Exclusive production of pions and the pion distribution amplitude*

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- The pion distribution amplitude (DA) at leading twist:

$$\varphi_\pi(x) = \frac{N_c}{4\pi^2 f_\pi^2} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_0^{\infty} du \, \frac{F(u + i\lambda\bar{x}, u - i\lambda x)}{D(u - i\lambda x) D(u + i\lambda\bar{x})} \times \left[x \Sigma(u + i\lambda\bar{x}) + \bar{x}\Sigma(u - i\lambda x)\right]$$

$\Rightarrow$ u-variable plays the role of the quark transverse momentum squared

$\Rightarrow$ λx, -$\lambda\bar{x}$: longitudinal projections of the quark momentum on the light cone directions ($\bar{x} = (1 - x)$)

$$D(u) \equiv u + \Sigma^2(u)$$

$\Rightarrow$ $\Sigma(u)$ is the dynamical quark mass

⇒ the function F (the momentum dependent part of the quark-pion vertex) can be approximated by

$$F(p^2,p'^2) = \sqrt{\Sigma(p^2)\Sigma(p'^2)}$$

⇒ the pion decay constant:

$$f_\pi^2 = \frac{N_c}{4\pi^2}\int_0^\infty du\,\frac{u\Sigma(u)}{D^2(u)}\left(\Sigma(u) - \frac{1}{2}u\Sigma'(u)\right)$$

where $\Sigma'(u) = d\Sigma(u)/du$

- The pion DA is normalized as

$$\int_0^1 dx\,\varphi_\pi(x) = 1$$

- Its is know that

$$\Sigma(p^2) \approx \Phi^{P}_{BS}(p,q)|_{q\to 0}$$

$\Rightarrow$ consequence of the fact that they are related through the Ward-Takahashi identity

- The homogeneous BSE can be, in general, written as

$$\Phi(k,P) = -i\int_q^{\infty} \frac{d^4q}{(2\pi)^4} K(k;q,P)\, S(q_+)\, \Phi(q;P)\, S(q_-)$$

$\Rightarrow$ the amplitude depends on the quarks total (P) and relative (q) momenta

$\Rightarrow$ K is the fully amputated quark-antiquark kernel

$\Rightarrow$ $S(q_i)$ are the dressed quark propagators

The pion DA from the BSE

$\Rightarrow q_+ = q + \eta P$ and $q_- = q - (1-\eta)P$, where $0 \leq \eta \leq 1$

$\Rightarrow$ the homogeneous BSE is valid on-shell (i.e. $P^2 = 0$ in the pion case)

- In QCD the fermion masses are dynamically generated along with bound state Goldstone bosons (pions)
- The homogeneous BSE can be transformed into a second order differential equation

  $\Rightarrow$ two solutions can be found; the first one is characterized by a soft asymptotic behavior

$$\Phi_\pi(p^2) \sim \Sigma(p^2 \gg \mu^2) \sim \frac{\mu^3}{p^2}$$

$\Rightarrow$ this solution leads to the standard DA $\varphi_\pi^{as}(x) = 6x(1-x)$

$\Rightarrow$ the second one is characterized by an extreme hard high-energy asymptotic behavior of a bound state wave function:

$$\Phi_\pi(p^2) \sim \Sigma(p^2 \gg \mu^2) \sim \mu \left[1 + bg^2\left(\mu^2\right)\ln\left(p^2/\mu^2\right)\right]^{-\gamma} \qquad (1)$$

where $b = (11N_c - 2n_f)/48\pi^2$, $c = 4/3$ and $\gamma = 3c/16\pi^2 b$

$\Rightarrow$ this solution satisfies the Callan-Symanzik equation

$\Rightarrow$ it is constrained by the BSE normalization condition: $\gamma > 1/2$ ($n_f > 5$) $\Rightarrow$ otherwise it is not consistent with a possible bound state solution in a $SU(3)$ non-Abelian gauge theory

- (1) also appears when using an improved RG approach in QCD [L.-N.Chang,N.-P.Chang,PRL**54**(1985)2407]
- (1) minimizes the vacuum energy as long as $n_f > 5$ [J.C.Montero *et al.*,PLB**161**(1985)151]

In sum:

$\Rightarrow$ (1) is the hardest (in momentum space) asymptotic behavior allowed for a bound state solution in a non-Abelian gauge theory

$\Rightarrow$ no matter this solution is realized in Nature or not, it will lead to the flattest pion DA

$\Rightarrow$ nowadays it is known that the chiral phase diagram for a non-Abelian theory may change considerably as we change the number of flavors

$\Rightarrow$ if $n_f \geq 6$ QCD may have a chiral broken phase whose self-energy is given by (1)

So, if (1) is a possible solution, how it affects the pion DA?

- In order to compute the pion DA we need perform an integral over the wave function in the full range of momenta
- To obtain the extreme field theoretical limit on the pion DA we adopt

$$\Sigma(p^2) = \mu \left[1 + bg^2\left(\mu^2\right) \ln\left(\frac{p^2+\mu^2}{\mu^2}\right)\right]^{-\gamma}$$

$\Rightarrow$ it is a simple interpolating expression that reflects the full behavior of (1)

$\Rightarrow$ the μ factor into the logarithm numerator leads to the right infrared behavior of $\Sigma(p^2 \to 0) = \mu$

The pion DA from the BSE

⇒ the coupling constant g^2 is calculated at the chiral symmetry breaking scale μ, and given by

$$g^2(k^2) = \frac{1}{b\,\ln[(k^2 + 4m_g^2)/\Lambda_{QCD}^2]}$$

⇒ it is an infrared finite coupling determined in QCD where gluons have an effective mass m_g

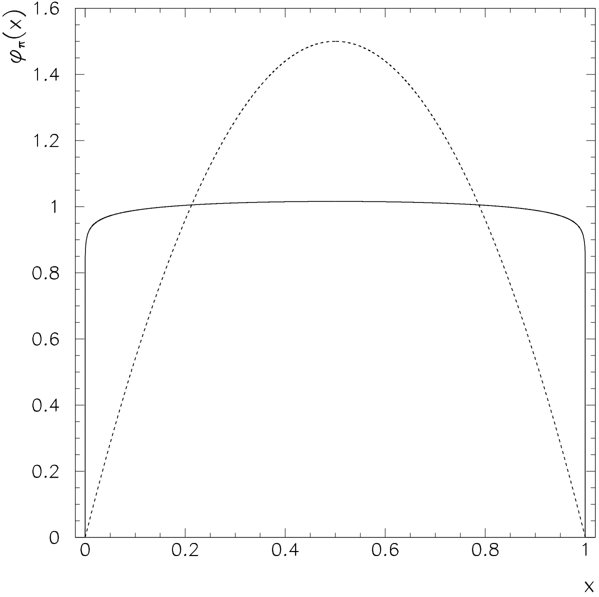
⇒ for the model calculations we take $\mu = 100$ MeV, $\Lambda_{QCD} = 300$ MeV and $m_g = 321.18$ MeV

⇒ the pion DA numerical result can be reproduced by using the normalized form

$$\varphi_\pi(x) = \frac{\Gamma\,(2+2\epsilon)}{\Gamma^2\,(1+\epsilon)}\,x^\epsilon(1-x)^\epsilon$$

where $\epsilon \approx 0.024802$

The pion DA from the BSE



The pion DA from the BSE

$\Rightarrow$ The pion DA turns out to be quite flat

$\Rightarrow$ we have not observed any significant variation as we change m_g and μ as long as we do not modify the f_π value

$\Rightarrow$ it should be noticed that the result is more dependent on the ratio m_g/Λ_{QCD} than the proper Λ_{QCD} value

$\Rightarrow$ the flat DA behavior is totally credited to the hard asymptotic self-energy behavior

$\Rightarrow$ the asymptotic behavior as $x \to 0$ is

$$\varphi_\pi(x \to 0) \sim \left(\ln \frac{1}{x}\right)^{-\gamma/2}$$

Pion transition form factor - LO

- At sufficiently high Q^2 it is expected that the standard factorization approach can be applied. Thus:

$$F_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2}\, f_\pi}{3} \int_0^1 dx\, \varphi_\pi(x) T_{\gamma\pi}^{H(LO)}(x, Q^2)$$

$\Rightarrow$ the hard scattering amplitude $T_{\gamma\pi}^{H(LO)}(x, Q^2)$ is

$$T_{\gamma\pi}^{H(LO)}(x, Q^2) = \frac{1}{xQ^2}$$

$\Rightarrow$ in this way:

$$F_{\gamma^*\gamma\pi}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx\, \frac{\varphi_\pi(x)}{xQ^2}$$

Pion transition form factor - LO

$\Rightarrow$ for a totally flat DA this integral should diverge...

... however, the finite size $R \approx 1/M$ of the pion provides a cut-off for the x integral [A.V.Radyushkin,PRD**80**(2009)094009]

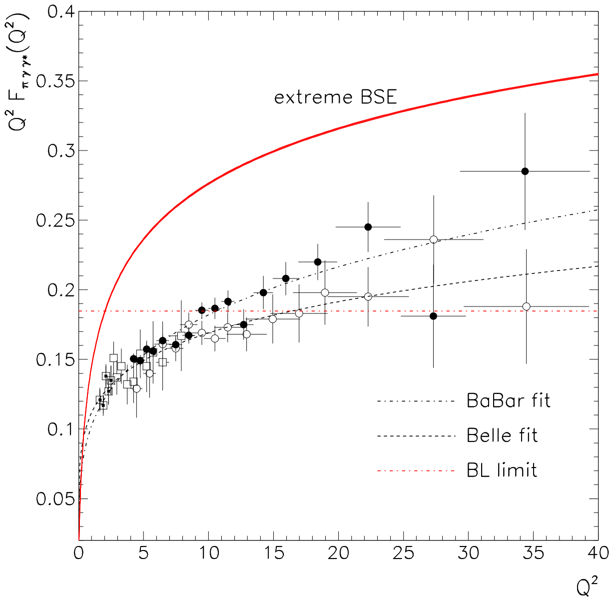
$\Rightarrow$ therefore the xQ^2 in the denominator will be changed as

$$xQ^2 \to xQ^2 + M^2$$

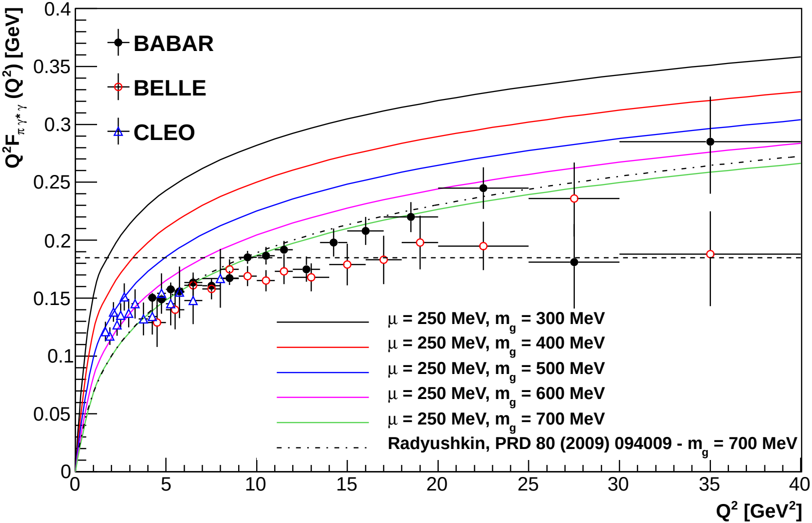
$\Rightarrow$ the parameter M in such modification is usually treated as the average transverse momentum of the propagating particle

$\Rightarrow$ it was proposed by Radyushkin that the factor M could be treated as an effective gluon mass

Pion transition form factor - LO



Pion transition form factor - LO



Pion transition form factor - NLO

- The one loop correction for the $\gamma^*\gamma \to \pi$ form factor is given by

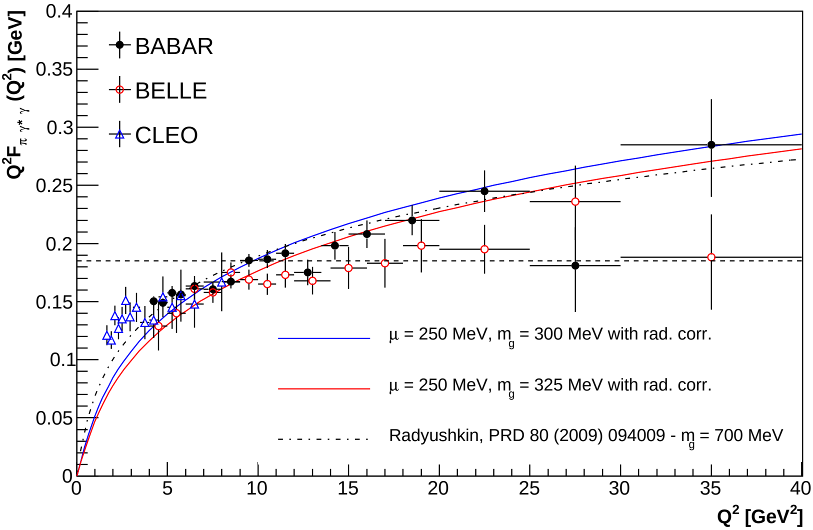
$$\int_0^1 dx\,\frac{\varphi_\pi(x)}{xQ^2} \to \int_0^1 dx\,\frac{\varphi_\pi(x,\mu)}{xQ^2}\left\{1+\frac{4}{3}\frac{\alpha_s}{2\pi}\left[\frac{1}{2}\left(\ln^2 x - 9 - \frac{x\ln x}{(1-x)}\right)+\left(\frac{3}{2}+\ln x\right)\ln\left(\frac{Q^2}{\mu^2}\right)\right]\right\}$$

$\Rightarrow$ if we take $\mu^2 = Q^2$:

$$T^{H(NLO)}_{\gamma\pi}(x,Q^2) = \frac{1}{xQ^2}\left(1+\frac{4}{3}\frac{\alpha_s}{2\pi}f(x)\right)$$

where

$$f(x) = \ln^2 x - \frac{x\ln x}{\bar{x}} - 9$$



BABAR
BELLE
CLEO
μ = 250 MeV, m_g = 300 MeV with rad. corr.
μ = 250 MeV, m_g = 325 MeV with rad. corr.
Radyushkin, PRD 80 (2009) 094009 - m_g = 700 MeV
Q²F_{π γ* γ} (Q²) [GeV]
Q² [GeV²]
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
5
10
15
20
25
30
35
40

- The pion form factor $F_\pi(Q^2)$ is also going to be changed if the pion DA is flat

  $\Rightarrow$ the QCD prediction for $F_\pi(Q^2)$ is also dependent on IR nonperturbative behavior of the gluon propagator

- The QCD expression for the pion factor is

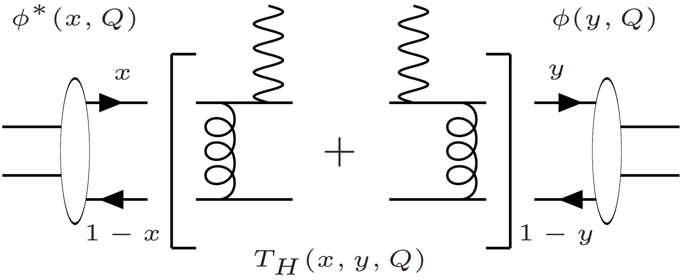
$$F_\pi(Q^2) = \frac{f_\pi^2}{12}\int_0^1 dx \int_0^1 dy\, \varphi^*(y,\tilde{Q}_y) T^H(x,y,Q^2)\varphi(x,\tilde{Q}_x)$$

where $\tilde{Q}_x = \min(x, 1-x)$

$\Rightarrow$ the function $\varphi(x,\tilde{Q}_x)$ is the momentum dependent pion DA

$\Rightarrow$ it gives the amplitude for finding the quark or antiquark within the pion carrying the fractional momentum x or $1-x$

$\Rightarrow$ $T^H(x, y, Q^2)$ is the hard scattering amplitude that is obtained by computing the following quark-photon scattering diagram:



The pion form factor

$\Rightarrow$ the lowest-order expression of $T^H(x,y,Q^2)$ is given by

$$T_H(x,y,Q^2) = \frac{64\pi}{3}\left[\frac{2}{3}\alpha_s(K^2)D(K^2) + \frac{1}{3}\alpha_s(P^2)D(P^2)\right]$$

where $K^2 = (1-x)(1-y)Q^2$ and $P^2 = xyQ^2$

$\Rightarrow$ $D(K^2)$ is related to the gluon propagator. In Landau gauge:

$$D_{\mu\nu}(q^2) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)D(q^2), \quad D(q^2) = \frac{1}{q^2}$$

$\Rightarrow$ we replace the perturbative $D(q^2) = \frac{1}{q^2}$ by

$$D(q^2) = \frac{1}{q^2 + M_g^2(q^2)}$$

where $M_g^2(q^2)$ is the dynamical gluon mass

The pion form factor

$\Rightarrow$ $M_g^2(q^2)$ is roughly given by

$$M_g^2(q^2) \approx \frac{m_g^4}{q^2 + m_g^2}$$

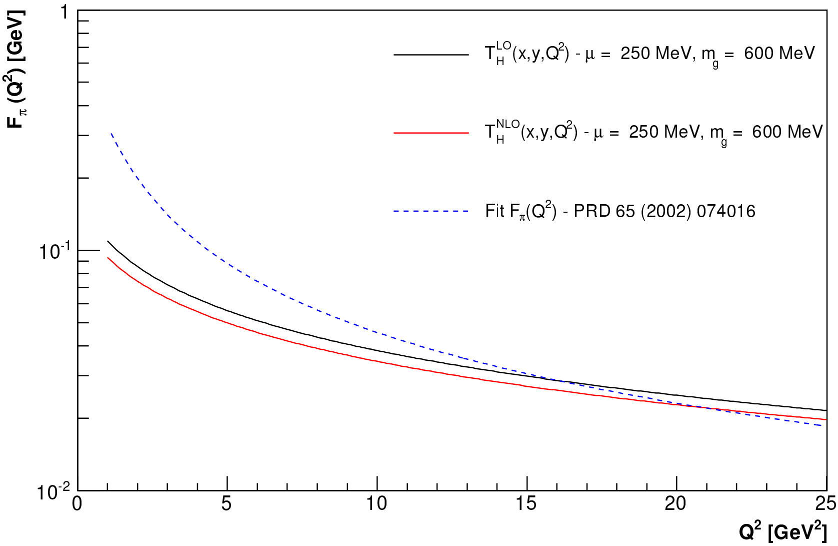
$\Rightarrow$ since the mass decays very fast with momentum we just assume

$$M_g^2(q^2) \approx m_g^2$$

$\Rightarrow$ the inclusion of radiative corrections imply that $T^H(x, y, Q^2)$ has to be multiplied by [F.del Aguila,M.K.Chase,NPB**193**(1981)517]

$$\left[1 - \frac{5}{6}\frac{\alpha_s(Q^2)}{\pi}\right]$$

Pion form factor



Hard exclusive two photon production

- In the area of exclusive processes, two-photon processes are of special interest since they can provide very clean tests of QCD
- Exclusive processes with hadronic final states test various model calculations motivated by perturbative and non-perturbative QCD
- Two-photon production of exclusive hadronic final states is particularly attractive due to the absence of strong interactions in the initial state
- We focus on photon-photon annihilation into two flavor-singlet helicity-zero mesons, $\gamma\gamma \rightarrow \pi^+\pi^-$

- The helicity amplitudes for a pion pair production in exclusive two photon collisions at high energies and large center of mass scattering angles θ_{CM} is given by

$$\mathcal{M}^{\lambda\lambda'} = \int_0^1 dx \int_0^1 dy\, \varphi^*(x,\tilde{Q}_x)\, \varphi^*(y,\tilde{Q}_y)\, T_H^{\lambda\lambda'}(x,y,Q^2) \qquad (2)$$

where $\tilde{Q}_x = \min(x, 1-x)\sqrt{s}\,|\sin\theta_{CM}|$ and where $s = W_{\gamma\gamma}^2$

- The spin-averaged cross section for producing the pion pair:

$$\frac{d\sigma}{dz} = \frac{1}{32\pi s}\langle|\mathcal{M}^{\lambda\lambda'}|^2\rangle$$

with $z = \cos\theta_{CM}$ and

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{4}|\mathcal{M}^{\lambda\lambda'}|^2$$

Hard exclusive two photon production

- The hard scattering amplitudes (at LO) for different helicity structures:

$$\left.\begin{array}{l} T_H^{(0)}(++) \\ T_H^{(0)}(--) \end{array}\right\} = \frac{16\pi\alpha_s}{3s} \frac{32\pi\alpha}{x(1-x)y(1-y)} \left[\frac{(e_1-e_2)^2 a}{1-z^2}\right]$$

$$\left.\begin{array}{l} T_H^{(0)}(+-) \\ T_H^{(0)}(-+) \end{array}\right\} = \frac{16\pi\alpha_s}{3s} \frac{32\pi\alpha}{x(1-x)y(1-y)} \left[\frac{(e_1-e_2)^2 a}{1-z^2} + \frac{e_1 e_2[x(1-x)+y(1-y)]}{a^2-b^2z^2} + \frac{(e_1^2-e_2^2)(x-y)}{2}\right]$$

where e_i are the quark charges and

$$\left.\begin{array}{l} a \\ b \end{array}\right\} = (1-x)(1-y) \pm xy$$

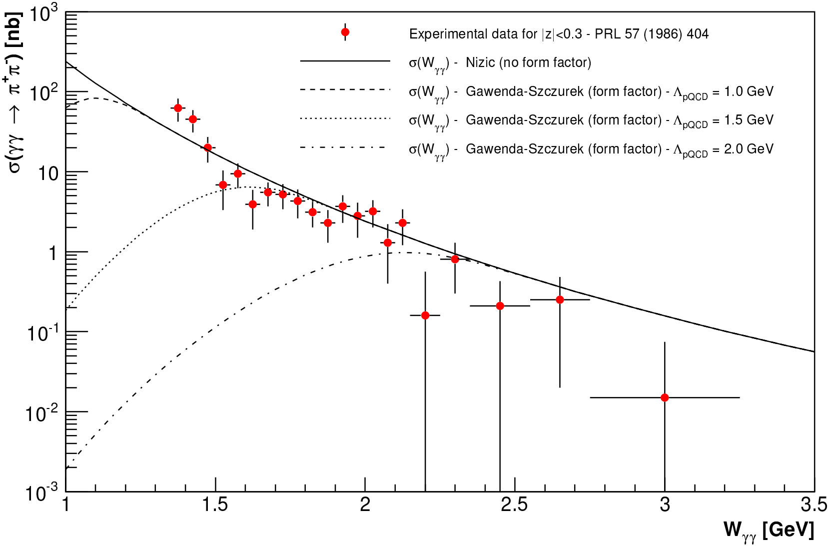
$\Rightarrow$ in order to restrain the calculation at the perturbative QCD level we multiply (2) by the factor

$$F^{pQCD}(s) = 1 - \exp\left(\frac{-(s-4m_\pi^2)^4}{\Lambda_{pQCD}^8}\right)$$

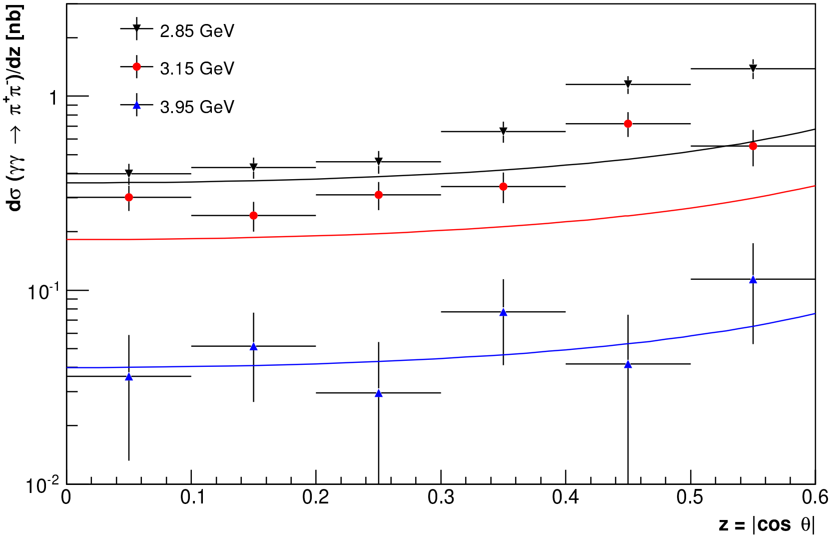
$\Rightarrow$ this factor smoothly switches off the pQCD contribution at low energies [M.K.-Gawenda,A.Szczurek,PRC**87**(2013)054908]

$\Rightarrow$ in this approach we can finally compute the total and differential cross sections for charged pion pair exclusive production:

Total cross section for charged pion pair exclusive production



Differential cross section for charged pion pair production



Conclusions

☐ The BaBar results suggested many authors to propose a (phenomenological) flat pion DA

$\Rightarrow$ we have observed that the pion DA can be related to the fundamental QCD Green's functions as a function of the quark self-energy and the quark-pion vertex...

$\Rightarrow$ ... which in turn are associated with the pion wave function through the Bethe-Salpeter equation

$\Rightarrow$ we provide a theoretical basis for the flat behavior

☐ In principle we did not may expect that the quark self-energy should follow the behavior (1)

$\Rightarrow$ thus the results that we could obtain with our "almost" flat DA would just give an extreme limit to the physical quantities that we have calculated

☐ However the description of the data is quite reasonable and seems to indicate that the pion wave function may be well approximated at large momentum by (1)

THANK YOU!