Heavy-Quarkonium Potential from the Lattice Gluon Propagator

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Introduction

There is interest in the study of heavy-quark systems since the decays of heavy-quarks can be used in the search of Beyond the Standard Model physics. A commonly used approximation to the spin-independent interquark potential for heavy quarks is the so-called Cornell (or Coulomb plus linear) potential, which interpolates the perturbative regime and non-perturbative regime by considering the potential a sum of two terms [1]. The first term is a Coulomb potential multiplied by a numerical factor and is obtainable through the one-gluon-exchange approximation (OGE), i.e. perturbation theory applied at first order only [2]. The second term is a linearly-rising potential and inspired by lattice simulations. It corresponds to the static quark potential from the Wilson loop at strong-coupling approximation. Recent results using lattice simulations and Bethe-Salpeter equation show agreement of the interquark potential with the Cornell potential in the infinitely heavy quark limit[3].

Lattice Gluon Propagator Potential

- One can verify that if we compute the scattering amplitude T_{fi} for a quark-antiquark collision, in the One-Gluon-Exchange (OGE) approximation, i.e. consider just the first order in perturbation theory, and apply a nonrelativistic approximation, it is possible to obtain a Coulomb-like potential through a Fourier transform (assuming that is used a free gluon propagator). A linearly-rising term is usually added, motivated by Lattice QCD simulations. This term is hoped to account for confinement [2].
- We expect that by using a gluon propagator obtained from lattice QCD, we can consider corrections not present in the free propagator.
- Propagator used comes from a SU(2) pure gauge theory [4]. It should differ from the gluon propagator just by a multiplicative factor

$$P^{ab}_{\mu\nu}(k) = \frac{C\,(s+k^2)}{t^2+u^2k^2+k^4}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab},$$
$$C = 0.784, \qquad s = 2.508\,\mathrm{GeV}^2,$$
$$u = 0.768\,\mathrm{GeV}, \quad t = 0.720\,\mathrm{GeV}^2.$$

- Due to the multiplicative factor already quoted, we have the freedom to set $C = 1$, so in the limit $k \to \infty$ we recover the free propagator.
- In the nonrelativistic approximation, $k_0 = 0$, meaning that the term $k_\mu k_\nu$ vanishes for P^{ab}_{00}. The scattering amplitude becomes

$$T_{fi} = \frac{4}{3}\frac{g_0^2}{(2\pi)^6}\frac{C\,(s+\boldsymbol{k}^2)}{t^2+u^2\boldsymbol{k}^2+\boldsymbol{k}^4}.$$

- A Fourier transform can be reduced to the calculation of residues for each one of the four poles of the propagator (one in each quadrant).

$$V_{\mathrm{LGP}}(r) = -(2\pi)^3\int T_{fi}(\boldsymbol{k})e^{(-i\boldsymbol{k}\cdot\boldsymbol{r})}d^3k = -\frac{4}{3}\frac{2\alpha_s}{r}\Re\left[\frac{C(s+k_{0,0}^2)e^{ik_{0,0}r}}{u^2+2k_{0,0}^2}\right],$$
$$k_{0,0} \equiv i\sqrt{t}\exp\left[i\frac{\phi}{2}\right], \quad \phi \equiv \arctan\left(\frac{\sqrt{4t^2-u^4}}{u^2}\right), \quad \alpha_s = \frac{g_0^2}{4\pi}.$$

Finding Eigenenergies

- In the nonrelativistic limit, finding the eigenergy means solving the Schrödinger equation.
- Since we are dealing with central potentials, we can use separation of variables to reduce the equation to

$$\frac{d^2}{dr^2}f(r) = -2\mu\left[E - V(r) - \frac{l(l+1)}{2\mu r^2}\right]f(r)$$
$$\psi(\boldsymbol{r}) = Y_l^m(\theta,\varphi)\frac{f(r)}{r}, \quad \mu = \frac{m}{2}$$

- Contour conditions: $f(0) = 0$ and $f(\infty) = 0$.
- Algorithm:

1. Guess a value for E
2. Integrate the wave function starting from $r = r_{\mathrm{MAX}}$ until $r = 0$ and record the sign of $f(0)$
3. Increase the value of E by dE and repeat step 2
4. Compare the sign of $f(0)$ found in step 3 with the one found in step 2

4.1 If the signs do not change, we check if $E + dE$ is smaller than a chosen E_{MAX} and in case so, we proceed. Otherwise we exit the eigenenergy search.
4.2 If the signs are different, then there is an eigenergy between E and $E + dE$. We repeat step 2 for an energy in the midpoint of the two pevious energy
4.3 If the sign of $f(0)$ at the midpoint is equal to its sign at the lower bound of the interval, we raise the lower bound to the middle point. Otherwise we lower the upper bound to the middle point.
4.4 If the steps 2 and 3 have been iterated less than $n = 10$ times, we repeat them. Otherwise we go to 3.

Fitting Procedure

- Quark masses and string tension are free parameters whose values can be set through a fit to the experimental states.
- The experimental spectrum of charmonium and bottomonium carries spin dependency, while the potential model in the Schrödinger does not. A comparison procedure needs to be established.
- The adopted procedure is, select the states with the same principal quantum number and consider as input for this energy level the midpoint between the lowest value and the highest value. We estimate the error as being half the distance between these values.
- We setup a grid of m_b, m_c and F_0. For each set of these values, we evaluate χ^2

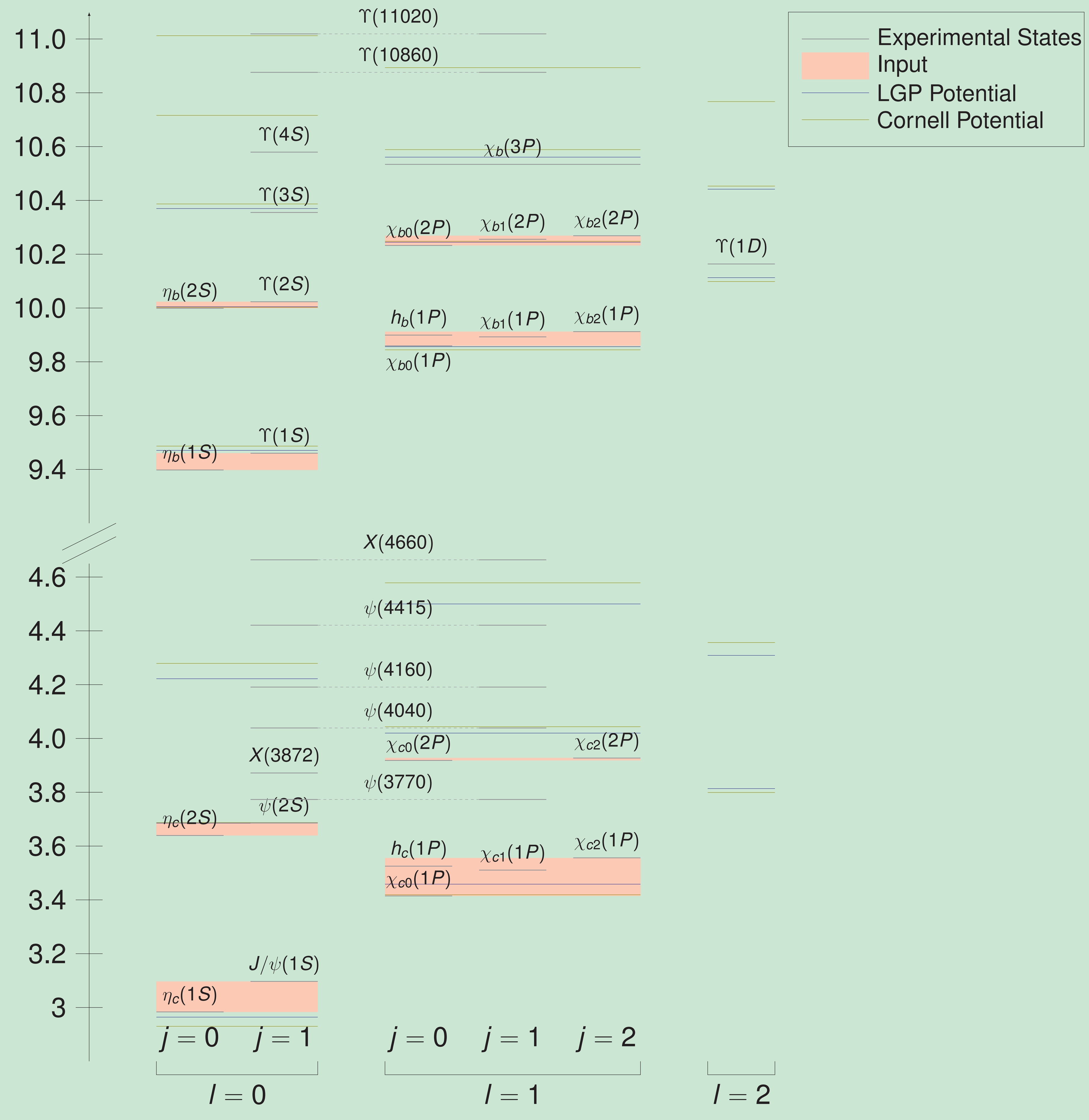
$$\chi^2 = \sum\frac{\left(M_{\mathrm{Computed}} - M_{\mathrm{Experimental}}\right)^2}{\left(\Delta M_{\mathrm{Experimental}}\right)^2}$$

Parameters Used

- $4.15\,\mathrm{GeV} < m_b < 4.85\,\mathrm{GeV}$ in steps of $0.01\,\mathrm{GeV}$
- $8.5\,\mathrm{GeV} < M_{b\bar{b}} < 12.5\,\mathrm{GeV}$ in steps of $0.04\,\mathrm{GeV}$
- $1.00\,\mathrm{GeV} < m_c < 2.00\,\mathrm{GeV}$
- $2.0\,\mathrm{GeV} < M_{c\bar{c}} < 6.0\,\mathrm{GeV}$ in steps of $0.04\,\mathrm{GeV}$
- $0.1\,\mathrm{GeV}^2 < F_0 < 0.5\,\mathrm{GeV}^2$ in steps of $0.01\,\mathrm{GeV}^2$
- $r_{\mathrm{MAX}} = 20.0\,\mathrm{GeV}^{-1}$

Results

LGP Potential	Cornell Potential
$m_c = 1.16(3)\,\mathrm{GeV}$	$m_c = 1.11^{+0.08}_{-0.02}\,\mathrm{GeV}$
$m_b = 4.61^{+0.02}_{-0.01}\,\mathrm{GeV}$	$m_b = 4.58^{+0.04}_{-0.01}\,\mathrm{GeV}$
$F_0 = 0.23(1)\,\mathrm{GeV}$	$F_0 = 0.26^{+0.01}_{-0.03}\,\mathrm{GeV}$
$\chi^2 = 6.196\,695$	$\chi^2 = 12.131\,939$



Conclusions

- We were able to obtain an spectrum that resembles the experimental
- We observed that the LGP plus linear potential presents small, but measurable improvements over the Cornell potential.
- The method makes possible access to wavefunction and consequently to average radius of states (results in process of being published)

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