Evidence of BRST-Symmetry Breaking in Lattice Minimal Landau Gauge

Attilio Cucchieri

http://lattice.ifsc.usp.br/

Instituto de Física de São Carlos – USP

Abstract

By evaluating the so-called Bose-ghost propagator, we present the first numerical evidence of BRST-symmetry breaking in minimal Landau gauge, i.e. due to the restriction of the functional integration to the first Gribov region in the Gribov-Zwanziger approach. We find that our data are well described by a simple fitting function, which can be related to a massive gluon propagator in combination with an infrared-free (Faddeev-Popov) ghost propagator. As a consequence, the Bose-ghost propagator, which has been proposed as a carrier of the confining force in Yang-Mills theories in minimal Landau gauge, presents a $1/p^4$ singularity in the infrared limit.

[A.C., D.Dudal, T.Mendes & N.Vandersickel, Phys.Rev. D 90 (2014)]

Color Confinement

Millennium Prize Problems by the Clay Mathematics Institute (US$1,000,000): Yang-Mills Existence and Mass Gap: Prove that for any compact simple gauge group G, a non-trivial quantum Yang-Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$.

Lattice simulations can solve QCD exactly (in discretized Euclidean space-time), allowing quantitative predictions for the physics of hadrons. But they can also help reveal the principles behind a central phenomenon of QCD: confinement. In fact, we can try to understand the QCD vacuum (the "battle for nonperturbative QCD", E.V. Shuryak, *The QCD vacuum, hadrons and the superdense matter*) by using inputs from lattice simulations and by testing numerically the approximations introduced in analytic approaches (Dyson-Schwinger equations, Bethe-Salpeter equations, Pomeron dynamics, QCD-inspired models, etc).

Pathways to Confinement

- How does confinement come about?
- Theories of quark confinement include: dual superconductivity (electric flux tube connecting magnetic monopoles), condensation of center vortices, etc.
- Proposal by Mandelstam (1979) linking linear potential to infrared behavior of gluon propagator as $1/p^4$.
- Green's functions carry all information of a QFT's physical and mathematical structure.
- Confinement given by behavior at large distances (small momenta) $\Rightarrow$ nonperturbative study of IR propagators and vertices $\longrightarrow$ it requires very large lattice volumes.
- Gribov-Zwanziger confinement scenario based on suppressed gluon propagator and enhanced ghost propagator in the IR.

Quantization and Gribov Copies

The invariance of the Lagrangian under local gauge transformations implies that, given a configuration $\{A(x), \psi_f(x)\}$, there are infinitely many gauge-equivalent configurations $\{A^g(x), \psi_f^g(x)\}$ (gauge orbits). In the path integral approach we integrate over all possible configurations

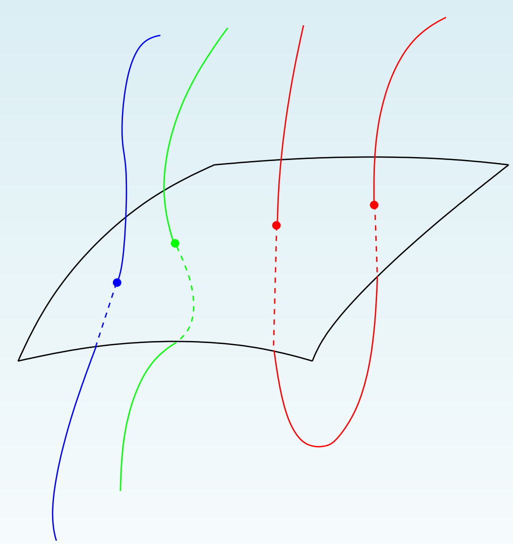
$$Z = \int DA \, \exp\left[-\int d^4x \, \mathcal{L}(x)\right]$$

There is an infinite factor coming from gauge invariance: $\int DA = \int D\overline{A}^g \, Dg$ and $\int Dg = \infty$.

To solve this problem we can choose a representative $\overline{A}$ on each gauge orbit (gauge fixing) using a gauge-fixing condition $f(\overline{A}) = 0$. The change of variable $A \to \overline{A}$ introduces a Jacobian in the measure.

Question: does the gauge-fixing condition select one and only one representative on each gauge orbit?

Answer: in general this is not true (Gribov copies).



Lattice Landau Gauge

In the continuum: $\partial_\mu \mathcal{A}_\mu(x) = 0$. On the lattice the Landau gauge is imposed by minimizing the functional

$$S[U;\omega] = -\sum_{x,\mu} Tr\, U_\mu^\omega(x)\ ,$$

where $\omega(x) \in SU(N)$ and $U_\mu^\omega(x) = \omega(x)\, U_\mu(x)\, \omega^\dagger(x + a\, e_\mu)$ is the lattice gauge transformation.

By considering the relations $U_\mu(x) = e^{i\, a\, g_0\, A_\mu(x)}$ and $\omega(x) = e^{i\, \tau\, \theta(x)}$, we can expand $S[U;\omega]$ (for small τ):

$$S[U;\omega] = S[U;\mathbb{1}] + \tau\, S^{'}[U;\mathbb{1}](b,x)\, \theta^b(x)$$

$$+ \frac{\tau^2}{2}\, \theta^b(x)\, S^{''}[U;\mathbb{1}](b,x;c,y)\, \theta^c(y) + \ldots$$

where $S^{''}[U;\mathbb{1}](b,x;c,y) = \mathcal{M}(b,x;c,y)[A]$ is a lattice discretization of the Faddeev-Popov operator $-D\cdot\partial$.

Constraining the Functional Integral

At a stationary point $S'[U;\mathbb{1}](b,x) = 0$, one obtains

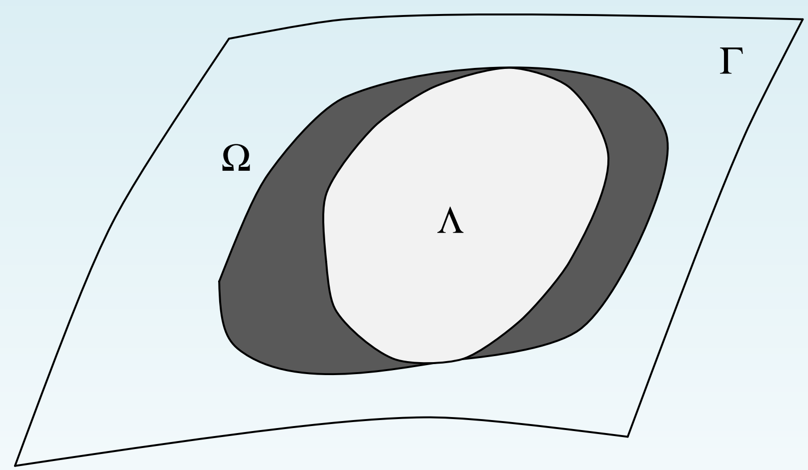
$$\sum_{\mu} A_{\mu}^{b}(x) - A_{\mu}^{b}(x - a\,e_{\mu}) = 0 \,,$$

which is a discretized version of the (continuum) Landau gauge condition. At a local minimum one also has $\mathcal{M}(b,x;c,y)[A] \geq 0$. This defines the first Gribov region (V.N. Gribov, 1978)

$$\Omega \equiv \{\, U : \partial \cdot A = 0,\, \mathcal{M} \geq 0 \,\} \equiv \text{ all local minima of } S[U;\omega] \,.$$

All gauge orbits intersect Ω (G. Dell'Antonio & D. Zwanziger, 1991) but the gauge fixing is not unique (Gribov copies).

Absolute minima of $S[U;\omega]$ define the fundamental modular region Λ, free of Gribov copies in its interior. (Finding the absolute minimum is a spin-glass problem.)



GZ Action and BRST Breaking

Analytically the restriction to the first Gribov region Ω can be achieved by adding a nonlocal term S_{h}, the horizon function (D. Zwanziger, 1993), to the usual Landau gauge-fixed Yang-Mills action:

$$S_{\mathrm{GZ}} = S_{\mathrm{YM}} + S_{\mathrm{gf}} + \gamma^4 S_{\mathrm{h}} \ ,$$

where the Gribov (massive) parameter γ is dynamically determined (in a self-consistent way) through the so-called horizon condition. The GZ action can be localized, using auxiliary fields (organized in BRST doublets), and can be written as

$$S_{\mathrm{GZ}} = S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_{\mathrm{aux}} + S_{\gamma} \ .$$

Under the usual nilpotent BRST variation s the localized GZ theory is not BRST-invariant. Indeed,

$$s\,(S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_{\mathrm{aux}}) = 0 \qquad \text{and} \qquad s\,S_{\gamma} \propto \gamma^2 \neq 0$$

but (M.A.L. Capri et al., 2015) $\quad s_{\gamma^2}\, S_{GZ} \;=\; \left(s + \delta_{\gamma^2}\right)\, S_{GZ} \;=\; 0$ (!!)

The Bose-ghost Propagator

Using the auxiliary fields $\omega_\mu^{ab}(x)$, $\overline{\omega}_\mu^{ab}(x)$, $\phi_\mu^{ab}(x)$, $\overline{\phi}_\mu^{ab}(x)$ one can consider the (BRST-exact) correlation function

$$Q_{\mu\nu}^{abcd}(x,y) = \langle\, s(\, \phi_\mu^{ab}(x)\, \overline{\omega}_\nu^{cd}(y))\,\rangle$$

$$= \langle\, \omega_\mu^{ab}(x)\, \overline{\omega}_\nu^{cd}(y) + \phi_\mu^{ab}(x)\, \overline{\phi}_\nu^{cd}(y)\,\rangle\,,$$

which (at tree level) is given by

$$Q_{\mu\nu}^{abcd}(k,k') = \gamma^4 \, \frac{(2\pi)^4\, \delta^{(4)}(k+k')\, g_0^2\, f^{abe} f^{cde} P_{\mu\nu}(k)}{k^2\, \left(k^4 + 2 g_0^2 N_c \gamma^4\right)}\,,$$

where $P_{\mu\nu}(k)$ is the usual transverse projector. [Extended to one loop by J.A. Gracey, JHEP 1002 (2010).]

The $Q(k^2)$ Propagator

We want to evaluate the scalar function $Q(k^2)$, defined through the relation

$$\gamma^{-4}\, Q^{abdb}_{\mu\mu}(k) \,\equiv\, \delta^{ad} N_c\, P_{\mu\mu}(k)\, Q(k^2)\;.$$

On the lattice one does not have direct access to the auxiliary fields $(\overline{\phi}^{ac}_{\mu}, \phi^{ac}_{\mu})$ and $(\overline{\omega}^{ac}_{\mu}, \omega^{ac}_{\mu})$. Nevertheless, since these fields enter the continuum action at most quadratically, we can integrate them out exactly. More precisely, one can

1. add sources to the (localized) GZ action,
2. explicitly integrate over the four auxiliary fields,
3. take the usual functional derivatives with respect to the sources, in order to obtain the chosen propagator.

The $Q(k^2)$ Propagator on the Lattice

This gives

$$\gamma^{-4}\, Q^{abcd}_{\mu\nu}(x-y) \,=\, \left\langle\, R^{ab}_{\mu}(x)\, R^{cd}_{\nu}(y)\,\right\rangle ,$$

where

$$R^{ab}_{\mu}(x) \,=\, \int d^4 z\, (\mathcal{M}^{-1})^{ae}(x,z)\, B^{eb}_{\mu}(z)$$

and $B^{eb}_{\mu}(z)$ is given by the covariant derivative $D^{eb}_{\mu}(z)$. Alternatively, by neglecting at the classical level the total derivatives $\partial_\mu(\phi^{aa}_{\mu} + \overline{\phi}^{aa}_{\mu})$ in the action S_γ, we find

$$B^{eb}_{\mu}(x) \,=\, g_0\, f^{ecb}\, A^{c}_{\mu}(x)\, .$$

The above expessions can be easily evaluated on the lattice.

Numerical Simulations

We evaluate the Bose-ghost propagator $Q(k^2)$ —modulo the global factor γ^4— using Monte Carlo simulations in the four-dimensional case for the SU(2) gauge group.

In order to check for discretization effects, we considered four different values of the lattice coupling β, corresponding to a lattice spacing a of about $0.210\,fm$, $0.140\,fm$, $0.105\,fm$ and $0.0841\,fm$. The lattice volumes V considered have physical volumes ranging from about $(3.366\,fm)^4$ to $(13.44\,fm)^4$.

The $B_\mu^{eb}(x)$ Vectors on the Lattice

We consider three different lattice $B_\mu^{eb}(x)$ vectors:

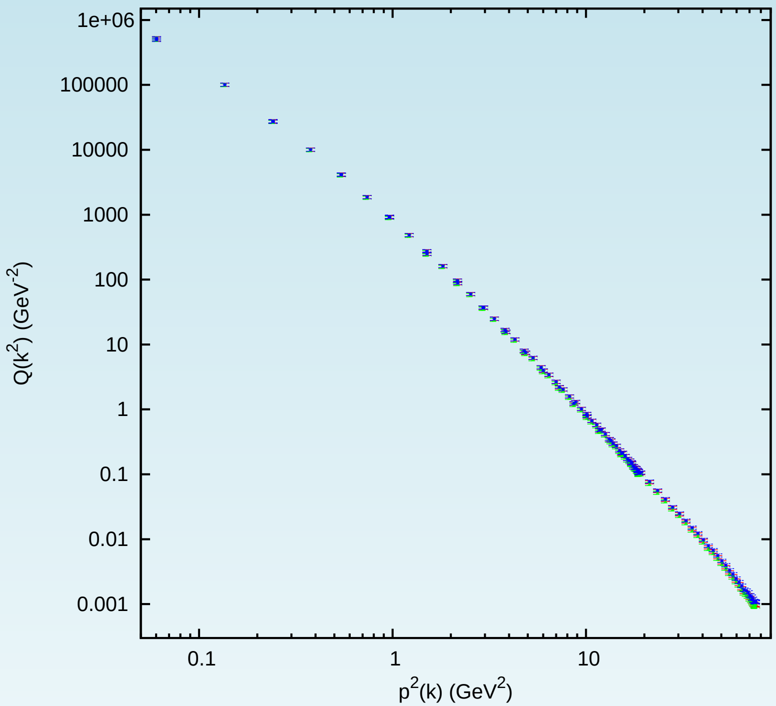
$$B_\mu^{bc}(x) = \delta^{bc} \frac{\mathrm{Tr}}{2} \left[U_\mu(x) \, - \, U_\mu(x - e_\mu) \right]$$

$$+ f^{cdb} \left[\, A_\mu^d(x) \, + \, A_\mu^d(x - e_\mu) \, \right] \, ,$$

which is a lattice discretization of the covariant derivative, the above equation without the diagonal part in color space (i.e. only the second line), and

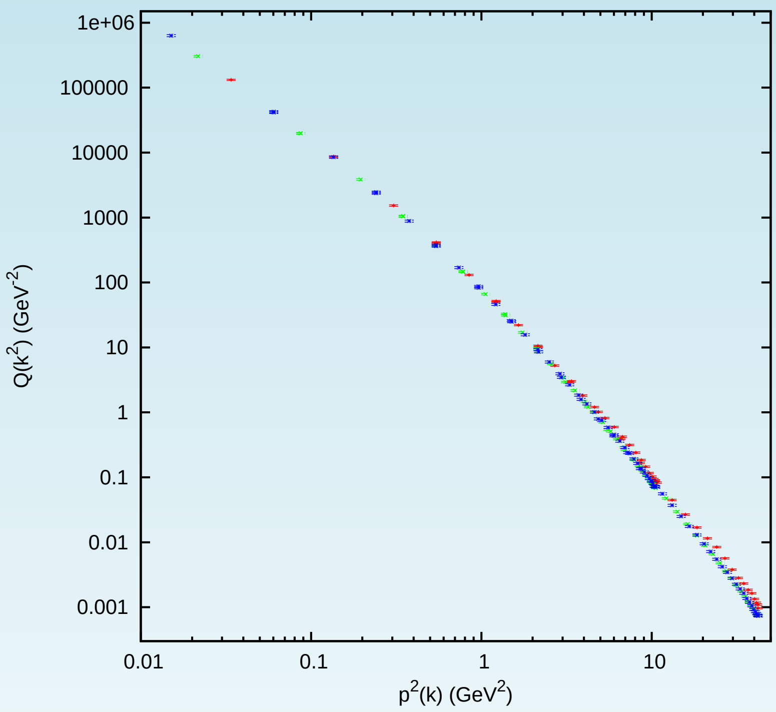
$$B_\mu^{bc}(x) \, = \, f^{bdc} \, A_\mu^d(x) \, .$$

Different $B_\mu^{eb}(x)$ Vectors



Plot of $Q(k^2)$ (lattice volume $V = 96^4$ at $\beta \approx 2.44$) as a function of the improved momentum squared $p^2(k)$ for the first (red, $+$), second (green, $\times$) and third (blue, $*$) different discretization of the sources $B_\mu^{bc}(x)$. For the latter case the data are multiplied by a factor 4. Note the logarithmic scale on both axes.

Finite-Volume Effects

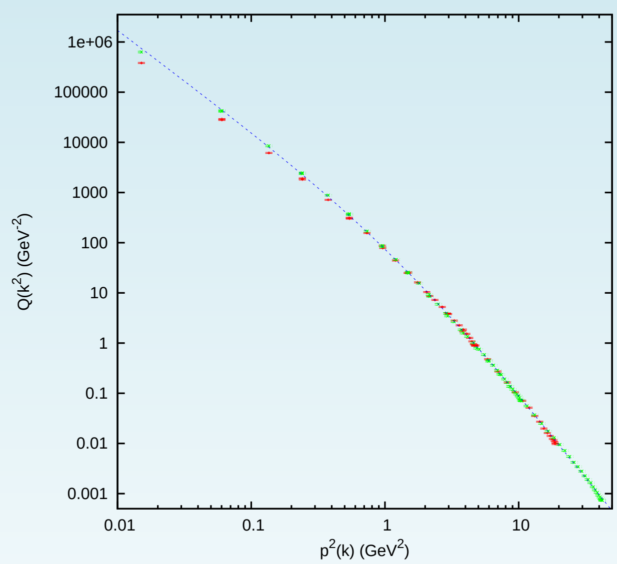


Plot of $Q(k^2)$ (at $\beta \approx 2.35$) as a function of the improved momentum squared $p^2(k)$ for the lattice volumes $V = 48^4$ (red, $+$), 60^4 (green, $\times$) and 72^4 (blue, $*$), using the third discretization formula for the sources $B_\mu^{bc}(x)$. Note the logarithmic scale on both axes.

Scaling and Fit (I)

Plot of $Q(k^2)$ at $\beta = 2.2$ and lattice volume $V = 48^4$ (+) matched with data at $\beta \approx 2.35$ and $V = 72^4$ (×). We also show the fitting function

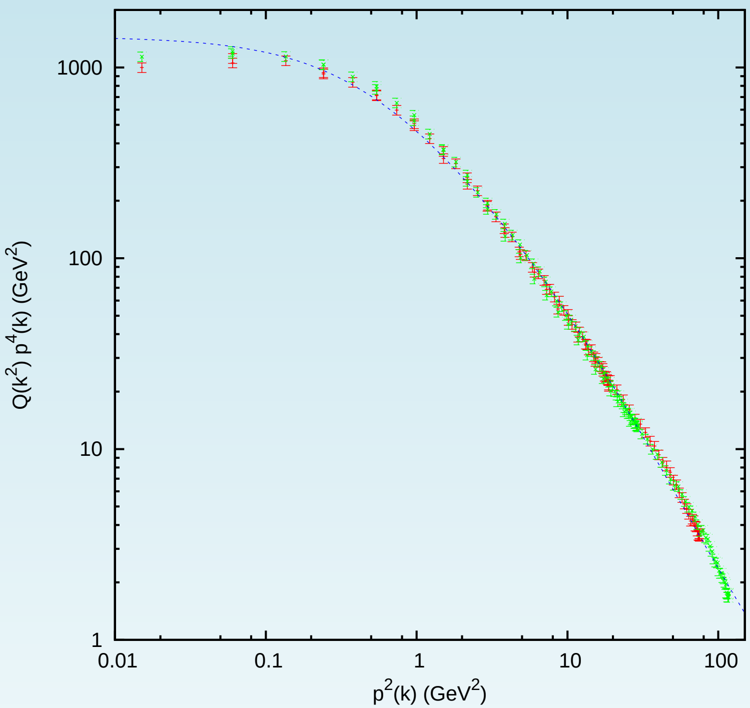
$$f(k^2) = \frac{c}{k^4} \frac{k^2 + s}{k^4 + u^2 k^2 + t^2} \sim G^2(k^2)\, D(k^2)$$



with $t = 3.2(0.3)(GeV^2)$, $u = 3.6(0.4)(GeV)$, $s = 49(14)(GeV^2)$ and $c = 37(4)$.

Note: $Q(k^2) \sim 1/k^4$ in the IR limit and $\sim 1/k^6$ in the UV limit.

Scaling and Fit (II)



Plot of $p^4(k)\,Q(k^2)$ at $\beta \approx 2.44$ and lattice volume $V = 96^4$ ($+$) matched with data at $\beta \approx 2.51$ and $V = 120^4$ ($\times$). We also show the fitting function

$$f(k^2) \,=\, c\,\frac{k^2 + s}{k^4 \,+\, u^2 k^2 \,+\, t^2}$$

with $t = 3.3(0.2)(GeV^2)$, $u = 4.8(0.3)(GeV)$, $s = 121(21)(GeV^2)$ and $c = 132(11)$. Note the logarithmic scale on both axes.

Poles of $Q(k^2)$

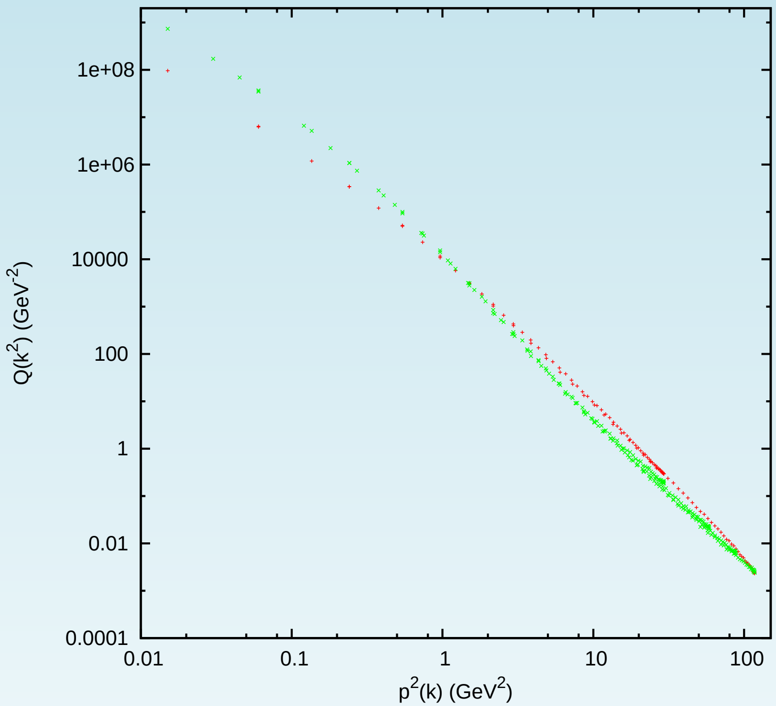
We can write the fitting function as

$$f(p^2) = \frac{c}{p^4}\left(\frac{\alpha_+}{p^2+\omega_+^2} + \frac{\alpha_-}{p^2+\omega_-^2}\right)$$

and the poles can be complex-conjugate, i.e. $\alpha_\pm = 1/2 \pm ib/2$ and $\omega_\pm^2 = v \pm iw$, or they can be real, i.e. $\alpha_\pm, \omega_\pm^2 = v \pm w \in \mathbb{R}$.

$V = N^4$	β	$v\,(GeV^2)$	$w\,(GeV^2)$	b or α_+	type
48^4	β_0	1.1(0.3)	2.0(0.2)	4.8(0.1)	$\mathbb{C}$
64^4	β_0	1.0(0.3)	1.9(0.2)	4.0(0.1)	$\mathbb{C}$
72^4	β_1	6.5(1.4)	5.6(0.2)	4.27(0.03)	$\mathbb{R}$
96^4	β_2	7.6(0.8)	6.99(0.04)	4.091(0.007)	$\mathbb{R}$
120^4	β_3	11.5(1.4)	11.04(0.06)	5.460(0.009)	$\mathbb{R}$

$Q(k^2)$ vs. $g_0^2\, G^2(k^2)\, D(k^2)$



Plot of $Q(k^2)$ (red, $+$) and of the product $g_0^2\, G^2(p^2)\, D(p^2)$ (green, $\times$) as a function of the improved momentum squared $p^2(k)$ for the lattice volume $V = 120^4$ at $\beta \approx 2.51$. The data of $Q(k^2)$ have been rescaled in order to agree with the data of the product $g_0^2\, G^2(p^2)\, D(p^2)$ at the largest momentum. Note the logarithmic scale on both axes.

Conclusions

To-do list:

- Extend these studies to the SU(3) case.
- Consider also the $2d$ and the $3d$ cases.
- Consider other correlation functions.

Conceptual issue:

- How to evaluate the Gribov parameter γ on the lattice?