Uniwersytet Wrocławski

Dense quark matter in astrophysical applications. Effective model EsoS and their motivation from a Dyson-Schwinger perspective.

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NATIONAL SCIENCE CENTRE POLAND
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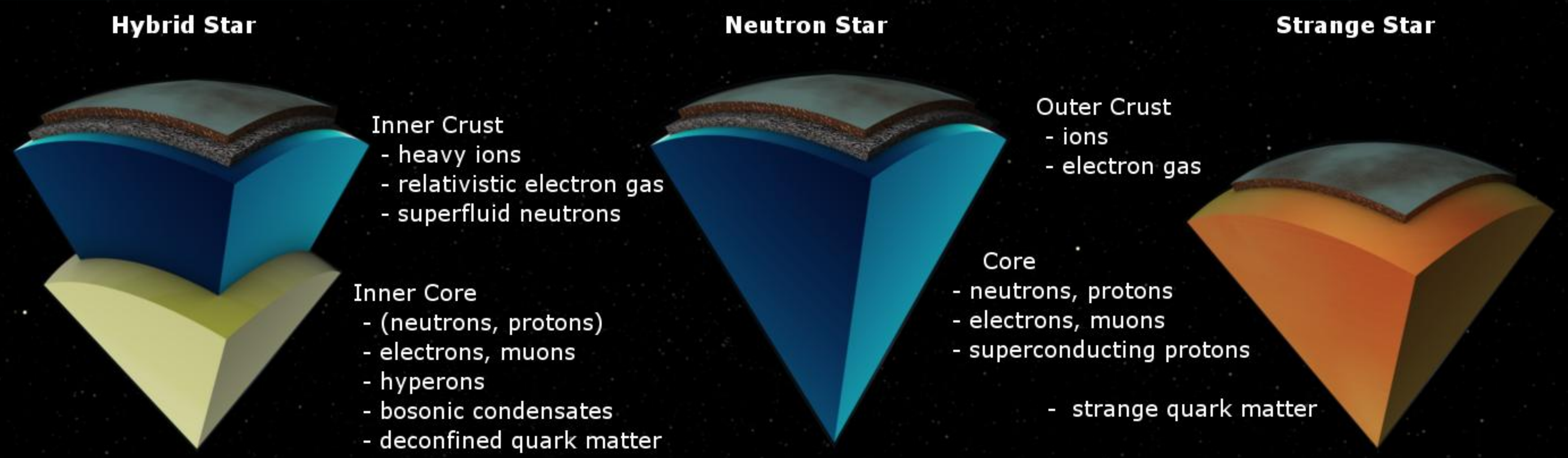
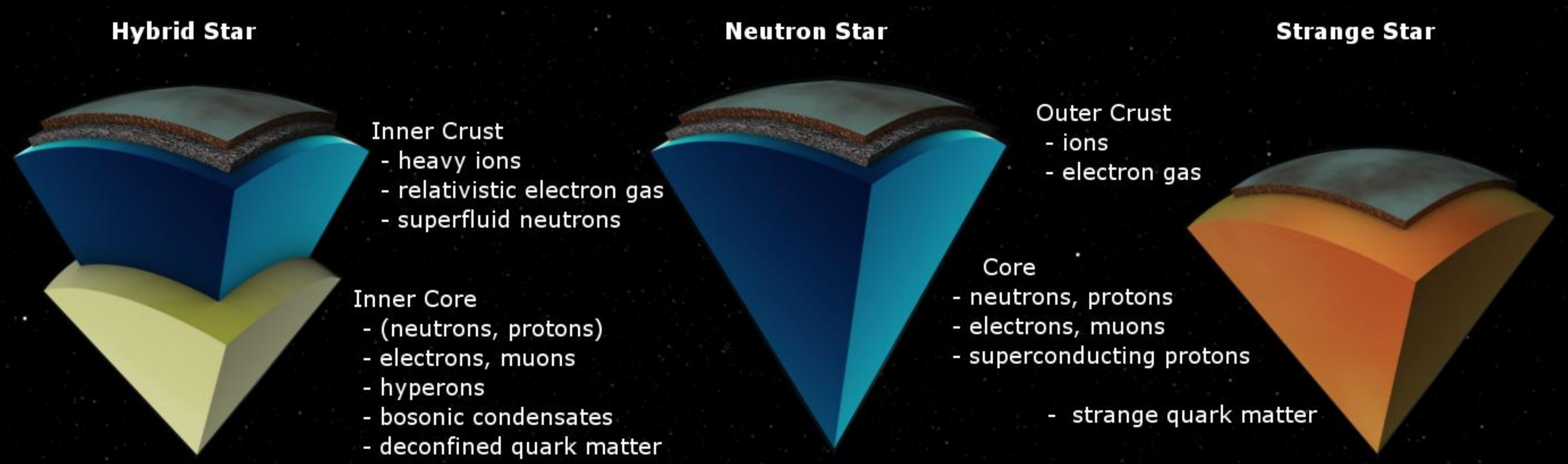
Neutron Stars = Quark Cores?

- Variety of scenarios regarding inner structure: with or without QM
- Question whether/how QCD phase transition occurs is not settled
- Most honest approach: take both (and more) scenarios into account and compare to available data



Neutron Stars = Quark Cores?



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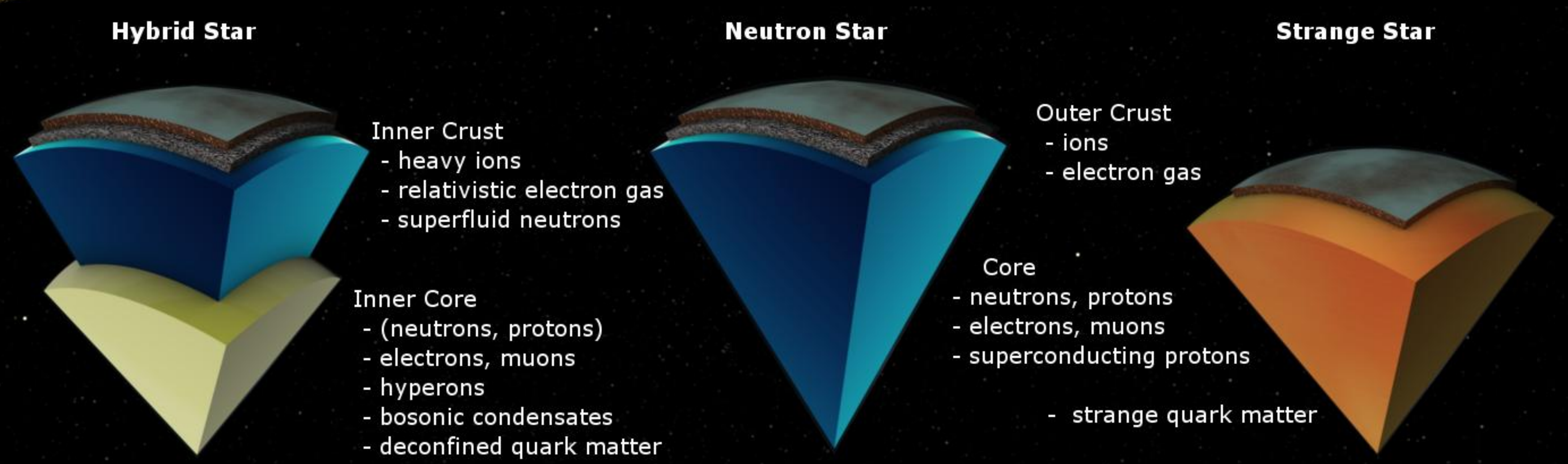

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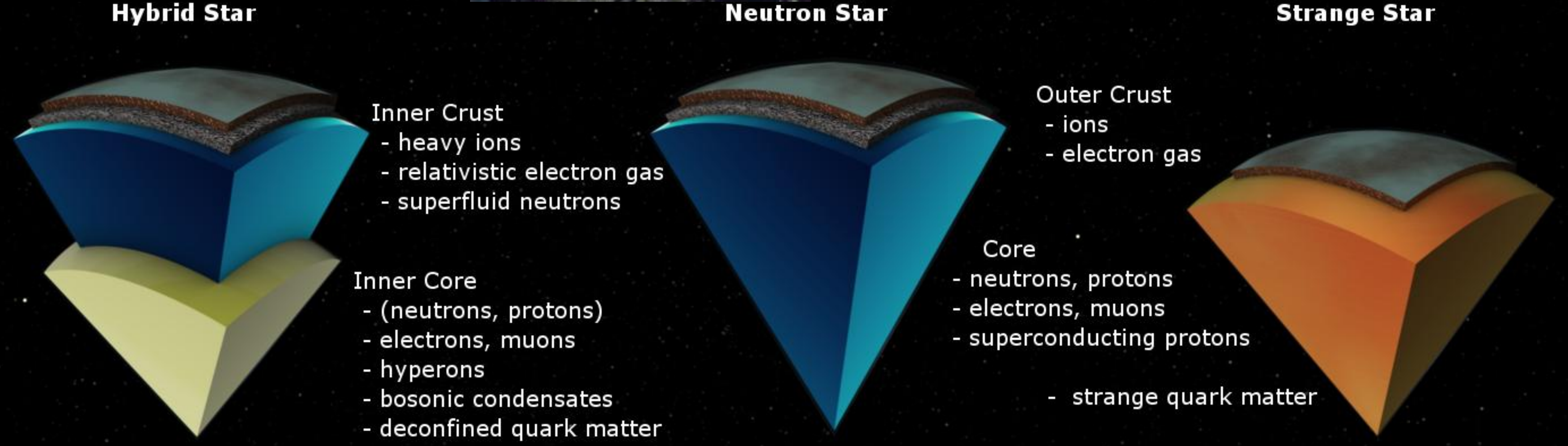
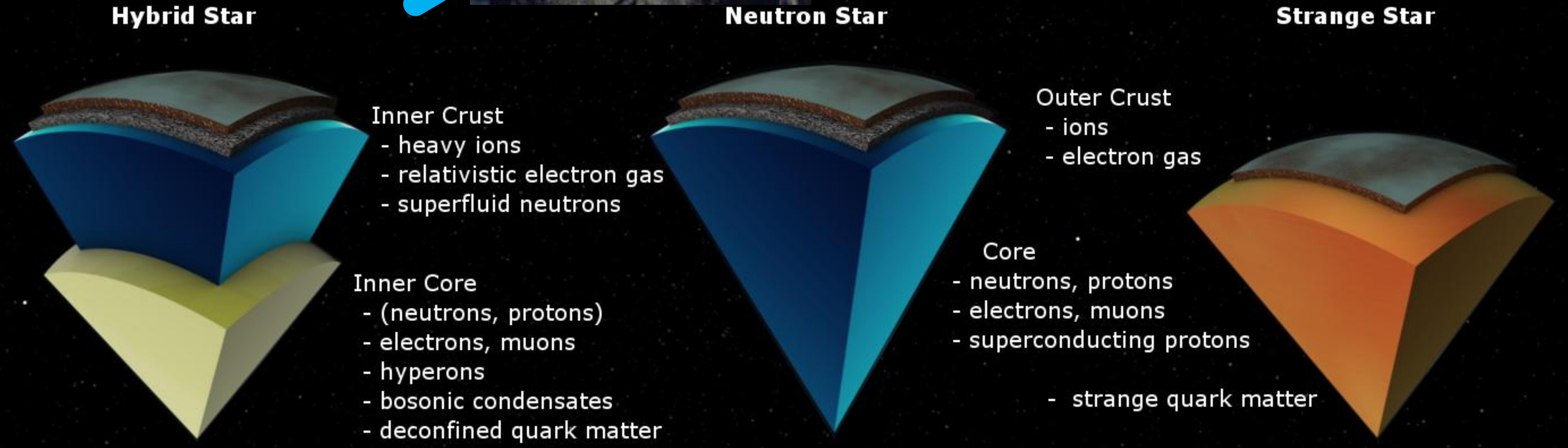


Neutron Stars = Quark Cores?



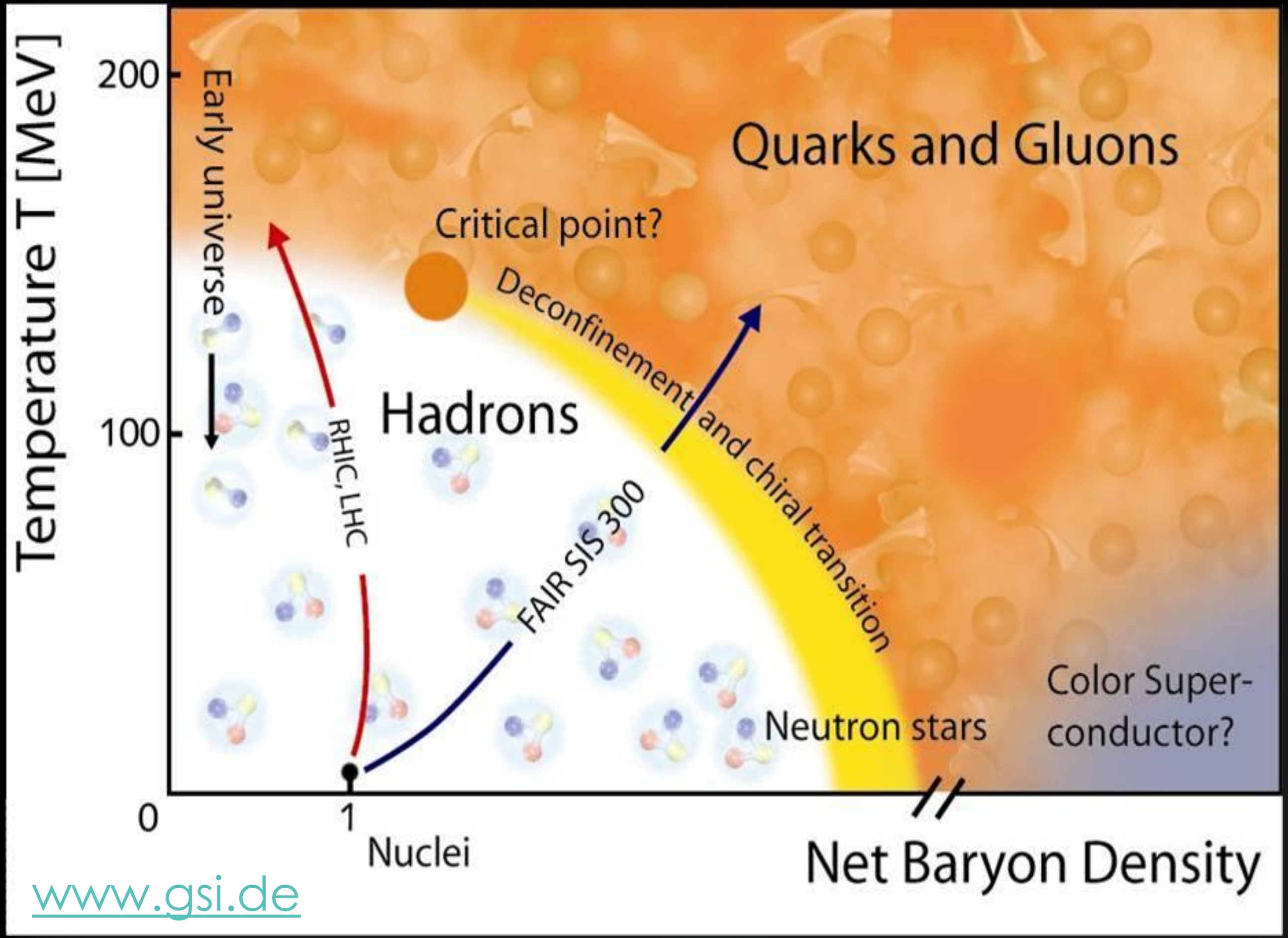
QCD Phase Diagram

- **dense hadronic matter**

<u>HIC in collider experiments</u>
Won't cover the whole diagram
Hot and 'rather' symmetric

<u>NS as a 2nd accessible option</u>
Cold and 'rather' asymmetric

Problem is more complex than
It looks at first gaze



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QCD Phase Diagram

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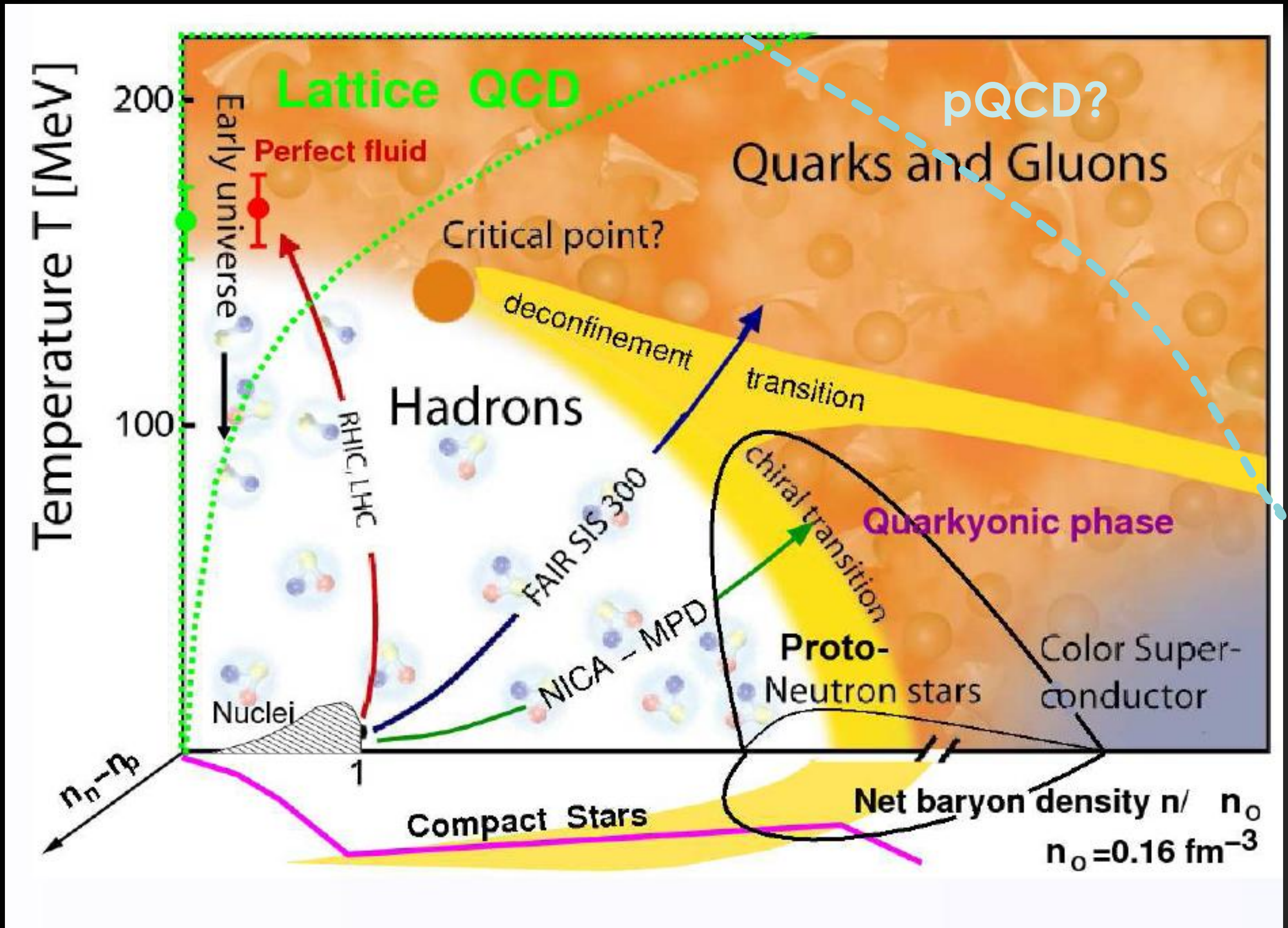
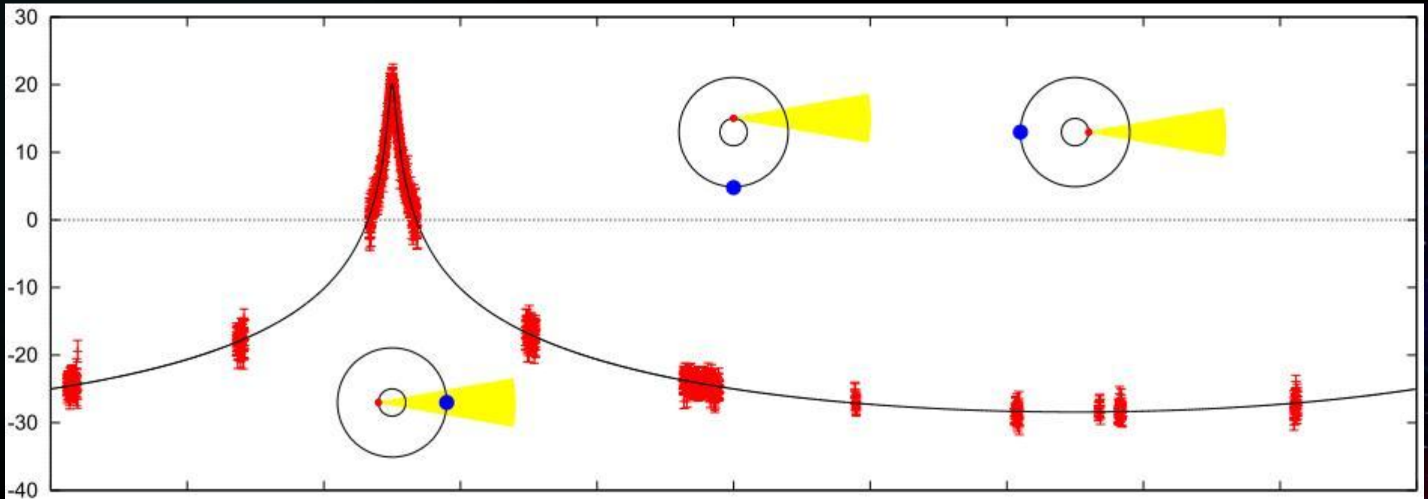


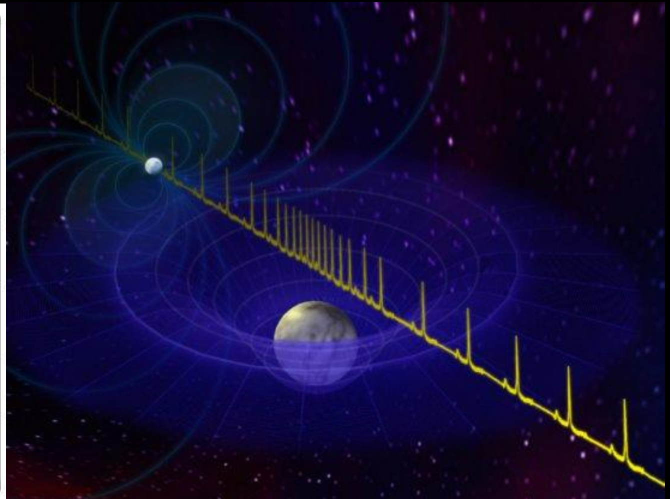
Neutron Star Data

- Data situation in general terms is good (masses, temperatures, ages, frequencies)
- Ability to explain the data with different models in general is good, too.

  ... sounds good, but becomes tiresome if everybody explains everything …
- For our purpose only a few observables are of real interest
- Most promising: High Massive NS with 2 solar masses (Demorest et al.(2010), Nature 467, 1081-1083)

  (Antoniadis et al.(2013), Science 340, 448)





Space, time and matter are related via Einsteins Field Equations

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

Einstein Tensor $G_{\mu\nu}$

defined by metric

Energy Momentum Tensor $T_{\mu\nu}$

defined by equation of state

Approximations

non rotating, spheric symmetry

$$ds^2 = g_{\mu\nu} dx^\mu dx^\mu$$

$$\rightarrow g_{00}(r)dt^2 + g_{11}(r)dr^2 + g_{22}(r)d\theta^2 + g_{33}(r,\theta)d\phi^2$$

hydrostatic equilibrium

$$-pg^{\mu\nu} + (p+\varepsilon)u^\mu u^\nu$$

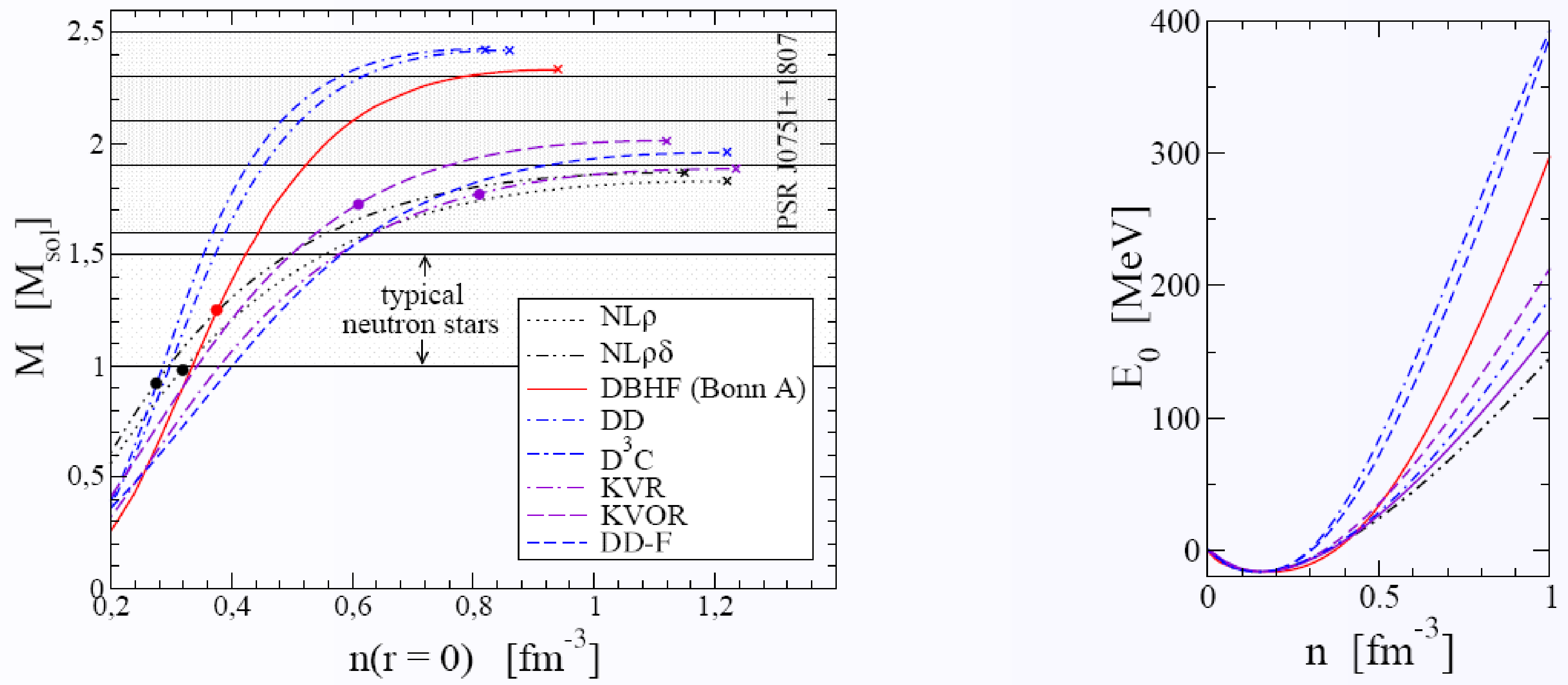
Tolman-Oppenheimer-Volkov (TOV) Equations (1939)

$$\frac{dp(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2}\left(1+\frac{p(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 p(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

$$m(r) = 4\pi\int_0^r dr'\, r'^2 \varepsilon(r')$$

NS masses and the (QM) Equation of State

- NS mass is sensitive mainly to the sym. EoS (In particular true for heavy NS)
- Folcloric:
  QM is soft, hence no NS with QM core
- Fact:
  QM is softer, but able to support QM core in NS
- Problem:
  (transition from NM to) QM is barely understood



$M(n)$ correlated to $E_0(n)$

stiff: higher M_{max} at smaller densities

soft: smaller M_{max} at higher densities

Quark Matter

What is so special about quarks?

Confinement: No isolated quark has ever been observed
Quarks are confined in baryons and mesons

Dynamical Mass Generation:

Proton 940 MeV, 3 constituent quarks with each 5 MeV
→ 98.4% from .... somewhere?

and then this:
eff. quark mass in proton: 940 MeV/3 ≈ 313 MeV
eff. quark mass in pion : 140 MeV/2 = 70 MeV

quark masses generated by interactions only
‚out of nothing'
interaction in QCD through (self interacting) gluons
dynamical chiral symmetry breaking (DCSB)
is a distinct nonperturbative feature!

Confinement and DCSB are connected. Not trivially seen from QCD Lagrangian.
Investigating quark-hadron phase transition requires nonperturbative approach.

Quark Matter

Confinement and DCSB are features of QCD.
It would be too nice to account for these phenomena
when describing QM in Compact Stars...

Current approaches mainly used to describe dense, deconfined QM:

Bag-Model :

While Bag-models certainly account for confinement (constructed to do exactly this)
they do not exhibit DCSB (quark masses are fixed - bare quark masses).

Chodos, Jaffe et al: Baryon Structure (1974)
Farhi, Jaffe: Strange Matter (1984)

NJL-Model :

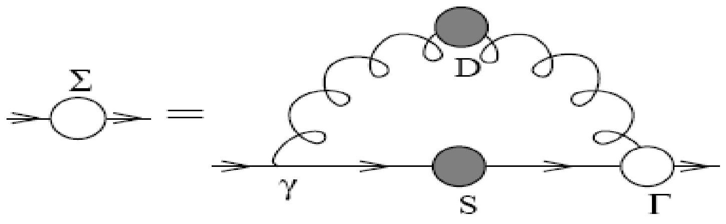
While NJL-type models certainly account for DCSB (applied, because they do)
they do not (trivialy) exhibit confinement.
Modifications to address confinement exist (e.g. PNJL) but are not entirelly satisfying
Both models: Inspired by, but not originally based on QCD.

Nambu, Jona-Lasinio (1961)

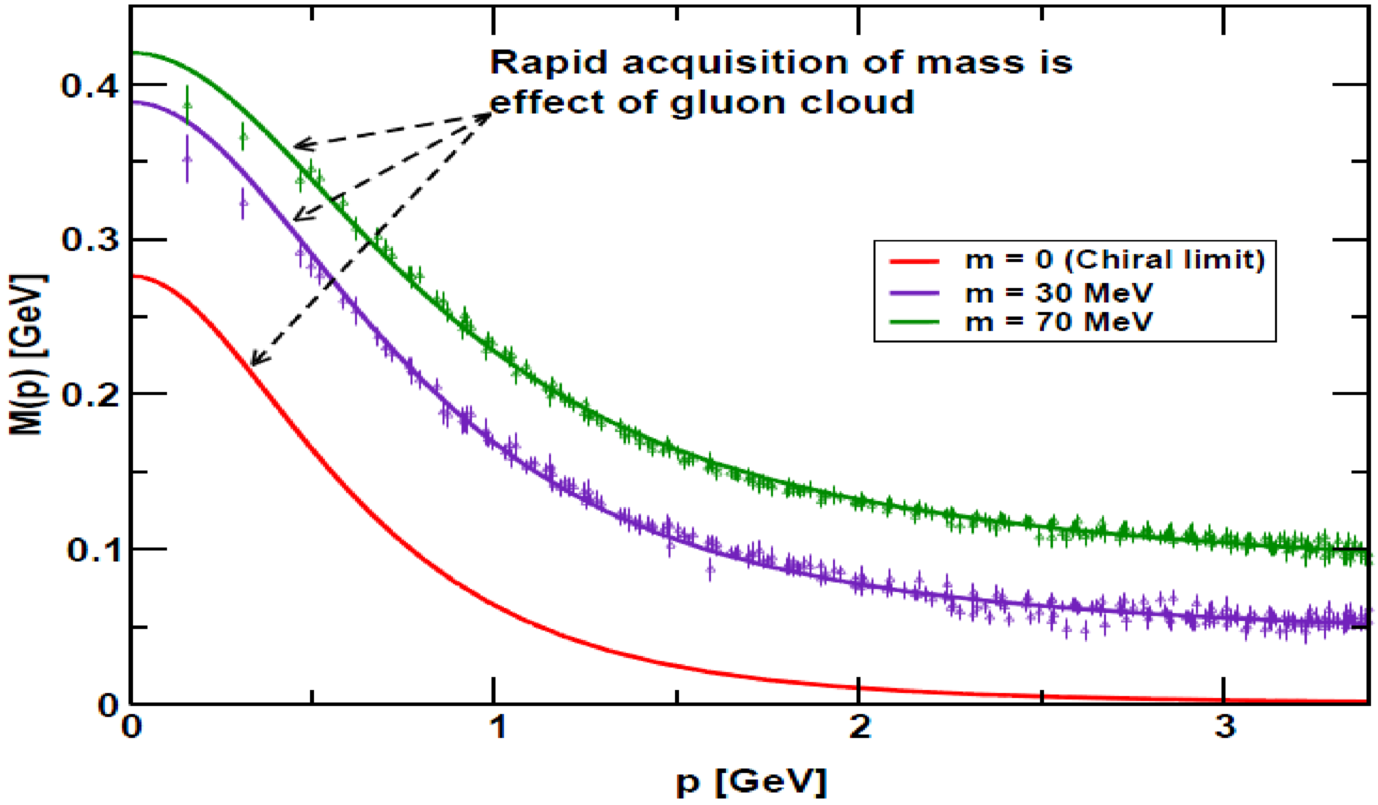
Lattice QCD still fails at T=0 and finite μ

Dyson-Schwinger Approach

Derive gap equations from QCD-Action. Self consistent self energies.
Successfuly applied to describe meson and hadron properties
Extension from vacuum to finite densities desirable
→ EoS within QCD framework



DSE : dynamical, momentum dependent mass generation



momentum dep. (here @ T=μ=0)
LQCD as benchmark

Neither NJL nor BAG have this!

How do momentum dependent gap solutions affect
- EoS of deconfined quark matter?
- EoS of confined quark matter?
- transport properties in medium?

Roberts (2011)
Bhagwat et al. (2003,2006,2007)
P. O. Bowman et al. (2005)

Bag model: bare quark mass ~5 MeV at all densities
NJL model: constant quark mass at all momenta, but changing dynamically with density/chemical potential

Quark Matter

Confinement and DCSB are features of QCD.
It would be too nice to account for these phenomena
when describing QM in Compact Stars...

Current reality is:

Bag-Model :

While Bag-models certainly account for confinement (constructed to do exactly this)
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While NJL-type models certainly account for DCSB (applied, because they do)
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Modifications to address these shortcomings exist (e.g. PNJL)
Still holds: Inspired by, but not based on QCD.

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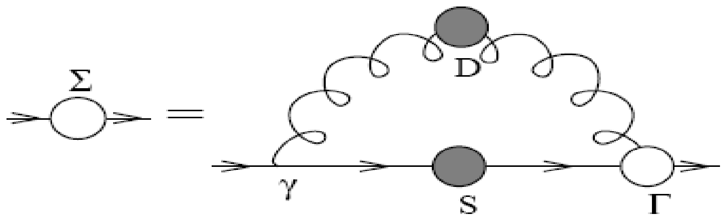
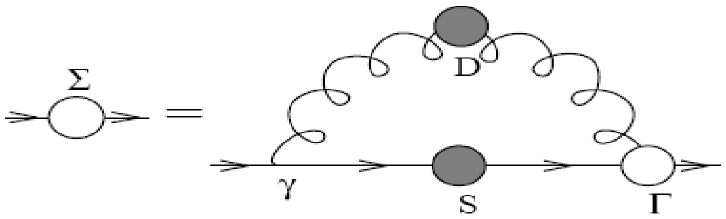
Derive gap equations from QCD-Action. Self consistent self energies.
Successfuly applied to describe meson and hadron properties
Extension from vacuum to finite densities desirable
→ EoS within QCD framework
→ **THIS TALK: Bag and NJL model as simple limits within DS approach**



Dyson Schwinger Perspective

One particle gap equation(s)

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p;\mu)$$



Self energy -> entry point for simplifications

$$\Sigma(p;\mu) = \int_\Lambda \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma_\rho \frac{\lambda^a}{2} S(q)\Gamma^a_\sigma(p;q)$$

General (in-medium) gap solutions

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p}A(p;\mu) + i\gamma_4(p_4 + i\mu)C(p;\mu) + B(p;\mu)$$

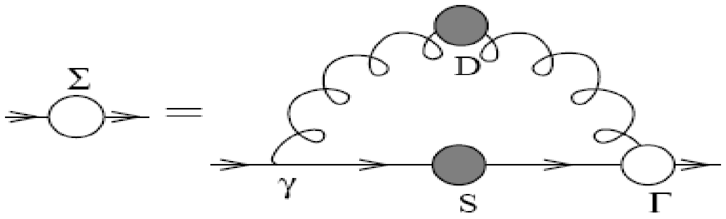
DSE -> NJL model

$$g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2}\delta_{\rho\sigma},$$

$$\Gamma_\rho^a(p;q) = \frac{\lambda^a}{2}\gamma_\rho.$$

Gluon contact interaction in configuration space (other models exist)

Rainbow approximation



$$\vec{p}^2 A_p = \vec{p}^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\vec{p}\vec{q}A_q}{\vec{q}^2A_q^2 + \widetilde{q}_4^2C_q^2 + B_q^2},$$

$$A = 1$$

$$B_p = m + \frac{16N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B_q}{\vec{q}^2A_q^2 + \widetilde{q}_4^2C_q^2 + B_q^2},$$

$$\widetilde{p}_4^2 C_p = \widetilde{p}_4^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\widetilde{p}_4\widetilde{q}_4C_q}{\vec{q}^2A_q^2 + \widetilde{q}_4^2C_q^2 + B_q^2},$$

$$B_\mu = m + \frac{4N_c}{9m_G^2}n_s(T,\mu^*,B),$$

$$\mu = \mu^* - \frac{2N_c}{9m_G^2}n_v(T,\mu^*,B),$$

$$\widetilde{p}_4 C = p_4 + i(\mu + \omega_\mu) \equiv \hat{p}_4$$

Thermodynamical Potential

DS: steepest descent

$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S].$$

$$P_{FG} = \mathrm{Tr}\ln S^{-1} = 2N_c \int_\Lambda \frac{d^4p}{(2\pi)^4} \ln(\vec{p}^2 + \hat{p}_4^2 + B_\mu^2)$$

$$P_I = -\frac{1}{2}\mathrm{Tr}\Sigma S = \frac{3}{4} m_G^2 \omega_\mu^2 - \frac{3}{8} m_G^2 \phi_\mu^2$$

Compare to NJL type model with following Lagrangian (interaction part only):

$$\mathcal{L}_\mathrm{I} = \mathcal{L}_\mathrm{S} + \mathcal{L}_\mathrm{V} = G_s \sum_{a=0}^{8} (\bar{q}\tau_a q)^2 + G_v(\bar{q}i\gamma_0 q)^2.$$

$$\Omega_q = \Omega_q^0 + \frac{\phi^2}{4G_s} - \frac{\omega^2}{2G_v} - \Omega_q(T = \mu = 0)$$

$$\phi_\mu = 2G_s N_c n_s(T, m_f^*, \mu_f^*)$$

$$\omega_\mu = -2G_s N_c n_v(T, m_f^*, \mu_f^*)$$

$$\frac{\partial \Omega_q}{\partial \phi_\mu} = \frac{\partial \Omega_q}{\partial \omega_\mu} = 0.$$

Thermodynamical Potential

DS: steepest descent

$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S].$$

NJL model is easily understood as a particular approximation of QCD’s DS gap equations

$$P_{FG} = \mathrm{Tr}\ln S^{-1} = 2N_c \int_\Lambda \frac{d^4p}{(2\pi)^4} \ln(\vec{p}^2 + \hat{p}_4^2 + B_\mu^2)$$

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$$\frac{\partial \Omega_q}{\partial \phi_\mu} = \frac{\partial \Omega_q}{\partial \omega_\mu} = 0.$$

NJL type models

- S: DCSB
- V: renormalizes μ
- D: diquarks → 2SC, CFL
- TD Potential minimized
  in mean-field approximation
- Effective model by its nature;
  can be motivated (1g-exchange)
  doesn't have to though and can
  be extended (KMT, PNJL)
- possible to describe hadrons

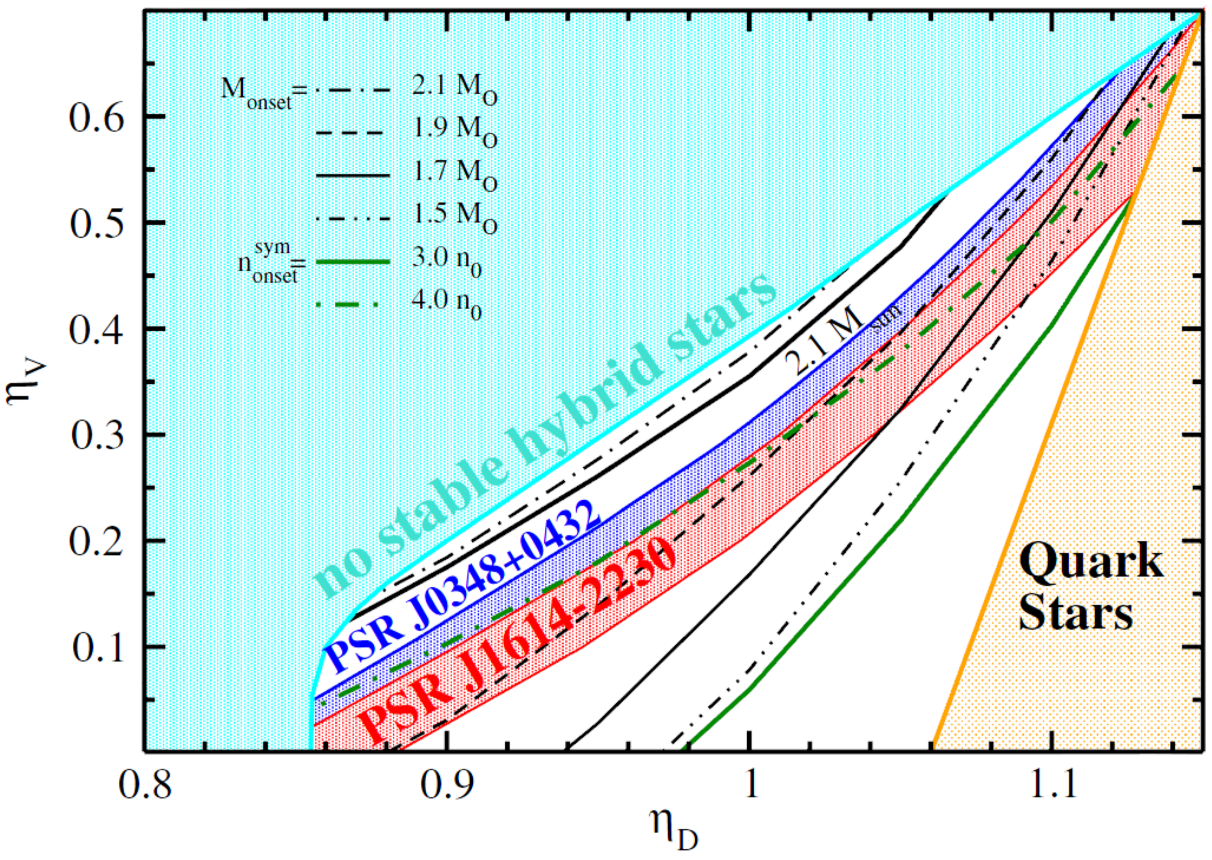
Effective Lagrangian

$$\mathcal{L}_{int} = G_S \eta_D \sum_{a,b=2,5,7} (\bar{q} i \gamma_5 \tau_a \lambda_b C \bar{q}^T)(q^T C i \gamma_5 \tau_a \lambda_a q) + G_S \sum_{a=0}^{8} \left[(\bar{q} \tau_a q)^2 + \eta_V (\bar{q} i \gamma_0 q)^2 \right]$$

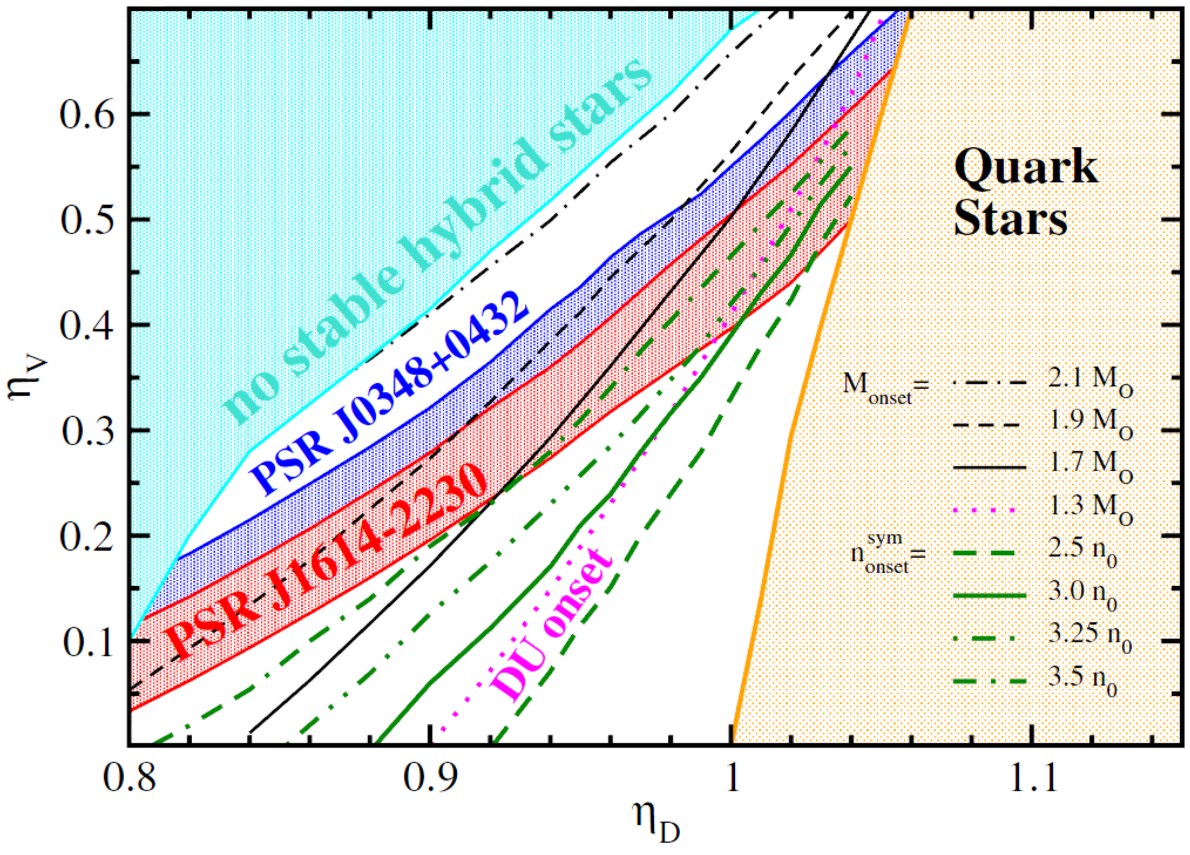
Thermodynamical potential

$$\Omega(T,\mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} - \frac{\omega_u^2 + \omega_d^2 + \omega_s^2}{8G_V} + \frac{\Delta_{ud}^2 + \Delta_{us}^2 + \Delta_{ds}^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3} \sum_{n=1}^{18} \left[E_n + 2T ln \left(1 + e^{-E_n/T} \right) \right] + \Omega_l - \Omega_0$$

NJL model study for NS (TK, R.Łastowiecki, D.Blaschke, PRD **88**, 085001 (2013))



Set A



Set B

Conclusion: NS may or may not support a significant QM core.
additional interaction channels won't change this if coupling strengths are not precisely known.

1st order phase transition observable in neutrino signal

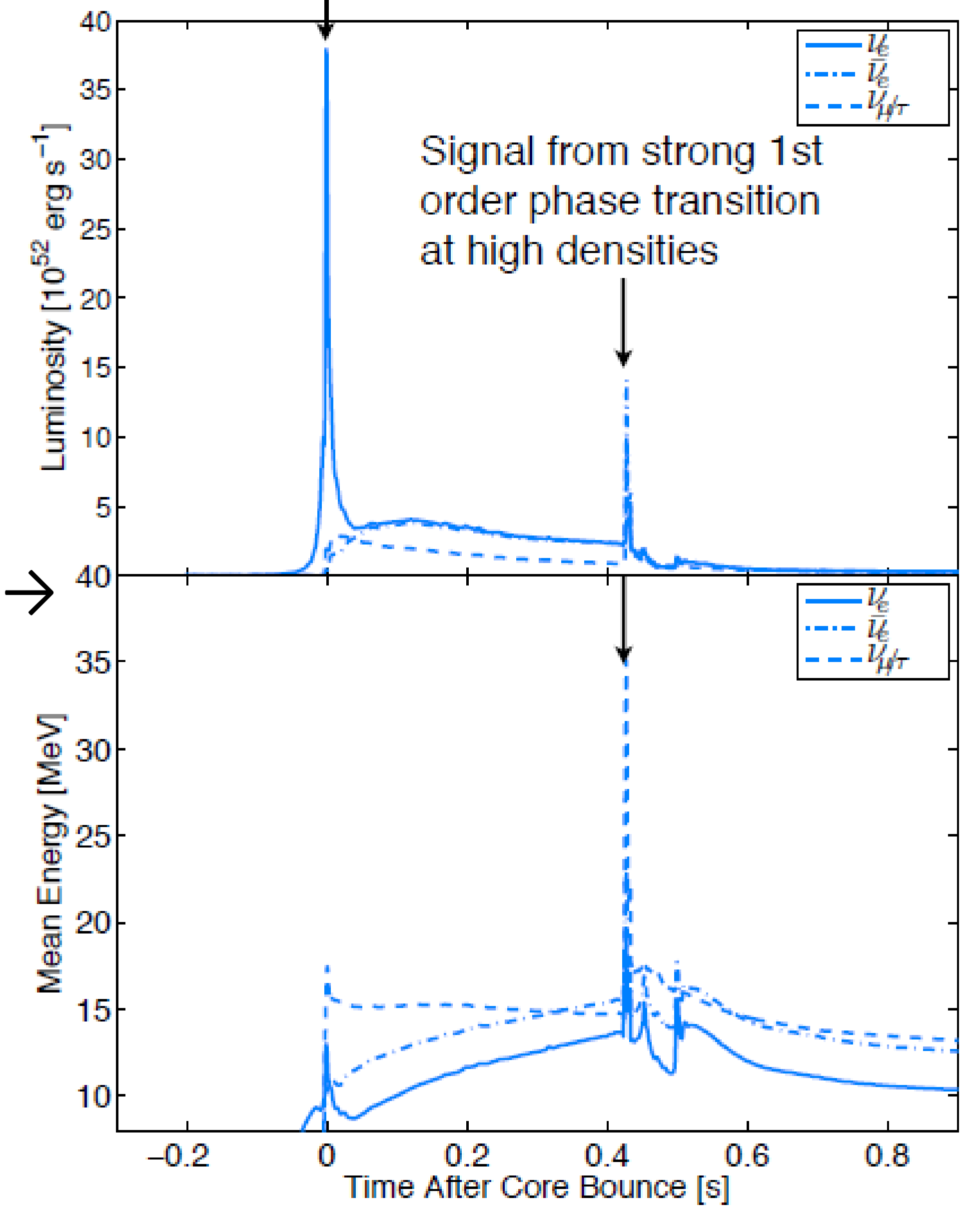
High mass NSs do not rule out QM cores

They are no evidence neither.

General problem:
Which observable would convince that QCD phase transition happens in nature?

Tobias Fischer (et al.) ? →

Standard feature: deleptonization burst form core bounce



1st neutrino burst shortly after core bounce, deleptonization burst, standard feature in all supernova models

Hard to detect because it comes in ν_e

2nd burst due to 2nd-shock propagation across neutrinospheres, dominated by:

$$\bar{\nu}_e \quad (\nu_{\mu/\tau}, \bar{\nu}_{\mu/\tau})$$

Neutrinos are emitted locally and come from low densities (hadronic phase)

2nd burst last only few milliseconds

Accompanied by significant rise of average neutrino energies

Observable for currently operating neutrino-detector facilities

Dasgupta & T.F. et al. (2010), PRD 81, 103005

Problem: Violation of current constraints from astrophysics

High mass NSs do not rule out QM cores

They are no evidence neither.

General problem:
Which observable would be convincing that QCD phase transition happens in nature?

Tobias Fischer (et al.) →

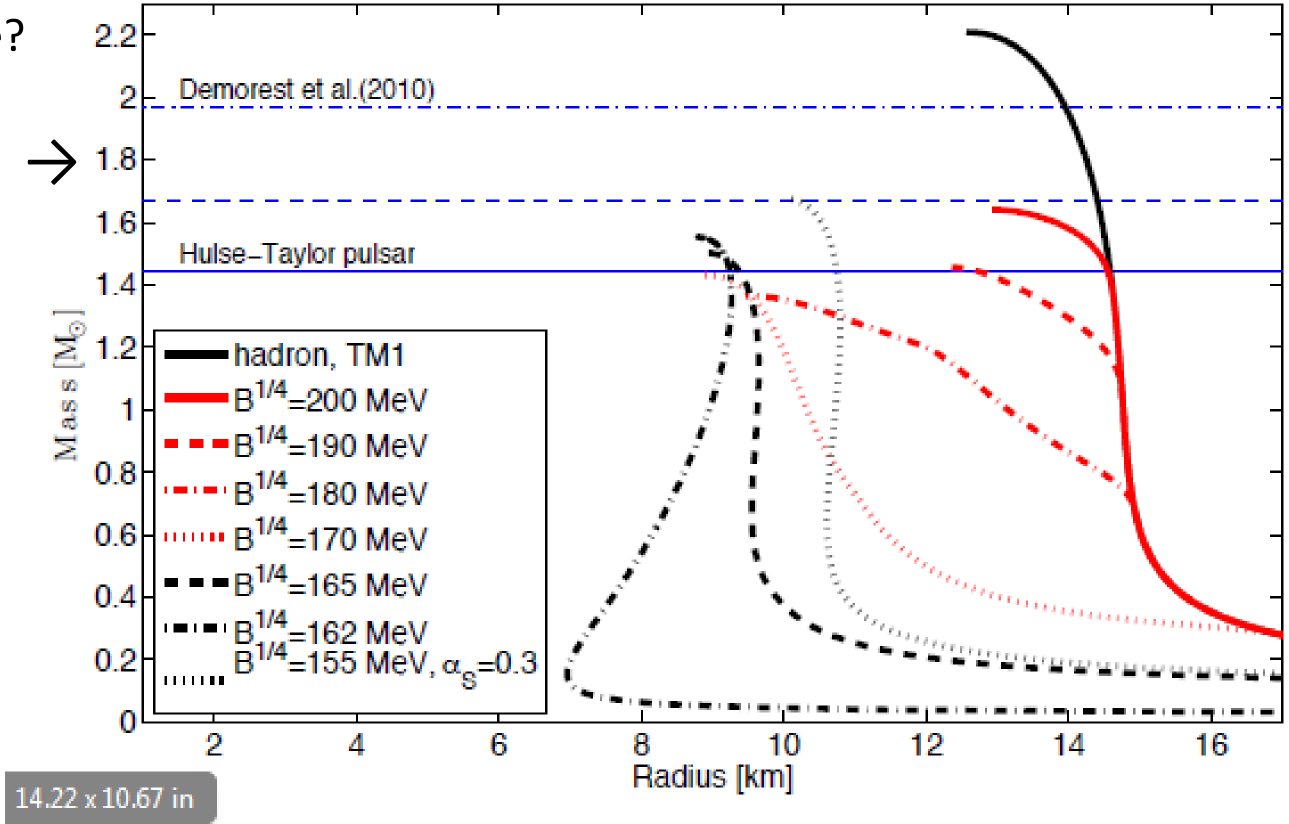
Demorest et al. (2010), Nature 09466, **J1614-2230**

Antoniadis et al. (2013), Science 340, 448, **J1614-2230**

Steiner et al. (2010), ApJ 722 (Bayesian analysis of few selected low-mass X-ray binary systems)

$$M_{\rm max} = (1.97 - 2.01) \pm 0.04\ {\rm M}_\odot$$
$$R|_{M=1.4\ {\rm M}_\odot} = 12 \pm 1\ {\rm km}$$

(see talk by Cole Miller)

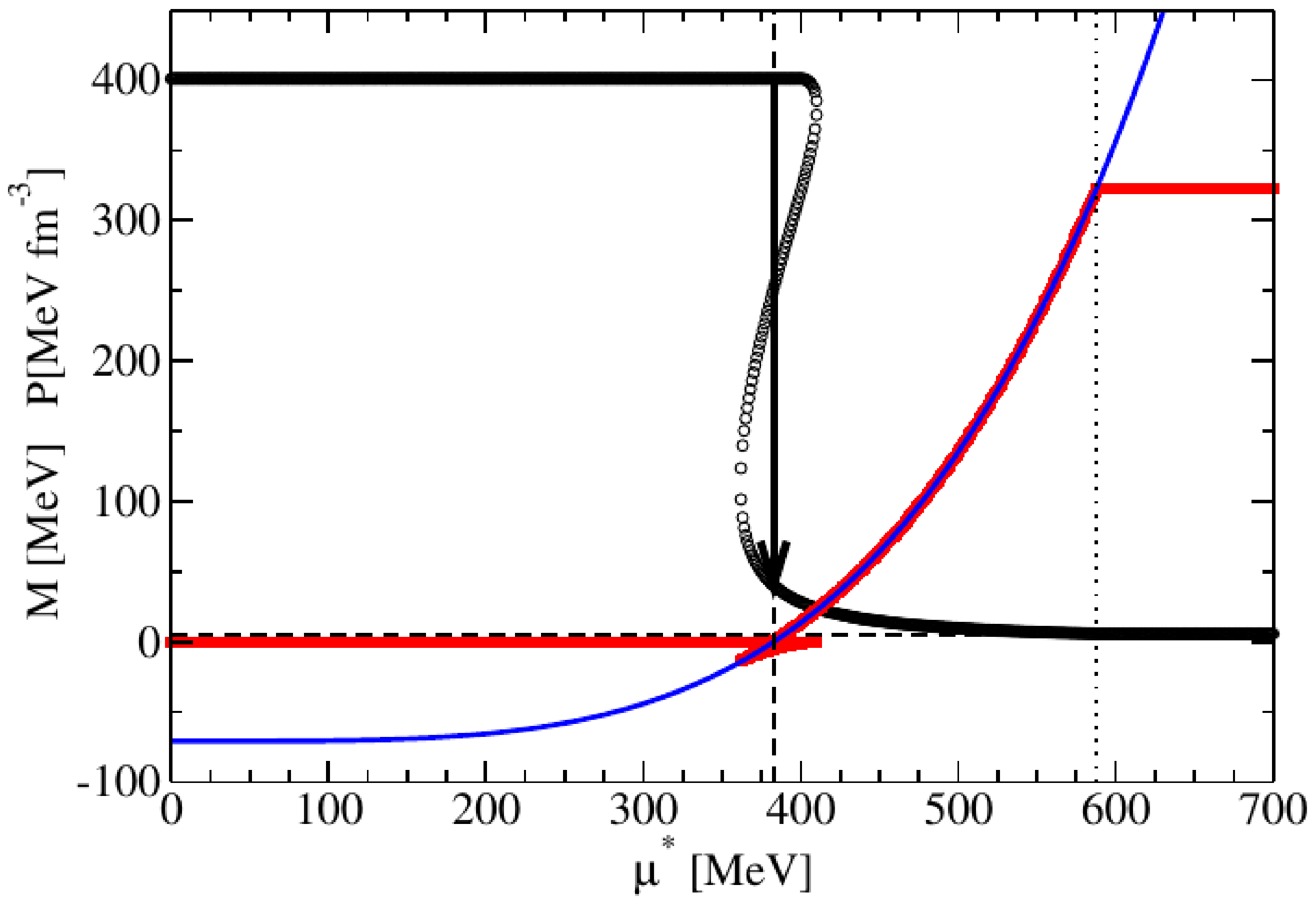


All quark-bag hybrid EOS tested are ruled out !

Bag Model from NJL perspective (TK, T.Fischer, ApJ, accepted)

obvious differences between NJL and Bag:

- **DχSB**
- confinement
- vector interaction



u,d-quark

Mass

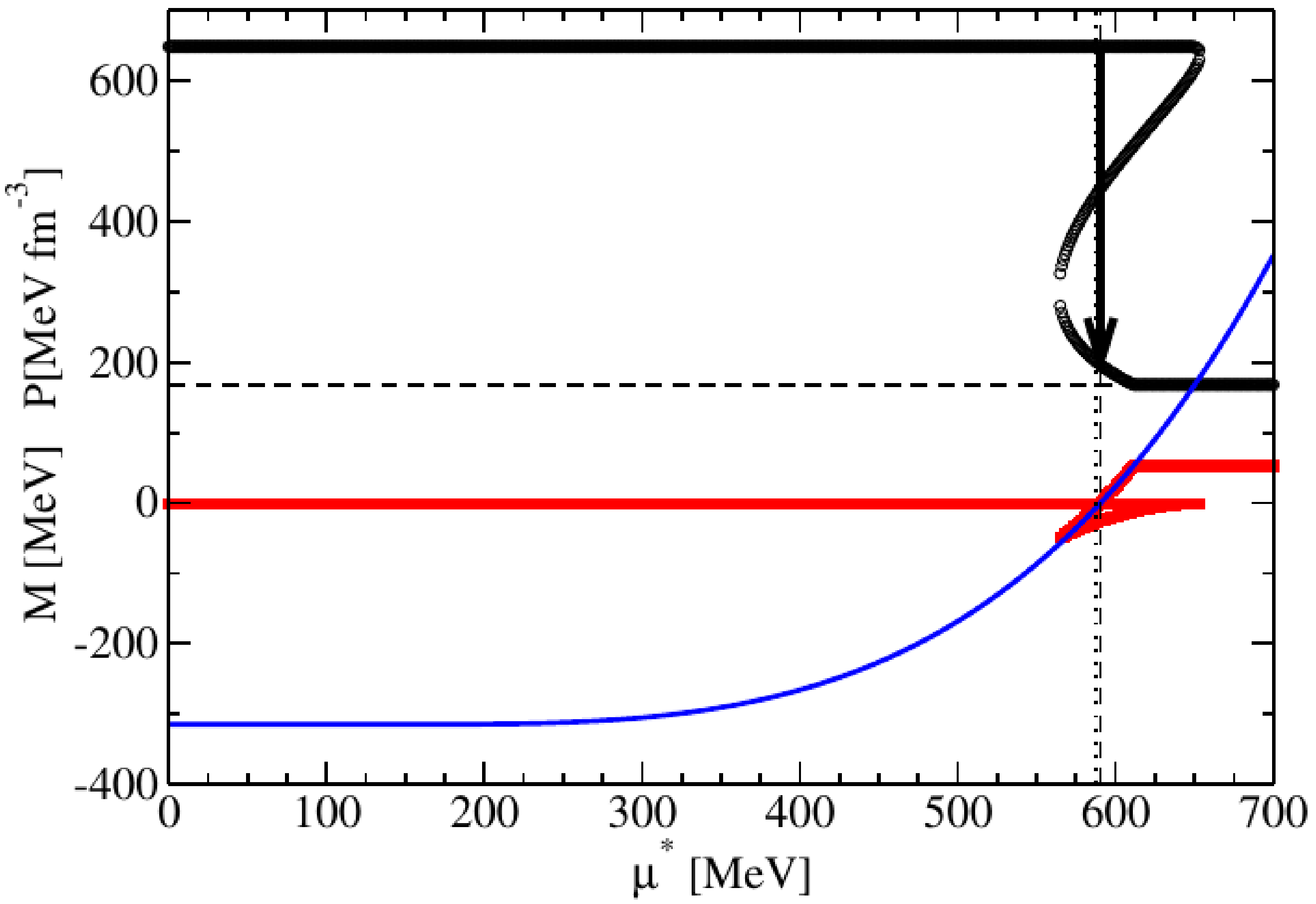
Pressure NJL

Pressure Ideal Gas - Bag

Bag Model from NJL perspective

obvious differences between NJL and Bag:

- **DχSB**
- confinement
- vector interaction



s-quark

Mass

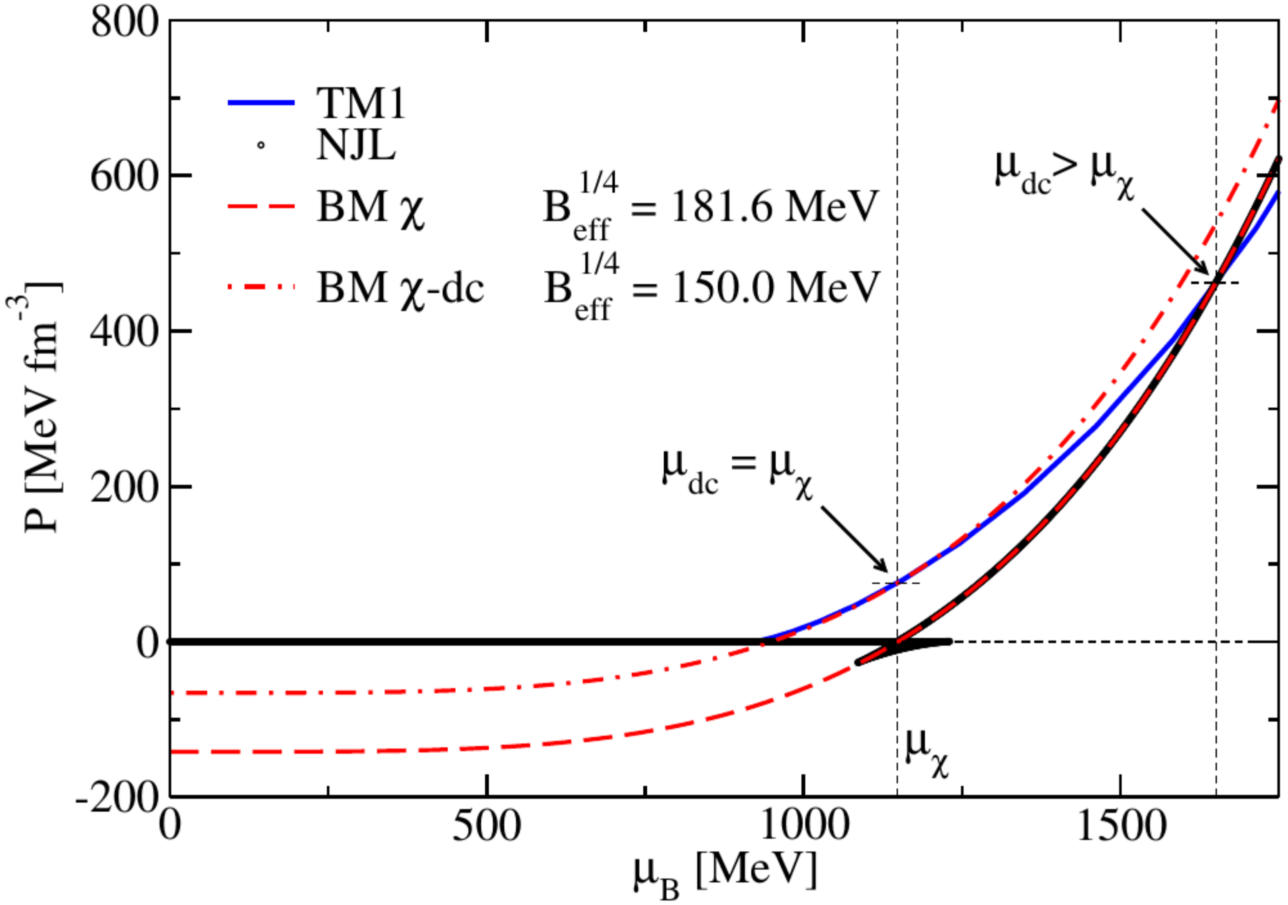
Pressure NJL

Pressure Ideal Gas - Bag

Bag Model from NJL perspective

obvious differences between NJL and Bag:

- DχSB
- **confinement**
- vector interaction



confinement

Pressure Quark NJL/Bag

Pressure Nuclear Matter

Obviously not zero at χ transition

Reduce χ bag pressure – by hand

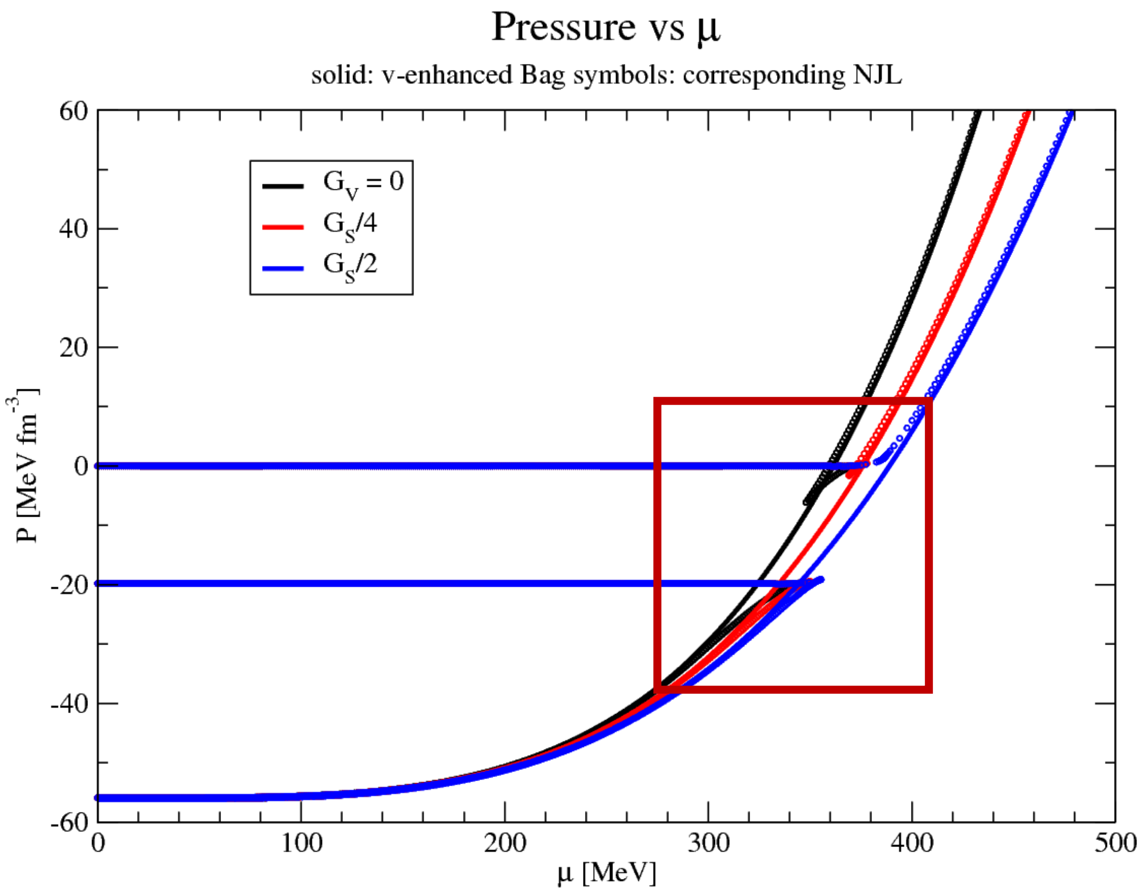
Bag Model from NJL perspective

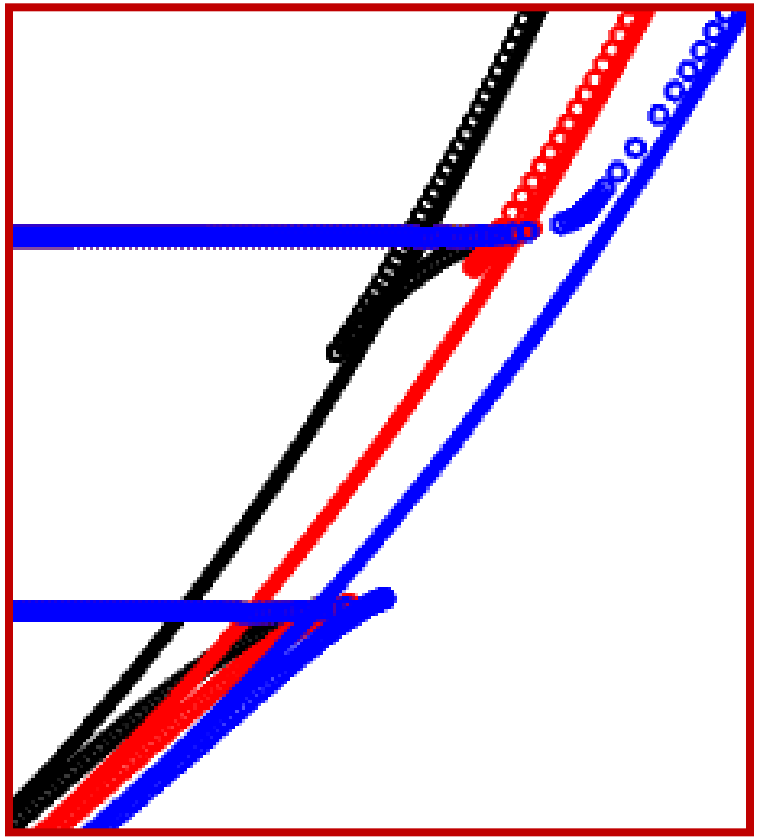
obvious differences between NJL and Bag:

- DχSB
- confinement
- **vector interaction**

$$B_\mu = m + \frac{4N_c}{9m_G^2} n_s(T, \mu^*, B),$$

$$\mu = \mu^* - \frac{2N_c}{9m_G^2} n_v(T, \mu^*, B),$$





vBag: vector interaction enhanced bag model

Chiral + Vector:

$$P^i_{BM}(\mu_i) = P_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) - P^i_{BAG}$$

$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) + P^i_{BAG}$$

$$\mu_i = \mu_i^* + K_v n_v(T, \mu_i^*)$$

‘Confinement’:

$$P = \sum_f P_f^{kin} - B_{eff} \text{ with } B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

And, of course, chiral+vector+’confinement’ (Klahn & Fischer arXiv:1503.07442 ApJ accepted)

Conclusions Part I

Vector enhanced bag like model can be derived from NJL - which can be obtained from DS gap equations

Bag model character: bare quark masses
effective bag pressure

Difference:
chiral bag pressure as consequence of DχSB, flavor dependence
confining bag pressure with opposite sign (binding energy)
accounts for vector interaction -> stiff EoS, promising for astrophysical applications

What NJL couldn't: bag pressure due to deconfinement -> subtracted by hand without harm to td consistence

Advantage of the model: extremely simple to use, no regularization required

$$P^i_{BM}(\mu_i) = P_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) - P^i_{BAG}$$

$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) + P^i_{BAG}$$

$$\mu_i = \mu_i^* + K_v n_v(T, \mu_i^*)$$

$$P = \sum_f P_f^{kin} - B_{eff} \text{ with } B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

Conclusions Part II

vBag: Bag-like model to reinvestigate … ‘everything’ … adding DχSB and vector interaction
application as simple as for the original bag model which omits these features

Neutron Stars

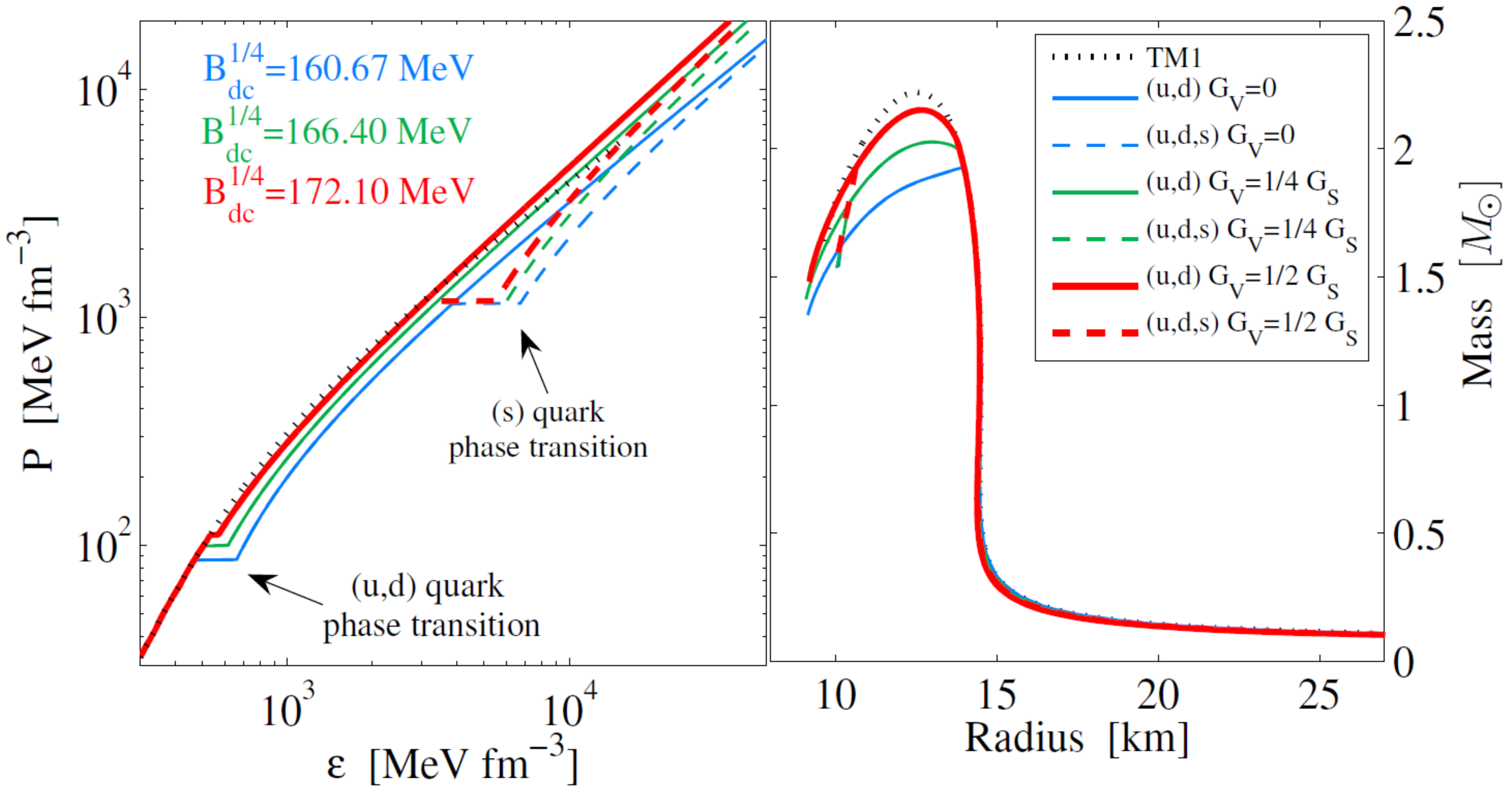
Mass Twin Solutions
Bayesian Analyses
Supernovae Simulations

Strange Matter

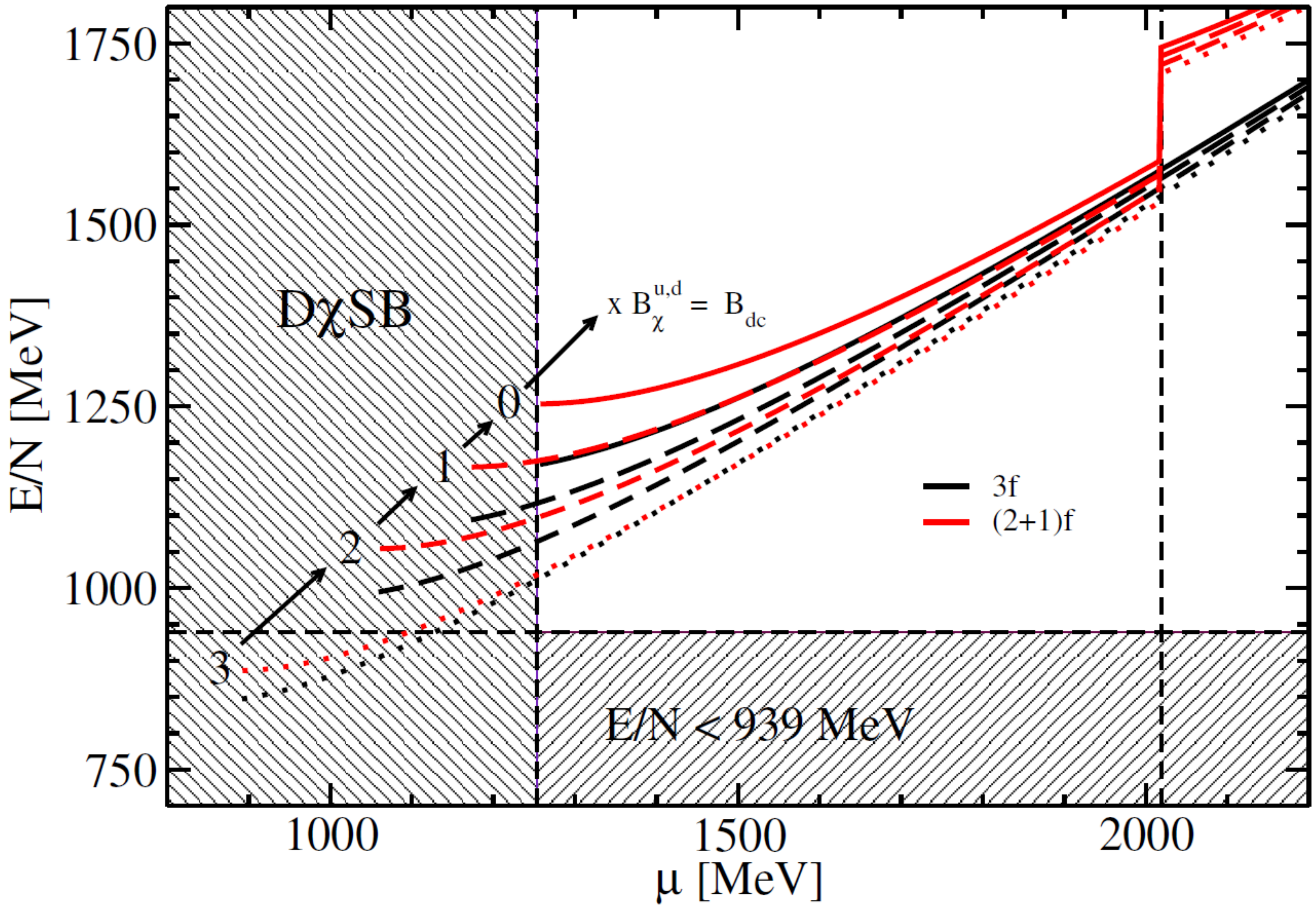
Iso-spin dependence
Heavy Ion Collisions
Critical Point

(work in progress)

Neutron Stars with QM core – vBAG vs BAG



Absolutely Stable Strange Matter?



Original BAG models prediction of absolutely stable strange quark matter for certain bag constants is an artifact of neglected dynamical chiral symmetry breaking (‘BAG quarks’ have bare quark mass)

Chodos et al. have been aware of this simplification.

NJL model and DS studies do not confirm ASSM hypothesis.

vBAG accounts for DχSB

Conclusions Part III

vBAG:

- vector interaction resolves the problem of too soft bag model EoS w/o perturbative corrections
- No problem at all to obtain stable hybrid neutron star configurations
- Standard BAG models bag constant is understood to mimic confinement, DχSB is absent
- vBAG introduces effective bag constant with similar values to original BAG

$$B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

- However, positive value due to chiral transition, deconfinement actually reduces B
- Absolutely stable strange matter likely ruled out due to DχSB

- NJL and Bag model result from particular approximations within Dyson-Schwinger approach
  rainbow approximation (quark-gluon vertex) + contact interaction (gluon propagator)
- Consequence: both models lack momentum dependent gap solutions

Effective gluon propagator

$$S(p;\mu)^{-1} = Z_2(i\,\vec{\gamma}\,\vec{p} + i\,\gamma_4(p_4 + i\mu) + m_{\mathrm{bm}}) + \Sigma(p;\mu)$$

$$\Sigma(p;\mu) = Z_1 \int_q^{\Lambda} g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma^a_\sigma(q,p;\mu)$$

Ansatz for self energy (rainbow approximation, effective gluon propagator(s))

$$Z_1 \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p) \to \int_q^{\Lambda} \mathcal{G}((p-q)^2)\, D^{\mathrm{free}}_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu$$

Specify behaviour o $\mathcal{G}(k^2)$

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D \delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2} \ln\left[\tau + \left(1 + k^2/\Lambda^2_{\mathrm{QCD}}\right)^2\right]} \mathcal{F}(k^2)$$

Infrared strength (zero width + finite width contribution) | running coupling for large k

EoS (finite densities):

1st term (Munczek/Nemirowsky (1983)) delta function in momentum space → Klähn et al. (2010)

2nd term → Chen et al.(2008,2011)

NJL model: $g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2}\delta_{\rho\sigma}$ delta function in configuration space = const. In mom. space

Munczek/Nemirowsky -> NJL‘s complement

Wigner Phase

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \mathcal{F}(k^2)$$

$B_W = 0, A_W = C_W$:

$$C_W(p,\mu) = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2\eta^2}{p_3^2 + (p_4 + i\mu)^2)}}\right)$$

Nambu Phase

$A_N = C_N$.

$\Re(\tilde{p}^2) < \frac{\eta^2}{4}$:

$$\begin{aligned} B_N(p,\mu) &= \sqrt{\eta^2 - 4(p_3^2 + (p_4 + i\mu)^2))} \\ C_N(p,\mu) &= 2 \end{aligned}$$

$\Re(\tilde{p}^2) > \frac{\eta^2}{4}$:

$$A_N = A_W, B_N = B_W, C_N = C_W.$$

MN antithetic to NJL
NJL:contact interaction in x
MN:contact interaction in p

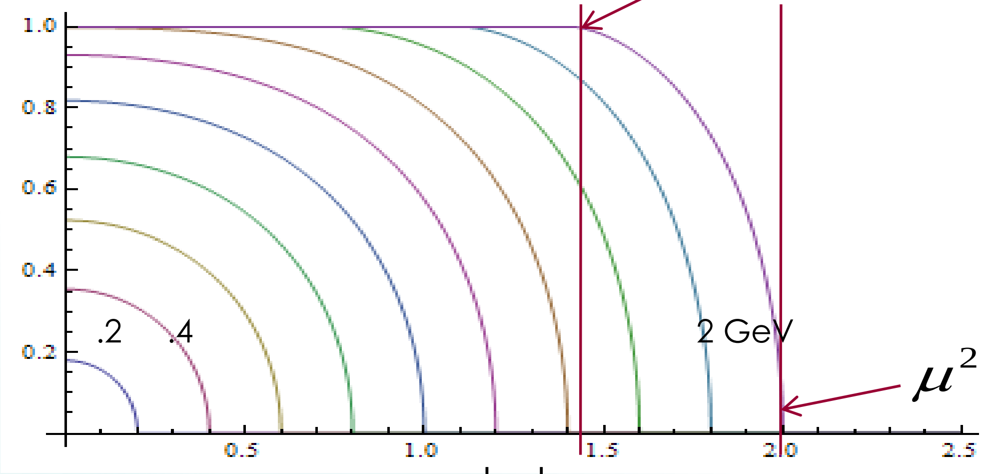
Munczek/Nemirowsky

$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr_D}(-\gamma_4)S(p;\mu)$$

$$P(\mu<\eta) = P_0 + \int_0^{\mu} d\mu'\, n(\mu') \propto P_0 + const \times \mu^5$$

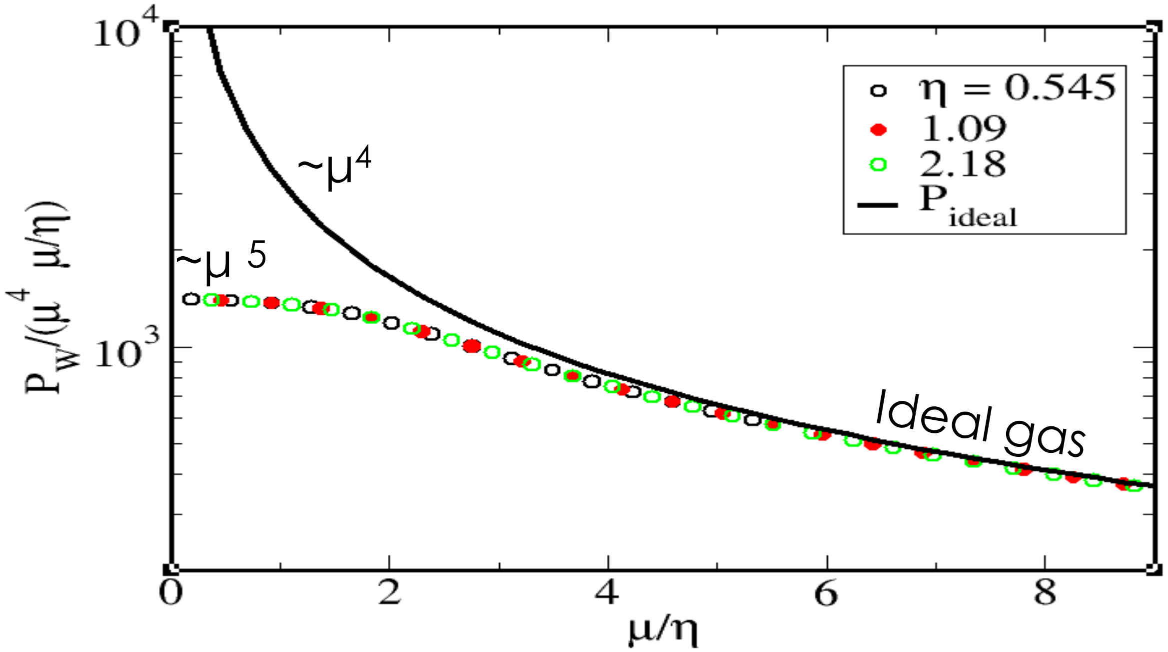
Wigner Phase

$\vec{p}^{\,2} = \mu^2 - 2\eta^2$



μ^2

$|\vec{p}|$



$\mu^2 \geq 2\eta^2$ to obtain $f_1(\vec{p}^{\,2}=0)=1$ model is scale invariant regarding μ/η

$P(\mu) \propto \mu^5$ well satisfied up to $\mu/\eta \approx 1$

$(\eta = 1.09\,\mathrm{GeV})$

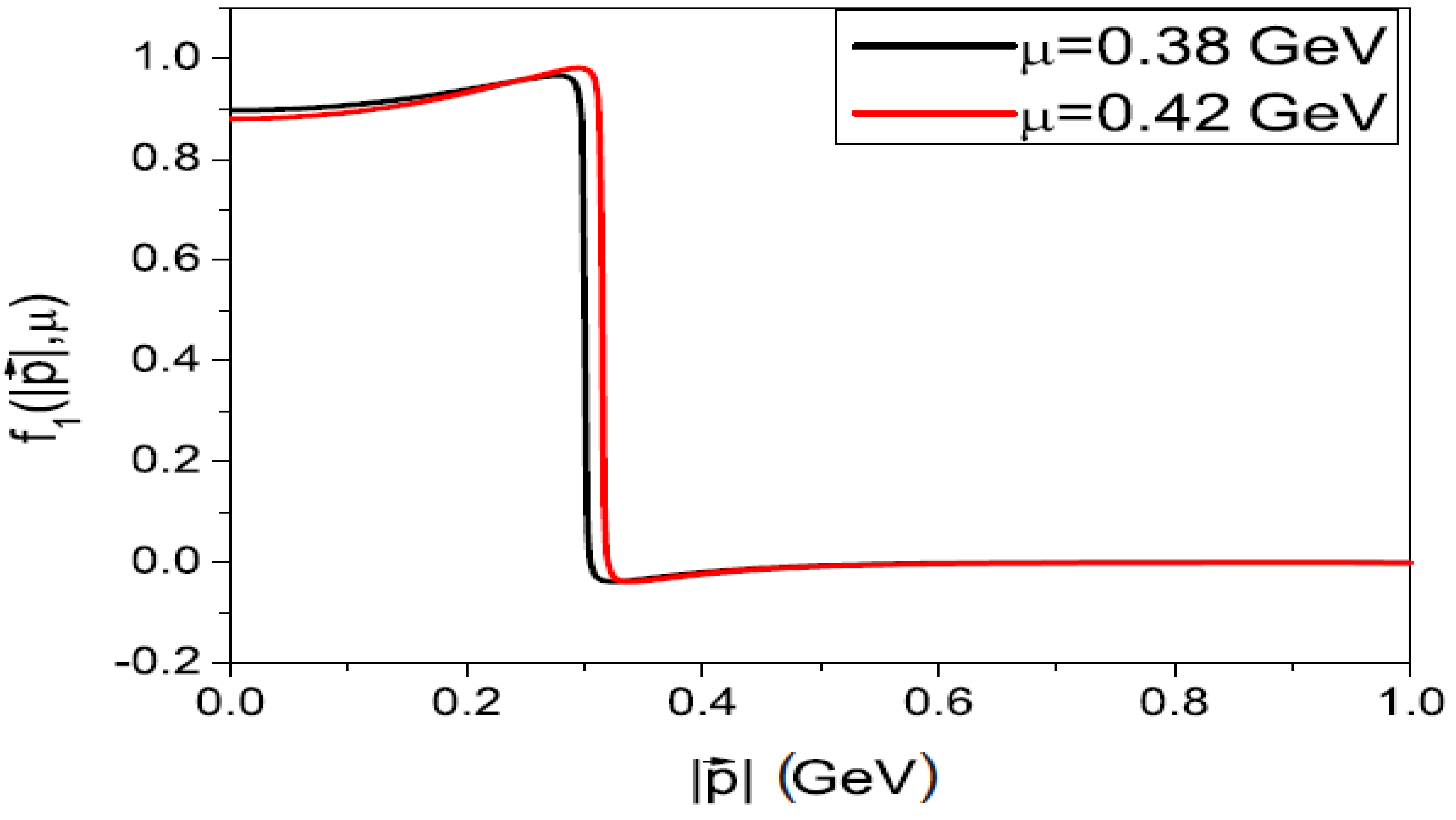
‚small‘ chem. Potential: $f_1(\vec{p}^{\,2}=0, \mu<\eta) \propto \mu \leftarrow$ $n(\mu<\eta) = \frac{2N_cN_f}{2\pi^2}\int d^3\vec{p}\;\, f_1(|\vec{p}|) \propto \mu^4$

T. Klahn, C.D. Roberts, L. Chang, H. Chen, Y.-X. Liu PRC 82, 035801 (2010)

DSE – simple effective gluon coupling

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \mathcal{F}(k^2)$$

<u>Wigner Phase</u> Less extreme, but again, 1particle number density distribution different from free Fermi gas distribution



Chen et al. (TK) PRD 78 (2008)

Conclusions

Thank you!

QCD in medium (near critical line):

- Task is difficult
- Not addressable by LQCD
- Not addressable by pQCD
- DSE are promising tool to tackle non-perturbative in-medium QCD
- Qualitatively very different results depending on effective gluon coupling
- Bag model mostly a simple limiting case of NJL model
- NJL model a simple contact interaction model in the gluon sector
- vBag connects them, other models exist