The universal auxiliary function of QCD

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Introduction

Our understanding of the infrared (IR) properties of the fundamental QCD Green's functions has improved considerably in the last few years, due to a variety of parallel efforts in lattice simulations [1, 2, 3] and Schwinger-Dyson equations (SDEs) [4, 5]. The majority of the aforementioned studies have focused on the low-momentum behavior of the quark, gluon and ghost propagators, which can be directly or indirectly related to some of the most fundamental nonperturbative phenomena of QCD, such as quark confinement, dynamical mass generation, and chiral symmetry breaking.

In the case of the SDEs, several major results were derived within the framework provided by the synthesis of the pinch technique (PT) with the background field method (BFM), known in the literature as the PT-BFM scheme [4]. Most notably, the formulation of the gluon SDE in this scheme furnishes considerable advantages, because it allows for a systematic truncation that respects manifestly, and at every step, the gauge invariance [5].

In this formalism, an auxiliary function, denoted by $G(q^2)$ plays a central role. This function is the $g_{\mu\nu}$ co-factor in the Lorentz decomposition of a special Green's function, denoted by $\Lambda_{\mu\nu}(q)$, which appears in a variety of field-theoretic contexts. In particular, $\Lambda_{\mu\nu}(q)$ enters in all "background-quantum" identities (BQIs), i.e. the infinite tower of non-trivial relations connecting the PT-BFM Green's functions to the conventional ones [4], as shown in Fig. 1.

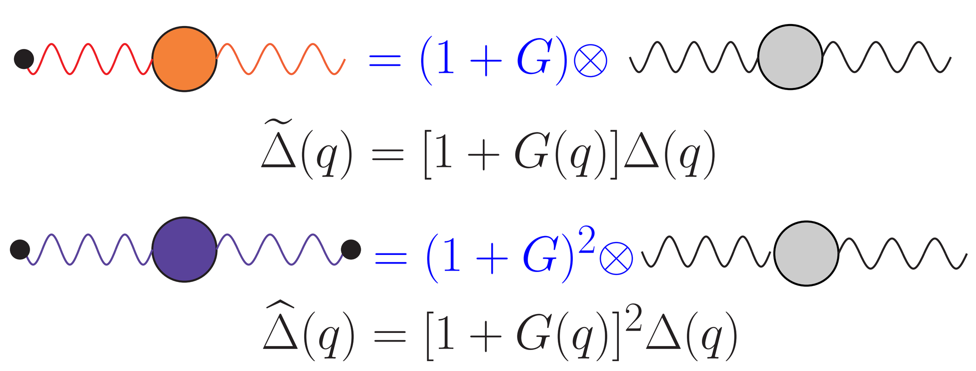

FIGURE 1: The "background-quantum" identities (BQIs) connecting the conventional gluon propagator $\Delta(q^2)$ with the PT-BFM gluon propagators $\widehat{\Delta}(q^2)$ and $\widetilde{\Delta}(q^2)$.

Most importantly, the same auxiliary function, jointly with the gluon propagator and the constant coupling $\alpha_s(\mu^2) = g^2/4\pi$, enter into the definition of the renormalization group invariant (RGI) - μ-independent quantity [6]

$$\widehat{d}(q^2) = \alpha_s(\mu^2)\frac{\Delta(q^2,\mu^2)}{[1+G(q^2,\mu^2)]^2}, \tag{1}$$

which is a fundamental ingredient in many phenomenological applications [7]. In this poster we will present our first steps towards the derivation of an improved calculational scheme for the dynamical equation governing the behavior of the auxiliary function $1+G(q^2)$.

The Formalism

Let us first introduce the notation and define some of the basic quantities appearing in the problem under study.

In the Landau gauge, the gluon propagator $\Delta_{\mu\nu}(q)$ has the form

$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2), \qquad P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2 \tag{2}$$

In addition, the full ghost propagator, $D(q^2)$, and its dressing function, $F(q^2)$, are related by

$$D(q^2) = \frac{iF(q^2)}{q^2}. \tag{3}$$

Moreover, the all-order ghost vertex (after factoring out the color structure and the coupling constant g) will be denoted by $\boldsymbol{\Gamma}_\mu(k,q)$, with k representing the momentum of the gluon and q that of the anti-ghost. Its tensorial structure is given by [6]

$$-\boldsymbol{\Gamma}_\mu(-k-q,k,q) = B_1(-k-q,k,q)q_\mu + B_2(-k-q,k,q)k_\mu. \tag{4}$$

Thus, at tree-level $\boldsymbol{\Gamma}^{(0)}_\mu(-k-q,k,q) = -q_\mu$.

The central quantity of the present study, namely $G(q^2)$, originates from the two-point function $\Lambda_{\mu\nu}(q)$, represented in Fig. 2, defined as

$$\begin{aligned}\Lambda_{\mu\nu}(q) &= -ig^2C_A\int_k H^{(0)}_{\mu\rho}D(k+q)\Delta^{\rho\sigma}(k)\,H_{\sigma\nu}(-k-q,k,q),\\ &= g_{\mu\nu}G(q^2) + \frac{q_\mu q_\nu}{q^2}L(q^2),\end{aligned} \tag{5}$$

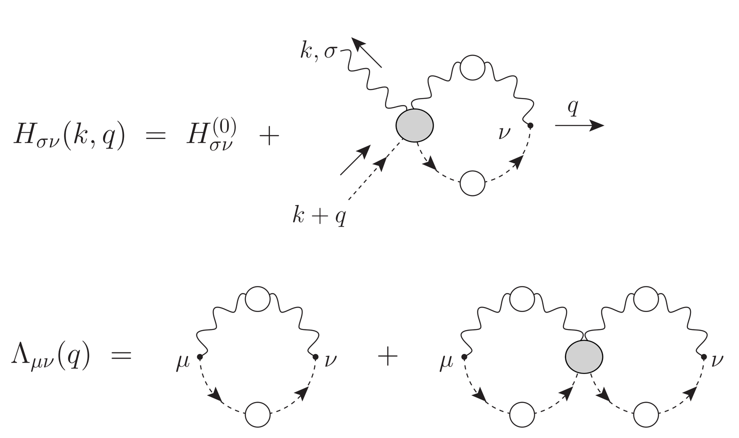

FIGURE 2: Diagrammatic representation of the functions H and Λ.

The function $H_{\mu\nu}(k,q)$ (see Fig. 2 for a diagrammatic definition) is in fact a familiar object: it appears in the all-order Slavnov-Taylor identity satisfied by the standard three-gluon vertex, and is related to the full gluon-ghost vertex by

$$q^\nu H_{\mu\nu}(-k-q,k,q) = -i\boldsymbol{\Gamma}_\mu(-k-q,k,q). \tag{6}$$

At tree-level, $H^{(0)}_{\mu\nu} = ig_{\mu\nu}$. Finally, using the most general Lorentz decomposition of $H_{\mu\nu}$ [6],

$$-iH_{\mu\nu}(-k-q,k,q) = A_1g_{\mu\nu} + A_2q_\mu q_\nu + A_3k_\mu k_\nu + A_4q_\mu k_\nu + A_5k_\mu q_\nu,$$

we obtain from Eq. (4) and Eq. (6) two constrains for the various form-factors, namely

$$B_1 = A_1 + q^2A_2 + (k\cdot q)A_4, \qquad B_2 = (k\cdot q)A_3 + q^2A_5. \tag{7}$$

where $A_i = A_i(-k-q,k,q)$ and $B_j = B_j(-k-q,k,q)$.

Let us study the functions $G(q^2)$ and $L(q^2)$ more closely. From Eq. (5) we have that (in d dimensions)

$$\begin{aligned}G(q^2) &= C_d\left[\int_k \Delta^{\rho\sigma}(k)\,H_{\sigma\rho}D(k+q) + i\frac{1}{q^2}\int_k q^\rho\Delta_{\rho\sigma}(k)\,\boldsymbol{\Gamma}^\sigma D(k+q)\right],\\ L(q^2) &= -C_d\left[i\frac{d}{q^2}\int_k q^\rho\Delta_{\rho\sigma}(k)\,\boldsymbol{\Gamma}^\sigma D(k+q) + \int_k \Delta^{\rho\sigma}(k)\,H_{\sigma\rho}D(k+q)\right].\end{aligned}$$

where we have defined $C_d = g^2C_{\rm A}/(d-1)$, with $C_{\rm A}$ the Casimir of the adjoint representation [$C_{\rm A} = N$ for $SU(N)$].

Inserting the decomposition of Eq. (4) and Eq. (7) in the above equation, we obtain [6]

$$\begin{aligned}G(q^2) &= C_d\int_k\left\{(d-1)A_1 - \left[1 - \frac{(k\cdot q)^2}{k^2q^2}\right]\left[B_1 - q^2A_2\right]\right\}\Delta(k)D(k+q),\\ L(q^2) &= C_d\int_k\left\{(1-d)A_1 + \left[1 - \frac{(k\cdot q)^2}{k^2q^2})\right]\left[dB_1 - q^2A_2\right]\right\}\Delta(k)D(k+q),\end{aligned} \tag{8}$$

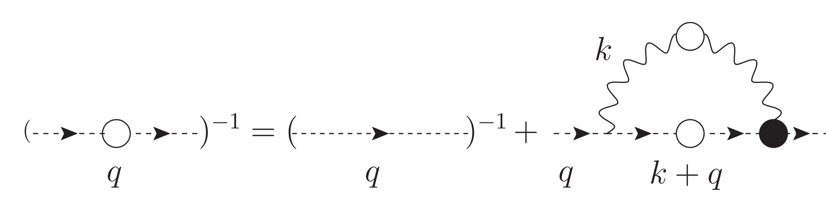

FIGURE 3: The SDE for the ghost.

At this point, let us consider the standard SDE of the ghost propagator, given in Fig. 3; substituting in it the decomposition of Eq. (4), we obtain

$$F^{-1}(q^2) = 1 + g^2C_{\rm A}\int_k\left[1 - \frac{(k\cdot q)^2}{k^2q^2}\right]B_1(-k-q,k,q)\Delta(k)D(k+q). \tag{9}$$

Then, it is straightforward to show that Eqs. (9) and (8) satisfies the relation [6]

$$F^{-1}(q^2) = 1 + G(q^2) + L(q^2). \tag{10}$$

This special relation, which is valid only in the Landau gauge, may also be formally derived within the framework of the Batalin-Vilkovisky quantization formalism [8].

Turning to the actual evaluation of the above quantities, notice that the integrals appearing in Eq. (8) involve the form factors A_1, A_2 and B_1, whilst in the integral of Eq. (9) enters only B_1. The form factor B_1 was determined from its own SDE in the limit of vanishing ghost momentum ($q = 0$) [9] (See Fig. 4).

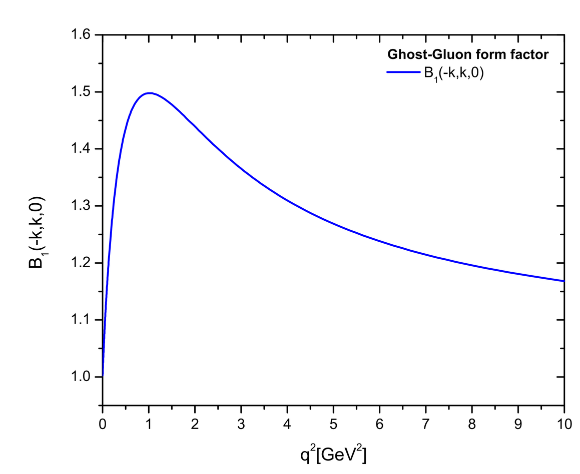

FIGURE 4: The form factor $B_1(-k,k,0)$ obtained in Ref. [9] from its own SDE, in the limit of vanishing ghost momentum ($q = 0$).

On the other hand, the form factors A_1 and A_2 are not known. In order to proceed further, we will consider the first relation in Eq. (7), and we will neglect the contributions of A_2 and A_4, thus obtaining $A_1 = B_1$. Implementing these approximations, one obtains the final set of equations [6]

$$\begin{aligned}1 + G(q^2) &= Z_c + C_d\int_k\left[(d-2) + \frac{(k\cdot q)^2}{k^2q^2}\right]B_1(-k,k,0)\Delta(k)D(k+q),\\ F^{-1}(q^2) &= Z_c + (d-1)C_d\int_k\left[1 - \frac{(k\cdot q)^2}{k^2q^2}\right]B_1(-k,k,0)\Delta(k)D(k+q),\\ L(q^2) &= C_d\int_k\left[1 - \frac{(k\cdot q)^2}{k^2q^2}\right]B_1(-k,k,0)\Delta(k)D(k+q),\end{aligned} \tag{11}$$

where the renormalization constant Z_c is determined from the MOM condition $F(\mu^2) = 1$.

Numerical Results

The numerical results for Eqs. (11) were obtained using the lattice data for $\Delta(k)$ given in the Ref. [2], while for $D(k)$ we use the solution obtained from the ghost SDE. The renormalization constant was fixed at $\mu = 4.3$ GeV, and $\alpha_s(\mu) = 0.22$ throughout.

In Fig. 5 and Fig. 6, we compare our numerical results for $[1+G(q^2)]^{-1}$, $L(q^2)$ and $F(q^2)$ given by Eq. (11) in two different scenarios: (i) $B_1(-k,k,0) = 1$, which corresponds to the tree level approximation for the gluon-ghost vertex (red dashed curve), and (ii) an improved version of the vertex, where $B_1(-k,k,0)$ is given as in Fig. 4 (black continuous curve).

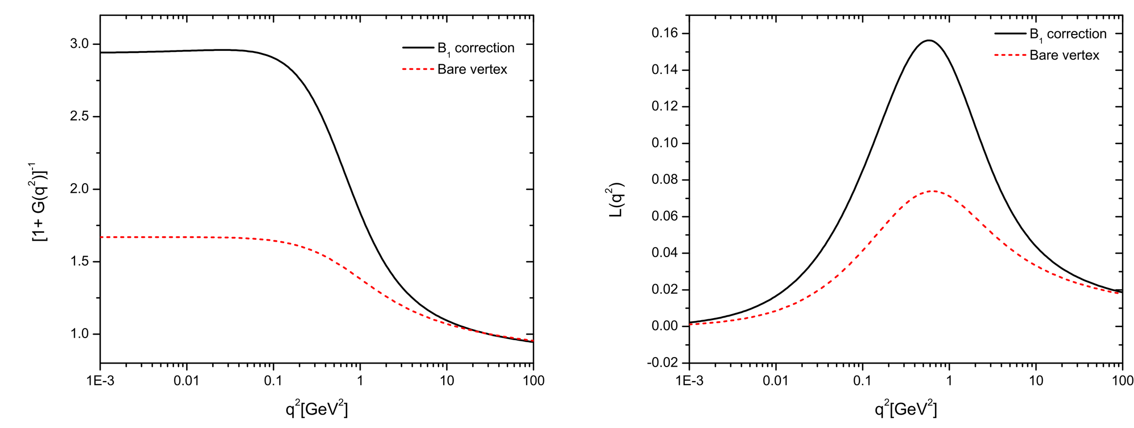

FIGURE 5: Comparison of the solutions for $1+G(q^2)$ (right panel) and $L(q^2)$ (left panel) determined from Eq. (11) using the form factor correction $B_1(-k,k,0)$ (black curve) and the bare vertex (red curve).

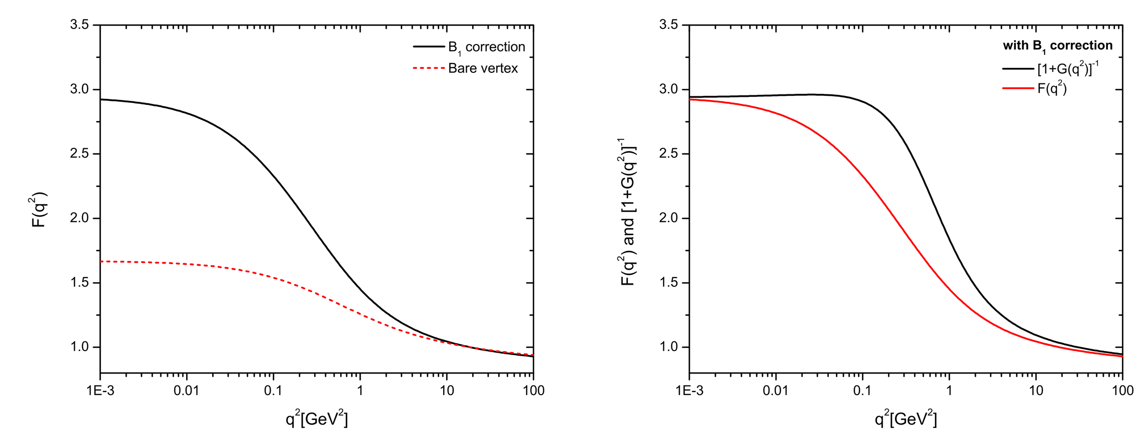

FIGURE 6: Comparison of the solutions for $F(q^2)$ (right panel) determined from Eq. (11) using the form factor correction $B_1(-k,k,0)$ (black curve) and the bare vertex (red curve). In the right panel we compare the solutions for $F(q^2)$ and the inverse of $1+G(q^2)$ in the presence of the form factor B_1.

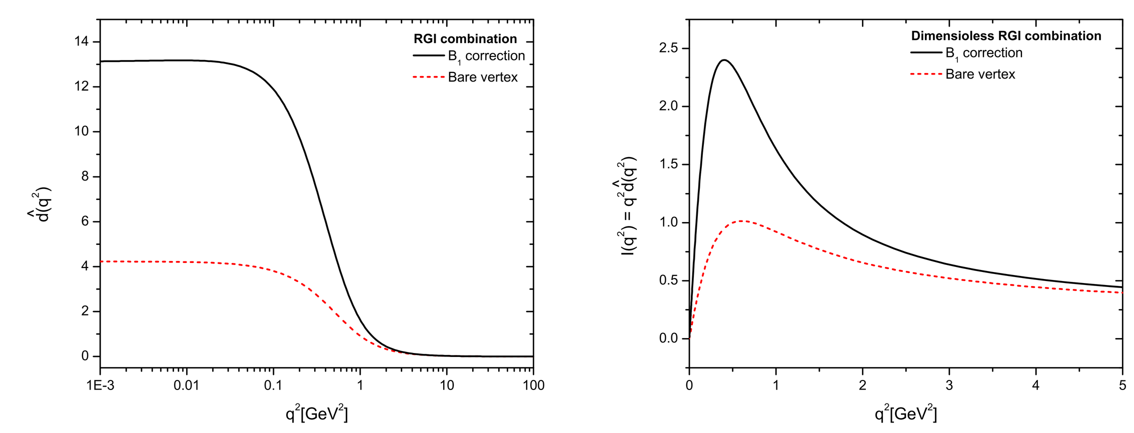

FIGURE 7: The corresponding dimensionful RGI combination, $\widehat{d}(q^2)$, given by Eq (1) (left panel), and the dimensionless quantity, $q^2\widehat{d}(q^2)$ (right panel), for the cases where $1+G(q^2)$ was computed with the form factor correction $B_1(-k,k,0)$ (black curve) or just with the bare vertex (red curve).

Conclusions

We have clearly established that $B_1(-k,k,0)$ has a sizable effect on both $[1+G(q^2)]$ and $L(q^2)$. We have also checked that the fundamental relation given by Eq. (10) is numerically satisfied in both approximations. In addition, we notice that even though $L(q^2)$ vanishes at the origin, it has a non-vanishing support in the region of physical interest (see the right panel of Fig.5). Therefore, the approximation $F^{-1}(q^2) \approx 1+G(q^2)$, often employed in the literature, is not sufficiently accurate in this case.

In Fig. 7 we present the same comparison for the dimensionful $\widehat{d}(q^2)$ and the dimensionless $q^2\widehat{d}(q^2)$ RGI quantities given by Eq. (1). Again, we notice the significant impact caused by the form factor B_1 in the interaction strengths. It would be interesting to quantify what would be the effect on the $1+G(q^2)$ caused by form factors A_2 and A_4 which were neglected our analysis. Most certainly, these corrections will reflect in changes in the RGI interaction strengths. Calculations in this direction are already in progress.

References

[1] A. Cucchieri and T. Mendes, PoS LAT **2007**, 297 (2007).

[2] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, PoS LAT **2007**, 290 (2007).

[3] O. Oliveira and P. J. Silva, Phys. Rev. D **86**, 114513 (2012).

[4] D. Binosi and J. Papavassiliou, Phys. Rept. **479**, 1 (2009).

[5] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008).

[6] A. C. Aguilar, D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D **80**, 085018 (2009).

[7] D. Binosi, L. Chang, J. Papavassiliou and C. D. Roberts, Phys. Lett. B **742**, 183 (2015).

[8] P. A. Grassi, T. Hurth and A. Quadri, Phys. Rev. D **70**, 105014 (2004).

[9] A. C. Aguilar, D. Ibáñez and J. Papavassiliou, Phys. Rev. D **87**, no. 11, 114020 (2013).

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