Unquenching the gap equation

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Introduction

The most appropriate nonperturbative tool for studying chiral symmetry breaking and the subsequent dynamical quark mass generation is the Schwinger-Dyson equation (SDE) for the quark propagator. It is well known that the main ingredients entering in this equation are the full gluon propagator and the quark-gluon vertex [1, 2].

In the last few years our knowledge on the nonperturbative behavior of the gluon propagator, in the Landau gauge, has improved substantially [3, 4, 5, 6, 7]. Now, our main challenge is to achieve the same level of understanding for the quark-gluon vertex. One formalism that is suitable for pursuing this objective is based on the synthesis of the pinch technique (PT) with the background field method (BFM), known in the literature as the PT-BFM scheme [6].

It is known that in the PT-BFM scheme there are two different quark-gluon vertices. The first one obeys a Slavnov Taylor identity (STI), whereas the second satisfies an Abelian-like Ward Identity (WI). Interestingly enough, both quantities are related to each other through the so-called Background Quantum Identities (BQI) [8].

In this work we take the first steps towards a truncation scheme where the Abelian-like quark-gluon vertex is employed in the quark SDE. Within this approximation, we study how the behavior of the quark dynamical mass is affected by the presence of a different number of quark flavors.

The gap equation

The SDE for the quark propagator is diagrammatically depicted in Fig. 1. The gray circles represent the full quark propagator and the full gluon propagator, while the black circle represent the full quark-gluon vertex.

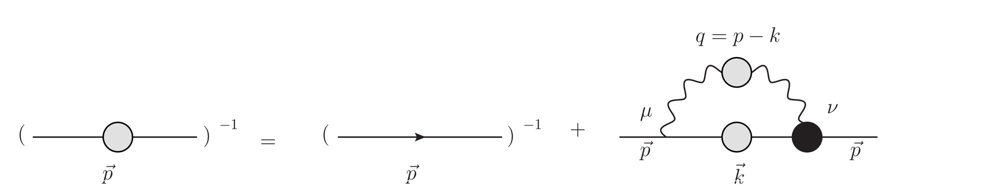

FIGURE 1: The SDE for the quark propagator. The gray circles represent full propagators and the black circle the full vertex.

Using the moment flow and the Lorentz indices indicated in Fig. 1, the renormalized gap equation in the chiral limit, $m_0 \to 0$, can be written as

$$S^{-1}(p) = Z_F \not{p} - Z_1 C_F g^2 \int_k \gamma_\mu S(k) \Gamma_\nu(-p,k,q) \Delta^{\mu\nu}(q) \,, \quad (1)$$

where $Z_1(\mu)$ and $Z_F(\mu)$ are the vertex and the quark wave-function renormalization constants respectively. C_F is the Casimir eigenvalue for the fundamental representation.

The full quark propagator can be written as

$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)\mathbb{I} = A(p^2)[\not{p} - \mathcal{M}(p^2)\mathbb{I}] \,, \quad (2)$$

where $A(p^2)$ and $B(p^2)$ are scalar functions, and we defined the dynamical quark mass function as $\mathcal{M}(p^2) = B(p^2)/A(p^2)$. The full gluon propagator in the Landau gauge, quenched or unquenched, is given by

$$\Delta^{\mu\nu}(q) = -i\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right]\Delta(q) \,. \quad (3)$$

The PT-BFM quark vertex

In the PT-BFM formalism there exist two types of the quark-gluon vertices, *i.e.*, the conventional vertex and the background vertex [8]. The first one is formed by a quantum gluon (Q) entering into $\psi\bar{\psi}$ pair and it is usually denoted by Γ^a_μ (see left vertex of Fig. 2); the second one corresponds to the three-point function with a background gluon ($\widehat{A}$) entering into $\psi\bar{\psi}$ pair, and it will be denoted by $\widehat{\Gamma}^a_\mu$ (see right vertex of Fig. 2). Choosing the flux of the momenta such that $p_1 = q + p_2$, we then define

$$\Gamma^a_\mu(q,p_2,-p_1) = g t^a \Gamma_\mu(q,p_2,-p_1) \,; \qquad \widehat{\Gamma}^a_\mu(q,p_2,-p_1) = g t^a \widehat{\Gamma}_\mu(q,p_2,-p_1) \,,$$

where $t^a = \lambda^a/2$ are the standard generating matrices of the $SU(3)$ group for the fundamental representation, and λ^a are the Gell-Mann matrices. It is important to remark that Γ_μ and $\widehat{\Gamma}_\mu$ coincide only at tree-level, where one has $\Gamma^{(0)}_\mu = \widehat{\Gamma}^{(0)}_\mu = \gamma_\mu$.

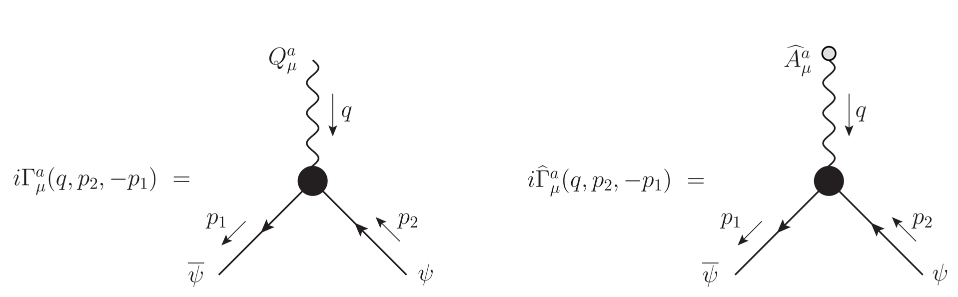

FIGURE 2: The conventional and background full quark-gluon vertices.

The main difference between these two vertices is that $\widehat{\Gamma}_\mu$ obeys the QED-like Ward identity (WI) [8]

$$q^\mu \widehat{\Gamma}_\mu(q,p_2,-p_1) = [S^{-1}(p_1) - S^{-1}(p_2)] \,, \quad (4)$$

while the vertex Γ_μ satisfies the fundamental STI

$$q^\mu \Gamma_\mu(q,p_2,-p_1) = F(q)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)S^{-1}(p_2)] \,, \quad (5)$$

here $H^a = t^a H$ is the quark-ghost scattering kernel and $\overline{H}$ its "conjugate", shown in Fig. 3. The function $F(q)$ is the ghost dressing function, defined from the ghost propagator $D(q) = iF(q)/q^2$.

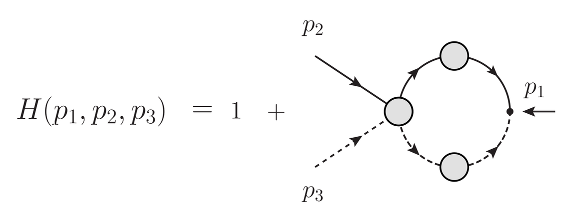

FIGURE 3: The quark-ghost scattering kernel $H(p_1,p_2,p_3)$.

The conventional and background vertices are related by means of the BQI, that in Landau gauge, can be written as [8]

$$[1 + G(q^2)]\Gamma_\mu(q,p_2,-p_1) = \widehat{\Gamma}_\mu(q,p_2,-p_1) + \cdots \,, \quad (6)$$

where the ellipses denote some auxiliary three point functions, whose effects will be neglected here.

The function $G(q)$ appearing above is particular to the PT-BFM formalism [6]; specifically, $G(q)$ is the form factor associated with the $g_{\mu\nu}$ component of the special two-point function $\Lambda_{\mu\nu}$, shown in Fig. 4 [9].

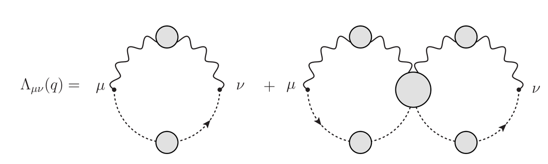

FIGURE 4: Definition of the auxiliary function $\Lambda_{\mu\nu}$.

In this work we will employ Eq. (6) using a particular choice for $\widehat{\Gamma}_\mu$, namely

$$\Gamma_\mu(q,p_2,-p_1) = \frac{\widehat{\Gamma}^{BC}_\mu(q,p_2,-p_1)}{[1+G(q)]} \,, \quad (7)$$

where $\widehat{\Gamma}^{BC}_\mu$ is the well-known Ball-Chiu vertex (which satisfies the WI given by Eq. (4)) ,

$$\widehat{\Gamma}^\mu_{BC} = \frac{A_1 + A_2}{2}\gamma^\mu + \frac{(p_1 - p_2)^\mu}{p_1^2 - p_2^2}\left\{[A_1 - A_2]\frac{\not{p}_1 - \not{p}_2}{2} + [B_1 - B_2]\right\} \,, \quad (8)$$

with $A_i = A(p_i)$ and $B_i = B(p_i)$, with $i = 1, 2$.

In order to proceed further, we need first to discuss the renormalization of the quark SDE given by Eq. (1). We know that the STI imposes that $Z_1 = Z_c^{-1} Z_F Z_H - 1$, where Z_c and Z_H are the renormalization of the ghost propagator and the quark-ghost scattering kernel, respectively. Now, in the Landau gauge, both the quark self-energy and the quark-ghost kernel are finite at one-loop; thus, at that order, $Z_F = Z_H = 1$, and, therefore, $Z_1 = Z_c^{-1}$. Imposing the above relations in Eq. (1), we obtain

$$S^{-1}(p) = \not{p} - Z_c^{-1} C_F g^2 \int_k \gamma_\mu S(k) \Gamma_\nu(-p,k,q)\Delta^{\mu\nu}(q) \,, \quad (9)$$

Using the propagators and the vertex given by Eq. (7) and taking the appropriate traces, we obtain from Eq. (1) the following system of integral equations

$$A(p) = 1 + Z_c^{-1} g^2 C_F \int_k \frac{\mathcal{K}_A(p,k)}{A^2(k)k^2 + B^2(k)}\left[\frac{\Delta(q)}{1+G(q)}\right] \,,$$
$$B(p) = Z_c^{-1} g^2 C_F \int_k \frac{\mathcal{K}_B(p,k)}{A^2(k)k^2 + B^2(k)}\left[\frac{\Delta(q)}{1+G(q)}\right] \,, \quad (10)$$

where $K_A(p,k)$ and $K_B(p,k)$ are scalar functions.

At this point it is important to recall that $1+G$ and F renormalized through the same renormalization constant, namely $F^{-1}(q^2,\mu^2) = Z_c F^{-1}(q^2,\Lambda_{UV})$ and $1+G(q^2,\mu^2) = Z_c[1+G(q^2,\Lambda_{UV})]$. Then, to enforce the correct renormalization group behavior of the dynamical quark mass, we will introduce the following modification $Z_c^{-1}\mathcal{K}_{A,B}(p,k) \to \mathcal{K}_{A,B}(p,k)[1+G(q^2)]^{-1}$. Then, Eq. (10) becomes

$$A(p) = 1 + C_F \int_k \widehat{d}(q^2)\frac{\mathcal{K}_A(p,k)}{A^2(k)k^2 + B^2(k)} \,,$$
$$B(p) = C_F \int_k \widehat{d}(q^2)\frac{\mathcal{K}_B(p,k)}{A^2(k)k^2 + B^2(k)} \,, \quad (11)$$

where $\widehat{d}(q^2)$ is the renormalization-group invariant quantity

$$\widehat{d}(q^2) = \frac{g^2\Delta(q)}{[1+G(q)]^2} \,. \quad (12)$$

Unquenching the gluon propagator

The next step it is to include the quark effects in the gluon propagator. To do that, we use the same procedure as in Ref. [9], which expresses the unquenched gluon propagator $\Delta_Q(q)$ as a deviation from the unquenched one, $\Delta(q)$, *i.e.*

$$\Delta_Q(q) = \frac{\Delta(q)}{1 + \left\{\widehat{X}(q)\,[1+G_Q(q)]^{-2} + \lambda^2\right\}\Delta(q)} \,, \quad (13)$$

where the quark loop contribution $\widehat{X}^{\mu\nu}(q) = \widehat{X}(p)P^{\mu\nu}(q)$ is shown in Fig. 5. The subscript Q denotes the presence of quark loops in the corresponding function, and λ^2 is the difference of the dynamical gluon mass in the quenched and unquenched cases.

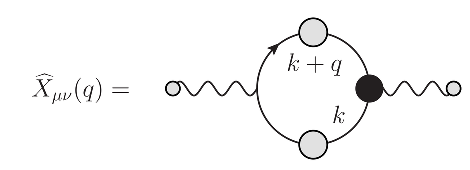

FIGURE 5: The quark loop contribution in the PT-BFM scheme.

Numerical Analysis and Conclusions

In Fig. 6 we compare the results for $\Delta(q)$ (left panel) and $[1+G(q)]^{-1}$ (right panel) for the quenched case (black curve) and the unquenched with $N_f = 2$ (red curve) and $N_f = 2+1+1$ (blue curve). All curves are renormalized at $\mu = 4.3$ GeV. The results for $[1+G(q)]^{-1}$ was obtained using its own dynamical equation from Ref. [10]. The values of $\alpha_s(\mu) = g^2/4\pi$ were fixed such that the ghost lattice data of Ref. [4] were reproduced. The values we obtain using this procedure are: $\alpha_s(\mu) = 0.22$ ($N_f = 0$), $\alpha(\mu) = 0.28$ ($N_f = 2$) and $\alpha(\mu) = 0.31$ ($N_f = 2+1+1$).

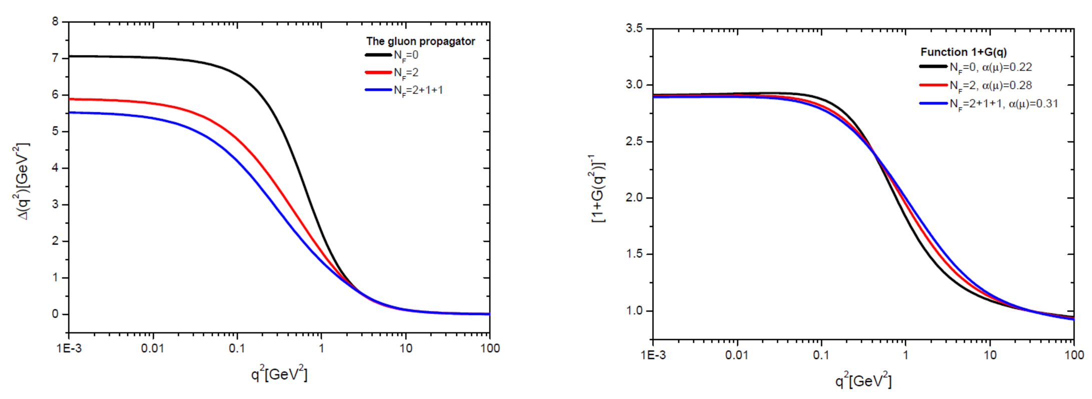

FIGURE 6: (i) Left panel: The gluon propagator $\Delta(q)$, and (ii) right panel: The function $[1+G(q)]^{-1}$.

Notice that the inclusion of the quark loop into Eq. (13) suppresses both the IR and the intermediate momenta regions of the gluon propagator. On the other hand, the changes in the $[1+G(q)]^{-1}$ are extremely mild.

In Fig. 7, we show the corresponding $A^{-1}(p^2)$ (left panel) and the $\mathcal{M}(p^2)$ (right panel) obtained when we solve the system formed by Eq. (10) using the results of Fig. 6. It is interesting to notice that for the three cases we obtain similar values for the dynamical quark masses, around $\mathcal{M}(0) = 270$ MeV.

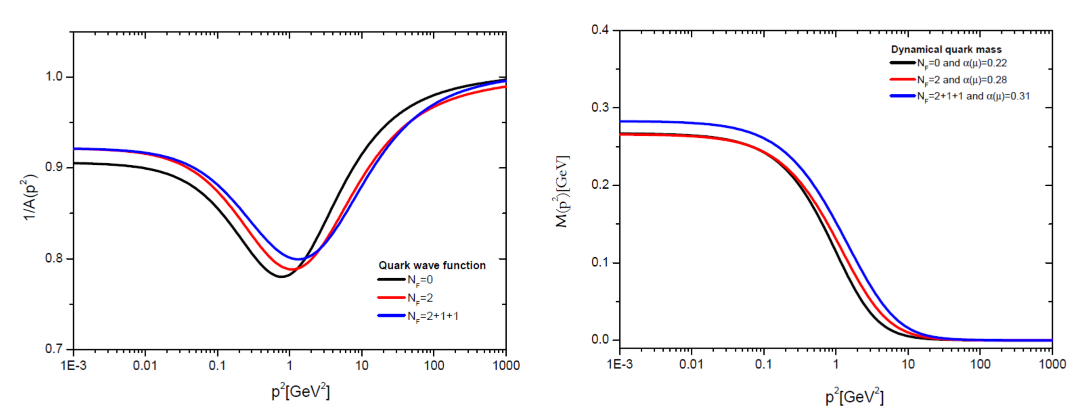

FIGURE 7: Quark wave function (left panel) and $N_F = 0$ (black curve), $N_F = 2$ (red curve) and $N_F = 2+1+1$ (blue curve).

In conclusion, we have presented a preliminary study of the quark mass generation using the quenched and unquenched gluon propagators. We have shown that it is possible to generate a dynamical mass of around $\mathcal{M}(0) = 270$ MeV, using the minimal Ansatz for the Abelian-like quark-gluon vertex, $\widehat{\Gamma}_\mu$. Of course, the Ansatz employed here can be improved using the transverse tensorial structures or by adding the neglected corrections appearing in the BQI relating the PT-BFM and the conventional vertex.

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