$R_{e+\,e-}$ and the infrared value of the QCD coupling constant

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1) Phenomenology and the IR QCD coupling

2) SDE and an infrared finite QCD coupling

3) $R_{e+\,e-}$

4) “Smearing“ $R_{e+\,e-}$

5) Conclusion

John Gomez & AN

1) Phenomenology and the IR QCD coupling

Strong interaction at low energies $\Rightarrow$ Models

Evidences for a finite coupling:

1) **Jet shapes observables → $\alpha(0) \approx 0.63$**
   [Dokshitzer, Weber,... PLB 352 (1995)451, Nucl.Phys.B 469(1996)93]

2) **Quarkonium potential models → $\alpha(0) \approx 0.6$**
   [Godfrey, Isgur, PRD 32(1985)189; Eichten et al. PRL 34(1975)369; PRD 21(1980)203; ...]

3) **Optmized perturbation theory ($R_{e+\,e-}$) → $\alpha(0) \approx 0.8$**
   [Mattingly and Stevenson, PRL 69(1992)1320, PRD 49(1994)437]

4) Quarkonium fine structure → $\alpha(0) \approx 0.4$

[Badalian, Simonov, Baker,...PRD 60(1990)116008, PRD 62(2000)094031]

5) QCD-inspired models (total hadronic cross sections) → $\alpha(0) \approx 0.5$

[Block, Halzen, Margolis, Pancheri,... PRD 45(1992) 839; PRD 60(1999)054024]

6) Unpolarized proton structure function → $\alpha(0) \approx 0.4 – 0.56$

[Courtoy, Liuti, PLB 726(2013)320]

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HERA data → "... frontier between perturbative and non-perturbative physics may occur at relatively small momenta...."

[E. Levin and I. Potashnikova, JHEP 1402(2014)089]

Theoretical arguments:

Functional Schrodinger equation $\alpha(0) \approx 0.5$

[J. Cornwall, PRD 76(2007)025012]

Gribov papers on confinement $\alpha(0) \approx 0.44$

[see Eur.Phys.J. C 10(1999)91]

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Many evidences for an infrared finite QCD coupling

... and the value is moderately small.... !!!!

Lattice QCD and Schwinger-Dyson equations

consistent with IR finite gluon propagator

⇩

QCD -- If any dynamical mass scale is generated for the gluons (or... scale in the gluon propagator)

→ the coupling freezes in the IR

[Aguilar, Silva, AN, PRL 90(2003)152001]

→ non-perturbative IR fixed point

2) SDE and an infrared finite QCD coupling

SDE + Pinch Technique → gauge invariant solutions for propagators and vertices

[Cornwall, Aguilar, Binosi, Papavassiliou, et al.... ....]

an effective infrared finite QCD charge:

$$\boxed{\alpha_s(Q^2) = [4\pi\beta_0 \ln(\frac{Q^2 + 4m_g^2(Q^2)}{\Lambda_{QCD}^2})]^{-1}}$$

Cornwall, PRD 26(1982)1453

evolution of the SDE calculation....

Aguilar, Binosi, Papavassiliou, Quintero, PRD 80(2009)085018

$$\boxed{\alpha(q^2) = \left[4\pi\beta_0 \ln\left(\frac{q^2 + h(q^2, m^2(q^2))}{\Lambda_{QCD}^2}\right)\right]^{-1}}$$

$$h(q^2, m^2(q^2)) = \rho_1 m^2(q^2) + \rho_2 \frac{m^4(q^2)}{q^2 + m^2(q^2)}$$

m_g = 500 MeV → ρ_1 = 4.5 , ρ_2 = -2

m_g = 600 MeV → ρ_1 = 2.2 , ρ_2 = -1.25

simple approximation for α_s

$$\alpha_s(Q^2) = [4\pi\beta_0 \ln(\frac{Q^2 + \rho m_g^2(Q^2)}{\Lambda_{QCD}^2})]^{-1}$$

$$m_g^2(Q^2) \approx \frac{m_g^4}{Q^2 + m_g^2}$$

ρ → 3 – 4
phenomenology
Courtoy, Liuti,...

Aguilar, AN, JHEP 0408 (2004) 057

NOTE:

This is not your favorite α_s ?

Worried about non-perturbative α_s definition?

$$\alpha_s(Q^2) = [4\pi\beta_0 \ln(\frac{Q^2 + \rho m_g^2(Q^2)}{\Lambda_{QCD}^2})]^{-1}$$

→ this expression maps any IR frozen coupling...

(is the possibility α_s (0) → 0 alive?)

Remember: IR mass scale → α_s IR freezing

3) $R_{e+\,e-}$

$$R_{e^+e^-}(s) \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_i^{n_f} q_i^2$$

Parton model

QCD

$$R_{e^+e^-}(s) = 3\sum_i^{n_f} q_i^2\left(1+\mathfrak{R}(s)\right)$$

n_f → number of flavours

$$\mathfrak{R}(s) = a(1 + r_1 a + r_2 a + \ldots)$$

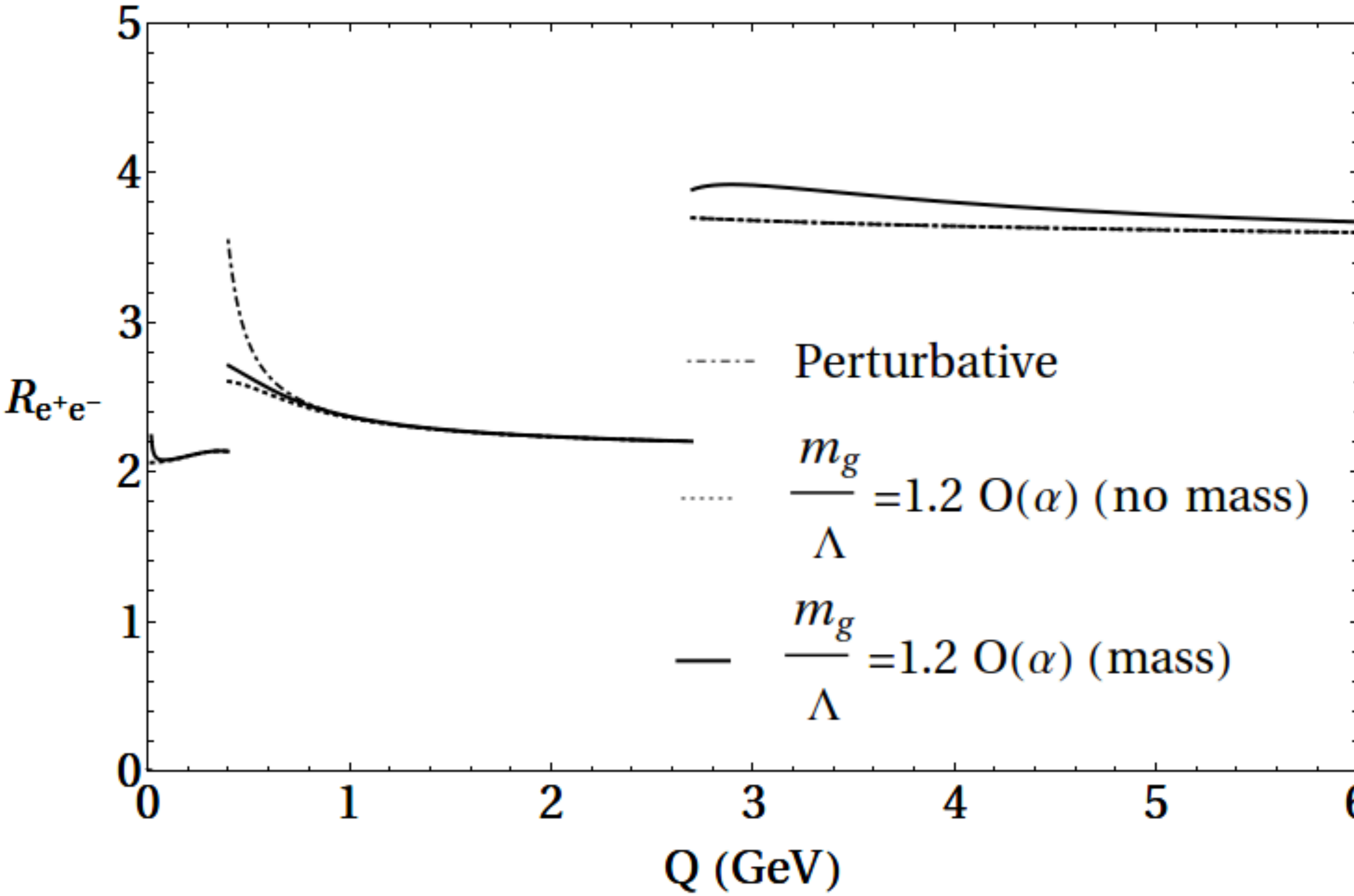
$\overline{MS}$

$$r_1 = 1.986 - 0.1153 n_f^2$$

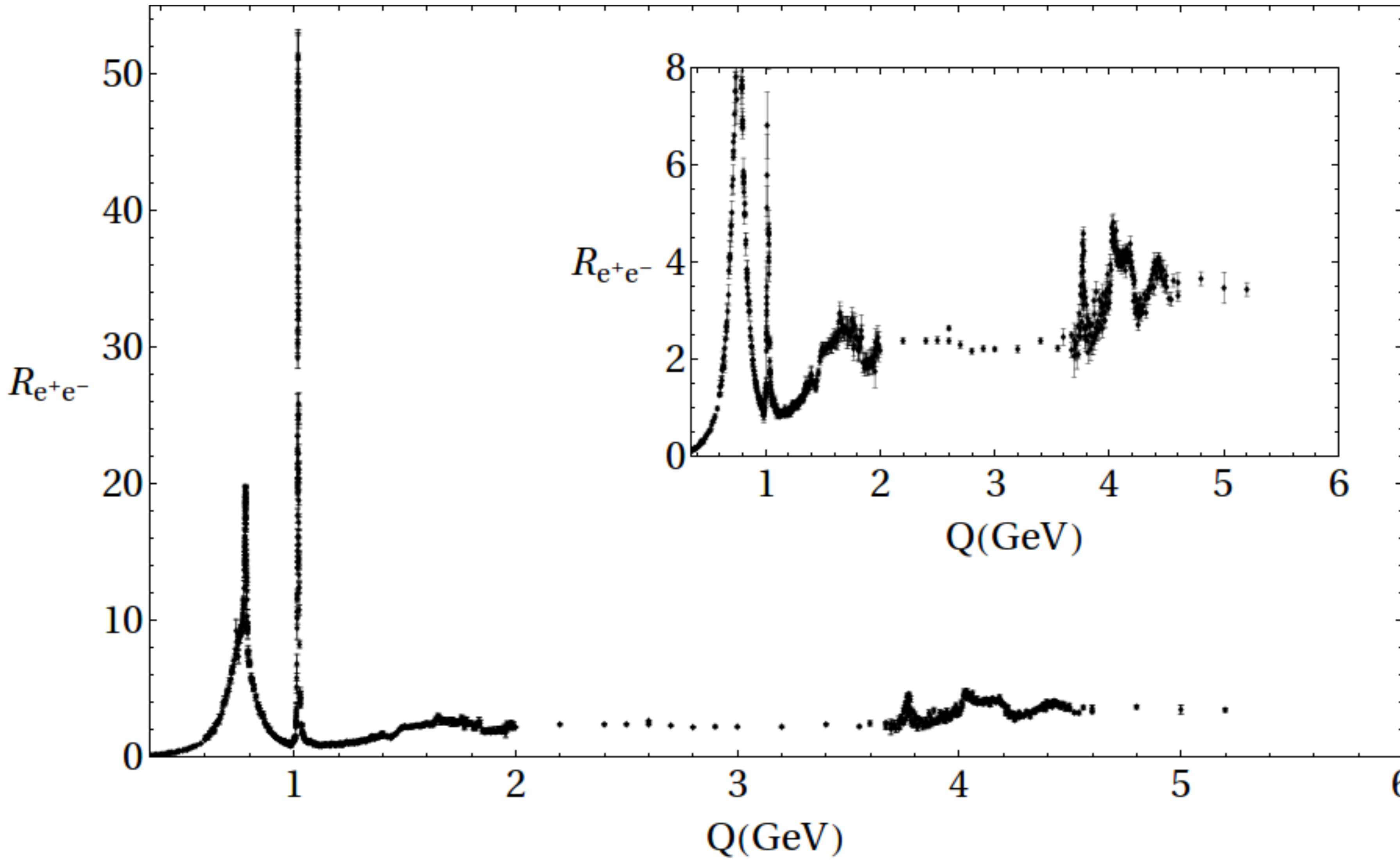
$$r_2 = -6.637 - 1.200 n_f - 0.00518 n_f^2$$

$$a = \frac{\alpha_s}{\pi}$$

$R_{e+\ e-}$ --- Parton model



Experimental data (PDG)



[OPT →Mattingly and Stevenson, PRL 69(1992)1320, PRD 49(1994)437, recent Stevenson, NPB 875(2013)63]

4) "Smearing" R_{e+e-}

Experiment X Theory → Smearing

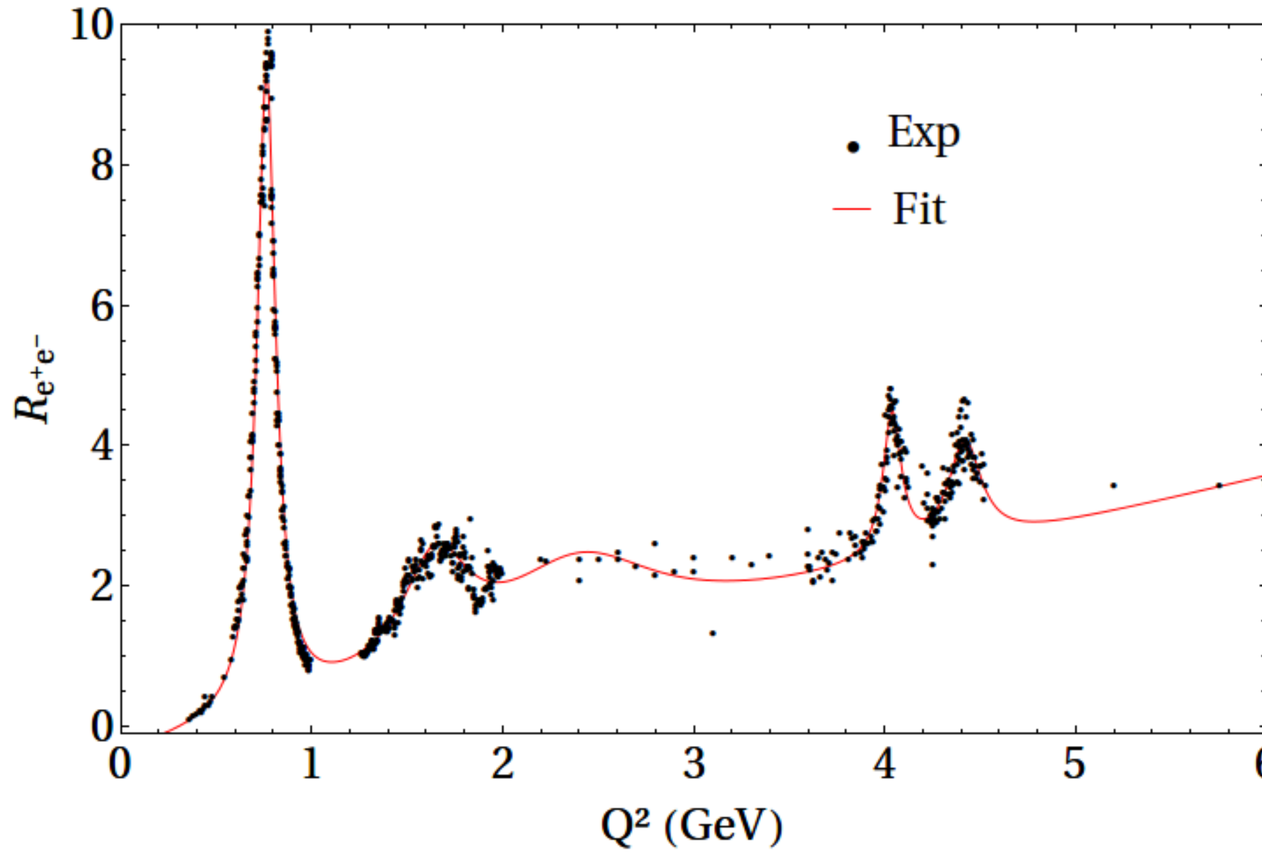
$$\overline{R}_{pqw}(Q^2;\Delta) = \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R_{e^+e^-}(\sqrt{s'})}{\left(s'-Q^2\right)^2+\Delta^2}$$

Poggio, Quinn and Weinberg, PRD 13(1976)1958

→ smooth curves

Data fit

PDG data



Red curve →
$$\boxed{\sum_{i=1}^{4} \frac{A_i B_i}{\left(Q^2 - M_i^2\right)^2 + A_i} + C + DQ^2}$$

ρ, ϕ, J/ψ, ψ(3686), ψ(3770) included latter (BW → δ functions)

Exp. → fit + narrow resonances

$$\overline{R}_{pqw}(Q^2;\Delta) = \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R_{e^+e^-}(\sqrt{s'})}{\left(s'-Q^2\right)^2+\Delta^2}$$

Th.→ QCD with effective charge up α^3

$$R_{e^+e^-}(s) = 3\sum_i^{n_f} q_i^2\,(1+\mathbb{R}(s))$$

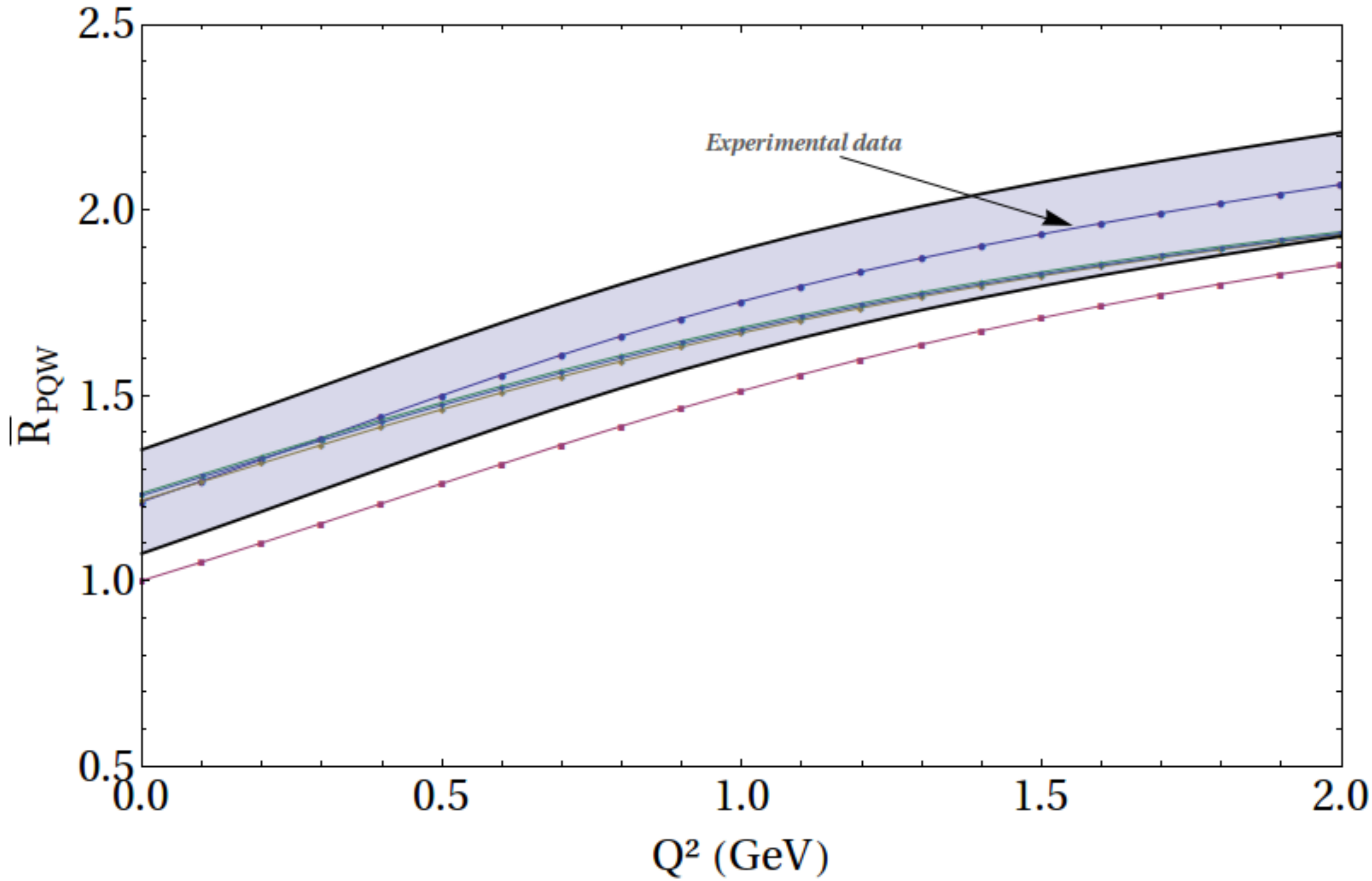
Integration: threshold by threshold - m_u to Q_{max}

fit + resonances

$0 - m_u \rightarrow R_{e+\,e-} = 0$

$Q_{max} - \infty \rightarrow R_{e+\,e-}$ constant

Exp. data – Δ = 1.5 GeV^2
shaded area - ± 7%
Theoretical curves – different m_g/Λ (0.7 magenta – excluded)



7% - ad hoc // different Δ

Best fit $\rightarrow\chi 2$ test

$$\chi^2(\theta) = \sum_i \frac{(O_i^{exp} - O_i^{th}(\theta))^2}{\left(\Delta O_i^{exp}\right)^2}$$

Considering:

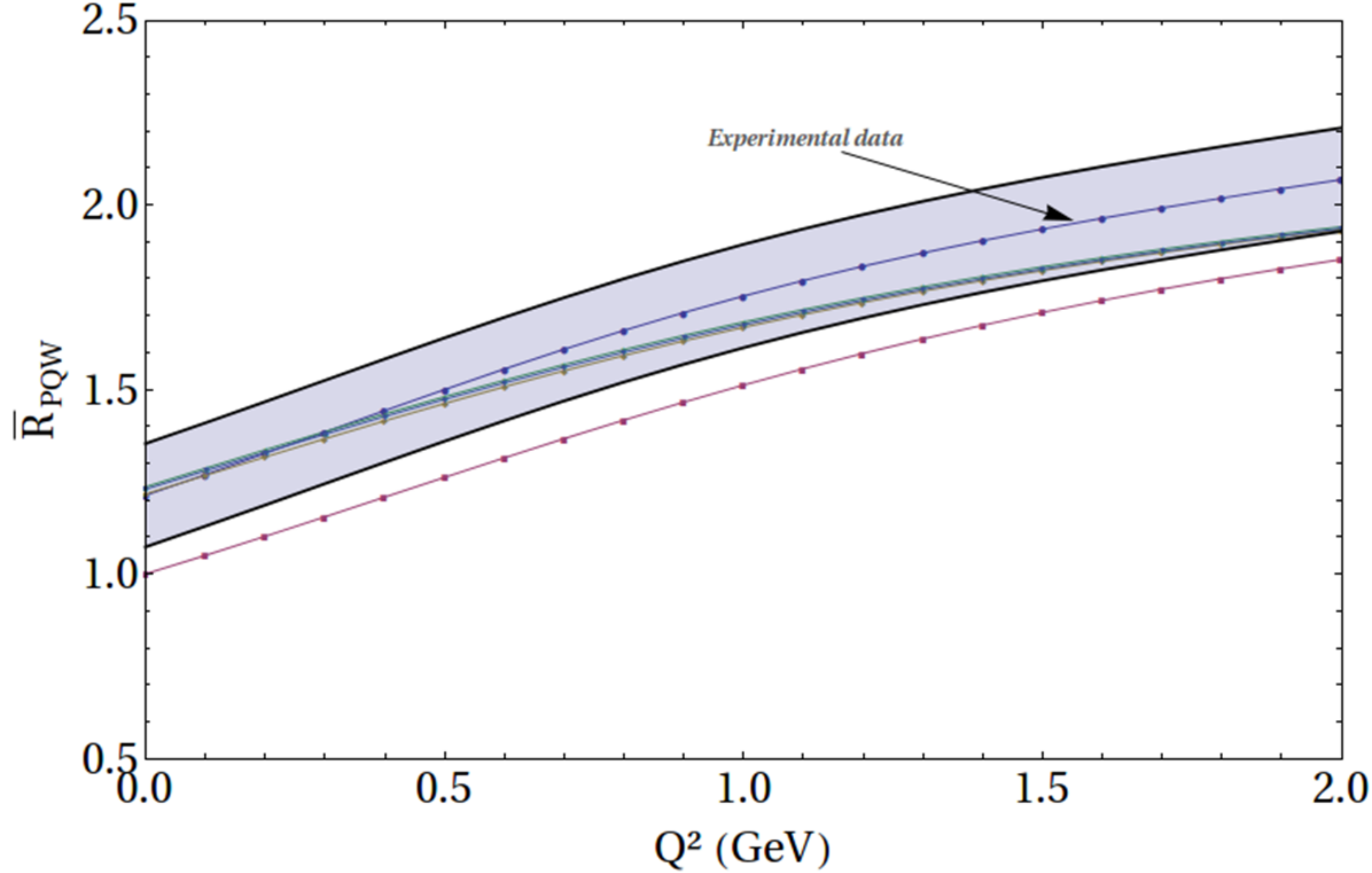
$$\alpha_s(Q^2) = [4\pi\beta_0 \ln(\frac{Q^2 + \rho m_g^2(Q^2)}{\Lambda_{QCD}^2})]^{-1}$$

$$\boxed{m_g^2(Q^2) \approx \frac{m_g^4}{Q^2 + m_g^2}}$$

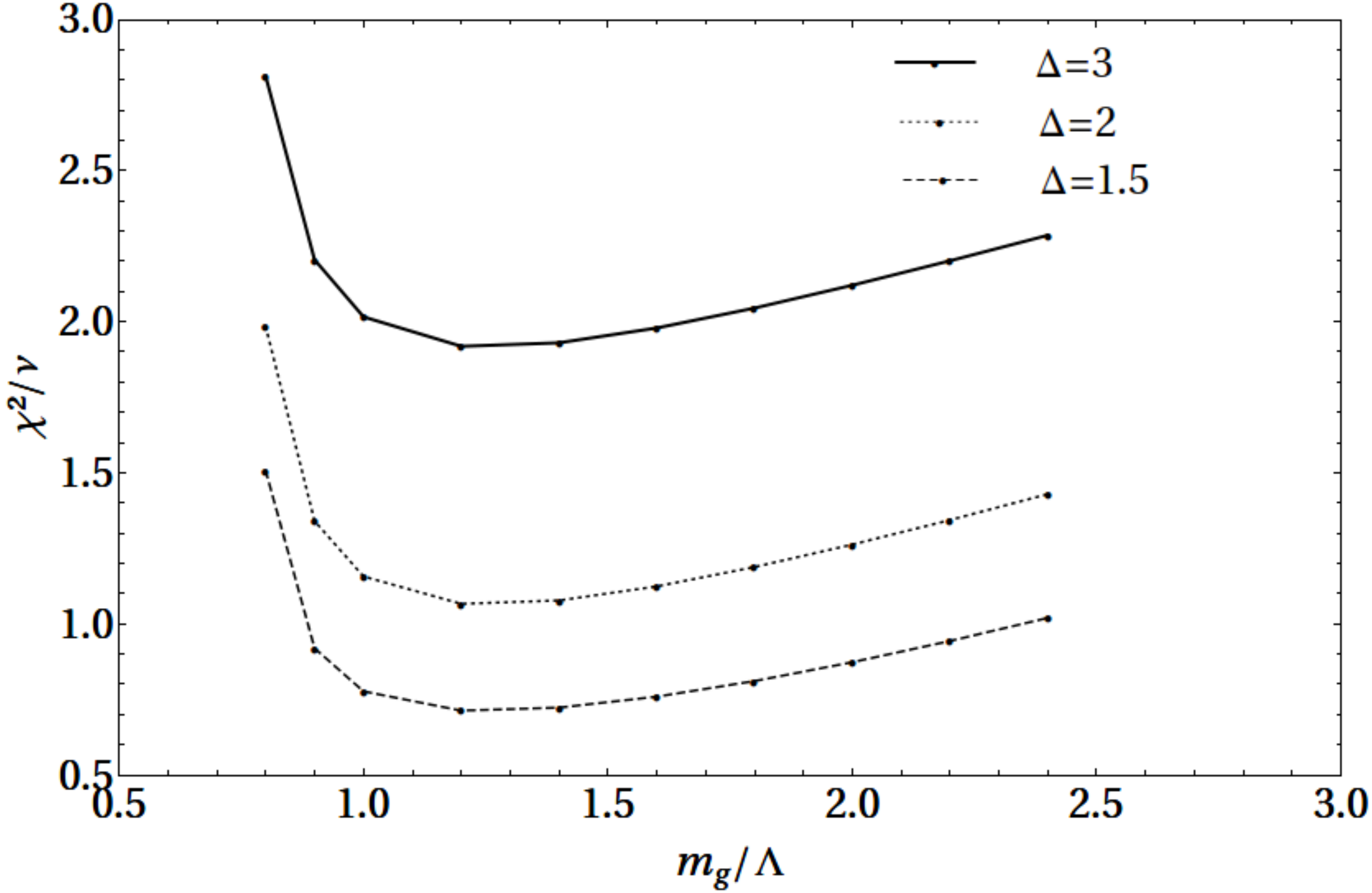
$\rho = 3 - 4$; $\Delta = 1.5 - 3\ \mathrm{GeV}^2$; $m_g/\Lambda = 0.5 - 2.5$

R_{PQW} up to $Q^2 = 2\ \mathrm{GeV}^2$

χ2 exp X th (different curves, Δ, ...)



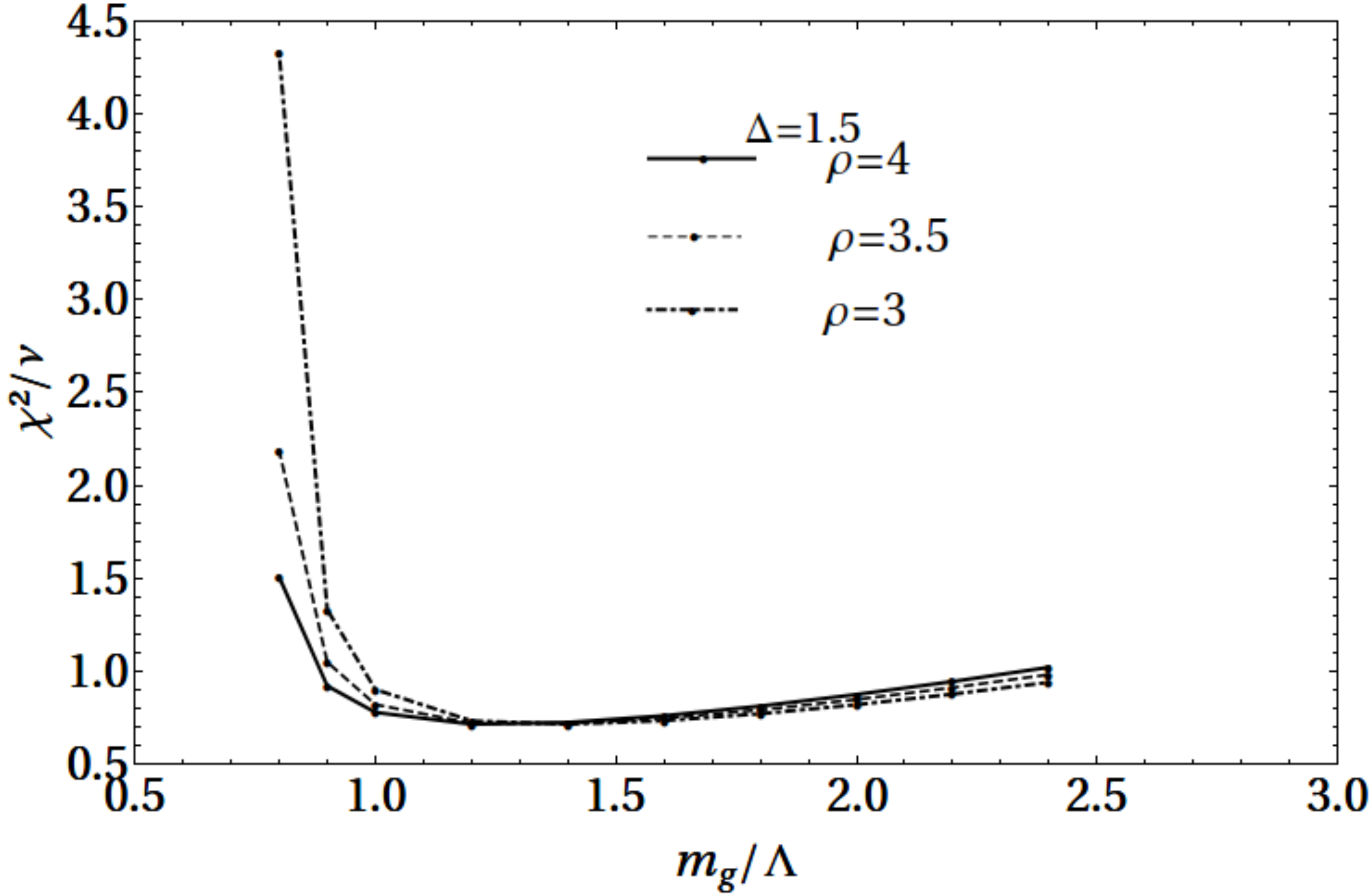
ρ = 4 , different Δ



Minimum → $m_g/\Lambda \approx 1.2$

Δ = 1.5 GeV^2

ρ → 3 - 4

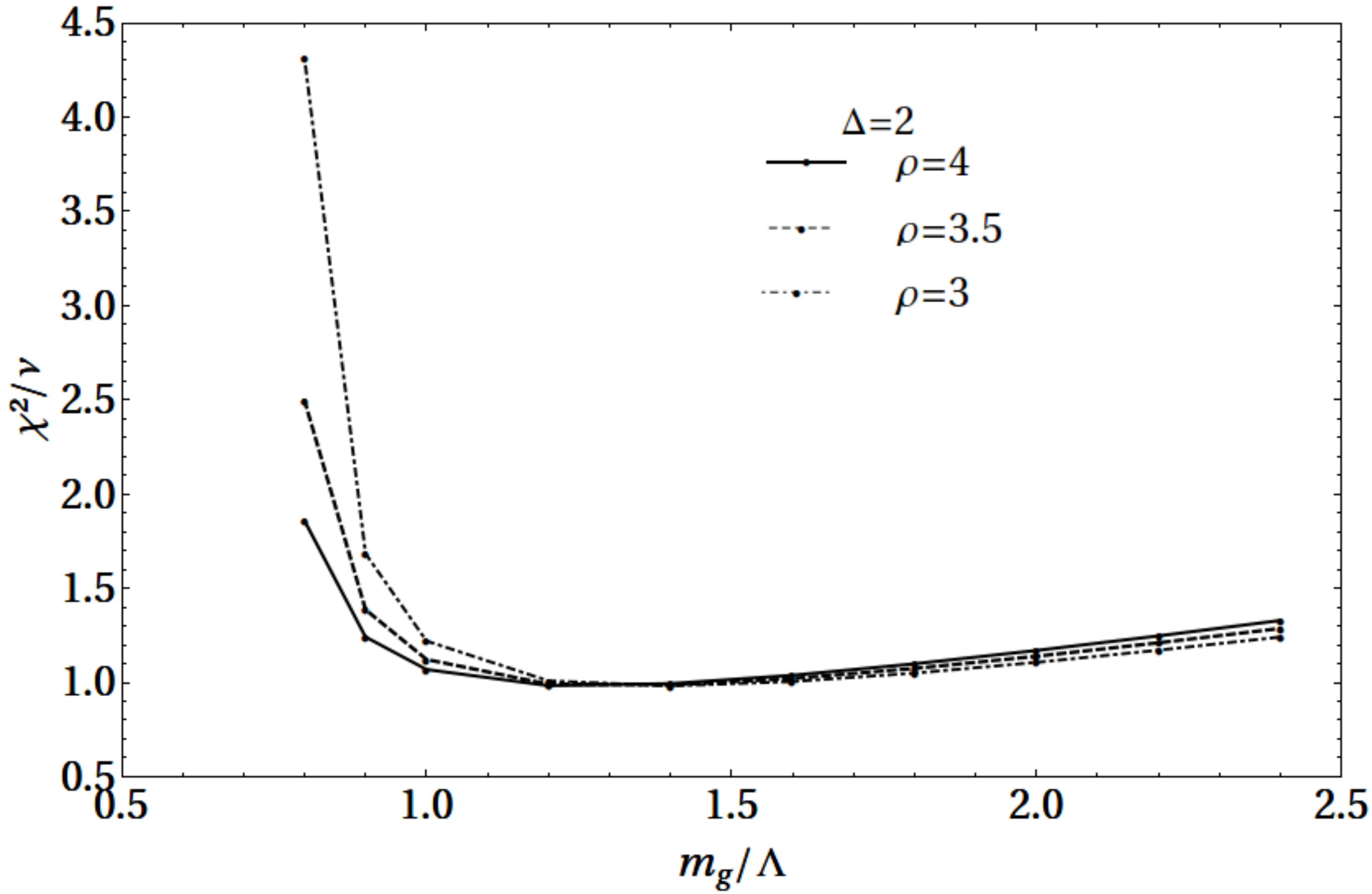


Minimum→ $m_g/\Lambda \approx$ 1.4

ρ = 3

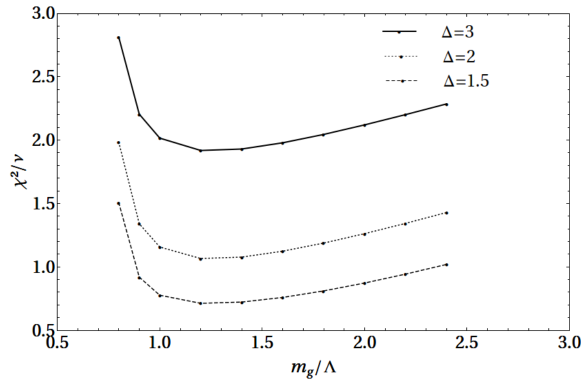
Δ = 2 GeV^2

$\rho \rightarrow$ 3 - 4



Minimum$\rightarrow$ $m_g/\Lambda \approx 1.4$

ρ = 3



If $m_g/\Lambda < 0.8$ → bad χ2

Above $m_g/\Lambda = 1.6$, slow increase of χ2 value

Minimum $3 < \rho < 4$

→ $1.2 < m_g/\Lambda < 1.4$

approximately constant with the product -- $\rho\, m_g/\Lambda$

$$\alpha_s(Q^2) = [4\pi\beta_0 \ln(\frac{Q^2 + \rho m_g^2(Q^2)}{\Lambda_{QCD}^2})]^{-1}$$

$$\beta_0 = (33 - 2n_f)/48\pi^2$$

Other consequences...

Global duality

[Bertlmann, Launer, de Rafael, NPB 250(1985)61]

$$\int_0^{s_0} dt \, \frac{1}{\pi} \, \mathrm{Im}\Pi(t)_{\exp} = \int_0^{s_0} dt \, \frac{1}{\pi} \, \mathrm{Im}\Pi(t)_{pQCD}$$

matching between long and short distances **(s_0= 1.5 GeV^2)**

(perturbative α)

Imply:

$$\boxed{\int_0^{s_0} ds \, \overline{R}_{\exp} = \int_0^{s_0} ds \, \overline{R}_{th}}$$

Surface of s_0 and ρ, m_g /Λ

What to expect? Smaller s_0

Define $$\varepsilon\left(Q^2, s_0, m_g/\Lambda, \Delta\right) = \int_0^{s_0} ds\,\overline{R}_{\exp} - \int_0^{s_0} ds\,\overline{R}_{th}$$

$\Delta=1.5$; $m_g/\Lambda=1.2$

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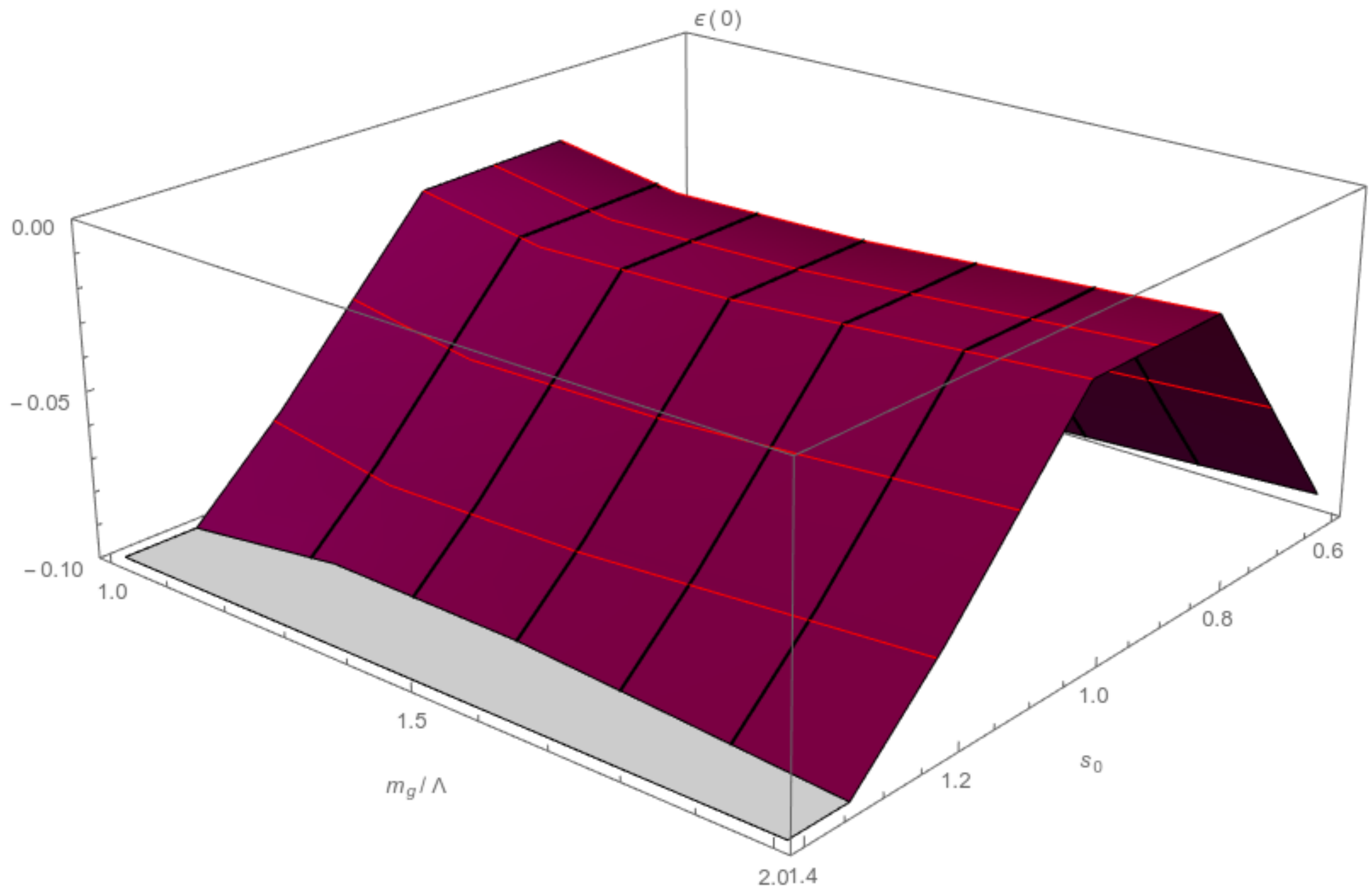
s_0	$\varepsilon(0)$
0.6	-0.0987
0.8	-0.0276
0.87	-0.0060
1.0	-0.0291
1.2	-0.0812
1.4	-0.1264



5) Conclusion

$R_{e+\,e-}$ → information about the IR effective QCD charge

Assuming a simplified expression for the effective charge:

$$\alpha_s(Q^2) = [4\pi\beta_0 \ln(\frac{Q^2 + \rho m_g^2(Q^2)}{\Lambda_{QCD}^2})]^{-1}$$

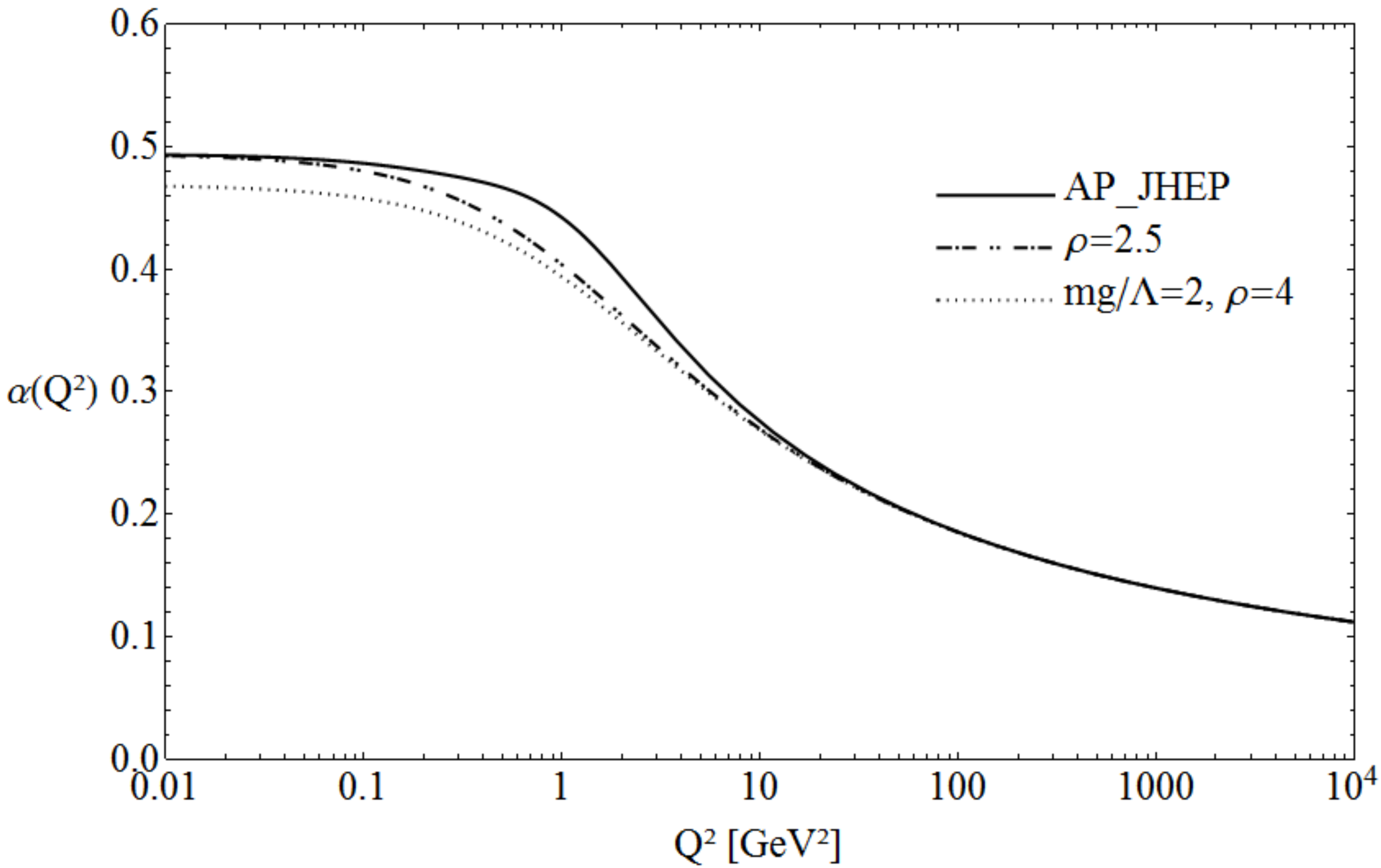
we obtain:

$\alpha(0)$ basically dependent on the product $\rho\, m_g / \Lambda$

$\alpha\,(0) \approx 0.7$

Smaller scale s_0
→ frontier between
pert. and non-pert. physics

In agreement with Mattingly and Stevenson
Improved (or different) expressions for α_s ... ?
How it will change at next order?
....


0.6
0.5
0.4
0.3
0.2
0.1
0.0
α(Q²)
0.01
0.1
1
10
100
1000
10⁴
Q² [GeV²]
AP_JHEP
ρ=2.5
mg/Λ=2, ρ=4