SM vacuum stability with an IR frozen QCD coupling constant

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Abstract

Several phenomenological and theoretical arguments favor a freezing of the QCD coupling constant in the infrared region at one moderate value. This coupling can be parameterized in terms of an effective dynamical gluon mass which is determined through Schwinger-Dyson equations, whose solutions are compatible with QCD lattice simulations. We find that the existence of such non-perturbative infrared fixed point moves the λ evolution towards stability. For the phenomenological preferred IR value of the QCD coupling constant the standard model Higgs potential is stable up to the Planck scale [1].

Introduction

One of the techniques to study QCD at low energies are the Schwinger-Dyson Equations (SDE). This is an analytic method but with the complication of dealing with an infinite tower of coupled integral equations. Because of this, when using the SDE, one have to make a truncation in the system of equations, which has to be done in a way that the symmetries of the theory are not broken, in particular, a rough truncation in the SDE can cause a violation of the gauge symmetry. In the recent years an enormous progress has been made in solving SDE in a gauge invariant way using the so called Pinch Technique [2], as a result it has been found the existence of a dynamical gluon mass in the propagator of the gluon field, as suggested many years ago by Cornwall [3]. Another non-perturbative technique is Lattice QCD [4], which implies heavy numerical calculations requiring a considerably high amount of computer power. Results obtained in QCD lattice simulations are in agreement with the SDE in what concerns dynamical gluon mass generation [5, 6].

The infrared QCD coupling turns out to be infrared finite when gluons develop a dynamically generated mass. This point was already emphasized in Ref.[3]; was also discussed at length in Ref.[7, 8], and leads to an infrared fixed point, which is a property of dynamical mass generation in non-Abelian theories [9].

SM Higgs potential stability

The QCD α_s coupling that we shall consider will not depend on the renormalization point μ but on the dynamical gluon mass $m_g(k^2)$ and, of course, on the QCD characteristic scale $\Lambda_{QCD} \equiv \Lambda$.

The first calculation of the IR frozen QCD coupling in the presence of a dynamically generated gluon mass was obtained in Ref.[3], leading to the following coupling:

$$g^2(k^2) = \frac{1}{\beta_0 \ln\left[\frac{k^2+4\cdot m_g^2}{\Lambda^2_{QCD}}\right]} = 4\pi\alpha_s(k^2), \tag{1}$$

where $\beta_0 = (11N - 2n_q)/48\pi^2$ with n_q quark flavors and $N = 3$. m_g is the IR value of the dynamical gluon mass $m_g(k^2)$, which naturally goes to zero at high energies.

Another approach can be obtained just assuming the following simple fit for the coupling constant

$$\begin{aligned} 4\pi\alpha_s(k^2) &\approx \frac{1}{\beta_0 \ln\left[\frac{4\cdot m_g^2}{\Lambda^2_{QCD}}\right]}\theta(1\mathit{GeV}^2 - k^2) \\ &+ \frac{\kappa}{\beta_0 \ln\left[\frac{k^2+4\cdot m_g^2/k^2}{\Lambda^2_{QCD}}\right]}\theta(k^2 - 1\mathit{GeV}^2), \end{aligned} \tag{2}$$

where κ is a constant that provides the interpolation between the constant behavior of the IR coupling with its high energy behavior, where the dynamical gluon mass falloff as $1/k^2$ [10].

In any case it is important to stress that the non-perturbative QCD coupling associated to the phenomenon of dynamical gluon mass generation matches exactly with the perturbative one at high energies.

Recent analysis of experimental data on the unpolarized structure function of the proton indicates that [11]:

$$0.13 \leq \alpha_{s,NLO}(scale \to 0)/\pi \leq 0.18 \; . \tag{3}$$

Other phenomenological calculations considering a finite IR QCD coupling can be found in Ref.[12], and a compilation of some results can be seen in Ref.[13].

For a dynamical gluon mass $m_g = 1.2\Lambda_{QCD}$ we obtain an IR QCD coupling of order of 0.8. Therefore, according to the many phenomenological determinations of the $\alpha_s(0)$ values described previously we will consider the following range for the IR value of this coupling

$$0.4 \leq \alpha_s(0) \leq 0.8 \; , \tag{4}$$

Note that, according to Eq.(1), the possible values of the ratio m_g/Λ_{QCD} for two quark flavors, to be in agreement with Eq.(4), are in the range:

$$1.2 \leq \frac{m_g}{\Lambda_{QCD}} \leq 2.86 \; . \tag{5}$$

The evolution of the scalar self coupling λ appearing in the SM, where the scalar field may acquire a vacuum expectation value $v \approx 246.2$ GeV, are giving by the standard RG equations where each SM ordinary coupling α_i is governed by the respective β function

$$\beta_i(\alpha_i) = \mu^2 \frac{d}{d\mu^2}\alpha_i(\mu), \tag{6}$$

where α_i represents λ and any gauge or Yukawa SM couplings. The SM stability up to the Planck scale requires $\lambda \geq 0$, and the evolution of this coupling is determined solving the coupled system of differential equations given by Eq.(6).

The SM RG equations with the one loop β functions in the ($\overline{\mathrm{MS}}$) scheme (up to Eq.(10)) are given by

$$\begin{aligned} \beta_\lambda &= \frac{1}{(4\pi)^2}\Big[24\lambda^2 - 6y_t^4 + \frac{3}{8}\big(2g_2^4 + (g_2^2+g_1^2)^2\big) \\ &-(9g_2^2 + 3g_1^2 - 12y_t^2)\lambda\Big], && (7) \\ \beta_{y_t} &= \frac{y_t}{(4\pi)^2}\Big[-\frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 - 8g_3^2 + \frac{9}{2}y_t^2\Big], && (8) \\ \beta_{g_1} &= \frac{1}{(4\pi)^2}\frac{41}{6}g_1^3, && (9) \\ \beta_{g_2} &= \frac{1}{(4\pi)^2}\frac{-19}{6}g_2^3, && (10) \\ \beta_{g_3} &= -\beta_0 g^3 \frac{e^t}{e^t + 4\frac{m_g^2(t)}{\Lambda^2}}\Big(1 - \frac{4}{e^t + \frac{m_g^2}{\Lambda^2}}\frac{m_g^2(t)}{\Lambda^2}\Big), && (11) \end{aligned}$$

where $t = \log\frac{k^2}{\Lambda^2}$, $\frac{m_g^2(t)}{\Lambda^2} = \frac{(m_g^4/\Lambda^4)}{[e^t + m_g^2/\Lambda^2]}$, and $\beta_0 = \frac{11N-2n_q}{48\pi^2}$ is the first coefficient of the QCD β function. Note that β_λ, β_{y_t}, β_{g_1}, β_{g_2} and β_{g_3} are respectively the scalar, Yukawa top quark, $U(1)$, $SU(2)$ and $SU(3)$ β functions. However β_{g_3} has been changed by the non-perturbative QCD β function generated by the non-perturbative coupling described previously.

To solve the RG equations we will use the same one-loop (and three-loop) SM β functions used in Ref.[14, 15]. In this case our results are shown in Fig.(2) indicated by 1 and 3-loops (respectively small and large dashed curve and the continuous one), which agree with the ones of Ref.[14, 15] and allow us to check the numerical code. We have used exactly the same initial conditions shown in Table 1 of Ref.[15] at $\mu = M_t$, where the top mass is $M_t = 172.9 \pm 0.6 \pm 0.9$ GeV, $M_H = 125.7$ GeV, and

$$\alpha_s(M_Z) = 0.1185 \pm 0.0007 \; . \tag{12}$$

For the λ evolution at very high energies we assumed $n_q = 6$, and no particular attention has been done to the low energy evolution and the various quark thresholds below the top quark scale, what was also not considered in [14, 15].

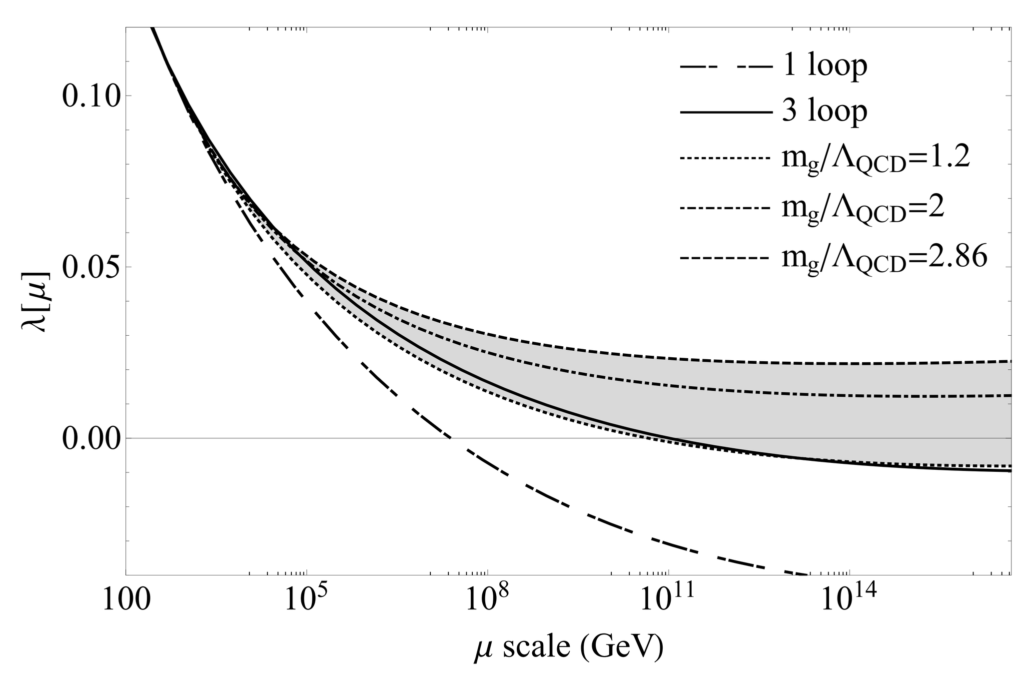

Figure 2: Scalar coupling (λ) evolution. The shaded area is obtained with the IR frozen couplings. Its lower and upper boundaries correspond respectively to $m_g/\Lambda_{QCD} \approx 1.5$ and 4.6 (or $\alpha_s(0) \approx 0.8$ and 0.4). The curve in the middle of the shaded area correspond to the phenomenologically preferred ratio of m_g/Λ_{QCD}.

Conclusions

We have computed the SM scalar coupling evolution with a very particular QCD resummation scheme, and, in this scheme, the scalar coupling evolution is positive up to the Planck mass for a certain range of the parameter that determine the scheme, which is the dynamically generated gluon mass.

Acknowledgements

The authors thank to Fundação de Amparo à Pesquisa do Estado de São Paulo FAPESP for supporting this work.

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