for RL truncation because for $\Gamma_{\mu}(k,p) = \gamma_{\mu} \implies \Lambda_{5\mu\beta}^{fg}(k,q;P) = 0$

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+ $g^2 \int_q^{\Lambda} D^{\alpha\beta}(k-q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k,q;P)$
L. Chang & C.D. Roberts (2009)

 $\Lambda_{5\mu\beta}^{fg}(k,q;P)$ is a 4-point Schwinger function; completely defined by the dressed quark propagator.

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- Development of DSE/BSE kernels in beyond rainbow-ladder approaches with models for the quark-gluon interaction.
- Combining constraints from lattice-QCD gluon, ghost and quark propagators to study the DSE kernels and DCSB.
- To produce sufficient dynamical chiral symmetry breaking (DCSB) the transverse vertex components are mandatory.
- Using constraints from Slavnov-Taylor identities in the longitudinal AND transverse vertex tensor structure.

Nonperturbative quark-gluon vertex: restrictions

- Must satisfy Slavnov-Taylor identities.
- $\Gamma_{\mu}(k,p) \text{ must be free of kinematic}$ singularities for $k^2 \rightarrow p^2$.
- Must transform as bare vertex γ_{μ} under
 C, P and T transformations.



- Correct weak-coupling limit: must reduce to the perturbative limit when coupling is small.
- Vertex ansatz should lead to gauge independent physical observables,
 i.e. fermion mass and condensate, meson masses and decay constants.

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Nonperturbative quark-gluon vertex: tensor structure

The fermion-gauge-boson vertex can be decomposed into longitudinal and transverse components: $\Gamma_{\mu}(p_1, p_2, p_3) = \Gamma^{L}_{\mu}(p_1, p_2, p_3) + \Gamma^{T}_{\mu}(p_1, p_2, p_3)$.



$$\Gamma^{\rm L}_{\mu}(p_1, p_2, p_3) = \sum_{i=1}^{4} \lambda_i(p_1, p_2, p_3) L^i_{\mu}(p_1, p_2)$$

$$\Gamma^{\rm T}_{\mu}(p_1, p_2, p_3) = \sum_{i=1}^{8} \tau_i(p_1, p_2, p_3) T^i_{\mu}(p_1, p_2)$$

$$\Gamma_{\mu}(p_1, p_2, p_3) \big|_{p_1^2 = p_2^2 = p_3^2 = \mu^2} = \gamma_{\mu}$$
$$p_3 \cdot \Gamma^{\mathrm{T}}(p_1, p_2, p_3) = 0$$

Nonperturbative quark-gluon vertex: tensor structure

Which independent tensor structures to specify the longitudinal and transverse vertex? Following Ball and Chiu (1980), one can write:

$$\begin{split} L^{1}_{\mu}(p_{1},p_{2}) &= \gamma_{\mu} \\ L^{2}_{\mu}(p_{1},p_{2}) &= \gamma \cdot (p_{1}-p_{2}) (p_{1}-p_{2})_{\mu} \\ L^{3}_{\mu}(p_{1},p_{2}) &= i (p_{1}-p_{2})_{\mu} \mathbb{I}_{D} \\ L^{4}_{\mu}(p_{1},p_{2}) &= \sigma_{\mu\nu} (p_{1}-p_{2})^{\nu} \\ \end{split}$$

$$\begin{split} T^{1}_{\mu}(p_{1},p_{2}) &= i [p_{1\mu} (p_{2} \cdot p_{3}) - p_{2\mu} (p_{1} \cdot p_{3})] \mathbb{I}_{D} \\ T^{2}_{\mu}(p_{1},p_{2}) &= i T^{1}_{\mu} \gamma \cdot (p_{1}-p_{2}) , \\ T^{3}_{\mu}(p_{1},p_{2}) &= p^{2}_{3} \gamma_{\mu} - p_{3\mu} \gamma \cdot p_{3} := p^{2}_{3} \gamma^{T}_{\mu} \\ T^{4}_{\mu}(p_{1},p_{2}) &= -i T^{1}_{\mu}(p_{1},p_{2}) \sigma_{\alpha\beta} p^{\alpha}_{2} p^{\beta}_{1} \\ T^{5}_{\mu}(p_{1},p_{2}) &= -\gamma_{\mu} (p^{2}_{1}-p^{2}_{2}) - (p_{1}-p_{2})_{\mu} \gamma \cdot p_{3} \\ T^{7}_{\mu}(p_{1},p_{2}) &= \frac{i}{2} (p^{2}_{2}-p^{2}_{1}) \left[\gamma_{\mu} \gamma \cdot (p_{1}-p_{2}) - (p_{1}-p_{2})_{\mu} \mathbb{I}_{D} \right] \\ &+ (p_{1}-p_{2})_{\mu} \sigma_{\alpha\beta} p^{\alpha}_{1} p^{\beta}_{2} \\ T^{8}_{\mu}(p_{1},p_{2}) &= i \gamma_{\mu} \sigma_{\alpha\beta} p^{\alpha}_{1} p^{\beta}_{2} - (p_{1\mu} \gamma \cdot p_{2} + p_{2\mu} \gamma \cdot p_{1}) \end{split}$$

Nonperturbative quark-gluon vertex: Rainbow-Ladder

Since the Dyson-Schwinger Equation for QCD imply an infinite tower of non-linear integral equations, a symmetry preserving truncation scheme must be employed. The leading term in such a scheme is the Rainbow-Ladder (RL) truncation (Abelian approach).

$$K_{tu}^{rs}(q,k;P) = -4\pi\alpha(Q^2)D_{\rho\sigma}^{\text{free}}(Q) \left[\frac{\lambda^a}{2}\gamma_{\rho}\right]_{ts} \left[\frac{\lambda^a}{2}\gamma_{\sigma}\right]_{ru}$$

RL truncation satisfies vector and flavour non-singlet axial-vector Ward-Takahashi identities but has bad gauge dependence \Rightarrow Landau gauge!

Nonperturbative quark-gluon vertex: symmetries

Which steps to remedy this gauge dependence?

Clearly must be beyond the RL — bare vertex violates gauge variance:

$$iq^{\mu}\gamma_{\mu} \neq i\gamma \cdot k A(k^2) + B(k^2) - i\gamma \cdot p A(p^2) - B(p^2)$$

Best "prepared" with Landau gauge to minimize dependence.

First step is the proposal for the longitudinal vertex by Ball and Chiu:

$$\begin{split} \Gamma^{\rm L}_{\mu}(p_1, p_2, p_3) &= \sum_{i=1}^4 \lambda_i(p_1, p_2, p_3) L^i_{\mu}(p_1, p_2) \\ \lambda_1(p_1, p_2, p_3) &= \frac{1}{2} \left[A(p_1^2) + A(p_2^2) \right] \qquad \lambda_2(p_1, p_2, p_3) = \frac{1}{2(p_2^2 - p_1^2)} \left[A(p_2^2) - A(p_1^2) \right] \\ \lambda_3(p_1, p_2, p_3) &= \frac{1}{p_1^2 - p_2^2} \left[B(p_1^2) - B(p_2^2) \right] \qquad \lambda_4(p_1, p_2, p_3) = 0 \end{split}$$

Widely employed in phenomenology and satisfies WGTI; however transverse part remains undetermined — what about gauge covariance?

Nonperturbative quark-gluon vertex: ansätze

A step further is the Curtis-Pennington ansatz:

$$\Gamma_{\mu}^{\rm CP}(p_1, p_2, p_3) = \Gamma_{\mu}^{\rm BC}(p_1, p_2, p_3) + \Gamma_{\mu T}^{\rm CP}(p_1, p_2, p_3)$$

$$\Gamma_{\mu T}^{CP}(p_1, p_2, p_3) = \tau_6(p_1^2, p_2^2) T_{\mu}^6(p_1, p_2)$$

$$\tau_6(p_1^2, p_2^2) = \frac{1}{2} \frac{A(p_1^2) - A(p_2^2)}{d(p_1^2, p_2^2)}$$

$$d(p_1^2, p_2^2) = \frac{(p_1^2 - p_2^2)^2 + [\mathcal{M}^2(p_1^2) + \mathcal{M}^2(p_2^2)]^2}{p_1^2 + p_2^2}$$

Gauge dependence reduced about 50% compared to RL truncation in neighborhood of the Landau gauge (fix point) for critical coupling in QED. A. Bashir, R. Bermudez, L. Chang, C.D. Roberts (2009)

Beyond rainbow-ladder approximations

Systematic truncation scheme; extends the rainbow-ladder approximation and ensures axial-vector Ward-Takahashi identity (employs model gluon propagator but proof is independent).

$$\Gamma^g_\mu(k,p) = \gamma_\mu + \frac{1}{6} \int \frac{d^4l}{(2\pi)^4} g^2 D_{\rho\sigma}(p-l) \gamma_\rho S(l+k-p) \gamma_\mu S(l) \gamma_\sigma$$

A. Bender, L. von Smekal & C. D. Roberts, Phys. Lett. B380, 7 (1996)

Dominant non-Abelian contributions to the dressed quark-gluon vertex stemming from the gluon self-interaction have been taken into account. The corresponding Bethe-Salpeter kernel satisfies the axial-vector Ward-Takahashi identity \Rightarrow allows to investigate the influence of the gluon self-interaction in (light) mesons.

C. S. Fischer & R. Williams, Phys. Rev. Lett. 103, 122001 (2009)

Nonperturbative quark-gluon vertex: ansätze

Bashir, Bermúdez, Chang & Roberts (2012)

Guided by perturbation theory; draws on comparison with structural dependence of the Ball-Chiu vertex on $A(p^2)$ and $B(p^2)$; perturbative limit of the transverse vertex conforms with its one loop expansion in the asymptotic limit of $k^2 \gg p^2$.

$ au_1(k^2,p^2)$	=	$\frac{a_1 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \qquad \qquad k = p_1$ $p = p_2$
$ au_2(k^2,p^2)$	=	$\frac{a_2 \Delta_A(k^2, p^2)}{(k^2 + p^2)}$
$\tau_3(k^2, p^2)$	=	$a_3 \Delta_A(k^2, p^2)$
$ au_4(k^2,p^2)$	=	$\frac{a_4 \Delta_B(k^2, p^2)}{[k^2 + M^2(k^2)[p^2 + M^2(p^2)]}$
$\tau_5(k^2, p^2)$	=	$a_5 \Delta_B(k^2, p^2)$
$\tau_6(k^2, p^2)$	=	$\frac{a_6(k^2+p^2)\Delta_A(k^2,p^2)}{[(k^2-p^2)^2+(M^2(k^2)+M^2(p^2))^2]}$
$\tau_7(k^2, p^2)$	=	$\frac{a_7 \Delta_B(k^2, p^2)}{(k^2 + p^2)}$
$\tau_8(k^2, p^2)$	=	$a_8 \Delta_A(k^2, p^2)$

$$\begin{aligned} \tau_1(k^2, p^2) &= \frac{a_1 \Delta_B(k^2, p^2)}{(k^2 + p^2)} & k = p_1 \\ p = p_2 \end{aligned}$$

$$\begin{aligned} \tau_2(k^2, p^2) &= \frac{a_2 \Delta_A(k^2, p^2)}{(k^2 + p^2)} \\ \tau_3(k^2, p^2) &= a_3 \Delta_A(k^2, p^2) \end{aligned}$$

$$\begin{aligned} \tau_4(k^2, p^2) &= \frac{a_4 \Delta_B(k^2, p^2)}{[k^2 + M^2(k^2)[p^2 + M^2(p^2)]]} \\ \tau_5(k^2, p^2) &= a_5 \Delta_B(k^2, p^2) \end{aligned}$$

$$\begin{aligned} \tau_6(k^2, p^2) &= \frac{a_6(k^2 + p^2) \Delta_A(k^2, p^2)}{[(k^2 - p^2)^2 + (M^2(k^2) + M^2(p^2))^2]} \\ \tau_7(k^2, p^2) &= \frac{a_7 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\ \tau_8(k^2, p^2) &= a_8 \Delta_A(k^2, p^2) \end{aligned}$$

Bashir, Bermúdez, Chang & Roberts (2012)

Guided by perturbation theory; draws on comparison with structural dependence of the Ball-Chiu vertex on $A(p^2)$ and $B(p^2)$; perturbative limit of the transverse vertex conforms with its one loop expansion in the asymptotic limit of $k^2 \gg p^2$.

Qin, Chang, Liu, Roberts & Schmidt (2013)

Make use of longitudinal + transverse WGTI to constrain the Abelian gauge-boson-fermion vertex. Agreement of $\tau_3 \tau_5$ and τ_8 with functional form derived by Bashir et al.

Based on Abelian longitudinal and transverse vertex of Qin et al. in the background field method and all-order relation connecting this background-field vertex with the quark-gluon vertex.

A.C. Aguilar, D. Binosi, D. Ibañez & J. Papavassiliou, Phys.Rev. D90, 6 (2014)

And thence from Athens turn away our eyes to seek new friends and stranger companies?

William Shakespeare, A Midsummer Night's Dream

PART II THENCE

Comparison of top-down and bottom-up interactions



Binosi, Chang, Papavassiliou & Roberts, Phys. Lett. B742 (2015)

Abelian Ward-Takahashi identities: divergence and curl

Ward-Takahashi identity:

$$q_{\mu}i\Gamma_{\mu}(k,p) = S^{-1}(k) - S^{-1}(p)$$

Transverse Ward-Takahashi identities:

$$q_{\mu}\Gamma_{\nu}(k,p) - q_{\nu}\Gamma_{\mu}(k,p) = S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu}S^{-1}(k) + 2im\Gamma_{\mu\nu}(k,p) + t_{\lambda}\varepsilon_{\lambda\mu\nu\rho}\Gamma^{A}_{\rho}(k,p) + A^{V}_{\mu\nu}(k,p)$$

$$q_{\mu}\Gamma^{A}_{\nu}(k,p) - q_{\nu}\Gamma^{A}_{\mu}(k,p) = S^{-1}(p)\sigma^{5}_{\mu\nu} - \sigma^{5}_{\mu\nu}S^{-1}(k) + t_{\lambda}\varepsilon_{\lambda\mu\nu\rho}\Gamma_{\rho}(k,p) + V^{A}_{\mu\nu}(k,p)$$

What is the origin of these transverse identities?

What is the origin of these transverse identities?

$$\psi(x) \longrightarrow \psi'(x) = \psi(x) + ig\alpha(x)\psi(x), \ \bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x),$$

$$q_{\mu}\Gamma^{\mu}_{V}(p_{1},p_{2})=S_{F}^{-1}(p_{1})-S_{F}^{-1}(p_{2}),$$

$$\delta_T \psi(x) = rac{1}{4} g lpha(x) \epsilon^{\mu
u} \sigma_{\mu
u} \psi(x), \quad \delta_T ar{\psi}(x) = rac{1}{4} g lpha(x) \epsilon^{\mu
u} ar{\psi}(x) \sigma_{\mu
u},$$

Infinitesimal Lorentz transformation

H.-x. He Phys.Rev. D80 (2009)

$$\begin{split} & iq^{\mu}\Gamma_{V}^{\nu}(p_{1},p_{2}) - iq^{\nu}\Gamma_{V}^{\mu}(p_{1},p_{2}) \\ & = S_{F}^{-1}(p_{1})\sigma^{\mu\nu} + \sigma^{\mu\nu}S_{F}^{-1}(p_{2}) + 2m\Gamma_{T}^{\mu\nu}(p_{1},p_{2}) \\ & + (p_{1\lambda} + p_{2\lambda})\varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_{1},p_{2}) - \int \frac{d^{4}k}{(2\pi)^{4}}2k_{\lambda}\varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_{1},p_{2};k), \end{split}$$

Non-Abelian Ward-Takahashi identities: divergence and curl

Slavnov-Taylor identity:

$$q_{\mu} i \Gamma^{a}_{\mu}(k,p) = F(q^{2}) \left[S^{-1}(k) H^{a}(k,p) - \bar{H}^{a}(k,p) S^{-1}(p) \right]$$

Transverse Slavnov-Taylor identities:

$$q_{\mu}\Gamma^{a}_{\nu}(k,p) - q_{\nu}\Gamma^{a}_{\mu}(k,p) = F(q^{2}) \left[S^{-1}(p)\sigma_{\mu\nu}H^{a}(k,p) + \bar{H}^{a}(k,p)\sigma_{\mu\nu}S^{-1}(k) \right] + 2im\Gamma^{a}_{\mu\nu}(k,p) + t_{\lambda}\varepsilon_{\lambda\mu\nu\rho}\Gamma^{aA}_{\rho}(k,p) + A^{aV}_{\mu\nu}(k,p)$$

$$q_{\mu}\Gamma_{\nu}^{aA}(k,p) - q_{\nu}\Gamma_{\mu}^{aA}(k,p) = F(q^{2}) \left[S^{-1}(p)\sigma_{\mu\nu}^{5}H^{a}(k,p) - \bar{H}^{a}(k,p)\sigma_{\mu\nu}^{5}S^{-1}(k) \right] + t_{\lambda}\varepsilon_{\lambda\mu\nu\rho}\Gamma_{\rho}^{a}(k,p) + V_{\mu\nu}^{aA}(k,p)$$

Slavnov-Taylor Identity

One can relate the longitudinal form factors λ_i to the quark propagator's scalar and vector pieces, $B(p^2)$ and $A(p^2)$ via an STI:

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Ghost dressing function
Quark-ghost scattering kernel

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Ghost dressing function
Quark-ghost scattering kernel

Decomposition of H(k,p) and its conjugate in terms of Lorentz covariants:

$$H(p_1, p_2, p_3) = X_0 \mathbb{I}_D + i X_1 \gamma \cdot p_1 + i X_2 \gamma \cdot p_2 + i X_3 \sigma_{\alpha\beta} p_1^{\alpha} p_2^{\beta}$$

$$\overline{H}(p_2, p_1, p_3) = \overline{X}_0 \mathbb{I}_D - i \overline{X}_2 \gamma \cdot p_1 - i \overline{X}_1 \gamma \cdot p_2 + i \overline{X}_3 \sigma_{\alpha\beta} p_1^{\alpha} p_2^{\beta}$$

$$X_i(p, k, q) = \overline{X}_i(k, p, q) \qquad \qquad X_i \equiv X_i(p_1, p_2, p_3)$$

Davydychev, Osland & Saks (2001)

Since the transverse part of the quark-gluon vertex does not contribute to the "longitudinal" STI, the longitudinal form factors λ_i may be written in terms of A(p²), B(p²), F(p²) and the quark-ghost scattering functions X_i.

$$\begin{split} \lambda_{1}^{\text{QCD}} &= \frac{F(q^{2})}{2} \Big\{ A(k^{2}) \left[X_{0} + \left(k^{2} - k \cdot p\right) X_{3} \right] + A(p^{2}) \left[\overline{X}_{0} + \left(p^{2} - k \cdot p\right) \overline{X}_{3} \right] \\ &+ B(k^{2}) \left[X_{1} + X_{2} \right] + B(p^{2}) \left[\overline{X}_{1} + \overline{X}_{2} \right] \Big\} , \\ \lambda_{2}^{\text{QCD}} &= \frac{F(q^{2})}{2(p^{2} - k^{2})} \Big\{ A(k^{2}) \left[\left(k^{2} + k \cdot p\right) X_{3} - X_{0} \right] + A(p^{2}) \left[\overline{X}_{0} - \left(p^{2} + k \cdot p\right) \overline{X}_{3} \right] \\ &+ B(k^{2}) \left[X_{2} - X_{1} \right] + B(p^{2}) \left[\overline{X}_{1} - \overline{X}_{2} \right] \Big\} , \\ \lambda_{3}^{\text{QCD}} &= \frac{F(q^{2})}{k^{2} - p^{2}} \Big\{ A(k^{2}) \left[k^{2} X_{1} + k \cdot p X_{2} \right] - A(p^{2}) \left[p^{2} \overline{X}_{1} + k \cdot p \overline{X}_{2} \right] \\ &+ B(k^{2}) X_{0} - B(p^{2}) \overline{X}_{0} \Big\} , \\ \lambda_{4}^{\text{QCD}} &= \frac{-F(q^{2})}{2} \Big\{ A(k^{2}) X_{2} - A(p^{2}) \overline{X}_{2} + B(k^{2}) X_{3} - B(p^{2}) \overline{X}_{3} \Big\} \end{split}$$

Aguilar & Papavassiliou (2011)

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 $X_1(q^2), X_2(q^2), X_3(q^2) \to 0$

$$\begin{split} \lambda_{1}^{\text{QCD}} &= \frac{X_{0}(q^{2}) F(q^{2})}{2} \left[A(k^{2}) + A(p^{2}) \right] \\ \lambda_{2}^{\text{QCD}} &= \frac{X_{0}(q^{2}) F(q^{2})}{2 (k^{2} - p^{2})} \left[A(k^{2}) - A(p^{2}) \right] \\ \lambda_{3}^{\text{QCD}} &= \frac{X_{0}(q^{2}) F(q^{2})}{k^{2} - p^{2}} \left[B(k^{2}) - B(p^{2}) \right] \\ \lambda_{4}^{\text{QCD}} &= 0 \end{split}$$

Aguilar & Papavassiliou (2011)

Decoupling the transverse STIs

Consider the transverse STI which involves the axialvector vertex:

$$q_{\mu}\Gamma_{\nu}^{aA}(k,p) - q_{\nu}\Gamma_{\mu}^{aA}(k,p) = F(q^{2}) \left[S^{-1}(p)\sigma_{\mu\nu}^{5}H^{a}(k,p) - \bar{H}^{a}(k,p)\sigma_{\mu\nu}^{5}S^{-1}(k) \right] + t_{\lambda}\varepsilon_{\lambda\mu\nu\rho}\Gamma_{\rho}^{a}(k,p) + V_{\mu\nu}^{aA}(k,p)$$

Contract it with the two tensors:

$$T^{1}_{\mu\nu} = \frac{1}{2} \varepsilon_{\alpha\mu\nu\beta} t_{\alpha} q_{\beta} \mathbf{I}_{\mathrm{D}} \qquad t = k + p$$
$$T^{2}_{\mu\nu} = \frac{1}{2} \varepsilon_{\alpha\mu\nu\beta} \gamma_{\alpha} q_{\beta}$$

and use the relations:

$$T^{1}_{\mu\nu} t_{\lambda} \varepsilon_{\lambda\mu\nu\rho} \Gamma_{\rho}(k,p) = t^{2} q \cdot \Gamma(k,p) - q \cdot t t \cdot \Gamma(k,p)$$

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H.-x. He (2009); S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

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$$\begin{aligned} q \cdot t \, t \cdot \Gamma^{a}(k,p) &= T^{1}_{\mu\nu} \, F(q^{2}) \left[S^{-1}(p) \sigma^{5}_{\mu\nu} H^{a}(k,p) - \bar{H}^{a}(k,p) \sigma^{5}_{\mu\nu} S^{-1}(k) \right] \\ &+ t^{2} \, q \cdot \Gamma^{a}(k,p) + T^{1}_{\mu\nu} V^{aA}_{\mu\nu}(k,p), \\ q \cdot t \, \gamma \cdot \Gamma^{a}(k,p) &= T^{2}_{\mu\nu} \, F(q^{2}) \left[S^{-1}(p) \sigma^{5}_{\mu\nu} H^{a}(k,p) - \bar{H}^{a}(k,p) \sigma^{5}_{\mu\nu} S^{-1}(k) \right] \\ &+ \gamma \cdot t \, q \cdot \Gamma^{a}(k,p) + T^{2}_{\mu\nu} V^{aA}_{\mu\nu}(k,p). \end{aligned}$$

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These two identities involve ONLY the vector vertex!

$$\begin{aligned} q \cdot t \, t \cdot \Gamma^{a}(k,p) &= T^{1}_{\mu\nu} F(q^{2}) \left[S^{-1}(p) \sigma^{5}_{\mu\nu} H^{a}(k,p) - \bar{H}^{a}(k,p) \sigma^{5}_{\mu\nu} S^{-1}(k) \right] \\ &+ t^{2} \, q \cdot \Gamma^{a}(k,p) + T^{1}_{\mu\nu} V^{aA}_{\mu\nu}(k,p), \\ q \cdot t \, \gamma \cdot \Gamma^{a}(k,p) &= T^{2}_{\mu\nu} F(q^{2}) \left[S^{-1}(p) \sigma^{5}_{\mu\nu} H^{a}(k,p) - \bar{H}^{a}(k,p) \sigma^{5}_{\mu\nu} S^{-1}(k) \right] \\ &+ \gamma \cdot t \, q \cdot \Gamma^{a}(k,p) + T^{2}_{\mu\nu} V^{aA}_{\mu\nu}(k,p). \end{aligned}$$

The unfamiliar complicated components in these identities can simply be decomposed:

 $iT^{1}_{\mu\nu}V^{aA}_{\mu\nu}(k,p) = \mathbf{I}_{\mathrm{D}}Y_{1}(k,p) + \gamma \cdot q Y_{2}(k,p) + \gamma \cdot t Y_{3}(k,p) + [\gamma \cdot q,\gamma \cdot t]Y_{4}(k,p)$ $iT^{2}_{\mu\nu}V^{aA}_{\mu\nu}(k,p) = \mathbf{I}_{\mathrm{D}}Y_{5}(k,p) + \gamma \cdot q Y_{6}(k,p) + \gamma \cdot t Y_{7}(k,p) + [\gamma \cdot q,\gamma \cdot t]Y_{8}(k,p)$

S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

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S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)

Not known for now until some *Ansatz/Model* for them is found:

$$V^{aA}_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \, 2k_\lambda \, \varepsilon_{\lambda\mu\nu\rho} \, \Gamma^a_\rho(k,p)$$

Transverse form factors of the quark-gluon vertex from transverse STIs

$$\begin{split} \tau_3^{\rm QCD} &= F(q^2) X_0(k,p) \frac{A(k^2) - A(p^2)}{2(k^2 - p^2)} + \frac{2Y_2 \left(k^2 - p^2\right) - (Y_3 - Y_5)(k+p)^2}{8(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}, \\ \tau_5^{\rm QCD} &= -F(q^2) X_0(k,p) \frac{B(k^2) - B(p^2)}{k^2 - p^2} - \frac{Y_1}{8(k^2 p^2 - (k \cdot p)^2)} - \frac{2Y_4 + Y_6}{2(k^2 - p^2)}, \\ \tau_8^{\rm QCD} &= -F(q^2) X_0(k,p) \frac{A(k^2) - A(p^2)}{k^2 - p^2} - \frac{2Y_8}{k^2 - p^2} - \frac{(k-p)^2 Y_2 + (k^2 - p^2) Y_3}{4(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)} \end{split}$$

$$\begin{split} \tau_1^{\rm QCD} &= -\frac{Y_1}{2\,(k^2 - p^2)\,(k^2p^2 - (k \cdot p)^2)} \\ \tau_2^{QCD} &= -\frac{Y_5 - 3Y_3}{4\,(k^2 - p^2)\,(k^2p^2 - (k \cdot p)^2)}, \\ \tau_4^{QCD} &= \frac{Y_1 - (6Y_4 + Y_6)\,(k^2 - p^2) - Y_7\,(k + p)^2}{2\,(k^2 - p^2)^2\,(k^2p^2 - (k \cdot p)^2)}, \\ \tau_6^{QCD} &= \frac{2Y_2\,(k - p)^2 - (Y_3 - Y_5)(k^2 - p^2)}{8\,(k^2 - p^2)\,(k^2p^2 - (k \cdot p)^2)}, \\ \tau_7^{QCD} &= \frac{Y_1\,(k - p)^2 - 4Y_7(k^2p^2 - (k \cdot p)^2)}{4\,(k^2 - p^2)\,(k^2p^2 - (k \cdot p)^2)} \end{split}$$

Quark-ghost and ghost-gluon scattering



The functions X_i are known in pQCD at one-loop order, see Davydychev, Osland & Saks (2001). Nonperturbative approach to quark-ghost scattering kernel by Aguilar & Papavassiliou (2011) with the approximation $X_{1,2,3} = 0$. This yields a "ghost-improved" Ball-Chiu vertex.

$$\tilde{\Gamma}^{BC}_{\mu} = X_0(p_3^2) F(p_3^2) \Gamma^{BC}_{\mu}$$
 \checkmark $X_0 = 1 + \mathcal{O}(g^2) \text{ and } X_i = \mathcal{O}(g^2), i = 1, 2, 3$

$$\lambda_{1}(p_{1}, p_{2}, p_{3}) = \frac{X_{0}(p_{3}^{3}) F(p_{3}^{2})}{2} \left[A(p_{1}^{2}) + A(p_{2}^{2})\right]$$

$$\lambda_{2}(p_{1}, p_{2}, p_{3}) = \frac{X_{0}(p_{3}^{3}) F(p_{3}^{2})}{2 (p_{2}^{2} - p_{1}^{2})} \left[A(p_{2}^{2}) - A(p_{1}^{2})\right]$$

$$\lambda_{3}(p_{1}, p_{2}, p_{3}) = \frac{X_{0}(p_{3}^{3}) F(p_{3}^{2})}{p_{1}^{2} - p_{2}^{2}} \left[B(p_{1}^{2}) - B(p_{2}^{2})\right]$$

$$\lambda_{4}(p_{1}, p_{2}, p_{3}) = 0$$



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$$\begin{aligned} \lambda_1(p_1, p_2, p_3) &= \frac{X_0(p_3^3) F(p_3^2)}{2} \left[A(p_1^2) + A(p_2^2) \right] \\ \lambda_2(p_1, p_2, p_3) &= \frac{X_0(p_3^3) F(p_3^2)}{2 (p_2^2 - p_1^2)} \left[A(p_2^2) - A(p_1^2) \right] \\ \lambda_3(p_1, p_2, p_3) &= \frac{X_0(p_3^3) F(p_3^2)}{p_1^2 - p_2^2} \left[B(p_1^2) - B(p_2^2) \right] \\ \lambda_4(p_1, p_2, p_3) &= 0 \end{aligned}$$



One still needs a nonperturbative model for $X_0(p_1, p_2, p_3)$!

The quark-quark-ghost-ghost scattering amplitude is given by connected and a disconnected diagrams.



Connected diagrams in terms of dressed vertices:



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Connected diagrams in terms of dressed vertices:



No tree-level contributions! Perturbative expansion starts with $\mathcal{O}(g^4)$ two-gluon exchange!

To obtain $H(p_1,p_2,p_3)$, an outgoing quark-ghost pair of the scattering amplitude must be contracted forming an non-standard QCD vertex.



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A tractable calculation of the functions $X_1(p_1,p_2,p_3)$ can be based on the dressed one-loop approximation.

$$H^{[1]}(p_1, p_2, p_3) = 1 + i \frac{g^2 C_F}{2} \int_k^{\Lambda} D(p_3 - k) G_\nu(p_3 - k) \Delta^{\mu\nu}(k) S(k + p_2) \Gamma_\mu(p_2, -p_2 - k, k)$$

Aguilar & Papavassiliou (2011)

$$X_{0} = \frac{1}{4} \operatorname{Tr} \{H\}$$

$$X_{1} = \frac{p_{2}^{2} \operatorname{Tr} \{\gamma \cdot p_{1} H\} - (p_{1} \cdot p_{2}) \operatorname{Tr} \{\gamma \cdot p_{2} H\}}{4 [p_{1}^{2} p_{2}^{2} - (p_{1} \cdot p_{2})^{2}]}$$

$$X_{2} = \frac{p_{1}^{2} \operatorname{Tr} \{\gamma \cdot p_{2} H\} - (p_{1} \cdot p_{2}) \operatorname{Tr} \{\gamma \cdot p_{1} H\}}{4 [p_{1}^{2} p_{2}^{2} - (p_{1} \cdot p_{2})^{2}]}$$

$$X_{3} = \frac{\operatorname{Tr} \{\tilde{\sigma}_{\alpha\rho} p_{1}^{\alpha} p_{2}^{\rho} H\}}{4 [(p_{1} \cdot p_{2})^{2} - p_{1}^{2} p_{2}^{2}]}$$

Aguilar & Papavassiliou (2011)

Result for X₀



Form factor contributions from X_1 , X_2 and X_3 are suppressed relative to X_0 .

E. Rojas, J.P.B.C. de Melo, B. El-Bennich, O. Oliveira, T. Frederico (2013)

Propagators from Lattice-regularized QCD

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Ayala, Bashir, Binosi, Cristoforetti & Rodríguez-Quintero (2012)



Ayala, Bashir, Binosi, Cristoforetti & Rodríguez-Quintero (2012)

Use with STI derived quark-gluon vertex in the limit:

Use with STI derived quark-gluon vertex in the limit:

 $X_1(q^2), X_2(q^2), X_3(q^2) \to 0$ and $Y_1(q^2) \dots Y_8(q^2) \to 0$

$$\begin{split} \lambda_1^{\text{QCD}} &= \frac{X_0(q^2) F(q^2)}{2} \left[A(k^2) + A(p^2) \right] \\ \lambda_2^{\text{QCD}} &= \frac{X_0(q^2) F(q^2)}{2 (k^2 - p^2)} \left[A(k^2) - A(p^2) \right] \\ \lambda_3^{\text{QCD}} &= \frac{X_0(q^2) F(q^2)}{k^2 - p^2} \left[B(k^2) - B(p^2) \right] \\ \lambda_4^{\text{QCD}} &= 0 \end{split}$$

$$\tau_3^{\text{QCD}} = F(q^2)X_0(k,p)\frac{A(k^2) - A(p^2)}{2(k^2 - p^2)}$$

$$\tau_5^{\text{QCD}} = -F(q^2)X_0(k,p)\frac{B(k^2) - B(p^2)}{k^2 - p^2}$$

$$\tau_8^{\text{QCD}} = -F(q^2)X_0(k,p)\frac{A(k^2) - A(p^2)}{k^2 - p^2}$$

36



Concluding remarks

- We did not solve the inhomogeneous Bethe-salpeter equation for the quark-gluon vertex.
- Instead, derived a quark-gluon vertex from symmetries.
- No free parameters, only ingredients are the N_f = 4 gluon and ghost propagators from lattice QCD.
- Current status of DCSB not satisfying when only known terms are kept.
- Next step: use Y_i form factors from nonlocal tensors.
- If the same vertex form is employed with a phenomenological interaction, (Qin, Chang & Roberts) then DCSB is large \Rightarrow M(0) \approx 370 MeV.