The Gribov Tapestry

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Unraveling the organization of the QCD tapestry
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- ◊ by presenting us with QCD, nature shows us how a nontrivial mass spectrum can be dynamically generated
- ◊ QCD is the gold standard for what a natural and UV complete continuum theory looks like

- ◊ by comparison the Higgs sector of the standard model fails in all these respects
- ◊ the theory of quantum gravity is similarly lacking

- ◊ the QCD tapestry is interesting not just for its own sake
- ◊ new insights, even of just a qualitative nature, may be useful for fundamental issues beyond QCD

- ◊ at least three modes of attack represented at this workshop

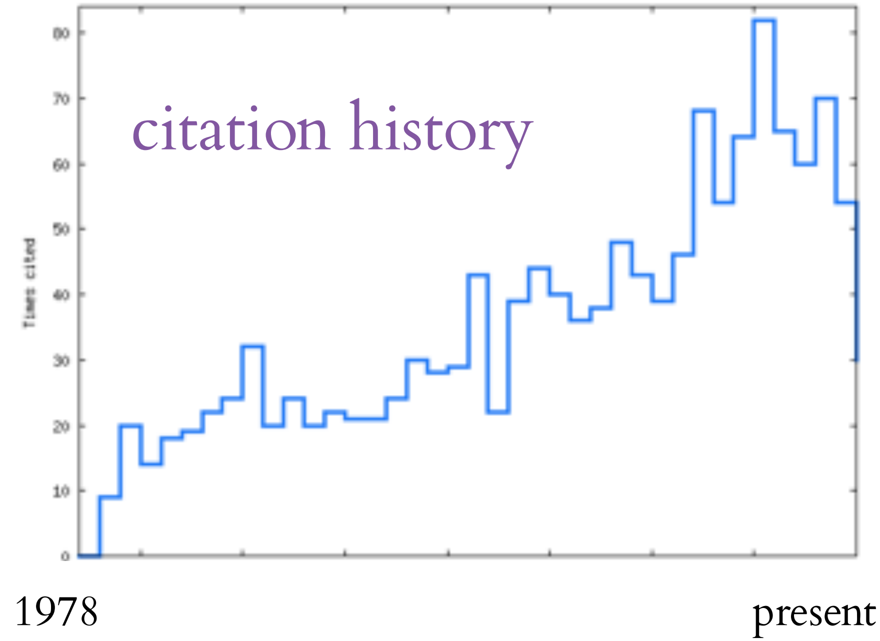
- ◊ study dynamics directly in the continuum
  - ◊ Schwinger-Dyon equations and related approaches

- ◊ Gribov copies
  - ◊ these are effects that are explicitly built into the theory
  - ◊ nondynamical but nonperturbative and nonlocal

- ◊ lattice
  - ◊ opening up the black box by gauge fixing
  - ◊ compare to and check the continuum studies
  - ◊ combines both dynamical and nondynamical effects

◊ focus on Gribov copies

◊ steady increase of interest



◊ but still largely ignored by the mainstream

◊ common reaction — "Gribov copies are gauge dependent and so physics cannot depend on them"

◊ response — in any gauge that has copies, copies are a necessary part of the correct (nonperturbative) description

◊ in this sense physics does depend on Gribov copies

- some gauges are more sensible for nonperturbative studies
  - covariant gauge (e.g. Landau)
  - Coulomb gauge — has at least rotational invariance, reflects the space and time split of Hamiltonian dynamics, and makes Gauss law manifest in the path integral

- that these sensible gauges have Gribov copies should be viewed as a bonus
- these gauges are giving a direct view into nonperturbative features that underlie confinement

- in both gauges gluon propagation is suppressed in infrared

- ◊ continuum path integral for gauge theory should sample each gauge orbit once
- ◊ integrate over a suitable slice in configuration space
- ◊ after Gribov, such a slice became known as the fundamental modular region (FMR)

$$Z = \int \mathcal{D}A_{\rm FMR} e^{iS(A)}$$

- ◊ the boundary of the FMR is highly nontrivial, and so this definition is not too useful
- ◊ an insertion of unity converts this into an integral over the whole configuration space (FMR times gauge transformations)

◊ the resolution of unity before Gribov (Faddeev-Popov)

$$1 = \int \mathcal{D}U \, \delta(F(A^U)) \det \left(\frac{\delta F(A^U)}{\delta U} \right)$$

◊ the resolution of unity after Gribov

$$1 = \frac{1}{1 + N(A)} \int \mathcal{D}U \, \delta(F(A^U)) \left| \det \left(\frac{\delta F(A^U)}{\delta U} \right) \right|$$

◊ $N(A)$ is the number copies on a gauge orbit picked by A

◊ 37 years old but still not found in textbooks

- note in passing — the naïve FP measure might also be correct
  - explicit simple model (Freedberg Lee Pang Ren 1996)
  - explicit counting $N(A)$ in SU(2) gauge theory in Coulomb gauge with spherically symmetric configurations
- $N(A)$ is even and copies come in pairs with alternating sign for det(FP)

- so all copies cancel out if absolute value is removed and then can replace $1/(1+N(A))$ with unity
- but then you are stuck with an alternating sign measure

- restricting to first Gribov region keeps a set of positive sign copies
- the set of negative sign copies that would cancel these are ignored
  - in this sense, a maximal error is made

- ◊ in Coulomb gauge, |det(FP)| is in fact canceled, and so in this gauge the absolute value is properly accounted for
- ◊ the resulting action is nonlocal and still sensitive to the FP operator via the instantaneous color Coulomb potential
- ◊ IR enhancement of the FP operator (⇒ confinement)
- ◊ can be understood as a clumping together of the copies …

- ◊ the factor $1/(1+N(A))$ is left to suppress gluon propagation in IR in both gauges

- ◊ what do we know about $N(A)$?

- ◊ $N(A)$ is scale invariant — two orbits related by a scale transformation have the same number of copies
- ◊ consider a scale invariant strongly coupled gauge theory (eg. sitting at some fixed point)
- ◊ Gribov copies would be expected to have nontrivial effects on all scales

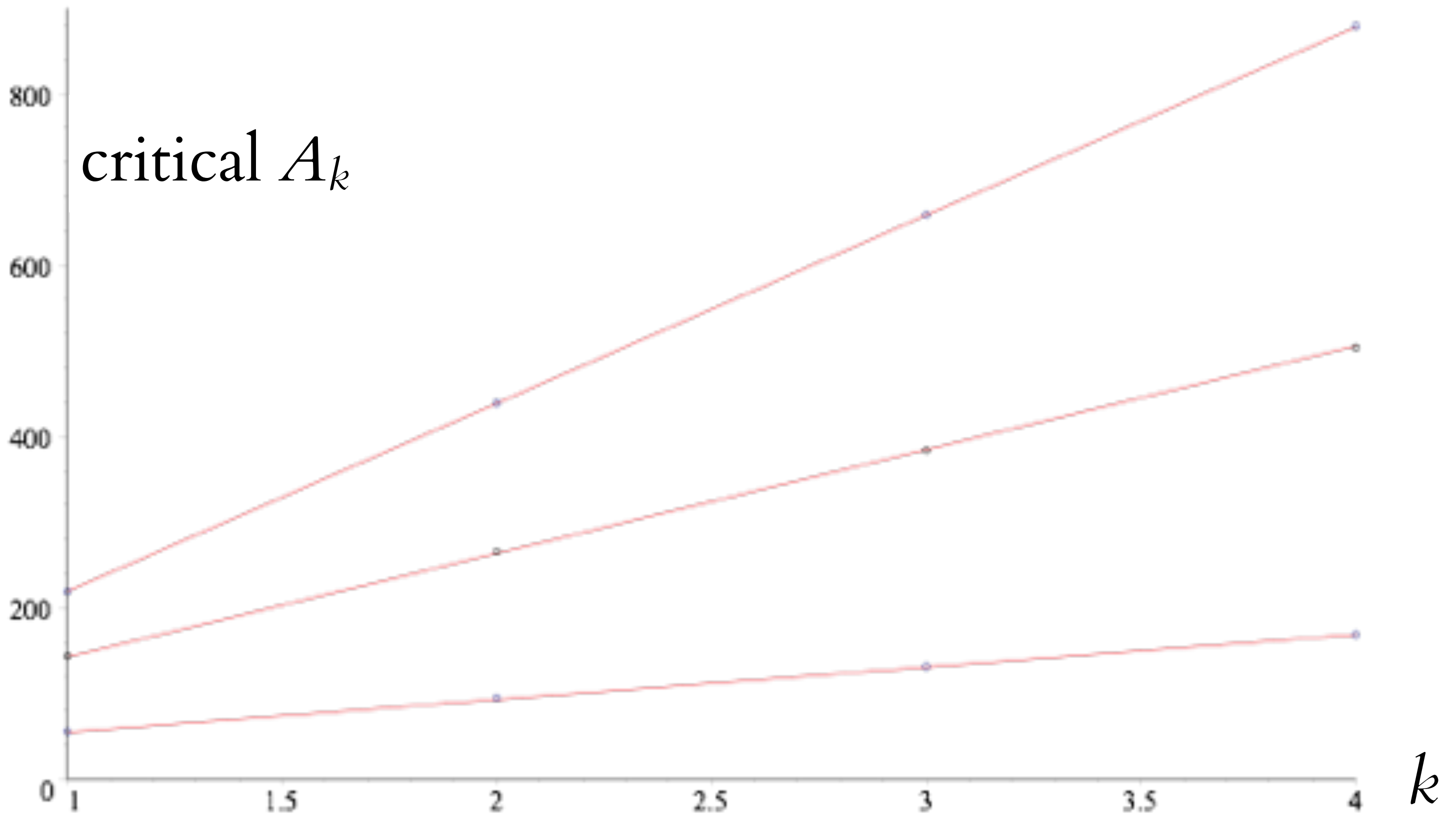
- ◊ on the other hand, approximations to deal with Gribov copies usually end up with a mass parameter in some modified action
  - ◊ contradiction?

- ◊ in QCD the effects of copies turn on at the strong coupling scale
  $\Rightarrow \Lambda$ does enter the description of Gribov copies in QCD

- ◊ $N(A)$ is an integer
  - ◊ $N(A) \equiv 0$ for a certain bounded region within the FMR
  - ◊ this region includes the perturbative regime

- ◊ consider a configuration with typical momentum k and with amplitude A_k
- ◊ since $N(A)$ is scale invariant, it should depend on the scale invariant ratio A_k/k
- ◊ $N(A)$ becomes nonzero at some critical value of $A_k \propto k$

◊ can see that critical $A_k \propto k$ directly from the Gribov equation



different lines are for different models of spherical plane waves

- typical size of a gauge field fluctuation is

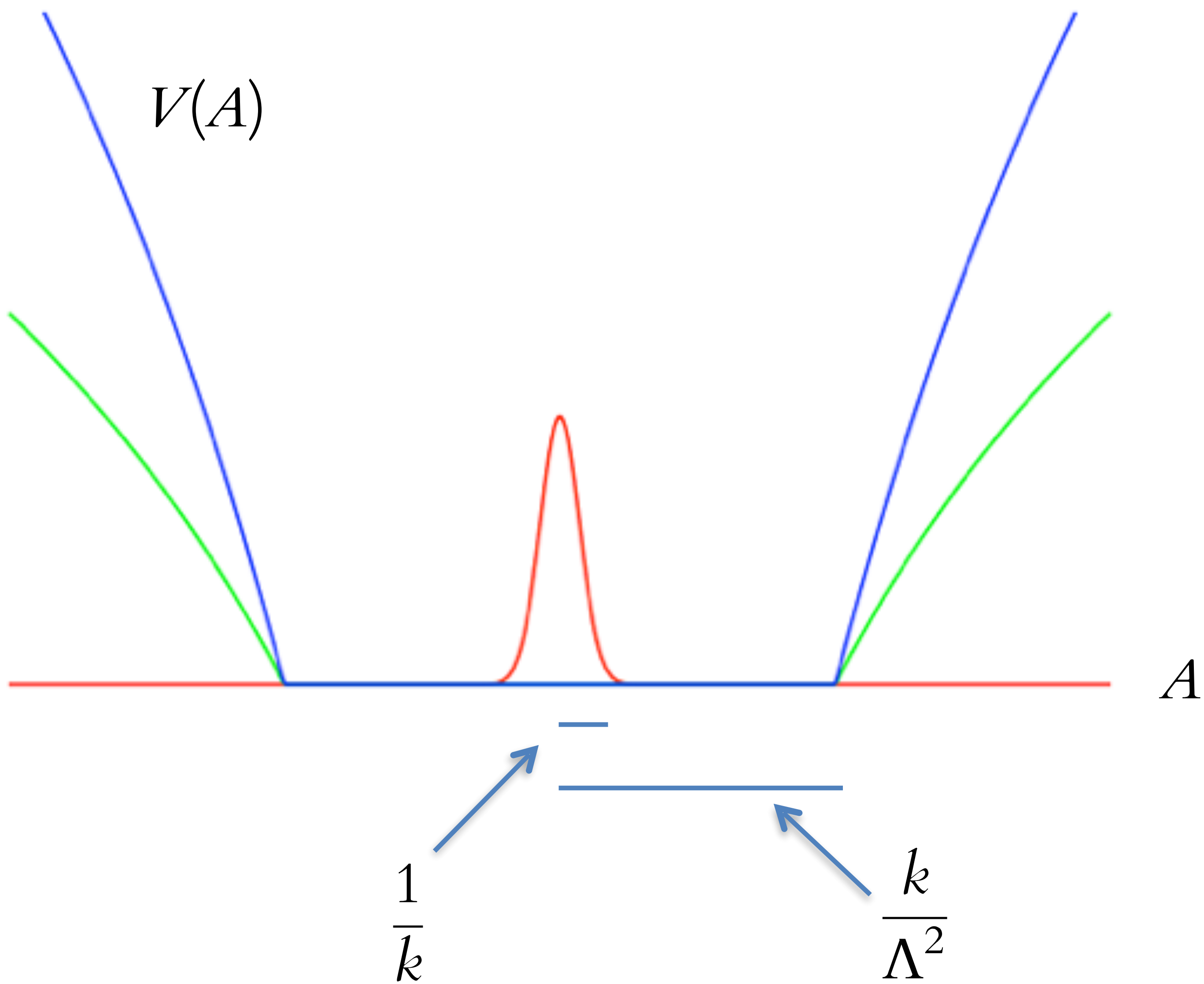
$$A_k^2 \approx \frac{1}{k^2}$$

- Gribov copies are only important when

$$A_k^2 \gtrsim \frac{k^2}{\Lambda^4}$$

- at large $k^2 \gg \Lambda^2$ this is much larger than typical
- the path integral suppresses the required large fluctuations by an exponential factor, in this case ~ $\exp(-k^4/\Lambda^4)$
- thus the corrections to the propagator at high k^2 are exponentially small

$$\text{define } e^{-V(A)} = \frac{1}{1+N(A)}$$



◊ consider path integral with toy version of Gribov measure over free part of action
(again ignore Lorentz structure)

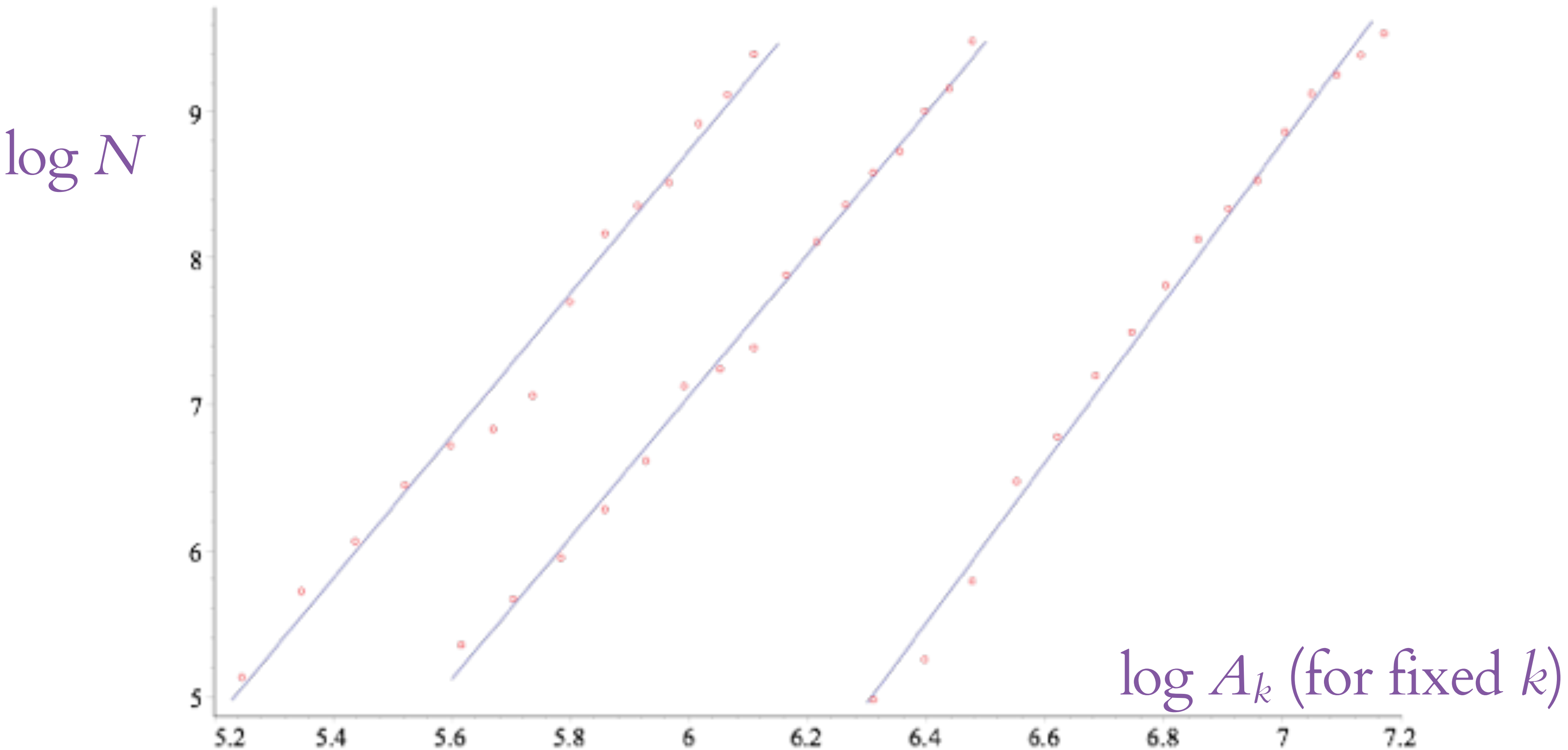
$$\int \mathcal{D}A e^{S_0} = \left(\prod_k \int dA_k \frac{1}{1+N(A_k)}\right) \exp(\sum_k -\frac{1}{2}k^2 A_k^2 + j_k A_k)$$

◊ if $N(A_k) \approx \left(\frac{\Lambda^2 A_k}{k}\right)^\alpha$ for $A_k > \frac{k}{\Lambda^2}$ then:

$$G(k^2 \to \infty) = \frac{1}{k^2}\left(1 - \alpha\sqrt{\frac{2}{\pi}}\frac{\Lambda^2}{k^2}\exp(-\frac{1}{2}\frac{k^4}{\Lambda^4}) + \ldots\right)$$

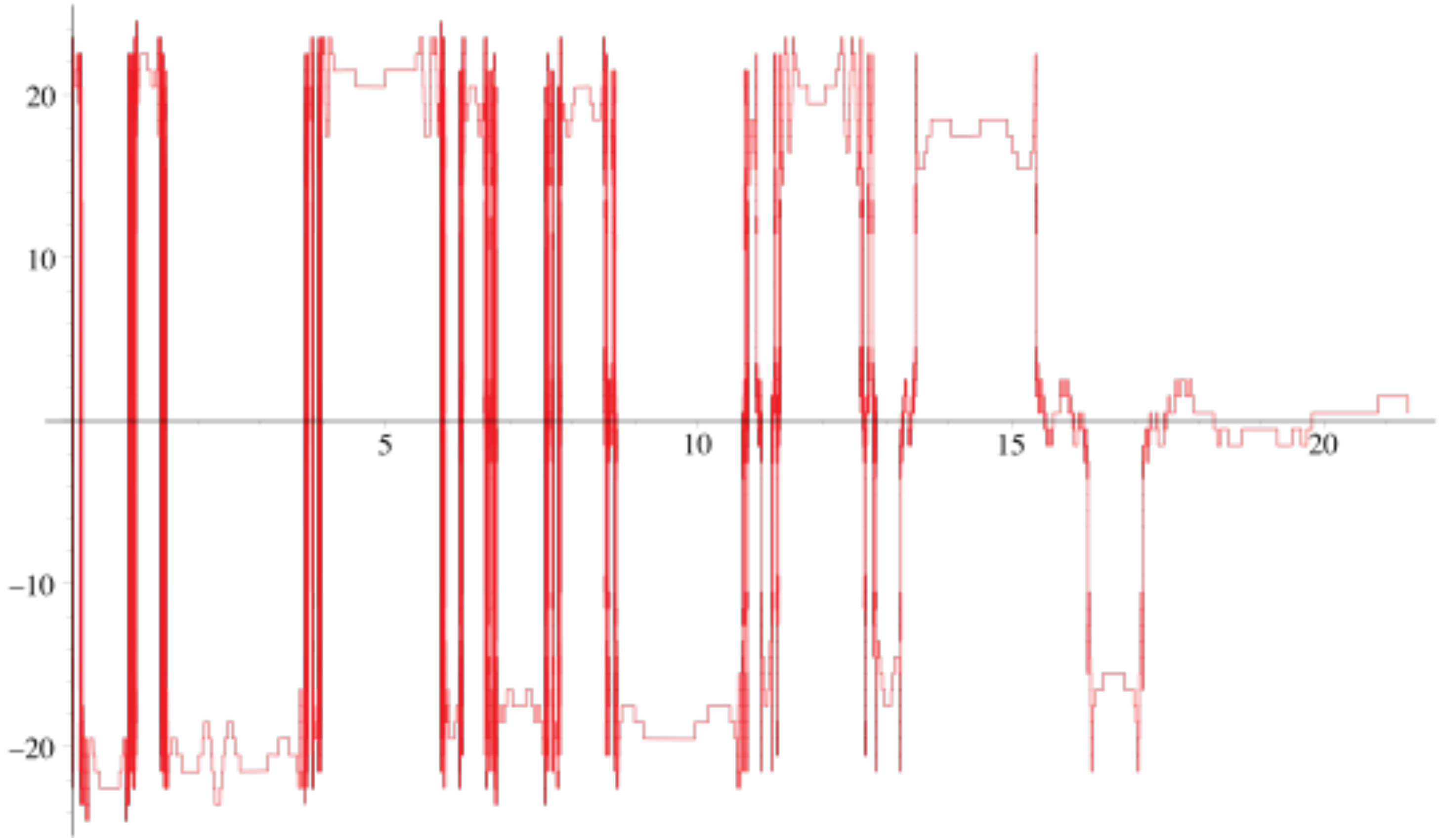
$$G(k^2 \to 0) = \frac{1}{3}\frac{\alpha - 1}{\alpha - 3}\frac{k^2}{\Lambda^4} \quad (\alpha > 3)$$

◊ for spherically symmetric configurations, copies can be counted from the Gribov equation



◊ we see power law growth (N ranges up to 14000)

◊ clumping together of copies on gauge orbit



◊ a copy every time red line has integer value

◊ 13856 copies

- “$N(A)$ is an integer” is a sign of extreme nonlocality
- all the proposed approximations for dealing with Gribov copies produce power law rather than exponential corrections
- these approximations are more local than the real thing

- power law corrections are associated with condensates via the operator product expansion
- effects of Gribov copies are not sufficiently local to produce condensates
- the physics of confinement (Gribov copies) may not be the physics of condensates

- ◊ another implication of "$N(A)$ is an integer"
- ◊ except for exceptional points in configuration space, a variation in the field does not produce a change in $N(A)$
- ◊ on the other hand a variation of the fields in the path integral is used to derive the Schwinger-Dyson equations
- ◊ in this sense SD equations are blind to $N(A)$

- ◊ SD equations capture conventional quantum effects not included in our discussion of Gribov copies
- ◊ for example SD equations capture the origin of chiral condensates

- ◊ where does all this leave the gluon condensate?
- ◊ if it is not associated with confinement physics then do conventional quantum effects generate it?
- ◊ thus use SD equations to study the asymptotic UV behavior of the gluon mass function $m^2(p^2)$
- ◊ linked to the gluon condensate via the OPE

$$\lim_{-p^2\to\infty} \Sigma(p^2) = c_1(p/\mu)\, m_\psi(\mu) + \frac{c_2(p/\mu)\langle\overline{\psi}\psi\rangle_\mu}{p^2} + \ldots$$

$$\lim_{\substack{-p^2 \to \infty \\ m_\psi \to 0}} p^2 m^2(p^2) = a_1(p/\mu)\langle G_{\alpha\beta}G^{\alpha\beta}\rangle_\mu + \frac{a_2(p/\mu)\langle\overline{\psi}\psi\rangle_\mu^2}{p^2} + \ldots$$

- ◊ explored SD equations of Aguilar, Binosi, Ibanez, Papavassiliou
- ◊ found rapid fall off of $m^2(p^2)$ and no evidence of gluon condensate

- ◊ how do we know that a gluon condensate exists in the limit of zero quark masses?
- ◊ fundamental question, since it is related to our understanding (or lack thereof) of vacuum energy

$$\Theta^{\alpha}_{\alpha} = \frac{\beta}{2g} G_{\alpha\beta} G^{\alpha\beta} + m_{\psi}(1 + \gamma_m)\bar{\psi}\psi$$

◊ lattice studies of the gluon propagator in Landau gauge indicate $G(k^2) \Rightarrow$ finite value at $k^2 = 0$

◊ thus consider a simple interpolation between the IR and UV (set Λ and constants to 1)

$$G(k^2) = \frac{1}{k^2}\left(1 - f(k^2)\exp(-\frac{1}{2}k^4)\right)$$

$$\text{where } f(0) = 1 \text{ and } f(k^2 \to \infty) = \frac{1}{k^2}$$

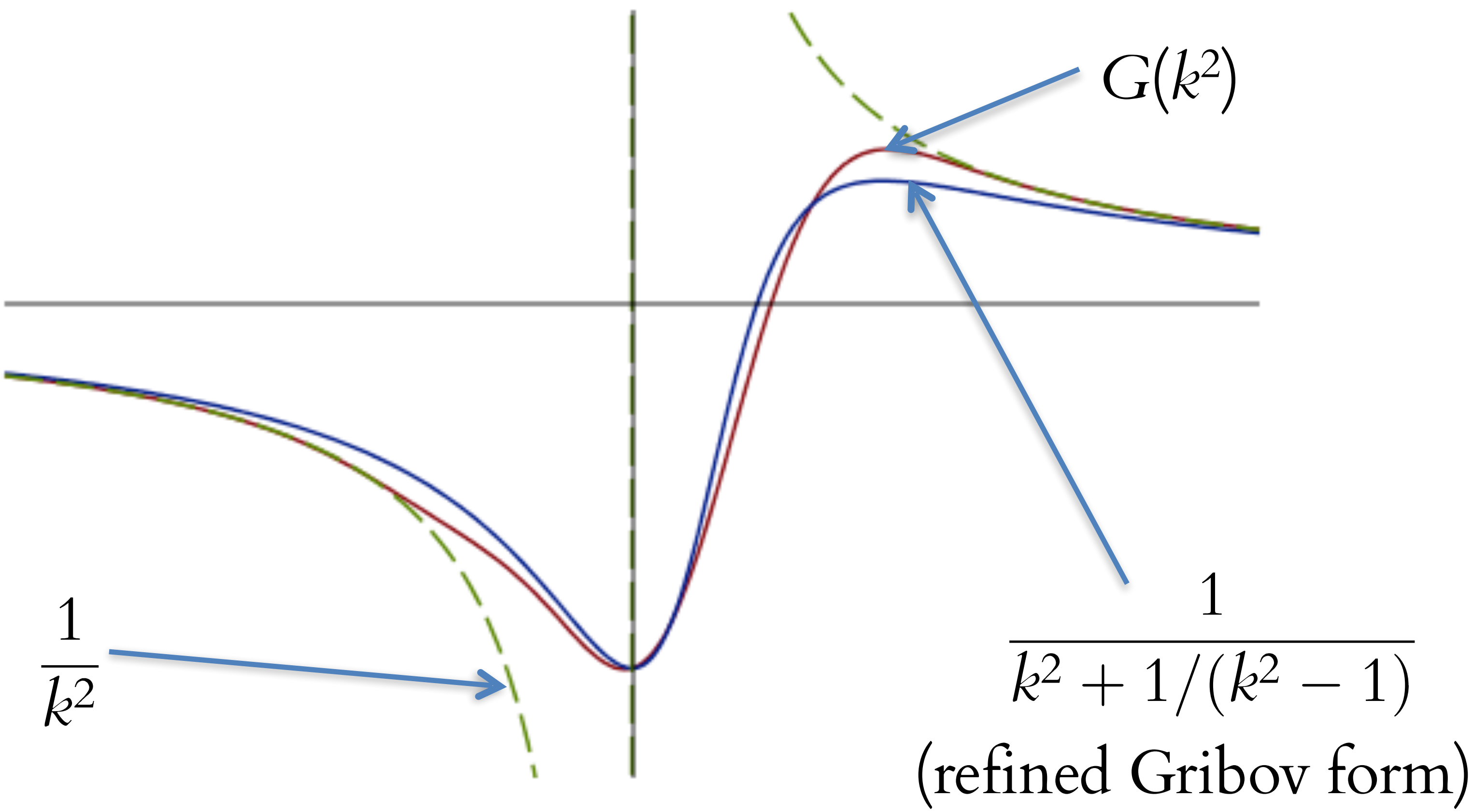
◊ $f(0) = 1$ eliminates the pole at $k^2 = 0$

◊ lack of power law corrections implies no complex conjugate poles

◊ $G(k^2)$ could be an entire function on the complex k^2 plane

◊ for example:

$$f(k^2) = \frac{1}{k^2}\left(1 - [1 - k^2 - bk^4 - ck^6]e^{-\frac{1}{2}k^4}\right)$$



- ◊ turn to gravity
- ◊ consider pure quadratic gravity with action

$$S = \int d^4x \sqrt{g} \left[\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} \right]$$

- ◊ PRO — can be asymptotically free and renormalizable
- ◊ CON — $1/k^4$ graviton propagator, while if Einstein term is added then massive spin-2 ghost emerges

- ◊ but this is in perturbation theory
- ◊ perturbative QCD is similarly misleading since it predicts a propagating gluon

- asymptotically free gravity can also grow strong at some scale
- as for QCD, the true infrared behavior can completely change

- like the gauge theory, a standard gauge fixing procedure is required
- if so then the possible affects of Gribov copies need to be considered

- a resolution of unity in gravity, given a gauge fixing $F(g_{\mu\nu})=0$

$$1 = \frac{1}{1 + N(g_{\mu\nu})} \int \mathcal{D}U \, \delta(F(g^U_{\mu\nu})) \left| \det \left(\frac{\delta F(g^U_{\mu\nu})}{\delta U} \right) \right|$$

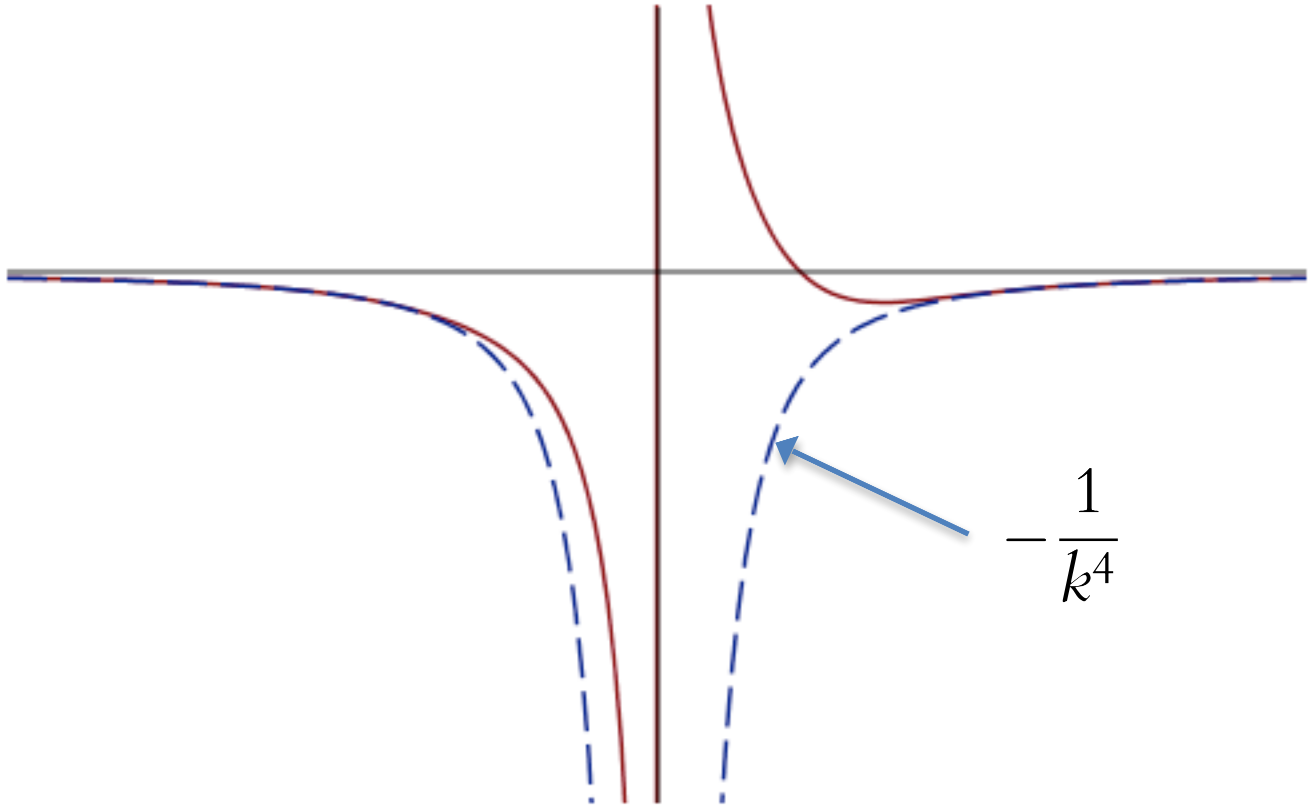
- now U represents a coordinate transformation

- now suppose the effect of Gribov copies is similar to gauge theories
- it is most similar if we simply replace $1/k^2 \to -1/k^4$ (we are again suppressing the tensor structure)

$$G(k^2) = -\frac{1}{k^4}\left(1 - f(k^2)\exp(-\frac{1}{2}k^4)\right)$$

$$\text{where } f(0) = 1 \text{ and } f(k^2 \to \infty) = \frac{1}{k^2}$$

- for $f(k^2)$ we choose the same parameters as before



- $-1/k^4$ graviton pole has been softened to $1/k^2$ pole
- this is the only singularity on the complex k^2 plane
- there is a zero instead of a massive ghost pole

◊ expanding the inverse propagator:

$$G(k^2)^{-1} = \frac{1}{1+2b}k^2 + 4\frac{1-c}{(1+2b)^2}k^4 + O(k^6)$$

◊ this points to an effective theory with the Einstein term appearing as the leading term in a derivative expansion

◊ but in UV, quadratic gravity is approached exponentially quickly

◊ theory is weakly coupled in the UV and IR limits, with a strong intermediate regime — just like QCD/chiral Lagrangian

◊ the difference is that gravity has no mass gap, confinement or chiral symmetry breaking, and the same field $g_{\mu\nu}$ appears in both UV and IR

- ◊ look for evidence of Gribov copies in gravity
- ◊ the gravity analog of the Gribov equation is difficult
  - ◊ look instead for the existence of Gribov horizons
  - ◊ i.e. look for zero eigenvalues of the analog of the FP operator

- ◊ write spherically symmetric metric as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(r)$
- ◊ consider an infinitesimal coordinate transformation $x'_{\mu} = x_{\mu} + \xi_{\mu}(r)$
- ◊ consider corresponding shift $\delta_{\xi} h_{\mu\nu}$
- ◊ choose a gauge fixing condition $F(h_{\mu\nu}) = 0$
- ◊ then $F(\delta_{\xi} h_{\mu\nu}) = 0$ is analog of a zero eigenvalue of FP operator

- ◊ we have found solutions of $F(h_{\mu\nu}) = 0$ and $F(\delta_\xi h_{\mu\nu}) = 0$ for some $\xi_\mu(r)$ and $h_{\mu\nu}(\mathrm{r})$

- ◊ gauge fixing condition has become degenerate in a certain gauge direction

  $\Rightarrow$ pair of Gribov copies connected by the infinitesimal coordinate transformation $\xi_\mu(r)$

- ◊ then also expect pairs of copies connected by finite coordinate transformations on either side of horizon

- ◊ structure of horizons, relation to FMR etc. is unknown

Conclusions

- ◊ gauge dependent views of the world are useful
- ◊ the Gribov measure introduces extremely nonlocal, and thus extremely UV soft, features into gauge theories
  - ◊ gives picture of confinement but not condensates
- ◊ Gribov may motivate a picture of gravity that leverages our understanding of QCD
  - ◊ Einstein emerges in the infrared due to the Gribov measure of an asymptotically free theory (quadratic gravity)