Physics within the Gribov horizon: applications of the (R)GZ theory

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1. Introduction

2. Applications
   - Physical spectrum
   - Phases of gauge theories
   - Gribov and SUSY

Introductory remarks

- The (R)GZ type theories, that takes into account the presence of Gribov copies in non-abelian gauge theories, provide an effective description of systems displaying confinement.
- These are remarkable theories that are written in terms of fields whose fundamental excitations cannot be associated with physically propagating degrees of freedom.
- An outstanding task in this context is to make sense of the theory as consistent description of physical reality.
  $\Rightarrow$ We have to study the predictions of the theory.
- The purpose of this talk is to provide an overview of some of the applications of the (R)GZ theories developed over the last few years.

The Gribov-Zwanziger theory

- The action has a non-local term that can be localized with auxiliary fields, resulting in the Gribov-Zwanziger action

$$S_{GZ} = \int d^D x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

$$+ \int d^D x \left(-\bar{\varphi}^{ac}_\mu \partial_\nu D^{ab}_\nu \varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu \partial_\nu D^{ab}_\nu \omega^{bc}_\mu + g f^{amb} (\partial_\nu \bar{\omega}^{ac}_\mu)(D^{mp}_\nu c^p) \varphi^{bc}_\mu \right)$$

$$+ \int d^D x \left(\gamma^2 \, g \, f^{abc} A^a_\mu (\varphi^{bc}_\mu - \bar{\varphi}^{bc}_\mu) - D(N^2 - 1)\gamma^4 \right) \quad \longleftarrow \quad \text{Gribov's restriction}$$

- A very important aspect of this formulation is that the Gribov parameter γ is not free, but determined by the gap equation.

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0$$

where

$$Z = e^{-V \mathcal{E}(\gamma)} = \int \mathcal{D}\Phi \; e^{-S_{GZ}}$$

where Φ stands for all fields. This is a fully quantum mechanical statement. The Gribov restriction is a modification of the path integral measure.

The Refined Gribov-Zwanziger theory

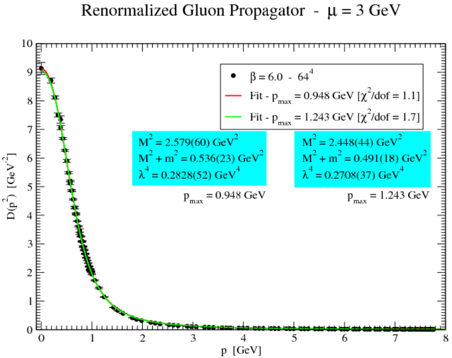
- The GZ action describes a system that allows for the Horizon function condensation (gap equation or horizon condition). In fact, the GZ action is unstable with respect to the dynamical formation of other condensates. These condensates can be taken into account by modifying (refining) the GZ action.

$$
\begin{aligned}
S_{RGZ} &= \int d^Dx \left(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right) \\
&+ \int d^Dx \left(-\bar{\varphi}^{ac}_\mu \partial_\nu D^{ab}_\nu \varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu \partial_\nu D^{ab}_\nu \omega^{bc}_\mu + gf^{amb}(\partial_\nu \bar{\omega}^{ac}_\mu)(D^{mp}_\nu c^p)\varphi^{bc}_\mu\right) \\
&+ \int d^Dx \left(\gamma^2\, g\, f^{abc} A^a_\mu(\varphi^{bc}_\mu - \bar{\varphi}^{bc}_\mu) - D(N^2-1)\gamma^4\right) \quad \longleftarrow \quad \text{Gribov's restriction} \\
&+ \int d^Dx \left(\frac{m^2}{2}A^a_\mu A^a_\mu - M^2\left(\overline{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \overline{\omega}^{ab}_\mu \omega^{ab}_\mu\right)\right)\,, \longleftarrow \text{Dimension } 2 \text{ condensates}
\end{aligned}
$$

- This action is supposed to effectively implement the restriction of the Yang-Mills system to the first Gribov region and is “refined” in the sense that it incorporates the effects of dimension two condensates. It is renormalizable and breaks the perturbative BRST symmetry.

The pure gauge field propagator

- The RGZ theory reproduces very well the recent lattice data on the low energy gluon propagator in the $4D$ SU(3) system.



- More recent results confirm this behavior. P. J. Silva, O. Oliveira, P. Bicudo and N. Cardoso, Phys. Rev. D **89**, no. 7, 074503 (2014) [arXiv:1310.5629 [hep-lat]].

D. Dudal, O. Oliveira, N. Vandersickel, Phys. Rev. **D81**, 074505 (2010).

The gluon propagator

- The (R)GZ gluon propagator has the general analytic structure ($\theta^4 = \gamma^4 g^2 N$)

$$D(k) = \frac{k^2 + M^2}{k^4 + (M^2 + m^2)k^2 + M^2 m^2 + 2\theta^4} = \frac{R_+}{k^2 + M_+^2} + \frac{R_-}{k^2 + M_-^2} \, ,$$

Where

$$M_\pm^2 = \frac{m^2 + M^2}{2} \pm \sqrt{\frac{(m^2 - M^2)^2}{4} - 2\theta^4}$$

and

$$R_\pm = \pm \frac{(M^2 - M_\pm^2)}{M_-^2 - M_+^2}$$

The gluon propagator

- One can see that for any value of the parameters, as long as $\theta \neq 0$ there will be problems in making sense of $D(k)$ as a propagator of physical excitations
  - if $(m^2 - M^2)^2 < 8\theta^4$, the poles are complex
  - if $(m^2 - M^2)^2 \geq 8\theta^4$, the poles are real but there are negative residues.
- $\Rightarrow$ No Källén-Lehmann representation
- $\Rightarrow$ No way to go back to Minkowski with a physical particle interpretation
- Lattice data points to complex poles (in GeV^2)

$$M_{\pm}^2 = 0.352 \pm 0.513i.$$

Scalar toy model

- In order to explore the analytical properties of this kind of propagator, it is profitable to look at a toy model version displaying the same propagator

$$S = \int d^D x \, \frac{1}{2}\phi \left(-\partial^2 + m^2 + \frac{2\theta^4}{-\partial^2 + M^2} \right) \phi$$

such that

$$\begin{aligned} \langle \phi(k)\phi(-k) \rangle &= \frac{k^2 + M^2}{k^4 + (M^2 + m^2)k^2 + M^2 m^2 + 2\theta^4} \\ &= \frac{R_+}{k^2 + M_+^2} + \frac{R_-}{k^2 + M_-^2} \, , \end{aligned}$$

i-particles

- In order to simplify the discussion, we choose $M = m = \mu$, corresponding to the region of complex poles.

$$M_\pm^2 = \mu^2 \pm i\sqrt{2}\theta^2; \quad R_\pm = \frac{1}{2}$$

The action can be localized, diagonalized and written as

$$S = \int d^D x \, \eta \left(-\partial^2 + \mu^2 + i\sqrt{2}\theta^2\right) \eta + \bar{\eta} \left(-\partial^2 + \mu^2 - i\sqrt{2}\theta^2\right) \bar{\eta}$$

such that

$$\langle \eta(k)\eta(-k) \rangle = \frac{1}{k^2 + \mu^2 + i\sqrt{2}\theta^2}$$
$$\langle \bar{\eta}(k)\bar{\eta}(-k) \rangle = \frac{1}{k^2 + \mu^2 - i\sqrt{2}\theta^2}$$

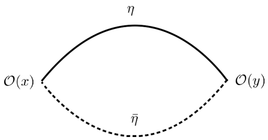
i-particles

- Is it possible to identify physical states in this model?

- Physical states can only appear as composite states of i-particles. See M. A. L. Capri, D. Dudal, M. S. Guimaraes, L. F. Palhares and S. P. Sorella, Int. J. Mod. Phys. A **28**, 1350034 (2013) [arXiv:1208.5676 [hep-th]]. for an interacting model where physical bound states can be identified.

- But even in the free i-particle theory we can study states that resemble physical bound states. They are in fact the lowest order approximation to the bound states.

- We look for two point functions of wanna-be physical excitations, such as $\mathcal{O}(x) = \eta(x)\bar{\eta}(x)$



Spectral representation

- In momentum space the expression has the general form

$$\langle \mathcal{O}(q)\mathcal{O}(-q)\rangle = \int \frac{d^D p}{(2\pi)^D}\,\frac{1}{(q-p)^2+m_1^2}\frac{1}{p^2+m_2^2}$$

This is a well known integral that can be put in a Källén-Lehmann spectral representation

$$\langle \mathcal{O}(q)\mathcal{O}(-q)\rangle = \int_{\tau_0}^{\infty} d\tau\,\frac{\rho(\tau)}{\tau+q^2}\,.$$

with

$$\rho(\tau) = \frac{\pi^{\frac{D-3}{2}}}{(2\pi)^{D-2}}\frac{1}{2^{(D-1)}\Gamma(\frac{D-1}{2})}\frac{\left[(\tau-m_1^2-m_2^2)^2-4m_1^2m_2^2\right]^{\frac{D-3}{2}}}{\tau^{\frac{D-2}{2}}}$$

$$\tau_0 = (m_1+m_2)^2$$

Spectral representation

- It turns out that these expressions work for complex masses as well

  D. Dudal, M. S. Guimaraes, Phys. Rev. **D83**, 045013 (2011).

  In our case

$$m_1^2 = \mu^2 + i\sqrt{2}\theta^2$$
$$m_2^2 = \mu^2 - i\sqrt{2}\theta^2$$

  Therefore in $D = 4$

$$\rho(\tau) = \frac{1}{(4\pi)^2}\sqrt{1 - \frac{4\mu^2}{\tau} - \frac{8\theta^4}{\tau^2}}$$
$$\tau_0 = 2\left(\mu^2 + \sqrt{\mu^4 + 2\theta^4}\right)$$

- $\Rightarrow$ It follows that $\rho(\tau)$ is positive for $\tau > \tau_0$ and we can have a particle interpretation!

Spectral representation

- The same reasoning can be applied to more general operators, such as

$$\partial_\mu \mathcal{O}(x), \;\; \partial^2 \mathcal{O}(x), \;\; \cdots$$

The two point function of such operators

$$\langle \mathcal{O}_i(q)\mathcal{O}_i(-q)\rangle = \int \frac{d^4p}{(2\pi)^4} \; \frac{1}{(q-p)^2+m_+^2} \frac{1}{p^2+m_-^2} f(p,q-p)$$

where $f(p,q-p)$ is a polynomial in the scalar products of the momenta (q,p), also lead to a physically consistent Källén-Lehmann spectral representation.

Extracting physical properties

- The properties of a physical state will be encoded in the analytical properties of the correlation functions of composite operators $\mathcal{O}(x)$.

$$\Pi(q^2) = \int d^4x \, e^{iqx} \langle \mathcal{O}(x)\mathcal{O}(0) \rangle \ ,$$

- A truly non-perturbative evaluation of $\Pi(q^2)$ would enable us to write the Källén-Lehmann spectral representation

$$\Pi(q^2) = \frac{1}{\pi} \int_0^\infty d\tau \, \frac{Im\Pi(\tau)}{\tau + q^2} \ ,$$

- The factor $Im\Pi(\tau)$ carries information about the physical spectrum

$$\frac{Im\Pi(\tau)}{\pi} = \sum_i \mathcal{R}_i \, \delta(\tau - m_i^2) + \theta(\tau - \tau_0)\sigma(\tau) \ ,$$

- m_i are masses of stable particles and τ_0 marks the threshold for multiparticle production. $\mathcal{R}_i$ and $\sigma(\tau)$ are positive quantities.

Infra-red moments

- There are some methods to extract information about the spectrum from the spectral function. (see M. A. L. Capri, A. J. Gomez, M. S. Guimaraes, V. E. R. Lemes, S. P. Sorella, D. G. Tedesco, Eur. Phys. J. **C71**, 1525 (2011) and Phys. Rev. D **85**, 085012 (2012). for a SVZ-inspired sum rule approach)
- Here we focus on a modified moment problem, adapted to the infrared, to obtain estimates of the glueball masses.
- In this approach we aim at extracting the pole, in the infrared (low q^2) region, of the two-point function

$$\langle \mathcal{O}(q)\mathcal{O}(-q)\rangle\Big|_{1-loop} = \Pi(q^2) = \int_{\tau_0}^{\infty} d\tau \, \frac{\rho(\tau)}{\tau + q^2} \, .$$

- Since we want to analise the infrared region, we expand for small q^2

$$\Pi(q^2) = \sum_{n=0}^{\infty} \int_{\tau_0}^{\infty} d\tau \, \left(\frac{1}{\tau}\right)^{n+1} \rho(\tau)(-1)^n (q^2)^n = \sum_{n=0}^{\infty} \nu_n (-1)^n (q^2)^n \, .$$

Where $\nu_n = \int_{\tau_0}^{\infty} d\tau \, \left(\frac{1}{\tau}\right)^{n+1} \rho(\tau)$ are the moments associated to the espectral function $\rho(\tau)$. There are theorems setting the conditions for a reconstruction of $\rho(\tau)$ from its moments.

Padé approximants

- The idea now is to express $\Pi(q^2)$ as a rational function of the external momentum in a approximation known as the Padé approximant. From the poles of this function the value of the mass can be extracted.

$$\Pi(q^2) = \frac{P_{N-1}(\frac{1}{q^2})}{Q_N(\frac{1}{q^2})} + \mathcal{O}(q^{2N}) \; .$$

  where P_N and Q_N are N-degree polynomials in their arguments. These functions are determined by the moments ν_n

- For low momenta we need only the first moments, and we obtain

$$\Pi(q^2) \approx \frac{\frac{\nu_0^2}{\nu_1}}{q^2 + \frac{\nu_0}{\nu_1}} \Rightarrow m^2 = \sqrt{\frac{\nu_0}{\nu_1}}$$

  leading to the determination of the mass at the pole

- There are some details concerning the definition of the moments which sometimes demand subtracted spectral representations and the consequent introduction of a scale. We demand that the result depends minimally on this scale (principle of minimal sensitivity).

Glueballs

- Glueballs are colorless, composite gluon states and are classified by the total angular momentum J, Parity P and charge conjugation C. They are thus associated with gauge invariant, composite operators carrying J^{PC} quantum numbers.
- Lowest mass dimension glueball operators

$$\mathcal{O}_{0^{++}}(x) = F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) \; ,$$
$$\mathcal{O}_{0^{-+}}(x) = \varepsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x) \; .$$

- The higher spin states may mix with the lower ones and have to be projected to be pure. We use

$$[\mathcal{O}_{2^{++}}(x)]_{\mu\nu} = \partial^4 t_{\mu\nu} - \partial^2\partial_\mu\partial_\alpha t_{\alpha\nu} - \partial^2\partial_\nu\partial_\alpha t_{\alpha\mu} + P_{\mu\nu}\partial_\alpha\partial_\beta t_{\alpha\beta}$$
$$[\mathcal{O}_{2^{-+}}(x)]_{\mu\nu} = \partial^2 q_{\mu\nu} - \frac{1}{3}P_{\mu\nu}q_{\alpha\alpha}$$

where $P_{\mu\nu} = \delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu$, $t_{\mu\nu} = F^a_{\mu\sigma}(x)F^a_{\nu\sigma}(x) - \frac{1}{4}\delta_{\mu\nu}F^a_{\alpha\beta}(x)F^a_{\alpha\beta}(x)$ and $q_{\mu\nu} = \partial\alpha\partial_\beta\left(F^*_{\mu\alpha}F_{\nu\beta} +^* F_{\mu\alpha}F_{\nu\beta}\right)$

Glueballs

- Comparison of our results with other methods

J^{PC}	confining gluon propagator
0^{++}	2.27
2^{++}	2.34
0^{-+}	2.51
2^{-+}	2.64

J^{PC}	Lattice	Flux tube model	Hamiltonian QCD	ADS/CFT
0^{++}	1.71	1.68	1.98	1.21
2^{++}	2.39	2.69	2.42	2.18
0^{-+}	2.56	2.57	2.22	3.05
2^{-+}	3.04	–	–	–

D. Dudal, M. S. Guimaraes and S. P. Sorella, Phys. Lett. B **732**, 247 (2014) [arXiv:1310.2016 [hep-ph]].

D. Dudal, M. S. Guimaraes, S. P. Sorella, Phys. Rev. Lett. **106**, 062003 (2011).
-Lattice: (1) Y. Chen *et al.* PRD **73**, 014516 (2006)
-Flux tube model: M. Iwasaki *et al.* PRD **68**, 074007 (2003).
-Hamiltonian QCD: A. P. Szczepaniak and E. S. Swanson, PLB **577**, 61 (2003).
-AdS/CFT: K. Ghoroku, K. Kubo, T. Taminato and F. Toyoda, arXiv:1111.7032.

-More information in the review: V. Mathieu, N. Kochelev, V. Vento, Int. J. Mod. Phys. **E18**, 1-49 (2009).

Gribov Horizon and Yang-Mills-Higgs phases

- We know that the no-pole condition can be implemented as a gap equation for the vacuum energy obtained from an action functional

$$Z = e^{-V\mathcal{E}(\gamma)} = \int \mathcal{D}A\, \delta(\partial A)\ \det \mathcal{M}\ e^{-\left(S_{YM}+\gamma^4 H(A)-\gamma^4 VD(N^2-1)\right)}$$

so that

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A)\rangle_{1PI} = VD(N^2-1)\ ,$$

- The presence of matter will only slightly modify this, most notably matter dynamics will contribute to the gap equation.

$SU(2)$ Yang-Mills-Higgs

- Consider the $SU(2)$ action in the presence of the Higgs field, both the fundamental and adjoint representations

$$S^{SU(2)} = \int d^D x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + |D_\mu \Phi|^2 + \frac{\lambda}{2} \left(|\Phi|^2 - v^2 \right)^2 + b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) \ ,$$

where the covariant derivative is defined by

$$D^{ij}_\mu \Phi^j = \partial_\mu \Phi^i - ig \frac{(\tau^a)^{ij}}{2} A^a_\mu \Phi^j \ , \quad \text{Fundamental}$$

$$(D_\mu \Phi)^a = \partial_\mu \Phi^a + g \epsilon^{abc} A^b_\mu \Phi^c \ , \quad \text{Adjoint}$$

- In what follows we will always work in the limit $\lambda \to \infty$.

$$\langle \Phi^i \rangle = v \delta^{i2} \ , \quad \text{Fundamental}$$

$$\langle \Phi^a \rangle = v \delta^{a3} \ , \quad \text{Adjoint}$$

Phases of Yang-Mills-Higgs systems

- The fundamental question we would like to address is the transition between the Higgs and confinement regimes in Yang-Mills-Higgs systems.

  E. H. Fradkin and S. H. Shenker, "Phase Diagrams of Lattice Gauge Theories with Higgs Fields," Phys. Rev. D 19, 3682 (1979).

- There are important differences between the case of the Higgs in the fundamental and in the adjoint representations of $SU(N)$ gauge theories in $4D$:
  - In the case of fundamental representation, there is a path in the phase diagram such that the Higgs and confining phases are continuously connected.
  - There may be a discontinuity in the case of adjoint representation.

Gribov vs Higgs

- In what follows we shall work only in the fundamental representation for illustrative purposes.
- We want to analyze the phase structure of this theory as a function of the gauge coupling g and the Higgs vacuum expectation value v.
- Perturbatively we know that the theory is in a Higgs phase characterized by massive vector boson excitations (Higgs mechanism).
- On the other hand the Gribov horizon modifies the gauge propagators rendering the excitations unphysical, confined.

  $\Rightarrow$ How can we reconcile these views?

Scalars in the fundamental

- We work in the quadratic approximation, such that the Horizon function reads

$$H(A) = Ng^2 \int \frac{d^D q}{(2\pi)^D} \frac{A^a_\mu(q) A^a_\mu(-q)}{q^2} \ .$$

so that the quadratic part of the action becomes

$$S = \frac{1}{2} \int \frac{d^D q}{(2\pi)^D} \int \frac{d^D p}{(2\pi)^D} A^a_\mu(p) P^{ab}_{\mu\nu}(p,q) A^a_\nu(q) \ .$$

where

$$P^{ab}_{\mu\nu}(p,q) = \delta^{ab}\delta(p+q)\delta_{\mu\nu}\left(q^2 + \frac{g^2 v^2}{2} + \frac{2N\gamma^4 g^2}{q^2}\right) \ .$$

Scalars in the fundamental

- In this approximation we have

$$\int \mathcal{D}A e^{-S} \sim \left(\det P^{ab}_{\mu\nu}(p,q)\right)^{-\frac{1}{2}} = e^{-\frac{1}{2} Tr \ln P^{ab}_{\mu\nu}(p,q)}$$

It follows that the vacuum energy has the form

$$V\mathcal{E}(\gamma) = \frac{1}{2} Tr \ln P^{ab}_{\mu\nu}(p,q) - \gamma^4 V D(N^2-1) + \text{terms indep. from } \gamma$$

and

$$Tr \ln P^{ab}_{\mu\nu}(p,q) = V(D-1)(N^2-1) \int \frac{d^D q}{(2\pi)^D} \left(q^2 + \frac{g^2 v^2}{2} + \frac{2N\gamma^4 g^2}{q^2}\right)$$

- Therefore, the gap equation, $\frac{\partial \mathcal{E}}{\partial \gamma^4} = 0$, becomes

$$g^2 N \frac{D-1}{D} \int \frac{d^D q}{(2\pi)^D} \frac{1}{\left(q^2 + \frac{g^2 v^2}{2} + \frac{2N\gamma^4 g^2}{q^2}\right)} = 1$$

This equation fixes γ^4 as a function of g and v.

Gauge propagator and phases

- For $D = 4$, $N = 2$ one can solve the gap equation, obtaining

$$R_+ \ln\left(\frac{m_+^2}{\bar{\mu}^2}\right) + R_- \ln\left(\frac{m_-^2}{\bar{\mu}^2}\right) = 1 - \frac{32\pi^2}{3g^2}$$

where $\bar{\mu}$ is the scale in the $\overline{MS}$ renormalization scheme in $d = 4 - \varepsilon$.

- Note that if $\gamma = 0$, then $m_+ = \frac{g^2 v^2}{2}$; $m_- = 0$; $R_+ = 1$; $R_- = 0$; and the equation becomes

$$\ln\left(\frac{g^2 v^2}{2\bar{\mu}^2}\right) = 1 - \frac{32\pi^2}{3g^2} \ .$$

This motivates the definition of the parameter

$$a = \frac{g^2 v^2}{4\bar{\mu}^2 e^{\left(1 - \frac{32\pi^2}{3g^2}\right)}} \ .$$

gap equation, propagator and phases

- The gap equation reads

$$2\ \ln(a) = g(x)$$

where

$$x = \frac{32\gamma^4 g^2 N}{g^4 v^4}$$

and

$$g(x) = \frac{1}{\sqrt{1-x}}\left(-\left(1+\sqrt{1-x}\right)\ \ln\left(1+\sqrt{1-x}\right)\right.$$
$$\left.+\left(1-\sqrt{1-x}\right)\ \ln\left(1-\sqrt{1-x}\right)\right)\ .$$

Results in 4D with scalars in the fundamental

It turns out that $g(x) \leq -2\ln 2$ for all $x \geq 0$ and is strictly decreasing.
We can distinguish three regions connected by a line in the plane g, v, parametrized by a, namely

- i) when $a > \frac{1}{2}$, the gap equation has no solution for x. This means that the restriction to the Gribov region cannot be consistently implemented in the first place. As a consequence, **the standard Higgs mechanism** takes place, yielding three massive gauge fields, according to

$$\left\langle A_\mu^a(q) A_\nu^b(-q) \right\rangle = \delta^{ab} \frac{1}{q^2 + \frac{g^2 v^2}{2}} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) .$$

For sufficiently weak coupling g^2, we underline that a will unavoidably be larger than $\frac{1}{2}$.
This is an important result. It means that the Gribov copies are not relevant for the perturbative Higgs phenomenon, as expected!

Results in 4D with scalars in the fundamental

- ii) when $\frac{1}{e} < a < \frac{1}{2}$, there are solutions for $0 \leq x < 1$ (masses are real). In this region, the gauge field propagator decomposes into the sum of two terms of the Yukawa type:

$$\left\langle A^a_\mu(q) A^b_\nu(-q) \right\rangle = \delta^{\alpha\beta} \left(\frac{\mathcal{F}_+}{q^2 + m^2_+} - \frac{\mathcal{F}_-}{q^2 + m^2_-} \right) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) ,$$

where

$$\mathcal{F}_+ = \frac{m^2_+}{m^2_+ - m^2_-} , \qquad \mathcal{F}_- = \frac{m^2_-}{m^2_+ - m^2_-} ,$$

$$m^2_\pm = \frac{1}{2} \left(\frac{g^2 v^2}{2} \pm \sqrt{\frac{g^4 v^4}{4} - 8\gamma^4 g^2 N} \right) \in \mathbb{R} .$$

Note that even though the masses are real, one of the residues is negative (note the relative minus sign).

Results in 4D with scalars in the fundamental

- iii) for $a < \frac{1}{e}$, there is a solution for $x > 1$. This scenario will always be realized if g^2 gets sufficiently large, that is, at strong coupling. In this region the roots (m_+^2, m_-^2) become complex conjugate and the gauge boson propagator is of the Gribov type, displaying complex poles. We are in the **Gribov-confining regime**.

In summary, we clearly notice that at sufficiently weak coupling, the standard Higgs mechanism will definitely take place, as $a > \frac{1}{2}$, whereas for sufficiently strong coupling, we always end up in a confining phase because then $a < \frac{1}{2}$.
It is nice to observe that the transition between Higgs and confined phases seems to be continuous, from the point of view of the propagators, in tune with Fradkin and Shenker's findings.

Phases of Yang-Mills-Higgs systems

- More details can be found in:
  - M. A. L. Capri, D. Dudal, A. J. Gomez, M. S. Guimaraes, I. F. Justo and S. P. Sorella, "A study of the Higgs and confining phases in Euclidean SU(2) Yang-Mills theories in 3d by taking into account the Gribov horizon," Eur. Phys. J. C **73**, 2346 (2013) [arXiv:1210.4734 [hep-th]].
  - M. A. L. Capri, D. Dudal, A. J. Gomez, M. S. Guimaraes, I. F. Justo, S. P. Sorella and D. Vercauteren, "Semiclassical analysis of the phases of 4d SU(2) Higgs gauge systems with cutoff at the Gribov horizon," Phys. Rev. D **88**, 085022 (2013) [arXiv:1212.1003 [hep-th]].
  - M. A. L. Capri, D. Dudal, M. S. Guimaraes, I. F. Justo, S. P. Sorella and D. Vercauteren, "$SU(2) \times U(1)$ Yang-Mills theories in $3d$ with Higgs field and Gribov ambiguity," Eur. Phys. J. C **73**, 2567 (2013) [arXiv:1305.4155 [hep-th]].
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Gribov and SUSY

- Supersymmetric gauge theories are very special theories with strong analytic properties (holomorphicity) that allows us to investigate many of its non-perturbative properties.

- But, as gauge theories, they also are afflicted by the Gribov problem. In fact the $\mathcal{N} = 1$ Super-Yang-Mills gauge theory feels the presence of Gribov copies. Constraints from supersymmetry (zero vacuum energy) and the constraints imposed by the Gribov gap equation (non-zero expectation value for horizon function) are compatible and allows us to reproduce well known non-perturbative properties of this theory, such as a non-zero gluino condensate and confinement.

  See the details in “Implementing the Gribov-Zwanziger framework in $\mathcal{N} = 1$ Super-Yang-Mills in the Landau gauge,” Eur. Phys. J. C **74**, 2961 (2014) [arXiv:1404.2573 [hep-th]].

- An important problem: Gribov copies seems to be related to confinement as we have seen, so what about the superconformal $\mathcal{N} = 4$ gauge theories?

Gribov and the superconformal $\mathcal{N} = 4$ SYM

- The gap equation of the Gribov formalism is a fundamental aspect of the theory. It defines the Gribov mass parameter γ as a dynamical mass parameter.

- But in a theory without a renormalization group invariant mass scale it is impossible to attach a dynamical meaning to the parameter γ and the only possible solution is to impose $\gamma = 0$.

- It is all because of the renormalization group equation for the vacuum energy:

$$\left(\bar{\mu}\frac{\partial}{\partial\bar{\mu}} + \gamma_{\gamma^2}\gamma^2\frac{\partial}{\partial\gamma^2} + \beta(g^2)\frac{\partial}{\partial g^2}\right)\mathcal{E} = 0 \ .$$

where $\bar{\mu}$ is the renormalization scale, γ_{γ^2} is the renormalization factor of the Gribov parameter, and $\beta(g^2)$ is the β-function.

M. A. L. Capri, M. S. Guimaraes, I. F. Justo, L. F. Palhares and S. P. Sorella, Phys. Lett. B **735**, 277 (2014) [arXiv:1404.7163 [hep-th]].

Gribov and the superconformal $\mathcal{N} = 4$ SYM

- In a theory with $\beta(g^2) \neq 0$ (such as QCD) the theory exhibits a genuine renormalization group invariant scale

$$\left(\bar{\mu}\frac{\partial}{\partial\bar{\mu}} + \beta(g^2)\frac{\partial}{\partial g^2}\right)\Lambda_{QCD} = 0 \ .$$

and imposing the Gribov gap equation

$$\frac{\partial\mathcal{E}}{\partial\gamma^2} = 0 \ .$$

just tells us that the vacuum energy is RG invariant, a well known result

$$\left(\bar{\mu}\frac{\partial}{\partial\bar{\mu}} + \beta(g^2)\frac{\partial}{\partial g^2}\right)\mathcal{E} = 0 \Rightarrow \mathcal{E} \sim \Lambda^4_{QCD} \ .$$

Gribov and the superconformal $\mathcal{N} = 4$ SYM

- But in a theory with $\beta(g^2) = 0$ there are no renormalization group invariant scale. If we use the general form for the vacuum energy in this case, computed with the restriction to the Gribov region

$$\mathcal{E} = \gamma^4 f\left(\frac{\gamma^2}{\bar{\mu}^2}\right)$$

Imposing the Gribov gap equation we have

$$\frac{\partial \mathcal{E}}{\partial \gamma^2} = 0 \;\Rightarrow \gamma^2 \left(2f\left(\frac{\gamma^2}{\bar{\mu}^2}\right) + \frac{\gamma^2}{\bar{\mu}^2} f'\left(\frac{\gamma^2}{\bar{\mu}^2}\right)\right) = 0.$$

Therefore, either $\gamma = 0$ or it is a function of $\bar{\mu}$. But if it is a function of $\bar{\mu}$, the vacuum energy will also be a function of $\bar{\mu}$, which contradicts its RG equation in this case.

$$\left(\bar{\mu}\frac{\partial}{\partial \bar{\mu}}\right) \mathcal{E} = 0$$

Gribov and the superconformal $\mathcal{N} = 4$ SYM

- The only consistent solution is to have $\gamma = 0$ and $\mathcal{E} = 0$. The solution $\gamma = 0$ means that the theory does not demand the Gribov region restriction. It does not necessarily mean that there are no Gribov copies, only that they don't cause problems in the quantization of the theory.
- In fact we can also show that the Faddeev-Popov operator does not change sign as the momenta are varied, meaning that we never leave the Gribov region.
  see details in M. A. L. Capri, M. S. Guimaraes, I. F. Justo, L. F. Palhares and S. P. Sorella, "On the irrelevance of the Gribov issue in N=4 Super Yang-Mills in the Landau gauge," Phys. Lett. B **735**, 277 (2014) [arXiv:1404.7163 [hep-th]].

Conclusions

- Gribov framework has provided many interesting results about the non-perturbative structure of non-abelian gauge theories.
- There are still many problems to be addressed...
  - More precise computation of the spectrum...
  - Phases of SUSY gauge theories...
  - Gribov's confinement (exclusion of the fundamental degrees of freedom from spectrum) seems to take care of gluons...
  - What about matter confinement? Have to understand better the proposal for Gribov matter confinement $\rightarrow$ origin of "matter Horizon function", vertex modification? What about heavy fundamental matter (how to make sense of the area law)?
  - Connection with topological defects? Finite temperature? Polyakov Loops...
  - What are the consequences of the BRST resurrection in its new non-perturbative life?!?!

Conclusions

Thank you!