Chiral symmetry breaking in continuum QCD

(in the “quenched” limit)

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Ilhabela, September 4, 2015

Der Wissenschaftsfonds.

GEFÖRDERT VOM

Bundesministerium
für Bildung
und Forschung

talk based on:
MM, J. Pawlowski, N. Strodthoff, Phys.Rev. D91 (2015) 054035

fQCD collaboration:
J. Braun, A. K. Cyrol, L. Fister, W. J. Fu, T. K. Herbst, MM
N. Müller, J. M. Pawlowski, S. Rechenberger, F. Rennecke, N. Strodthoff

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Functional approaches to QCD at $T \neq 0$, $\mu = 0$

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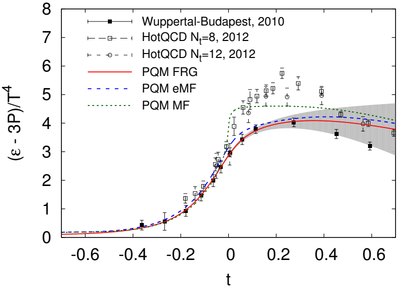
  - interaction measure
  - $2+1$ flavor Polyakov loop extended quark-meson model
  - functional renormalization group

[Herbst, MM, Pawlowski, Schaefer, Stiele, 2013]



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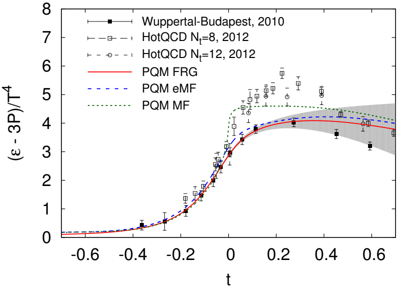

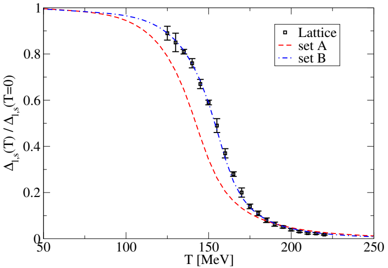

  - chiral condensate
  - $2+1$ flavor quark propagator Dyson-Schwinger equation

[Luecker, Fischer, Welzbacher, 2014]

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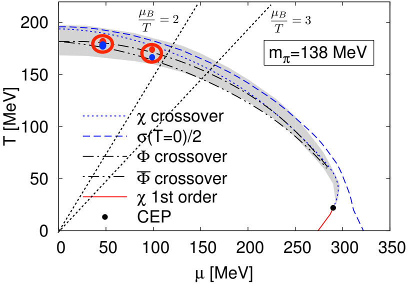
Functional appr. to QCD phase diagram



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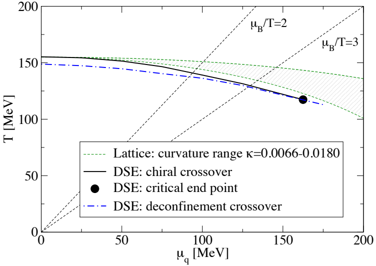

Functional appr. to QCD phase diagram II

- work well at $\mu = 0$: agreement with lattice
- disagreement at large μ (possibly already at small μ)
- shown results used model input:
  - quark-meson model:
    - inital values at $\Lambda \approx \mathcal{O}(\Lambda_{\text{QCD}})$
    - deconfinement dynamics via Polyakov loop potential
  - quark propagator DSE:
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possible explanation:

- $\mu \neq 0$: relative importance of diagrams changes
  $\Rightarrow$ summed contributions vs. individual contributions

Back to QCD in the vacuum

- use only perturbative QCD input
  - $\alpha_S(\Lambda = \mathcal{O}(10)\ \text{GeV})$
  - $m_q(\Lambda = \mathcal{O}(10)\ \text{GeV})$

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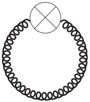
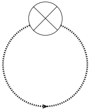
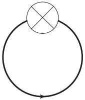
$\partial_k \Gamma_k = \frac{1}{2}$  $-$  $-$ 

$\Rightarrow$ effective action $\Gamma[\Phi] = \lim\limits_{k \to 0} \Gamma_k[\Phi]$

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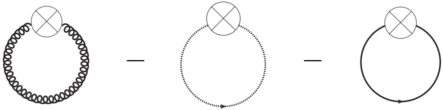

$\Rightarrow$ effective action $\Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$

- derivatives $\Rightarrow$ equations for 1PI n-point functions
- similar to DSE - but different resummation scheme: importance of diagrams can be different

Effective action

[MM, Strodthoff, Pawlowski, 2014]

- $\lim_{k\to 0} \Gamma_k[\phi; \alpha(\Lambda), m_q(\Lambda)]$ depends on approximation

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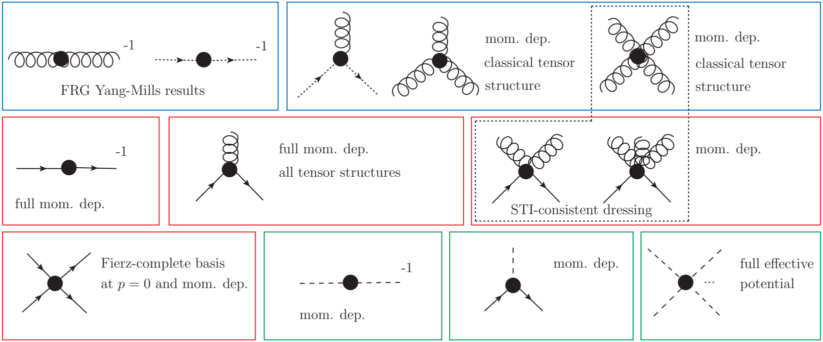
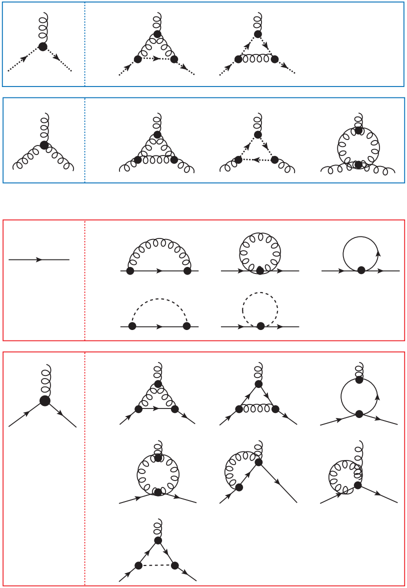
- where we are:

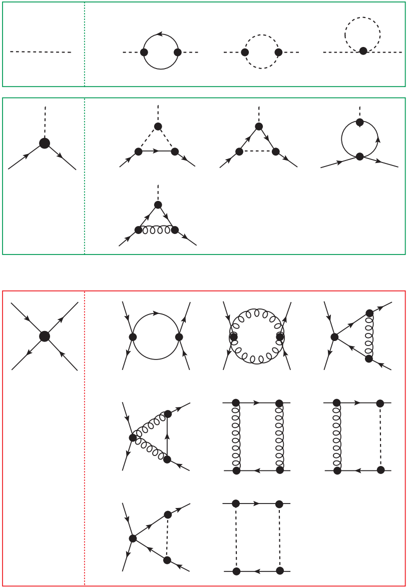


- Landau gauge: transversally projected n-point functions

Contributions

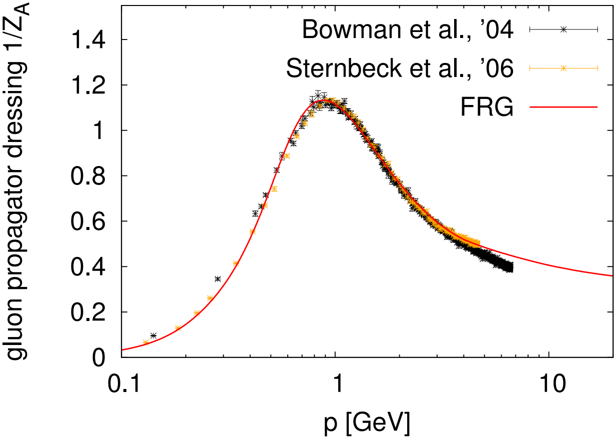
[MM, Strodthoff, Pawlowski, 2014]





Quenched gluon

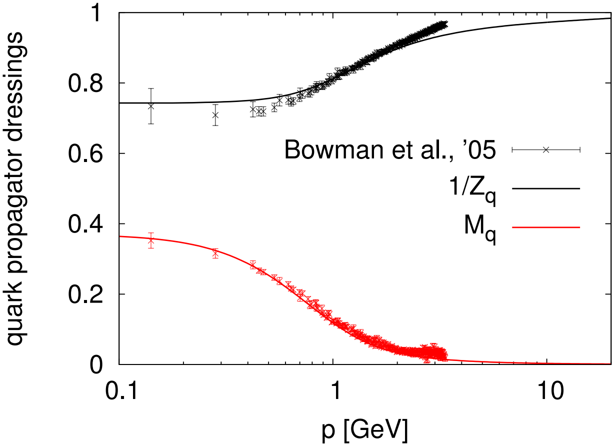
[Fischer, Maas, Pawlowski, 2009], [Cyrol, Fister, Pawlowski, in prep.]



- FRG result ⇒ self-consistent calculation within FRG approach
- sets the scale in comparison to lattice QCD

Quark propagator

[MM, Pawlowski, Strodthoff, 2014]



- FRG bare mass vs. lattice bare mass
- FRG-quenched vs. lattice quenched
- FRG scale vs. lattice scale

Quark-gluon interactions I

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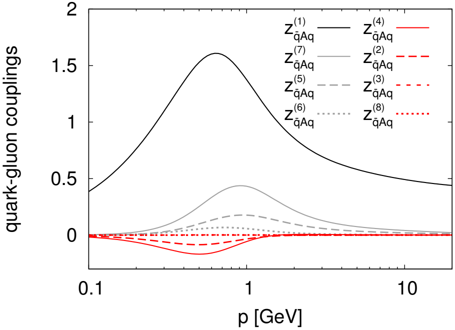

- vertex strength: reflects gluon gap
- 8 tensors (transversally projected):
  - classical tensor
  - chirally symmetric
  - break chiral symmetry

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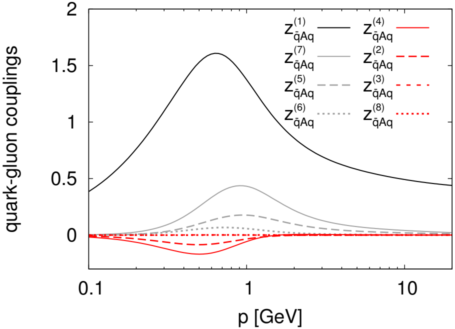

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- important non-classical tensors: c.f., [Hopfer et al., 2012], [Williams, 2014], [Aguilar et al., 2014]
  - $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$ $(\frac{1}{2}\mathcal{T}^{(5)}_{\bar{q}Aq} + \mathcal{T}^{(7)}_{\bar{q}Aq})$: increases Z_q/decreases M_q considerably
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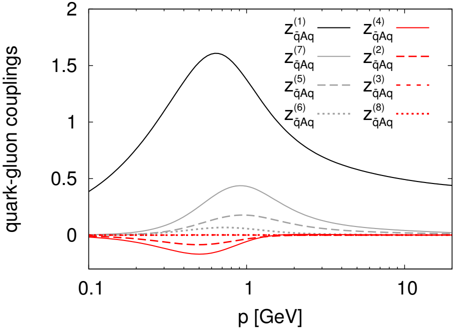

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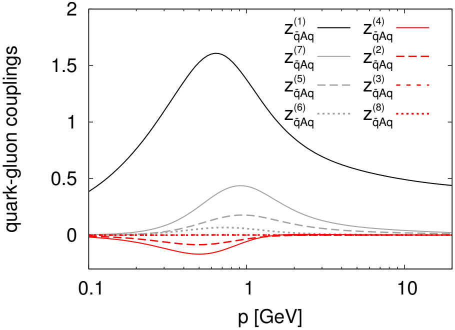

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- also important ingredient for bound-state equations

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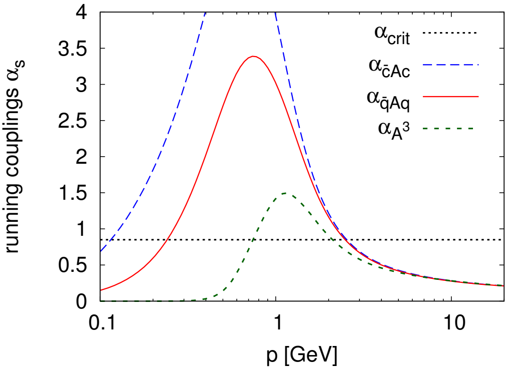
- explicit calculations of $AA\bar{q}q$-vertex: [MM, Pawlowski, Strodthoff, in prep.]
  - full basis: 63 chirally symmetric tensor elements
  - 15 chirally symmetric tensor elements ($\bar{\psi}\not{D}^3\psi$):
    - ⋆ all seem important
    - ⋆ order of effect similar to $\bar{q}\gamma_5\gamma_\mu\epsilon_{\mu\nu\rho\sigma}\{F_{\nu\rho}, D_\sigma\}q$
    - ⋆ why? underlying principle?

Effective running couplings

[MM, Pawlowski, Strodthoff, 2014]



- agreement in perturbative regime required by symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking

4-Fermi interaction

- chiral symmetry breaking $\Leftrightarrow$ resonance in 4-Fermi interaction

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- without momentum dependencies: [Braun,2011]
  resonance in one (pion) channel ⇒ singularities in all channels:
  - 4 symmetric channels: $(\text{S-P})_+$, V, AV, $(\text{V-A})^{\text{adj}}$
  - 2 $SU(N_f)_A$-breaking channels
  - 2 $U(1)_A$-breaking channels: $(\text{S+P})_-^{(\text{adj})}$ ('t Hooft determinant(s))
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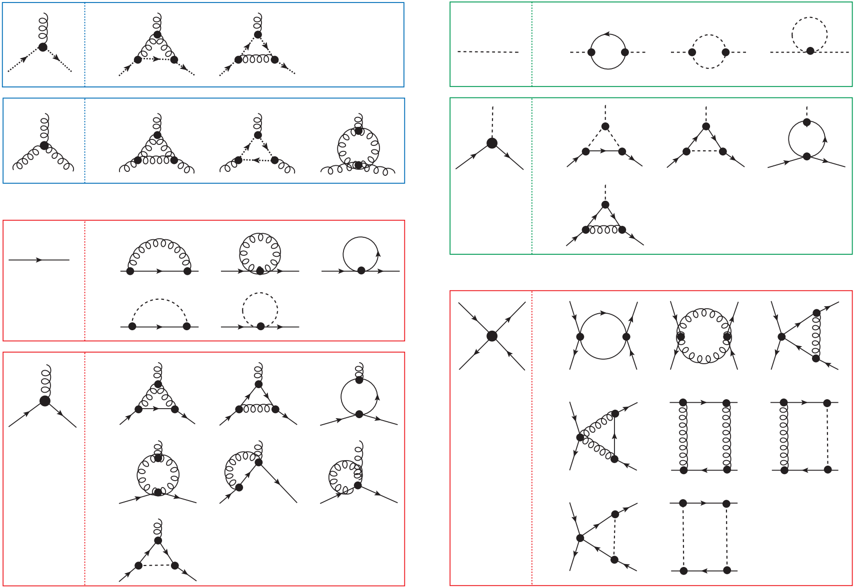
  - 2 $U(N_f)_A$-breaking channels
- what should happen with momentum dependence: [Braun,2011]
  - non-perturbative momenta: α_s grows
  - large contribution to four-fermi vertex due to two-gluon exchange
  - resonance four-fermi vertex due to quark-loop
  - infinitesimally small quark-mass grows through tadpole
  - system is stabilized by quark-gap
  - connection to DχSB in DSEs?

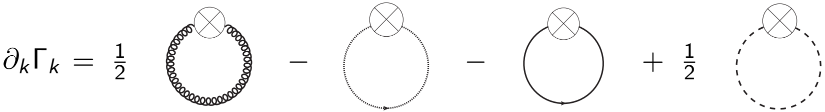
Contributions again

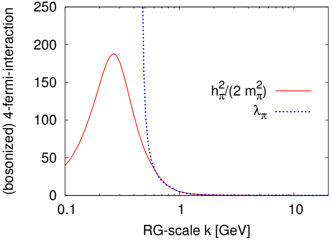
[MM, Strodthoff, Pawlowski, 2014]



4-Fermi vertex via dynamical hadronization [Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels → meson exchange
- efficient inclusion of momentum dependence ⇒ no singularities
- identifies relevant effective low-energy dofs from QCD





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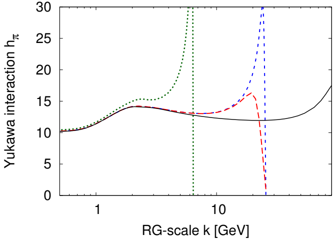


[Braun, Fister, Haas, Pawlowski, Rennecke, 2014]

[MM, Strodthoff, Pawlowski, 2014]

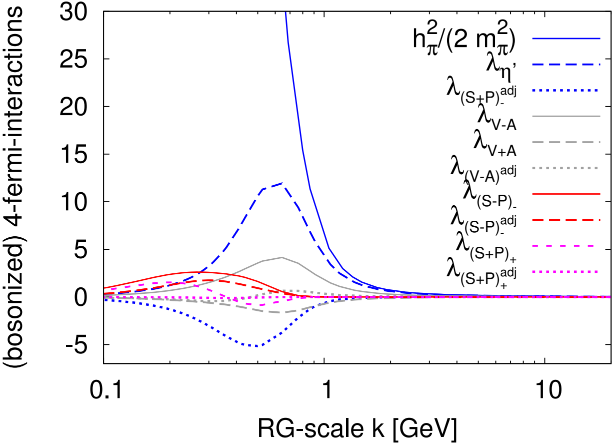
Dynamical hadronization and bound states

- dynamical hadronization: $\Gamma^{(4)}_{(\bar{q}q)^2} \to \frac{\Gamma^{(3)}_{\bar{q}q\pi}\bar{\Gamma}^{(3)}_{\bar{q}q\pi}}{\Gamma^{(2)}_{\pi}}$
- at pion pole: $\Gamma^{(3)}_{\bar{q}q\pi} \propto$ Bethe-Salpeter amplitude
- $\Rightarrow \Gamma^{(3)}_{\bar{q}q\pi}$ and $\Gamma^{(2)}_{\bar{q}q}$: $f_\pi \approx 90$ MeV (PRELIMINARY!)

[MM, Pawlowski, Strodthoff, in prep.]

other 4-Fermi channels

[MM, Pawlowski, Strodthoff, 2014]



- bosonized only σ-π-channel $\Rightarrow$ sufficient
  diquark momentum configuration more important
- other channels: do not feed back

't Hooft determinant

[MM, Pawlowski, Strodthoff, in preparation]

- $m_{\eta'} - m_\pi \propto$ 't Hooft determinant
- two contributions to 't Hooft determinant:
    - $U_A(1)$-anomaly
    - (explicit and spontaneous) chiral symmetry breaking

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- chiral symmetry breaking: large contribution to 't Hooft determinant
  - $m'_\eta \approx 800 - 900$ MeV (screening mass)
  - lattice ($N_f = 2$): 880 MeV [Hashimoto, 2008]
- effect of $U_A(1)$-anomaly on $m_{\eta'}$ small?

η'-meson (screening) mass at chiral crossover

- small η'-meson mass above chiral crossover? [Kapusta, Kharzeev, McLerran, 1998]
- experiment: drop in η' mass at chiral crossover [Csörgo et al., 2010]

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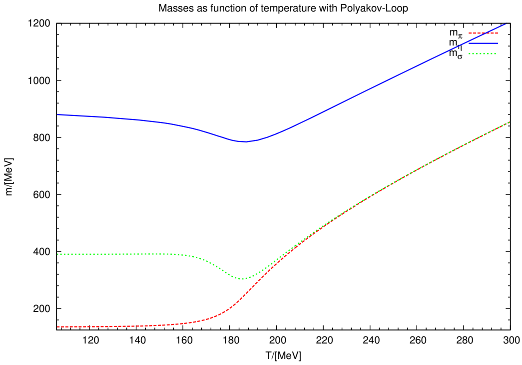

[Heller, MM, in preparation]

- Polyakov-Loop extended Quark-Meson model $N_f = 2$
- (RG-)scale dependent 't Hooft determinant
- match RG-scale to T via chiral crossover
- screening masses!

- chiral symmetry restoration alone:
  $\Rightarrow$ drop in $m_{\eta'}$

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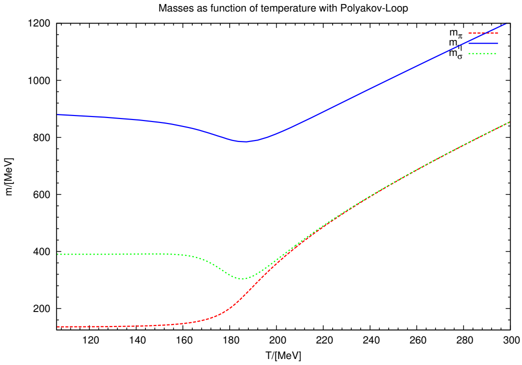

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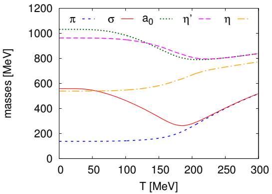
- QM-Model $N_f = 2 + 1$:



[MM, Schaefer, 2013]

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Stability of truncation

[MM, Strodthoff, Pawlowski, 2014]

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  - 4-gluon vertex approximated via STI from 3-gluon vertex
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  - 2-quark-2-gluon vertex approximated from quark-gluon vertex
  - field dependence of Yukawa interaction: 5-10% [Pawlowski, Rennecke, 2014]

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- possible effects of neglected vertices:
  - fermionic 6- and 8-point functions: (partially) included via mesons
  - ghost and ghost-gluon 4-point functions: small in first checks

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- possible effects of neglected vertices:
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  - ghost and ghost-gluon 4-point functions: small in first checks
- effect of $U(1)_A$-anomaly: comparably small in first checks [Pawlowski, 1996]

How we treat this large set of equations

used/written programs - fQFT framework

- Derivation of diagrams: DoFun/ERGE [Huber, Braun, 2011]/[Fister, unpublished]
- tracing: FORMTracer [Cyrol, MM, Strodthoff, in preparation]
- automatic c++ code for kernels: CreateKernels [Cyrol, MM, Strodthoff, unpublished]
- parallelized c++ framework: frgsolver [Cyrol, MM, Strodthoff, unpublished]

more automatization in development

- automatic (GPU-)parallelized c++ code from given action

Summary and Outlook

FRG with dynamical hadronization

- sole input $\alpha_S(\Lambda = \mathcal{O}(10)\text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10)\text{ GeV})$: (too?) good agreement with lattice simulations
- (non-perturbative) results:
  - quark-propagator
  - quark-gluon vertex
  - 4-Fermi interaction channels
- phenomenology:
  - Bethe-Salpeter Amplitude $\Rightarrow f_\pi, \ldots$
  - η'-meson and pion mass splitting

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wish list

- unquenching (no confinement without DχSB?)
- (more) bound-state properties (form factor,PDA. . .)
- finite temperature/chemical potential
- more checks on convergence of vertex expansion