Perspectives on the BRST symmetry and Gribov region in the Landau gauge

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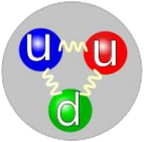
Ilhabela, 31/08/2015

Overview

1. Brief survey of the (Refined) Gribov-Zwanziger approach (GZ+RGZ)
2. (Softly broken) standard BRST
3. Construction of an “order parameter” for the standard BRST symmetry
4. A new BRST, its localization and nonperturbative definition

Overview

QCD



Our goal?

- We wish to start from "elementary" knowledge: the basic ingredients of YM: gluon & ghost (& quark) propagators and the interactions between them
- We shall work in a specific gauge, and try to describe/understand nonperturbative effects
- Could we get some (good) results on "physical" QCD quantities: spectrum, phase transition, Wilson/Polyakov loop, etc? See e.g. Justo's poster and Guimarães' & Palhares' talks.

Faddeev-Popov quantization of QCD

The Faddeev-Popov action

- Faddeev and Popov implemented a gauge choice as follows

$$\begin{aligned} Z_{FP} &= \int [dA]\delta(\partial A)\det \mathcal{M}^{ab} e^{-S_{YM}} \\ \mathcal{M}^{ab} &= -\partial_\mu(\partial_\mu \delta^{ab} - g f^{abc} A_\mu^c) \quad = \text{Faddeev-Popov operator} \end{aligned}$$

- The FP trick is based on the functional version of

$$\int dx \delta[f(x)-y]g(x) = \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x)=y}$$

- Notice that this *assumes* that $f(x) = y$ only has a single solution! Otherwise one needs

$$\int dx \delta[f(x)-y]g(x) = \sum_i \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x_i)=y}$$

Faddeev-Popov quantization of QCD

The Faddeev-Popov action

- Faddeev and Popov implemented a gauge choice as follows

$$Z_{FP} = \int [dA] \underbrace{\delta(\partial A) \det \mathcal{M}^{ab}}_{\to \text{ unity}} e^{-S_{YM}}$$

- $\delta(\partial A) \Rightarrow \partial A = 0 \equiv$ Landau gauge
- Faddeev-Popov determinant $\det \mathcal{M}^{ab}$ is corresponding Jacobian
- This form is not suitable to work/compute with (we want Feynman rules from local action)

Faddeev-Popov quantization of QCD

The Faddeev-Popov action in the Landau gauge

- We shall work with Landau gauge $\partial_\mu A_\mu = 0$.
- Very popular gauge, as it has many nice (quantum) properties.
- The eventual gauge fixed action reads

$$S_{YM} + S_{gf} = \int \mathrm{d}^4 x \left(\frac{1}{4} F_{\mu\nu}^2 + b^a \partial_\mu A_\mu^a + \overline{c}^a \partial_\mu D_\mu^{ab} c^b \right)$$

- We lifted the δ-constraint into the action with the b-field, and introduced anti-commuting (ghost) scalars for the lifting of the det.

The BRST symmetry

an important (crucial) symmetry

- Quantum action enjoys nilpotent BRST symmetry, $s(S_{YM}+S_{gf})=0$,

$$sA_\mu^a = -D_\mu^{ab}c^b\,, \qquad sc^a = \frac{g}{2}f^{abc}c^bc^c$$
$$s\bar{c}^a = b^a\,, \qquad sb^a = 0\,, \qquad s^2 = 0$$

- Quantum replacement for classical gauge invariance
- Used for proofs of
  - perturbative renormalizability via *Slavnov-Taylor identity*
  - perturbative unitarity: ghost, antighost, longitudinal and timelike gluon polarizations cancel, only 2 transverse gluon degrees of freedom survive!
  - BRST symmetry is an important concept

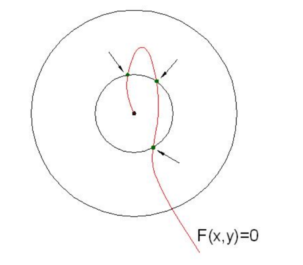
Potential flaw in FP quantization

The Gribov problem

- Take A_μ in Landau gauge $\Leftrightarrow \partial_\mu A_\mu = 0$
- Consider (infinitesimal) gauge transform: $A'_\mu = A_\mu + D_\mu \omega$
- $\partial_\mu A'_\mu = 0$ if $\partial_\mu D_\mu \omega = 0$
  GAUGE COPY if FP operator has (normalizable) zero modes!
- Was investigated (and drawn attention to) by Gribov in late seventies.

Potential flaw in FP quantization

The Gribov problem



- There is still some overcounting when using FP action (which is mathematically seen "wrong")
- Solution: use the more correct functional version of δ-function where the argument has multiple zeros.
- Unfortunately: cannot be put in useful partition function

Treating the copy problem

From Faddeev-Popov to Gribov-Zwanziger

- A class of copies was related to zero modes of Faddeev-Popov operator $M = -\partial D$
- Let us restrict path integral to region Ω where $\partial A = 0$ and $M > 0$.
- Ω corresponds to local minima of the functional $\int d^4xA_\mu^2$!
- $\Rightarrow$ This is already an improvement of Faddeev-Popov!
- Compare with lattice where one seeks for (in theory) global minima of $\int d^4xA_\mu^2$
- How to implement restriction to Ω in continuum???

The Gribov-Zwanziger action

Gribov-Zwanziger

- Gribov and later on Zwanziger worked out this problem and proved many properties of region Ω
- Example: every gauge orbit passes through Ω, it is convex and bounded in every direction.

Gribov-Zwanziger

Gribov→Zwanziger: formal version

- Restricts the integration to the Gribov region to all orders (work of Zwanziger)
- Implements the no-pole condition to all orders (see work of Sorella et al, positivity of FP operator)
- The Gribov-Zwanziger action is given by

$$S_h = S_{YM} + S_{gf} + \gamma^4 \int \mathrm{d}^4 x\, h(x)$$

  with the horizon function

$$h(x) = g^2 f^{abc} A_\mu^b \left(\mathcal{M}^{-1}\right)^{ad} f^{dec} A_\mu^e$$

- $$\text{horizon condition (= gap equation)} \ \langle h(x) \rangle = d(N^2-1)$$
- For $\gamma = 0$, everything reduces to Faddeev-Popov.

Gribov-Zwanziger

Gribov→Zwanziger: useful version

- We replace the action with a local (equivalent) action

$$S_{GZ} = S_{YM+GF} + S_h$$

with

$$\begin{aligned} S_h = & \int \mathrm{d}^4 x \left(\overline{\varphi}_\mu^{ac}\partial_\nu\left(\partial_\nu\varphi_\mu^{ac} + gf^{abm}A_\nu^b\varphi_\mu^{mc}\right) - \overline{\omega}_\mu^{ac}\partial_\nu\left(\partial_\nu\omega_\mu^{ac} + gf^{abm}A_\nu^b\omega_\mu^{mc}\right) - g\left(\partial_\nu\overline{\omega}_\mu^{ac}\right)f^{abm}\left(D_\nu c\right)^b\varphi_\mu^{mc}\right. \\ & \left. -\gamma^2 g\left(f^{abc}A_\mu^a\varphi_\mu^{bc} + f^{abc}A_\mu^a\overline{\varphi}_\mu^{bc} + \frac{4}{g}\left(N^2-1\right)\gamma^2\right)\right) \end{aligned}$$

- horizon condition (= gap equation)

$$\frac{\partial\Gamma}{\partial\gamma^2} = 0 \Leftrightarrow \underbrace{\left\langle gf^{abc}A_\mu^a(\varphi+\overline{\varphi})_\mu^{bc}\right\rangle = 2d(N^2-1)\gamma^2}_{d=2\,\text{condensate!!}}$$

Gribov-Zwanziger

Gribov-Zwanziger quantization

The GZ formalism is a geometrically inspired path-integral construction (cf. no-pole boundary condition to stay within the Gribov region) with good quantum properties that improves upon the standard FP quantization.

Gribov-Zwanziger quantization

Nice property: closely related to lattice formulation, as in both cases minimization of $\int A^2$ along the gauge orbit is used to define the (a) nonperturbative Landau gauge.

The Refined Gribov-Zwanziger action

Extra dynamical effects

- We included extra dynamical effects due to nonperturbative $d = 2$ condensates.
- $d = 2$ condensate was popularized in last decennium, especially in Landau gauge with $\langle A^2 \rangle$ (see work by Zakharov et al).
- Ghost propagator $G(p^2) \sim \frac{1}{p^2}$ for $p^2 \sim 0$.
- Gluon propagator
$$D(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4} .$$
  m^2 and M^2 are mass scales corresponding to condensates, in particular
$$m^2 \sim \langle A^2 \rangle , \qquad M^2 \sim \langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle$$
- Works pretty well to describe lattice data Oliveira et al, PRD81 (2010) 074505; Cucchieri, Mendes et al, PRD85 (2012) 094513 . See poster + other talks for applications.

Overview

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2. **(Softly broken) standard BRST**
3. Construction of an “order parameter” for the standard BRST symmetry
4. A new BRST, its localization and nonperturbative definition

The Gribov-Zwanziger action

What about the BRST symmetry?

- The (naturally extended) BRST symmetry

$$s\overline{\omega}_\mu^{ab} \;=\; \overline{\varphi}_\mu^{ab}\,, \qquad s\overline{\varphi}_\mu^{ab} = 0\,, \qquad s\varphi_\mu^{ab} = \omega_\mu^{ab}\,, \qquad s\omega_\mu^{ab} = 0\,,$$

  is softly broken

$$sS_{GZ} = g\gamma^2 \int d^4x \left(f^{abc} A_\mu^a \omega_\mu^{bc} - (D_\mu^{am} c^m)(\overline{\varphi}_\mu^{bc} + \varphi_\mu^{bc}) \right)$$

  softly means proportional to mass parameter γ^2, thus it can be controlled!

- Apparently: treating Gribov copy leads to soft breaking of BRST. Recall: Gribov copies $\leftrightarrow$ $\int A^2$ minimization (lattice)
- BRST breaking is perhaps also related to imposing boundary conditions on propagator(s) to single out (desired) solutions (cfr DSE).
- In effective Curci-Ferrari description (cf. Serreau & Tissier, PLB712 (2012) 97 , $\sim$ sampling over Gribov copies): BRST is also softly broken by $d = 2$

Back to the issue of BRST breaking

- Looks very harmful from the viewpoint of renormalization (**no!**), definition of subspace (**no!**), or positivity in subspace (**actually, the latter is not connected to the BRST itself!**)?
- From the viewpoint of gauge parameter independence of correlation functions of BRST invariant operators? (**we didn't know**)
  Gribov copies $\rightarrow$ nonperturbative definition of gauge is usually employed $\rightarrow$ as far as we know, this is usually coming from definitions like
  $$\min_{\text{gauge orbit}} \int d^4x\, \mathcal{F}(A_\mu^a),$$
  free of gauge parameters or so. Also, the GZ construction heavily relies on properties of Landau gauge. Extension to other gauges? see later and in particular talk of Sorella!
- There was some work in the noncovariant Coulomb gauge, and work by Reshetnyak et al ($\rightarrow$ very nonlocal actions)

A little history

- First 20 years of GZ formalism: not mentioned, no questions asked. Massive citations to GZ formalism, consistent with Kugo-Ojima, confinement understood (but all based on BRST symmetry, valid nonperturbatively??)
- It was however known, see original work of Zwanziger (1989-1993).
- 1994: Maggiore and Schaden came along with Phys.Rev. D50 (1994) 6616.
  The GZ action is invariant w.r.t. a new BRST, where
  $$s\overline{\omega}_\mu^{ab} = \overline{\varphi}_\mu^{ab} + \gamma^2 \delta^{ab} x_\mu$$
  so if $\gamma \neq 0$, spontaneous BRST breaking since $\langle \overline{\omega}_\mu^{ab} \rangle = \delta^{ab}\gamma^2 x_\mu$
- (1) x_μ-dependence? (2) problems with translational invariance/boundary conditions on the fields? (3) what about the Goldstone?
- Idea of spontaneous BRST breaking as a nonperturbative effect sounds appealing, but the MS implementation is unclear to us See also Schaden&Zwanziger, Phys.Rev. D92 (2015) 2, 025001 for recent thoughts using periodic box etc.

Excerpt from Zwanziger, NPB399(1993)

It is sometimes thought, erroneously, that BRS invariance is a form of gauge invariance, so some readers may conclude that the term in the action which violates BRS invariance means that a horrible gauge-violating error was made. To avoid possible confusion on this point, we emphasize that gauge invariance was in fact lost (more precisely, it was fully exploited) when the gauge was fixed in an optimal way, as described in the sect. 2. Moreover the theory with the parameter γ, whose renormalizability we wish to establish, does not represent a gauge theory at all, except when γ is assigned the value determined by the horizon condition, and this value is known only after calculations in the more general theory. On the other hand, BRS invariance is a new and useful symmetry that arises whenever a δ-function and its accompanying jacobian determinant are represented by integrals over a larger set of Bose and Fermi variables. The BRS transformation increases the fermion number by one, so it is defined only in the larger space. The BRS transformation may be isomorphic to an infinitesimal gauge transformation in some cases. The relevant issue is not whether the full action is BRS invariant but whether it is renormalizable. We shall see that it is sufficient that the dimension-4

The unitarity question

- Assume (as is common) that there is an exact BRST invariance, with $s^2 = 0$ (or charge $Q^2 = 0$).
- As Q generates a symmetry, the cohomology is time conserved.
- The physical operator subspace is then given by gauge invariant operators (let's take F^2 as example). The quantum version reads

$$O = F^2 + s(\text{something}) + EOM$$

  (first result by Zinn-Justin/Joglekar, later on generally proven by Henneaux et al)
- At level of correlation functions, as BRST is symmetry, one has

$$\langle OO \rangle = \langle F^2 F^2 \rangle$$

The unitarity question

- But, even if there would be a BRST, this does not prove anything on the positiveness (=“physicalness”) of the spectrum created by the gauge invariant operators.

$$\boxed{\text{BRST} \neq \text{unitarity}}$$

- The correct spectral properties of the eventual mesons, glueballs, ... need to be checked/proven. To our understanding, this is not always done, one is satisfied with finding the mass/decay, and there it stops [$\rightarrow$ already a tremendous effort!].
- This is absolutely not trivial!!!
- BRST is important to define an invariant subspace, but this does not prove the physicalness of that subspace.

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BRST exact correlator

paper in preparation

Define

$$\hat{\mathcal{F}}^{abcd}_{\mu\nu}(p^2) = \int d^4x\, e^{ip(x-y)}\, \mathcal{F}^{abcd}_{\mu\nu}(x-y)\,,$$

with

$$\mathcal{F}^{abcd}_{\mu\nu}(x-y) = \left\langle s\left(\bar{c}^a(x)\bar{c}^b(y)\int d^4z A^c_\mu(z)\int d^4z' g f^{dmn} A^m_\nu(z')c^n(z')\right)\right\rangle\,.$$

General idea

We will try to find a nonzero $\langle s(\ldots)\rangle$. Clear signal that s is not a symmetry generator.

Decomposition of BRST exact correlator

$$\hat{\mathcal{F}}^{abcd}_{\mu\nu}(p^2) = \hat{\mathcal{R}}^{abcd}_{\mu\nu}(p^2) + \hat{\mathcal{Q}}^{abcd}_{\mu\nu}(p^2)\ ,$$

$$\mathcal{R}^{abcd}_{\mu\nu} = \left\langle (b^a(x)\bar{c}^b(y) - \bar{c}^a(x)b^b(y)) \int d^4z A^c_\mu(z) \int d^4z' g f^{dmn} A^m_\nu(z') c^n(z') \right\rangle\ ,$$

and

$$\mathcal{Q}^{abcd}_{\mu\nu} = \left\langle \bar{c}^a(x)\bar{c}^b(y) \int d^4z\, g f^{cpq} A^p_\mu(z) c^q(z) \int d^4z' g f^{dmn} A^m_\nu(z') c^n(z') \right\rangle\ .$$

IR behaviour of BRST exact correlator

- We will argue that

$$\hat{\mathcal{R}}^{abcd}_{\mu\nu}(p^2)\Big|_{p\sim 0} \sim \frac{1}{p^2}\,, \qquad \hat{Q}^{abcd}_{\mu\nu}(p^2)\Big|_{p\sim 0} \sim \frac{1}{p^4}\,,$$

- $\hat{Q}^{abcd}_{\mu\nu}(p^2)$ dominating term in deep IR

$$\lim_{p^2\to 0}\left(p^4\,\hat{\mathcal{F}}^{abcd}_{\mu\nu}(p^2)\right) = \lim_{p^2\to 0}\left(p^4\,\hat{Q}^{abcd}_{\mu\nu}(p^2)\right) = \langle s(....)\rangle \sim \text{nonzero constant}\,,$$

- $\lim_{p^2\to 0}\left(p^4\,\hat{Q}^{abcd}_{\mu\nu}(p^2)\right)$ becomes "order parameter" for the breaking of the BRST symmetry.

A few technical comments

- We will employ RGZ, as this is compatible with lattice data. We need a partition function to discuss $\langle \ldots \rangle$.
- Though, our DSE or CF colleagues can, using their respective "language", consider the very same *s*-exact correlation function, its decomposition and try to establish the dominant IR behaviour [using the "massive/decoupling" solution; cf. IR power counting of Huber et al?]
- Major complication(s) (?): presence of (zero momentum) composite operator insertions related to *sA*; presence of *b*-field.

Lattice evidence for BRST breaking? See also talk of Cucchieri for more!

Nonlocal (equivalent) version of Q-correlator via FP operator

$$Q^{abcd}_{\mu\nu}(x-y) = \left\langle Q^{ab}_{\mu}(x) Q^{cd}_{\nu}(y) \right\rangle ,$$

$$Q^{ac}_{\mu}(x) = \int d^4z \, (\mathcal{M}^{-1})^{ad}(x,z) \, g f^{dec} A^{e}_{\mu}(z) ,$$

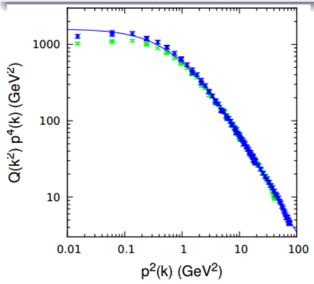

Figure : Cucchieri et al PRD90 (2014) 5, 051501

$$Q(p^2) \sim 1/p^4 \quad \text{for } p^2 \sim 0$$

The Q-correlator

Consistent with what we found. Combined with IR dominance of Q (not proven via lattice), would-be proof of BRST breaking.

Exploring the IR dominance of $\mathcal{Q}$

- We will show that, to all orders, $\mathcal{Q} \sim \frac{1}{p^4}$ is IR dominant over $\mathcal{R} \sim \frac{1}{p^2}$.
- using BPHZ(L)tools, which have rigorously proven IR power counting theorems.

  Piguet & Rouet review, PhysRep 76, 1 (1981).

  Similar results exist in “dimensional renormalization”
- BPHZ = momentum subtractions to UV-renormalize;

Exploring the IR dominance of Q

- Small issue: BPHZ is for massive theories. As ghost is massless, regulating ghost mass is needed. Specifically

$$
\begin{aligned}
S &= S_{RGZ} + \int d^4x\, s(\rho \bar{c}^a c^a) \\
&= S_{RGZ} + \int d^4x \left(\mu^2 (z^2 - 1) \bar{c}^a c^a - \rho b^a c^a + \rho \frac{g}{2} f^{abc} \bar{c}^a c^b c^c \right) ,
\end{aligned}
$$

  with

$$
s\rho = \mu^2 (z^2 - 1) , \qquad s\mu^2 = sz = 0 ,
$$

  Renormalizability is preserved, still Slavnov-Taylor identity Quadri J. Phys. G **30**, 677 (2004) .

- BPHZL = extra subtractions in z to treat IR, $z \to 1$ limit well-defined! Clean way to define massless DOFs.

  Notice: IR regularization is at action level, could also be implemented for DSEs?

Exploring the IR dominance of Q

- Reconsider

$$Q^{abcd}_{\mu\nu}(x-y) = \left\langle \bar{c}^a(x)\bar{c}^b(y) \int d^4z\, g f^{cpq} A^p_\mu(z) c^q(z) \int d^4z'\, g f^{dmn} A^m_\nu(z') c^n(z') \right\rangle .$$

This is equivalent to two-point correlator

$$\langle \bar{c}^a(x)\bar{c}^b(y)\rangle$$

with insertions of the composite operator $\int d^4z\, g f^{cpq} A^p_\mu(z) c^q(z)$.

- Can be formally defined (and consistent with BPHZL) via

$$S' = S_{RGZ} + g k^a_\mu \int d^4x f^{abc} A^b_\mu c^c ,$$

with k^a_μ a constant source with ghost charge -1. Leads to mixed gluon-ghost propagators to compute with.

Exploring the IR dominance of Q

Summary of the methodology

- At tree level

$$\hat{Q}^{abcd}_{\mu\nu}(p^2)\Big|_{\text{tree level}} \sim \langle \bar{c}(p)c(-p)\rangle\langle \bar{c}(p)c(-p)\rangle\langle A(p)A(-p)\rangle\Big|_{p\sim 0} \sim \frac{1}{p^4}\,.$$

- What at higher order?
- We will consider

$$\langle \bar{c}^a(x)\bar{c}^b(y)\rangle_{S'}$$

up to 2nd order in source k. This *defines* the Q-correlator!

Exploring the IR dominance of Q

Summary of the methodology

- Matrix propagator

$$R=\begin{pmatrix}
\langle A_\mu(p)A_\nu(-p)\rangle & \langle A_\mu(p)\varphi_\nu(-p)\rangle & \langle A_\mu(p)\bar\varphi_\nu(-p)\rangle & \langle A_\mu(p)b(-p)\rangle & \langle A_\mu(p)c(-p)\rangle & \langle A_\mu(p)\bar c(-p)\rangle \\
\langle \varphi_\mu(p)A_\nu(-p)\rangle & \langle \varphi_\mu(p)\varphi_\nu(-p)\rangle & \langle \varphi_\mu(p)\bar\varphi_\nu(-p)\rangle & \langle \varphi_\mu(p)b(-p)\rangle & \langle \varphi_\mu(p)c(-p)\rangle & \langle \varphi_\mu(p)\bar c(-p)\rangle \\
\langle \bar\varphi_\mu(p)A_\nu(-p)\rangle & \langle \bar\varphi_\mu(p)\varphi_\nu(-p)\rangle & \langle \bar\varphi_\mu(p)\bar\varphi_\nu(-p)\rangle & \langle \bar\varphi_\mu(p)b(-p)\rangle & \langle \bar\varphi_\mu(p)c(-p)\rangle & \langle \bar\varphi_\mu(p)\bar c(-p)\rangle \\
\langle b_\mu(p)A_\nu(-p)\rangle & \langle b_\mu(p)\varphi_\nu(-p)\rangle & \langle b_\mu(p)\bar\varphi_\nu(-p)\rangle & \langle b_\mu(p)b(-p)\rangle & \langle b_\mu(p)c(-p)\rangle & \langle b_\mu(p)\bar c(-p)\rangle \\
\langle c_\mu(p)A_\nu(-p)\rangle & \langle c_\mu(p)\varphi_\nu(-p)\rangle & \langle c_\mu(p)\bar\varphi_\nu(-p)\rangle & \langle c_\mu(p)b(-p)\rangle & \langle c_\mu(p)c(-p)\rangle & \langle c_\mu(p)\bar c(-p)\rangle \\
\langle \bar c(p)A_\nu(-p)\rangle & \langle \bar c(p)\varphi_\nu(-p)\rangle & \langle \bar c(p)\bar\varphi_\nu(-p)\rangle & \langle \bar c(p)b(-p)\rangle & \langle \bar c(p)c(-p)\rangle & \langle \bar c(p)\bar c(-p)\rangle
\end{pmatrix}$$

- IR asymptotic behaviour

$$R \xrightarrow{p\to 0} \begin{pmatrix}
\text{const.} & \text{const.} & \text{const.} & \frac{1}{p} & 0 & \frac{1}{p^2} \\
\text{const.} & \text{const.} & \text{const.} & \frac{1}{p} & 0 & \frac{1}{p^2} \\
\text{const.} & \text{const.} & \text{const.} & \frac{1}{p} & 0 & \frac{1}{p^2} \\
\frac{1}{p} & \frac{1}{p} & \frac{1}{p} & \frac{1}{p^2} & 0 & \frac{1}{p^2} \\
0 & 0 & 0 & 0 & 0 & \frac{1}{p^2} \\
\frac{1}{p^2} & \frac{1}{p^2} & \frac{1}{p^2} & \frac{1}{p^3} & \frac{1}{p^2} & \frac{1}{p^4}
\end{pmatrix}$$

Exploring the IR dominance of Q

Summary of the methodology

- 1PI decomposition ($\rightarrow$ BPHZL works at the level of 1PI correlators)

$$\langle \bar{c}(p)\bar{c}(-p)\rangle_{conn} = R_{\bar{c}\bar{c}} + \sum_{I,J} R_{\bar{c}I} M_{IJ} R_{J\bar{c}} + \sum_{I,J,K,L} R_{\bar{c}I} M_{IJ} R_{JK} M_{KL} R_{L\bar{c}} + \cdots$$

M_{IJ} = self-energy matrix

- BPHZL power counting analysis eventually leads to

$$M \xrightarrow{p\to 0} \begin{pmatrix} \text{const.} & \text{const.} & \text{const.} & p & \text{const.} & p^2 \\ \text{const.} & \text{const.} & \text{const.} & p & \text{const.} & p^2 \\ \text{const.} & \text{const.} & \text{const.} & p & \text{const.} & p^2 \\ p & p & p & p^2 & p & p^3 \\ \text{const.} & \text{const.} & \text{const.} & p & \text{const.} & p^2 \\ p^2 & p^2 & p^2 & p^3 & p^2 & p^4 \end{pmatrix}$$

Exploring the IR dominance of Q

Summary of the methodology

- We then have

$$MR \xrightarrow{p\to 0} \begin{pmatrix} \text{const.} & \text{const.} & \text{const.} & \frac{1}{p} & \text{const.} & \frac{1}{p^2} \\ \text{const.} & \text{const.} & \text{const.} & \frac{1}{p} & \text{const.} & \frac{1}{p^2} \\ \text{const.} & \text{const.} & \text{const.} & \frac{1}{p} & \text{const.} & \frac{1}{p^2} \\ p & p & p & \text{const.} & p & \frac{1}{p} \\ \text{const.} & \text{const.} & \text{const.} & \frac{1}{p} & \text{const.} & \frac{1}{p^2} \\ p^2 & p^2 & p^2 & p & p^2 & \text{const.} \end{pmatrix}$$

- Nice observation: one finds $(MR)^2 \sim MR, \quad \text{for } p \to 0$

$$\Rightarrow \langle \bar{c}(p)\bar{c}(-p)\rangle_{conn} \xrightarrow{p\to 0} \sim R_{\bar{c}\bar{c}} + \text{const.} \sum_{I,J} R_{\bar{c}I} M_{IJ} R_{J\bar{c}}$$

- Explicit analysis then yields

$$\langle \bar{c}(p)\bar{c}(-p)\rangle_{conn} \xrightarrow{p\to 0} \frac{1}{p^4}$$

what we wanted to show.

Uncovering the BRST breaking

Summary

- We introduced
$$\langle s(\ldots)\rangle = \mathcal{Q}+\mathcal{R}$$
- We showed, to all orders, that
$$\mathcal{Q}_{p^2\sim 0}\sim p^{-4}$$
Confirmed by lattice simulation.
- At tree level,
$$\mathcal{R}_{p^2\sim 0}\sim p^{-2}$$
Analysis underway: IR power counting to confirm this to all orders, via (lengthy) 1PI decomposition of $\mathcal{R}$.

  Comment: lattice simulation of $\mathcal{R}$ unfeasible, as depending on auxiliary b.
- Prominent signal of BRST breaking when using (lattice) Landau gauge

Uncovering the BRST breaking

Outlook

- Make the signal even more prominent $\rightarrow$ invitation to DSE colleagues.

  We used perturbative interactions vertices, albeit to all orders.
  But nonperturbative corrections to vertices?

  DSE estimates reported (mild) logarithmic singularities in some vertices Huber et al PRD89 (2014) 061703; Aguilar et al, PRD89, 085008 (2014). In principle no danger to IR power counting.

  Notice that these log-singularities can also appear in our case, similar to effective CF analysis of Tissier et al, PRD88, 125003 (2013). More refined IR power counting analysis exist in principle, see work of Breitenlohner & Maison.

- Genuine IR power counting via DSE?

Uncovering the BRST breaking

Outlook

- Other possible signal via

$$
\begin{aligned}
\mathcal{T} &= \mathcal{U}+\mathcal{S} \\
\mathcal{T} &= \frac{g}{2} f^{abc} \left\langle s\left(\bar{c}^a(x)\bar{c}^b(y)\int d^4z\, c^c(z)\right)\right\rangle \\
\mathcal{S} &= \frac{g^2}{4} f^{abc} f^{cmn} \left\langle \bar{c}^a(x)\bar{c}^b(y)\int d^4z\, c^m(z)c^n(z)\right\rangle \\
\mathcal{U} &= \frac{g}{2} f^{abc} \left\langle \left(b^a(x)\bar{c}^b(y)-\bar{c}^a(x)b^b(y)\right)\int d^4z\, c^c(z)\right\rangle
\end{aligned}
$$

- In the IR, S dominates and thus nonvanishing s-exact piece for $p^2 \to 0$.
- Link to lattice:

$$
\mathcal{S}(x-y) \propto \int d^4z\, (\mathcal{M}^{-1})^{am}(x,z)\, (\mathcal{M}^{-1})^{bn}(y,z)\,,
$$

Also clear distinction between scaling and decoupling by the way.

Overview

Motivation

- The standard BRST symmetry s is clearly broken when using (R)GZ.
- How to define a (physical) subspace?
- Without BRST, it is hard to "connect" different gauges (also with Gribov problem) while maintaining gauge invariance (or better said: gauge parameter independence).
- based on PRD92, 045039 (2015) and work in progress.

Preliminaries

- Consider A^2-functional

$$f_A[u] \equiv \mathrm{Tr}\int d^4x\, A^u_\mu A^u_\mu = \mathrm{Tr}\int d^4x \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u\right)\left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u\right)$$

and set $v = he^{ig\omega}$.

- Working up to 2nd order to identify minima:

$$f_A[v] = f_A[h] + 2\mathrm{Tr}\int d^4x\, (\omega\partial_\mu A^h_\mu) - \mathrm{Tr}\int d^4x\, \omega\partial_\mu D_\mu(A^h)\omega + O(\omega^3)\,,$$

$\Rightarrow$ $\partial_\mu A^h_\mu = 0$ & $-\partial_\mu D_\mu[A^h] > 0$

We recognize the Landau gauge and defining condition of the Gribov region (positive FP operator).

Preliminaries

- The "minimum configuration" can be solved for

$$A^h_\mu = A_\mu - \frac{1}{\partial^2}\partial_\mu\partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu,\partial_\nu\frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A,\frac{1}{\partial^2}\partial A\right] + ig\left[A_\mu,\frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A,\frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3)\,.$$

  It's transverse and gauge invariant order by order.

- Observation: if $\partial A = 0$, $A = A^h$. More precisely

$$A = A^h + \text{non-local power series in } (A,\partial A)$$

Rewriting GZ action

- $$A = A^h + \text{non-local power series in } (A, \partial A)$$

- Consider GZ action with non-local horizon action
  $H(A) = g^2 \int d^4x d^4y \; f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)$

$$\begin{aligned}
S_{GZ} &= S_{YM} + \int d^4x \left(b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A) \\
&= S_{YM} + \int d^4x \left(b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h) - \gamma^4 R(A)(\partial A) \\
&= S_{YM} + \int d^4x \left(b^h\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h) \; ,
\end{aligned}$$

  with a new field b^h

$$b^h = b - \gamma^4 R(A) \; .$$

Rewriting GZ action and identification of non-perturbative BRST

- Introduce auxiliary fields to obtain

$$
\begin{aligned}
S_{GZ} &= S_{YM} + \int d^4x \left(b^h \partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c \right) \\
&\quad + \int d^4x \ \left(\bar{\phi}\mathcal{M}(A^h)\phi - \bar{\omega}\mathcal{M}(A^h)\omega + \gamma^2 A^h(\bar{\phi}+\phi) \right)
\end{aligned}
$$

- This new (equivalent) GZ action in the Landau gauge enjoys a nilpotent (unbroken) BRST symmetry

$$
s_{\gamma^2} = s + \delta_{\gamma^2} \,, \qquad s_{\gamma^2} S_{GZ} = 0
$$

with

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b \,, \; sc^a = \frac{g}{2} f^{abc} c^b c^c \,, \; s\bar{c}^a = b^a \,, \; sb^a = 0 \,, \\
s\phi^{ab}_\mu &= \omega^{ab}_\mu \,, \; s\omega^{ab}_\mu = 0 \,, \; s\bar{\omega}^{ab}_\mu = \bar{\phi}^{ab}_\mu \,, \; s\bar{\phi}^{ab}_\mu = 0 \\
, \delta_{\gamma^2} \bar{c}^a &= -\gamma^4 R^a(A) \,, \quad \delta_{\gamma^2} b^a = \gamma^4 sR^a(A) \,, \\
\delta_{\gamma^2} \bar{\omega}^{ac}_\mu &= \gamma^2 g f^{kbc} A^{h,k}_\mu \left[\mathcal{M}^{-1}(A^h) \right]^{ba} \,, \; \delta_{\gamma^2}(\text{rest}) = 0 \,.
\end{aligned}
$$

Rewriting GZ action and identification of non-perturbative BRST

- Operator algebra

$$\{s, \delta_{\gamma^2}\} = s^2 = \delta_{\gamma^2}^2 = s_{\gamma^2}^2 = 0$$

and clearly, for $\gamma^2 \to 0$ (GZ$\to$FP) we have $s_{\gamma^2} \to s$.

- New BRST operator s_{γ^2} is genuinely non-perturbative, as it depends on $\gamma^2 \propto \Lambda_{QCD}^2$. γ^2 is a physical parameter, as it does not couple to s_{γ^2}-exact piece.

- Thanks to s_{γ^2}, new Ward identities

$$\langle s_{\gamma^2}(\bar{c}\Lambda)\rangle = 0 ,$$

where Λ has ghost number zero. Standard BRST operator s will always acquire a breaking term proportional to γ^2, namely

$$\langle s(\bar{c}\Lambda)\rangle = -\langle \delta_{\gamma^2}(\bar{c}\Lambda)\rangle .$$

Non-perturbative BRST and application to other gauges

Stay tuned for the talk of Sorella!

Non-perturbative BRST

2 potential criticisms

1. What about convergence of A^h? Non-perturbative lattice formulation possible?
2. Pretty non-local formulation. How to work with this?

Locality of the action (short)

- Using a clever rewriting with multipliers, everything can be brought into equivalent local formulation.

$$
\begin{aligned}
S^1_{loc} \quad &= \quad \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu(A) c^b \right. \\
&+ \quad \bar{\varphi}^{ac}_\nu \mathcal{M}^{ab}(B) \varphi^{bc}_\nu - \bar{\omega}^{ac}_\nu \mathcal{M}^{ab}(B) \omega^{bc}_\nu + \gamma^2 g f^{abc} B^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) \\
&\quad \left. + \lambda^a_\mu \partial^2 (B^a_\mu - (A^h)^a_\mu) + \tau^a \partial_\mu B^a_\mu \right\} .
\end{aligned}
$$

1st constraint: The Linear Covariant Gauge condition, i.e. $\partial_\mu A^a_\mu = i\alpha\, b^a$;
2nd constraint: $B^a_\mu = (A^h)^a_\mu$;
3rd constraint: The field B is transverse, i.e. $\partial_\mu B^a_\mu = 0$.

- Locality of A^h_μ via Stückelberg field

$$
A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \quad h = e^{ig\xi^a t^a}
$$

Can be combined with algebraic renormalization formalism, following

Dragon, Hürth & Van Nieuwenhuizen, Nucl.Phys.Proc.Suppl.56B (1997) 318.

Locality of the BRST (short)

s

$$\begin{aligned} sh^{ij} &= -igc^a(t^a)^{ik}h^{kj}\,, \quad s\phi=\omega\,, \quad s\omega=0\,, \\ s\overline{\omega} &= \overline{\phi}\,, \quad s\overline{\phi}=0\,, \quad s\lambda=s\tau=sB=0 \end{aligned}$$

δ_{γ^2}

$$\delta_{\gamma^2}\overline{\omega}=B\frac{1}{\mathcal{M}(B)}\,, \qquad \delta_{\gamma^2}(\text{rest})=0$$

s_{γ^2}

$$s_{\gamma^2}=s+\delta_{\gamma^2}\text{ generates symmetry of local action}\,, \quad s_{\gamma^2}S^1_{loc}=0\,, \quad s^2_{\gamma^2}=0$$

BRST is still nonlocal.

Locality of the BRST (short)

- Work with (following Dudal&Vandersickel, PLB700 (2011) 369)

$$S^2_{loc} = S^1_{loc} + \int d^4x \left\{ \beta \mathcal{M}(B)\bar{\beta} + \psi \mathcal{M}(B)\bar{\psi} - gB\bar{\beta} \right\}$$

with

$$s\overline{\psi} = 0, \qquad s\overline{\beta} = 0, \qquad s\beta = 0, \qquad s\psi = 0$$

- It can be shown that

$$\langle \ldots \rangle_{S^1_{loc}} = \langle \ldots \rangle_{S^2_{loc}}$$

- Introduce

$$\delta'_{\gamma^2}\overline{\beta} = \omega, \qquad \delta'_{\gamma^2}\overline{\omega} = \beta, \qquad \delta_{\gamma^2}(\text{rest}) = 0$$

- Then

$$s'_{\gamma^2} = s + \delta'_{\gamma^2}, \qquad s'^2_{\gamma^2} = 0, \qquad s'_{\gamma^2} S^2_{loc} = 0$$

is an equivalent 100% local formulation. Can be used to study Ward identities, renormalization etc. (underway)

Nonperturbative definition of A^h? (very short)

- We'd propose (inspired by Cucchieri & Mendes PRL103 (2009) 141602)

$$
\begin{aligned}
\mathcal{R}(A,B,U,V) \equiv & \operatorname{Tr}\int d^4x \left(A_\mu^U A_\mu^U + \frac{2}{g}\operatorname{Re}(iU\Lambda)\right) + \operatorname{Tr}\int d^4x \left(B_\mu^V B_\mu^V\right) \\
& + \operatorname{Tr}\int d^4x \left(B_\mu^V - P_{\mu\nu}A_\nu^U\right)^2
\end{aligned}
$$

with $P_{\mu\nu} = \delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}$.

- Gauging = minima of $\mathcal{R}(A,B,U,V)$, for a fixed function $\Lambda(x) = \Lambda^a(x)t^a$ in function of variable U, V.
- (Local) minimization leads to

$$
\begin{aligned}
B &= \text{transverse part of } A \\
\partial B &= 0\,, \qquad \partial A = \Lambda \text{ (linear covariant gauge)} \\
\mathcal{M}(B) &> 0
\end{aligned}
$$

(in principle) nonperturbative B corresponds to A^h! There is other positivity rule, but can be ignored. No time to explain, related to Sorella's talk and some manipulations.

Fim!

Obrigado!