Impacts of Dynamical Chiral Symmetry Breaking

Ian Cloët Argonne National Laboratory

QCD-TNT4

Unraveling the organization of the QCD tapestry

IlhaBela, São Paulo, Brazil

31 August – 4 September 2015



Office of Science



The Pion – Nature's strong messenger



- Hideki Yukawa in 1935 postulated a strongly interacting particle of mass ~ 100 MeV
 - Yukawa called this particle a "meson"
- Cecil Powell in 1947 discovered the π-meson from cosmic ray tracks in a photographic emulsion – a technique Cecil developed





- Cavendish Lab had said method is incapable of *"reliable and reproducible precision measurements"*
- The measured *pion* mass was: 130 150 MeV
- Both Yukawa & Powell received Nobel Prize in 1949 and 1950 respectively
- Discovery of pion was beginning of particle physics; before long there was the particle *zoo*

table of contents

QCD-TNT4 31 August to 4 September

3/32

- Today the pion is understood as both a bound state of a dressed-quark and a dressed-antiquark in QFT and the Goldstone mode associated with DCSB in QCD
 - This dichotomous nature has numerous ramifications, e.g.:

 $m_{
ho}/2 \sim M_N/3 \sim 350 \,\mathrm{MeV}$ however $m_{\pi}/2 \simeq 0.2 \times 350 \,\mathrm{MeV}$

- The pion is unusually light, the key is dynamical chiral symmetry breaking
 - in coming to understand the pion's lepton-like mass, DCSB (and confinement) has been exposed as the origin of more than 98% of the mass in the visible Universe
- QCD is characterized by two emergent phenomena: confinement & DCSB
 - it is also the only known example in nature of a fundamental QFT that is innately non-perturbative
- In the quest to understand QCD must discover the origin of confinement, its relationship to DCSB and understand how these phenomenon influence hadronic obserables





QCD's Dyson-Schwinger Equations



- The equations of motion of QCD \iff QCD's Dyson–Schwinger equations
 - an infinite tower of coupled integral equations
 - tractability \implies must implement a symmetry preserving truncation
 - The most important DSE is QCD's gap equation \implies quark propagator



• ingredients - dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i \not p + M(p^2)}$$

- S(p) has correct perturbative limit
 - mass function, $M(p^2)$, exhibits dynamical mass generation
- complex conjugate poles
 - no real mass shell \Longrightarrow confinement



Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame dependent concept
 - boost operators are dynamical, that is, they are interaction dependent
- In high energy scattering experiments particles move at near speed of light
 - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$



- Light-front quantization \implies light-front WFs; many remarkable properties:
 - frame-independent; probability interpretation as close as QFT gets to QM
 - boosts are kinematical not dynamical
- Parton distribution amplitudes (PDAs) are (almost) observables & are related to light-front wave functions

$$arphi(x) = \int d^2 \vec{k}_{\perp} \; \psi(x, \vec{k}_{\perp})$$

Light-Front Wave Functions

- In equal-time quantization a hadron wave function is a frame dependent concept
 - boost operators are dynamical, that is, they are interaction dependent
- In high energy scattering experiments particles move at near speed of light
 - natural to quantize a theory at equal light-front time: $\tau = (t+z)/\sqrt{2}$



- Light-front quantization \implies light-front WFs; many remarkable properties:
 - frame-independent; probability interpretation as close as QFT gets to QM
 - boosts are kinematical not dynamical
- Parton distribution amplitudes (PDAs) are (almost) observables & are related to light-front wave functions

$$arphi(x) = \int d^2 ec{k_\perp} \; \psi(x, ec{k_\perp}) \; ,$$

Pion's Parton Distribution Amplitude



- pion's PDA $\varphi_{\pi}(x)$: is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state
 - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2



PDAs enter numerous hard exclusive scattering processes

Pion's Parton Distribution Amplitude



- **pion's PDA** $\varphi_{\pi}(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
 - it's a function of the light-cone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
- The pion's PDA is defined by

$$f_{\pi} \, \varphi_{\pi}(x) = Z_2 \int \frac{d^4k}{(2\pi)^2} \, \delta\left(k^+ - x \, p^+\right) \operatorname{Tr}\left[\gamma^+ \gamma_5 \, S(k) \, \Gamma_{\pi}(k, p) \, S(k-p)\right]$$

• $S(k) \Gamma_{\pi}(k, p) S(k - p)$ is the pion's Bethe-Salpeter wave function

- in the non-relativistic limit it corresponds to the Schrodinger wave function
- φ_π(x): is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front [at twist-2 also pseudoscalar projection]
- Pion PDA is an essentially nonperturbative quantity whose asymptotic form is known; in this regime governs, e.g., Q² dependence of pion form factor

$$Q^2 F_{\pi}(Q^2) \xrightarrow{Q^2 \to \infty} 16 \pi f_{\pi}^2 \alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_{\pi}^{\text{asy}}(x) = 6 x (1-x)$$

QCD Evolution & Asymptotic PDA



ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu} \varphi(x,\mu) = \int_0^1 dy \ V(x,y) \, \varphi(y,\mu)$$

This evolution equation has a solution of the form

$$\varphi_{\pi}(x,Q^2) = \frac{6 x (1-x)}{1+\sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1)}$$

- α = 3/2 because in Q² → ∞ limit QCD is invariant under the collinear conformal group SL(2; ℝ)
- Gegenbauer- $\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$
- The coefficients of the Gegenbauer polynomials, a^{3/2}_n(Q²), evolve logarithmically to zero as Q² → ∞: φ_π(x) → φ^{asy}_π(x) = 6 x (1 − x)
- At what scales is this a good approximation to the pion PDA?

• E.g., AdS/QCD find $\varphi_{\pi}(x) \sim x^{1/2} (1-x)^{1/2}$ at $Q^2 = 1 \text{ GeV}^2$; expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2}/a_2^{3/2} \sim 10\%$

Pion PDA from the DSEs





Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA

- scale of calculation is given by renormalization point $\zeta = 2 \,\text{GeV}$
- A realization of DCSB on the light-front
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor

Pion PDA from lattice QCD





Standard practice to fit first coefficient of "asymptotic expansion" to moment

$$\varphi_{\pi}(x,Q^2) = 6 x (1-x) \left[1 + \sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1) \right]$$

- however this expansion is guaranteed to converge rapidly only when $Q^2 o \infty$
- this procedure results in a *double-humped* pion PDA
- Advocate using a generalized expansion

$$\varphi_{\pi}(x,Q^2) = N_{\alpha} x^{\alpha} (1-x)^{\alpha} \left[1 + \sum_{n=2,4,\dots} a_n^{\alpha+1/2}(Q^2) C_n^{\alpha+1/2}(2x-1) \right]$$

• Find $\varphi_{\pi} \simeq x^{\alpha}(1-x)^{\alpha}$, $\alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \sim 0.52$

table of contents

Pion PDA from lattice QCD





Standard practice to fit first coefficient of "asymptotic expansion" to moment

$$\varphi_{\pi}(x,Q^2) = 6 x (1-x) \left[1 + \sum_{n=2,4,\dots} a_n^{3/2}(Q^2) C_n^{3/2}(2x-1) \right]$$

- however this expansion is guaranteed to converge rapidly only when $Q^2 \rightarrow \infty$
- this procedure results in a double-humped pion PDA
- Advocate using a generalized expansion

$$\varphi_{\pi}(x,Q^2) = N_{\alpha} x^{\alpha} (1-x)^{\alpha} \left[1 + \sum_{n=2,4,\dots} a_n^{\alpha+1/2}(Q^2) C_n^{\alpha+1/2}(2x-1) \right]$$

• Find
$$\varphi_{\pi} \simeq x^{\alpha} (1-x)^{\alpha}$$
, $\alpha = 0.35^{+0.32}_{-0.32}$; good agreement with DSE: $\alpha \sim 0.52$

Updated Pion PDA from lattice QCD





Updated lattice QCD moment: [V. Braun et al., arXiv:1503.03656 [hep-lat]]

$$\int_{0}^{1} dx \, (2 \, x - 1)^{2} \varphi_{\pi}(x) = 0.2361 \, (41) \, (39) \, (?)$$

DSE prediction:

$$\int_0^1 dx \, (2\,x-1)^2 \varphi_\pi(x) = 0.251$$

When is the Pion's PDA Asymptotic





• Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100 \text{ GeV}^2$, compared with $\varphi_{\pi}^{\text{asy}}(x) = 6 x (1 - x)$

• Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors

• Importantly, $\varphi_{\pi}^{\text{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_{\pi}(x)$ when pion valence quark PDF satisfies: $q_{v}^{\pi}(x) \sim \delta(x)$

This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic





LO QCD evolution of momentum fraction carried by valence quarks

$$\left\langle x \, q_v(x) \right\rangle(Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \left\langle x \, q_v(x) \right\rangle(Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

• therefore, as $Q^2 \to \infty$ we have $\langle x q_v(x) \rangle \to 0$ implies $q_v(x) \propto \delta(x)$

At LHC energies valence quarks still carry 20% of pion momentum
 the gluon distribution saturates at (x g(x)) ~ 55%

• the gluon distribution saturates at $\langle x g(x) \rangle \sim$

Asymptotia is a long way away!

table of contents

Pion Elastic Form Factor

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2 F_{\pi}(Q^2)$ at $Q^2 \approx 6 \text{ GeV}^2$
 - magnitude of this product is determined by strength of DCSB at all accessible scales
- The QCD prediction can be expressed as

$${}^{2}F_{\pi}(Q^{2}) \overset{Q^{2} \gg \Lambda_{\text{QCD}}^{2}}{\sim} 16 \pi f_{\pi}^{2} \alpha_{s}(Q^{2}) \boldsymbol{w}_{\pi}^{2}; \qquad \boldsymbol{w}_{\pi} = \frac{1}{3} \int_{0}^{1} dx \, \frac{1}{x} \, \varphi_{\pi}(x)$$

- Within DSEs there is consistency between the direct pion form factor calculation and that obtained using the DSE pion PDA
 - 15% disagreement explained by higher order/higher-twist corrections
- We predict that QCD power law behaviour with QCD's scaling law violations sets in at $Q^2 \sim 8 \text{ GeV}^2$





Measuring onset of Perturbative scaling





To observe onset of perturbative power law behaviour – to differentiate from a monopole – optimistically need data at 8 GeV² but likely also at 10 GeV²

• this is a very challenging task experimentally

Scaling predictions are valid for both spacelike and timelike momenta

• timelike data show promise as the means of verifying modern predictions



The Nucleon

Nucleon Electromagnetic Form Factors





- Provide vital information on the distribution of charge and magnetization within the most basic element of nuclear physics
 - form factors also directly probe confinement at all energy scales
- Today accurate form factor measurements are creating a paradigm shift in our understanding of nucleon structure:
 - proton radius puzzle
 - $\mu_p G_{Ep}/G_{Mp}$ ratio and a possible zero-crossing
 - flavour decomposition and evidence for diquark correlations
 - meson-cloud effects
 - seeking verification of perturbative QCD scaling predictions & scaling violations

Nucleon Sachs Form Factors



- Experiment gives Sachs form factors: $G_E = F_1 \frac{Q^2}{4M^2}F_2$ $G_M = F_1 + F_2$
- Until the late 90s Rosenbluth separation experiments found that the $\mu_p G_{Ep}/G_{Mp}$ ratio was flat
- Polarization transfer experiments completely altered our picture of nucleon structure
 - distribution of charge and magnetization are not the same
 - Proton charge radius puzzle $[7\sigma]$

 $r_{Ep} = 0.84087 \pm 0.00039 \text{ fm}$

muonic hydrogen [Pohl et al. (2010)]

- one of the most interesting puzzles in hadron physics
- so far defies explanation



 $r_{Ep} = 0.8775 \pm 0.0051 \text{ fm}$

CODATA: e p + e-hydrogen

Nucleon Structure



- A robust description of the nucleon as a bound state of 3 dressed-quarks can only be obtained within an approach that respects Poincaré covariance
- Such a framework is provided by the Poincaré covariant Faddeev equation



- sums all possible interactions between three dressed-quarks
- much of the three-body interaction can be absorbed into renormalized two-body interactions
- A *prediction* of these approaches is that owing to DCSB in QCD strong diquark correlations exist within baryons
 - any interaction that describes colour-singlet mesons also generates *non-pointlike* diquark correlations in the colour- $\overline{3}$ channel
 - where *scalar and axial-vector diquarks* are most important for the nucleon

Nucleon Structure



- A robust description of the nucleon as a bound state of 3 dressed-quarks can only be obtained within an approach that respects Poincaré covariance
- Such a framework is provided by the Poincaré covariant Faddeev equation



- sums all possible interactions between three dressed-quarks
- much of the three-body interaction can be absorbed into renormalized two-body interactions
- A *prediction* of these approaches is that owing to DCSB in QCD strong diquark correlations exist within baryons
 - any interaction that describes colour-singlet mesons also generates *non-pointlike* diquark correlations in the colour- $\overline{3}$ channel
 - where scalar and axial-vector diquarks are most important for the nucleon

Nucleon EM Form Factors from DSEs



- A robust description of form factors is only possible if electromagnetic gauge invariance is respected; equivalently all relevant Ward-Takahashi identities (WTIs) must be satisfied

- transverse structure unconstrained
- Diagrams needed for a gauge invariant nucleon EM current in DSEs



• Feedback with experiment can shed light on elements of QCD via DSEs

Beyond Rainbow Ladder Truncation



Include "anomalous chromomagnetic" term in quark-gluon vertex

 $\frac{1}{4\pi} g^2 D_{\mu\nu}(\ell) \Gamma_{\nu}(p',p) \rightarrow \alpha_{\rm eff}(\ell) D_{\mu\nu}^{\rm free}(\ell) \left[\gamma_{\nu} + i \sigma^{\mu\nu} q_{\nu} \tau_5(p',p) \right]$

- In chiral limit *anomalous chromomagnetic* term can only appear through DCSB – since operator flips quark helicity
- EM properties of a spin- $\frac{1}{2}$ point particle are characterized by two quantities:
 - charge: e & magnetic moment: μ
- Expect strong gluon dressing to produce ^{0.6} non-trivial electromagnetic structure for a dressed quark
 - recall dressing produces from massless quark a $M \sim 400 \,\mathrm{MeV}$ dressed quark
- Large anomalous chromomagnetic moment in the quark-gluon vertex – produces a large quark anomalous electromagnetic moment
 - dressed quarks are not point particles





Quark anomalous magnetic moment required for good agreement with data

- important for low to moderate Q^2
- power law suppressed at large Q^2



- Illustrates how feedback with EM form factor measurements can help constrain the quark-photon vertex and therefore the quark-gluon vertex within the DSE framework
 - knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs \Leftrightarrow confinement

Proton G_E form factor and **DCSB**



Find that slight changes in $M(p^2)$ on the domain $1 \leq p \leq 3 \text{ GeV}$ have a striking effect on the G_E/G_M proton form factor ratio

• strong indication that position of a zero is very sensitive to underlying dynamics and the nature of the transition from nonperturbative to perturbative QCD

• Zero in
$$G_E = F_1 - \frac{Q^2}{4M_N^2}F_2$$
 largely determined by evolution of $Q^2 F_2$

- F₂ is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment *vanishes in perturbative limit*
- the quicker the perturbative regime is reached the quicker $F_2 \rightarrow 0$

Flavour separated proton form factors



Prima facie, these experimental results are remarkable

- u and d quark sector form factors have very different scaling behaviour
- However, when viewed in context of diquark correlations results are straightforward to understand
 - in proton (*uud*) the *d* quark is much more likely to be in a scalar diquark [*ud*] than a *u* quark; diquark ⇒ 1/Q²

Quero in F^d_{1p} a result of interference between scalar and axial-vector diquarks
 location of zero indicates relative strengths – correlated with d/u ratio as x → 1



Nucleon to Resonance Transitions



- Given the challenges posed by non-perturbative QCD it is insufficient to study hadron ground-states alone
- Nucleon transition form factors provide a critical extension to elastic form factors – providing more windows into and different perspectives on quark-gluon dynamics
 - e.g. nucleon resonances are more sensitive to long-range effects in QCD than the properties of ground states . . . analogous to exotic and hybrid mesons
- Important example is $N \to \Delta$ transition parametrized by three form factors
 - $\bullet \ G^*_E(Q^2), \ \ G^*_M(Q^2), \ \ G^*_C(Q^2)$
 - if both N and Δ were purely S-wave then $G_E^*(Q^2) = 0 = G_C^*(Q^2)$
- When analyzing the N → ∆ transition it is common to construct the ratios:

$$R_{EM} = -\frac{G_{E}^{*}}{G_{M}^{*}}, \quad R_{SM} = -\frac{|\mathbf{q}|}{2 M_{\Delta}} \frac{G_{C}^{*}}{G_{M}^{*}}$$



$N \rightarrow \Delta$ form factors from the DSEs





• For $R_{SM} = -\frac{|q|}{2M_{\Delta}} \frac{G_C^*}{G_M^*}$ DSEs reproduces rapid fall off with Q^2

- Find that $R_{EM} = -\frac{G_E^*}{G_M^*}$ is a particular sensitive measure of *quark orbital angular momentum* within the nucleon and Δ
- At large Q^2 helicity conservation demands: $R_{SM} \rightarrow \text{constant}, R_{EM} \rightarrow 1$
 - however these asymptotic results are not reached until incredibility large Q^2 which will not be accessible at any present or foreseeable facility
- Comparison with Argonne-Osaka results suggest that the pion cloud is masking expected zero in R_{EM}

$N \rightarrow \Delta$ magnetic form factor





Roper Resonance





The Excited Baryon Analysis Center (EBAC), resolved a fifty-year puzzle by demonstrating that the Roper resonance is the proton's first radial excitation

• its lower-than-expected mass owes to a dressed-quark core shielded by a dense cloud of pions and other mesons

[Decadal Report on Nuclear Physics: Exploring the Heart of Matter]

Roper Resonance from the DSEs





The Faddeev equation that produces the nucleon also gives its excited states

- amplitudes for the lightest excited state typically possess a zero
- therefore lightest nucleon excited state is a radial excitation \iff Roper resonance
- "quark core" mass: $M_R = 1.73$ GeV; c.f. Argonne-Osaka group $M_R = 1.76$ GeV

Now have a unified description of the nucleon, Delta and Roper baryons

• Find e.g. that the Roper charge radius is 80% larger than the nucleon's table of contents OCD-TNT4 31 August to 4 September

Nucleon \rightarrow **Roper transition form factors**





- Results agree well with data for $Q^2 \gtrsim 2 m_N^2$ & at the real photon point
- However contemporary kernels just produce a hadron's *dressed-quark core*
 - pion cloud contributions are absent from our calculation, however these are inferred from the deviation with data
 - on domain $0 < Q^2 \lesssim 2 m_N^2$ pion cloud contributions should be negative and deplete the transition form factors



Conclusion



- QCD and therefore hadron physics is unique:
 - must confront a fundamental theory in which the elementary degrees-of-freedom are confined and only hadrons reach detectors
- QCD will only be solved by deploying a diverse array of experimental and theoretical methods
 - must define and solve the problems of confinement and its relationship with DCSB



- These are two of the most important challenges in fundamental Science
- Determined the pion form factor for all spacelike momenta
 - $Q^2 F_{\pi}(Q^2)$ peaks at 6 GeV², with maximum directly related to DCSB
 - predict that QCD power law behaviour sets in at $Q^2 \sim 8 \,\mathrm{GeV^2}$
 - DSEs & lattice QCD agree pion PDA is much broader than asymptotic result
- Location of a zero in G_{Ep}/G_{Mp} is a sensitive measure of DCSB in QCD

Continuum-QCD approaches are essential; are at the forefront of guiding experiment & provide rapid feedback; building intuition & understanding