On the lattice Landau gauge quark propagator

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Gluon Propagator in Linear Covariant Gauges

arXiv:1505.05897

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On going effort to investigate the quark gluon vertex on the lattice

$$\Gamma_{\alpha\beta\ \mu} = \langle 0|\overline{\psi}_{\alpha}\,\psi_{\beta}\,A_{\mu}|0\rangle$$

12 form factors

quark propagator + gluon propagator

close to the chiral limit

$$S(p) = Z(p^2)\ \frac{-ia\not{k} + aM(p^2)}{a^2k^2 + M^2(p^2)}$$

quark wave function

running quark mass

several quenched calculations

Dynamical lattice simulations

improved staggered action

P. O. Bowman et al, Nucl. Phys. Proc. Suppl. 119, 323 (2003) [hep-lat/0209129]

P. O. Bowman et al, Phys. Rev. D71, 054507 (2005) [hep-lat/0501019]

2+1 flavors

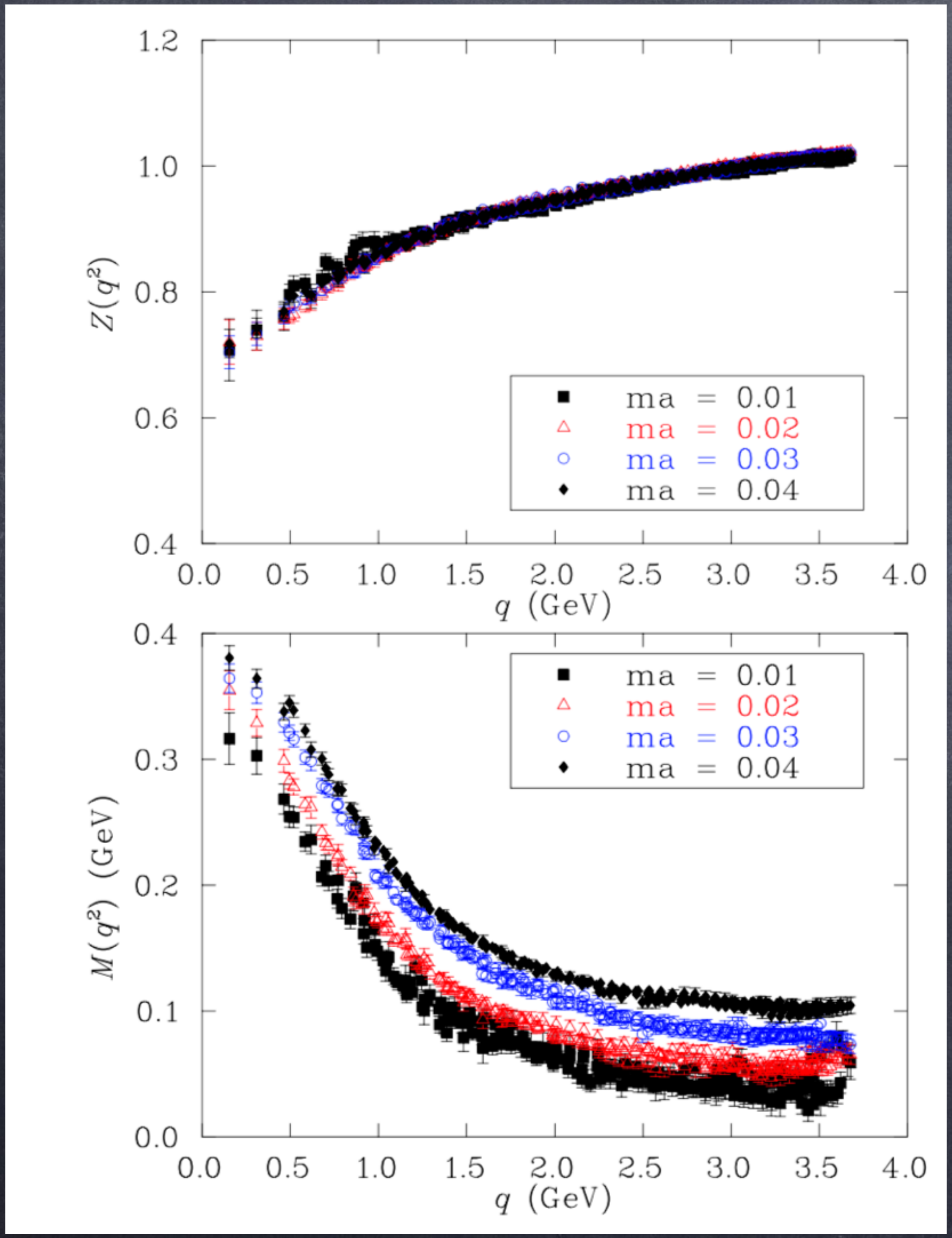
Bare quark masses:

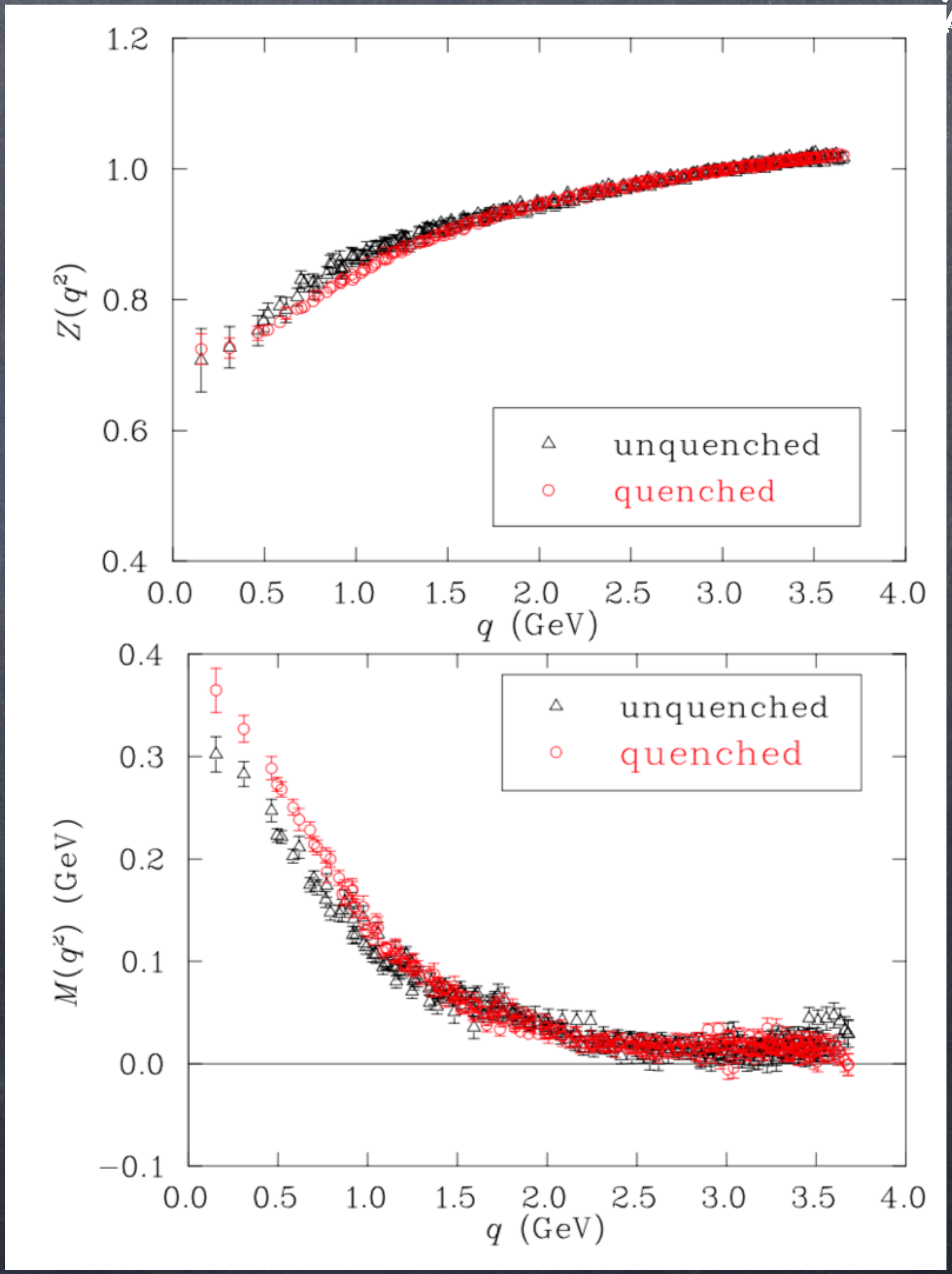
sea 16, 32, 47, 63 Mev + 79 MeV

$a = 0.125$ fm $20^3 \text{x} 64$

$Z(p^2)$ suppressed in the IR
$M(p^2)$ enhanced in the IR

chiral extrapolation





S. Furui, H. Nakajima, Phys. Rev. D73, 074503 (2006) [hep-lat/0511045]

$M(0) \simeq 380$ MeV (chiral extrapolation)

FLIC overlap quark action

W. Kamleh et al, Phys. Rev. D76, 094501 (2007) [arXiv:0705.4129]

$N_{sea} = 2$ flavors 50 configurations

	β	a(fm)	m_π(MeV)	
$12^3 \times 24$	4.00	0.120	806	$(1.44\ \mathrm{fm})^3 \times (2.88\ \mathrm{fm})$
$16^3 \times 32$	4.20	0.096	820	$(1.44\ \mathrm{fm})^3 \times (2.88\ \mathrm{fm})$

$M(0) < 300$ MeV (chiral extrapolation)

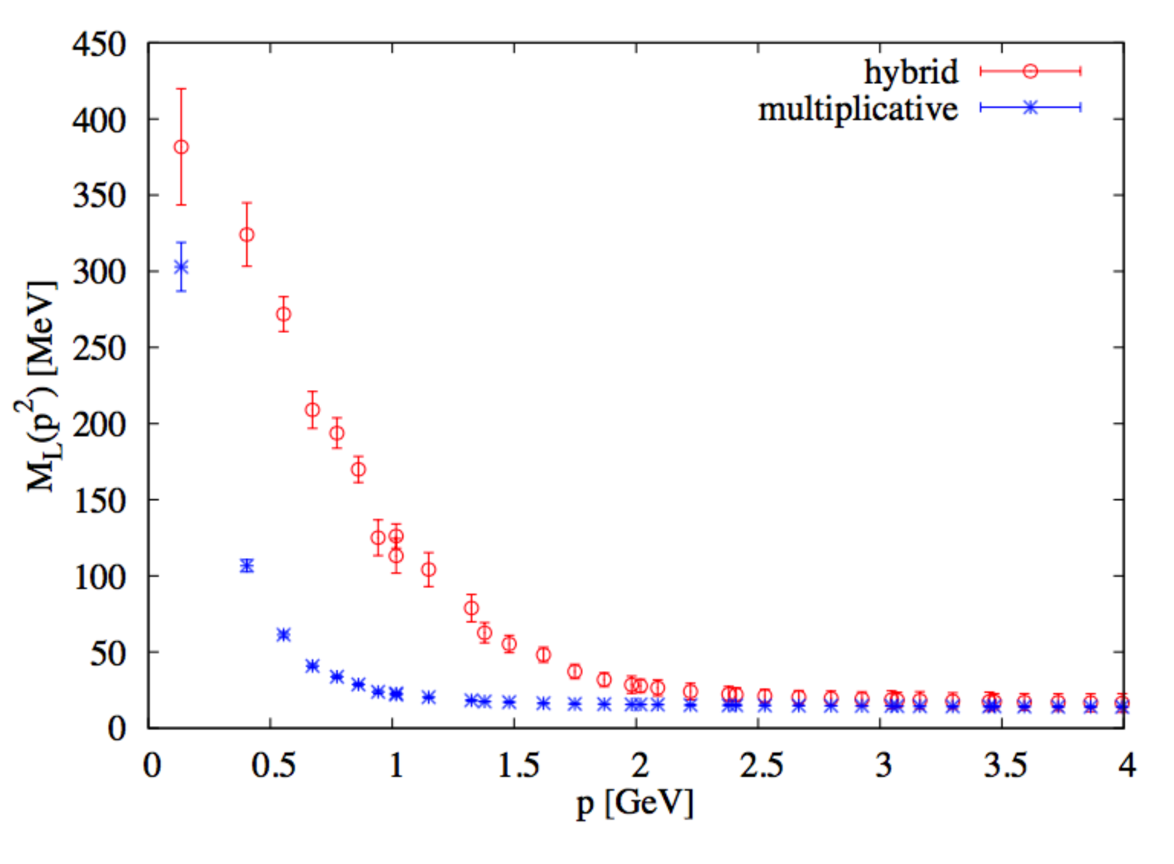
CI action

M. Schrok, Phys. Lett. B711, 217 (2012) [arXiv:1112.5107]

N_{sea} = 2 flavors   125 configurations

$16^3 \times 32$ a = 0.144 fm

m_{bare} = 15.3(3) MeV



$M(p^2_{min}) \simeq 300 - 380$ MeV

Clover action

($\mathcal{O}(a)$ improved Sheikholeslami-Wohlert action)

J. I. Skullerud et al, Phys. Rev. D63, 054508 (2001)
J. I. Skullerud et al, Phys. Rev. D64, 074508 (2001)
Ph. Boucaud et al, Phys. Lett. B575, 256 (2003)

m_{bare} = 50 MeV, 58 MeV, 102 MeV, 118 MeV

β = 6.0, 6.2 and 6.0 – 6.8

$(1.5\ \mathrm{fm})^3 \times (4.5\ \mathrm{fm})$
$(1.6\ \mathrm{fm})^3 \times (3.3\ \mathrm{fm})$

$$\sum_f \sum_x (4 + m_f)\overline{\psi}_f(x)\psi(x)$$

$$- \frac{1}{2} \sum_f \sum_{x,\mu} \Bigg[\overline{\psi}_f(x)(1 - \gamma_\mu)U_\mu(x)\psi_f(x + \mu)$$

$$+ \overline{\psi}_f(x + \mu)(1 + \gamma_\mu)U_\mu^\dagger(x)\psi_f(x) \Bigg]$$

$$- \frac{1}{2} c_{SW} \sum_f \sum_{x,\mu\nu} \overline{\psi}_f(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi_f(x)$$

$$\overline{\psi}(x)(i\partial\!\!\!/ + m)\psi(x) + \overline{\psi}(x)\partial^2\psi(x) + \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

O(a) nonperturbative improved action

$N_f = 2$ $\beta = 5.29$ $\kappa = 0.13632$

$a = 0.072$ fm

$m_{bare} = 6.66$ MeV

$m_\pi = 296$ MeV $m_N = 1078$ MeV

525 Landau gauge configurations

$$S(x,y) = \langle 0|\, \psi(x)\ \overline{\psi}(y)\, |0\rangle$$

$\mathcal{O}(a)$ improvement of quark propagator

$$\psi \longrightarrow (1 + b_q am)(1 - c_q a \not{D})\psi$$

$$\overline{\psi} \longrightarrow \overline{\psi}(1 + b_q am)(1 + c_q a \overleftarrow{\not{D}})$$

tree level values

$c_q = b_q = 1/4$

$$S(p) = \sum_x e^{-i\,p\cdot x}\ S(x,0)$$

$$= Z(p^2)\ \frac{-ia\not{k} + aM(p^2)}{a^2k^2 + a^2M^2(p^2)}$$

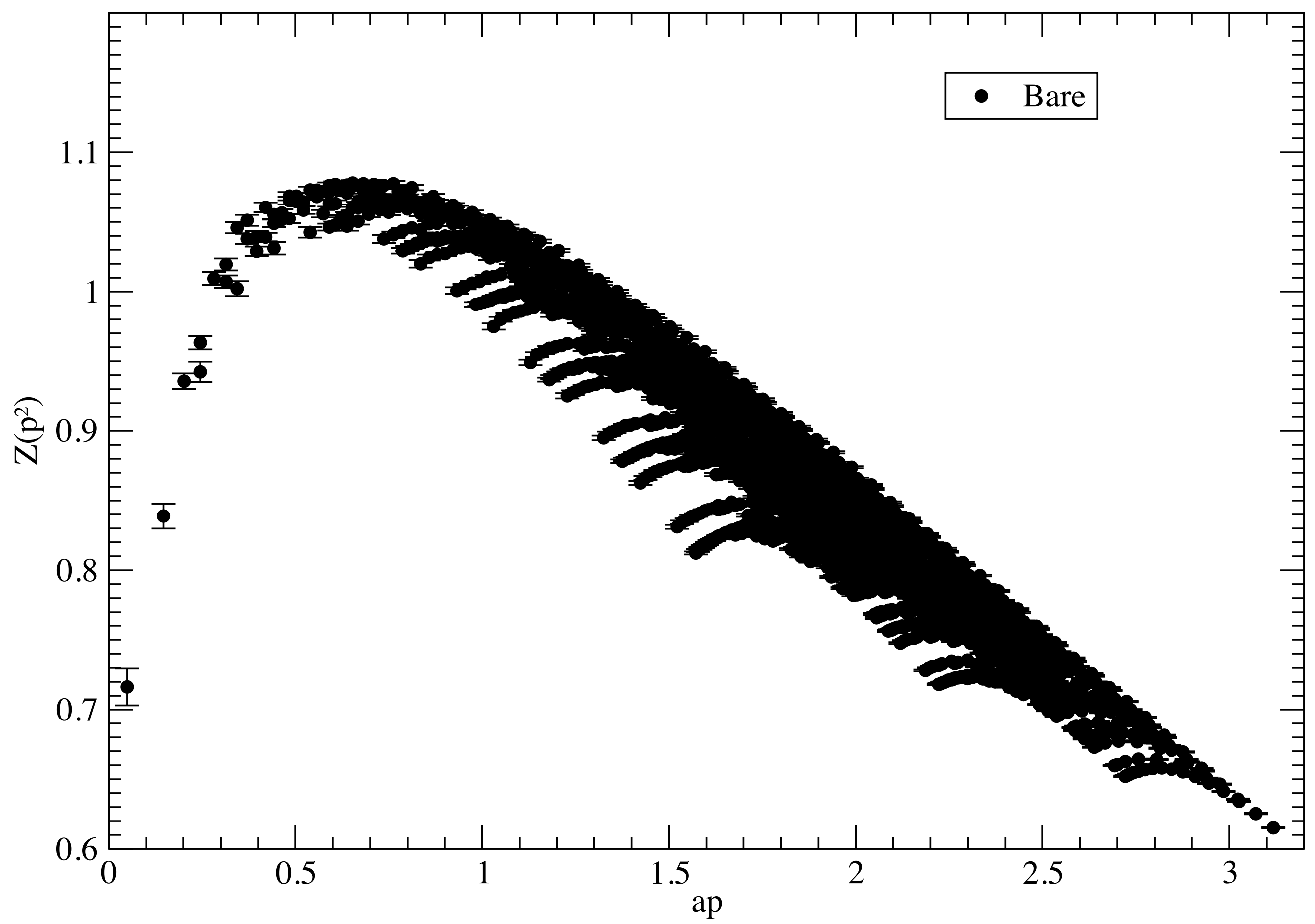
$$ak_\mu = \sin(p_\mu a)$$

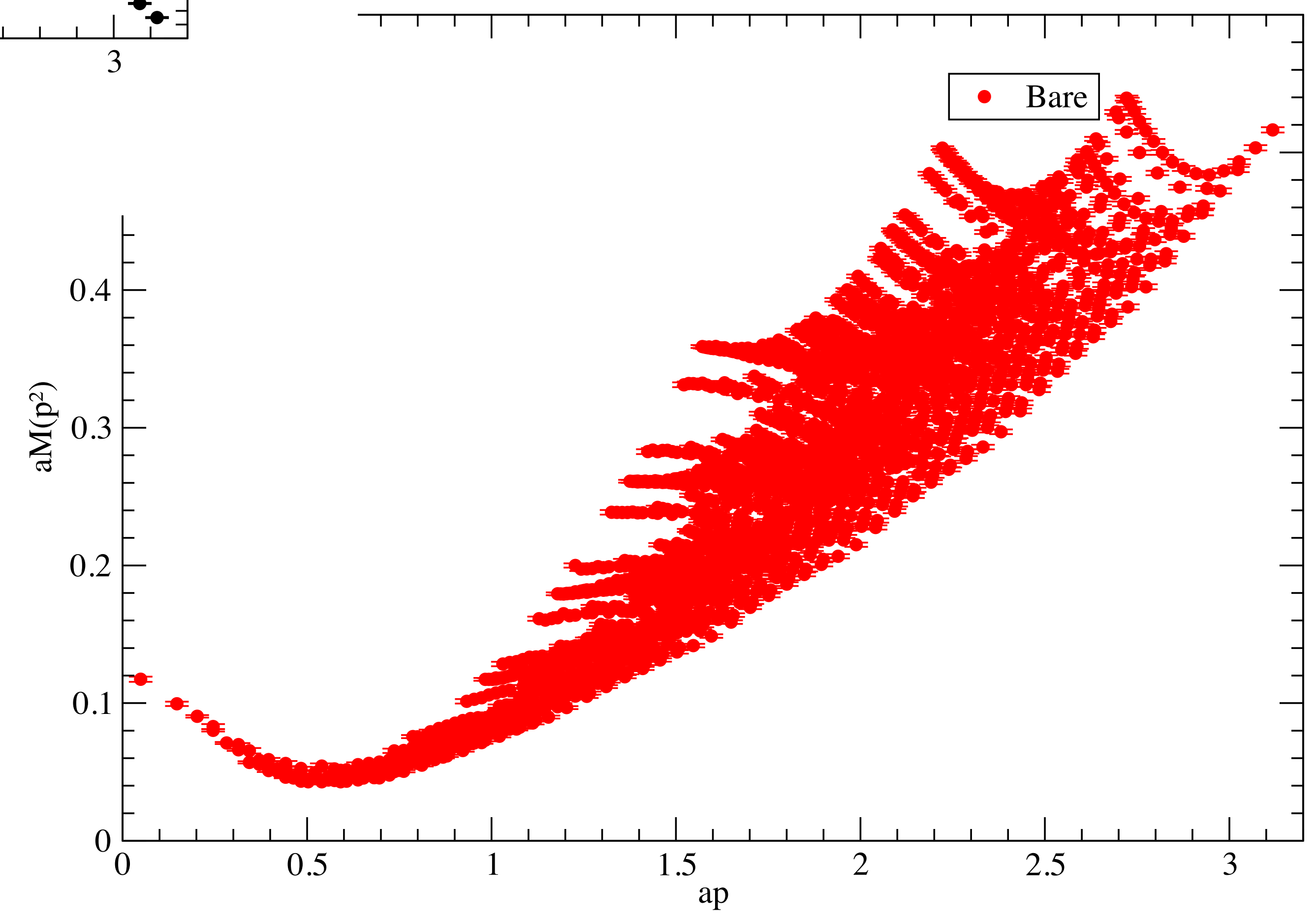
$$Z(p^2)\frac{-ia\not{k} + aM(p^2)}{a^2k^2 + a^2M^2(p^2)}$$

$$ak_\mu = \sin(p_\mu a)$$

$$Z(p^2) \approx 1 - \frac{a^2k^2}{16} + \mathcal{O}(a^2)$$

$$aM(p^2) \approx am\left(1 - \frac{am}{2} + \frac{a^2m^2}{16}\frac{k^2 + 4m^2 - 3k^4/m^2}{k^2 + m^2} - \frac{a^2k^2}{16}\right) + \mathcal{O}(a^3)$$





Preliminary data

H4 method

fundamental momenta invariants

$$p^{[2]} = \sum_\mu p_\mu^2, \qquad p^{[4]} = \sum_\mu p_\mu^4, \qquad p^{[6]} = \sum_\mu p_\mu^6, \qquad p^{[8]} = \sum_\mu p_\mu^8$$

$$Q(p^{[2]}, p^{[4]}, p^{[6]}, p^{[8]}) = Q(p^{[2]}, 0, 0, 0) + \frac{\partial Q(p^{[2]}, 0, 0, 0)}{\partial p^{[4]}}\, a^2\, \frac{p^{[4]}}{p^2} + \cdots$$

continuum value

a² corrections

$$a^2 \text{ corrections:} \qquad \frac{p^{[4]}}{p^2}, \qquad \frac{p^{[6]}}{(p^2)^2}, \qquad \frac{p^{[8]}}{(p^2)^3}$$

problem: what about p² corrections?

tree level quark propagator

$$S^{-1}(p) = \frac{ia\not{k}A(ap) + B(ap)}{z(ap)}$$

$$Z^{(0)}(ap) = z(ap)/A(ap)$$

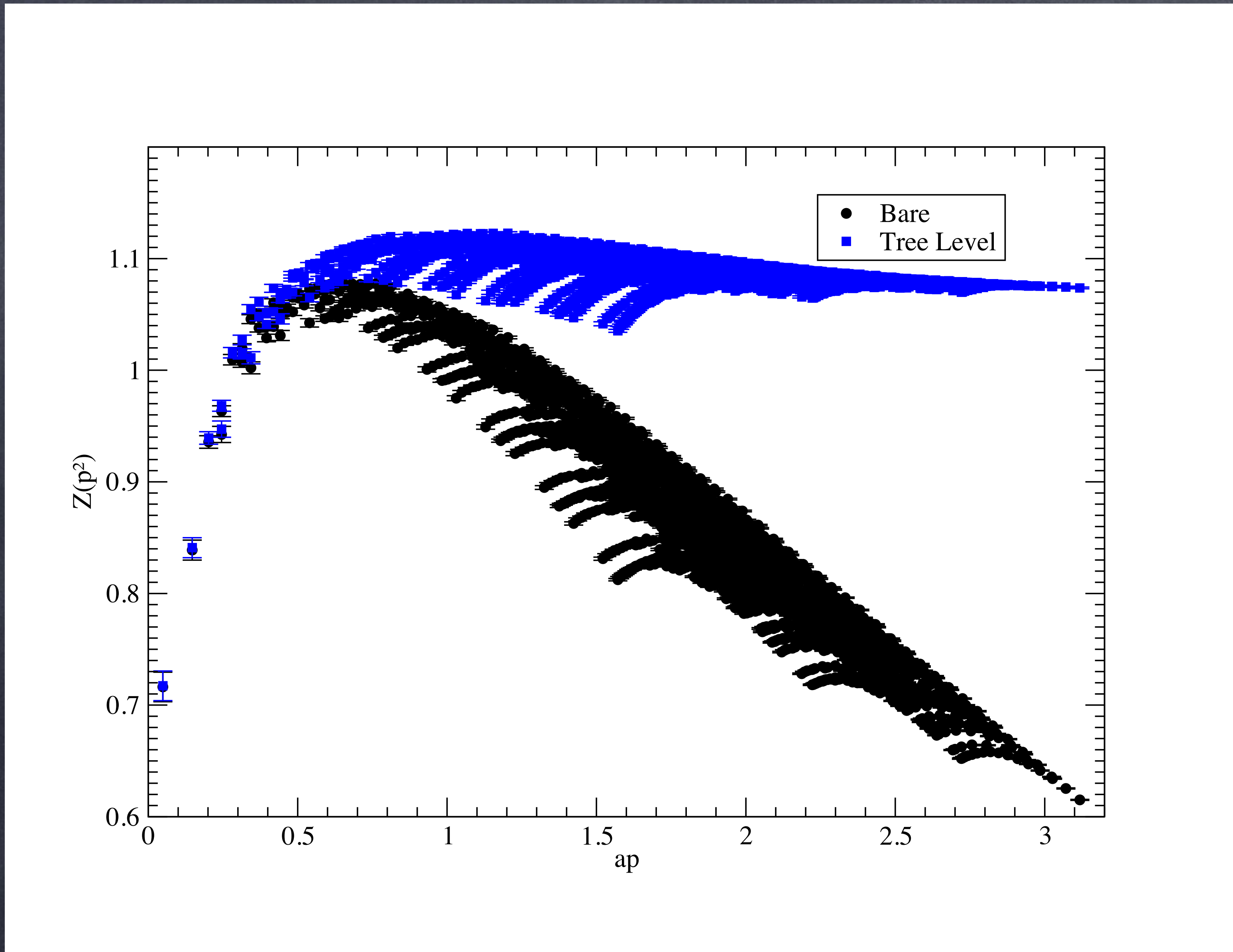
$$aM(ap) = \frac{B(ap)}{A(ap)} = am + a\Delta M^{(0)}(ap) = amZ_m^{(0)}(ap)$$

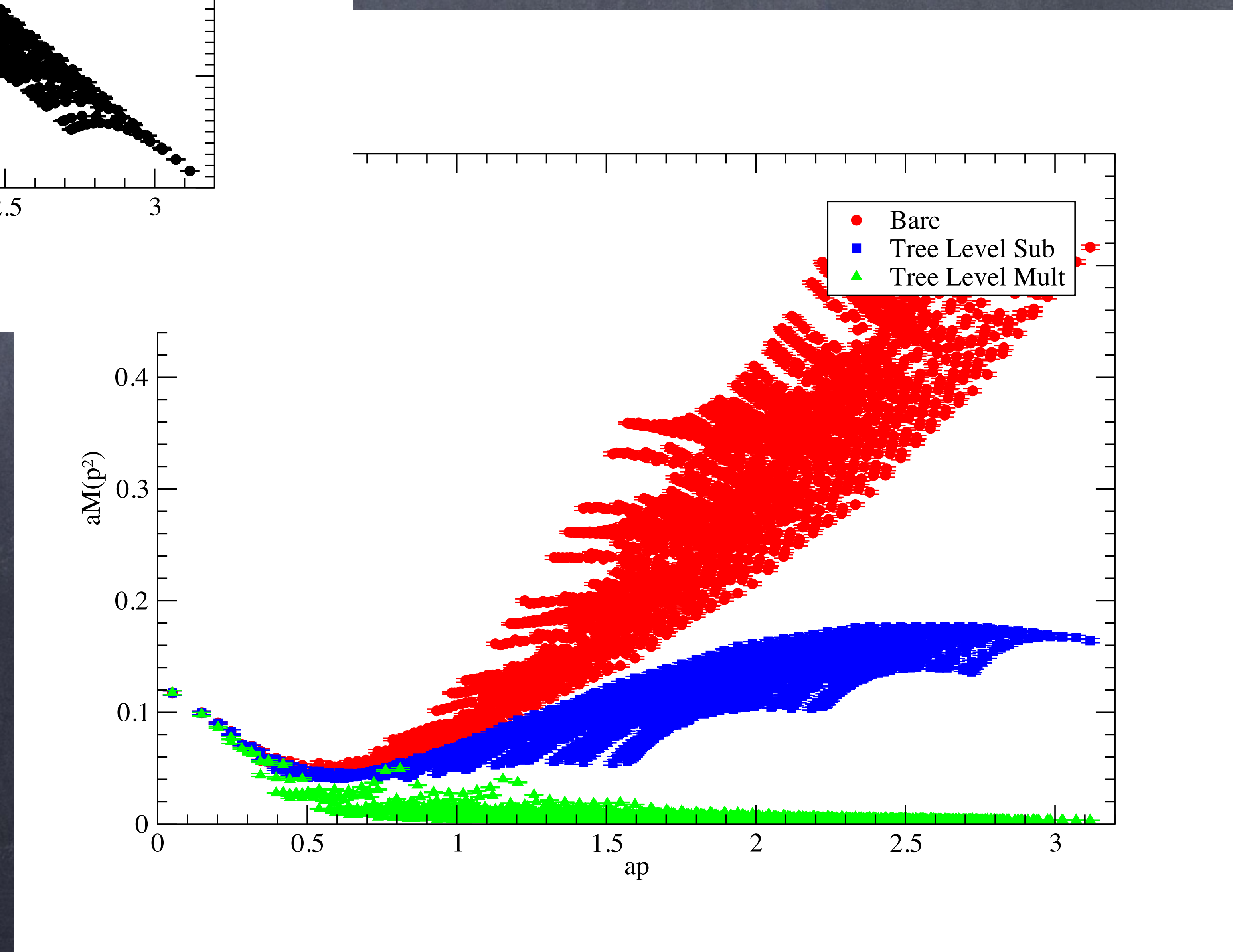
$$S^{-1}(p) = \frac{ia\not{k} + aM^L(ap)}{Z^L(ap)}$$

$$Z(ap) = \frac{Z^L(ap)}{Z^{(0)}(ap)}$$

$$aM(ap) = aM^L(ap) - a\Delta M^{(0)}(ap)$$

$$aM(ap) = \frac{aM^L(ap)}{Z_m^{(0)}}$$

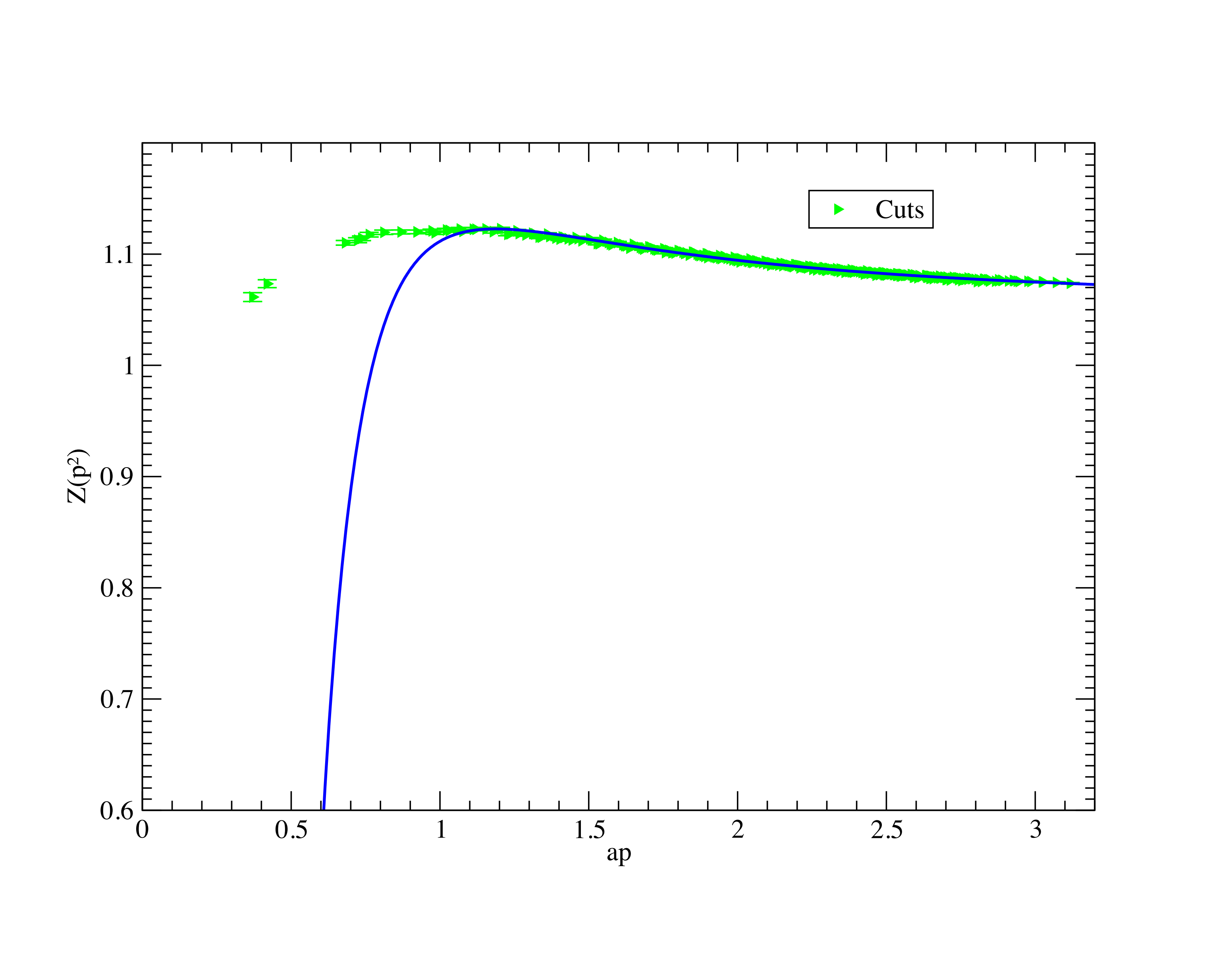




Preliminary data

Momentum Cuts

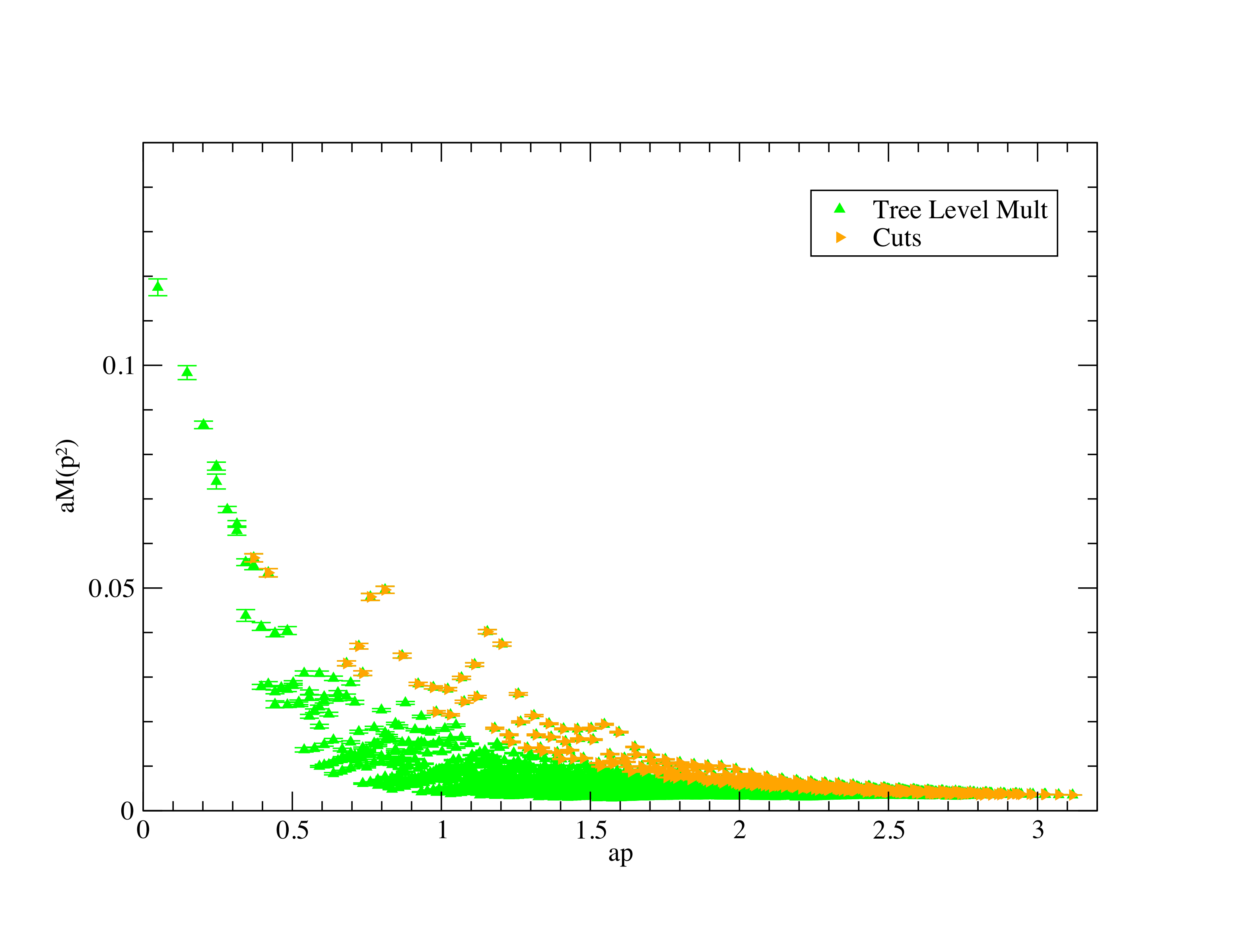
$$p = (1,1,1,1) \qquad \frac{p^{[4]}}{(p^{[2]})^2} = \frac{1}{4} \qquad \frac{p^{[4]}}{(p^{[2]})^2} < 0.3$$



$$Z(p^2) = z_0 + \frac{z_1}{p^2} + \frac{z_2}{p^4}$$

$$z_0 = 1.0555(2)$$
$$z_1 = 0.189(1)$$
$$z_2 = -0.133(2)$$
$$\chi^2 = 1.6$$
$$p > 1 (= 2.741 \text{ GeV})$$

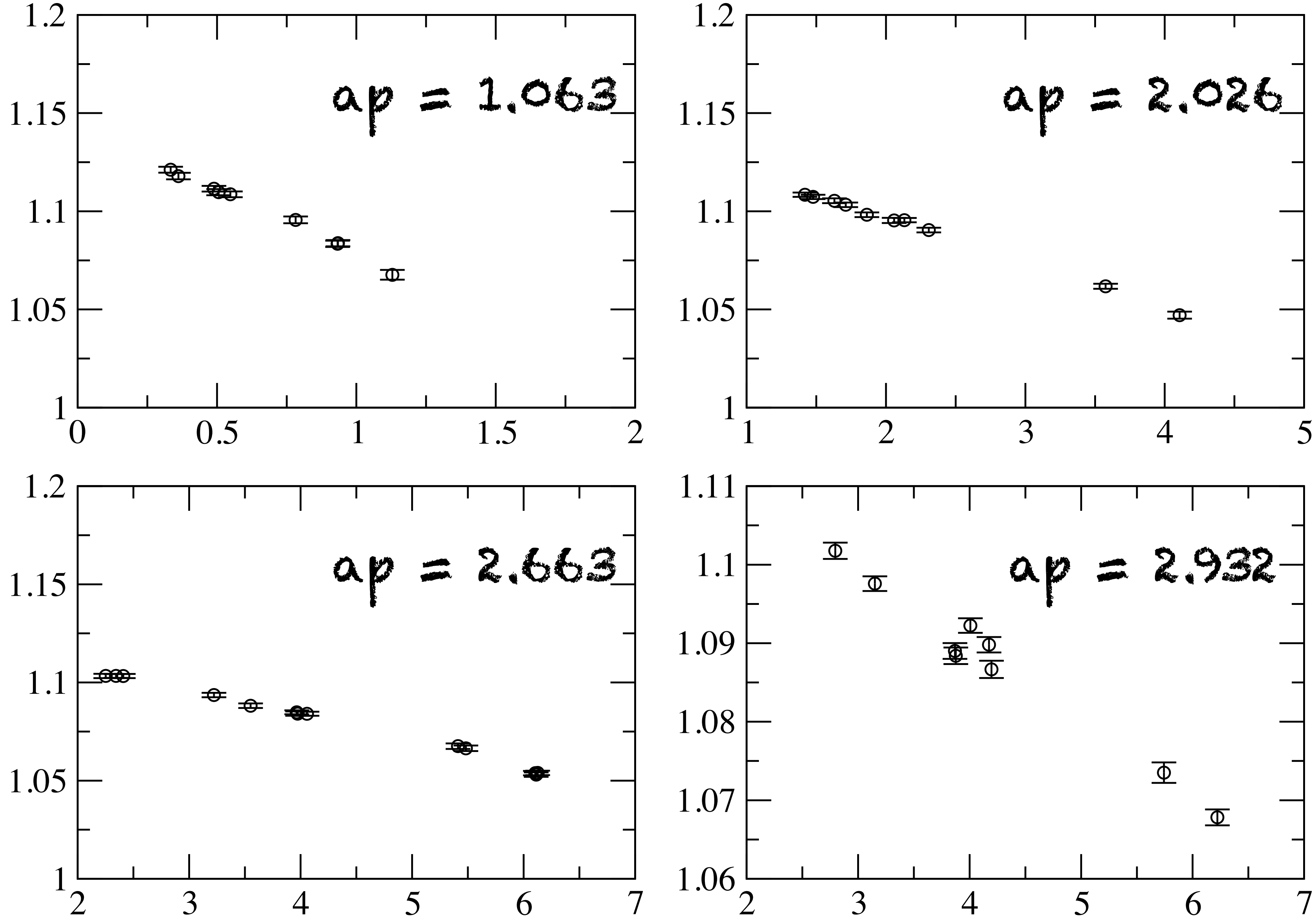
Preliminary data



$$am_q = (3.595 \pm 0.014)\times 10^{-3}$$
$$m_q = 9.854 \pm 0.038 \text{ MeV}$$

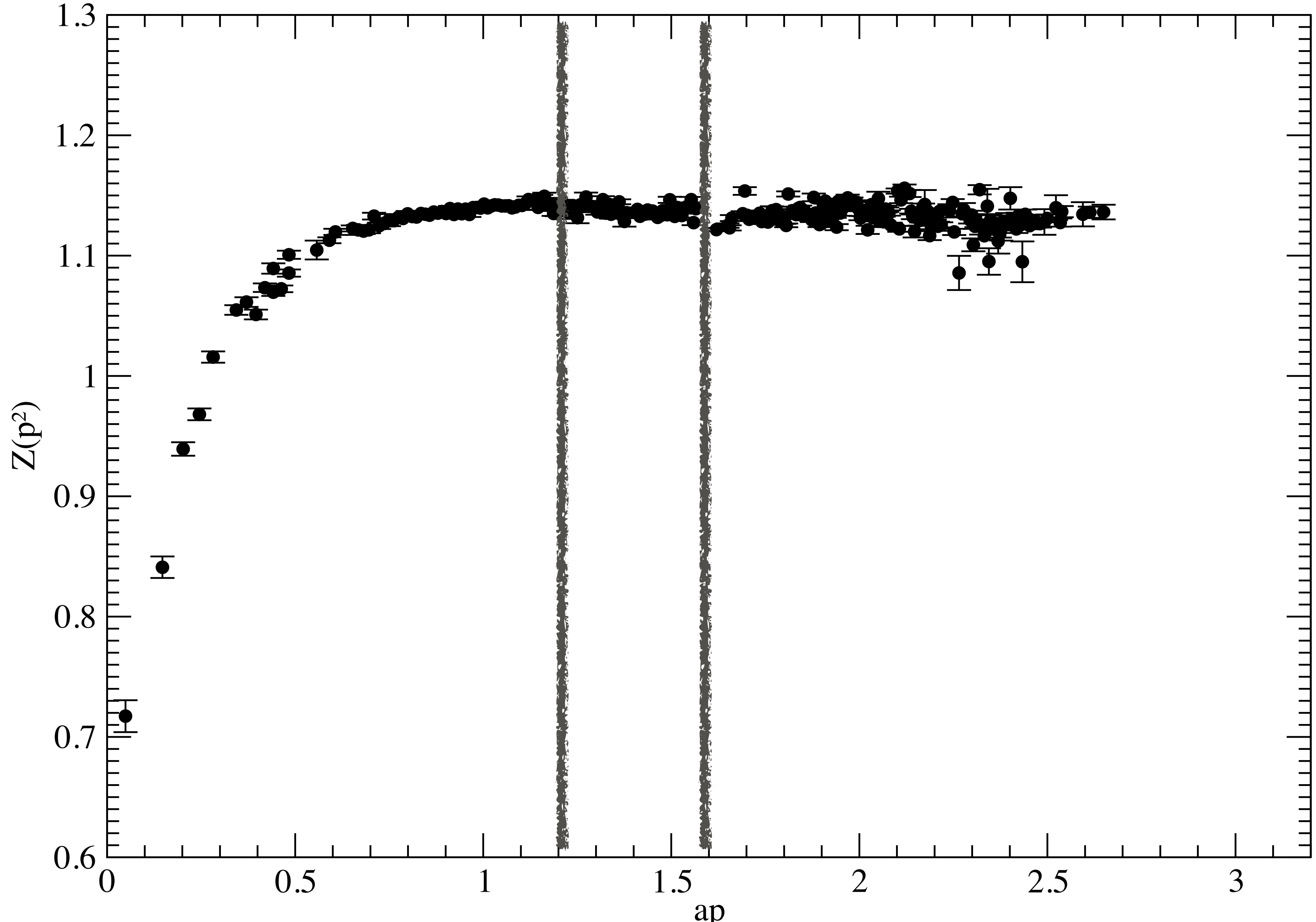
$$a^{-1} = 2.741 \text{ GeV}$$

Z(p^2) as function of $p^{[4]}$



Preliminary data

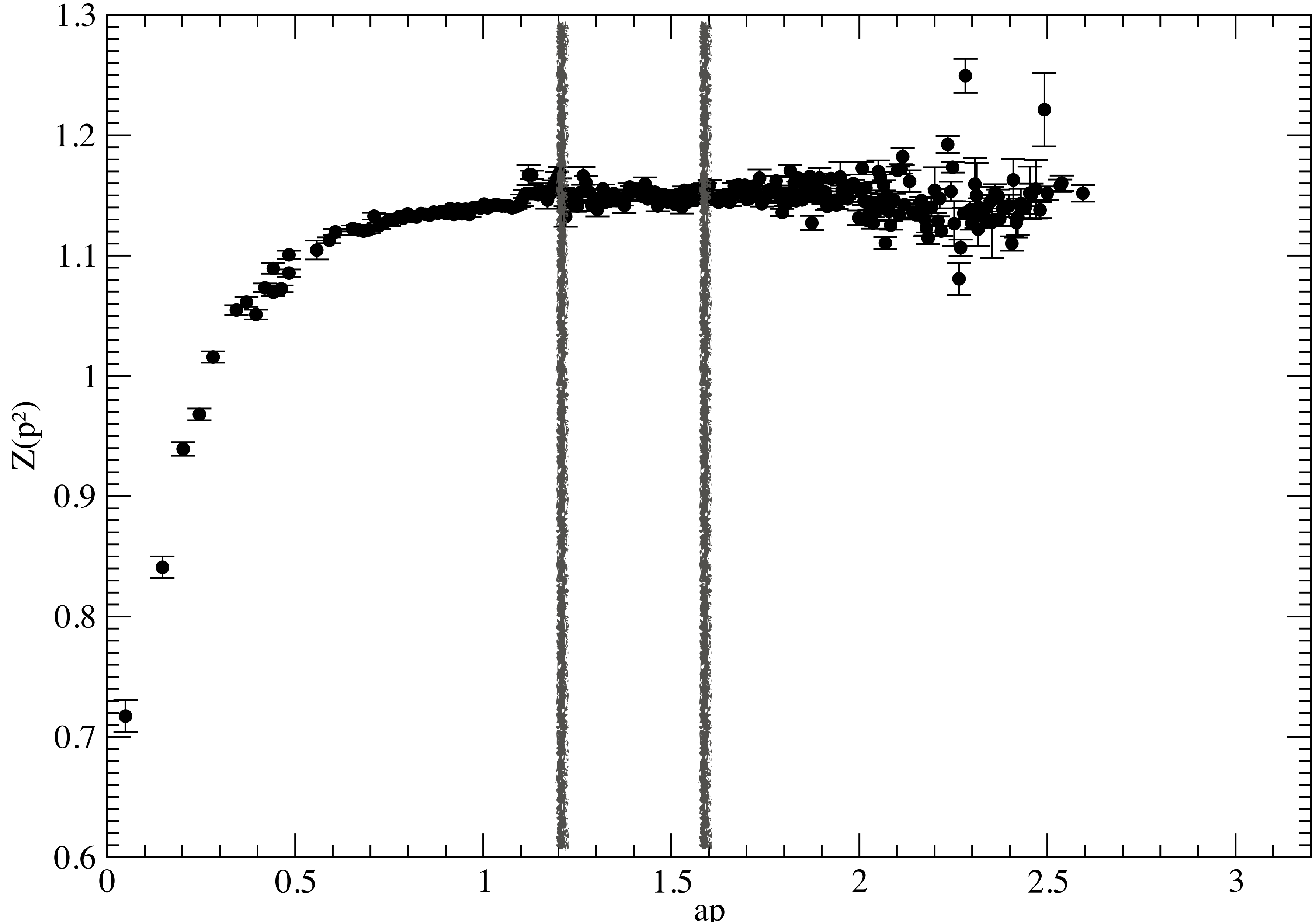
p^6 Ext.



$Z = 1.1480(40)$

Preliminary data

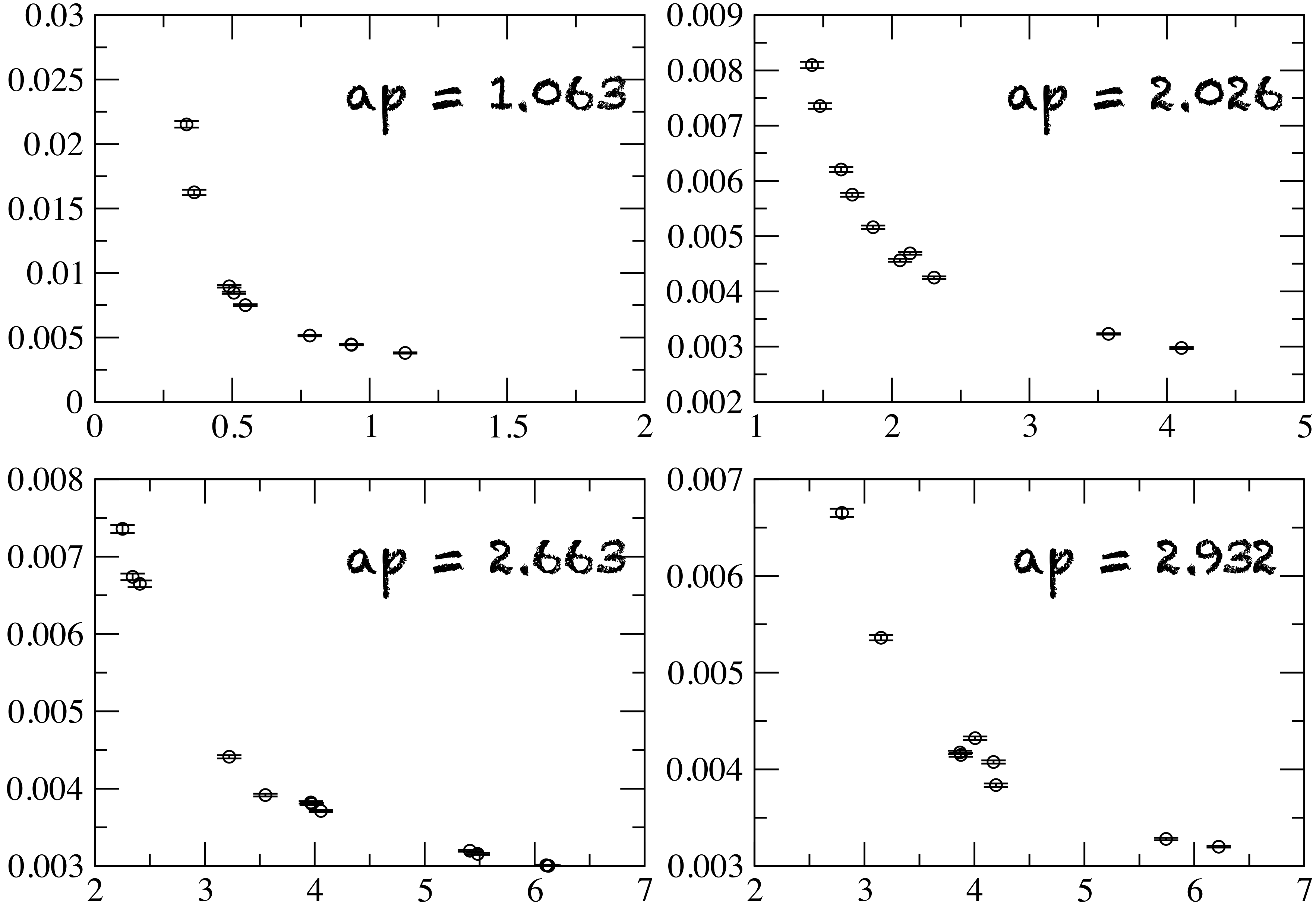
p^8 Ext.



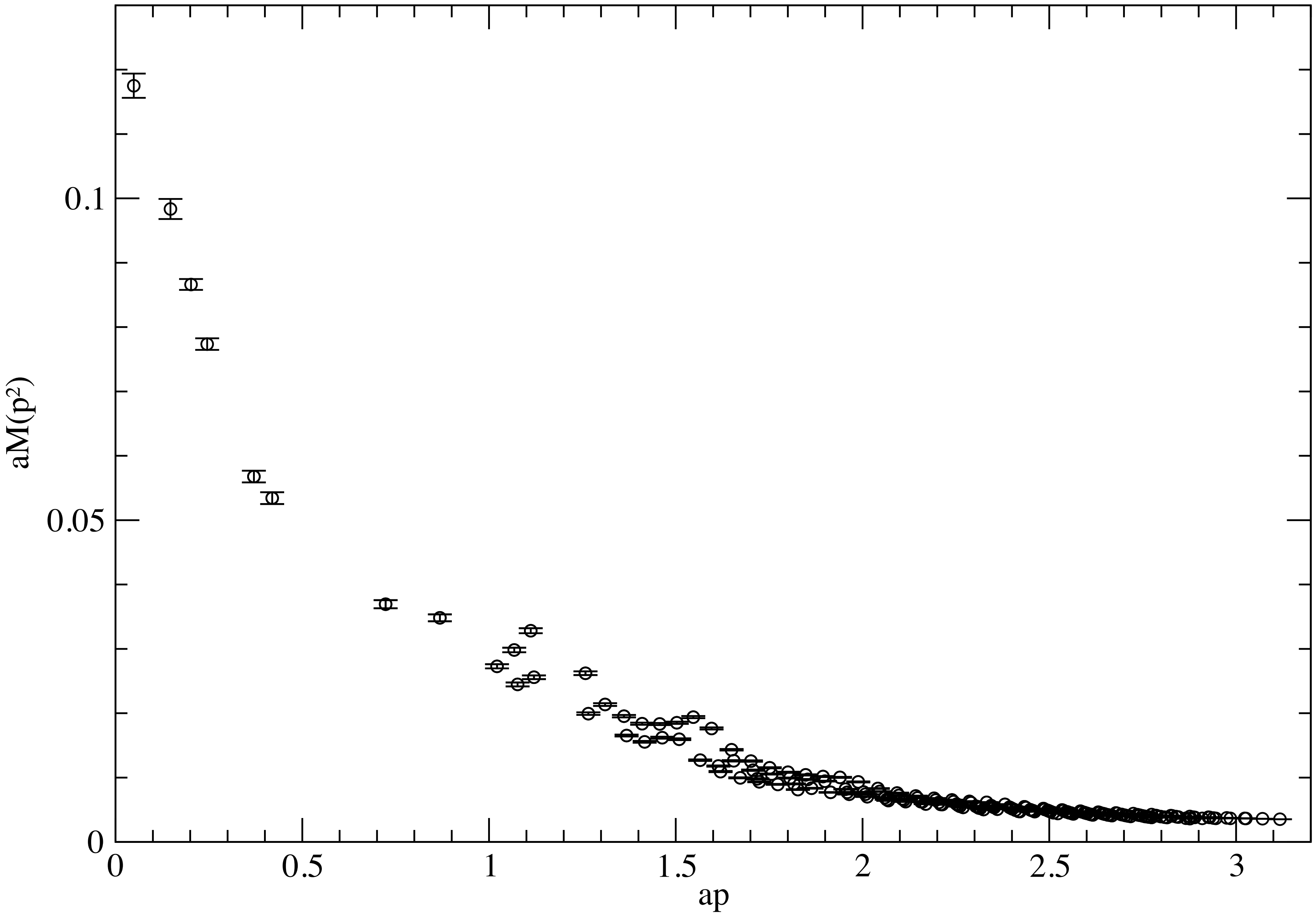
$Z = 1.1491(43)$

Preliminary data

aM(p²) as function of p[4]



Preliminary data

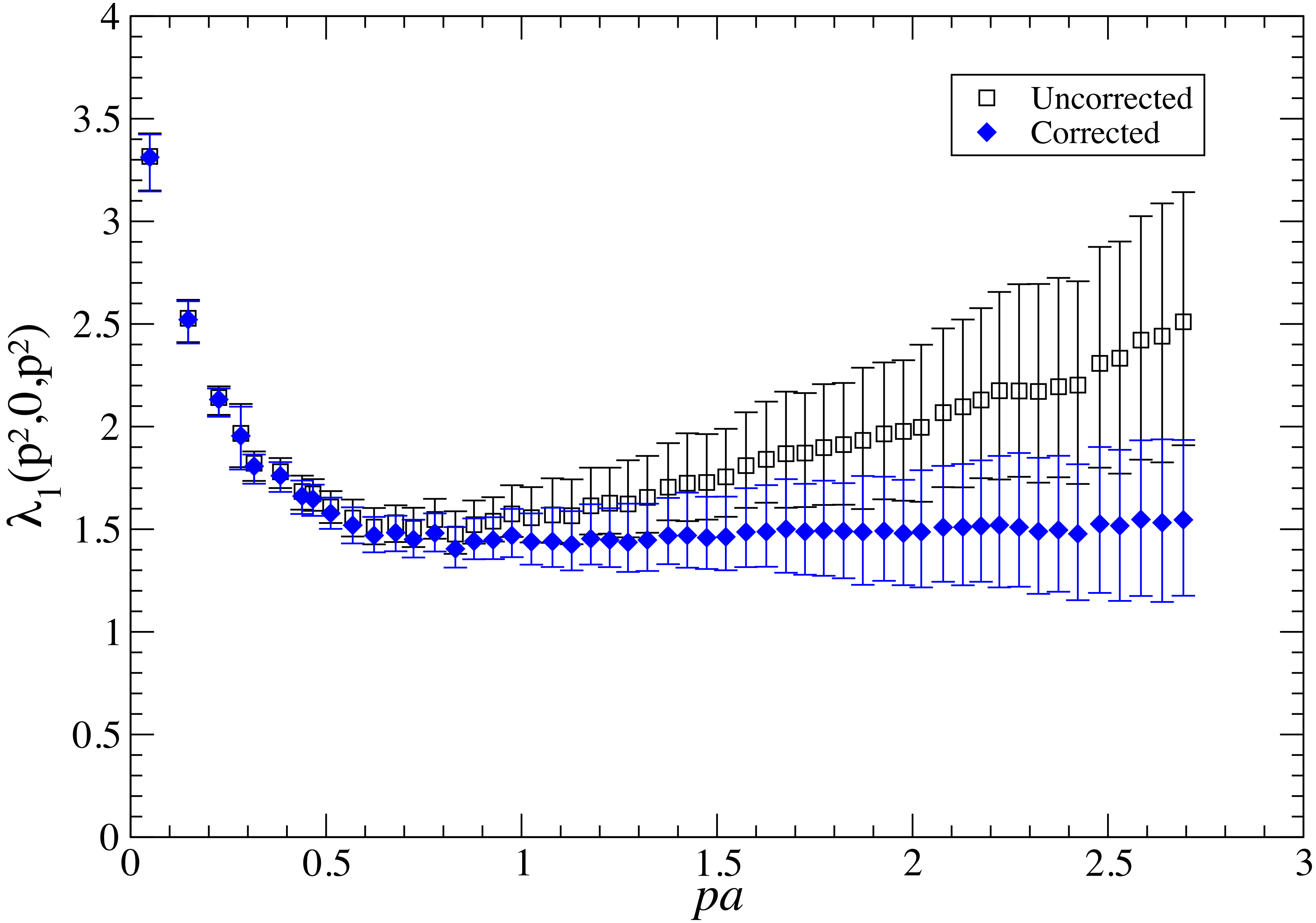


Preliminary data

Summary and outlook

- dont forget that it is a preliminary analysis
- closer to the chiral limit
- DCSB with a $M(0) \simeq 322.1 \pm 5.2$ MeV
- finite size effects for $Z(p^2)$ under control
- need to be understood better for $M(p^2)$
- check other m and lattice spacing

zero gluon momentum



γ_μ

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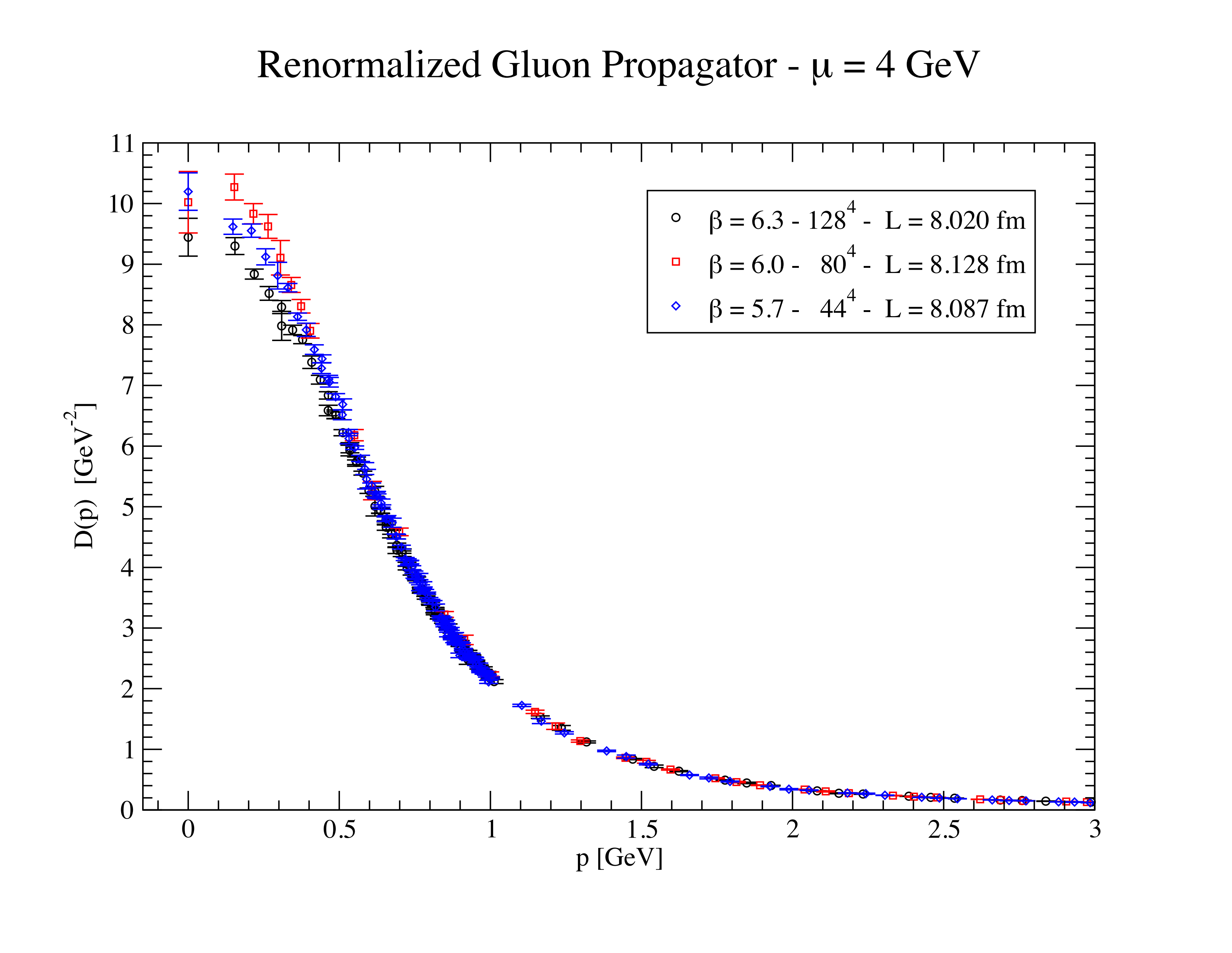
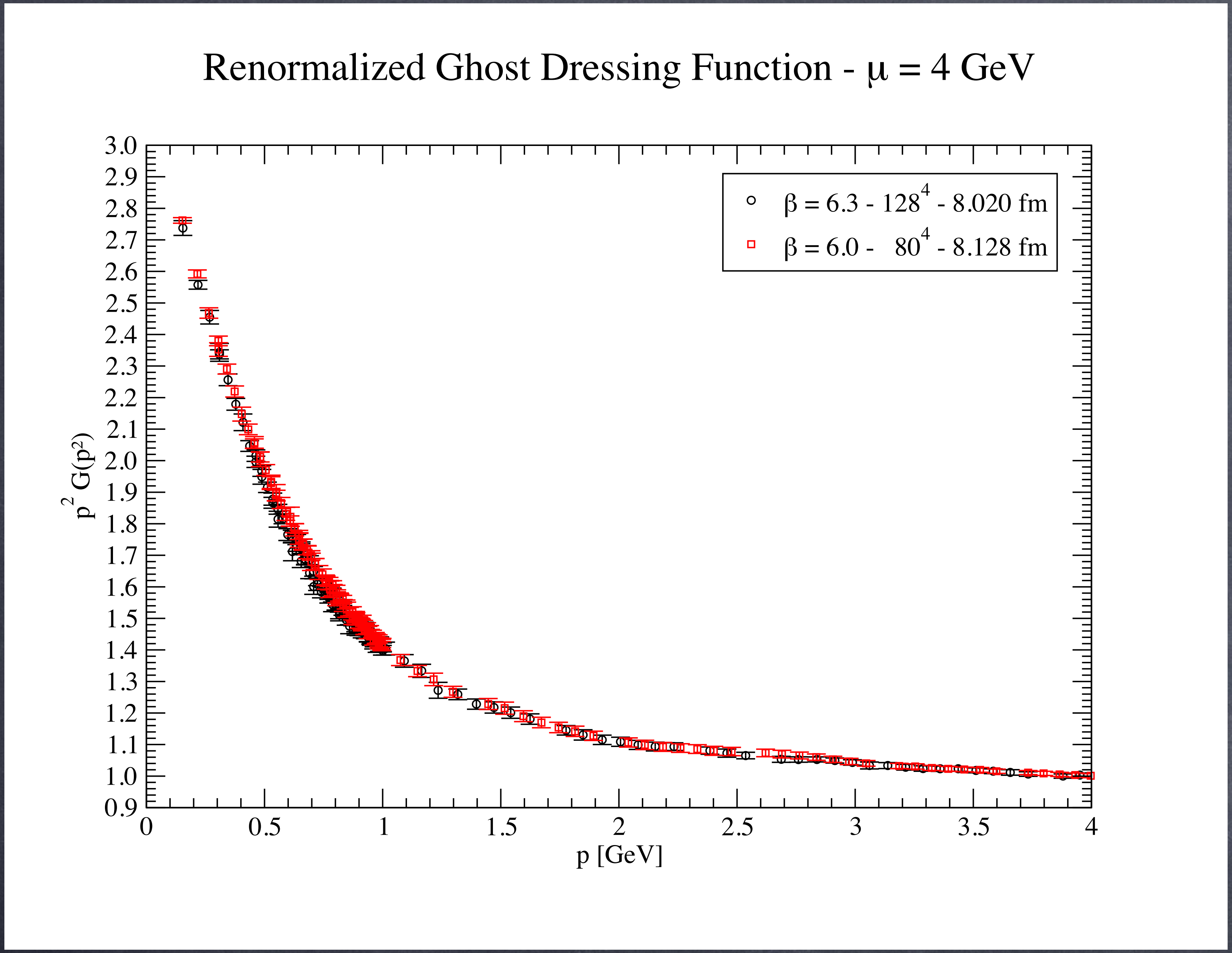
Landau gauge



$$M(p^2) = \frac{m_0^4}{m_0^2 + p^2}$$

O. Oliveira, P. Bicudo,
J Phys G38, 045003 (2011)

effective gluon mass



Renormalized Ghost Dressing Function - μ = 4 GeV
β = 6.3 - 128^4 - 8.020 fm
β = 6.0 - 80^4 - 8.128 fm
p^2 G(p^2)
p [GeV]
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0
0.5
1
1.5
2
2.5
3
3.5
4

Renormalizable R$_\xi$ gauges

A. Cucchieri, T. Mendes, and E. M. Santos,
Phys. Rev. Lett. 103, 141602 (2009)

- Two integrations: links and gaussian distributed matrices Λ

$$P[\Lambda^a(x)] \propto \exp\left\{ \frac{-1}{2\xi} \sum_a (\Lambda^a(x))^2 \right\}$$

$$F[g] = \sum_{x,\mu} \mathrm{Re}\ \mathrm{Tr}\left[g(x) U_\mu(x) g^\dagger(x+\mu) \right] + \sum_x \mathrm{Re}\ \mathrm{Tr}\left[i g(x) \Lambda(x) \right]$$

$$\partial A(x) = \Lambda(x) \qquad U_\mu(x) = \exp i a g_0 A_\mu(x+\mu/2)$$

- non trivial minimization problem

$$g = \Pi_i \delta g_i$$

$$\delta g_i = 1 + i \sum_j \omega^a(x) t^a$$

choosen to minimize F[g]

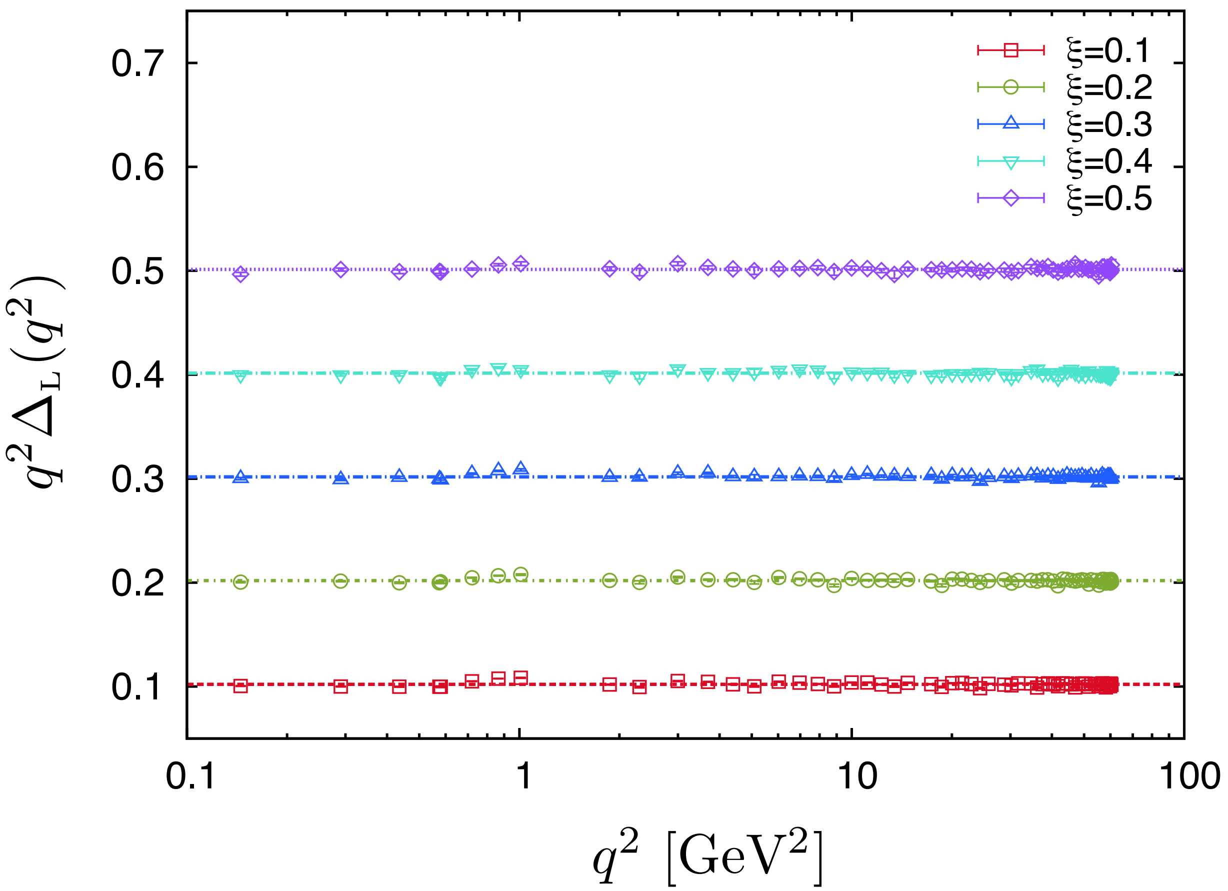
$$\Delta(x) = \sum_\mu g_0 \nabla A_\mu(x) - \Lambda(x)$$

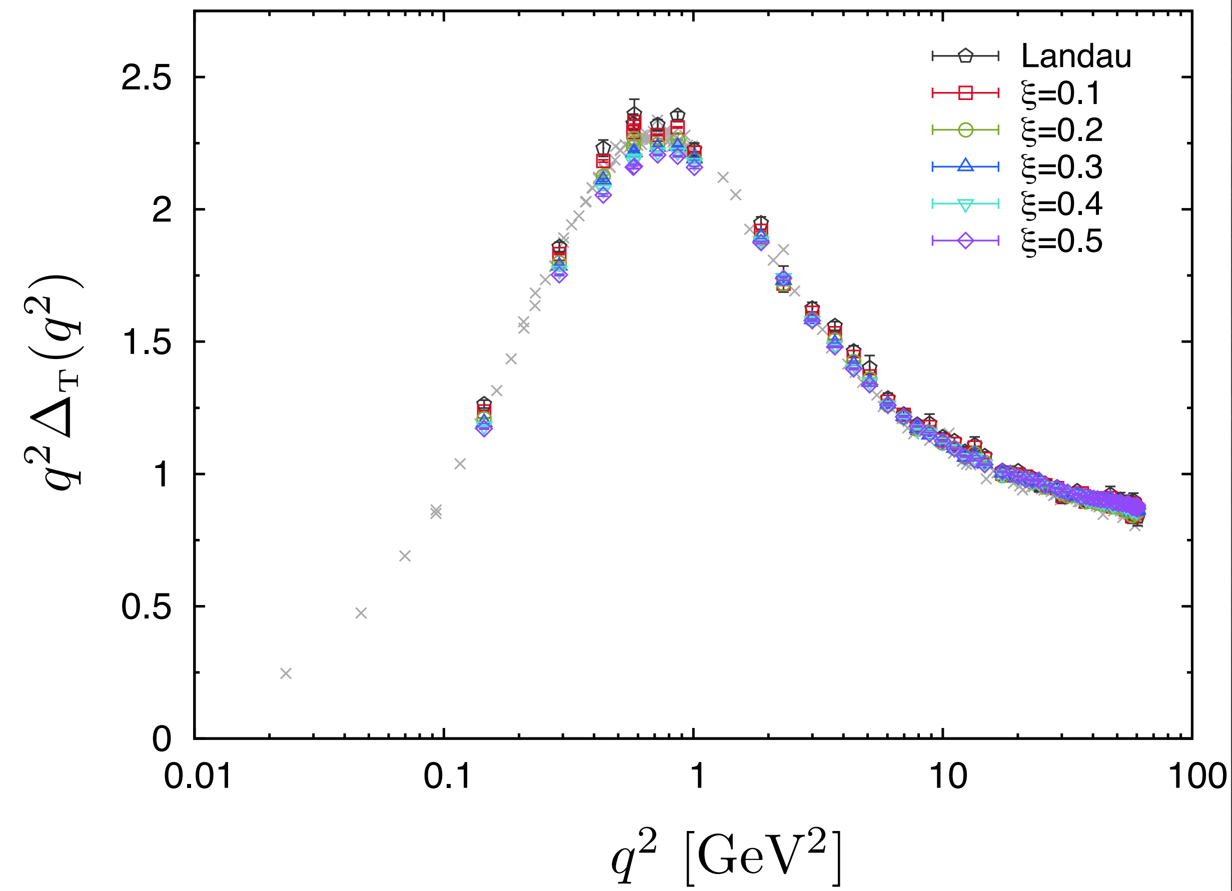
$$\theta = \frac{1}{NL^4} \sum_x \mathrm{Tr}\left[\Lambda(x)\Lambda^\dagger(x)\right] < 10^{-15}$$

$$D_{\mu\nu}(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D_T(p^2) + \frac{p_\mu p_\nu}{p^2} D_L(p^2)$$

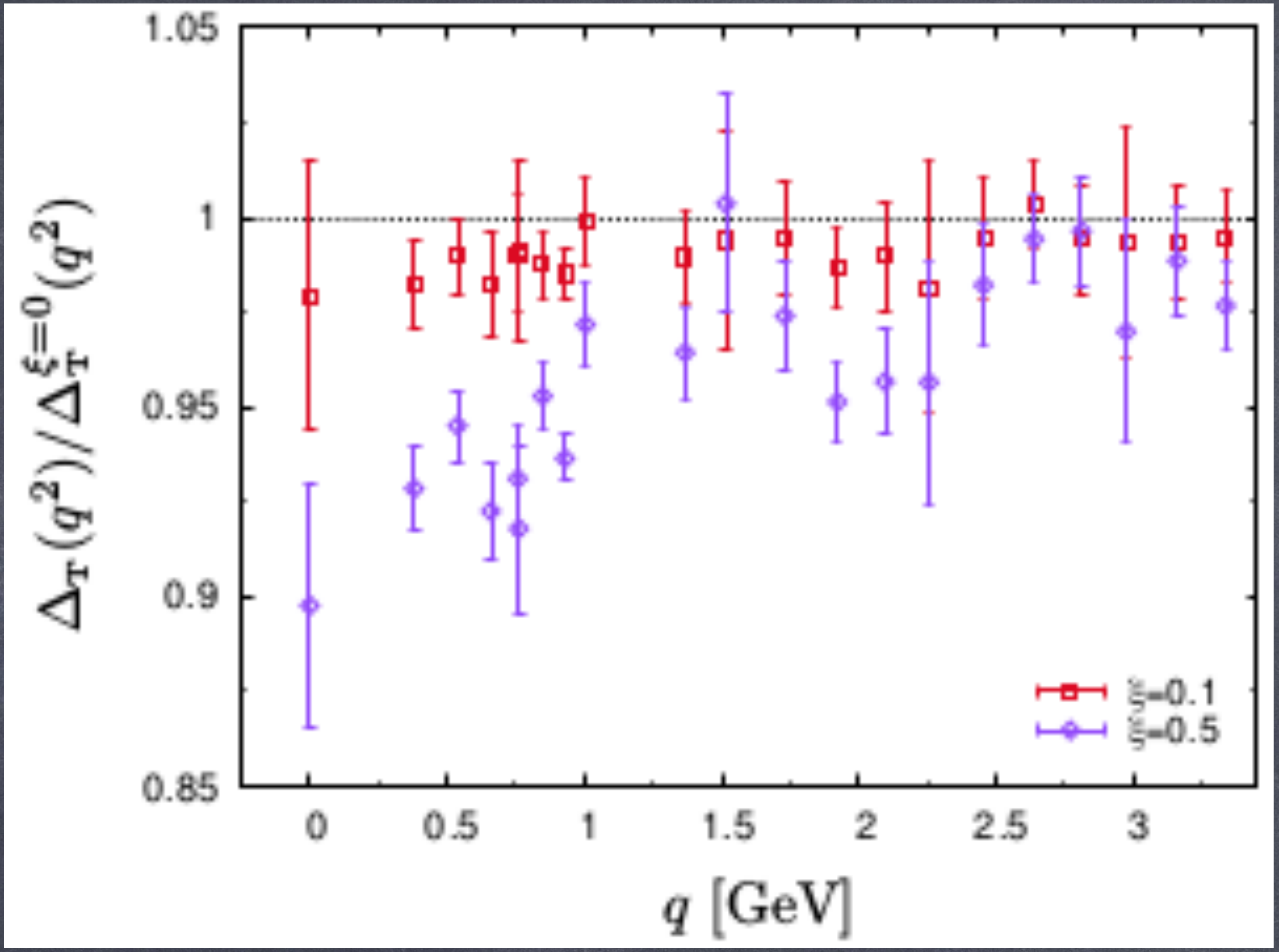
Lattice Setup + Gauge Fixing

- Wilson action (50 configurations)
- 32^4 $\beta = 6.0$ $a = 0.1016(25)$ fm
- 50 Λ configurations for each U_μ
- FFT steepest descent + overrelaxation + stochastic overrelation + restart after random gauge transformation
- $\xi = 0.1, 0.2, 0.3, 0.4, 0.5$
- data renormalised at $\mu = 4.317$ GeV

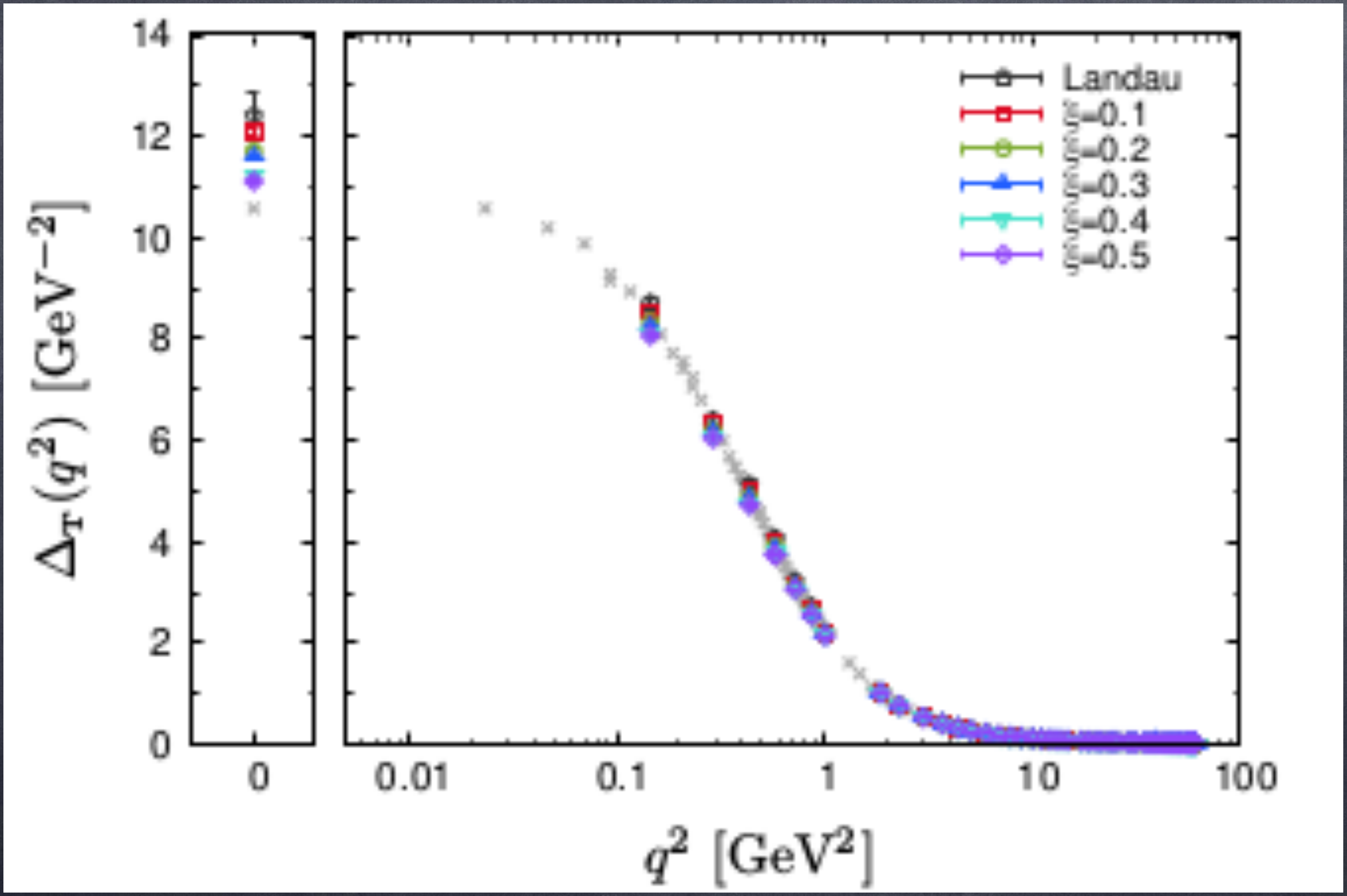




ξ = 0.103(2), 0.203(2), 0.302(3), 0.402(3), 0.502(3)



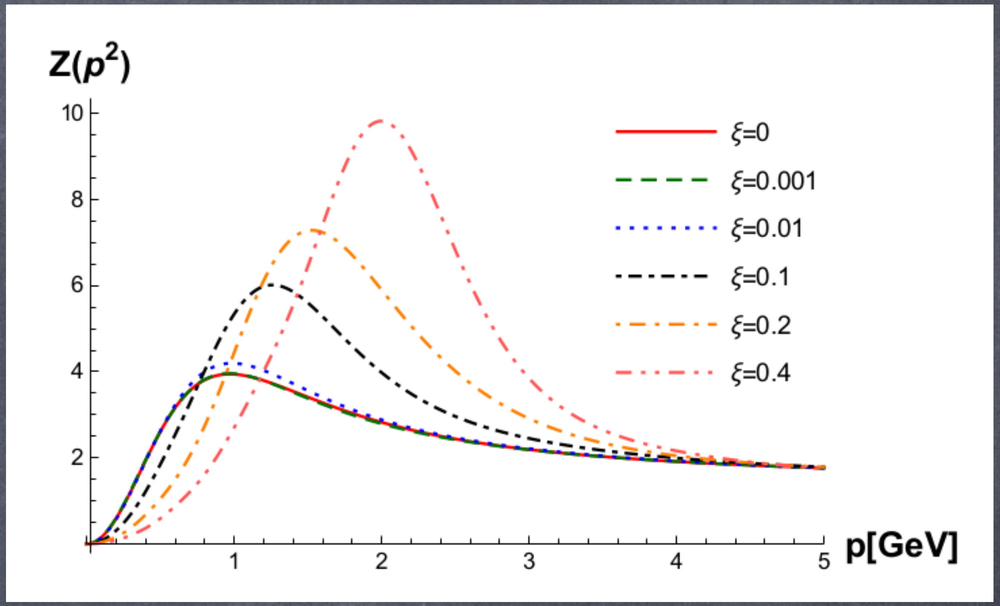
1.05
1
0.95
0.9
0.85
0
0.5
1
1.5
2
2.5
3
$\Delta_T(q^2)/\Delta_T^{\xi=0}(q^2)$
q [GeV]
ξ=0.1
ξ=0.5



Landau
ξ=0.1
ξ=0.2
ξ=0.3
ξ=0.4
ξ=0.5
$\Delta_T(q^2)$ [GeV^{-2}]
q^2 [GeV^2]

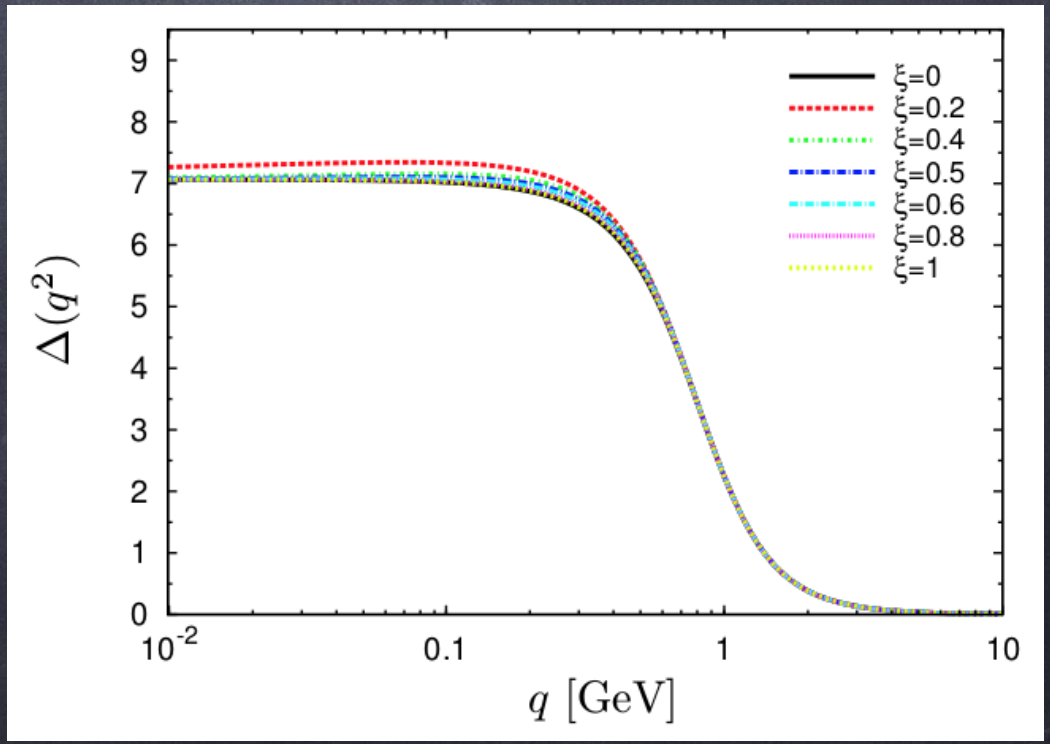
Some differences with

M. Q. Huber, Phys.Rev. D91, 085018 (2015)



Nice agreement

A. Aguilar, D. Binosi, and J. Papavassiliou, Phys. Rev. D91, 085014 (2015)

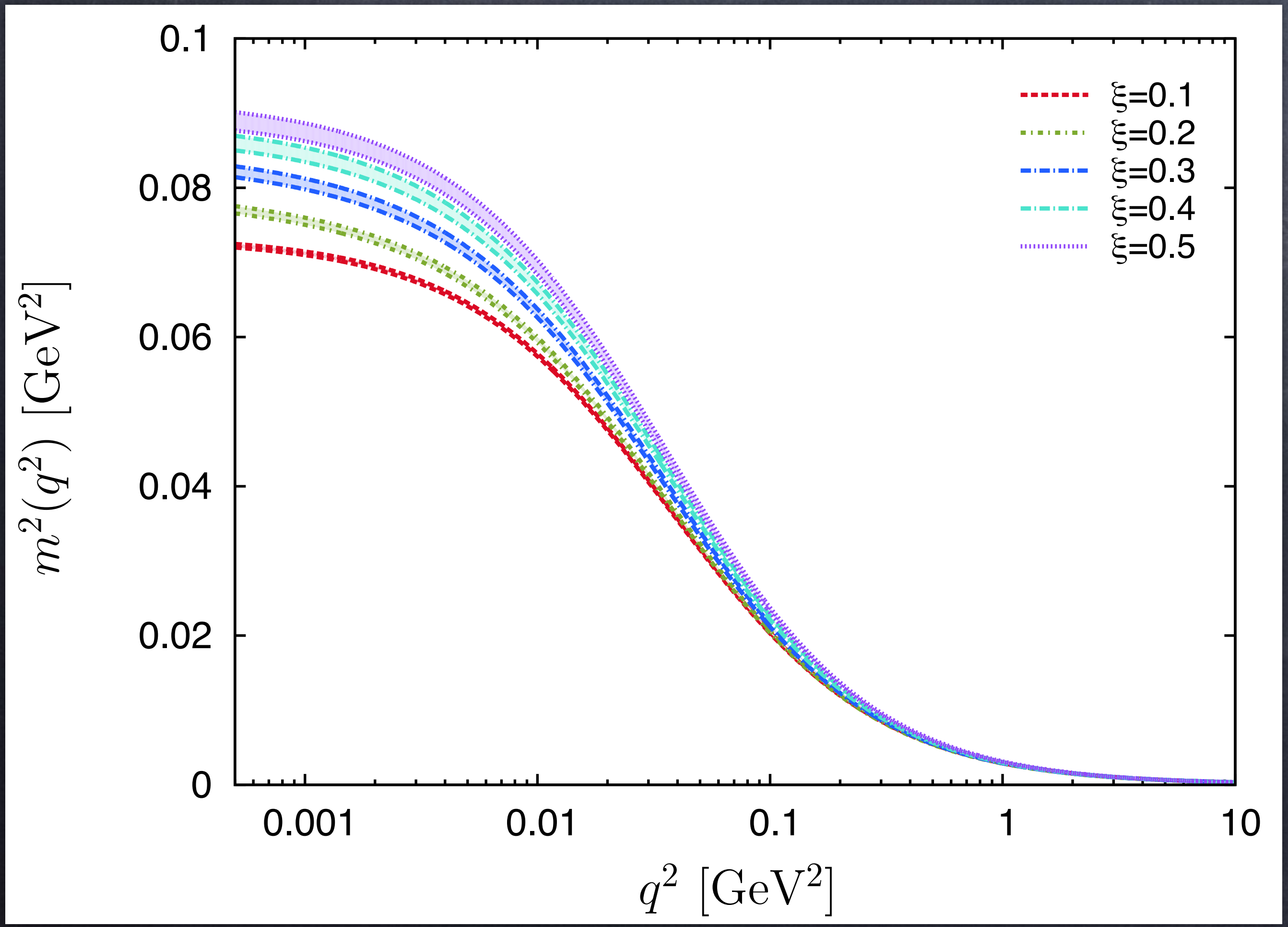


$$M^2(p^2) = \left[a(\xi) + c(\xi) \left(\frac{p^2}{\mu^2} \right)^{\xi} \log \frac{p^2}{\mu^2} \right] M^2_{\xi=0}(p^2)$$

$$a(\xi) = 1 + a_1 \xi$$

$$c(\xi) = c_{NI} \xi \qquad c_{NI} \approx 0.32$$

$a_1 \approx 0.26$ fit mass at $p^2 = 0$



Summary and Outlook

- Gauge fixing is the most intensive numerical part of the calculation
- The gluon propagators in R_ξ gauges for a lattice volume large enough to access the IR
- Data shows an inflection point (few hundreds MeV) and a saturation in the IR
- Suggests that a dynamical gluon mass generation is a common feature of all R_ξ gauges