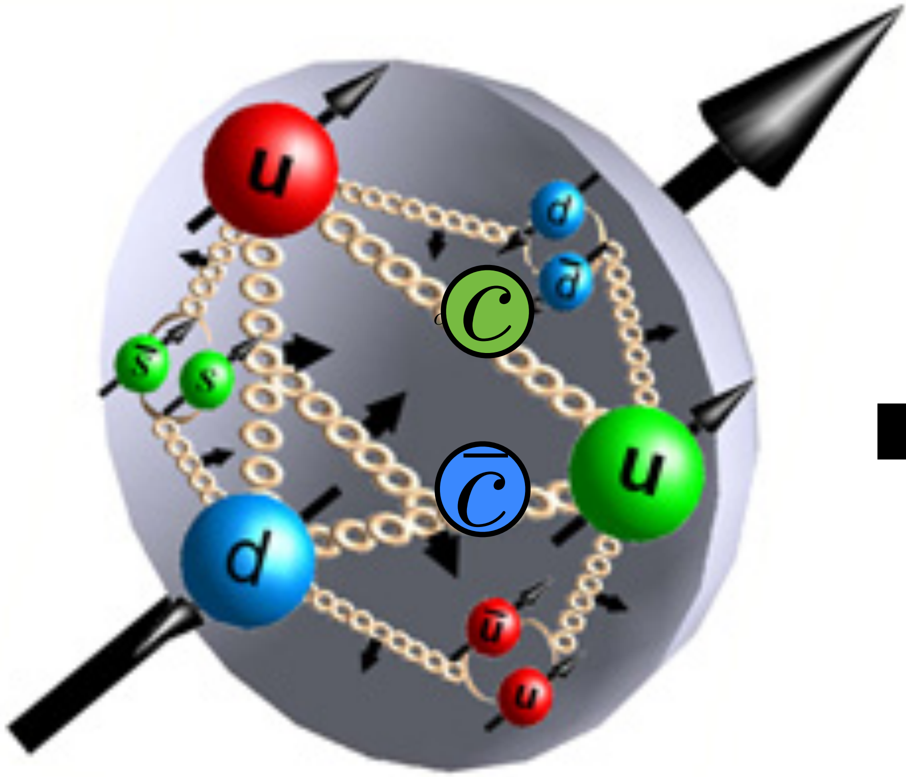
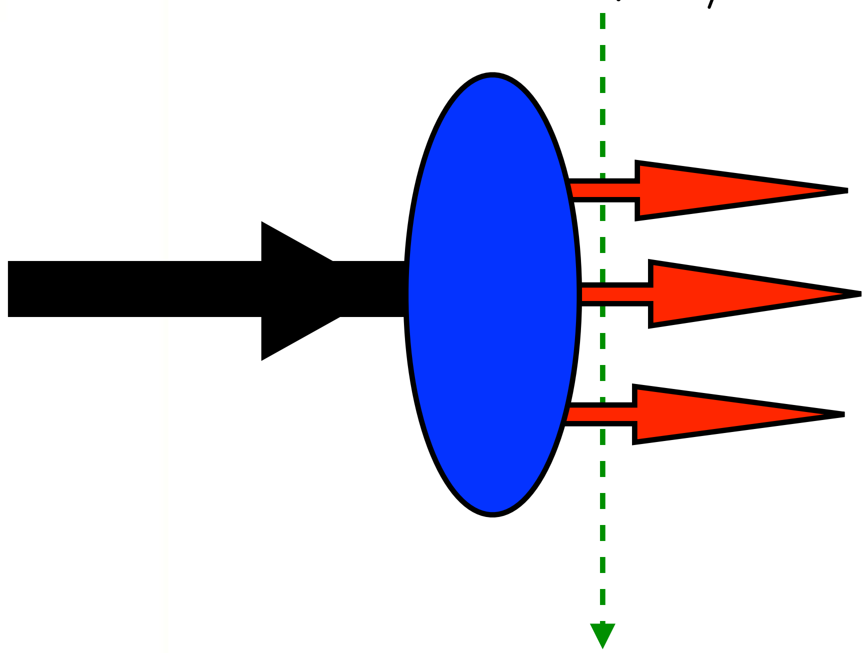
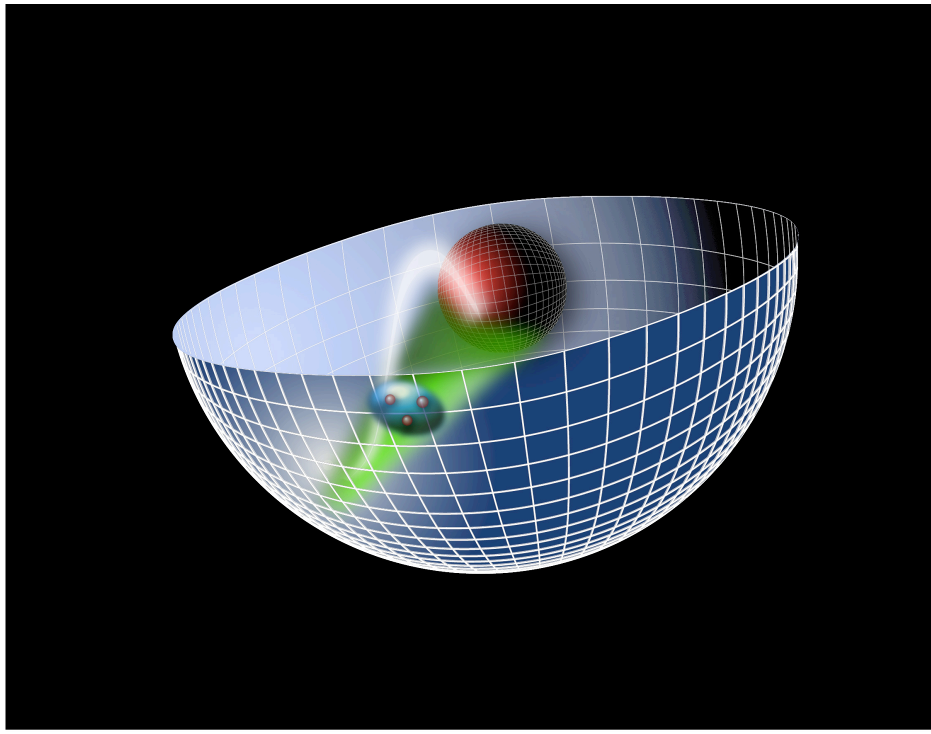
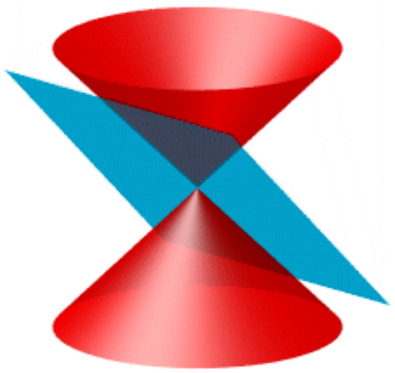
Light-Front Holographic QCD, Color Confinement, and Supersymmetric Features of QCD









Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

QCD-TNT4 *Unraveling the Organization of the Tapestry of QCD*

IhaBela, São Paulo, Brazil - August 31 to September 04, 2015

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

quark mass term

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if m_q=0

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

QCD Mass Scale from Confinement not Explicit

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for m_ρ.

$$m_p \simeq 3.21 \, \Lambda_{\overline{MS}}$$

$$m_\rho \simeq 2.2 \, \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving m_ρ / m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

$$(m_q = 0)$$

$$m_\pi = 0$$

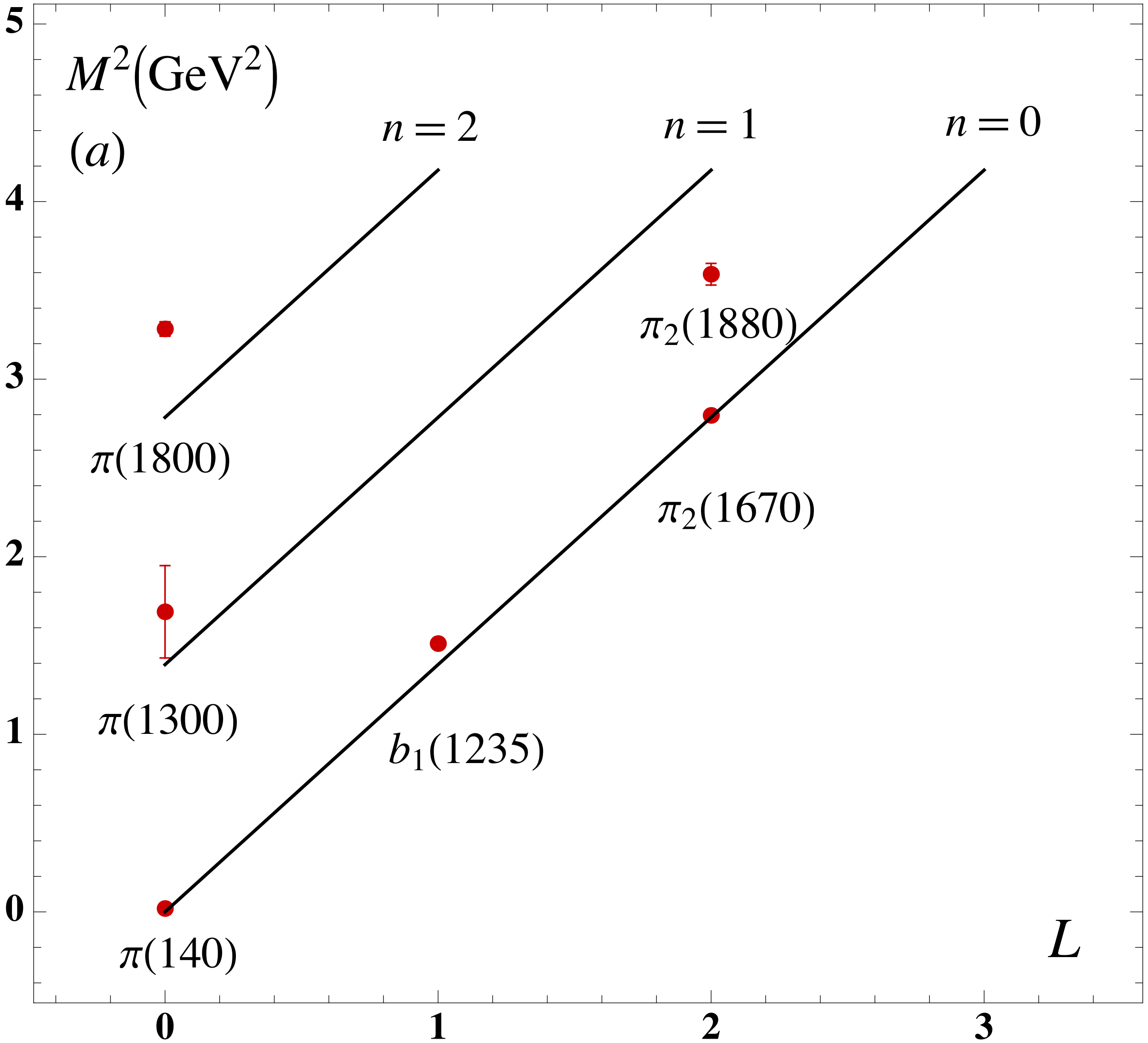
$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

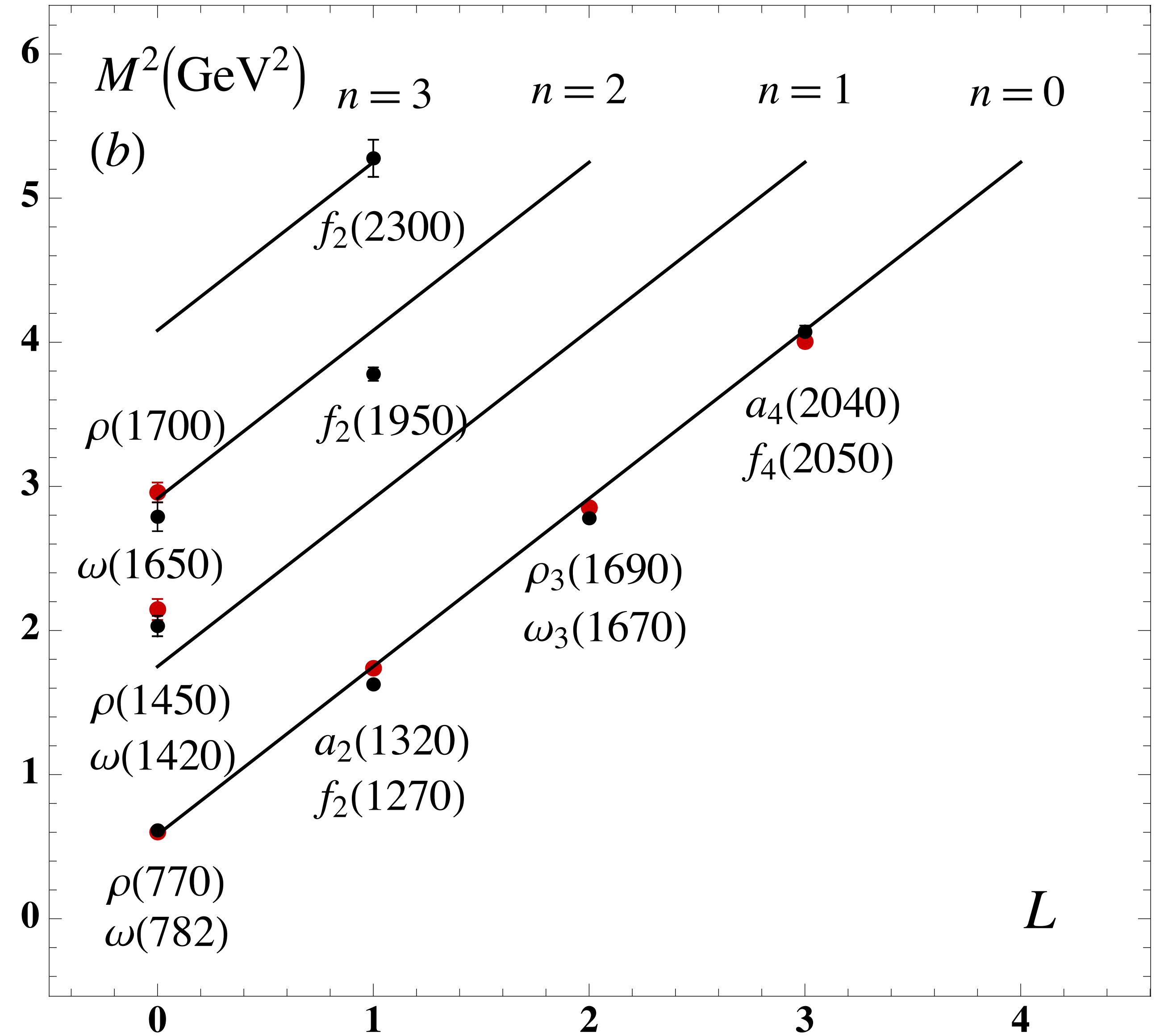
$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

$$m_u = m_d = 0$$

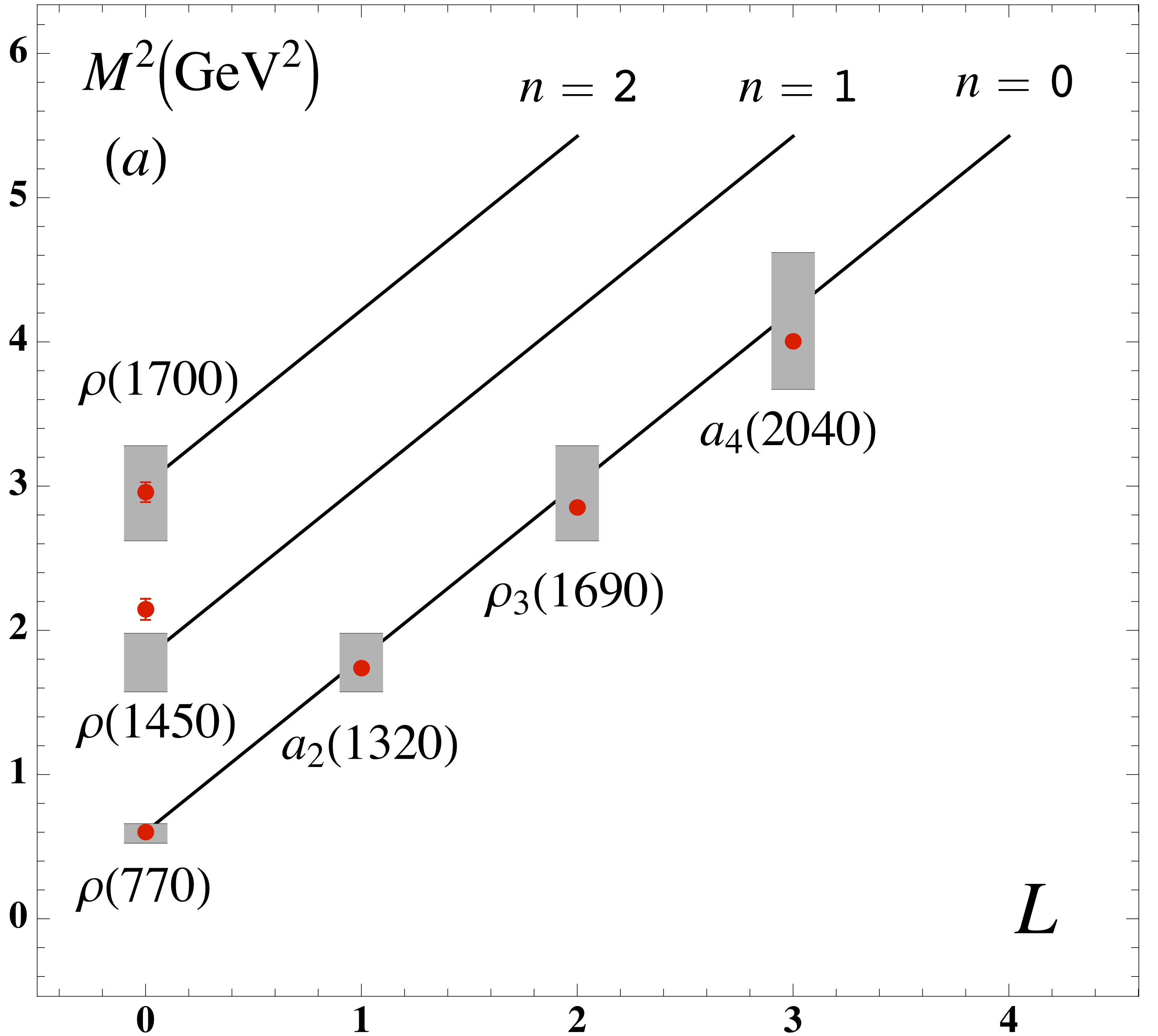
Preview

de Tèramond, Dosch, sjb





$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$



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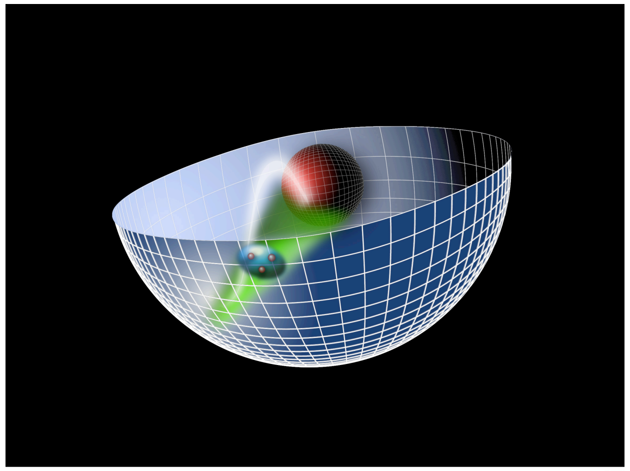
de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

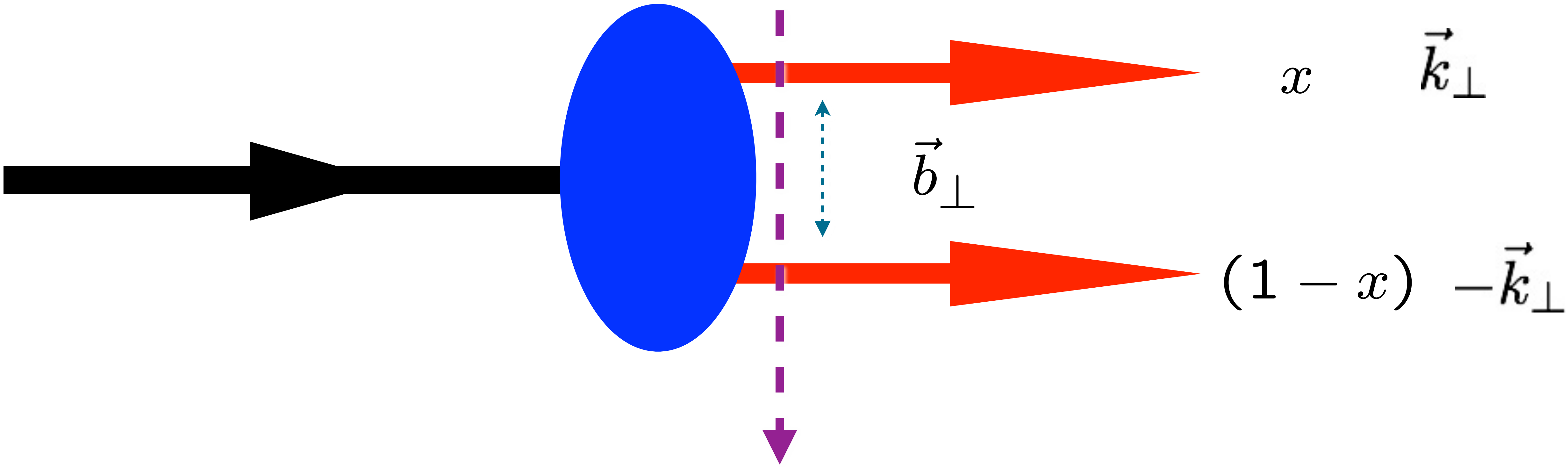
$$\kappa \simeq 0.6 \; GeV$$

$$1/\kappa \simeq 1/3 \; fm$$

- **de Alfaro, Fubini, Furlan:**
  - **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}^2_{q+\bar{q}}$$

$$\int dk^- \Psi_{BS}(P,k) \rightarrow \psi_{LF}(x, \vec{k}_\perp)$$

LF Holography

Baryon Equation

Superconformal Algebra

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+ = M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ !

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

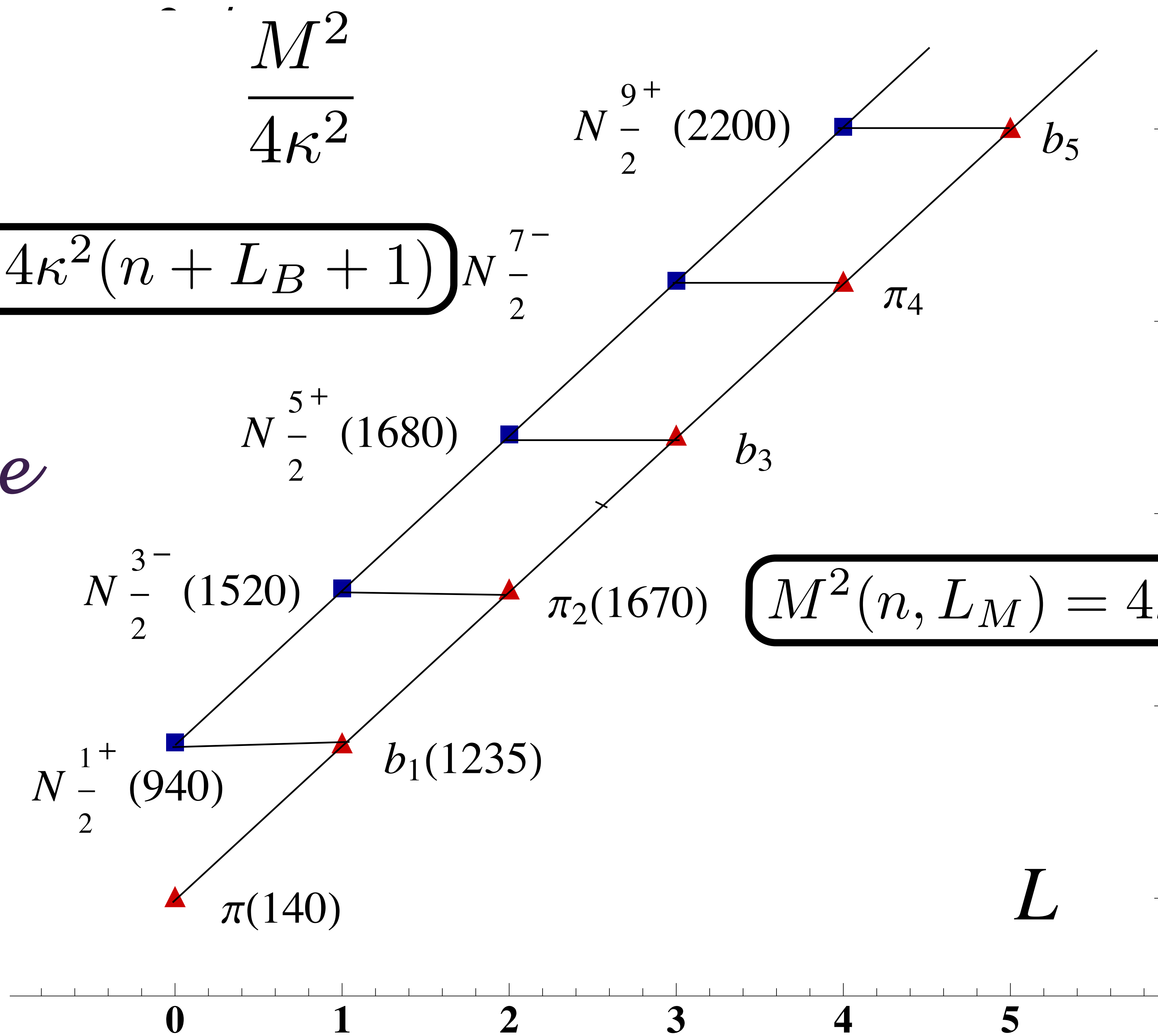
Meson-Baryon Degeneracy for $L_M=L_B+1$

Superconformal Algebra

de Tèramond, Dosch, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

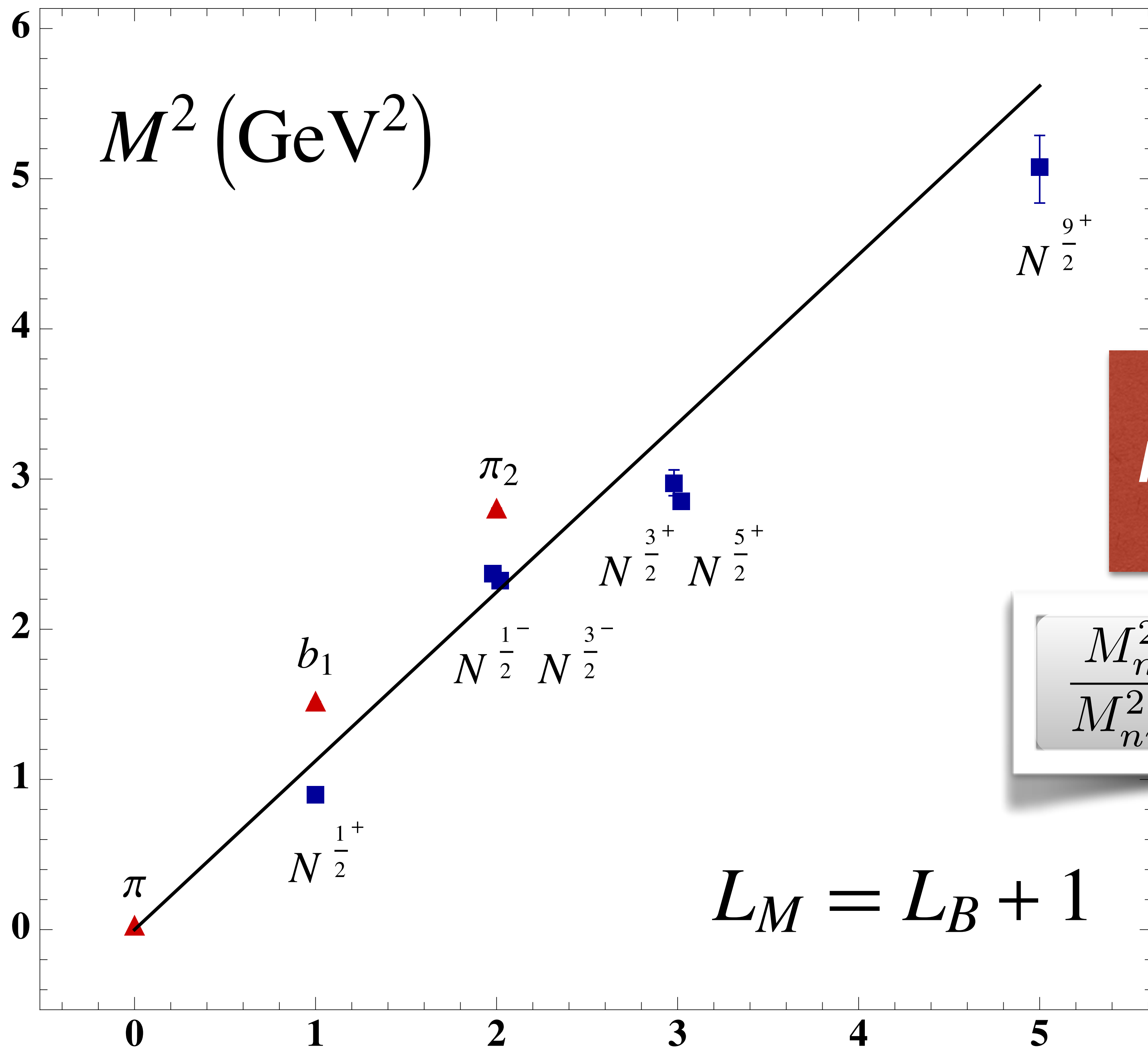


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

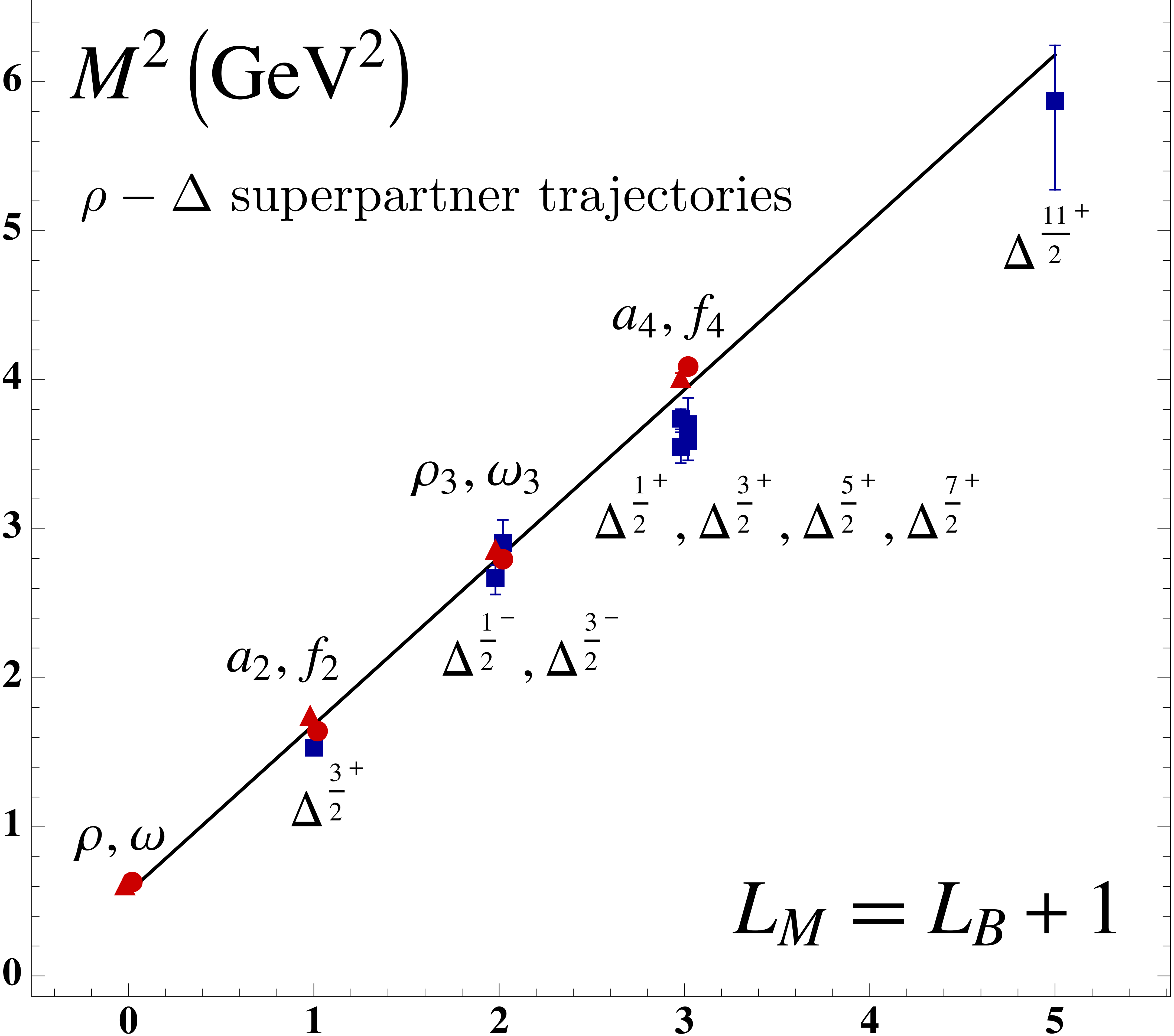
Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



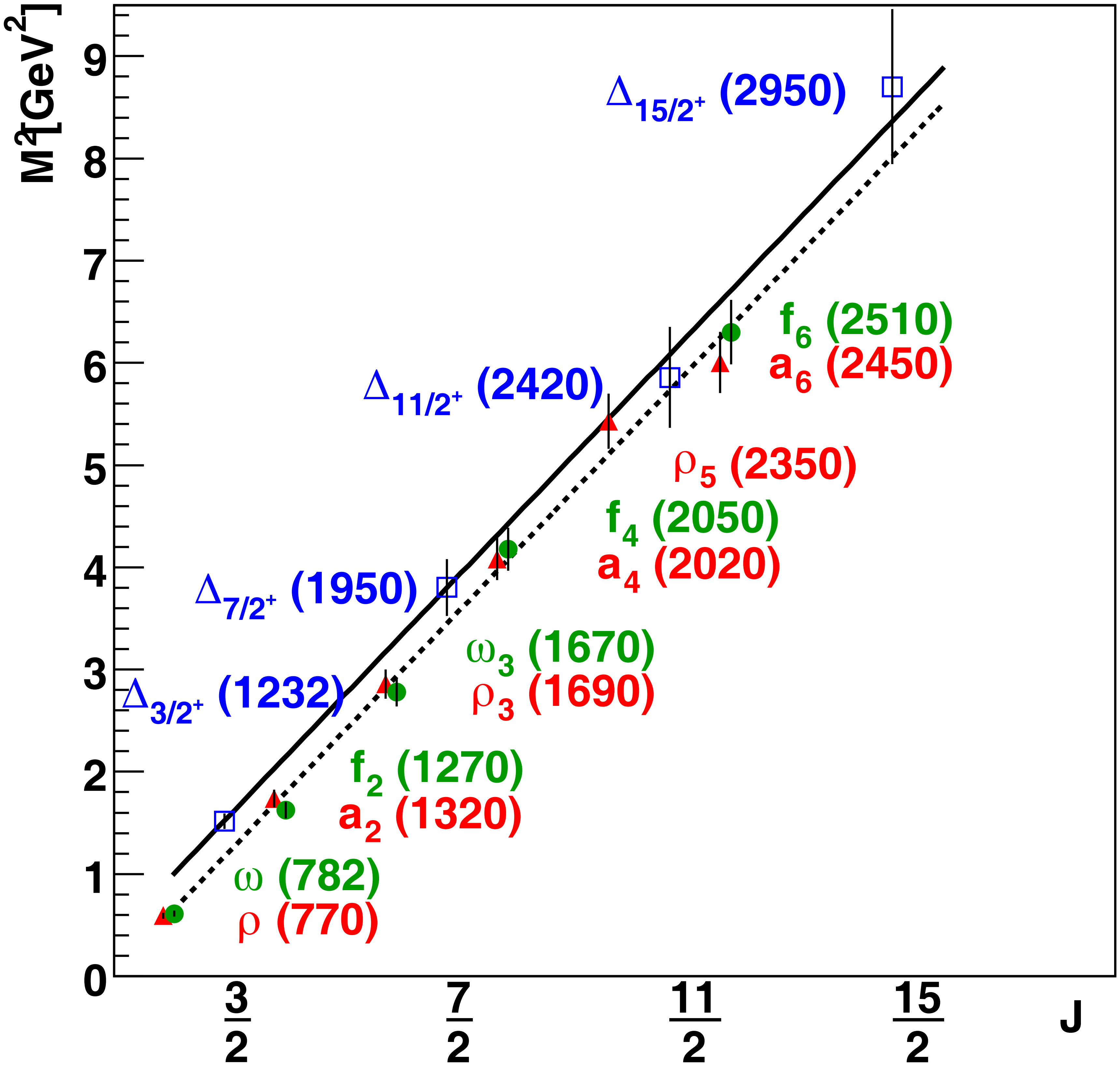
Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n+L_M}{n+L_B+1}$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon



Dosch, de Teramond, sjb



The leading Regge trajectory: Δ resonances with maximal J in a given mass range.
Also shown is the Regge trajectory for mesons with J = L+S.

Some Features of AdS/QCD

- ***Regge spectroscopy—same slope in n,L for mesons,***
- ***Chiral features for m_q=0: m_π=0, chiral-invariant proton***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for L_M=L_B+1

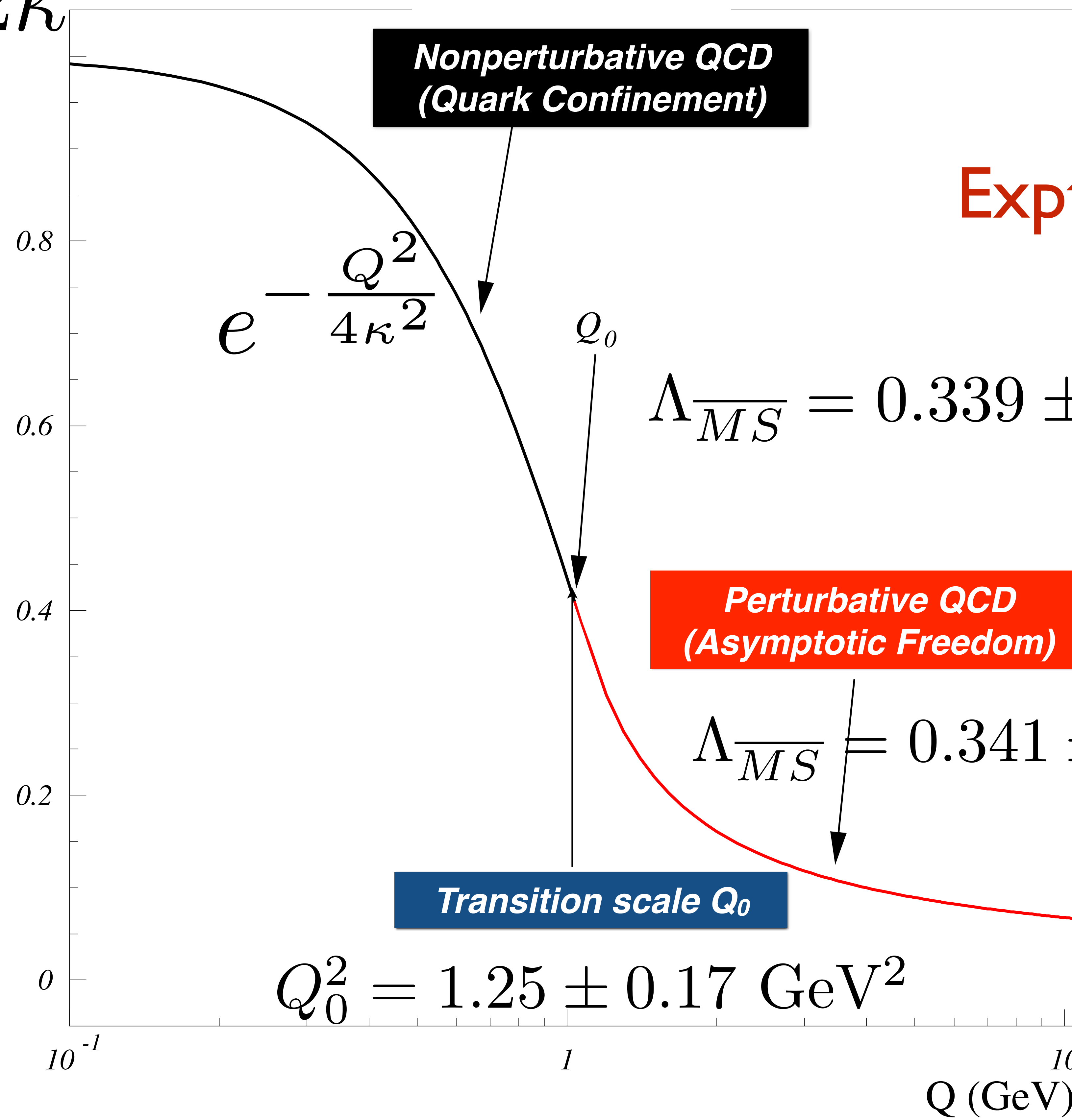
All-Scale QCD Coupling

Deur, de Tèramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



$$e^{-\frac{Q^2}{4\kappa^2}}$$

Expt:

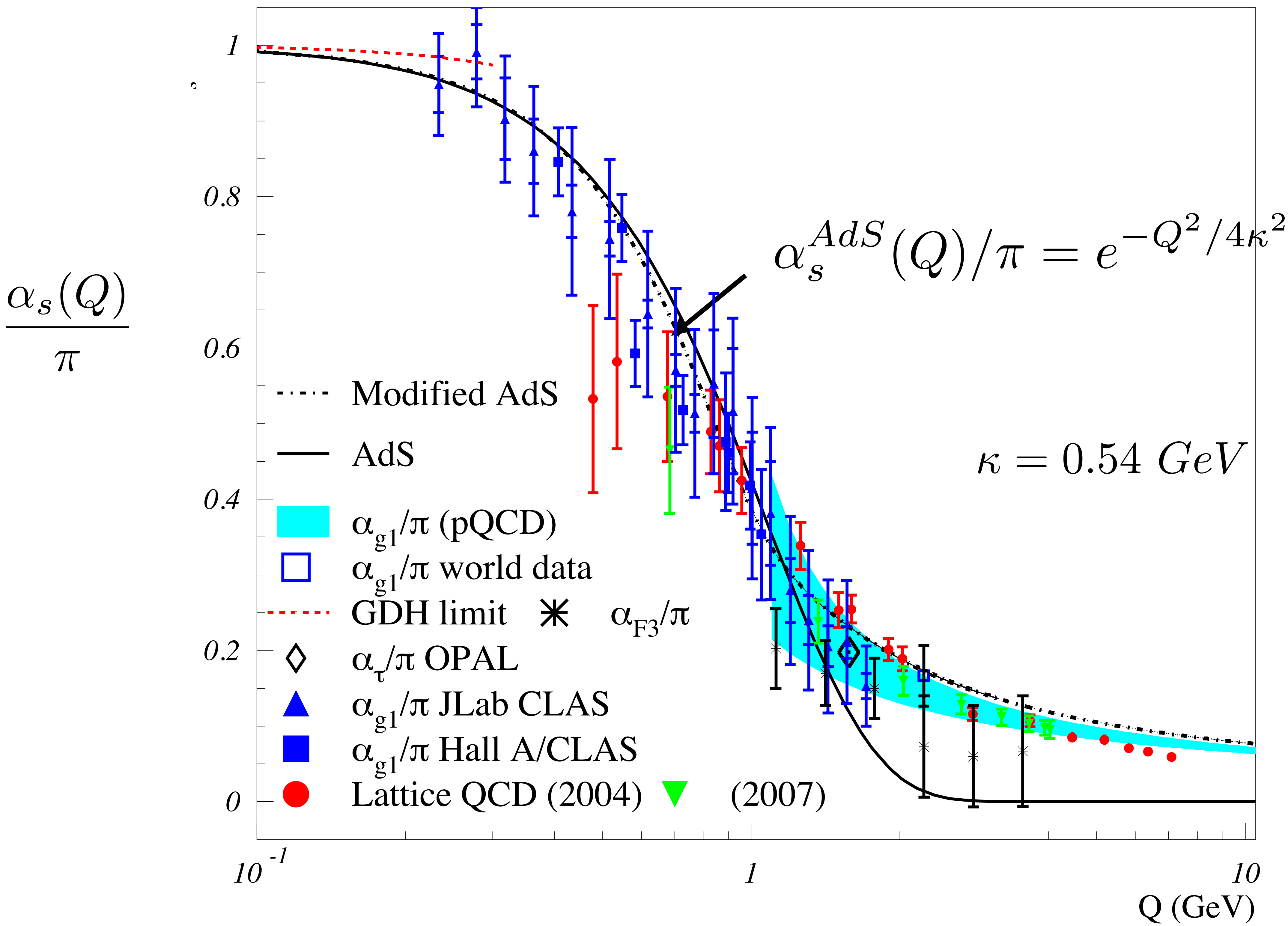
$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016 \ GeV$$

$$\Lambda_{\overline{MS}} = 0.341 \pm 0.024 \ GeV$$

$$\lambda \equiv \kappa^2$$

$$Q_0^2 = 1.25 \pm 0.17 \ \mathrm{GeV}^2$$

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$

All-Scale QCD Coupling

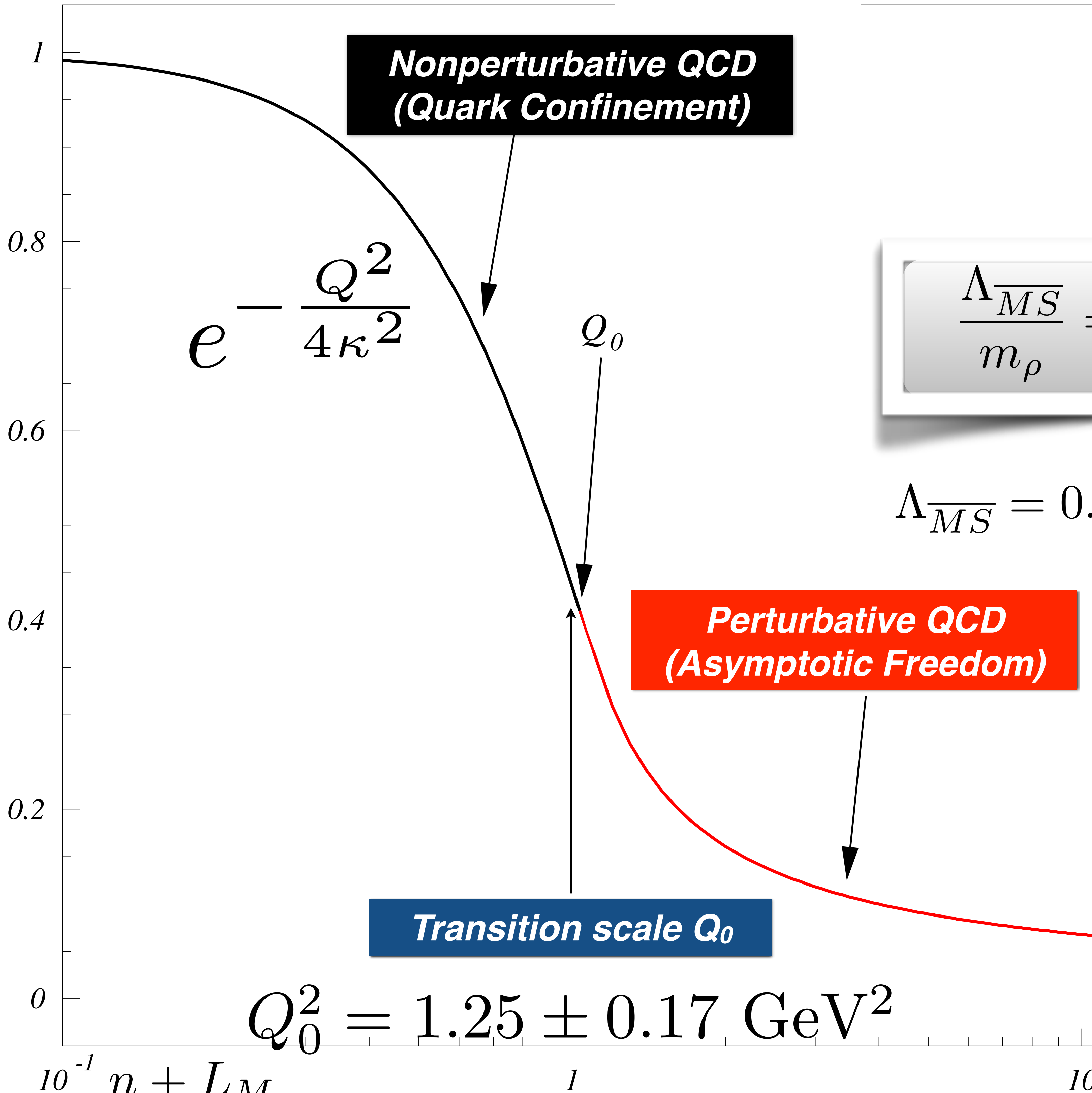
Deur, de Teramond, sjb

Prediction from AdS/QCD:

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

$$\Lambda_{\overline{MS}} = 0.351 \pm 0.024\ GeV$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



$$e^{-\frac{Q^2}{4\kappa^2}}$$

$$Q_0^2 = 1.25 \pm 0.17\ \mathrm{GeV}^2$$

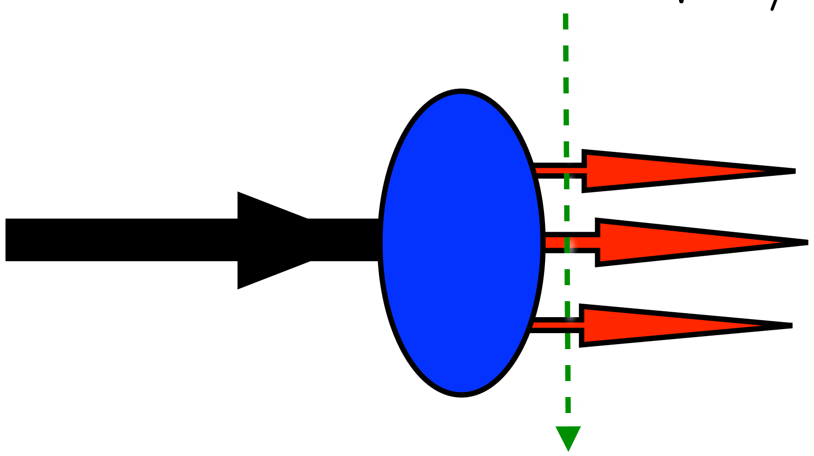
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\boxed{\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>}$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

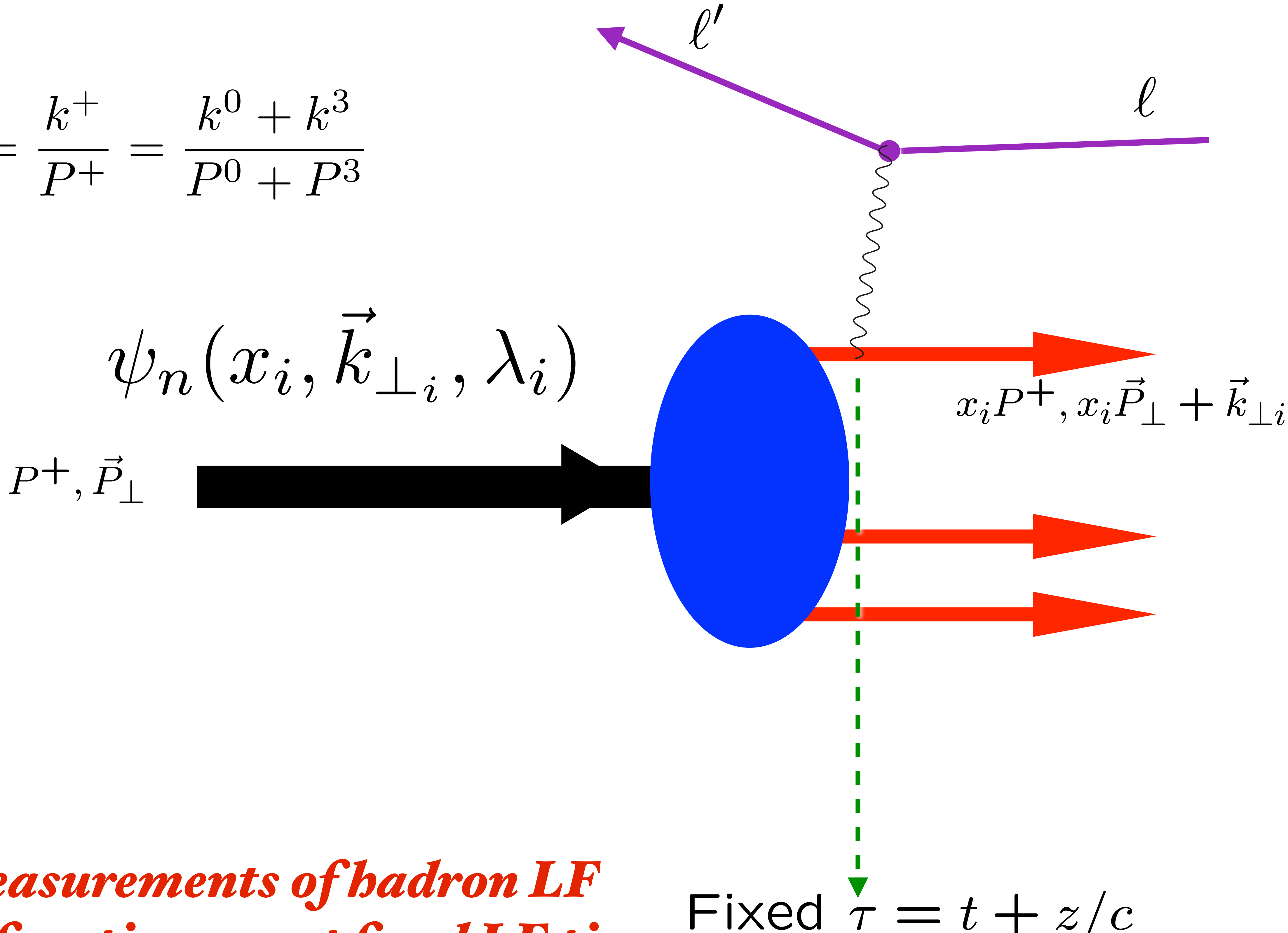
Eigenstate -- independent of τ

$$H_{LF} = P^+P^- - \vec{P}_\perp^2$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Measurements of hadron LF wavefunction are at fixed LF time

$$x_{bj} = x = \frac{k^+}{P^+}$$

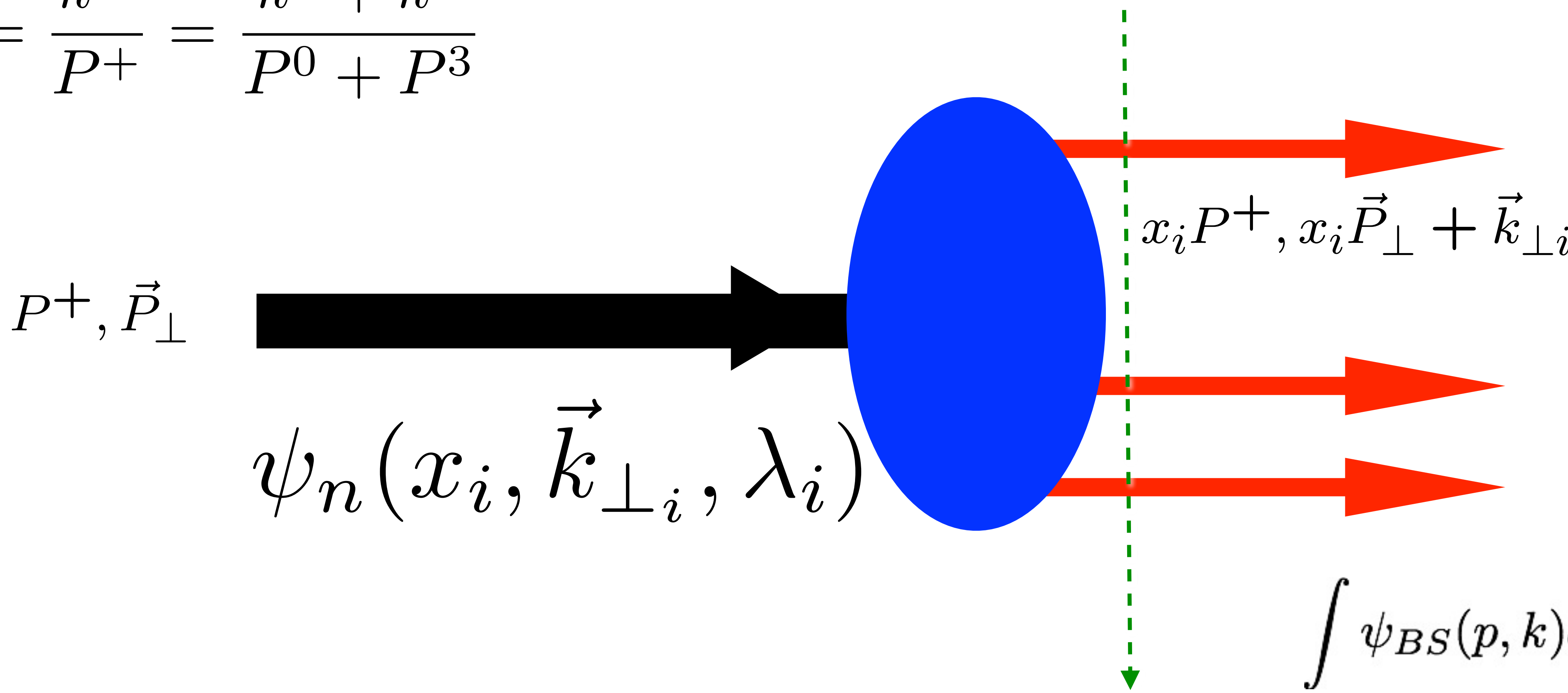
Like a flash photograph

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

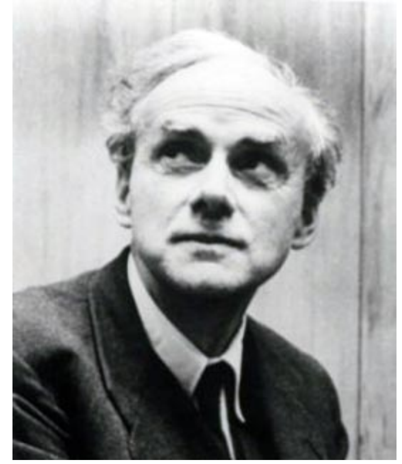
$$\int \psi_{BS}(p,k) dk^- \rightarrow \psi_{LF}$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; x_i, \vec{k}_{\perp i}, \lambda_i >$$

Invariant under boosts! Independent of P^μ

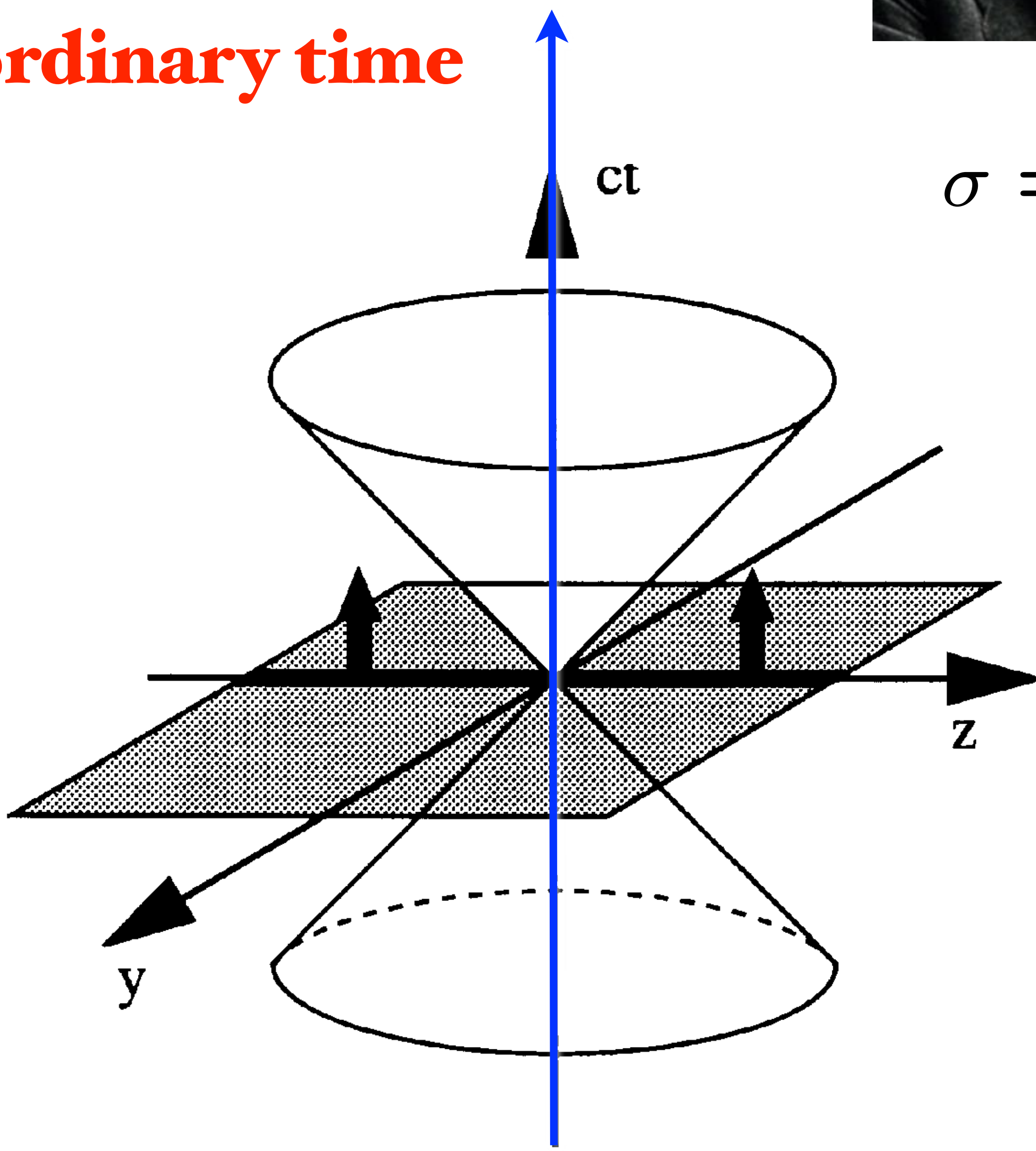
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Dirac's Amazing Idea: The "Front Form"



P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

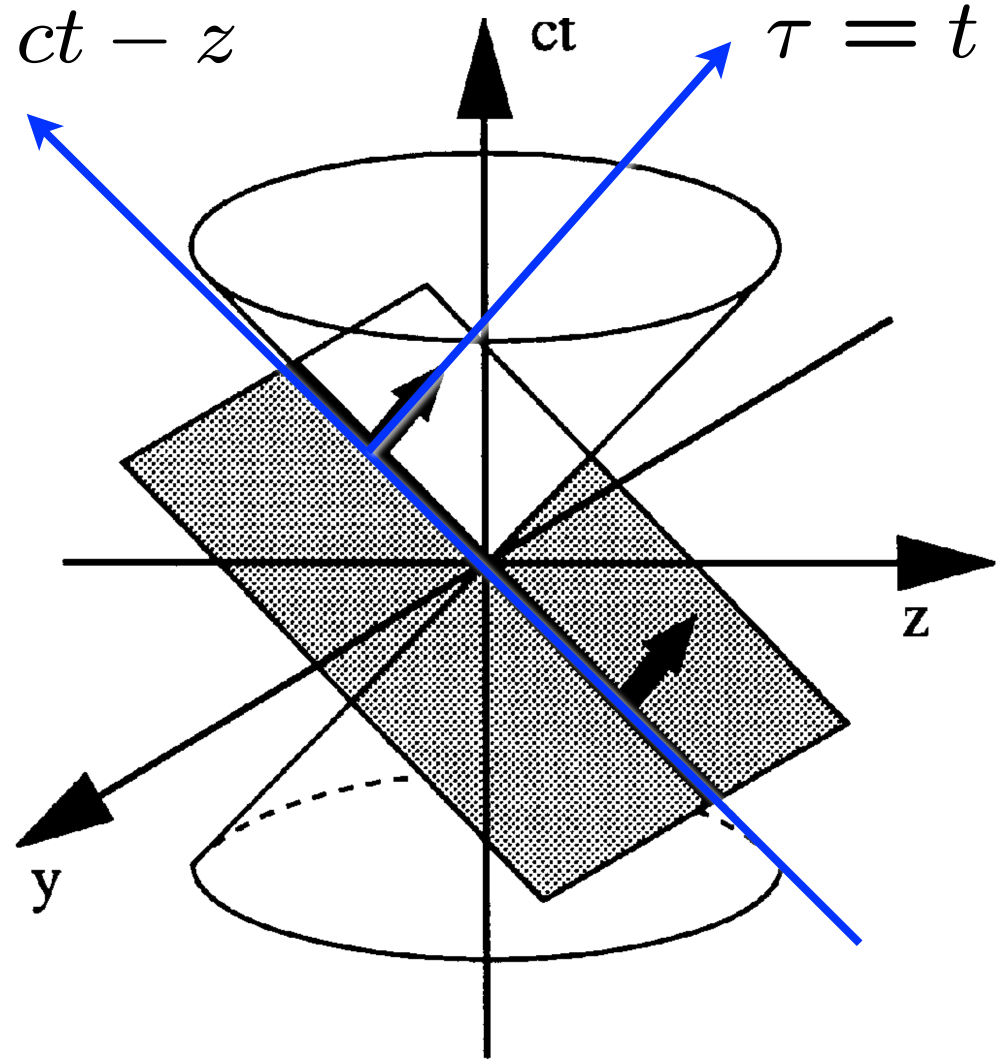
Evolve in ordinary time



Instant Form

Evolve in light-front time!

$\sigma = ct - z$ $\qquad$ $\tau = t + z/c$

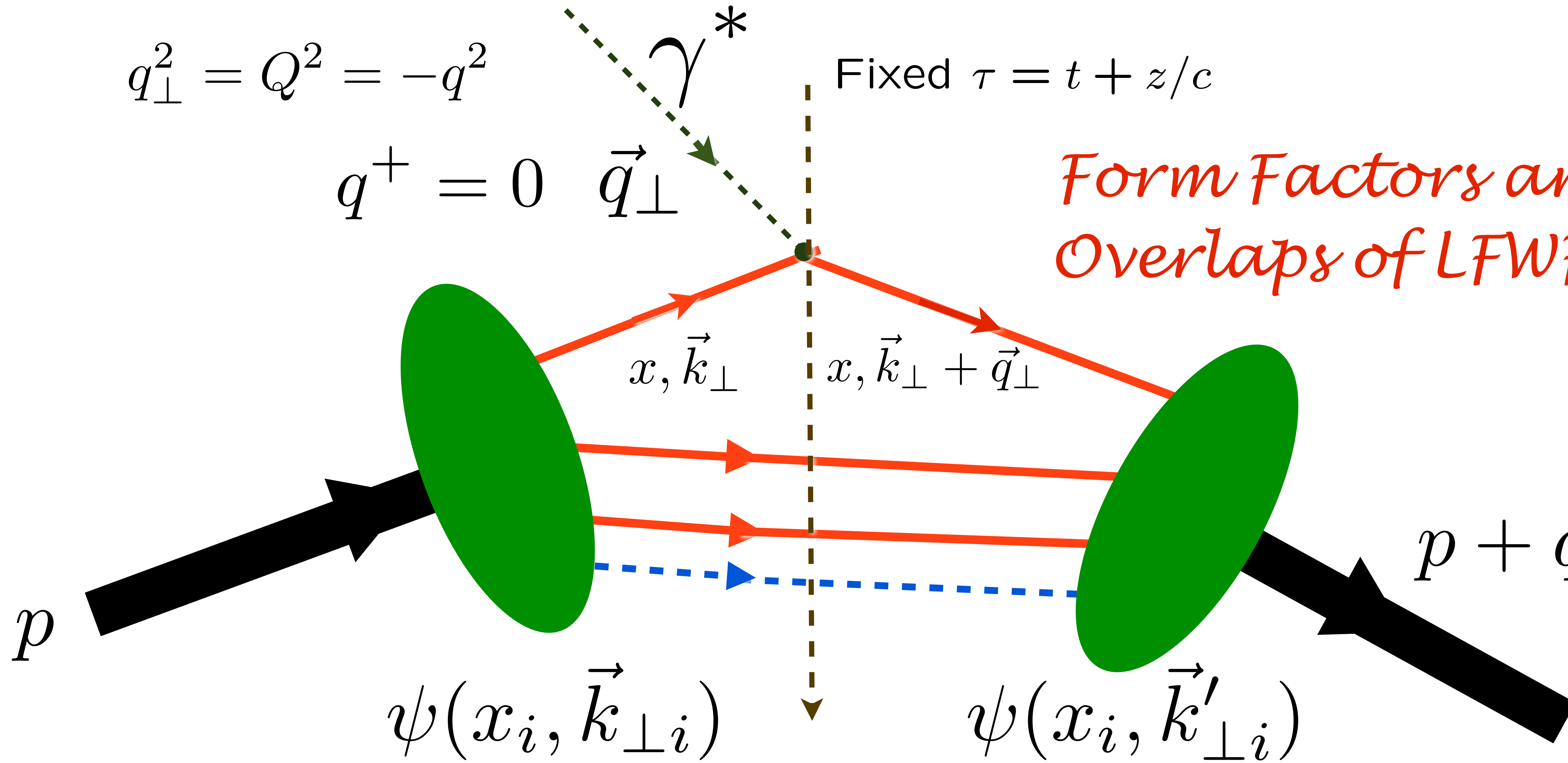


Front Form

Boost Invariant!

$$< p+q|j^+(0)|p >= 2p^+ F(q^2)$$

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

No comparable formula in instant form

Exact LF Formula for Pauli Form Factor

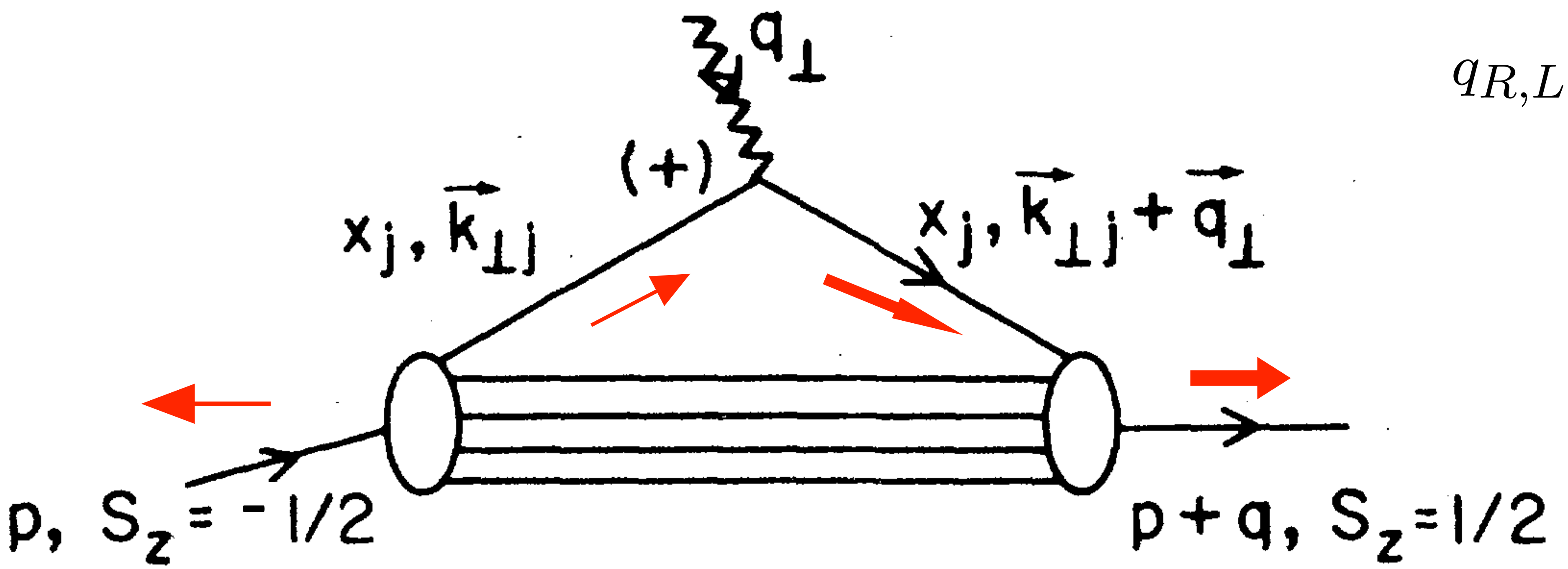
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



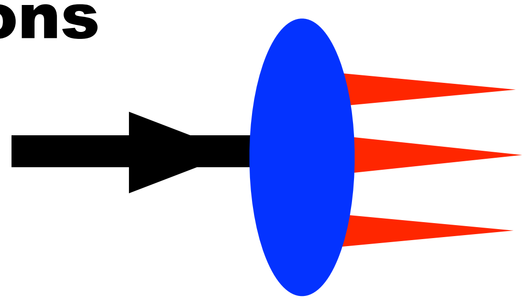
Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!
- Same structure function in e p collider and p rest frame
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant

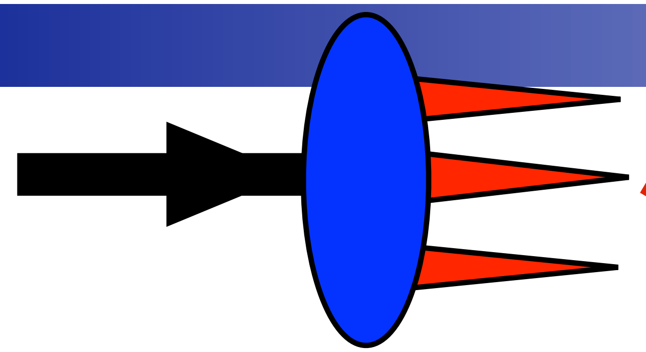
- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**

- *Light Front Wavefunctions:*



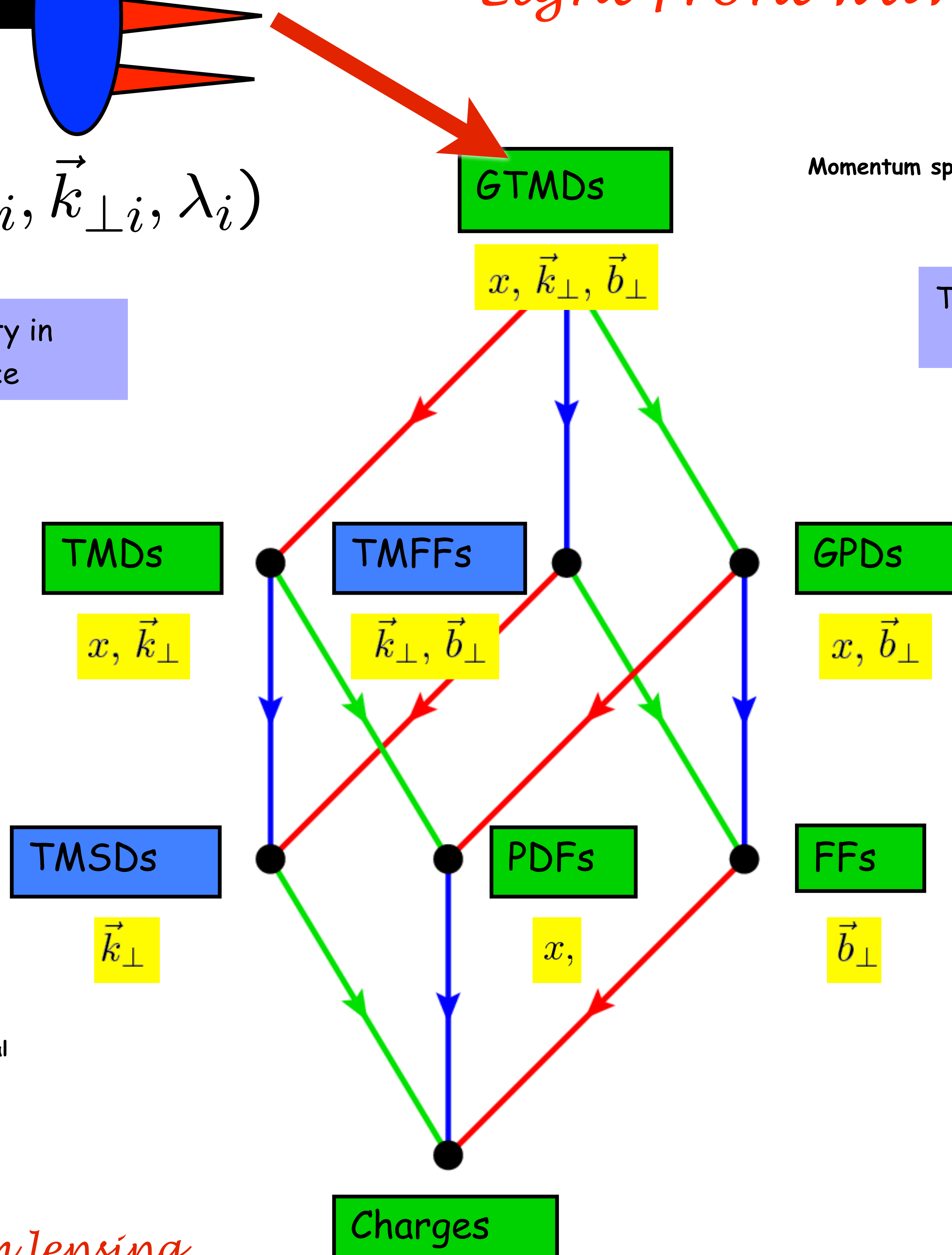
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

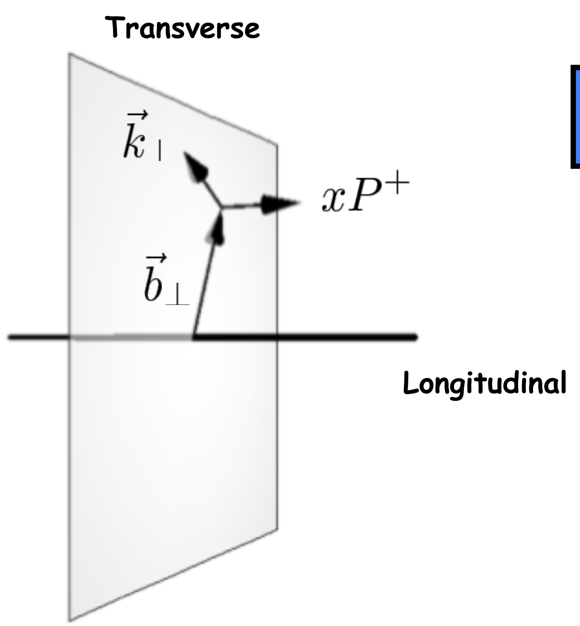
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in momentum space

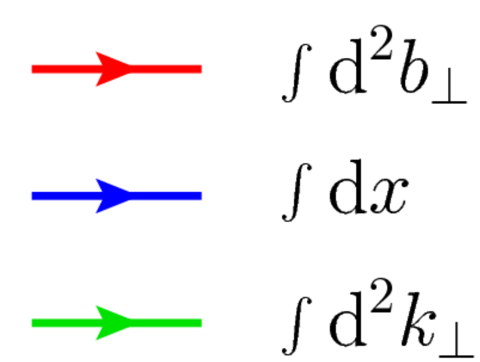
Transverse density in position space



Lorce, Pasquini



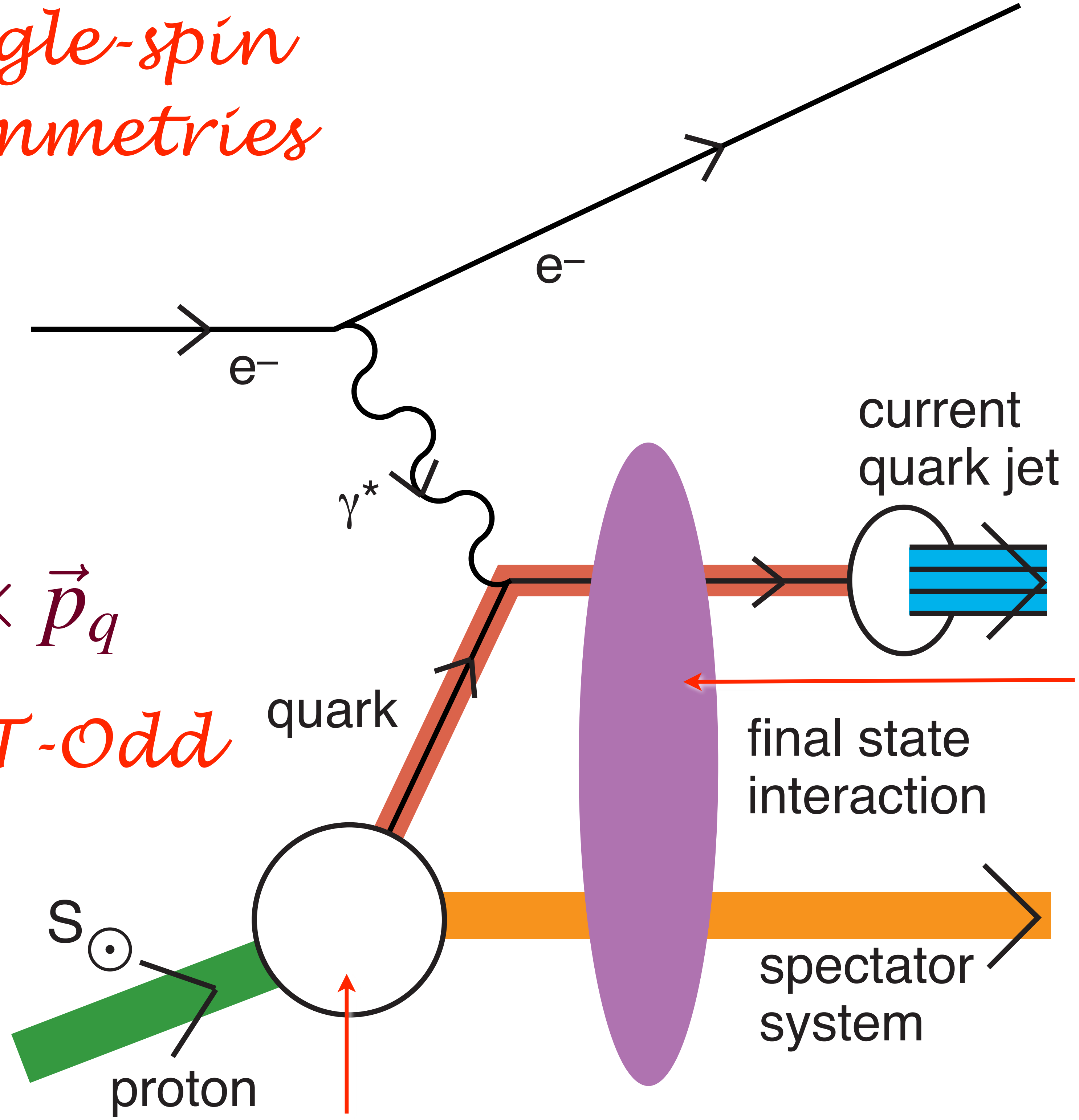
Sivers, T-odd from lensing



Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb



Light-Front Wavefunction S and P- Waves!

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

QCD S- and P- Coulomb Phases --Wilson Line

Pseudo- T-Odd

"Lensing Effect"

"Lensing" involves soft scales

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

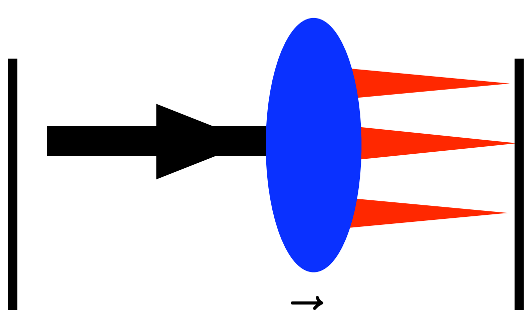
Hwang, Schmidt, sjb,

Mulders, Boer

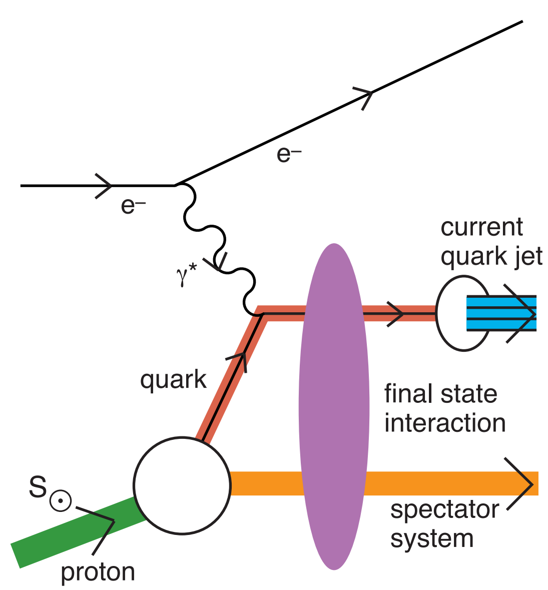
Qiu, Sterman

Collins, Qiu

Pasquini, Xiao, Yuan, sjb



$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$



Goal: An analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What is the analytic form of the confining interaction?
- What sets the QCD mass scale?
- QCD Running Coupling at all scales
- Hadron Spectroscopy-Regge Trajectories
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates
- Chiral Symmetry
- Systematically improvable

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

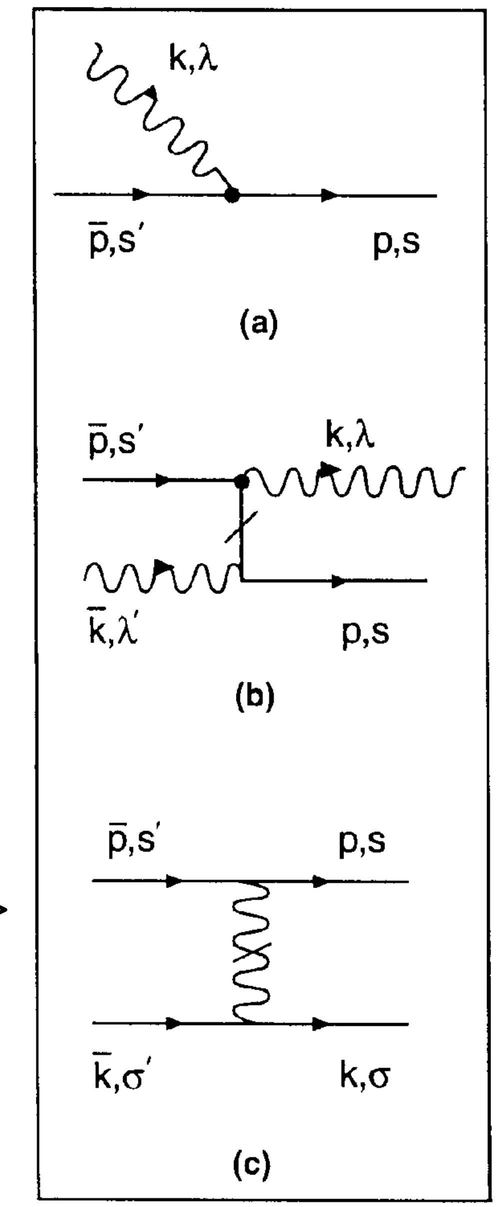
H^{int}_{LF}: Matrix in Fock Space

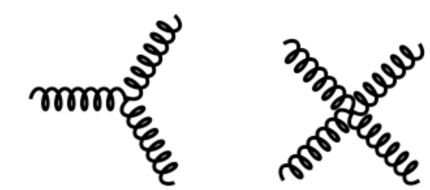
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass





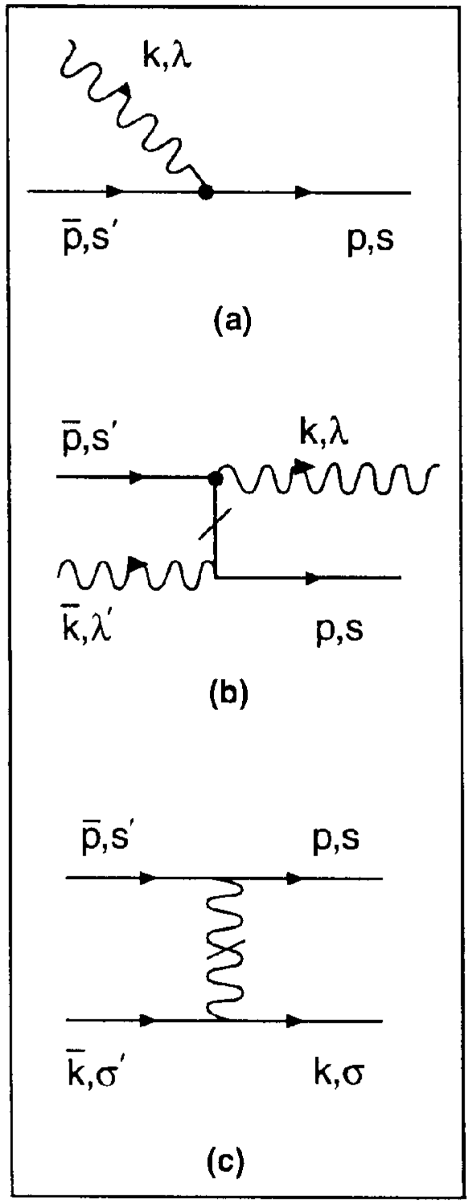
H^{int}_{LF}

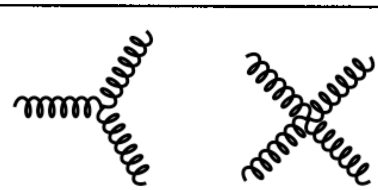
Light-Front QCD
Heisenberg Equation

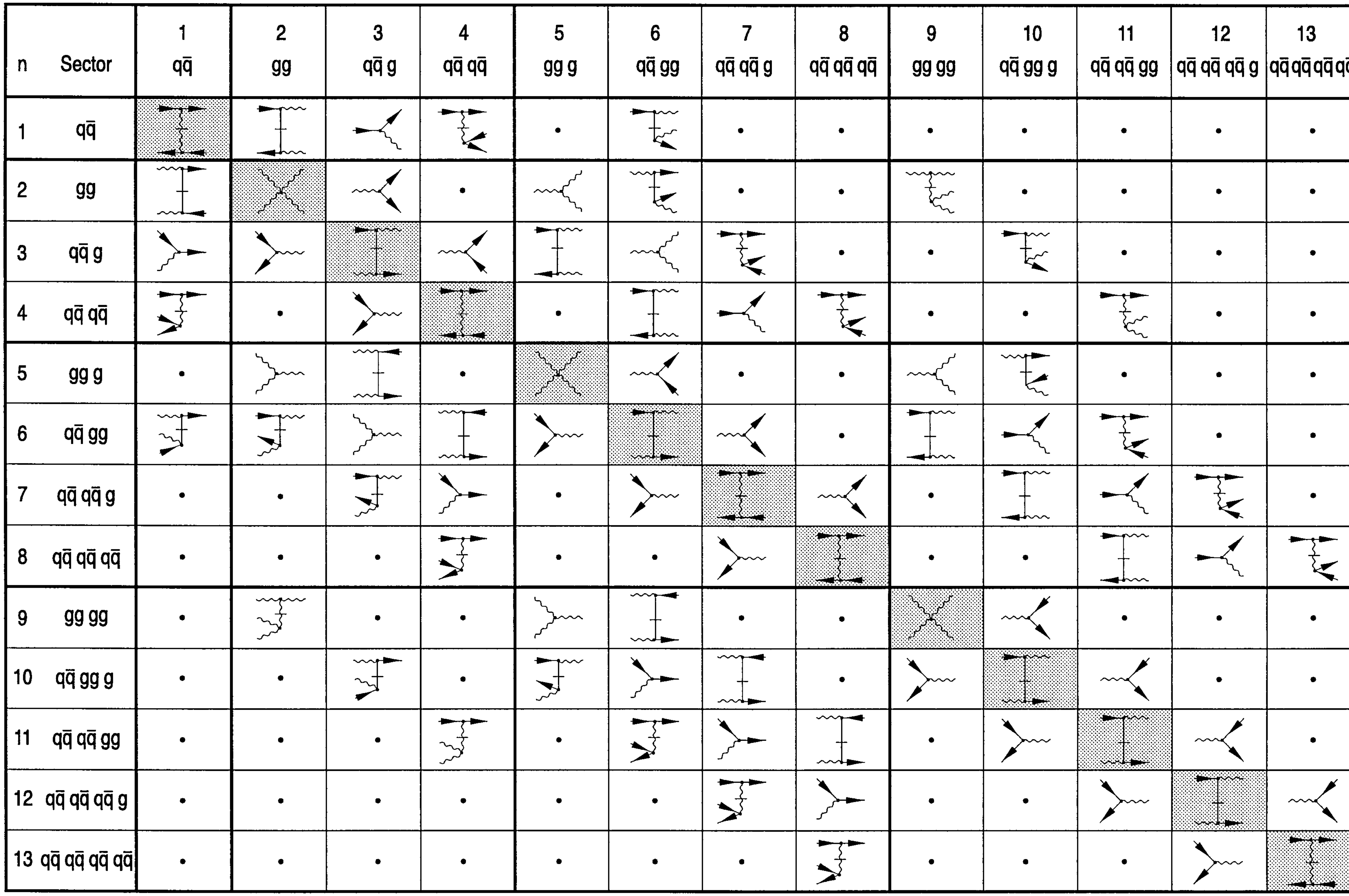
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







Minkowski space; frame-independent; no fermion doubling; no ghosts
trivial vacuum

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

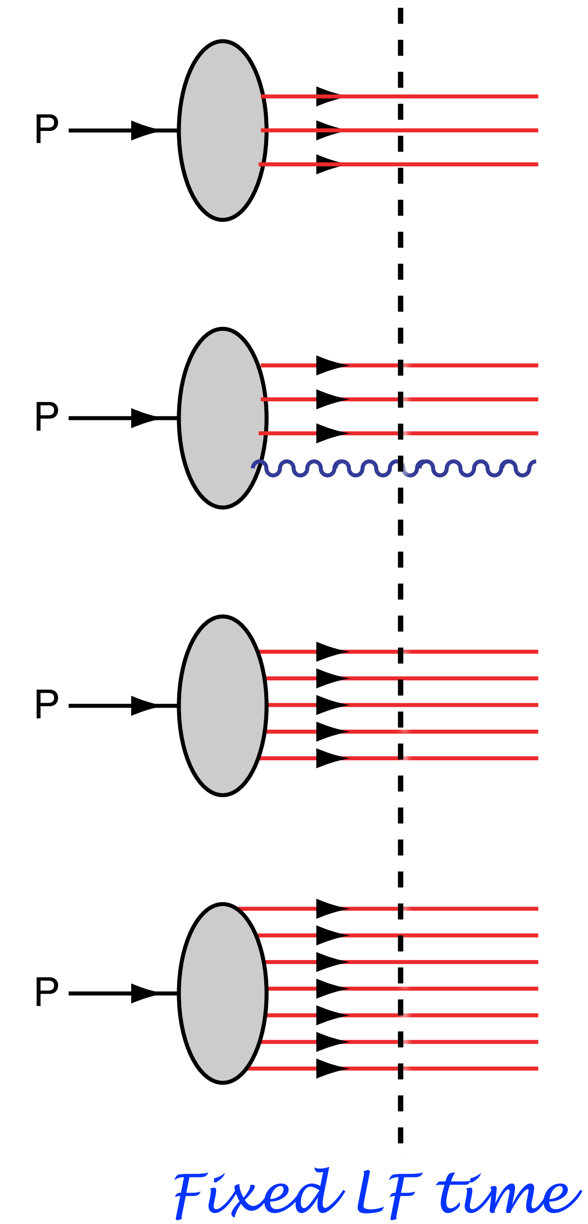
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



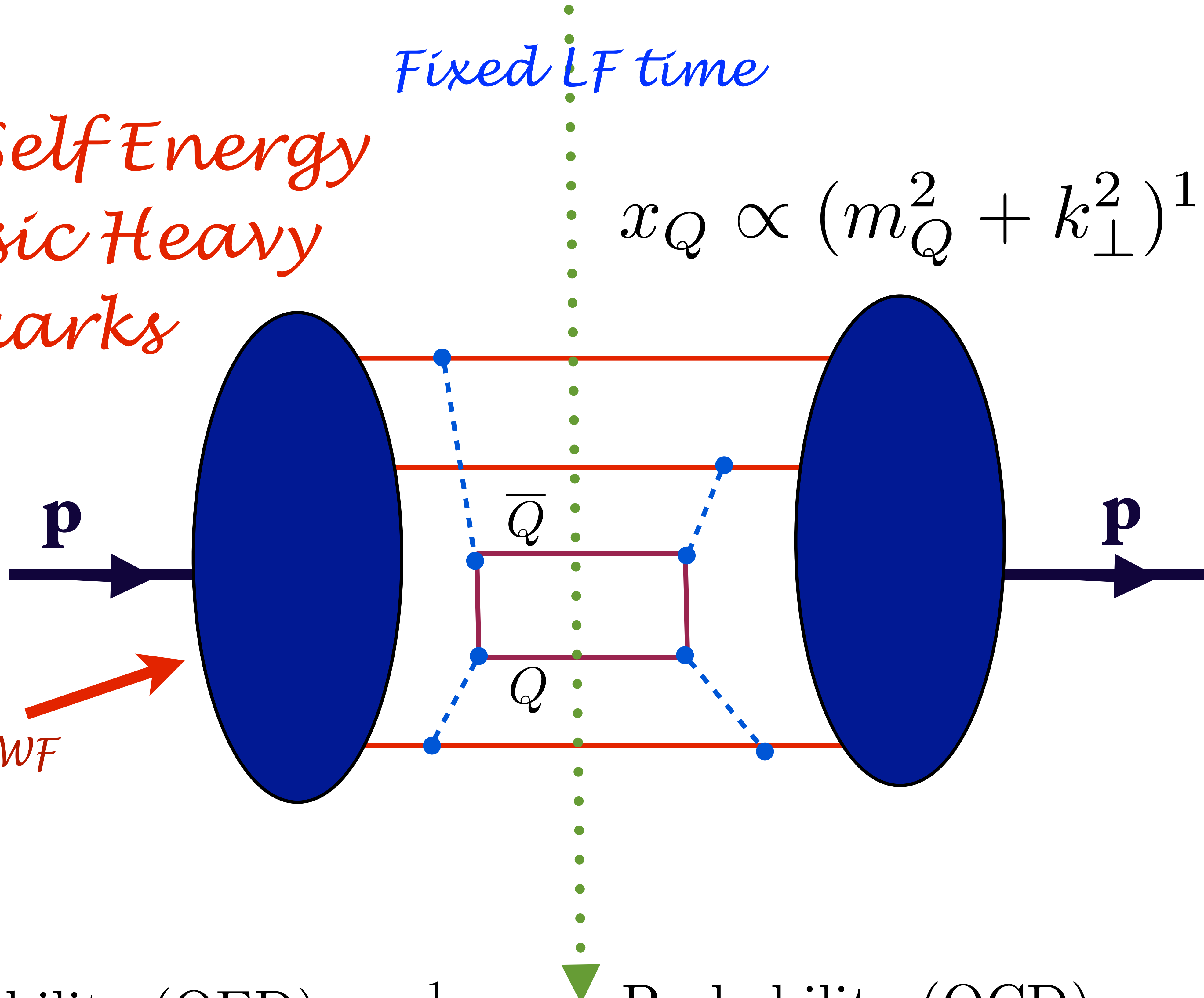
Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Hidden Color

Proton Self Energy Intrinsic Heavy Quarks

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$



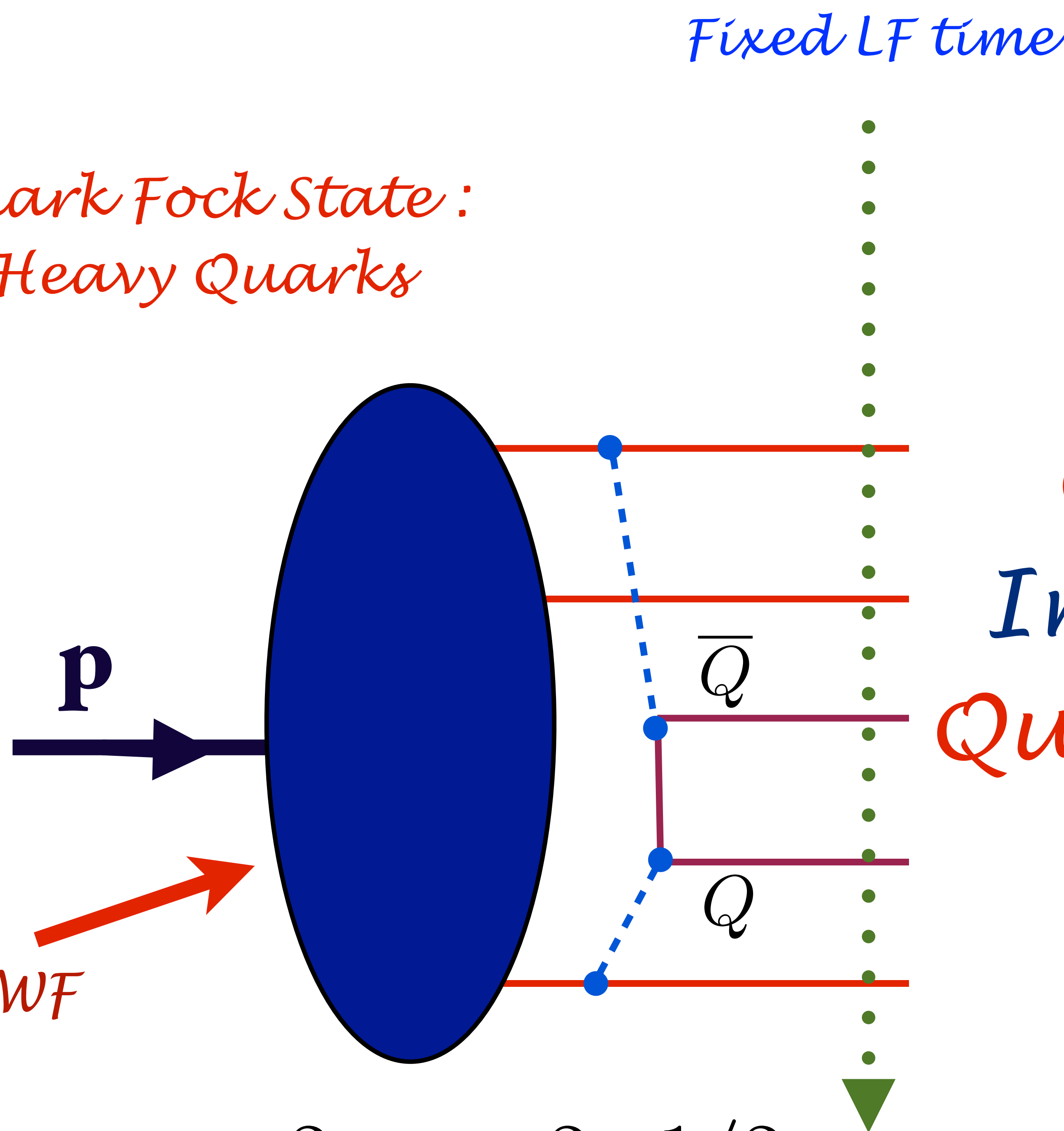
Use AdS/QCD LFWF

Probability (QED) $\propto \frac{1}{M_\ell^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

Proton 5-quark Fock State :
Intrinsic Heavy Quarks



Use AdS/QCD LFWF

QCD predicts Intrinsic Heavy Quarks at high x

Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

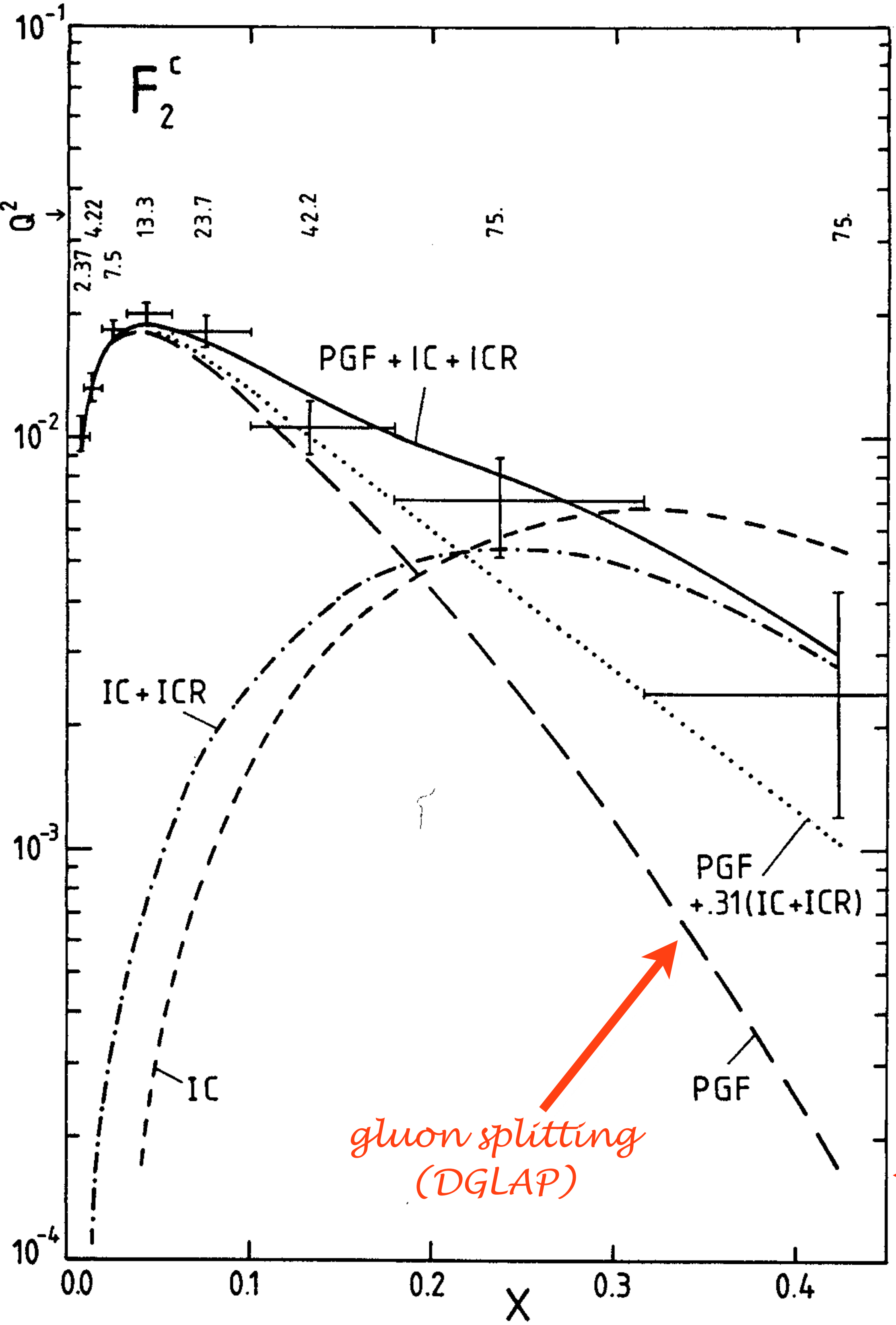
Probability (QED) $\propto \frac{1}{M_\ell^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

Measurement of Charm Structure Function!

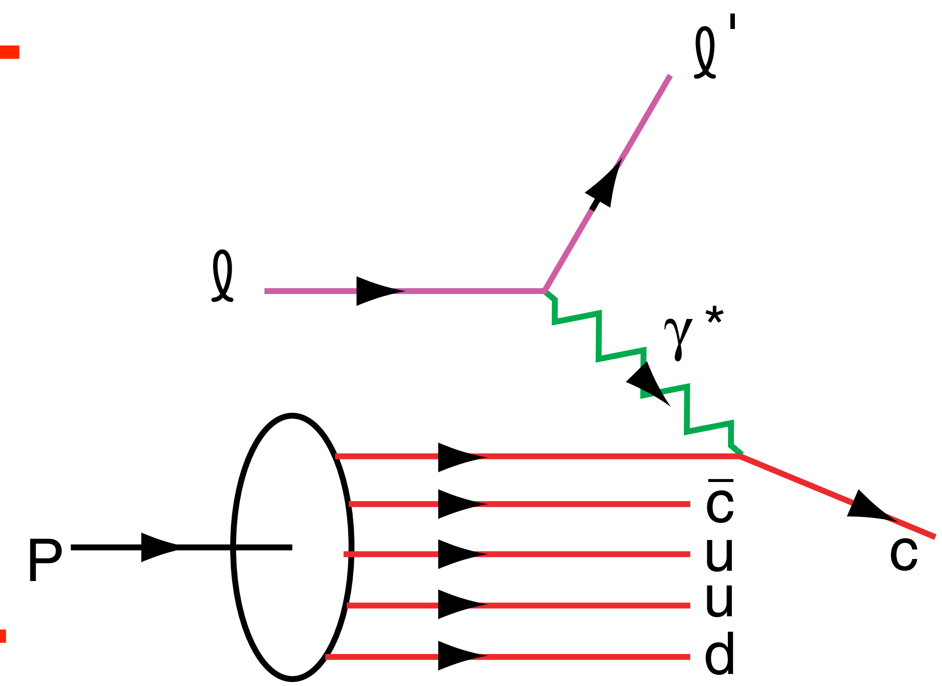
J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).



First Evidence for Intrinsic Charm

Hoyer, Peterson, Sakai, sjb

factor of 30 !

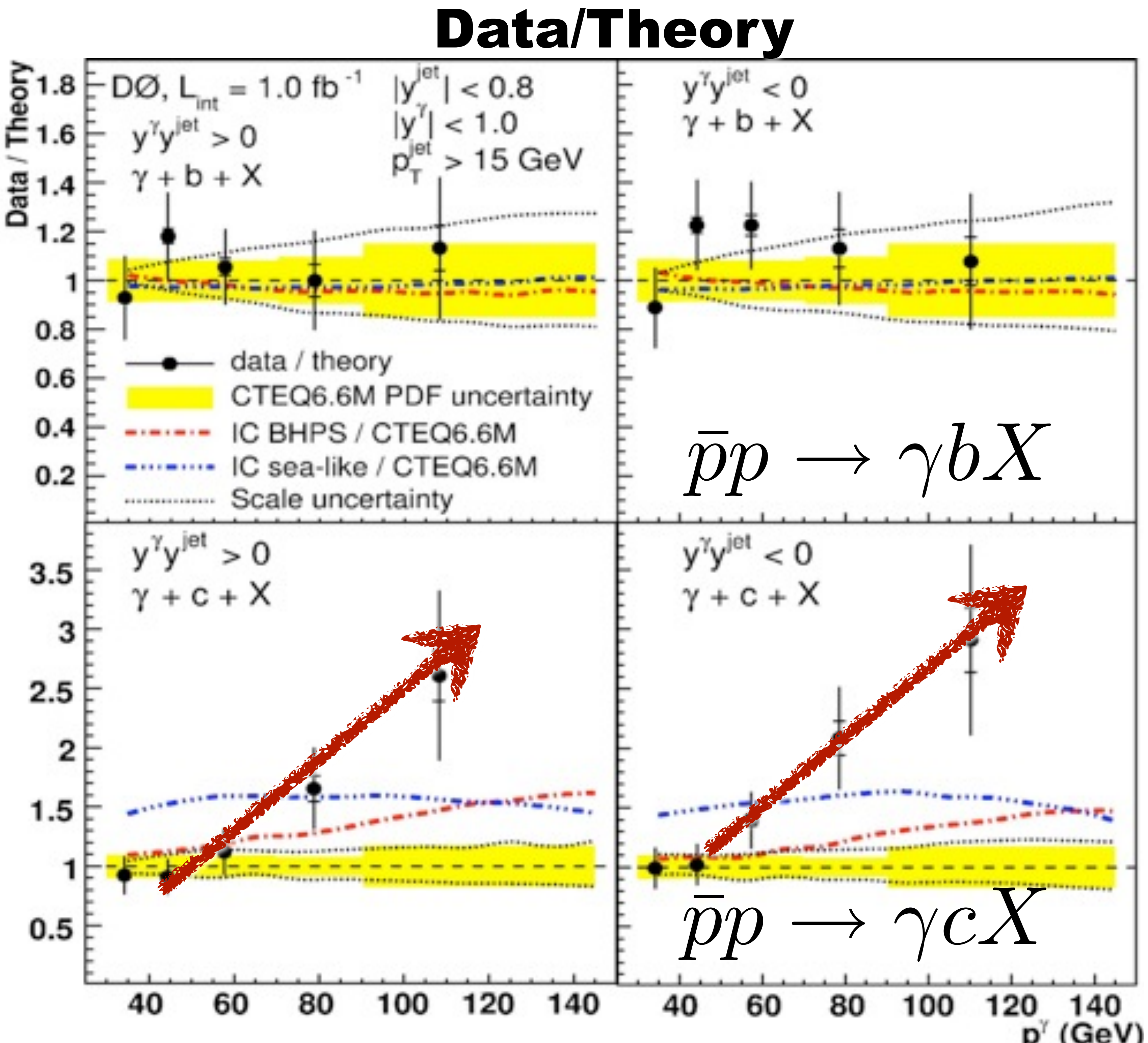


DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV



$$\frac{\Delta\sigma(\bar{p}p \to \gamma c X)}{\Delta\sigma(\bar{p}p \to \gamma b X)}$$

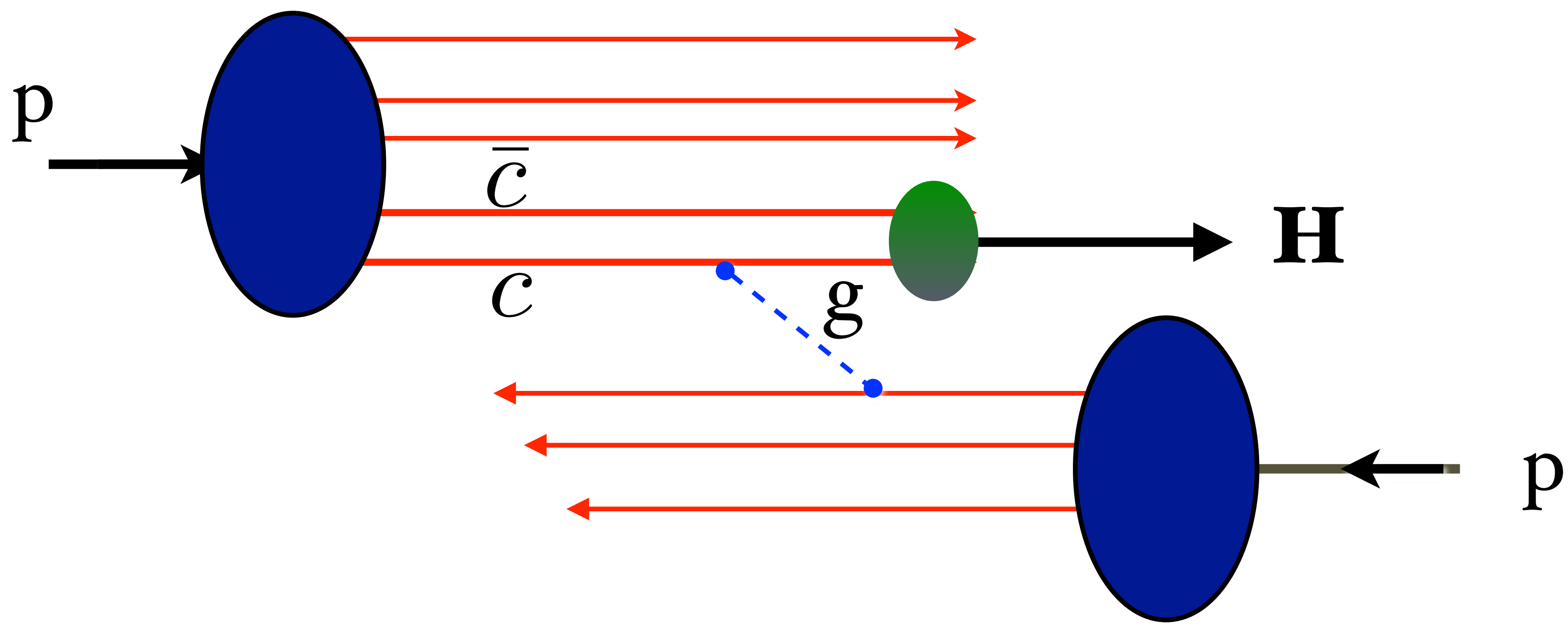
Ratio insensitive to gluon PDF, scales

Signal for significant IC at x > 0.1

Consistent with EMC measurement of charm structure function at high x

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



$$pp \to HX$$

Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

AFTER: Higgs production at threshold!

JLab: Charm production near threshold!

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi> = E\ |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

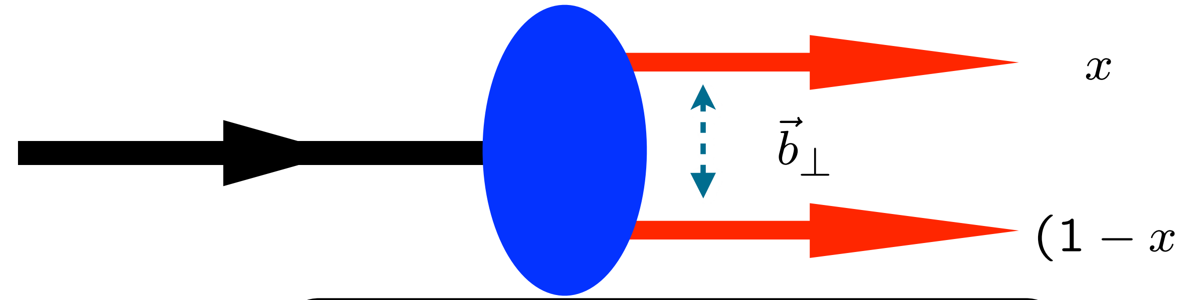
Bohr Spectrum

Semiclassical first approximation to QED

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states
and retarded interactions

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

$$\zeta, \phi$$

$$m_q = 0$$

AdS/QCD:

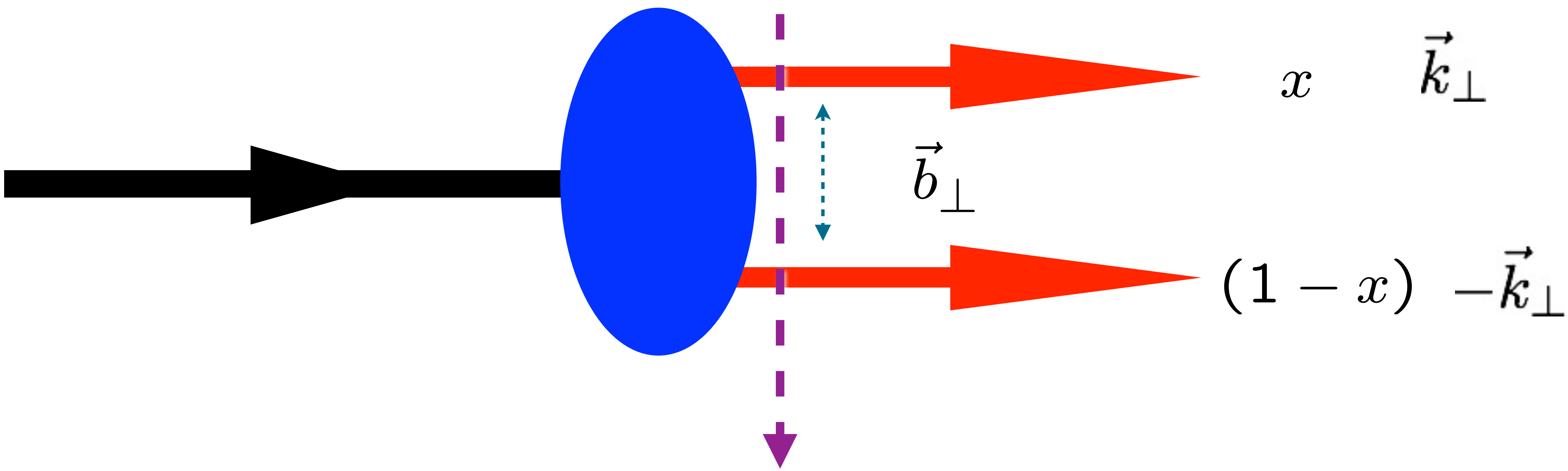
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD
potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Fixed $\tau = t + z/c$



$$\zeta^2 \equiv b_\perp^2 x(1-x)$$

Invariant transverse separation

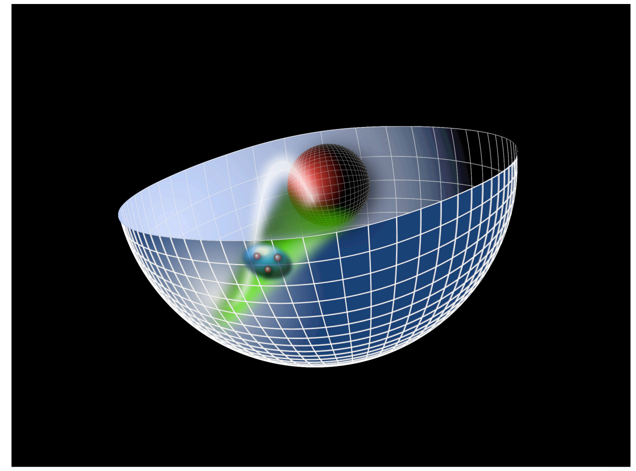
$$\zeta^2 \text{ conjugate to } \frac{k_\perp^2}{x(1-x)} = (p_q + p_{\bar{q}})^2 = \mathcal{M}^2_{q+\bar{q}}$$

$$\int dk^- \Psi_{BS}(P,k) \rightarrow \psi_{LF}(x, \vec{k}_\perp)$$

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6 \ GeV$$

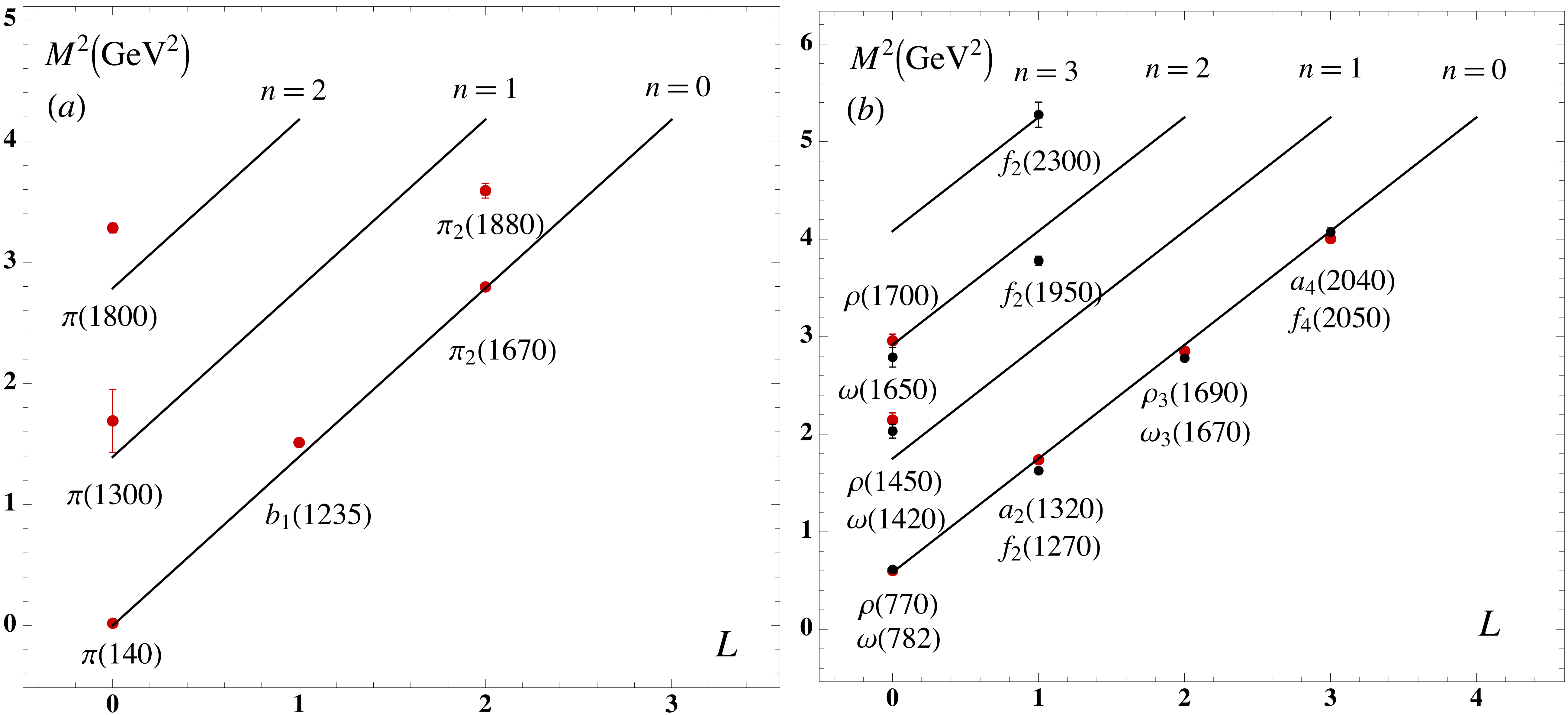
$$1/\kappa \simeq 1/3 \ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

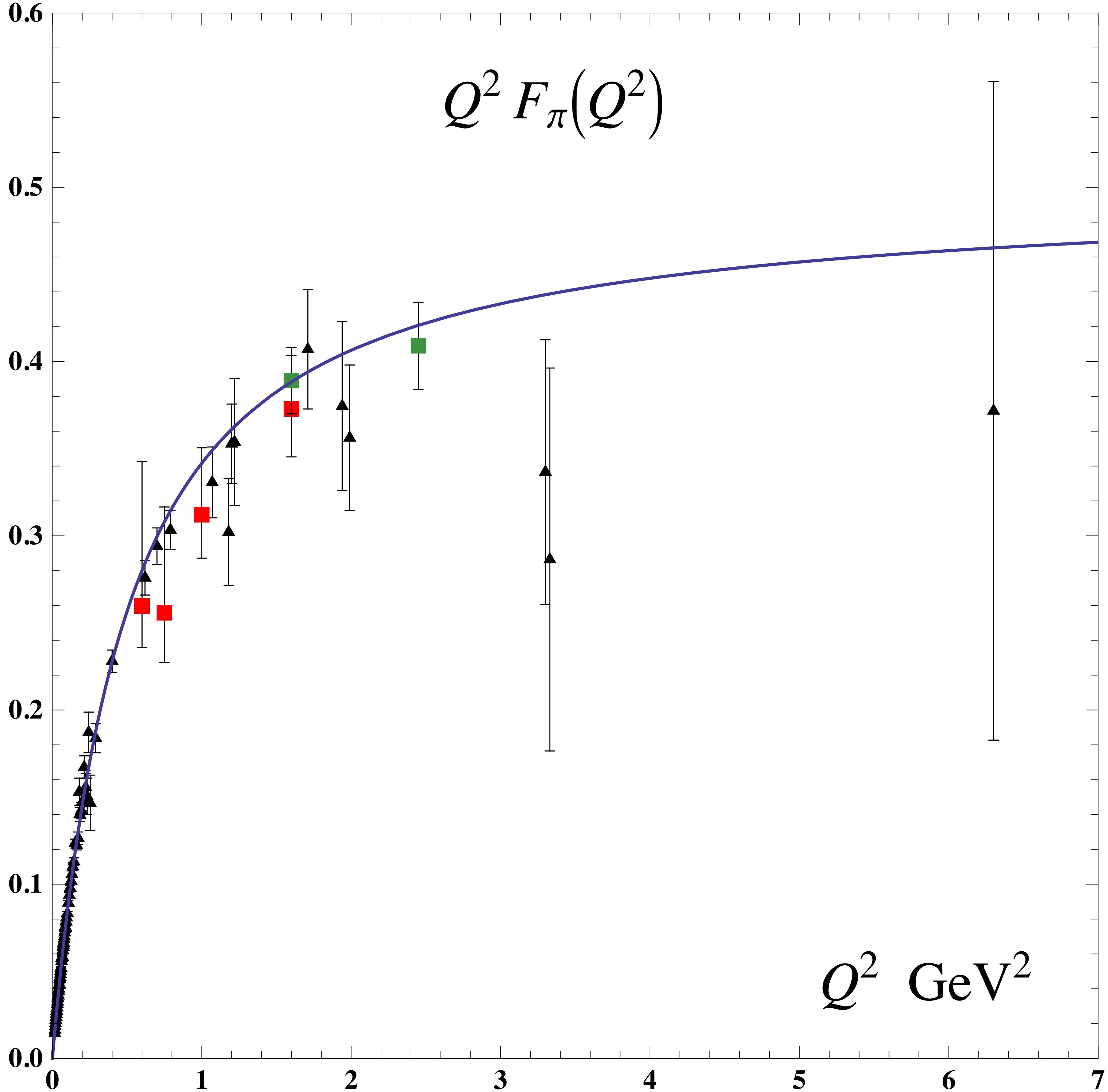
Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

$$m_u = m_d = 0$$

Preview

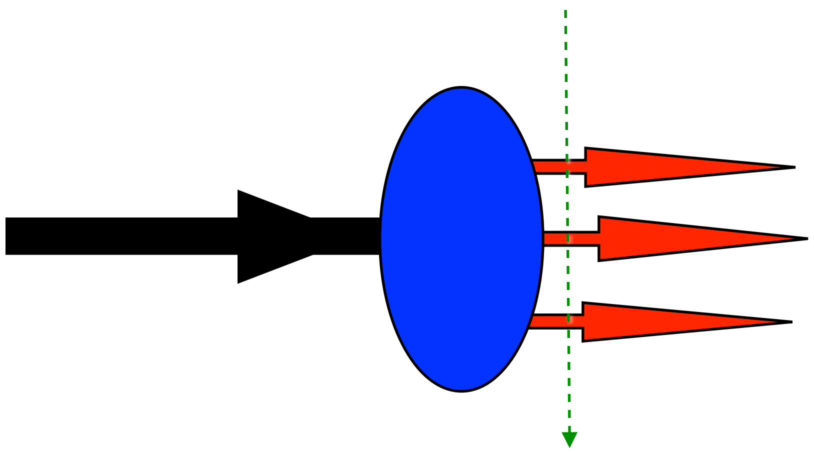


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i) \qquad x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

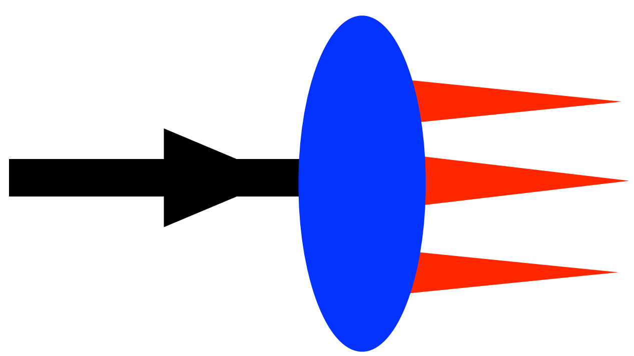
Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

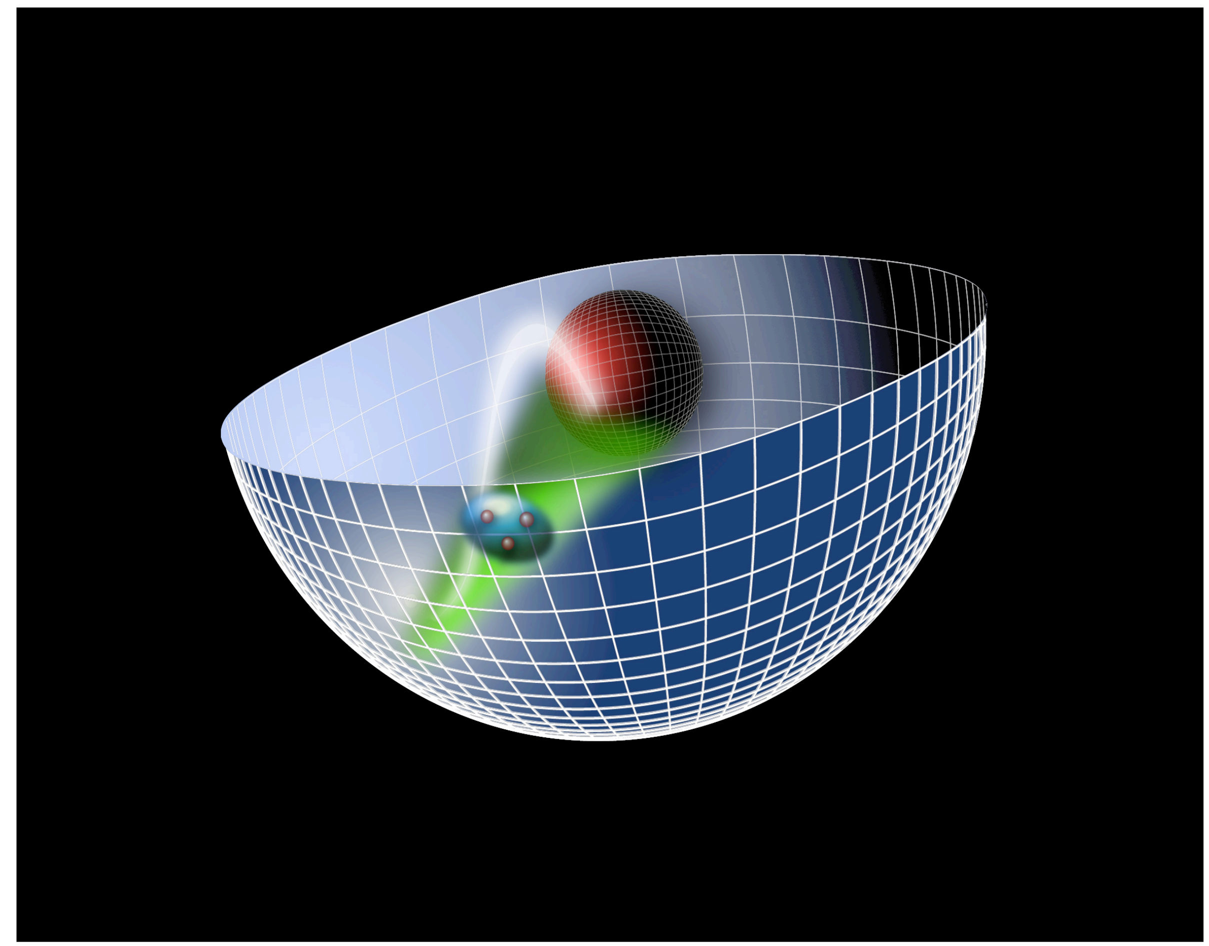
Light-Front Holography and Non-Perturbative QCD

Goal:
Use AdS/QCD duality to construct a first approximation to QCD

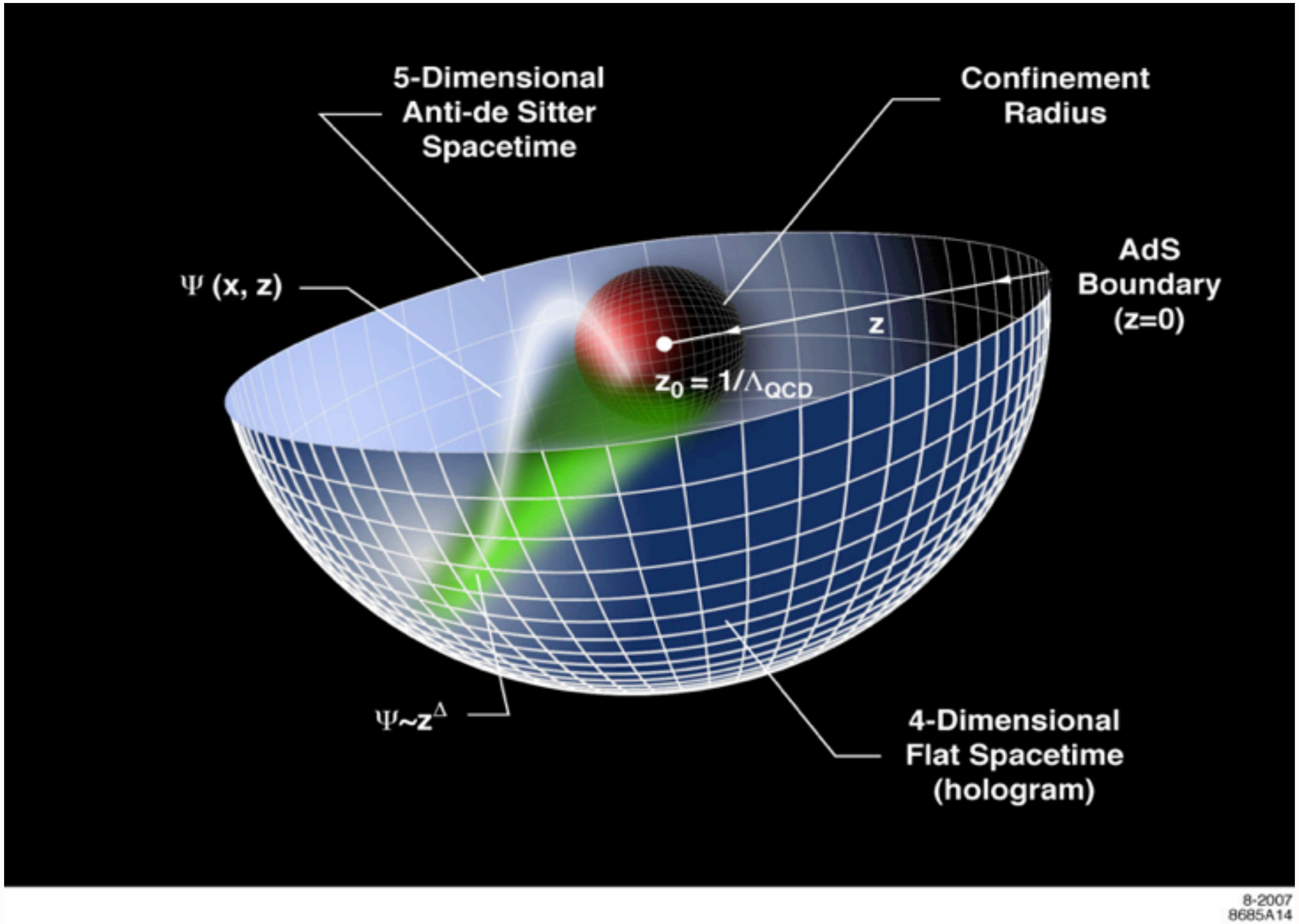
Hadron Spectrum
Light-Front Wavefunctions,
Form Factors, DVCS, etc



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with Guy de Teramond and H. Guenter Dosch



Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background $\boxed{\text{dilaton field } \varphi(z)}$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

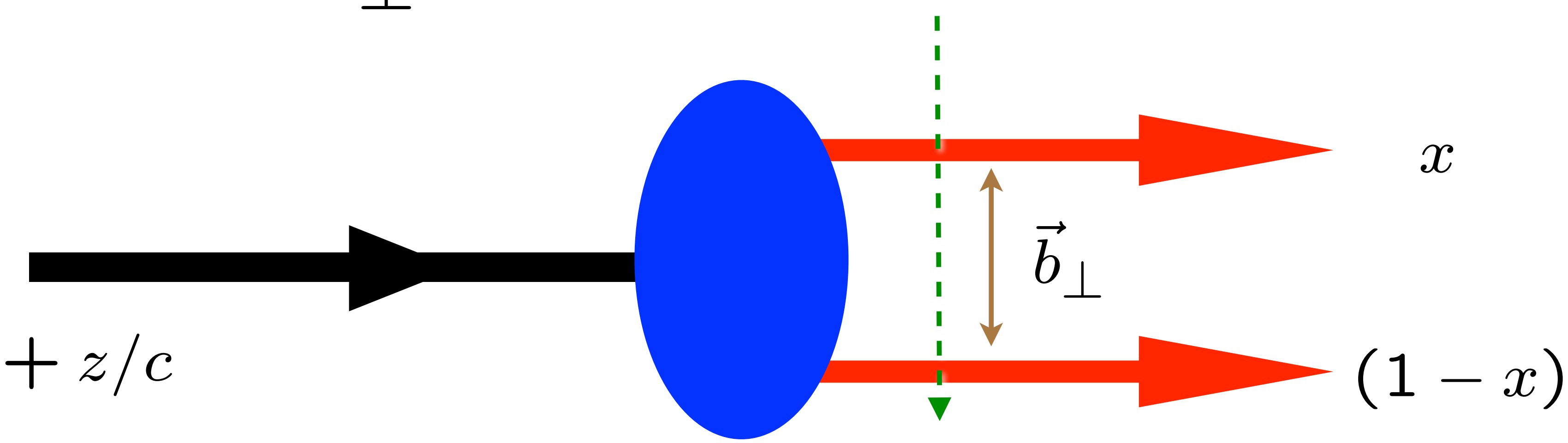
de Teramond, sjb

LF(3+1) $\longleftrightarrow$ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec b_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec b_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

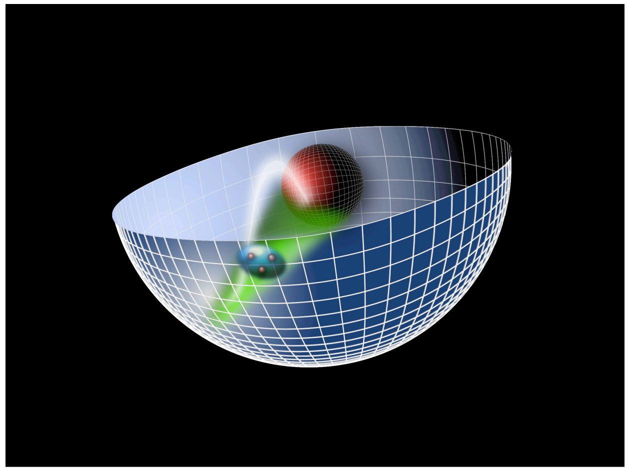
$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

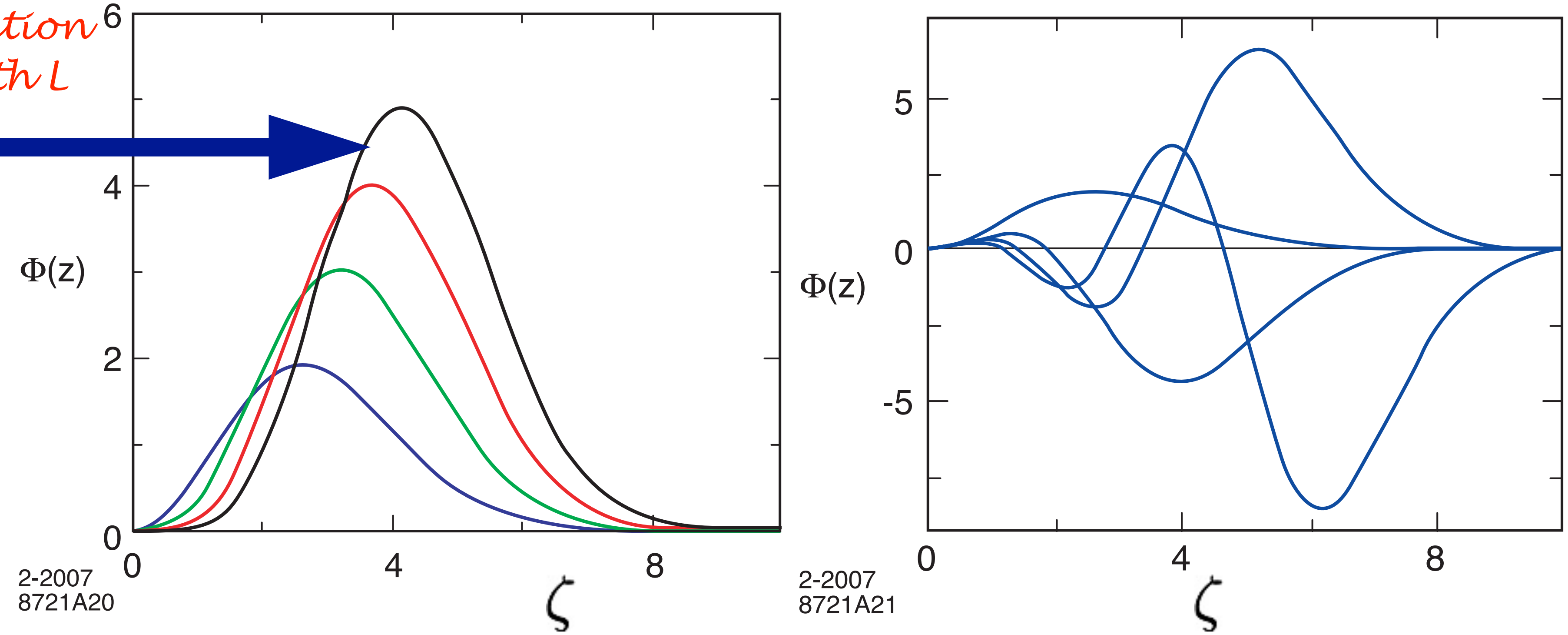
$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\boxed{\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)}$$

G. de Teramond, H. G. Dosch, sjb

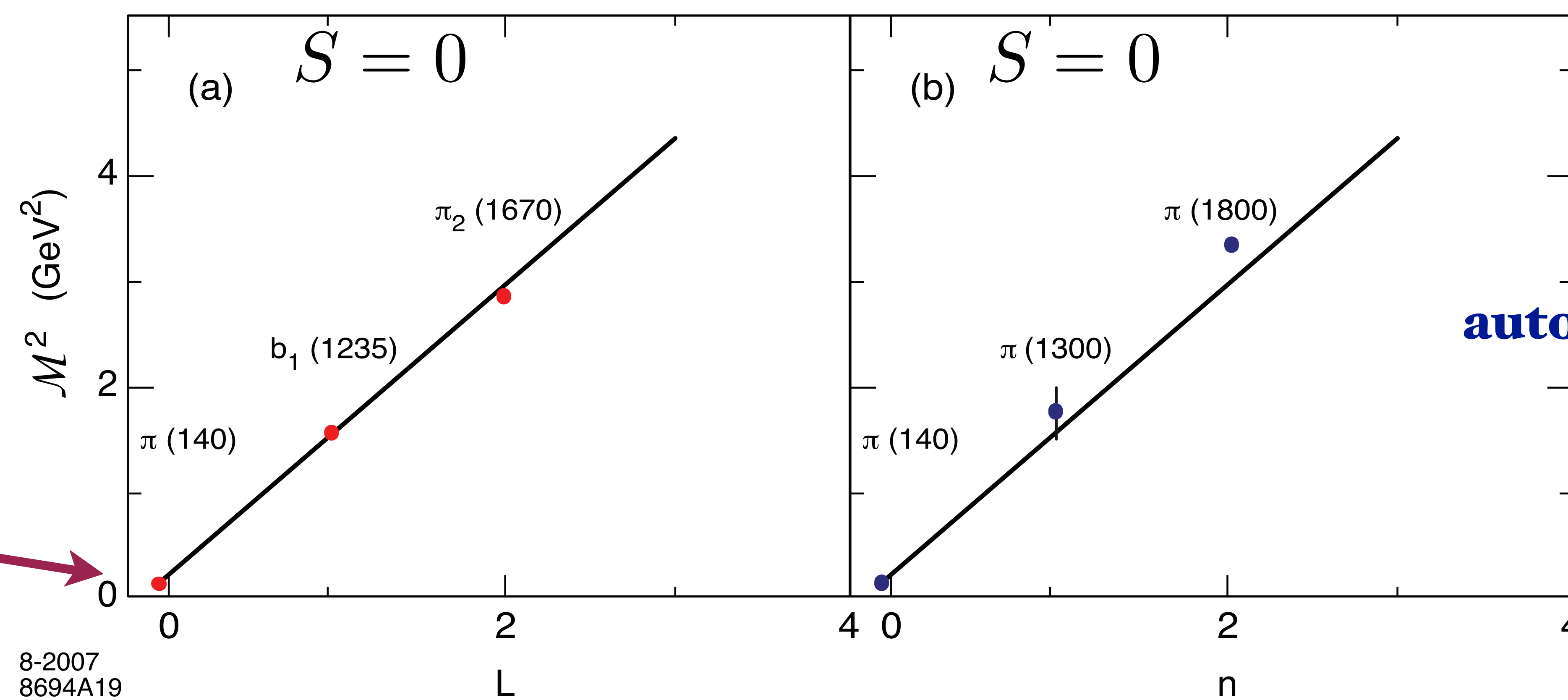
Quark separation increases with L



Soft Wall Model

Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Same slope in n and L!



Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

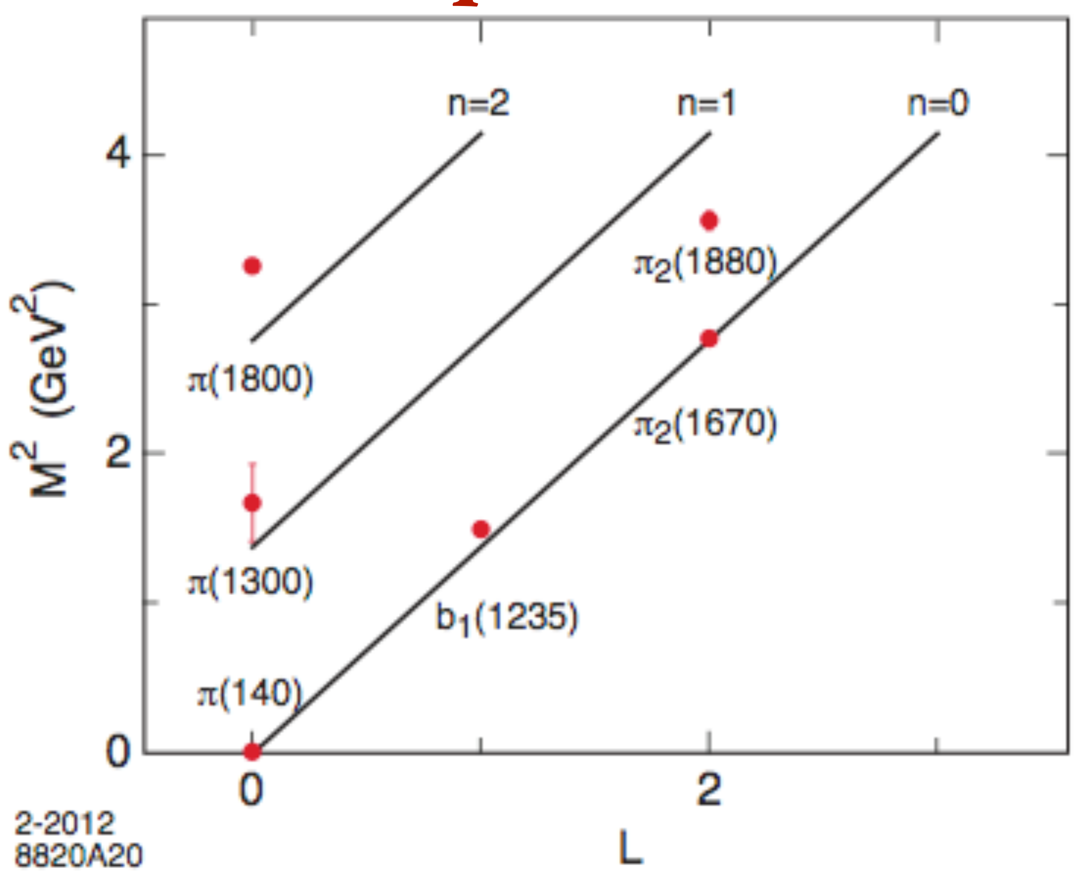
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

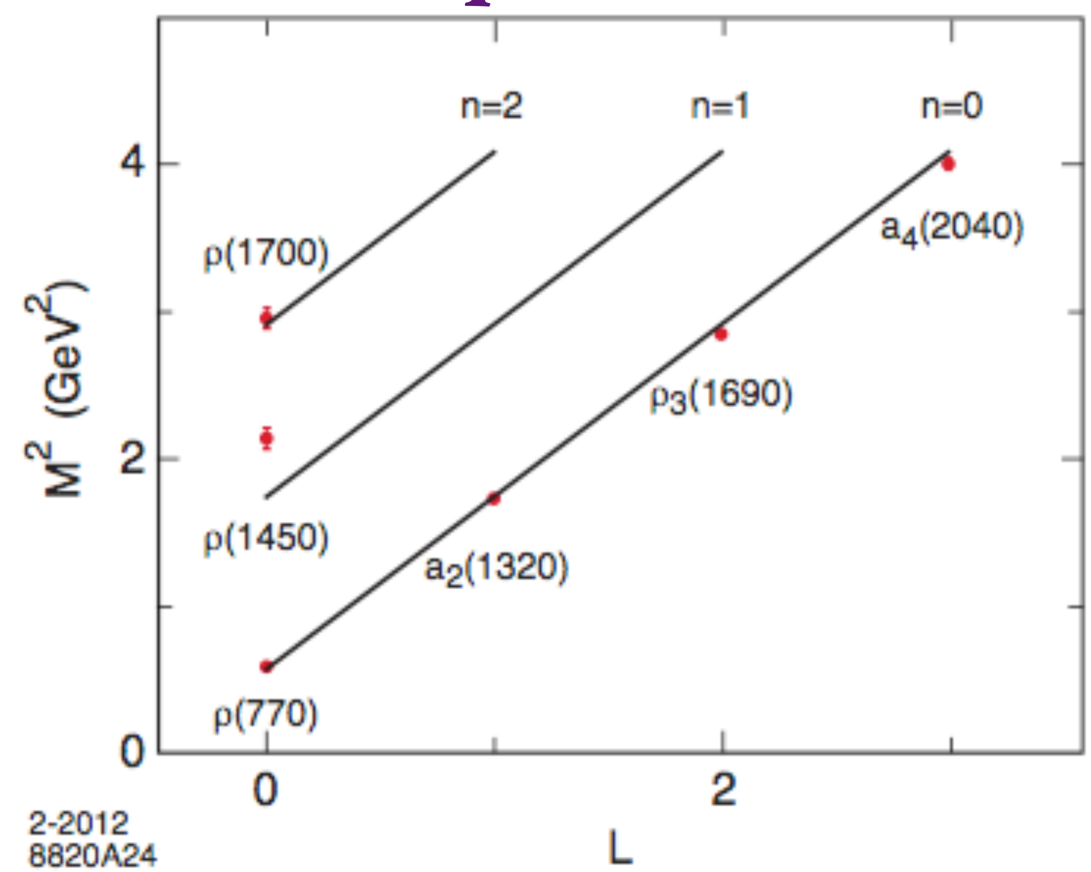
- $J = L + S, I = 1$ meson families $\boxed{\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)}$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$$\boxed{m_q = 0}$$

Massless pion in Chiral Limit! ***Same slope in n and L!***





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

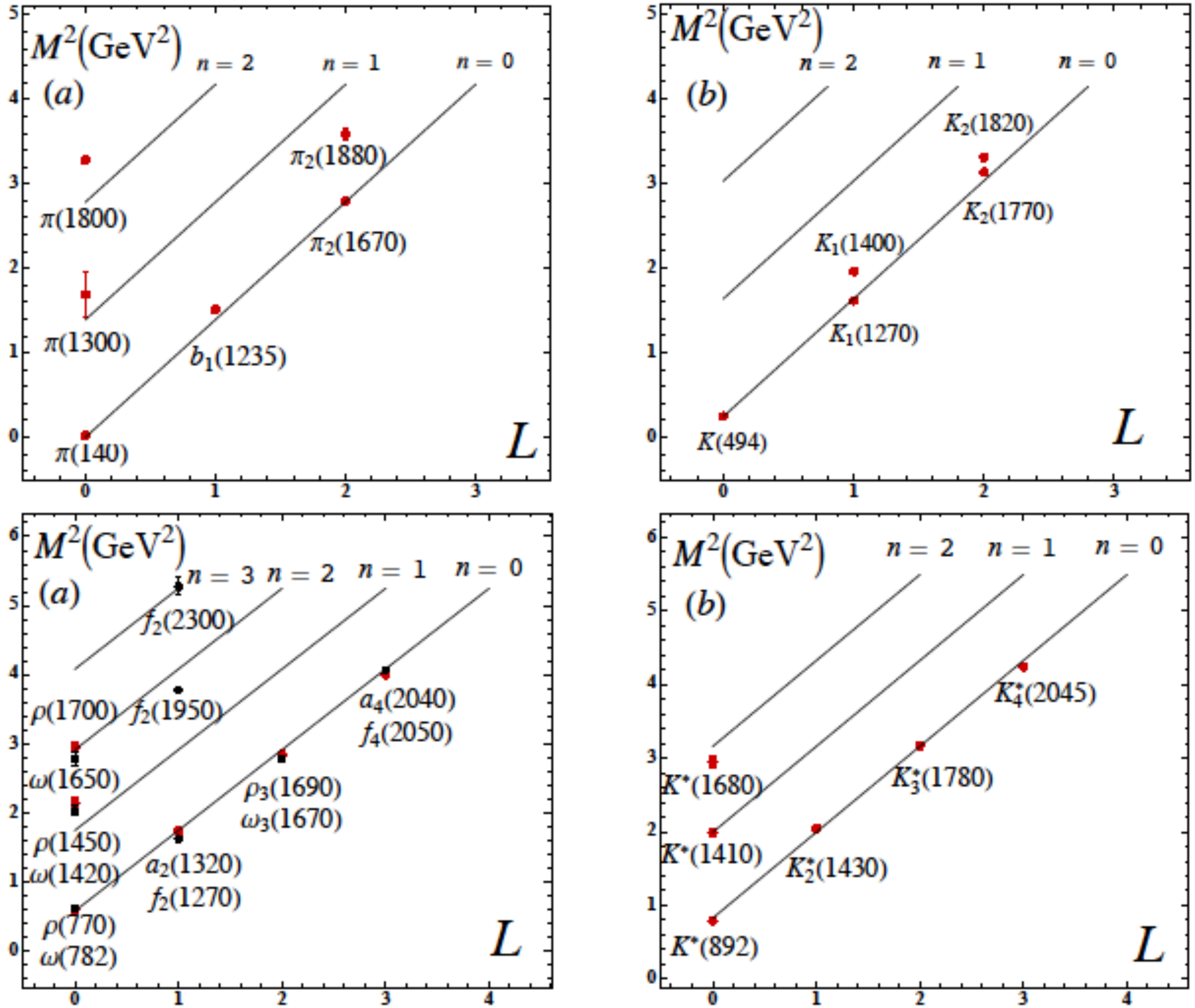
Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

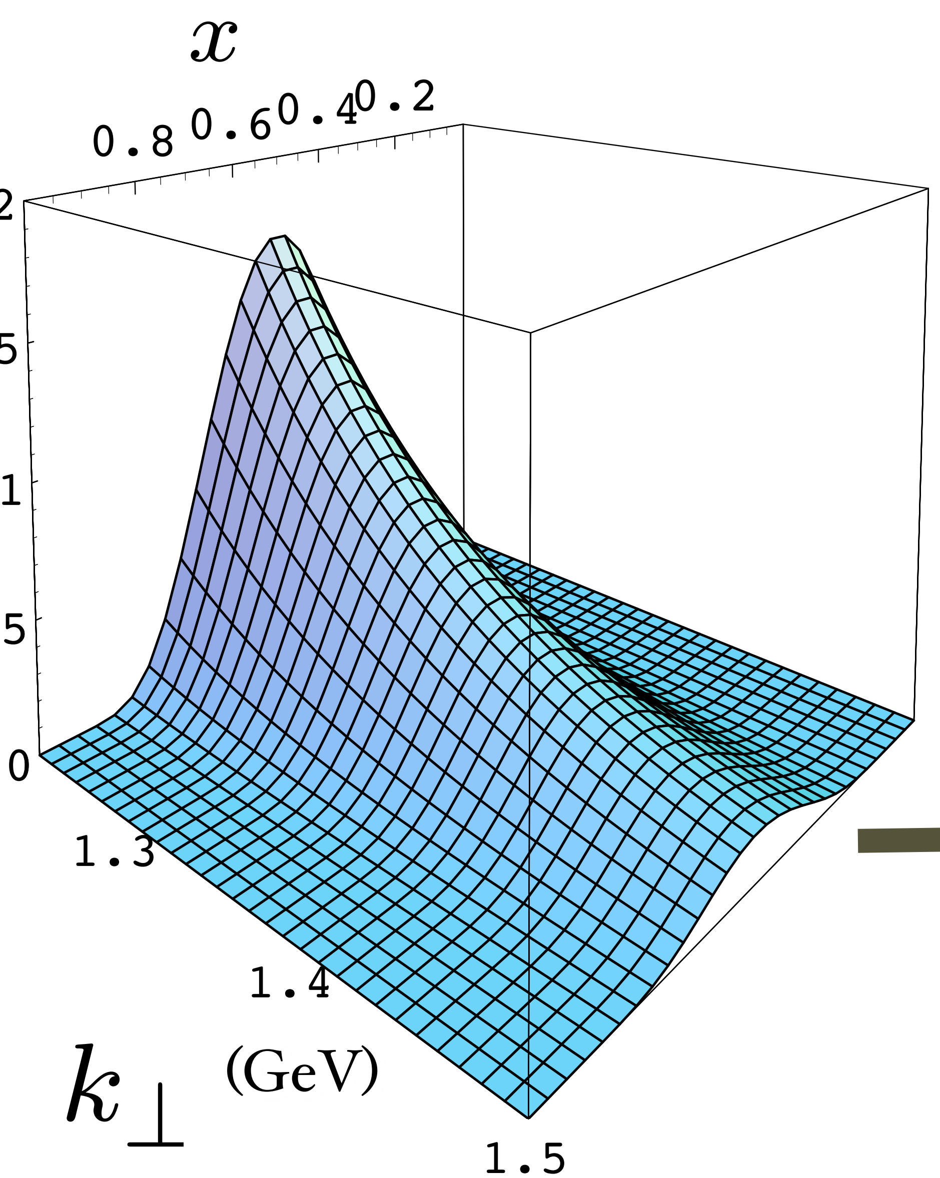
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



Prediction from AdS/QCD: Meson LFWF

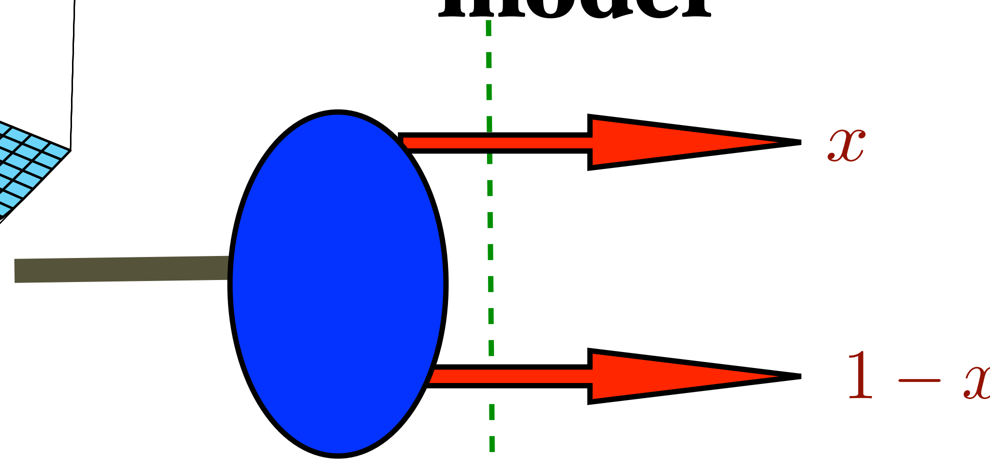
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE!

Provides Connection of Confinement to Hadron Structure

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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(Received 5 April 2012; published 20 August 2012)

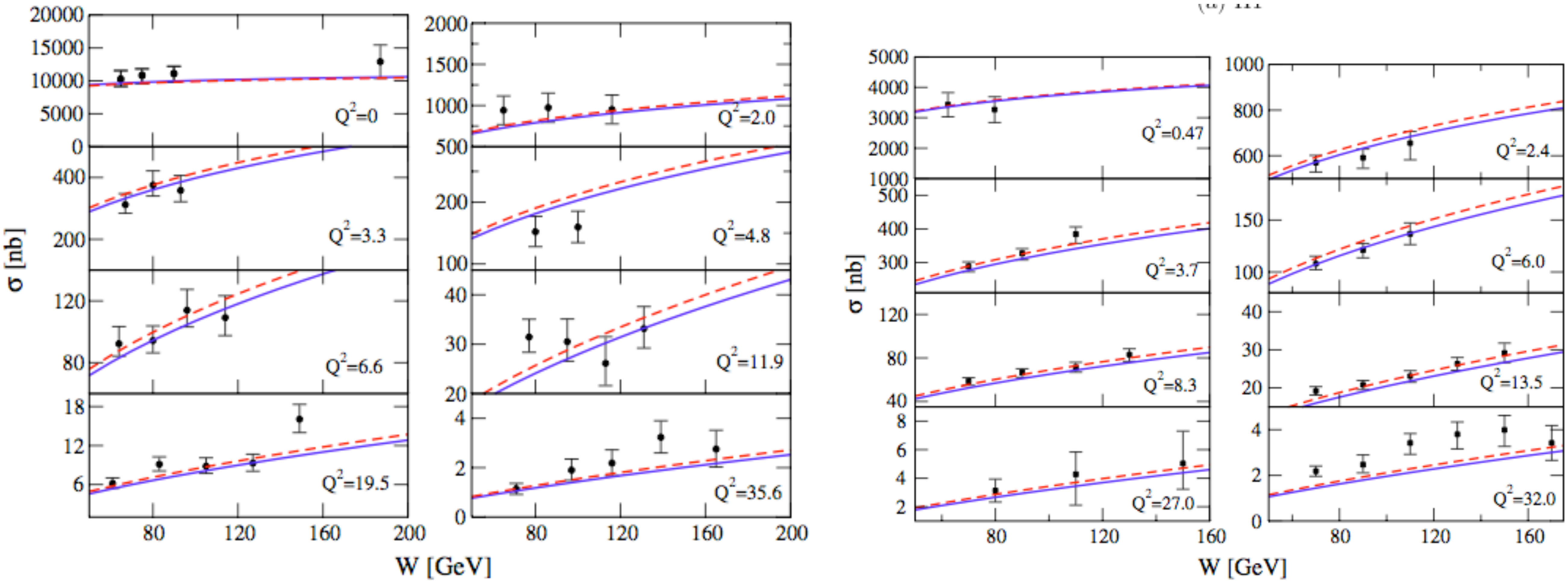
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x,k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

See also Ferreira and Dosch

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

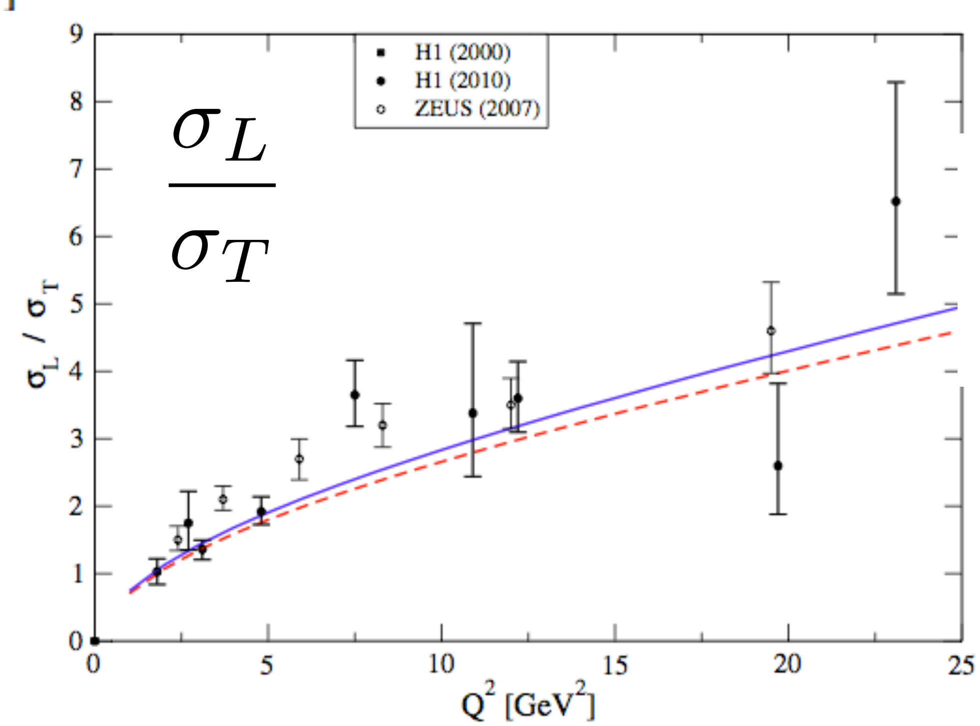


(a) H

(b) ZEUS

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

See also Ferreira and Dosch

Uniqueness

de Tèramond, Dosch, sjb

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \qquad e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **ζ^2 confinement potential and dilaton profile unique!**
- **Linear Regge trajectories in n and L: same slope!**
- **Massless pion in chiral limit!   No vacuum condensate!**
- **Conformally invariant action for massless quarks retained despite mass scale**
- **Same principle, equation of motion as de Alfaro, Furlan, Fubini, Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569**

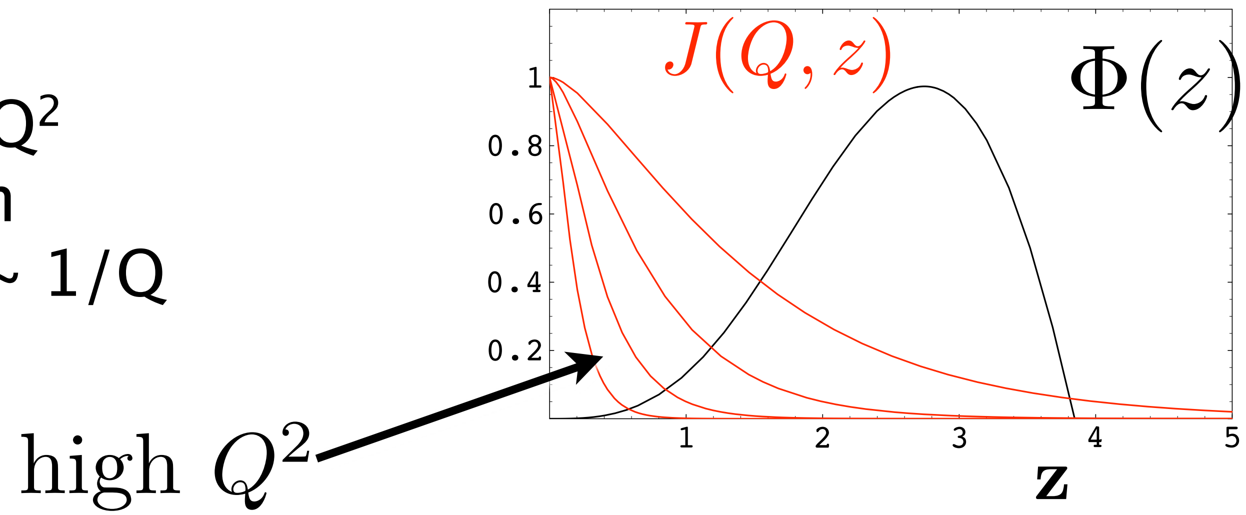
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

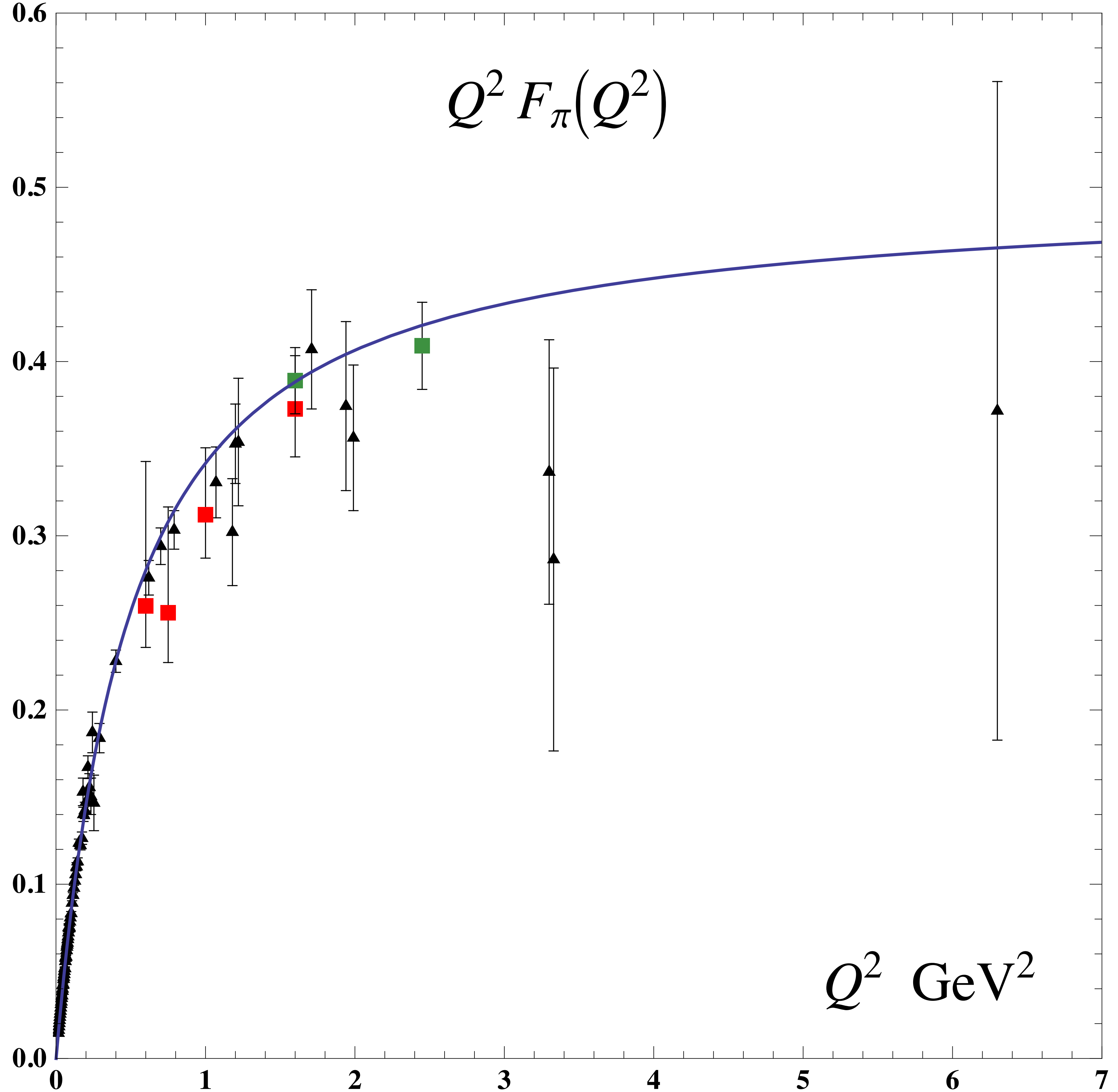
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$.

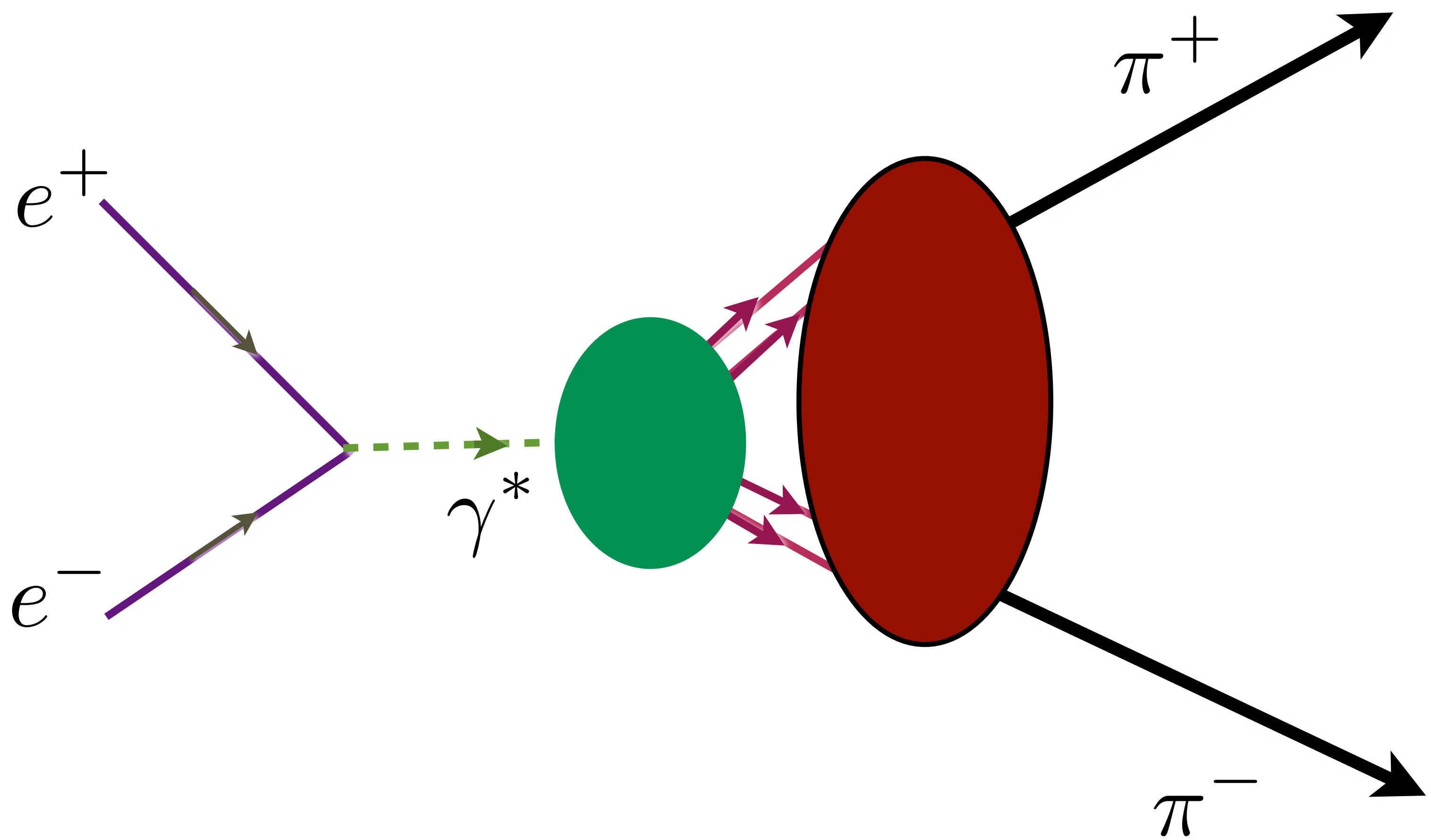
Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

$$\text{Twist } \tau = n + L$$

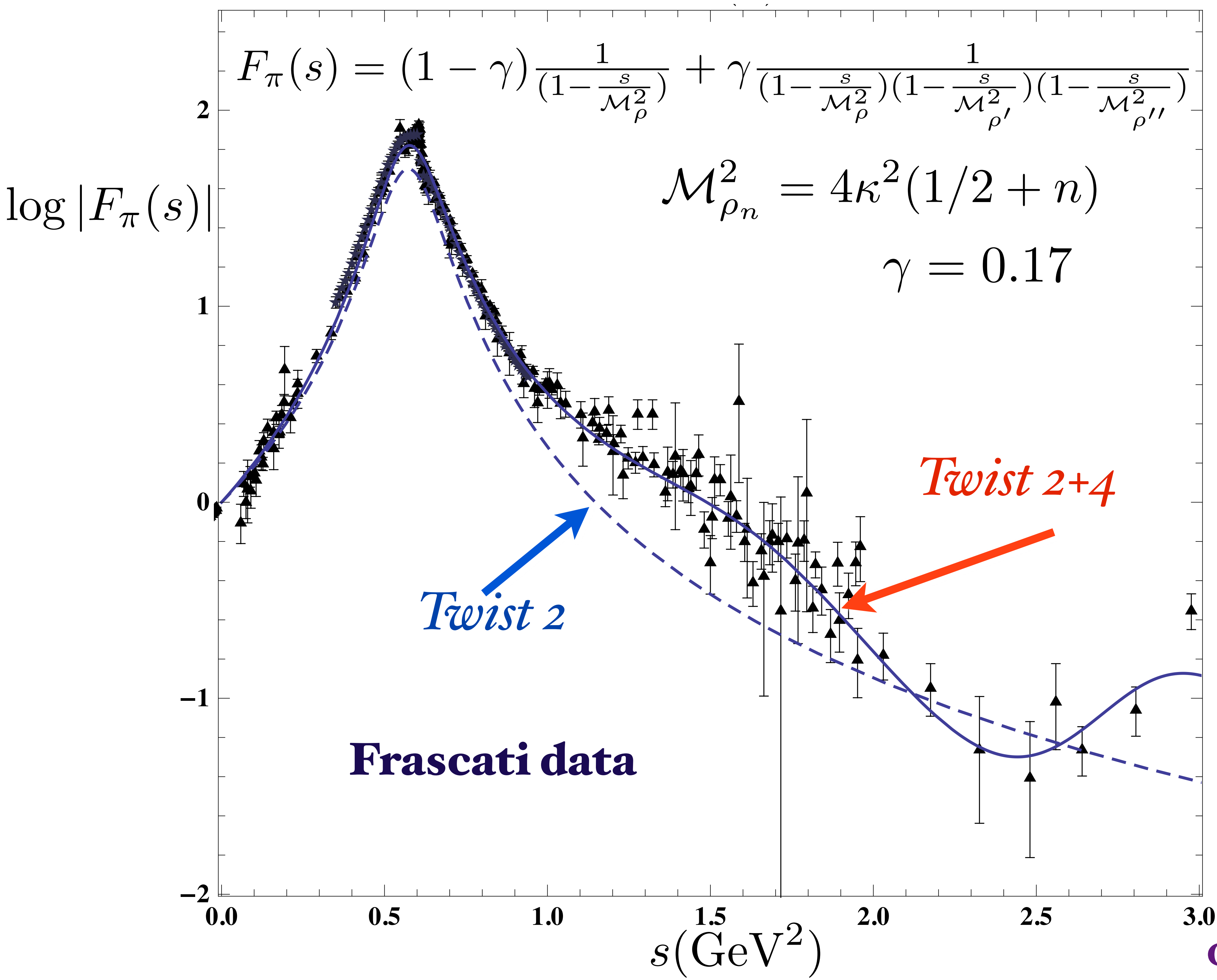


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



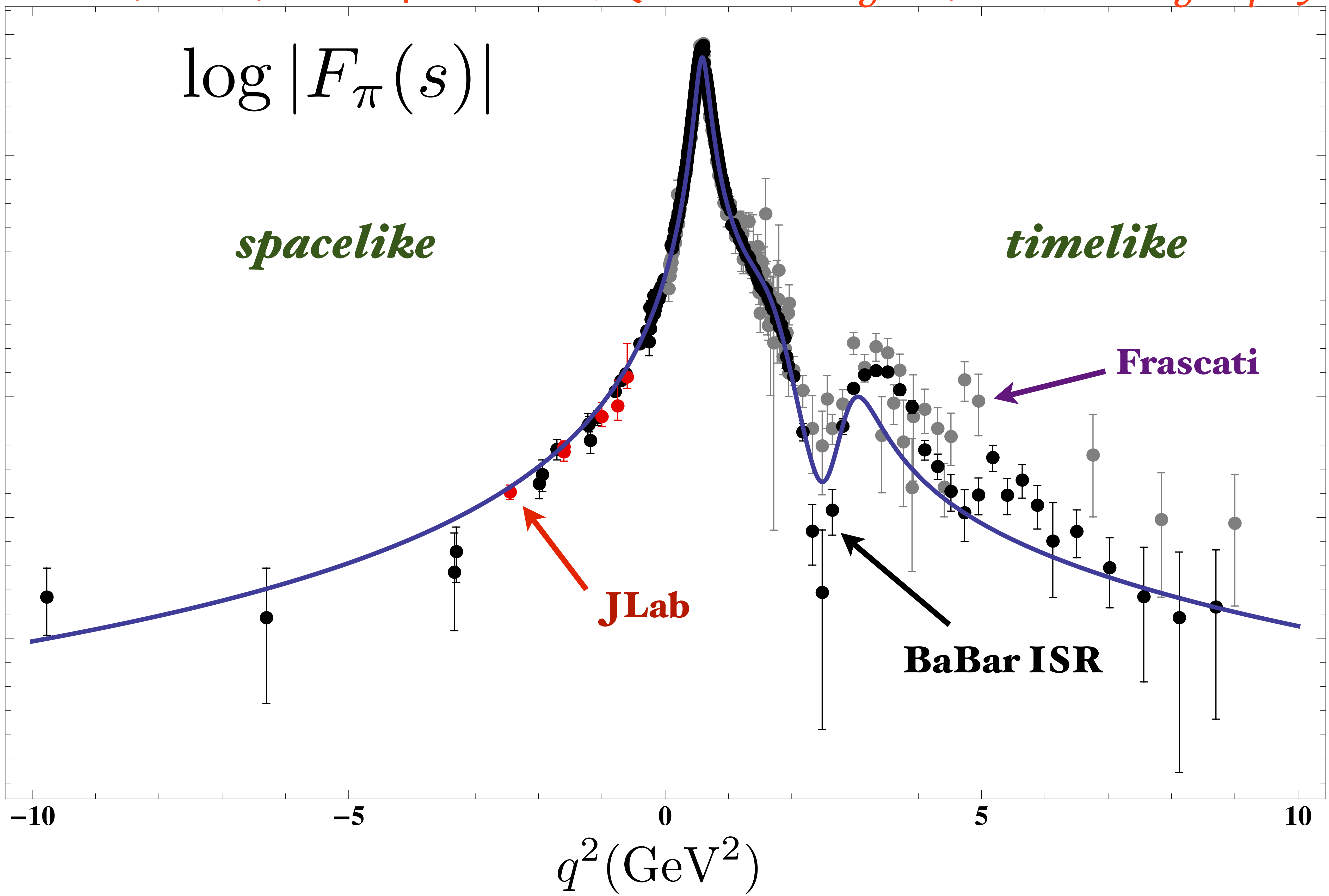
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography

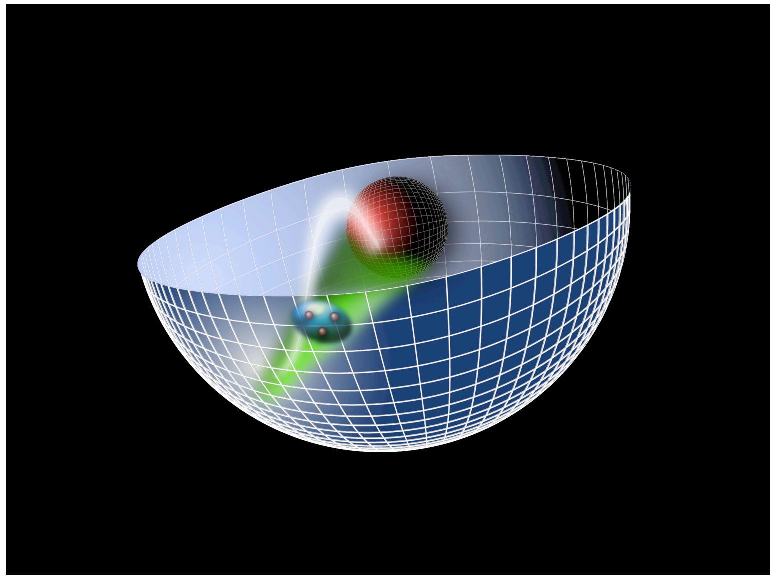


Remarkable Features of Light-Front Schrödinger Equation

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$\boxed{U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)}$$

$\phi(z)$

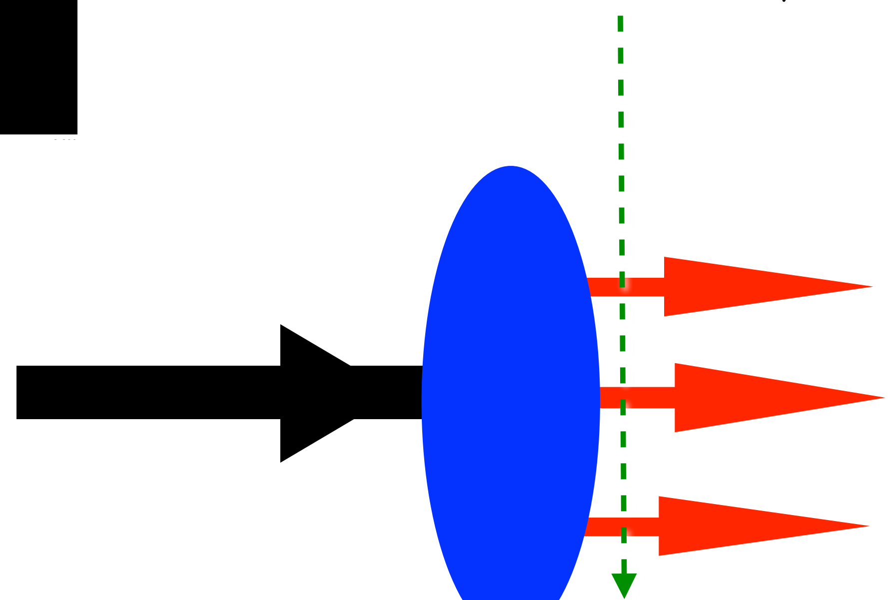


AdS5: Conformal Template for QCD

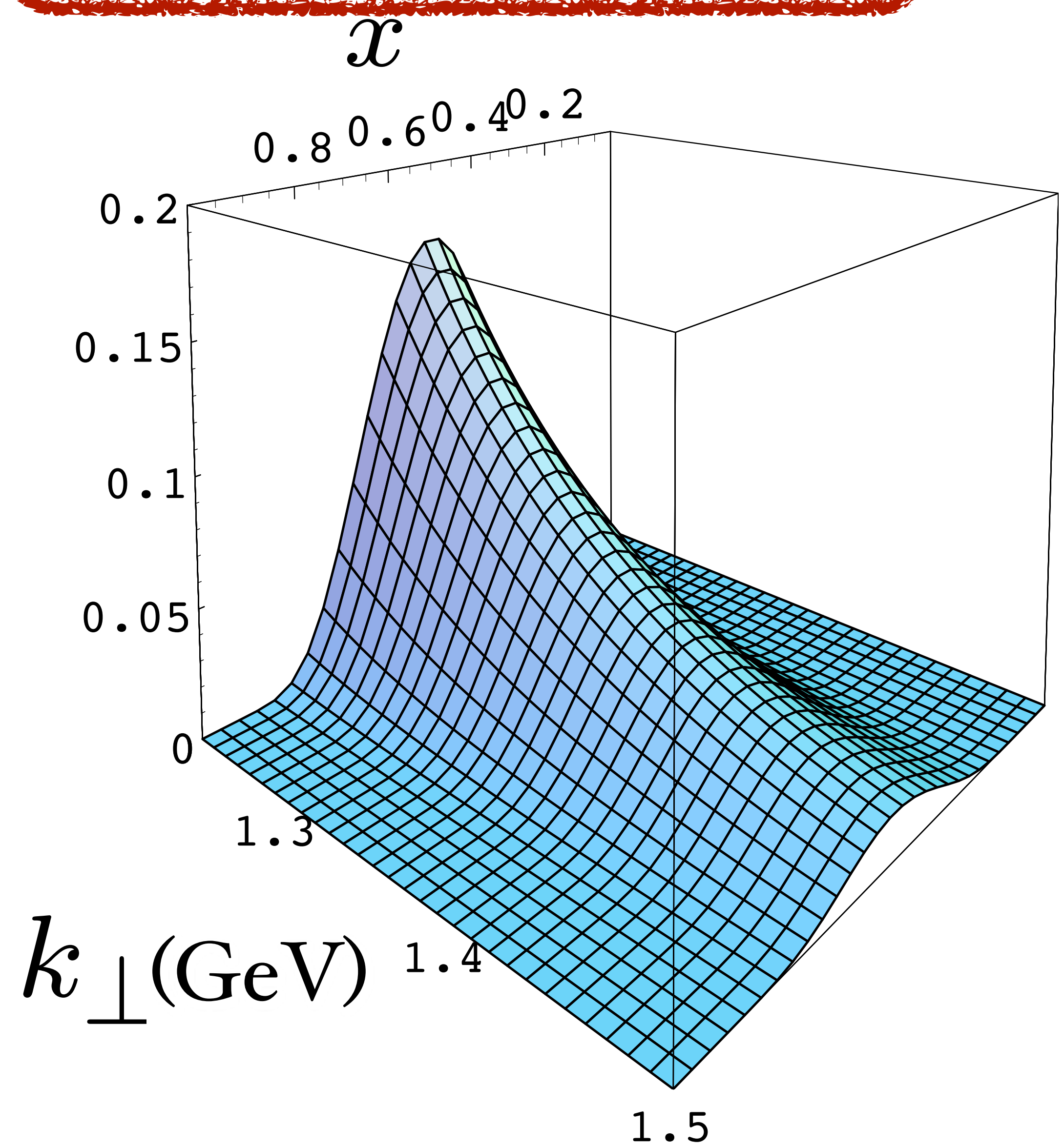
- *Light-Front Holography*

Duality of AdS$_5$ with LF Hamiltonian Theory

Fixed $\tau = t + z/c$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation

Spectroscopy and Dynamics

QCD Lagrangian

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

● **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\Big(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\Big)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

● **Dosch, de Teramond, sjb**

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $\quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

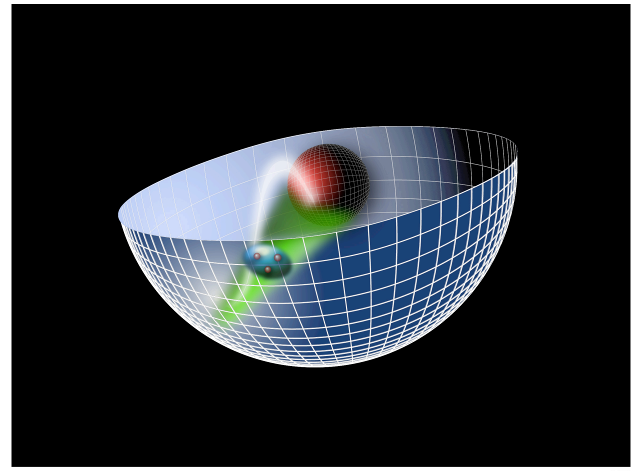
dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Conformal Symmetry
of the action

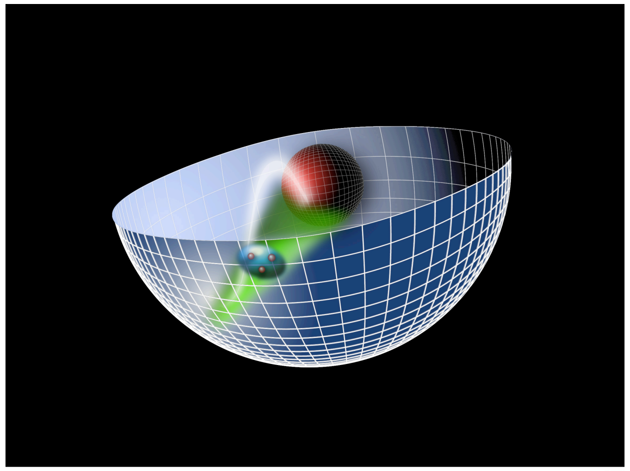
Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

de Teramond, Dosch, sjb



AdS/QCD Soft-Wall Model

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent
Unique color-confining potential
Zero mass pion for massless quarks
Regge trajectories with equal slopes in n and L
Light-Front Wavefunctions

Conformal Symmetry of the action

Light-Front Schrödinger Equation

Fubini and Rabinovici

Superconformal Algebra

de Teramond
Dosch
and SJB

1+1

$$\{\psi, \psi^+\} = 1$$

two anti-commuting fermionic operators

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \qquad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\boxed{Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

Superconformal Algebra

Baryon Equation

$\boxed{\text{Consider } R_w = Q + wS;}$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \big(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \big)$$

$$G_{22} = \big(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \big)$$

$$\boxed{\text{Identify } f - \tfrac{1}{2} = L_B \ , \quad w = \kappa^2}$$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

$x \to \zeta$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ \quad = \quad M^2\,\psi_J^+,$$

G_{22}

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- \quad = \quad M^2\,\psi_J^-.$$

G_{11}

$$\boxed{M_B^2(N, L_B) = 4\lambda_B(n + L_B + 1)}$$

S=1/2, P=+

both chiralities

Meson Equation

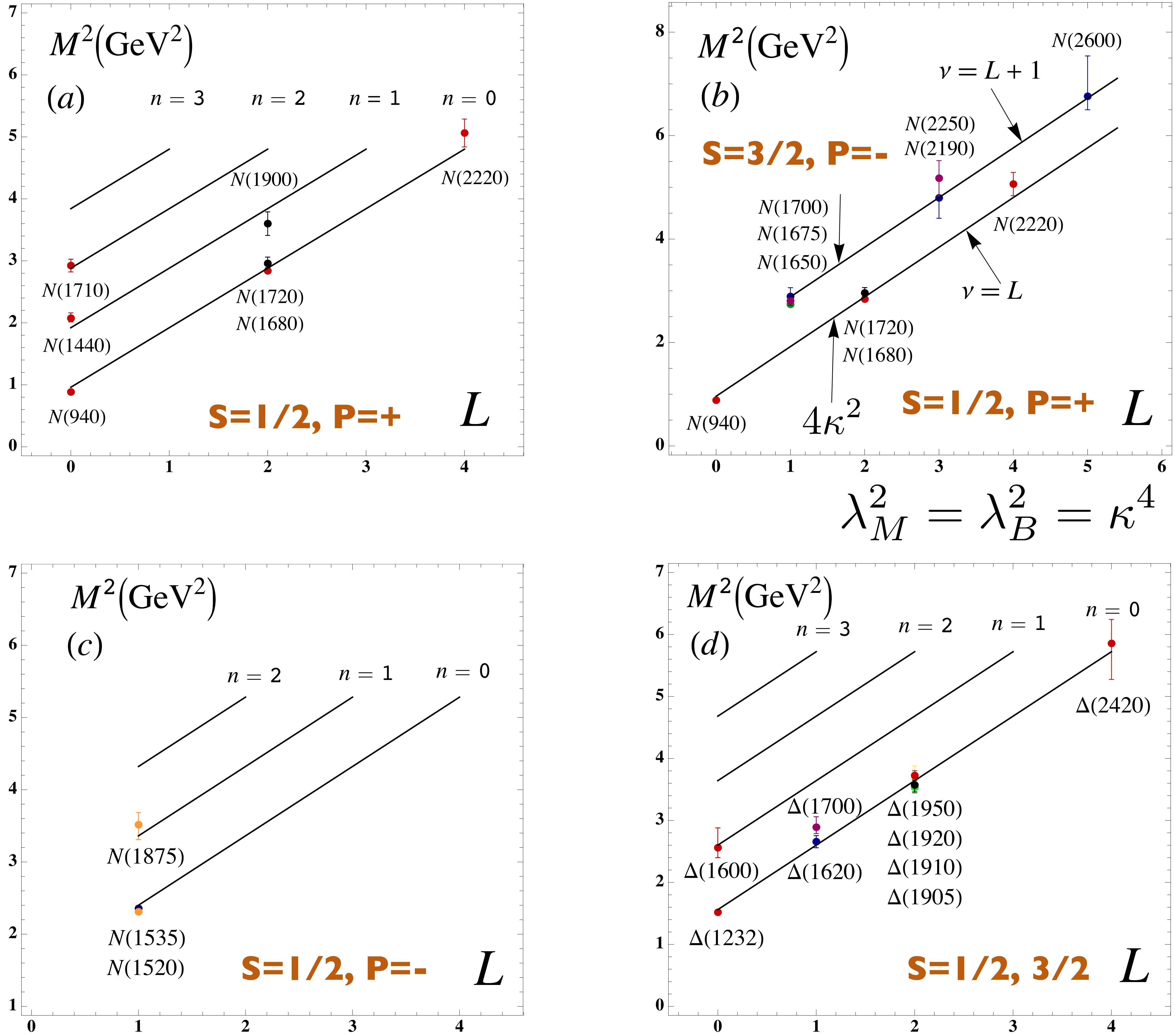
$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2\zeta^2 + 2\lambda_M(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_j$$

G_{11}

$$\boxed{M_M^2(N, L_M, S=0) = 4\lambda_M(n + L_M)}$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$ $\quad \lambda_M^2 = \lambda_B^2 = \kappa^4$

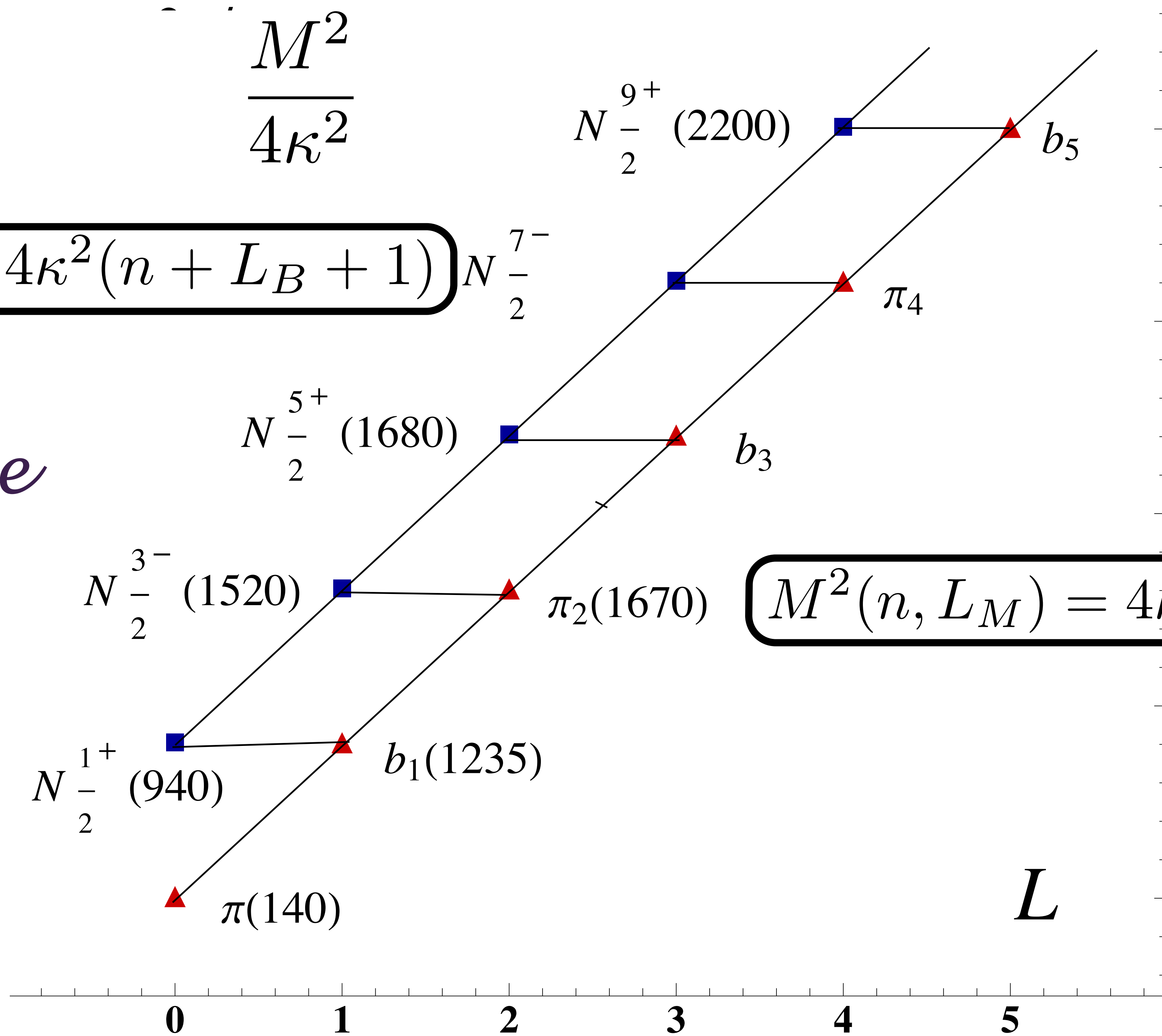


$M^2(\mathrm{GeV}^2)$
(a)
$n=3$
$n=2$
$n=1$
$n=0$
$N(1900)$
$N(2220)$
$N(1710)$
$N(1440)$
$N(1720)$
$N(1680)$
$N(940)$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
$\nu = L+1$
$N(2600)$
$N(2250)$
$N(2190)$
S=3/2, P=-
$N(1700)$
$N(1675)$
$N(1650)$
$N(2220)$
$\nu = L$
$N(1720)$
$N(1680)$
$N(940)$
$4\kappa^2$
S=1/2, P=+
L
$\lambda_M^2 = \lambda_B^2 = \kappa^4$
$M^2(\mathrm{GeV}^2)$
(c)
$n=2$
$n=1$
$n=0$
$N(1875)$
$N(1535)$
$N(1520)$
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
$n=3$
$n=2$
$n=1$
$n=0$
$\Delta(2420)$
$\Delta(1700)$
$\Delta(1950)$
$\Delta(1920)$
$\Delta(1910)$
$\Delta(1905)$
$\Delta(1600)$
$\Delta(1620)$
$\Delta(1232)$
S=1/2, 3/2
L

Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



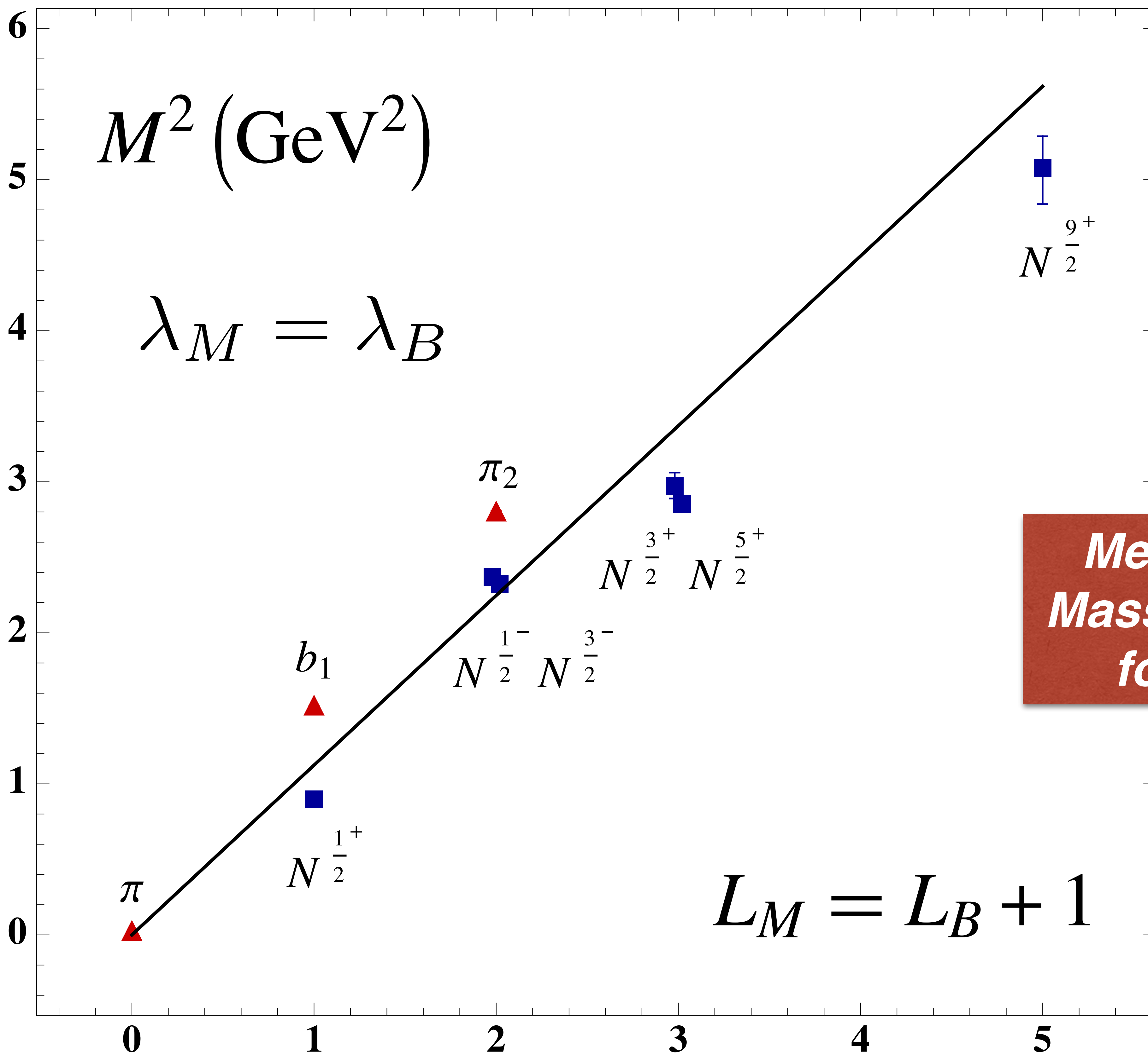
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

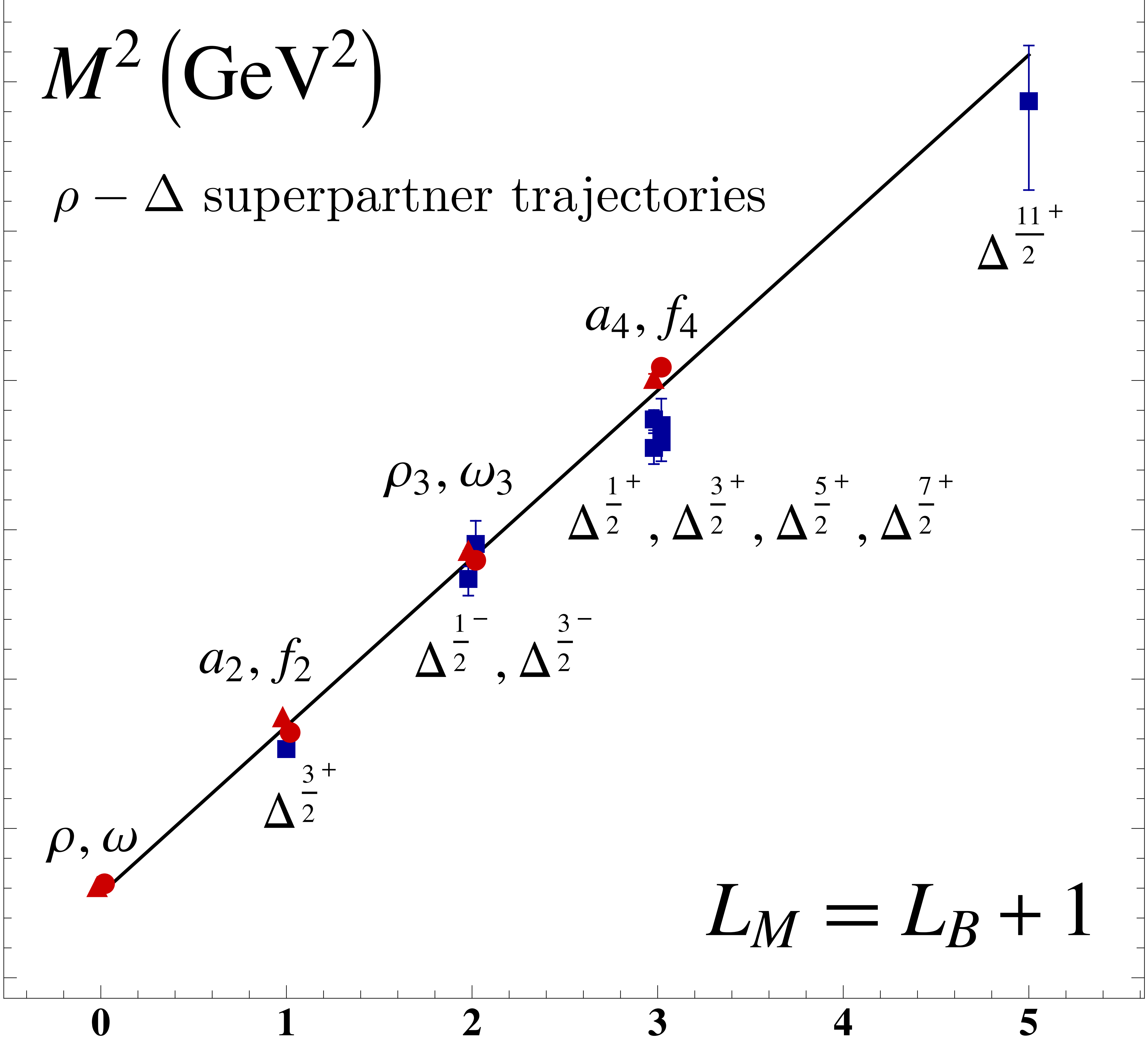
$$\lambda_M^2 = \lambda_B^2 = \kappa^4$$

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!

$\lambda = \kappa^2$



Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



Dosch, de Teramond, sjb

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$\boxed{J^z = +1/2 :< L^z >= 1/2, < S_q^z >= 0}$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

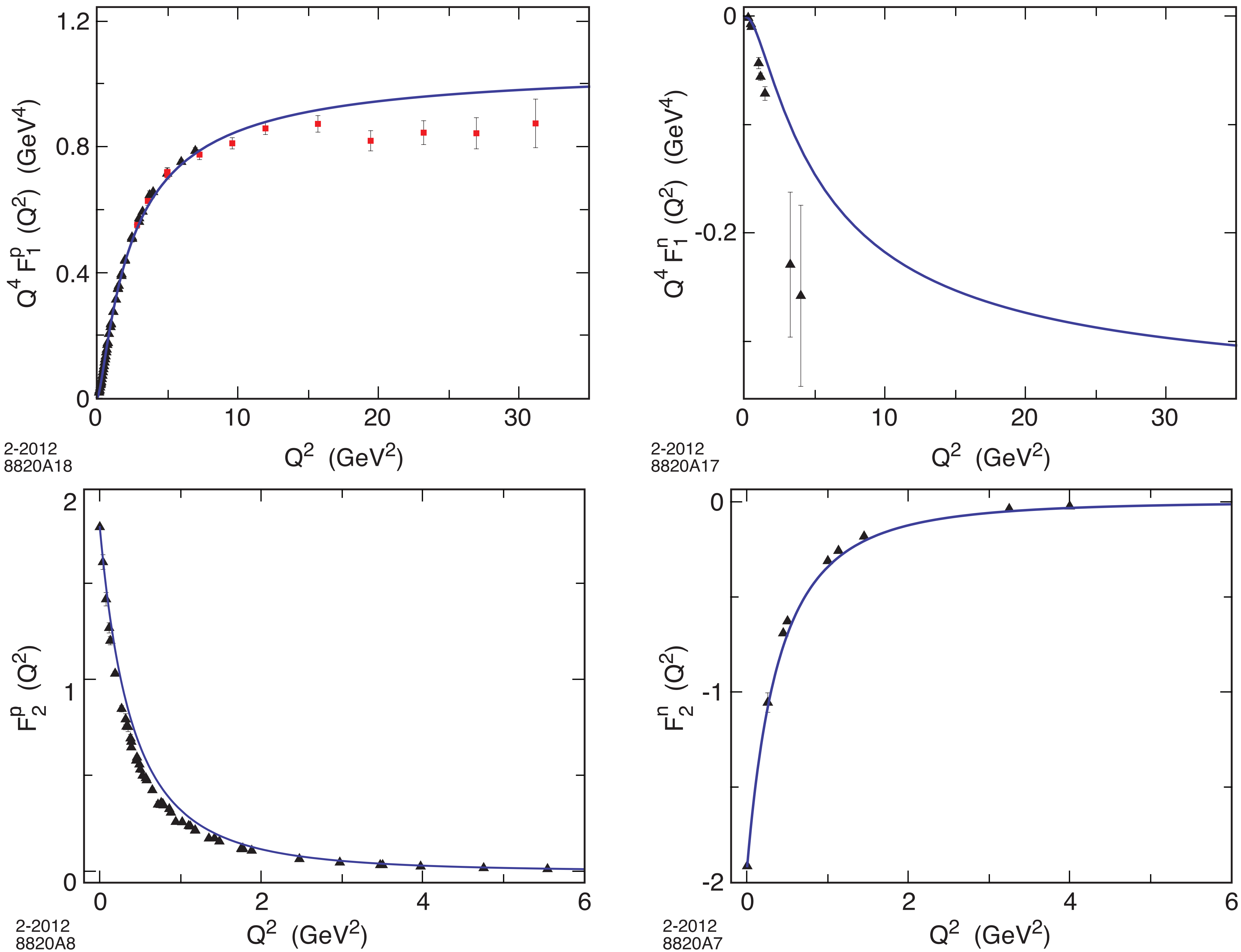
No mass -degenerate parity partners!

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue
- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\qquad S^z = \pm 1/2$
- Baryon spin carried by quark orbital angular momentum: $<J^z> = L^z + 1/2$
- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

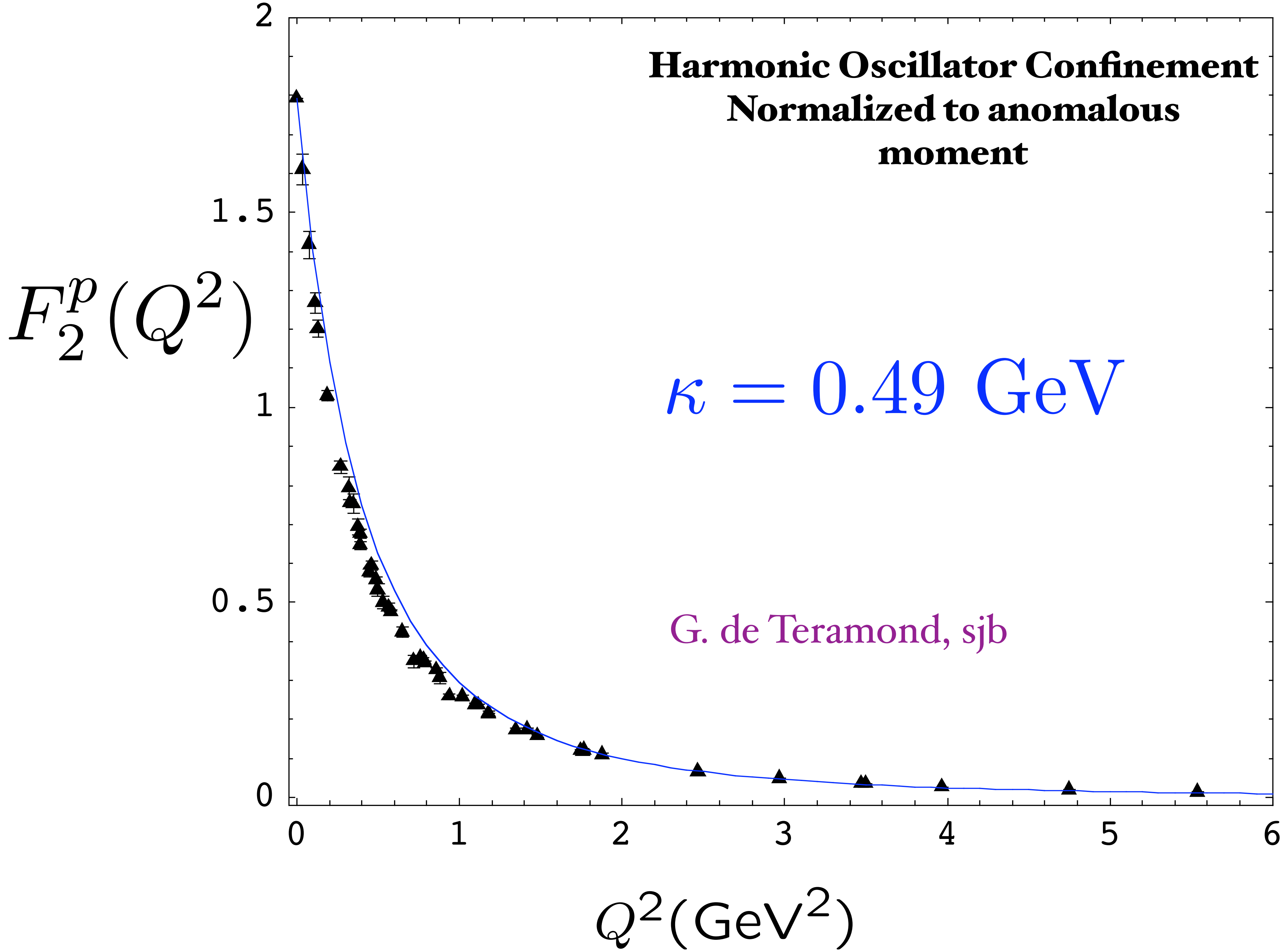
Meson and baryon have same κ !

Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

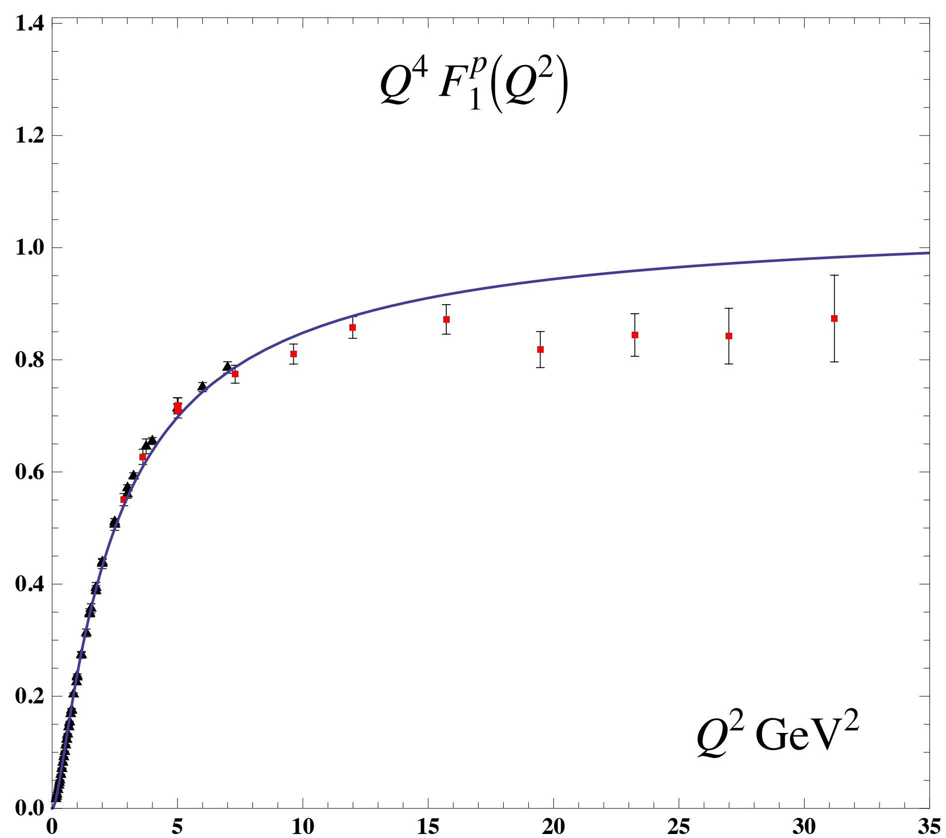
From overlap of $L = 1$ and $L = 0$ LFWFs

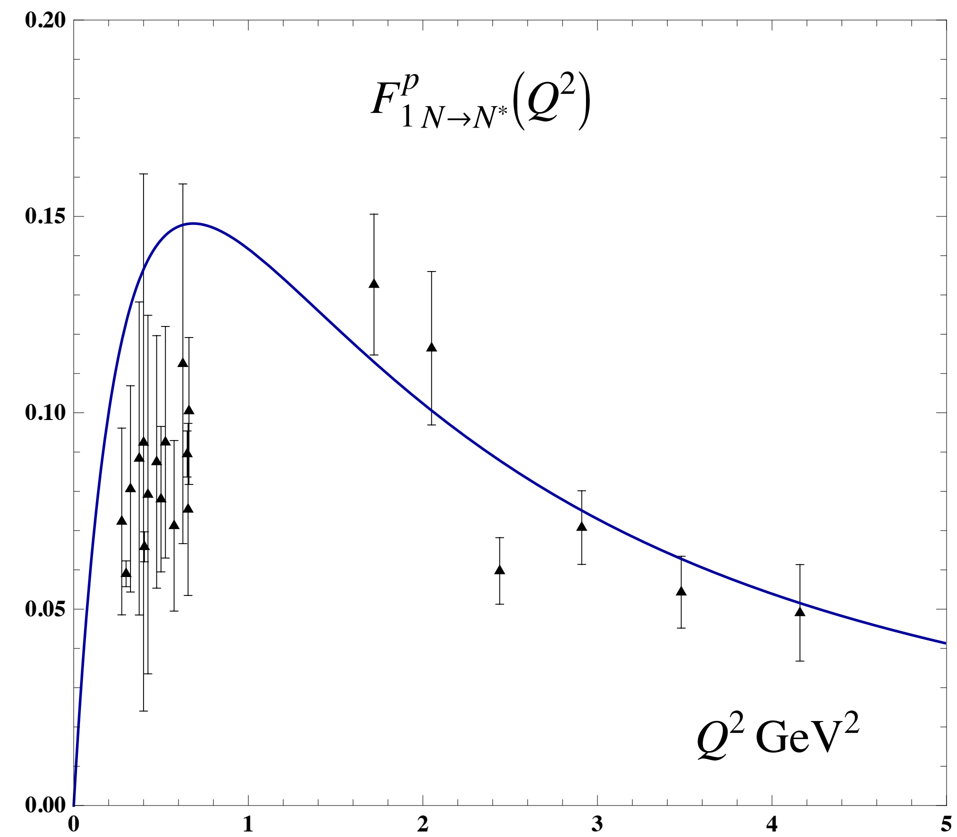


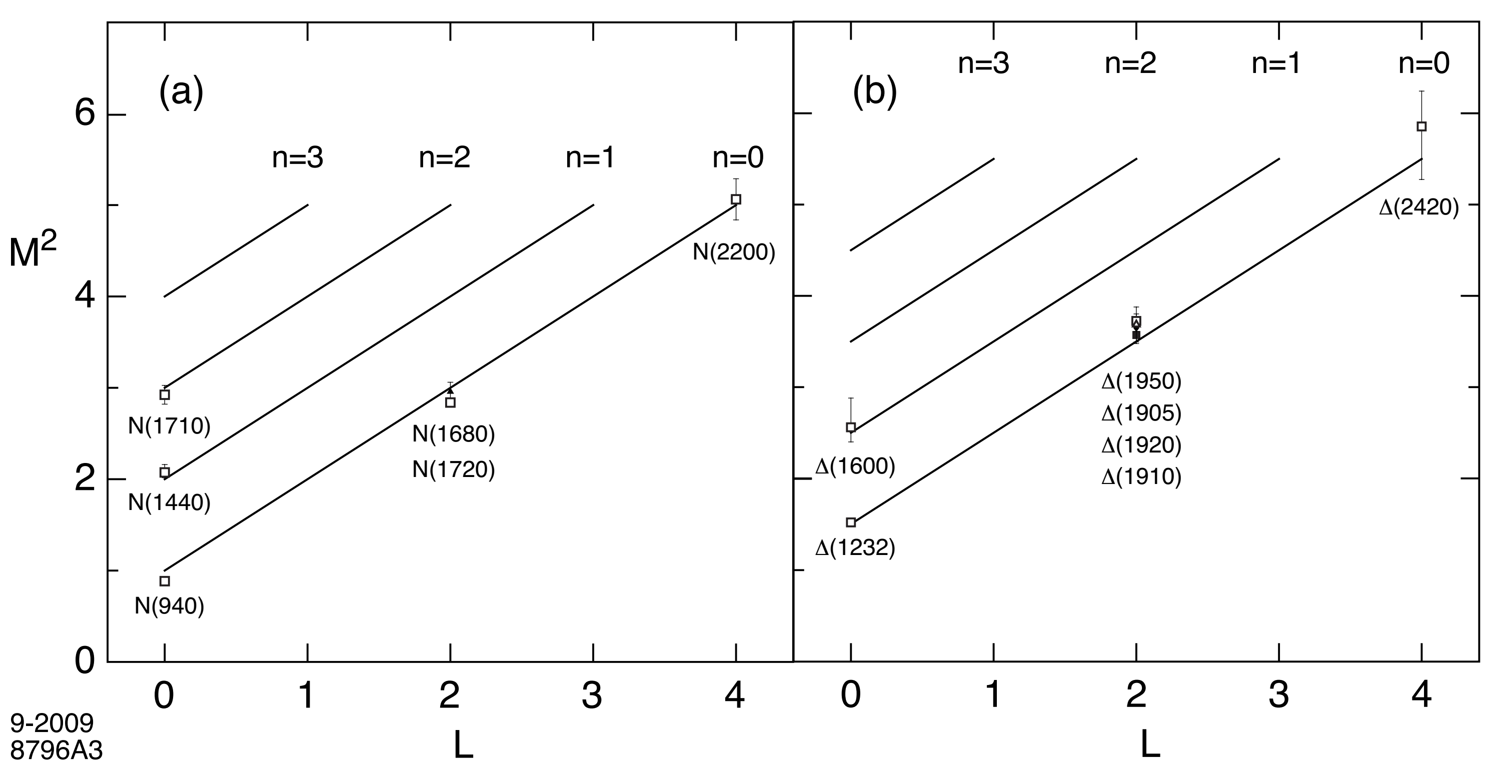
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb







Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_1{}^p_{N\to N^*}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\quad \left(F_1{}^p_{N\to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_1{}^p_{N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

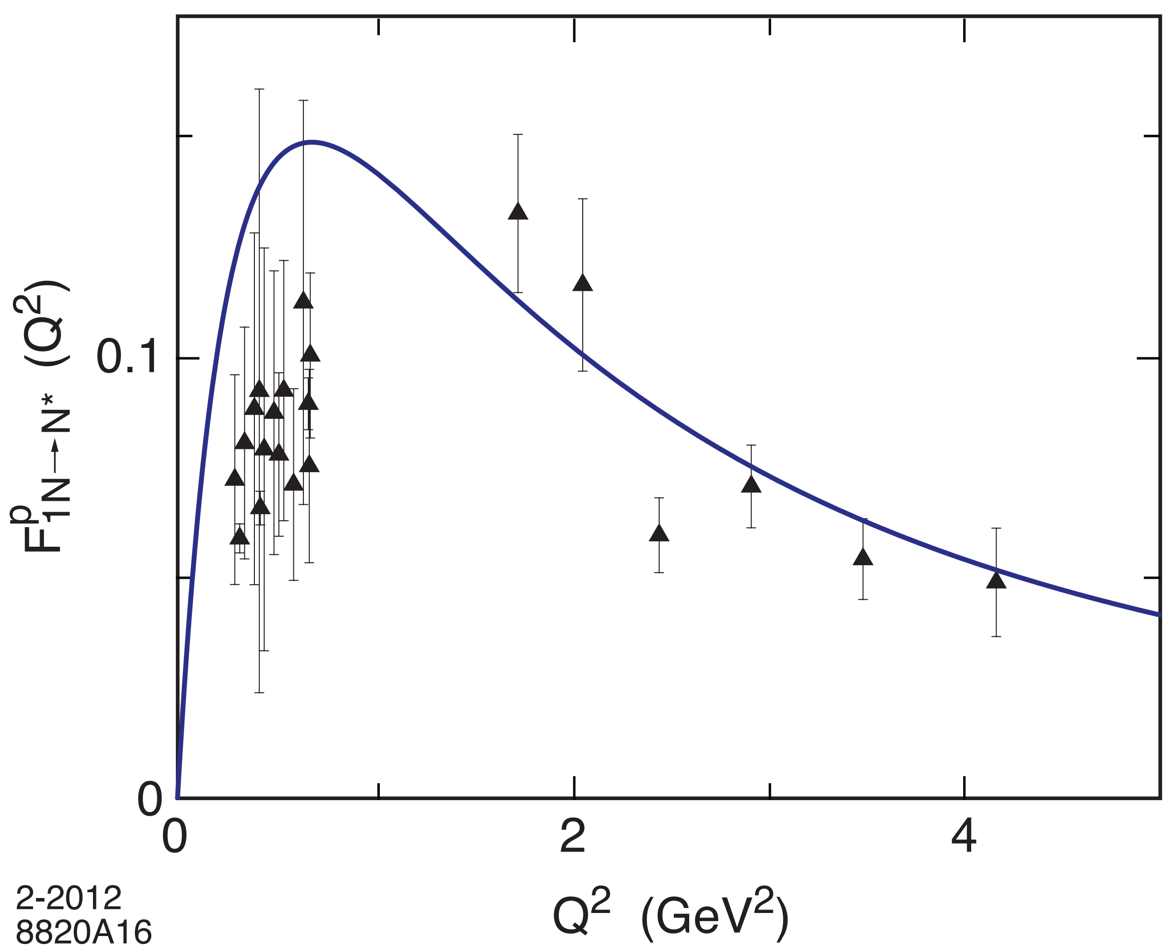
with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n + 1/2)$

de Teramond, sjb

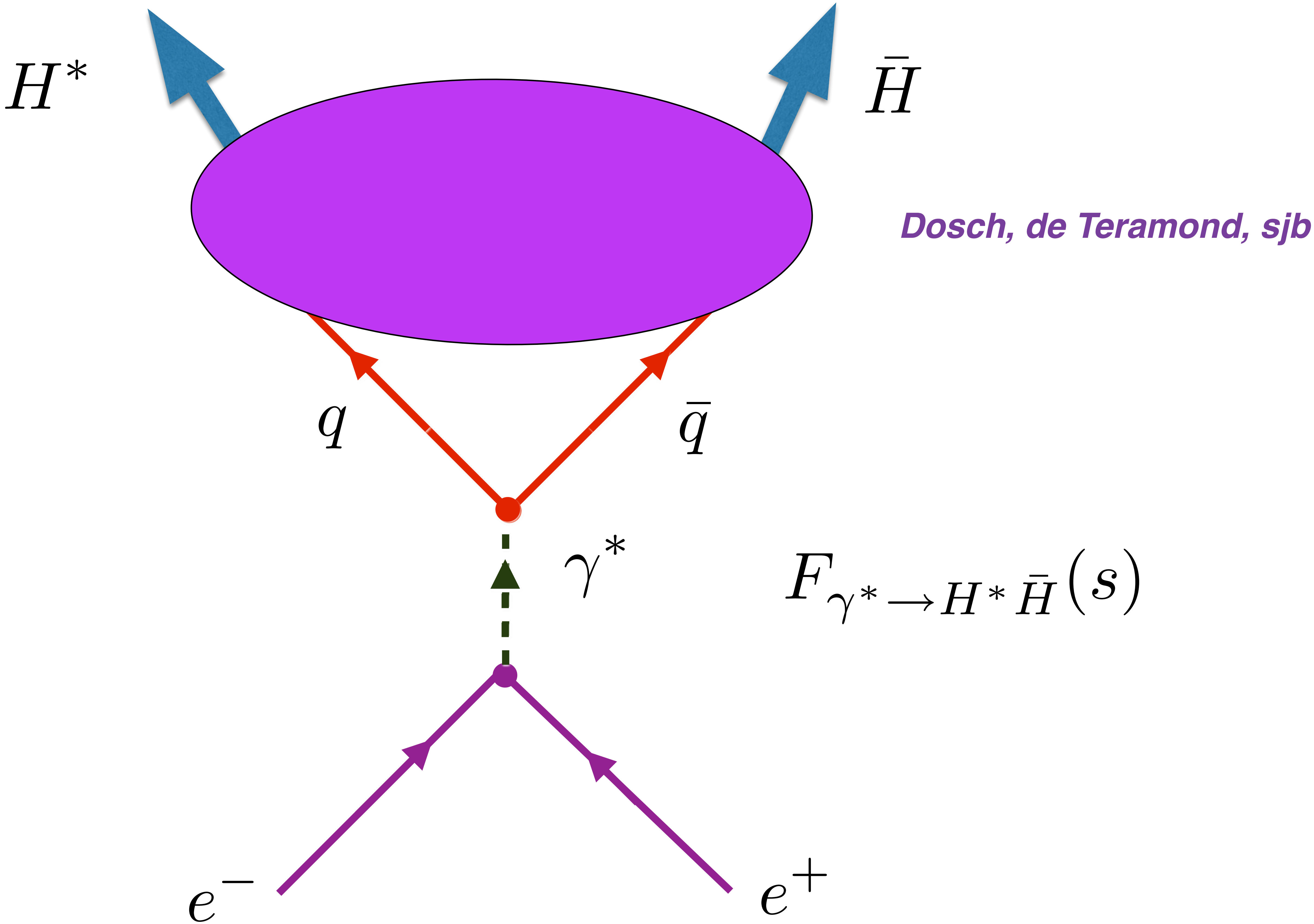
Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{\mathcal{M}^2_\rho}}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab



Prediction from Super Conformal AdS/QCD:
Same Form Factors for H= M and H=B if $L_M = L_B + 1$

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\,dz\,\sqrt{g}\,e^{\varphi(z)}\,\frac{1}{g_5^2}\,G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)}\frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

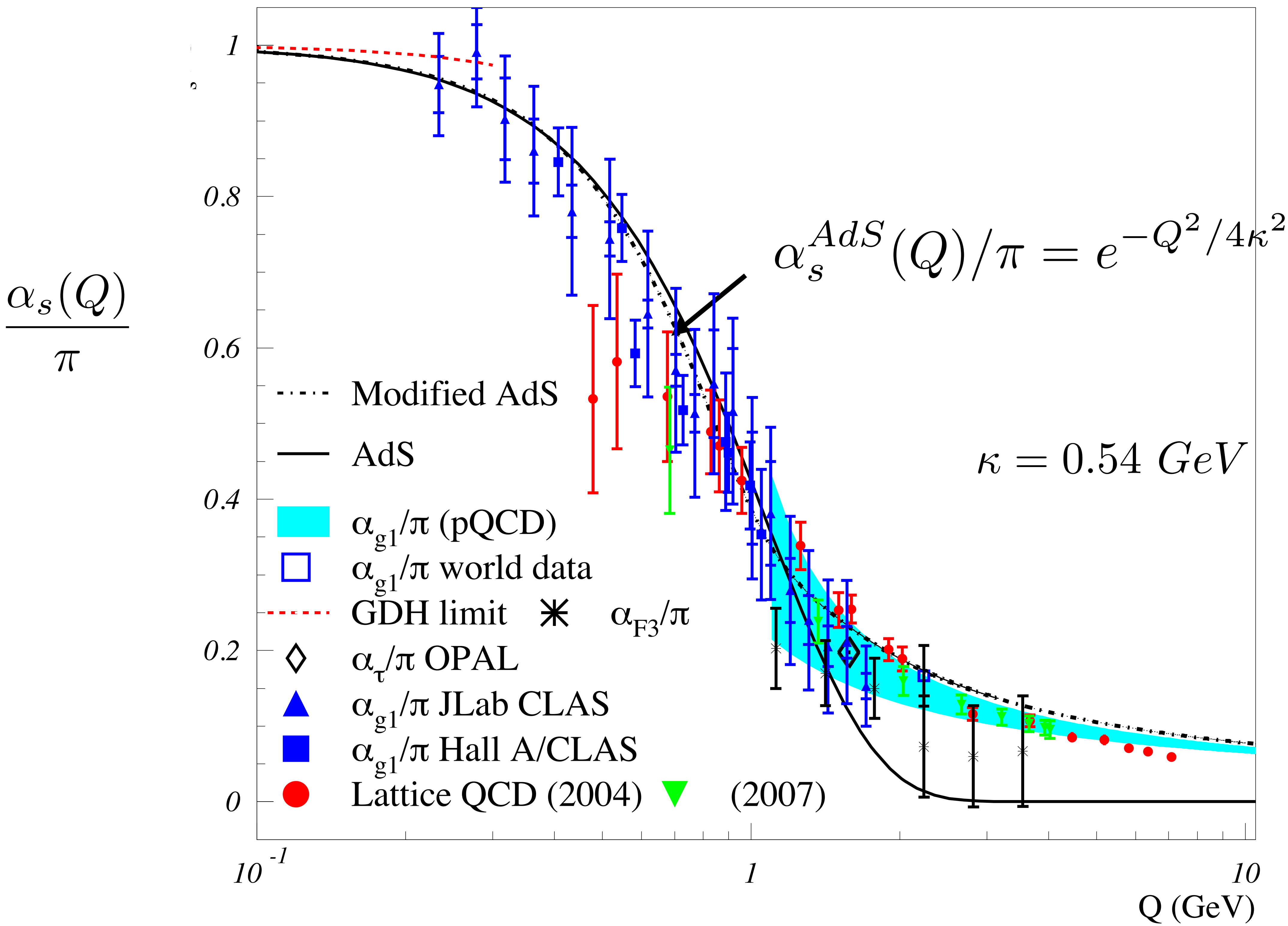
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\,\alpha_s^{AdS}(\zeta)$$

- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\,e^{-Q^2/4\kappa^2}.}$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

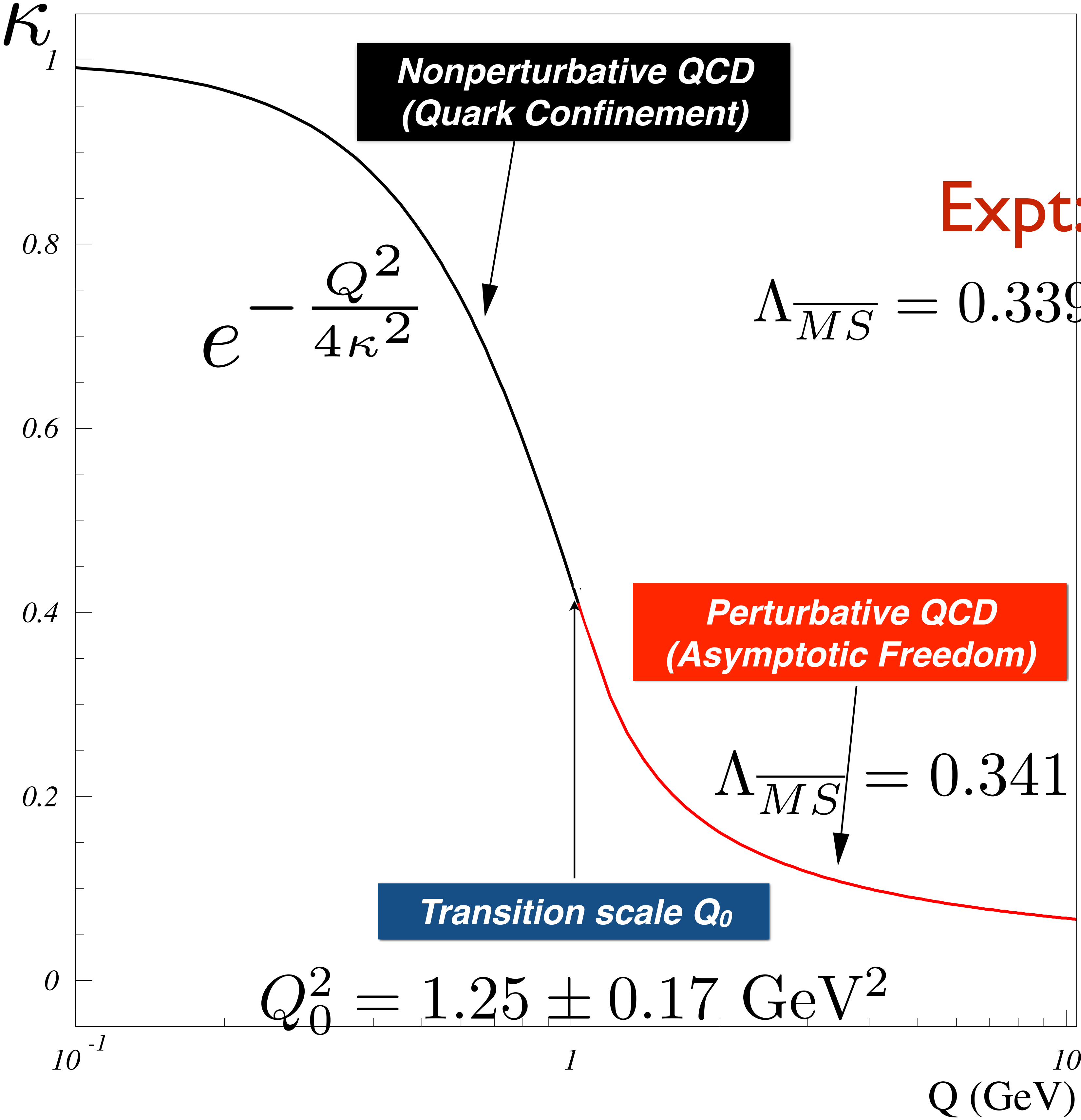
$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Teramond, sjb

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$

$$\Lambda_{\overline{MS}} = 0.341 \pm 0.024\ GeV$$

$$Q_0^2 = 1.25 \pm 0.17\ \mathrm{GeV}^2$$

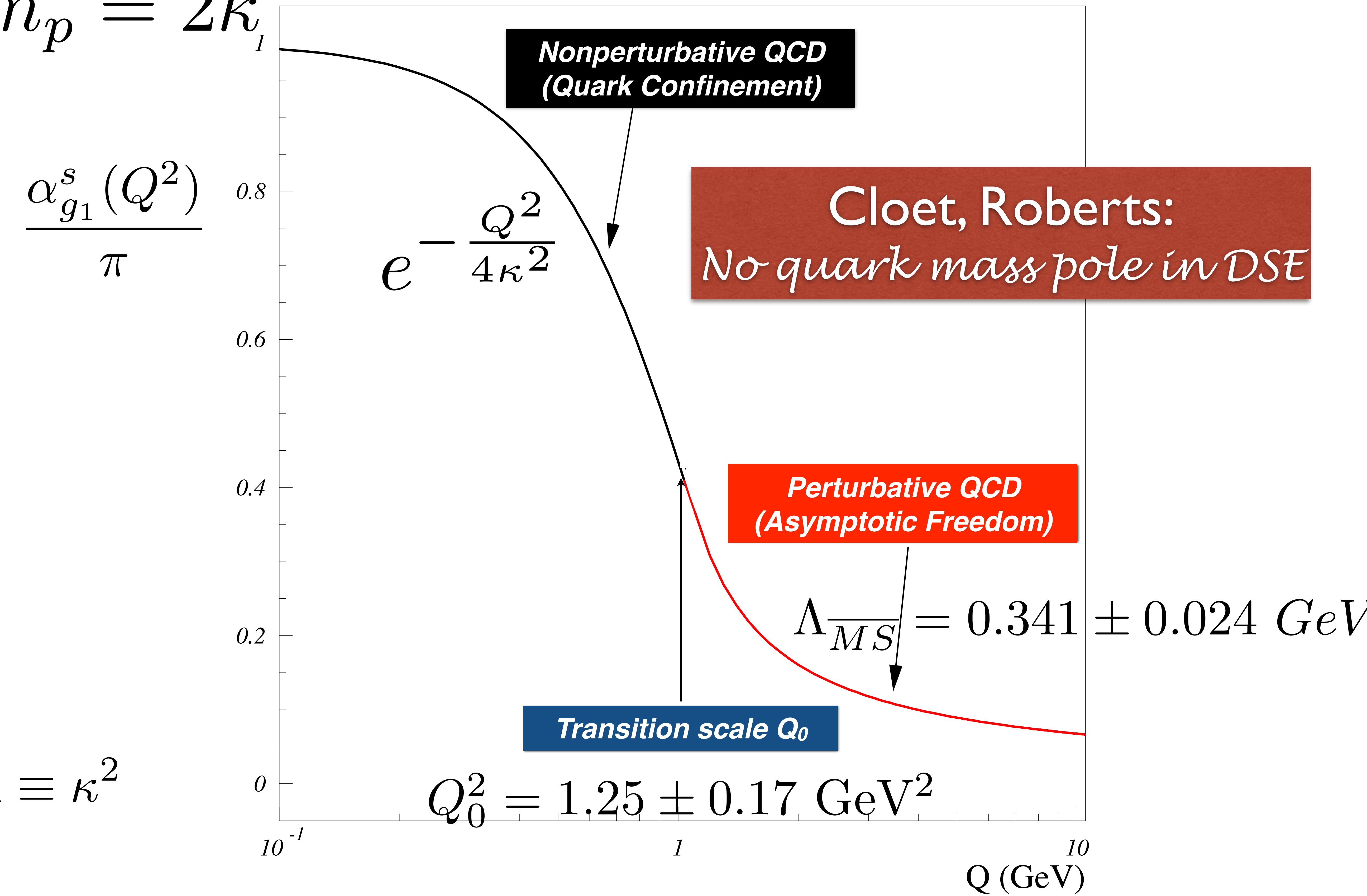
$$\lambda \equiv \kappa^2$$

$$m_\rho = \sqrt{2}\kappa$$

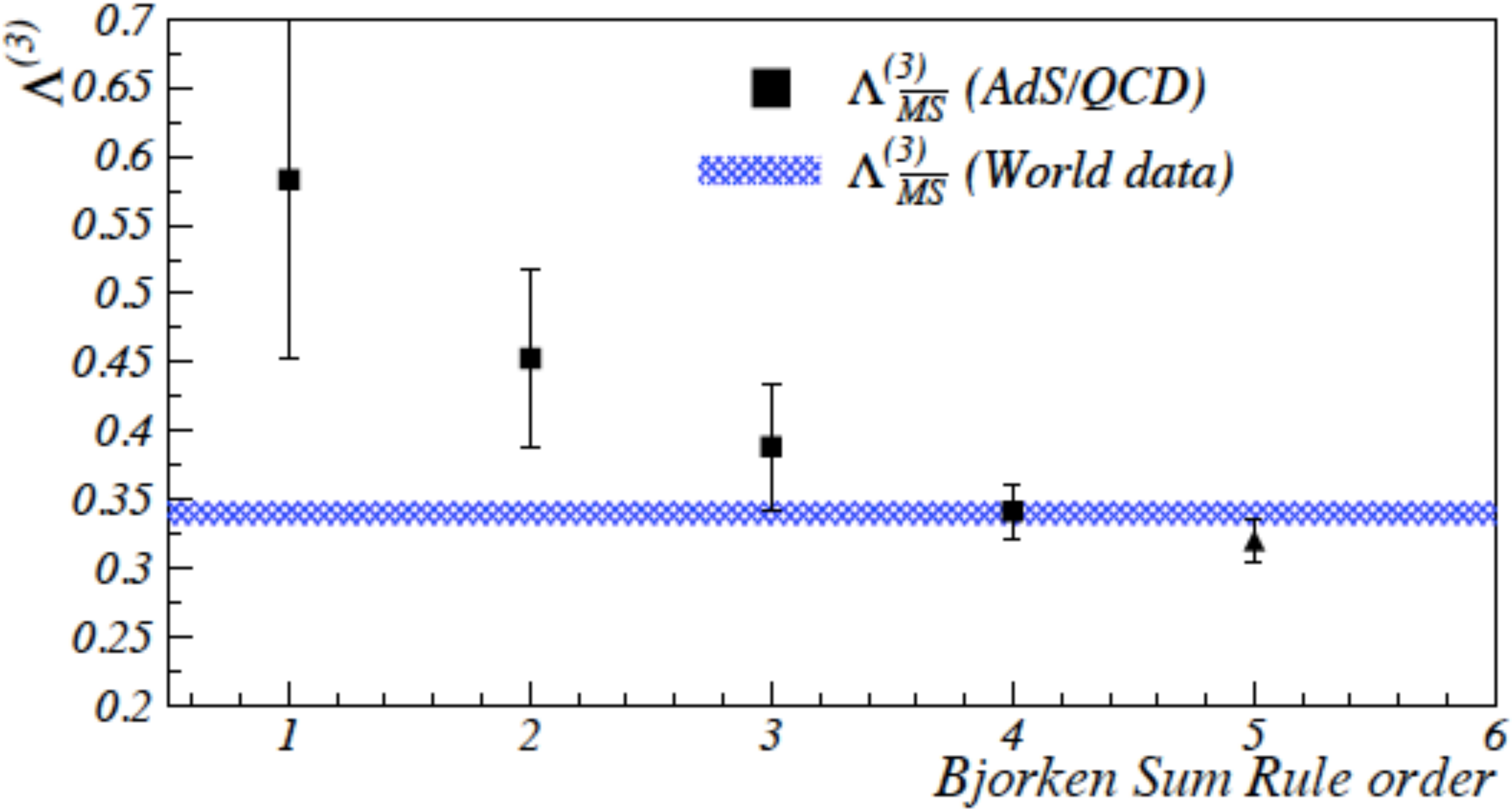
$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Teramond, sjb



$$\lambda \equiv \kappa^2$$



$$\Lambda_{\overline{MS}} = 0.341\ GeV = 0.440 m_\rho = 0.622\kappa$$

Connect $\Lambda_{\overline{MS}}$ to hadron masses!

Experiment: $M_\rho = 0.7753 \pm 0.0003\ GeV$

de Tèramond, Dosch, sjb

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of κ
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

AdS/QCD and Light-Front Holography

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

- **Zero mass pion for m_q =0 (n=J=L=0)**
- **Regge trajectories: equal slope in n and L**
- **Form Factors at high Q^2: Dimensional counting** $[Q^2]^{n-1} F(Q^2) \to \text{const}$
- **Space-like and Time-like Meson and Baryon Form Factors**
- **Running Coupling for NPQCD** $\alpha_s(Q^2) \propto e^{-\frac{Q^2}{4\kappa^2}}$
- **Meson Distribution Amplitude** $\phi_\pi(x) \propto f_\pi \sqrt{x(1-x)}$

Features of AdS/QCD

de Tèramond, Dosch, Deur, sjb

- **Color confining potential $\kappa^4 \zeta^2$ and universal mass scale from dilaton**
$$e^{\phi(z)} = e^{\kappa^2 z^2} \qquad \alpha_s(Q^2) \propto \exp - Q^2/4\kappa^2$$
- **Dimensional transmutation** $\Lambda_{\overline{MS}} \leftrightarrow \kappa \leftrightarrow m_H$
- **Chiral Action remains conformally invariant despite mass scale** *DAFF*
- **Light-Front Holography: Duality of AdS and frame-independent LF QCD**
- **Reproduces observed Regge spectroscopy — same slope in n, L, and J for mesons and baryons**
- **Massless pion for massless quark**
- **Supersymmetric meson-baryon dynamics and spectroscopy: $L_M = L_B + 1$**
- **Dynamics: LFWFs, Form Factors, GPDs**

Superconformal Algebra
Fubini and Rabinovici

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

de Tèramond, Dosch, Lorce, sjb

Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffractive dissociation of pion and proton to jets**
- **Identify the factorization Scale for ERBL, DGLAP evolution: Q_0**
- **Compute Tetraquark Spectroscopy Sequentially**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization**

We view the universe as light reaches us along the light-front at fixed

$$\tau = t + z/c$$

Front Form Vacuum Describes the Empty, Causal Universe

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD “Condensates”

(C) Higgs Light-Front Zero Mode

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0 >= E_0|\psi_0 >, E_0 = min\{E_i\}$$

Eigenstate defined at one time t over all space; Acausal! Frame-Dependent

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

$$H_{LF}|\psi_0 >_{LF}= M_0^2|\psi_0 >_{LF}, M_0^2 = 0.$$

Frame-independent eigenstate at fixed LF time τ = t+z/c within causal horizon

Frame-independent description of the causal physical universe!

Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state M= 0.
- Trivial up to k^+=0 zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD

P. Srivastava, sjb

Standard Model on the Light-Front

- **Same phenomenological predictions**
- **Higgs field has three components**
- **Real part creates Higgs particle**
- **Imaginary part (Goldstone) become longitudinal components of W, Z**
- ***Higgs VEV of instant form becomes $k^+=0$ LF zero mode!***
- **Analogous to a background static classical Zeeman or Stark Fields**
- **Zero contribution to $T^\mu{}_\mu$; zero coupling to gravity**

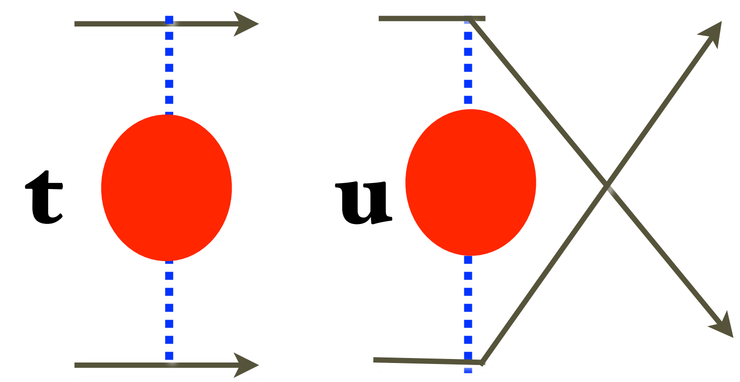
Goals

- **Test QCD to maximum precision at the LHC**
- **Maximize sensitivity to new physics**
- **High precision determination of fundamental parameters**
- **Determine renormalizations scales without ambiguity**
- **Eliminate scheme dependence**

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- **Two separate physical scales: t, u = photon virtuality**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!**
- **If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**

δ-$\mathcal{R}$enormalization Scheme ($\mathcal{R}_\delta$ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\rm MS}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

M. Mojaza, Xing-Gang Wu, sjb

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu^2_\delta = \mu^2_{\overline{\rm MS}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

M. Mojaza, Xing-Gang Wu, sjb

Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$-scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\mathrm{MS}}\ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS} \qquad \mu^2 = \mu^2_{\overline{\mathrm{MS}}} \exp(\ln 4\pi - \gamma_E)\ , \quad \mu^2_{\delta_2} = \mu^2_{\delta_1} \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \mathrm{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

Special Degeneracy in PQCD

There is nothing special about a particular value for δ, thus for any δ

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3$$
$$+ [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4$$

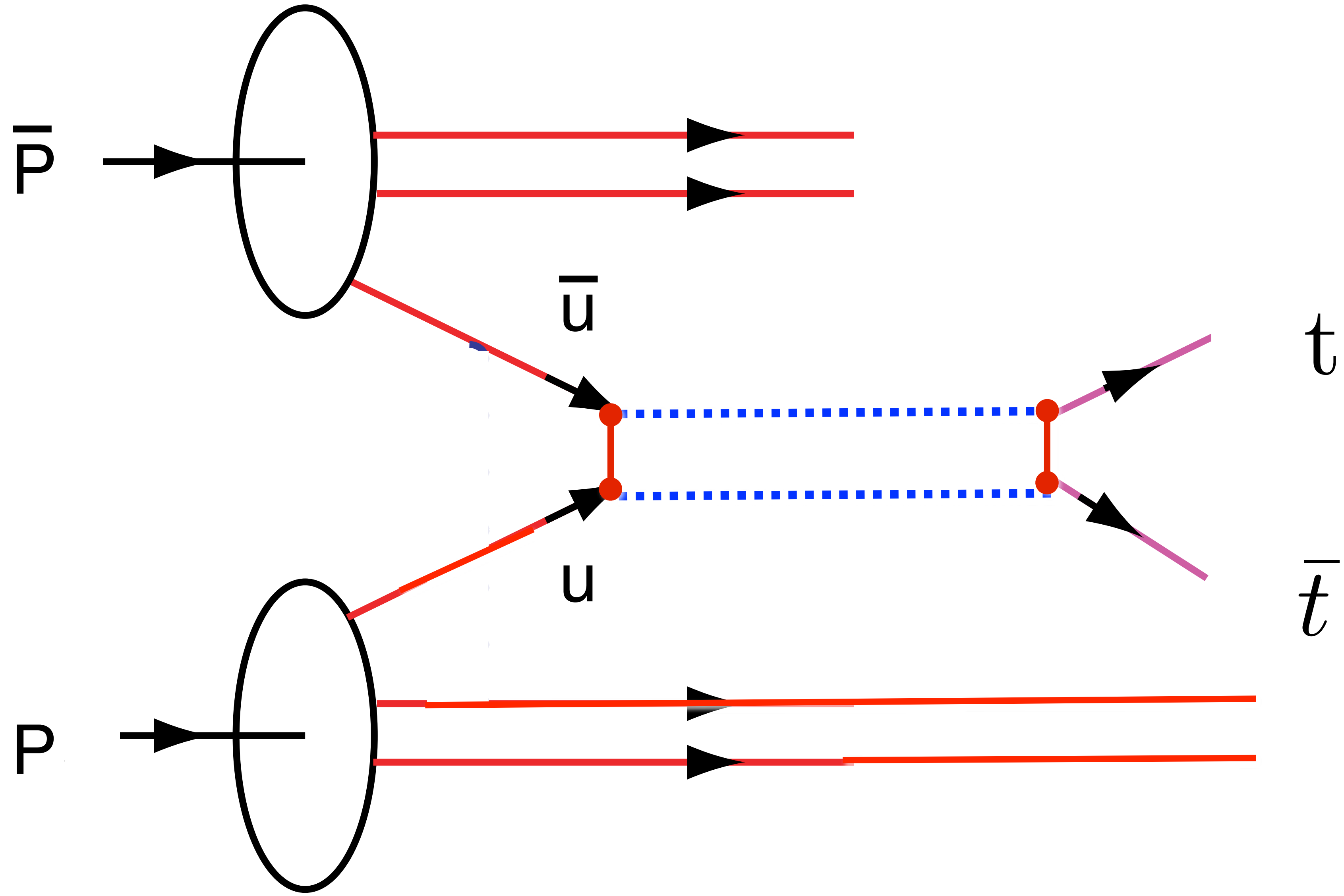
According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 \underline{+ \cdots)r_{2,1}}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 \underline{+ \cdots)r_{3,2}} + (\beta_0^3 + \cdots)r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 \underline{+ \cdots)r_{3,1}}$$
$$+ \cdots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron

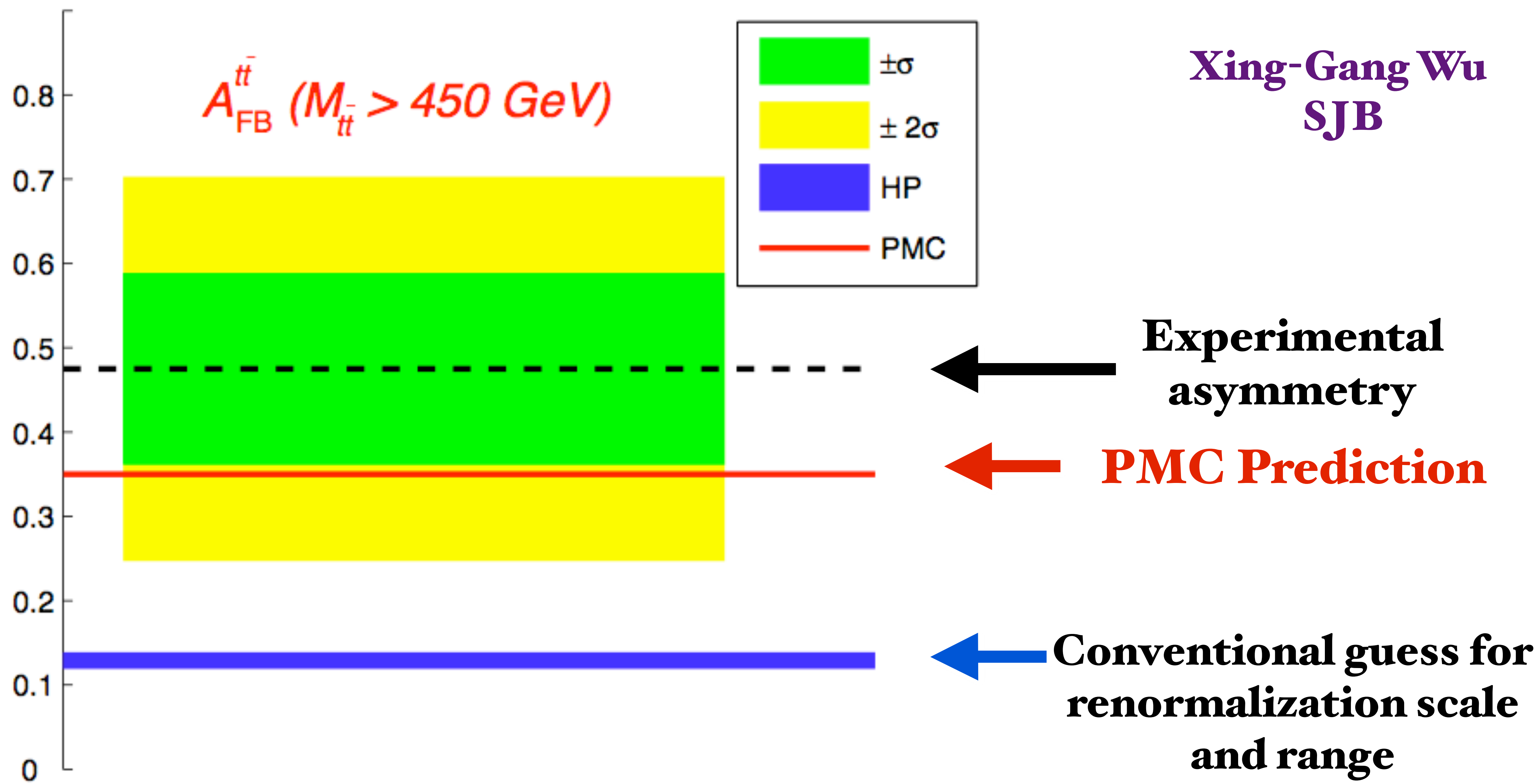


Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED

Xing-Gang Wu, sjb

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



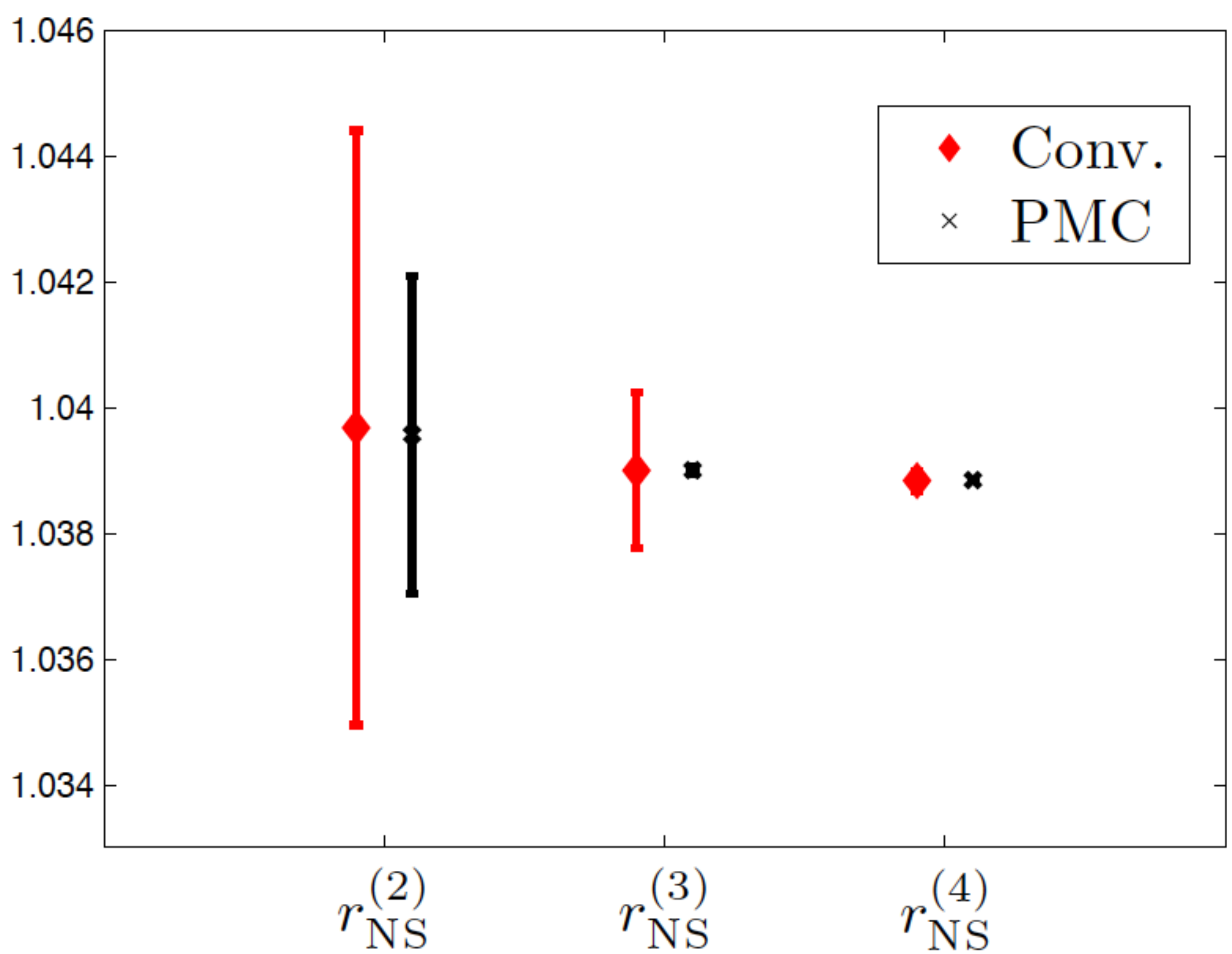
Xing-Gang Wu
SJB

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of $r_{\rm NS}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\rm NS} a_s^i$ and their errors $\pm|C_n^{\rm NS} a_s^n|_{\rm MAX}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{\rm init} = M_Z$.

What is PMC ?

Principle of Maximum Conformality

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\mathrm{init}})$

↓

Choose μ_R^{init}*; arbitrary initial renormalization scale*

↓

$Identify\ \{\beta_i^R\} - terms\ using\ n_f - terms$

through the $PMC - BLM$ *correspondence principle*

↓ order-by-order

Shift scale of α_s *to* μ_R^{PMC} *to eliminate* $\{\beta_i^R\} - terms$

↓

Conformal Series

Result is independent of μ_R^{init} *and scheme at fixed order*

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

PMC–BLM – one

Phys. Rev. Lett. **109**, 042002 (2012)

R_δ–scheme – two

Phys. Rev. Lett. **110**, 192001 (2013)

Eliminate β–terms

n_f dependence of pQCD series does not uniquely identify the β terms

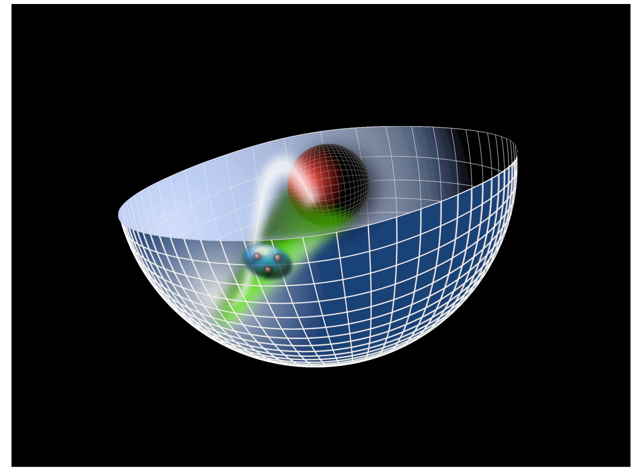
Features of BLM/PMC

- ***Predictions are scheme-independent***
- ***Matches conformal series***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation (*Kataev, Lu, Rathsman, sjb*)***
- ***No n! Renormalon growth***
- ***New scale at each order; n_F determined at each order***
- ***Multiple Physical Scales Incorporated***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Realistic Estimate of Higher-Order Terms***
- ***Eliminates unnecessary theory error***

QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks only from gluon splitting
- Renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- QCD gives 10^{42} to the cosmological constant

de Tèramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

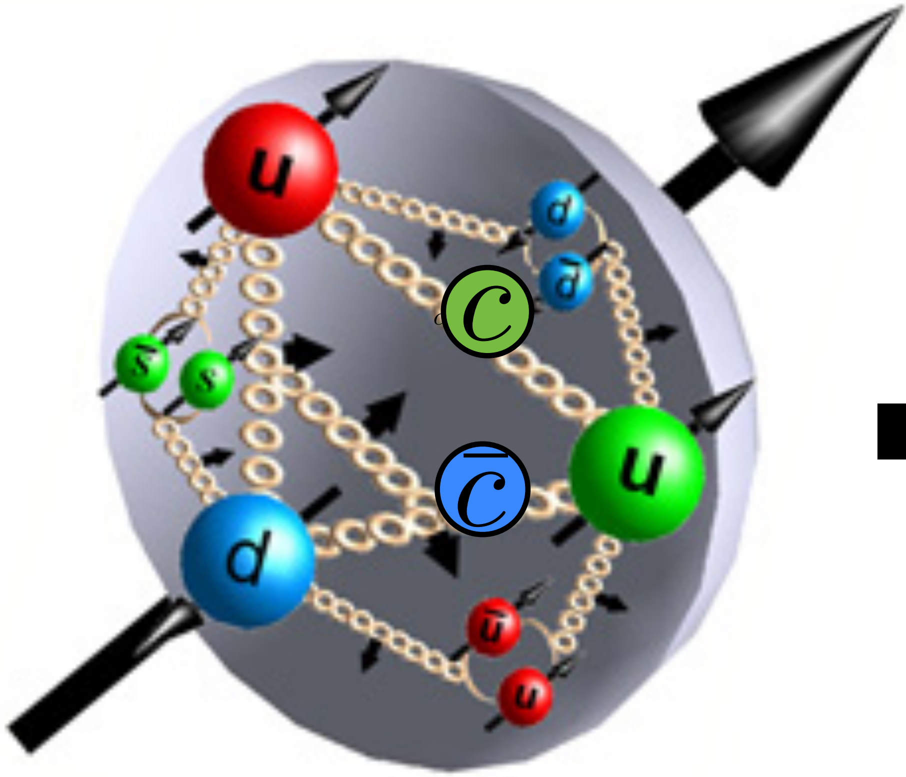
$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

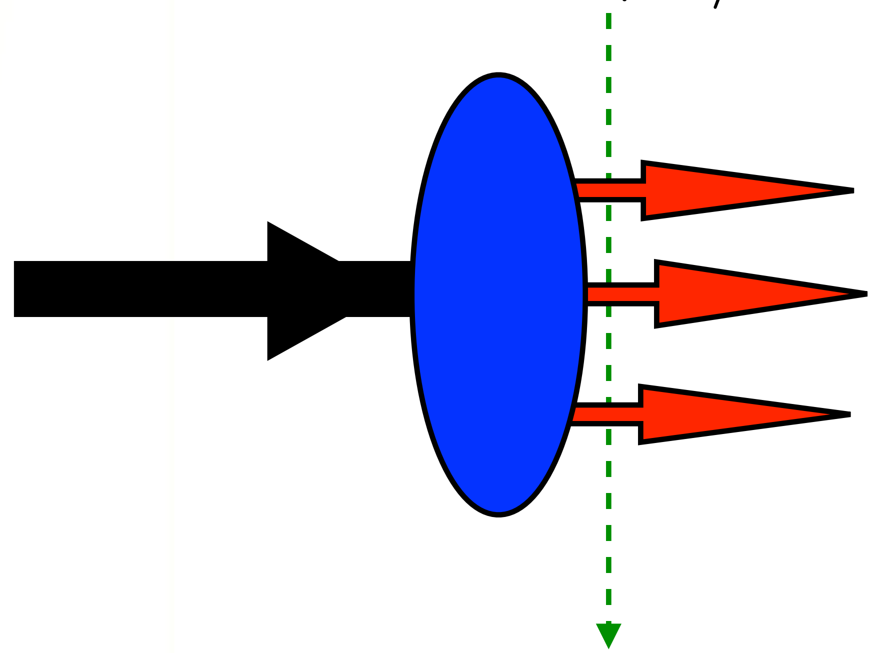
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

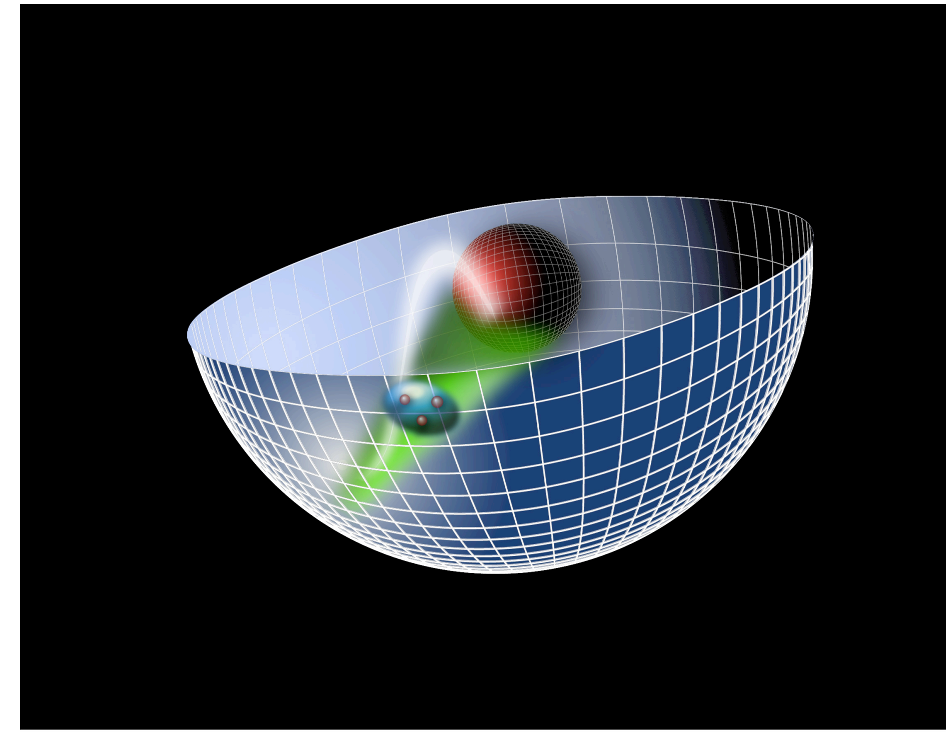
Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Light-Front Holographic QCD, Color Confinement, and Supersymmetric Features of QCD



Fixed $\tau = t + z/c$





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QCD-TNT4 Unraveling the Organization of the Tapestry of QCD

IlhaBela, São Paulo, Brazil - August 31 to September 04, 2015