Introduction

There is interest in the study of heavy-quark systems since the decays of heavy-quarks can be used in the search of Beyond the Standard Model physics. A commonly used approximation to the spin-independent interquark potential for heavy quarks is the so-called Cornell (or Coulomb plus linear) potential, which interpolates the perturbative and non-perturbative regime by considering the potential a sum of two terms [1]. The first term is a Coulomb potential multiplied by a numerical factor and is obtainable through the one-gluon-exchange approximation (OGE), i.e. perturbation theory applied at first order only [2]. The second term is a linearly-rising potential and inspired by lattice simulations. It corresponds to the static quark potential from the Wilson loop at strong-coupling approximation. Recent results using lattice simulations and Bethe-Salpeter equation show agreement of the interquark potential with the Cornell potential in the infinitely heavy quark limit [3].

Lattice Gluon Propagator Potential

One can verify if we compute the scattering amplitude $T_2$ for a quark-antiquark collision in the One-Gluon-Exchange (OGE) approximation, i.e. OGE is just the first order in perturbation theory, and apply a nonrelativistic approximation, it is possible to obtain a Coulomb-like potential through a Fourier transform (assuming that is used a free gluon propagator). A linearly-rising term is usually added, motivated by Lattice QCD simulations. This term is hoped to account for confinement [2].

We expect that by using a gluon propagator obtained from lattice QCD, we can consider corrections not present in the free propagator.

Propagator used comes from a SU(2) pure gauge theory [4]. It differs from the gluon propagator just by a multiplicative factor

$$P_{	ext{gluon}}(k) = \frac{C(s + k^2)}{(s + k^2)^2} \left(\frac{k}{k^2 + \frac{C}{s}}\right)^{\text{d}b}.$$ 

The scattering amplitude becomes

$$T_2 = \frac{4}{3} \frac{Q^2}{16 \pi^2} C(s + k^2).$$

A Fourier transform can be reduced to the calculation of residues for each of the four poles of the propagator (one in each quadrant).

$${\text{V}}_{\text{LGP}}(r) = -(2\pi)^3 \int d^3 k \frac{\text{d}k}{2\pi^3} \left(\frac{C(s + k^2)}{s + k^2}\right)^{\text{d}b} = \frac{4}{3} \frac{Q^2}{16 \pi^2} \int d^3 k \frac{\text{d}k}{2\pi^3} \left(\frac{C(s + k^2)}{s + k^2}\right)^{\text{d}b}.$$ 

Fitting Procedure

- Quark masses and string tension are free parameters whose values can be set through a fit to the experimental states.
- The experimental spectrum of charmonium and bottomonium carries spin dependency, while the potential model in the Schrödinger does not. A comparison procedure needs to be established.
- The adopted procedure is, select the states with the same principal quantum number and consider as input for this energy level the midpoint between the lowest value and the highest value. We estimate the error as being half the distance between these values.
- We setup a grid of $m_b, m_c$ and $F_0$. For each set of these values, we evaluate $\chi^2$

$$\chi^2 = \sum \left(\frac{M_{\text{Computed}} - M_{\text{Experimental}}}{\Delta M_{\text{Experimental}}^2}\right)^2$$

Parameters Used

$\eta_c, \eta_b \leq 4.85 \text{GeV}$ in steps of 0.01 GeV.

$8.5 \text{GeV} < M_\rho < 12.5 \text{GeV}$ in steps of 0.04 GeV.

$1.00 \text{GeV} < m_c < 2.00 \text{GeV}$

$2.00 \text{GeV} < m_b < 6.00 \text{GeV}$ in steps of 0.04 GeV.

$0.1 \text{GeV} < F_0 < 0.5 \text{GeV}$ in steps of 0.01 GeV.

$\Lambda = 20.0 \text{GeV}^{-1}$

Results

<table>
<thead>
<tr>
<th>Experimental States</th>
<th>LGP Potential</th>
<th>Cornell Potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_b = 1.16(3) \text{GeV}$</td>
<td>$m_b = 1.11(4) \text{GeV}$</td>
<td></td>
</tr>
<tr>
<td>$m_c = 4.61(5) \text{GeV}$</td>
<td>$m_c = 4.58(6) \text{GeV}$</td>
<td></td>
</tr>
<tr>
<td>$F_0 = 0.23(1) \text{GeV}$</td>
<td>$F_0 = 0.26(1) \text{GeV}$</td>
<td></td>
</tr>
<tr>
<td>$\chi^2 = 6.19 \pm 0.15$</td>
<td>$\chi^2 = 12.13 \pm 0.93$</td>
<td></td>
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</tbody>
</table>

Conclusions

- We were able to obtain an spectrum that resembles the experimental.
- We observed that the LGP plus linear potential presents small, but measurable improvements over the Cornell potential.

Bibliography


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