Effective models for confinement

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QCD succesfully describes:

- High Energy Physics
- Low Energy Physics:

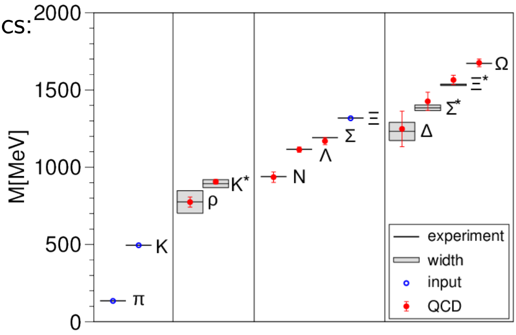


Figure:
Experimental masses compared with QCD ab-initio calculations (in red).
S. Dürr et al., Science, 2008

The heavy quark potential in lattice YM

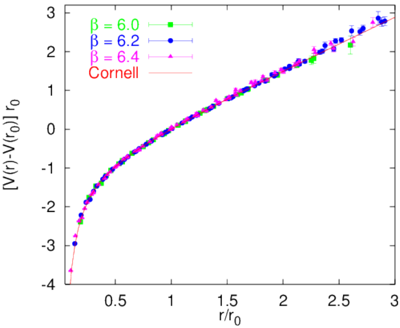


Figure: Asymptotic linearity. From Bali (2001)

- with string-like behavior ($1/R$ correction), Lüscher & Weisz (2002)

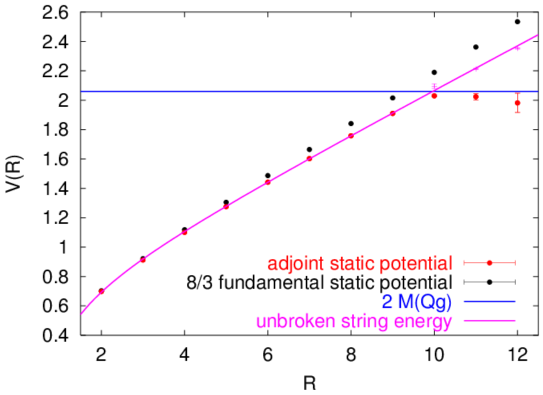


Figure: N-ality: at asymptotic distances, string tensions depend on how the center is realized in a given rep. of $SU(N)$.
From de Forcrand & Kratochvila (2003).

Dual superconductivity ('t Hooft, Nambu, Mandelstam)

- Understand: search for chromomagnetic quantum d.o.f. in pure YM, such as center vortices and monopoles, that could capture the path integral measure: Greensite, Reinhardt, Faber, Di Giacomo, Lucini...
- Describe: search for effective dual models in a Higgs phase where the chromoelectric confining string is represented by a classical vortex solution:

  $SU(N) \to Z(N)$ SSB $\;\Rightarrow\;$ center vortices $\;\Rightarrow\;$ N-ality

  - Adjoint external sources would not be confined

  - minimum number of adjoint Higgs fields is N

Effective dual model

M. Baker, J. S. Ball & F. Zachariasen, PRD (1997), the interquark potential from dual model with SU(3) gauge group and three adjoint Higgs fields.

The potential is defined on an ansatz, with the Higgs fields along λ_1, λ_2 ; λ_4, λ_5 ; λ_6, λ_7 off-diagonal directions

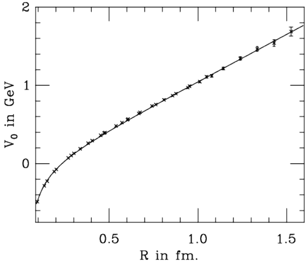


Figure: Dual model vs. Lattice. From Baker et al. 1997

Hybrid QCD spectrum

- Lattice calculations predict a rich spectrum of exotic mesons
- Some of them correspond to $qg\bar{q}'$ hybrid mesons. Nonsinglet colour pair and a valence gluon form a colourless state.

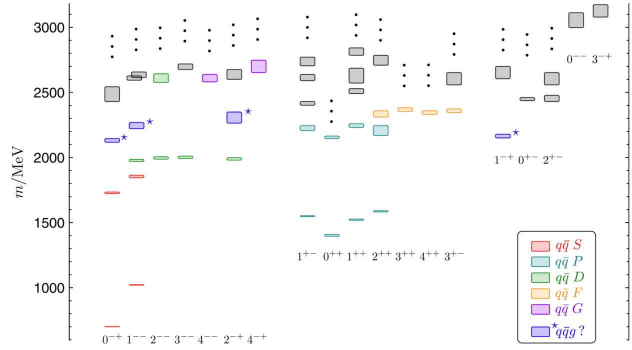
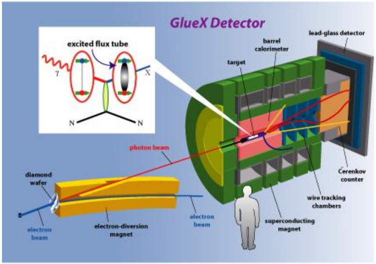
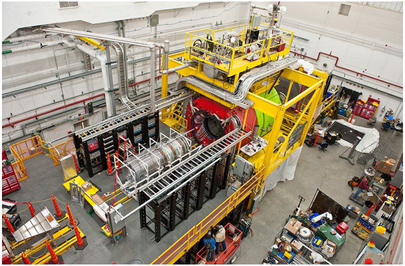


Figure: From B. Ketzer 2012

Gluex

This year, a collaboration based at the Jefferson Lab (GlueX) will start mapping gluonic excitations by searching hybrid meson states, generated by photoproduction





Heavy quark hybrid potentials in lattice YM

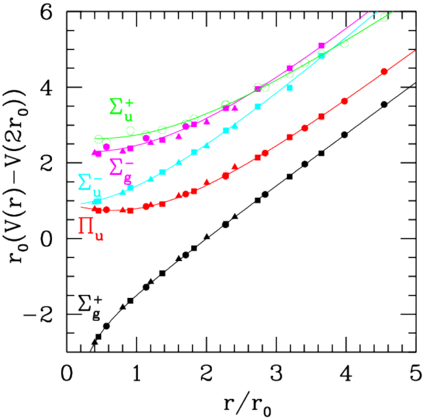


Figure:
Hybrid potentials. From K. J. Juge, J. Kuti & C. J. Morningstar 1997, Orlando Oliveira 2007

Effective dual model?

Phenomenological model that captures all the gluonic confining states?

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What's analogous to the Guinzburg-Landau model?

- We have recently proposed a class of Yang-Mills-Higgs (YMH) $4D$ effective models with $SU(N) \to Z(N)$

$$\frac{1}{2}\langle D_\mu \psi_I, D^\mu \psi_I \rangle + \frac{1}{4}\langle F_{\mu\nu}, F^{\mu\nu} \rangle - V_{\rm Higgs}(\psi_I)$$

$$D_\mu = \partial_\mu - ig[\Lambda_\mu, \ ] \quad , \quad F_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu - ig[\Lambda_\mu, \Lambda_\nu]$$

- $\psi_I \in \mathfrak{su}(N)$ is a set of adjoint Higgs fields

- Natural invariant terms to form $V_{\rm Higgs}(\psi_I)$

$$\langle \psi_I, \psi_J \rangle \quad , \quad \langle \psi_I, \psi_J \wedge \psi_K \rangle$$

$$\langle \psi_I \wedge \psi_J, \psi_K \wedge \psi_L \rangle \quad , \quad \langle \psi_I, \psi_J \rangle \langle \psi_K, \psi_L \rangle$$

- $$\psi_I \wedge \psi_J = -i[\psi_I, \psi_J]$$

Example

- $U(1) \to 1$: $\quad \phi = v\, e^{i\alpha} \;\to\; \bar{\phi}\phi - v^2 = 0 \;\to\; \frac{\lambda}{4}\,(\bar{\phi}\phi - v^2)^2$
- $SU(N) \to Z(N)$:

$$\phi_A = v\, R(S)|_{AB}\, T_B = v\, S T_A S^{-1} \;\to$$

$$\to\;\; \psi_A \wedge \psi_B - f_{ABC}\;\; v\, \psi_C = 0$$

$$\to\;\; V_{\rm Higgs} = \frac{\lambda}{4}\, \langle \psi_A \wedge \psi_B - f_{ABC}\;\; v\, \psi_C \rangle^2$$

$$\psi_A = \phi_A \quad , \quad \psi_A = 0\,.$$

- $I \to A = 1, \ldots, N^2 - 1$
- Flavour symmetric Higgs potential,

$$V_{\mathrm{Higgs}} = c + \frac{m^2}{2} \langle \psi_A, \psi_A \rangle$$
$$+ \frac{\gamma}{3} f_{ABC} \langle \psi_A \wedge \psi_B, \psi_C \rangle + \frac{\lambda}{4} \langle \psi_A \wedge \psi_B, \psi_A \wedge \psi_B \rangle$$

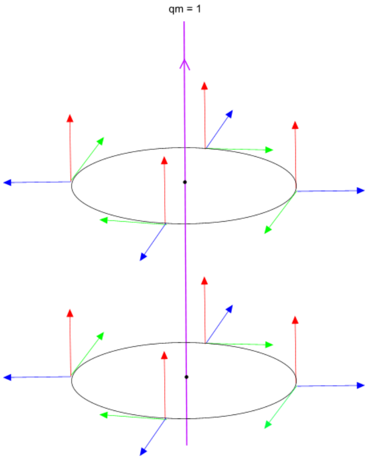
At $m^2 = \frac{2}{9} \frac{\gamma^2}{\lambda}$, $\gamma < 0$, defining $v = -\frac{2\gamma}{3\lambda}$: degenerate potential.

For $m^2 < \frac{2}{9} \frac{\gamma^2}{\lambda}$, the absolute minima are given by ϕ_A.

$SU(N) \to Z(N)$

- Manifold of absolute minima denoted by the tuple $(\phi_1, \phi_2, \dots) \in \mathcal{M}$
- $SU(N)$ symmetry $\Rightarrow (U\phi_1 U^{-1}, U\phi_2 U^{-1}, \dots)$ is also in $\mathcal{M}$
- a point in $\mathcal{M}$ is invariant iff $U \in Z(N)$
- $\mathcal{M} = SU(N)/Z(N) = Ad(SU(N))$
- $\Pi_1(Ad(SU(N))) = Z(N) \Rightarrow$ there are center vortices:
  - strings that confine quarks to form normal hadrons

$SU(2)$: the asymptotic Higgs fields for a center vortex



$SU(N)$: polar decomposition

- We can write $\psi_I = S q_I S^{-1}$ in term of "modulus" q_I and "phase" S variables,

$$\sum_I [q_I, u_I] = 0$$

$(u_1, u_2, \dots)$ is an x-independent reference point in $\mathcal{M}$

- $\Rightarrow (\phi_1, \phi_2, \dots)$, with $\phi_I = S\, u_I S^{-1}$, is the point in $\mathcal{M}$ closer to $(\psi_1, \psi_2, \dots)$

(the minimization of $\sum_I \langle \psi_I - \phi_I \rangle^2$ gives, $\sum_I [\psi_I, \phi_I] = 0$)

- For three adjoint fields ψ_1, ψ_2, ψ_3 in $SU(2)$ this corresponds to the usual polar decomposition of a 3×3 matrix

$SU(N)$: normal glue

Center vortices can be labelled by the weights of the different group representations, Konishi-Spanu (2001)

$$S = e^{i\varphi\, \vec{\beta}\cdot\vec{T}} \quad , \quad \vec{\beta} = 2N\, \vec{w}$$

A weight $\vec{w}$ is defined by the eigenvalues of diagonal generators corresponding to one common eigenvector.

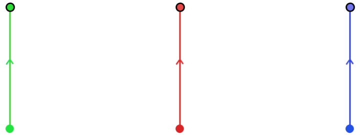
$$[T_q, T_p] = 0 \quad , \quad T_q\, eigenvector = \vec{w}|_q\, eigenvector$$

For the fundamental representation we have N weights $\vec{w}_i$ (fundamental colours), $\vec{\beta}_1 + \cdots + \vec{\beta}_N = \vec{0}$
They are associated with the simplest center vortices

$$e^{i2\pi\, \vec{\beta}_i\cdot\vec{T}} = e^{i\, 2\pi/N}\, I$$

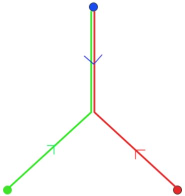
$SU(3)$: normal glue

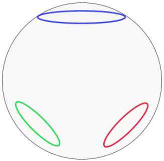
$\vec{\beta}_1, \vec{\beta}_2, \vec{\beta}_3$

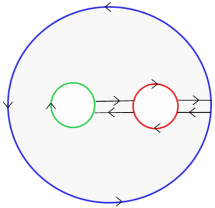


$$S = e^{i\chi_1 \vec{\beta}_1 \cdot \vec{T}} \, e^{i\chi_2 \vec{\beta}_2 \cdot \vec{T}}$$

$$\vec{\beta}_{c_1} + \vec{\beta}_{c_2} + \vec{\beta}_{c_3} = 0 \qquad (\vec{\beta}_1 + \vec{\beta}_2 = -\vec{\beta}_3)$$

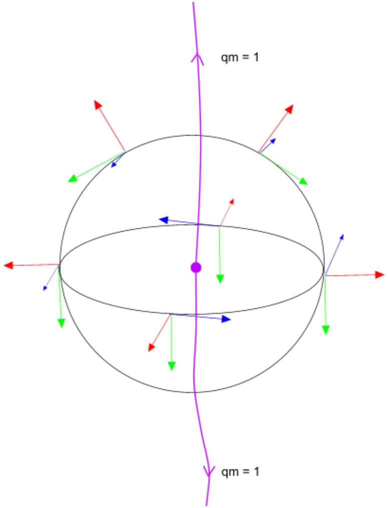






$SU(2)$: the non Abelian phase for a junction

$SU(2)$:



$SU(N)$: colour adaptor

Consider

$$S = e^{i\varphi\,\vec{\beta}_1\cdot\vec{T}}\,W(x) \quad , \quad W(x) = e^{i\theta\,\sqrt{N}T_\alpha}$$

Around the north pole,

$$S(x) \sim e^{i\varphi\,\vec{\beta}_1\cdot\vec{T}}$$

Around the south pole, $W(x) \sim W_\alpha = e^{i\pi\,\sqrt{N}T_\alpha}$, a Weyl reflection,

$$W_\alpha^{-1}\vec{\beta}\cdot\vec{T}\,W_\alpha = \vec{\beta}'\cdot\vec{T} \quad , \quad \vec{\beta}' = \vec{\beta} - 2\vec{\alpha}\,(\vec{\alpha}\cdot\vec{\beta})/\alpha^2$$

For $\vec{\alpha} = (\vec{\beta}_1 - \vec{\beta}_2)/2N$: $\vec{\beta}' = \vec{\beta}_2$, and the monopole charge is,

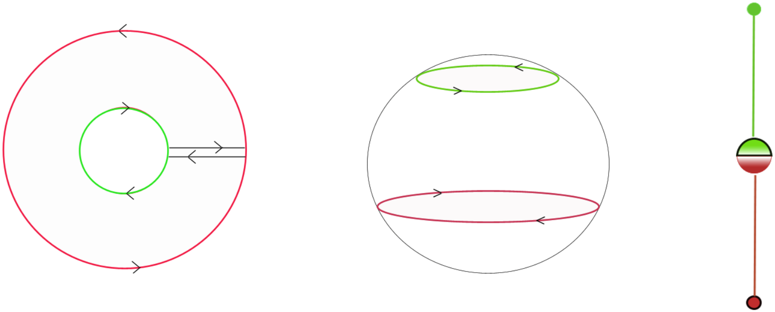
$$\vec{Q}_m = \frac{2\pi}{g} \, 2N \, \vec{\alpha} \; .$$

As the roots are the weights of the adjoint representation, which acts via commutators

$$[T_q, E_\alpha] = \vec{\alpha}|_q \, E_\alpha$$

the monopole can naturally be identified with a confined valence gluon with adjoint colour $\vec{\alpha}_{ij}$

$SU(3)$: colour adaptor



- We can start by giving a well defined configuration for the local Cartan directions on S^2

$$n_q = S T_q S^{-1} \quad , \quad q = 1, \ldots, N-1$$

- The set of n_q's can be identified with the quotient space $Ad(G)/Ad(H)$

$$G = SU(N) \quad , \quad H = U(1)^{N-1}$$

Exact sequence: $Im(f_1) = Ker(f_2)$, $Im(f_2) = Ker(f_3)$

$$\Pi_2(Ad(G)) \stackrel{f_1}{\to} \Pi_2\left(\frac{Ad\,(G)}{Ad\,(H)}\right) \stackrel{f_2}{\to} \Pi_1(Ad\,(H)) \stackrel{f_3}{\to} \Pi_1(Ad\,(G))$$

- $\Pi_2(Ad(G)) = 0 \;\Rightarrow\; Im(f_1) = 0 \;\Rightarrow\; Ker(f_2) = 0$

  (f_2 is injective)

- There is a one to one mapping between $\Pi_2\left(\frac{Ad\,(G)}{Ad\,(H)}\right)$ and $Im(f_2)$

- $Im(f_2) = Ker(f_3) =$ loops in $\Pi_1(Ad\,(H))$ that are trivial when seen as loops in $\Pi_1(Ad\,(G)) = Z(N)$

- This implies the condition

$$e^{i2\pi\,\vec{\beta}_{\mathrm{c}_1}\cdot\vec{T}}\,e^{i2\pi\,\vec{\beta}_{\mathrm{c}_2}\cdot\vec{T}}\ldots = I$$

  (closed loops in the fundamental rep of $SU(N)$, so they are contractible: $\Pi_1(SU(N)) = 0$)

- The loops in $\Pi_1(Ad(H))$ correspond to the composition of loops around the guiding centers

We described...

- the confinement of quarks to form normal states
- the confinement of quarks in a nonsinglet colour state plus a valence gluon to form hybrid states

  - the presence of local Cartan directions give a scale for the interpolating monopole

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  - the presence of local Cartan directions give a scale for the interpolating monopole
- what about (valence) gluon confinement?

  $\Pi_2(\mathcal{M}) = \Pi_2(Ad(SU(N))) = 0 \Rightarrow$ there are no isolated monopoles

  - valence gluons are confined

Discussion

- Phenomenological models to describe all the confining gluon states

  - the string tension as a function of N-ality?

  - the interquark potential in hybrid states?

- underlying phenomenological ensembles?

- effective adjoint Higgs fields and $SU(N) \to Z(N)$: N-ality with non Abelian monopole d.o.f. with adjoint charge?

- role played by center vortex d.o.f.? they are difficult to understand as they are two-dimensional objects in $4D$