Nonperturbative correlation functions in R_ξ gauges through gluon self-energy effects

Fátima A. Machado

QCD TNT 4 – IlhaBela - SP, Aug./Sept. 2015

SCIENCE WITHOUT BORDERS

University of Pittsburgh

PITTPACC

Outline

Motivations and Goals
- Why, What for, and How

Gluon Propagator
- Renormalization schemes
- Transversality: 3-gluon vertex and background field gauge
- Results: in and beyond Landau gauge
- Transversality: dealing with longitudinal component

Ghost propagator
- Landau gauge and comparison to lattice
- Beyond Landau gauge

Concluding Remarks

Why

- Problems with perturbative QCD [Natale's talk]
  - Series is asymptotic but not convergent;
  - IR effects? Dokshitzer(hep-ph/9812252)
- Still a lot to understand on IR QCD, even for correlation functions
  - SDEs [Binosi, Aguilar; Huber.]
  - Lattice [Oliveira;Cucchieri,Mendes.]
  - Nonperturbative Lagrangians [Sorella,Palhares,Guimarães,Dudal;Tissier]
- Bridging UV and IR domains: ?
- From exploring the IR domain to an actual understanding of confinement, description of hadron phenomena: ?

What for

Develop a simple tool to:

- Bridge the IR and UV domains
- Explore the IR one

Simple enough to be used in phenomenology, either

- corrections to higher order pQCD predictions (sensitivity to lower scales)
- hadron models and processes

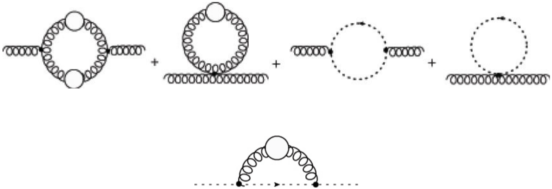
How

Incorporate nonperturbative features into loop expansions.

HOW

Incorporate nonperturbative features into loop expansions.
Start REALLY simple:

- Dress gluons
- Compute pure Yang-Mills correlation functions



Tissier,Wschebor,PRD84:045018(2011)

Effectively dressed gluon propagators:

$$\frac{1}{p^2+m^2} T_{\mu\nu(p)} + \frac{\xi}{p^2} L_{\mu\nu(p)} \ ,$$

$$T_{\mu\nu(p)} = \delta_{\mu\nu} - p_\mu p_\nu / p^2 \ , \ \ L_{\mu\nu(p)} = p_\mu p_\nu / p^2$$

Gluon propagator

$$\frac{1}{p^2+m^2} T_{\mu\nu(p)} + \frac{\xi}{p^2} L_{\mu\nu(p)} \implies$$

$$G^{-1}_{\mu\nu} = G^{-1}_{(0)\mu\nu} + \Pi_{\mu\nu} = \left(p^2 + m_0^2 + \Pi_T\right) T_{\mu\nu(p)} + \left(\frac{p^2}{\xi} + \Pi_L\right) L_{\mu\nu(p)} \,,$$

Gluon propagator

$$\frac{1}{p^2+m^2} T_{\mu\nu(p)} + \frac{\xi}{p^2} L_{\mu\nu(p)} \implies$$

$$G^{-1}_{\mu\nu} = G^{-1}_{(0)\mu\nu} + \Pi_{\mu\nu} = \left(p^2 + m_0^2 + \Pi_T\right) T_{\mu\nu(p)} + \left(\frac{p^2}{\xi} + \Pi_L\right) L_{\mu\nu(p)} \,,$$

$$\Pi_T(p,m) = p^2 \left[\bar{\alpha}\left(\frac{A}{\epsilon} + f(s)\right) + \delta_Z\right] + m^2 \left[\bar{\alpha}\frac{B}{\epsilon} + \delta_m\right] \,,$$

$\bar{\alpha} = N\alpha/48\pi.$

- Perturbative-like approach $\implies$ counterterms $\implies$ renormalization schemes (IR appropriate?)
- Comparison to lattice ($SU(3)$ Landau gauge)

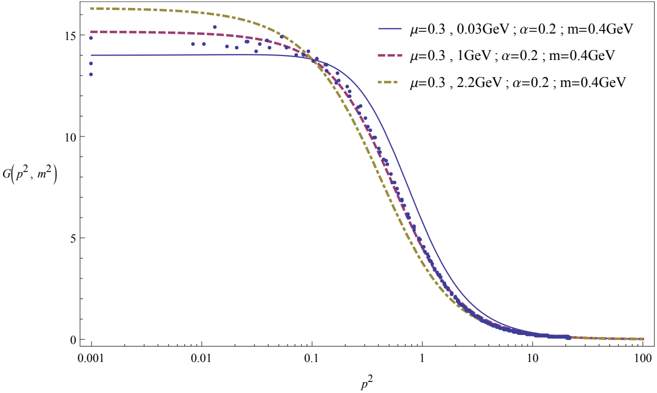
(Ilgenfritz, E.-M. et al. Braz. J. Phys. 2007, vol.37.1b)

Scheme: fixing $G(\mu_1)$ and $G(\mu_2)$

- UV and IR consistency; lattice data.

SCHEME: FIXING $G(\mu_1)$ AND $G(\mu_2)$

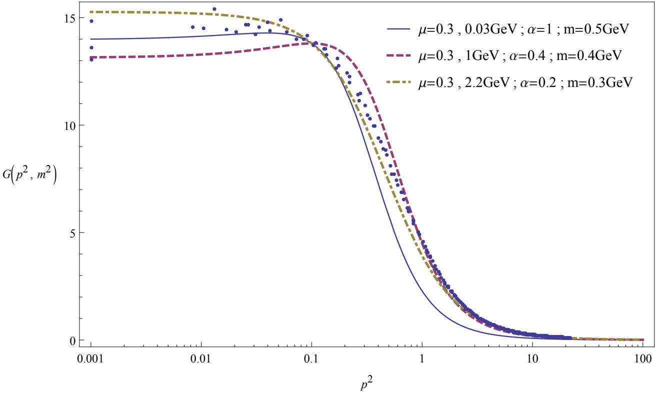
- UV and IR consistency; lattice data.



- $\therefore$ Can yield good fits. But: let's vary $\alpha(\mu)$ and $m(\mu)$...

SCHEME: FIXING $G(\mu_1)$ AND $G(\mu_2)$

- UV and IR consistency; lattice data.



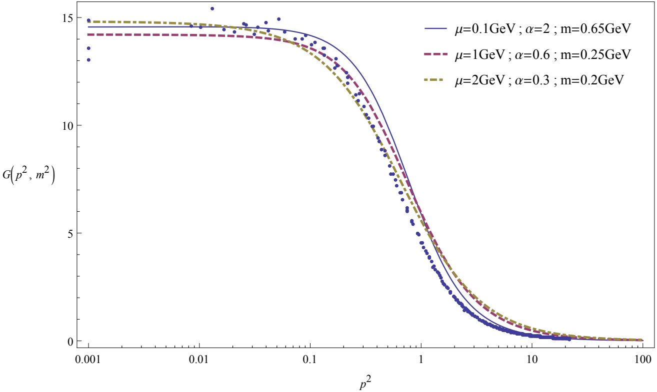
- Non-monotonic behavior, even for small α: another scheme?

Scheme: fixing $G(\mu)$ and $G'(\mu)$

- perturbative approach, RGEs.

Scheme: fixing $G(\mu)$ and $G'(\mu)$

- perturbative approach, RGEs.



$G(\mu) = \frac{1}{\mu^2+m^2(\mu^2)}$, $G'(\mu)$ adjusted accordingly; overall factor 6.3

- Good for G_T. But...

What about Π_L?

In Landau gauge:

$$\lim_{\xi \to 0} \frac{1}{\frac{p^2}{\xi} + \Pi_L} = 0 \,, \forall \Pi_L .$$

Still, $\Pi_L(p, m, \xi = 0) \neq 0$.

What about Π_L?

In Landau gauge:

$$\lim_{\xi \to 0} \frac{1}{\frac{p^2}{\xi} + \Pi_L} = 0 \,, \forall \Pi_L .$$

Still, $\Pi_L(p, m, \xi = 0) \neq 0$.

- What about other (R_ξ) gauges?
- How does dynamical mass generation go for the T and L components of the gluon?
- Can Π_L be identically $= 0$?
  (symmetries, spurious polarization states)

Pinch Technique and the three-gluon vertex

- Pinch algorithm $\longrightarrow$ effective Green functions satisfying WIs instead of STIs Binosi,Papavassiliou, Phys.Rept. 479 (2009)
- WI for the three-gluon vertex:

$$p^{\alpha}\mathbf{\Gamma}_{\alpha\mu\nu}(p,p_1,p_2)|_{p=-p_1-p_2} = \mathbf{\Delta}^{-1}_{\mu\nu}(p_1) - \mathbf{\Delta}^{-1}_{\mu\nu}(p_2)$$

$\Longrightarrow$ exactly transverse $\Pi_{\mu\nu}$ from SDEs

Pinch Technique and the three-gluon vertex

- Pinch algorithm $\longrightarrow$ effective Green functions satisfying WIs instead of STIs Binosi,Papavassiliou, Phys.Rept. 479 (2009)
- WI for the three-gluon vertex:

$$p^{\alpha}\mathbf{\Gamma}_{\alpha\mu\nu}(p,p_1,p_2)|_{p=-p_1-p_2} = \mathbf{\Delta}^{-1}_{\mu\nu}(p_1) - \mathbf{\Delta}^{-1}_{\mu\nu}(p_2)$$

$\Longrightarrow$ exactly transverse $\Pi_{\mu\nu}$ from SDEs

- Cornwall's $\mathbf{\Gamma} = \Gamma + V$: Cornwall, PRD80:096001(2009)

$$\begin{aligned} V_{\alpha\beta\gamma}(k_1,k_2,k_3) &= \left\{ \frac{k_{3\gamma}}{k_3^2}\left[\perp^{\mu}_{\alpha}(k_1)\Pi_{\mu\beta}(k_2) - \perp^{\mu}_{\beta}(k_2)\Pi_{\mu\alpha}(k_1)\right]\right. \\ &\quad \left. + \frac{k_{1\alpha}k_{2\beta}}{2k_1^2k_2^2}(k_1-k_2)^{\mu}\,\Pi_{\mu\gamma}(k_3)\right\} + \text{cyc. perm.} \\ &\equiv V\left(\Pi_{\mu\nu}\right) \end{aligned}$$

Pinch Technique and the Background Field Method

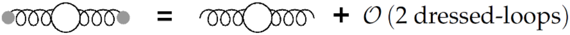
$$\mathbf{\Gamma} \to \mathbf{\Gamma}^{\text{PINCH}} \,,\ \text{with}\, V(\Pi_{\mu\nu})\,,\, \Pi_{\mu\nu} \leftrightarrow \mathbf{\Delta}_{\mu\nu}$$

$$\mathbf{\Delta}_{\mu\nu} \to \mathbf{\Delta}_{\mu\nu}^{\text{PINCH}} = ?$$

Pinch Technique and the Background Field Method

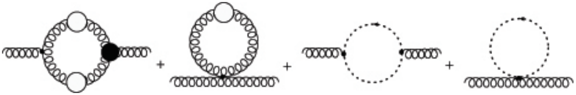
$$\mathbf{\Gamma} \to \mathbf{\Gamma}^{\text{PINCH}}\,,\ \text{with}\ V(\Pi_{\mu\nu})\,,\ \Pi_{\mu\nu} \leftrightarrow \mathbf{\Delta}_{\mu\nu}$$

$$\mathbf{\Delta}_{\mu\nu} \leftrightarrow \mathbf{\Delta}_{\mu\nu}^{\text{PINCH}} \equiv \mathbf{\Delta}_{\mu\nu}^{\text{BFG}}$$



Aguilar,B,P, JHEP0612:012,2006

∴ Implementable into Feynman diagrams:



$\mathbf{\Gamma} = \Gamma + \theta V(m^2 T_{\mu\nu})$, in both background field (BFG) and linear covariant (LCG) gauges.

Background field and linear covariant gauges

- In the BFG, $\Pi_L \equiv 0$

Background field and linear covariant gauges

- In the BFG, $\Pi_L \equiv 0$ for any ξ !

  Direct consequence of Ward ids. in the BFG.
- What about Π_T in BFG and LCG?

Background field and linear covariant gauges

- In the BFG, $\Pi_L \equiv 0$ for any ξ !

  Direct consequence of Ward ids. in the BFG.
- What about Π_T in BFG and LCG?

  Its dependence on θ is proportional to ξ or ξ^2

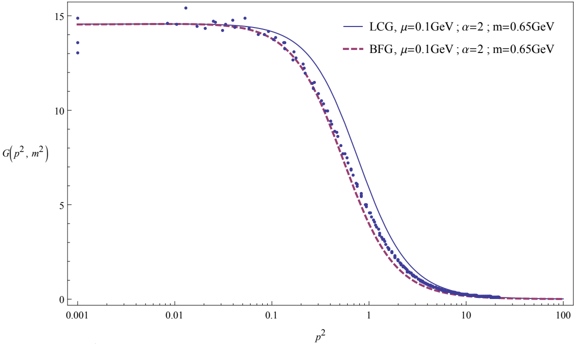
  $\therefore$ in Landau gauge, Π_T is independent of the dressing V,

  whether in BFG or CG.

Background field and linear covariant gauges

- In the BFG, $\Pi_L \equiv 0$ for any ξ !

  Direct consequence of Ward ids. in the BFG.
- What about Π_T in BFG and LCG?

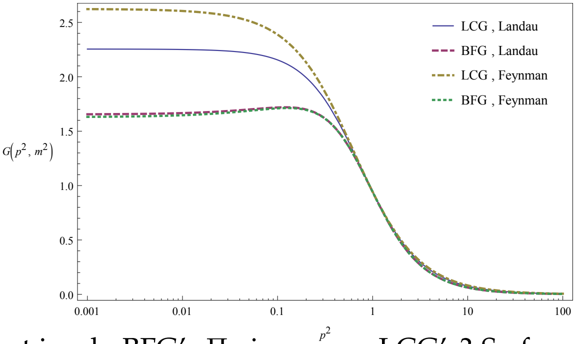


- Does that imply BFG's Π_T improves LCG's? No.

Background field and linear covariant gauges

- In the BFG, $\Pi_L \equiv 0$ for any ξ !

  Direct consequence of Ward ids. in the BFG.
- What about Π_T in BFG and LCG?



- Does that imply BFG's Π_T improves LCG's? So far, unlikely...

Beyond Landau gauge

While $\Pi_L \equiv 0$ for $\theta = 1$...

Beyond Landau gauge

While $\Pi_L \equiv 0$ for $\theta = 1$...

$\lim_{p^2\to 0} \Pi_T$:

- LCG: $\frac{3\bar{\alpha}}{2} m^2 \left(5(\xi+1) - 2\theta\xi^2\right)$: OK.

Beyond Landau gauge

While $\Pi_L \equiv 0$ for $\theta = 1$...

$\lim_{p^2 \to 0} \Pi_T$:

- LCG: $\frac{3\bar{\alpha}}{2} m^2 \left(5(\xi + 1) - 2\theta\xi^2\right)$: OK.
- BFG: $\frac{3\bar{\alpha}}{2} \left(-2\theta\xi(\xi + 2) + 2\theta\xi \log\left(\frac{p^2}{m^2}\right) + 5m^2(\xi - 1)\right)$: vanishing G_T

Beyond Landau gauge

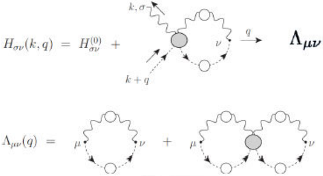
While $\Pi_L \equiv 0$ for $\theta = 1$...

$\lim_{p^2 \to 0} \Pi_T$:

- LCG: $\frac{3\bar{\alpha}}{2} m^2 \left(5(\xi+1) - 2\theta\xi^2\right)$: OK.
- BFG: $\frac{3\bar{\alpha}}{2} \left(-2\theta\xi(\xi+2) + 2\theta\xi \log\left(\frac{p^2}{m^2}\right) + 5m^2(\xi - 1)\right)$: vanishing G_T

Possible fixing: corrections from ghost-gluon kernel

$$\widehat{\Delta}(q) = [1 + \mathbf{G}(\mathbf{q})]^2 \Delta(q)$$



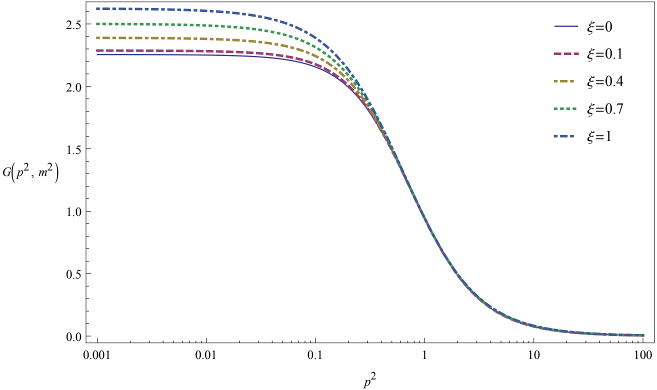
$$\Lambda_{\mu\nu}(q) = -ig^2 C_A \int_k H^{(0)}_{\mu\rho} D(k+q) \Delta^{\rho\sigma}(k)\, H_{\sigma\nu}(k,q)$$
$$= g_{\mu\nu} G(q) + \frac{q_\mu q_\nu}{q^2} L(q)$$

Beyond Landau gauge

Now for $\theta = 0$, LCG propagator: Aguilar,B,P, PRD91:085014(2015)

Beyond Landau gauge

Now for $\theta = 0$, LCG propagator: Aguilar,B,P, PRD91:085014(2015)



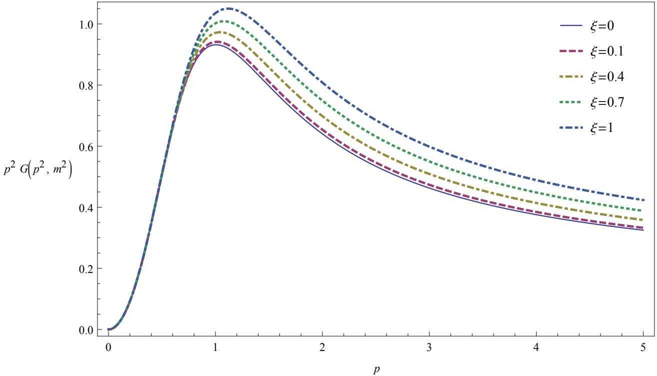
- Behavior of $m^2(0)$ under ξ: decreasing $m^2(0,\xi)$?
  $m^2(0,\xi) \equiv f(\mu^2/m^2, \text{scheme}, \xi)$
- Renormalization scheme $\leftrightarrow$ truncation scheme?

Beyond Landau gauge

Now for $\theta = 0$, LCG form factor: Huber, PRD91:085018(2015)

Beyond Landau gauge

Now for $\theta = 0$, LCG form factor: Huber, PRD91:085018(2015)



- Dependence on renormalization scheme and scale?
- Possibly better scheme on the way, involving Π_L...

Managing the longitudinal term

Acknowledge the nontrivial Π_L generation, and deal with it:

Managing the longitudinal term

Acknowledge the nontrivial Π_L generation, and deal with it:

$$\frac{1}{p^2 + m_T^2} T_{\mu\nu(p)} + \frac{\xi}{p^2 + m_L^2} L_{\mu\nu(p)}$$

$m_L^2 = \xi m_T^2$ recovers the local Lagrangian with $m_T^2 A^2$.

- Control, renormalize and analyze the longitudinal term, specially for R_ξ gauges.
- NJL-like approach (self-consistency condition)?

Managing the longitudinal term

Acknowledge the nontrivial Π_L generation, and deal with it:

$$\frac{1}{p^2 + m_T^2} T_{\mu\nu(p)} + \frac{\xi}{p^2 + m_L^2} L_{\mu\nu(p)}$$

$m_L^2 = \xi m_T^2$ recovers the local Lagrangian with $m_T^2 A^2$.

- Control, renormalize and analyze the longitudinal term, specially for R_ξ gauges.
- NJL-like approach (self-consistency condition)?

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{YM} + (1-\lambda)\,\mathcal{L}_{\mathrm{se}} \\ &= (\mathcal{L}_0 + \mathcal{L}_{\mathrm{se}}) + (\mathcal{L}_{\mathrm{i}} - \lambda \mathcal{L}_{\mathrm{se}}) + \mathcal{L}_{\mathrm{ct}} \end{aligned}$$

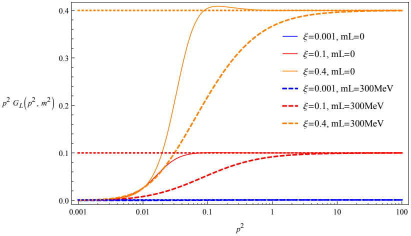
Ongoing...

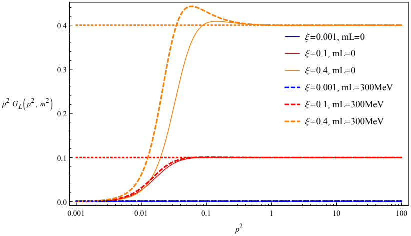
Longitudinal form factor

Compared to (massless) tree-level, $p^2 G_L(p^2, m_T^2, m_L^2, \mu^2)$, for $\lambda = 0$ (left) and $\lambda = 1$ (right):

Longitudinal form factor

Compared to (massless) tree-level, $p^2 G_L(p^2, m_T^2, m_L^2, \mu^2)$, for $\lambda = 0$ (left) and $\lambda = 1$ (right):





- Still nontrivial IR behavior... Price for simplicity?
- The farther from Landau, the greater the effects of m_L...
- Sensitivity to $m_L \to$ further tests: ghost.

Ghost propagator

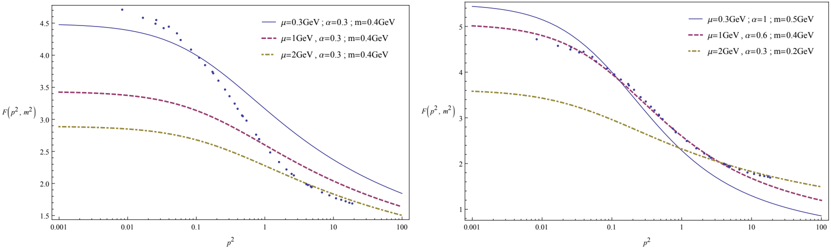
$$\tilde{G}(p^2, m^2) = \frac{F(p^2, m^2)}{p^2}$$

Landau gauge – contrast with lattice:

Ghost propagator

$$\tilde{G}(p^2, m^2) = \frac{F(p^2, m^2)}{p^2}$$

Landau gauge – contrast with lattice:



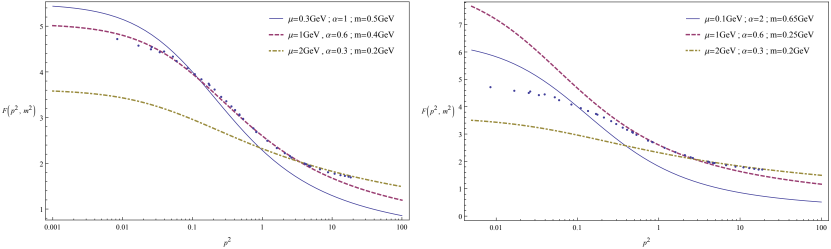
- Better for $\alpha(\mu), m(\mu)$: good for perturbative-like approach...

Ghost propagator

Landau gauge – contrast with ‘good-gluon’ parameters:

Ghost propagator

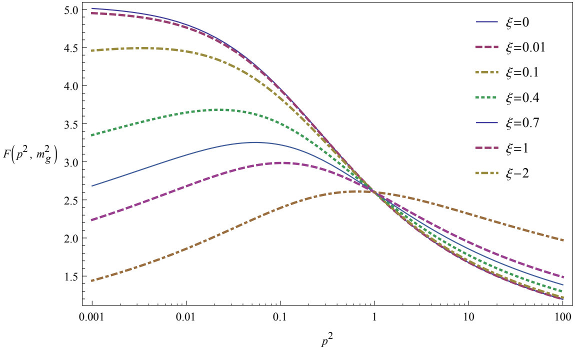
Landau gauge – contrast with 'good-gluon' parameters:



- Renormalization scheme and scale dependence:
  RG analysis!
- Consistent picture for all correlation functions considered.

Ghost dressing for $\xi \in [0, 2]$

GHOST DRESSING FOR $\xi \in [0, 2]$



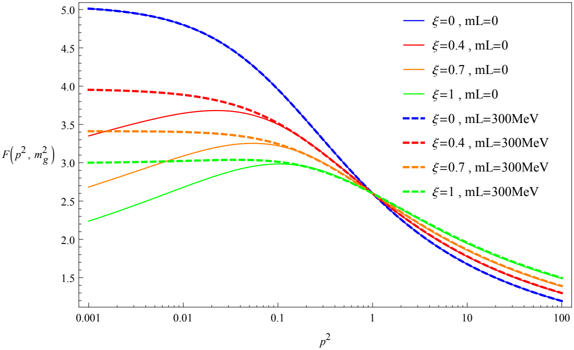
- Qualitative agreement with recent SDEs
- Asymptotic behavior: $\sim \xi \log(p^2/m^2)$ as $p^2 \to 0$

Ghost dressing for $\xi \in [0, 1]$

How does m_L affect it?

Ghost dressing for $\xi \in [0,1]$

How does m_L affect it?



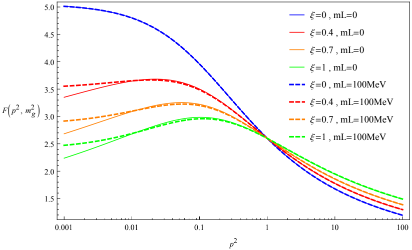
- Landau-like behavior: finite as $p^2 \to 0$.

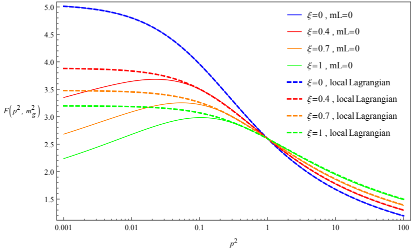
Ghost dressing for $\xi \in [0, 1]$

How sensitive to m_L?

Ghost dressing for $\xi \in [0, 1]$

How sensitive to m_L?





- Asymptotic behavior: finite as $p^2 \to 0$.
- Lattice and SDE calculations for $F(p^2, \xi)$: further corroborate purely transverse gluon mass generation. (still lacking for non-Pinch SDEs)

Concluding Remarks

- Renormalization scheme and scale dependence:
  RG analysis: requires tree-level masses – i.e., modeling.
  NJL-like approach?

- A simple dressed gluon expansion can be a tool to probe the IR domain:
  - vertices, kernels, quarks, pheno...
  - Analytically implementable approximation of SDEs
    $\times$
    Improved perturbation theory.
  - Consistent picture for all correlation functions
    $\times$
    Phenomenological applicability.

- Ways to explore IR QCD, yet there's a long way to go...

Concluding Remarks

- Renormalization scheme and scale dependence:
  RG analysis: requires tree-level masses – i.e., modeling.
  NJL-like approach?

- A simple dressed gluon expansion can be a tool to probe the IR domain:
  - vertices, kernels, quarks, pheno...
  - Analytically implementable approximation of SDEs
    $\times$
    Improved perturbation theory.
  - Consistent picture for all correlation functions
    $\times$
    Phenomenological applicability.

- Ways to explore IR QCD, yet there's a long way to go...

...so we all play with it... *Thank you!*