The Faddeev-Popov operator and a scenario for matter confinement

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Outline

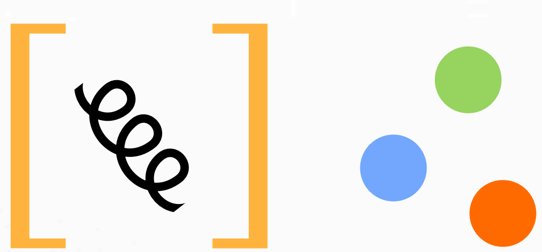
- Motivation: confinement
- Quantizing Yang-Mills theories: the Gribov approach
- An IR eff. action for non-Abelian theories: Refined-Gribov-Zwanziger theory
  - RGZ action and soft breaking of BRST
- Modelling color confinement through soft BRST breaking
  - quarks and adjoint scalars: tree-level propagators vs lattice data
  - absence from spectrum: positivity violation
  - physical spectrum of bound states
- Thermodynamics of confined quarks
- Final comments and perspectives

Motivation: the confinement problem

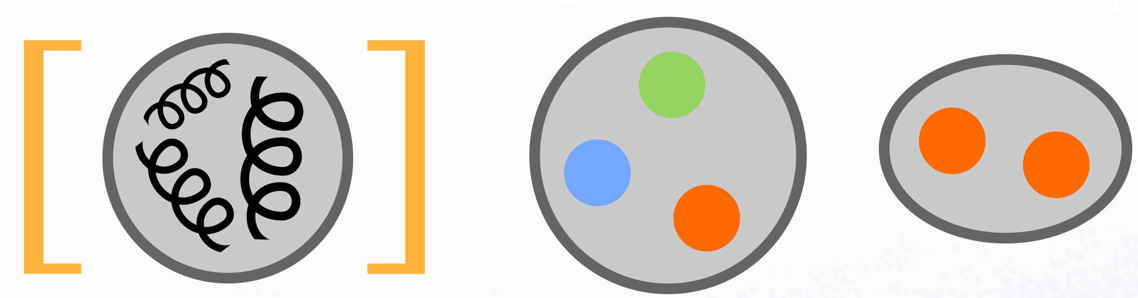
Fundamental degrees of freedom are ***unphysical***: not part of the spectrum; $\xrightarrow{\;?\;}$ **Physical spectrum of bound states** dynamically generated at low energies.

[Yang-Mills gauge theories] and QCD:

$$\mathcal{L}^{\mathrm{E}}_{\mathrm{YM}} = \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}\right] + \overline{\Psi}_f[\gamma_\mu D_\mu + m_f]\Psi_f$$

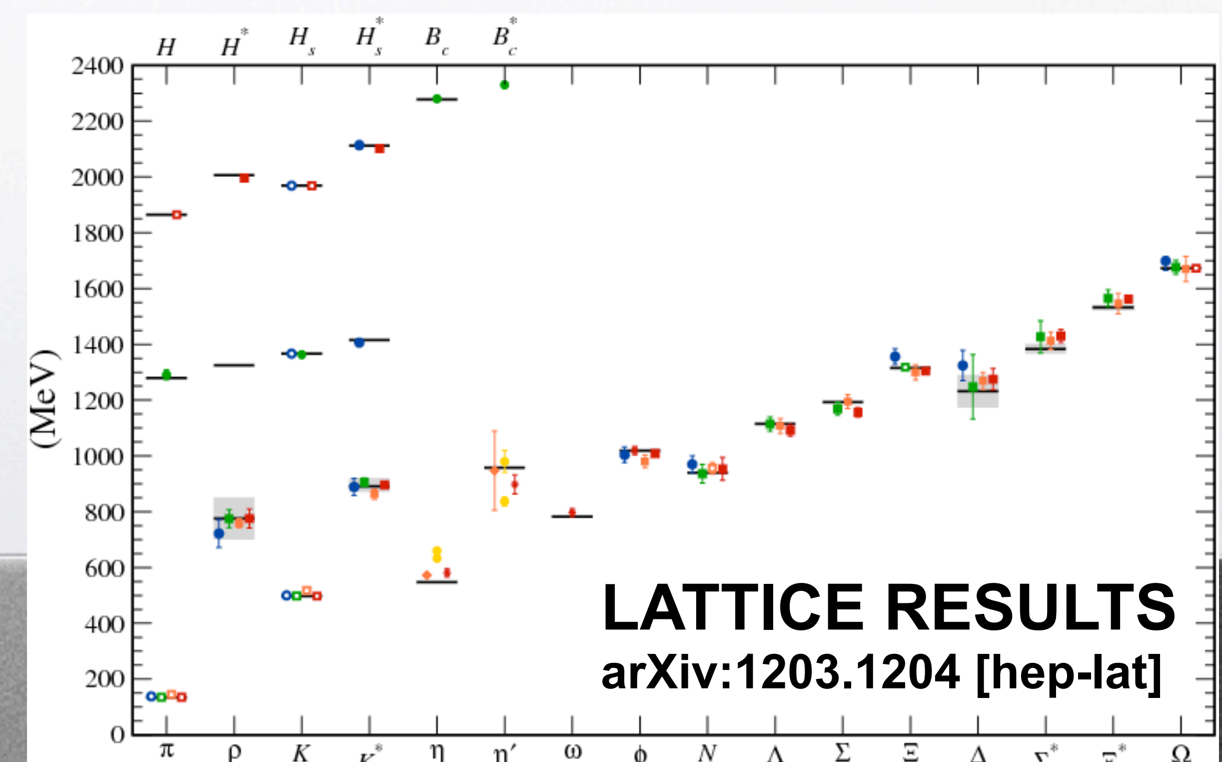


$\xrightarrow{\;?\;}$



[Gluons] and quarks — **[Glueballs,] baryons and mesons**

- **What is the mechanism??**
- **What happens to quarks and gluons in the IR?**



Yang-Mills theory

$$S_{YM} = \frac{1}{4}\int dx\, F^a_{\mu\nu} F^a_{\mu\nu},$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- It is not known how to formulate a quantum theory for this system.

$$\int \mathcal{D}A\, e^{-S_{YM}}$$

- Gauge redundancy must be fixed to properly define the path integral.

$$A_\mu \to U A_\mu U^\dagger - U \partial_\mu U^\dagger$$

- We only know how to do it perturbatively.

Faddeev-Popov procedure

- The procedure amounts to disentangle the gauge redundancy from the integral measure

$$\int \mathcal{D}A\, e^{-S_{YM}} \to \int \mathcal{D}\Omega \int \mathcal{D}A\, \delta[G(A)]\, \det \mathcal{M} e^{-S_{YM}}$$

supposing we can write

$$\int \mathcal{D}\Omega\, \delta[G(A)]\, \det \mathcal{M} = \mathbb{1}$$

with the Faddeev-Popov operator

$$\mathcal{M}^{ab}(A) = \left. \frac{\delta G^a[A^{(g)}]}{\delta \Omega^b} \right|_{\Omega=0}$$

The Gribov problem

[Gribov (1978)]

- In the Landau gauge, for instance, the theory assumes the form

$$\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}b\, e^{-S_{YM}+S_{gf}}$$

$$S_{gf} = b^a \partial_\mu A^a_\mu - \bar{c}^a \mathcal{M}^{ab} c^b \,, \qquad \mathcal{M}^{ab} = -\partial_\mu \left(\delta^{ab}\partial_\mu + g f^{abc} A^c_\mu\right)$$

- Gribov copies $\rightarrow$ zero eigenvalues of the Faddeev-Popov operator $\mathcal{M}^{ab}$.
- Copies cannot be reached by small fluctuations around $A = 0$ (perturbative vacuum) $\rightarrow$ pertubation theory works.
- Once large enough gauge field amplitudes have to be considered (non-perturbative domain) the copies will show up enforcing the effective breakdown of the Faddeev-Popov procedure.

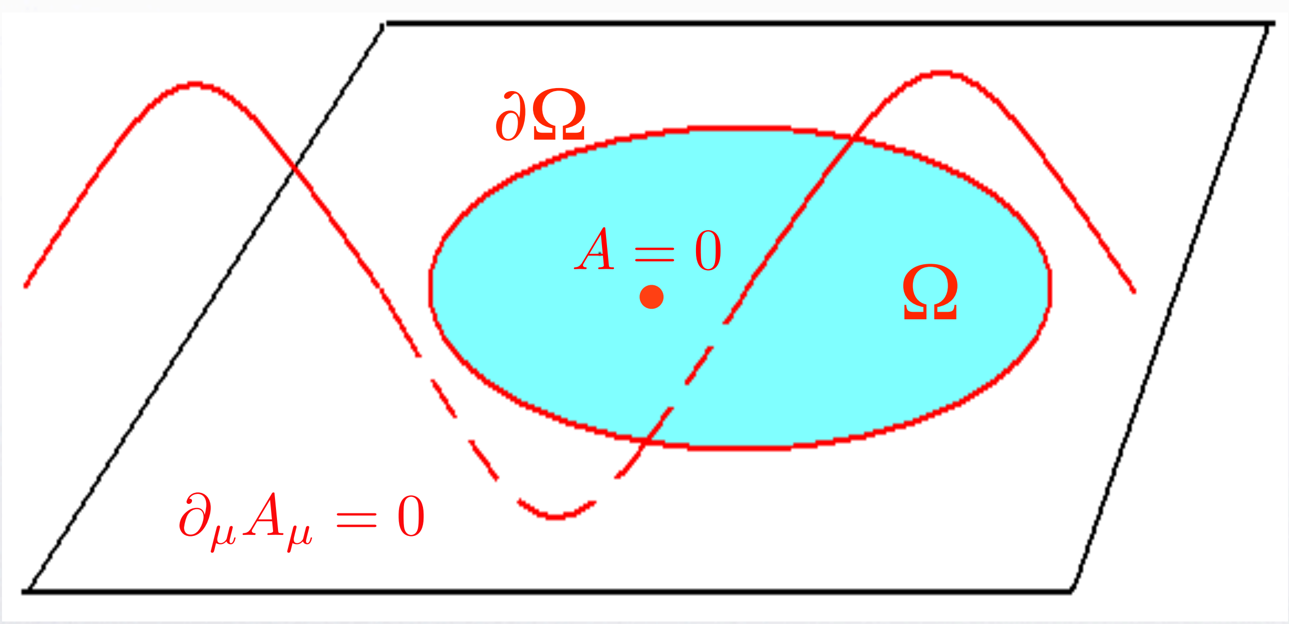
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \qquad S_{\mathrm{YM}} = \frac{1}{4}\int_x F^2$$

with $\Omega = \{A^a_\mu \;;\; \partial A^a = 0, \mathcal{M}^{ab} > 0\}$

$$\boxed{\mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu\right) = -\partial_\mu D^a_\mu}$$

(Faddeev-Popov operator)



- The FP operator is related to the ghost 2-point funtion:

$$\mathcal{G}^{ab}(k;A) = \langle k|c^a\bar{c}^b|k\rangle = \langle k|\left(\mathcal{M}^{ab}\right)^{-1}|k\rangle$$

positivity of $\mathcal{M}^{ab}$ $\longleftrightarrow$ ***No-pole condition*** for the ghost prop.

[Capri,Dudal,Guimaraes,LFP,Sorella, PLB(2013)]

- The **no-pole condition** can be computed to all orders for an external *A* field:

$$\langle \mathcal{G}(k;A)\rangle = \frac{1}{k^2}\left(1+\langle\sigma(k;A)\rangle\right) = \frac{1}{k^2}\left(\frac{1}{1-\langle\sigma(k;A)\rangle_{1PI}}\right)$$

$$\sigma(0,A) = -\frac{g^2}{VD(N^2-1)}\int\frac{d^Dp}{(2\pi)^D}\int\frac{d^Dq}{(2\pi)^D}A^{ab}_{\mu}(-p)\left(\mathcal{M}^{-1}\right)^{bc}_{pq}A^{ca}_{\mu}(q) = \frac{H(A)}{VD(N^2-1)}$$

where σ is a monotonically decreasing function of the momentum *k*, so that the absence of poles to all orders is guaranteed by:

$$\langle\sigma(0;A)\rangle_{1PI} = 1 \quad \text{or} \quad \boxed{\langle H(A)\rangle_{1PI} = VD(N^2-1)}$$

(No-pole condition)

$$H(A) = \int_p\int_q A^a_{\mu}(-p)\left(\mathcal{M}^{ab}\right)^{-1}A^b_{\mu}(q)$$

(Horizon Function)

The Gribov-Zwanziger action

- The **no-pole condition** can be implemented as a **gap equation** for the vacuum energy obtained as:

$$Z = e^{-V\mathcal{E}(\gamma)} = \int \mathcal{D}A\,\delta(\partial A)\ \det\mathcal{M}\ e^{-\left(S_{YM}\underbrace{+\gamma^4 H(A)-\gamma^4 VD(N^2-1)}_{=:\,+\gamma^4\mathcal{H}}\right)}$$

Gap equation:

$$\frac{\partial\mathcal{E}(\gamma)}{\partial\gamma} = 0 \Rightarrow \langle H(A)\rangle_{1PI} = VD(N^2-1)$$

- Using auxiliary fields, this can be cast in a *local* form: $Z = \int [\mathcal{D}\Phi]\ \delta(\partial A)\ \det\mathcal{M}\ e^{-S_{\mathrm{GZ}}}$

$$\begin{aligned} S_{GZ} = & \int d^Dx\left(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right) \\ & + \int d^Dx\left(\underbrace{-\bar{\varphi}^{ac}_\mu\partial_\nu D^{ab}_\nu\varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu\partial_\nu D^{ab}_\nu\omega^{bc}_\mu + gf^{amb}(\partial_\nu\bar{\omega}^{ac}_\mu)(D^{mp}_\nu c^p)\varphi^{bc}_\mu}_{\text{trivial aux. term}}\right) \\ & \underbrace{+ \int d^Dx\left(\gamma^2\, g\, f^{abc}A^a_\mu(\varphi^{bc}_\mu - \bar{\varphi}^{bc}_\mu) - D(N^2-1)\gamma^4\right)}_{\text{Gribov restriction}} \end{aligned}$$

The Refined Gribov-Zwanziger action

- The GZ theory is unstable against the formation of certain dimension 2 condensates, giving rise to a refinement of the effective IR action:

$$S_{\mathrm{YM}} \xrightarrow[\text{(UV}\to\text{IR)}]{\text{Gribov restriction}} S_{\mathrm{GZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H}$$

Dynamical generation of dim.2 condensates

$$\boxed{S_{\mathrm{RGZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H} + \frac{m^2}{2} AA - M^2 \left(\overline{\varphi}\varphi - \overline{\omega}\omega\right)}$$

Gap equation for the Gribov param.:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = VD(N^2 - 1)$$

The parameters *M* and *m* are obtained via minimization of an effective potential for:

$$\langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle \neq 0 \qquad \langle A^2 \rangle \neq 0$$

- Non-perturbative effects included: $(\gamma, M, m) \propto \mathrm{e}^{-\frac{1}{g^2}}$

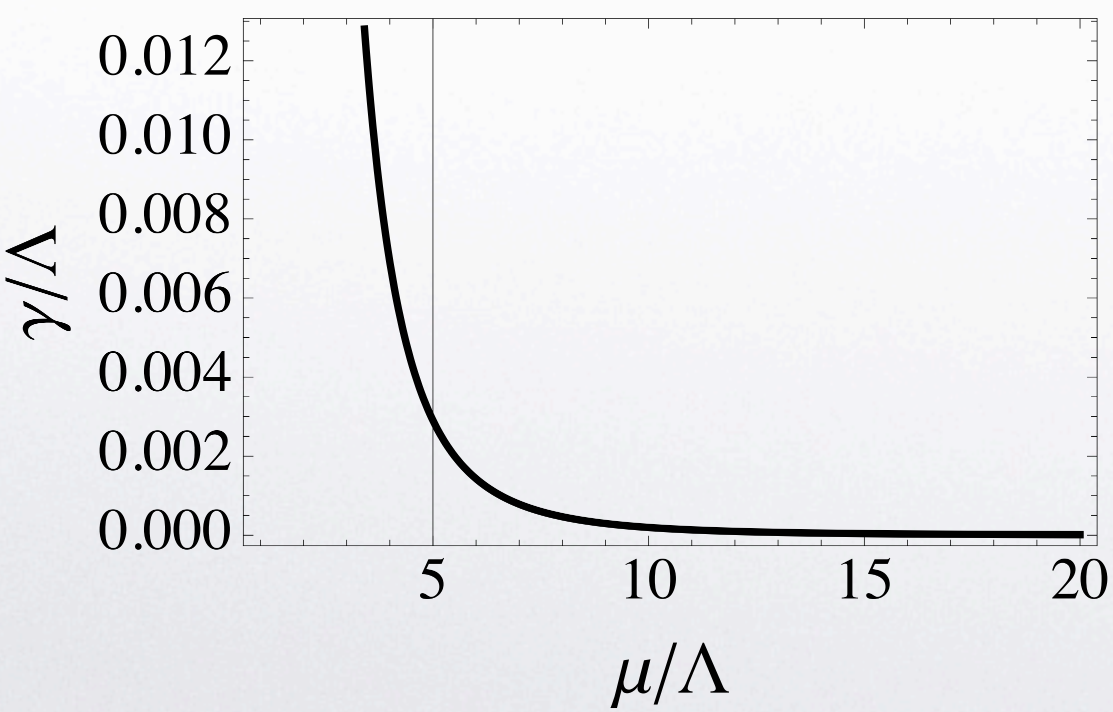
Gribov parameter in the UV

- The one-loop solution of the gap equation in the GZ theory gives:

$$2Ng^2\gamma^4 = \tilde{\gamma}^4 = \mu^4 e^{\frac{5}{3} - \frac{128\pi^2}{3Ng^2}}$$

- Using the definition of the MSbar YM scale Λ (RG-invariant scale):

$$\frac{\tilde{\gamma}^4}{\Lambda} = e^{5/12} \left[\frac{\Lambda}{\mu}\right]^{\frac{ab_0\pi}{2N}} \qquad \frac{ab_0\pi}{2N} \sim 3.9$$



$$\Lambda = 300\mathrm{MeV}$$

$$\tilde{\gamma}(\mu = 1\,\mathrm{GeV}) \sim 4\,\mathrm{MeV}$$
$$\tilde{\gamma}(\mu = 5\,\mathrm{GeV}) \sim 0.008\,\mathrm{MeV}$$

A checklist for RGZ

(see M. Guimaraes' talk!)

- ✓ *(can be cast in a) local and renormalizable action*
- ✓ *reduces to YM at high energies*
- ✓ *gluon confinement: confining propagator (no physical propagation; violation of reflection positivity)*
- ✓ *consistent with lattice IR results*
- ✓ *physical spectrum of bound states*
- ✓ *possible to compute with! Analytical approach to non-perturbative phenomena*

RGZ action and soft breaking of BRST symmetry

- The Faddeev-Popov action in the Landau gauge

$$S_{\rm YM} = \int_x \left(\frac{1}{4}F^2 + b^a\partial A^a + \bar{c}^a(\partial_\mu D_\mu)^{ab}c^b\right)$$

is invariant under BRST transformations:

$$sA^a_\mu = -D^{ab}_\mu c^b \qquad sc^a = \frac{g}{2}f^{abc}c^bc^c \qquad s\bar{c}^a = b^a \qquad sb^a = 0$$

- ***The restriction to the Gribov region breaks BRST symmetry:*** the horizon term is not BRST invariant

$$S_{\rm GZ} = S_{\rm YM} + \gamma^4\mathcal{H} \longrightarrow \left|\begin{array}{l} sS_{\rm GZ} = \gamma^2\Delta \\ \Delta = \int_x \left[-gf^{abc}(D^{am}_\mu c^m)(\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) + gf^{abc}A^a_\mu\omega^{bc}_\mu\right] \end{array}\right.$$

- The BRST breaking terms have dimension less than D=4, so that they correspond to ***SOFT*** terms.

- The softness of the BRST breaking is crucial for keeping the UV intact. ***Perturbative results recovered at high energies!***

- Can be understood as a **non-pertubative BRST symmetry**, that controls gauge-par. dep. and the extension to other gauges **[Capri et al, (2015)]**

- Direct signatures of BRST breaking on the lattice are related to: $\langle s(\cdots)\rangle \neq 0$
- The first proposal was for GZ: **[Cucchieri et al, PRD(2014)]**

$$\boxed{\tilde{\mathcal{Q}}(p^2) \;=\; \langle \tilde{\mathcal{R}}^a_\mu(p)\tilde{\mathcal{R}}^a_\mu(-p)\rangle \stackrel{\text{GZ}}{=} \langle s(\,\varphi\overline{\omega}\,)\rangle} \qquad \mathcal{R}^a_\mu(x) \;=\; g f^{abc}\int_y A^b_\mu(y)(\mathcal{M}^{-1}_{xy})^{ac}$$

➠ **A nonzero $\tilde{\mathcal{Q}}$ means BRST is broken in GZ (which predicts: $\tilde{\mathcal{Q}} \stackrel{p\sim 0}{\sim} 1/p^6$)**

- This correlation can also be written in RGZ as the IR dominant part of $\langle s(\,\text{fund. fields}\,)\rangle$:

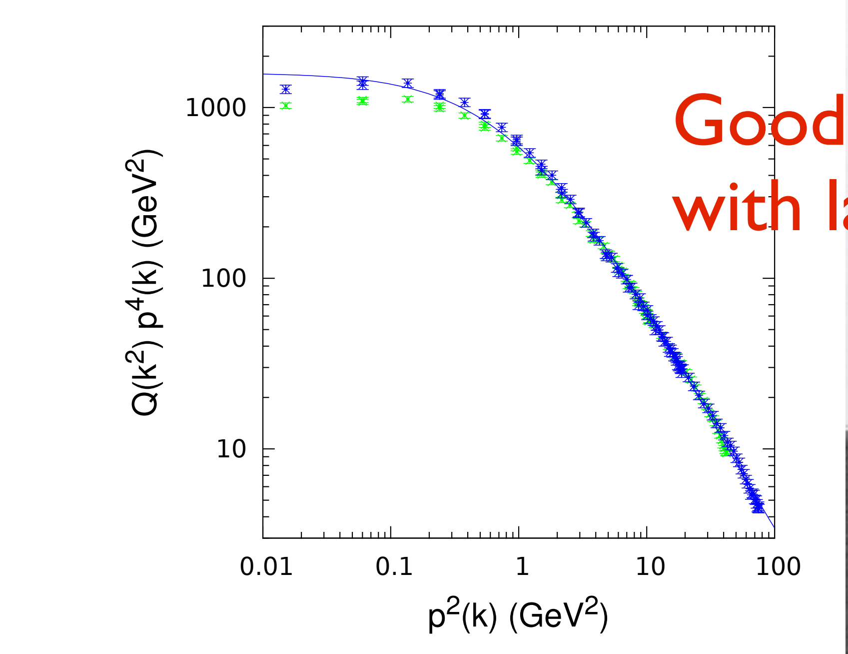
[Capri,Dudal,Fiorentini,Justo,Guimaraes,LFP,Sorella, *to appear*]

$$\langle s\left(\bar{c}^a(x)\bar{c}^b(y)\int_z A^a_\mu(z)\int_{z'} f^{bnp}A^n_\mu(z')c^p(z')\right)\rangle \;=\; \mathcal{G}(x-y)+\mathcal{Q}(x-y)$$

where: $$\mathcal{G}(x-y) \;=\; \langle\left(b^a(x)\bar{c}^b(y)-\bar{c}^a(x)b^b(y)\right)\int_z A^a_\mu(z)\int_{z'} f^{bnp}A^n_\mu(z')c^p(z')\rangle$$

In the deep IR, there is a clear hierarchy of the contributions in RGZ:

$$\tilde{\mathcal{G}} \stackrel{p\sim 0}{\sim} \frac{1}{p^2} \qquad\qquad \tilde{\mathcal{Q}} \stackrel{p\sim 0}{\sim} \frac{1}{p^4}$$



Good agreement with lattice

- The horizon term has a ***geometrical interpretation in terms of the Gribov restriction***, but also has the crucial property of ***soft BRST breaking***. It provides a successful scenario for the gluon sector.

  How can this be extended to the confined matter sector?

- The restriction to the Gribov region is a constraint on a quantum operator: $\mathcal{M}^{ab} \geq 0$

  ➠ **should naturally affect all correlation functions built with it...**

$$\boxed{\int [D\mu]\mathrm{e}^{-S} \overset{\mathrm{IR}}{\mapsto} \int [D\mu]_\Omega \mathrm{e}^{-S}} \quad \left\{ \begin{array}{l} \langle \tilde{R}\tilde{R} \rangle \sim 1/k^4 \\ \langle \mathcal{M}^{-1}\mathcal{M}^{-1} \rangle \sim 1/k^4 \\ \quad \cdots \end{array} \right. \qquad \mathcal{R} \approx \int A\mathcal{M}^{-1}$$

... including the ones with <u>matter fields</u>: $F^I = (\psi^i, \phi^a)$

$$\mathcal{R}_F = g \int_z (\mathcal{M}^{-1})^{ab}(x,z)(T^b)^{IJ} F^J(z) \qquad \langle \tilde{\mathcal{R}}_F \tilde{\mathcal{R}}_F \rangle$$

Can in principle be checked on the lattice!

How to construct an IR effective ***gauge-matter*** action that can account for these correlations?

Extension to the matter sector: COLOR confinement

PROPOSAL: Faddeev-Popov operator as a carrier of color confinement via soft BRST breaking.

A general gauge-matter IR action reads: $\boxed{S_{\rm IR} = S_{\rm YM} + \sum_F M_F \mathcal{H}_F}$

$$\mathcal{H}_F = -g^2 \int \frac{d^D p}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} \left(T^b\right)^{ij} F^j(-p) \left(\mathcal{M}^{-1}\right)^{bc}_{pq} \left(T^c\right)^{ik} F^k(q) \,.$$

where F^j stands for a fundamental confined field $= \{A^a_\mu, \phi^a, \psi^i\}$

➡ This generates a soft BRST breaking and a correlation of the form: $Q = \langle R_F R_F \rangle$

➡ Can be localized, resulting in a renormalizable action that ***reduces to QCD in the UV***.

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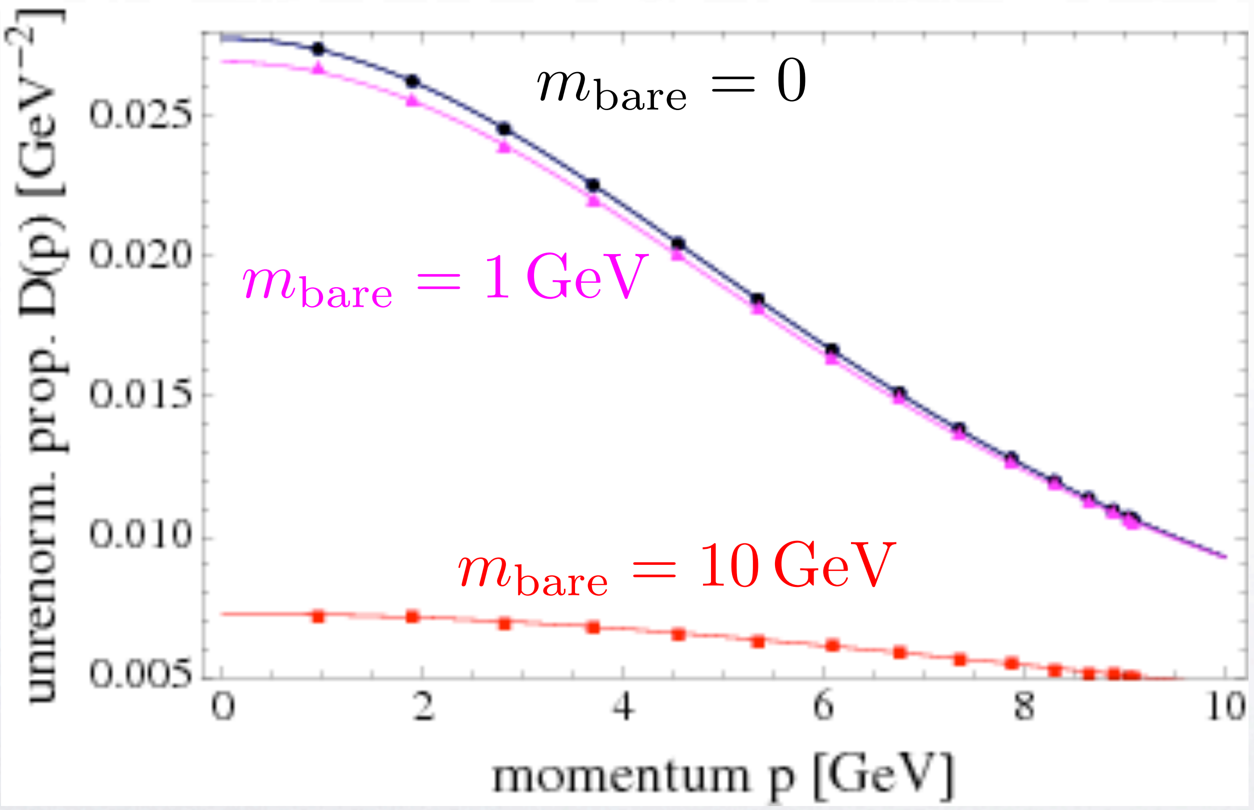
Let's check the predictions for confined matter:
Predicted propagators vs lattice? Positivity violation? Spectrum? Thermodynamics?

Adjoint scalar case:

$$\langle \phi^a(k)\phi^b(-k)\rangle = \delta^{ab}\frac{k^2+\mu_\phi^2}{k^4+(\mu_\phi^2+m_\phi^2)k^2+2Ng^2\sigma^4+\mu_\phi^2 m_\phi^2}\,.$$

where m_ϕ is the scalar mass and μ_ϕ is the mass parameter associated to the dimension 2 condensate.



Lattice data from [A. Maas (2010--2014)]

➡ With the obtained fit values, it is easy to show that the propagator displays complex-conjugated poles, implying ***positivity violation***!

m_{bare}	μ_ϕ^2	m_ϕ^2	σ^4	Z	χ^2/dof
0	120	0	4913	1.137	0.31
1	46	34	644	1.28	1.84
10	88	158	1267	1.26	0.10

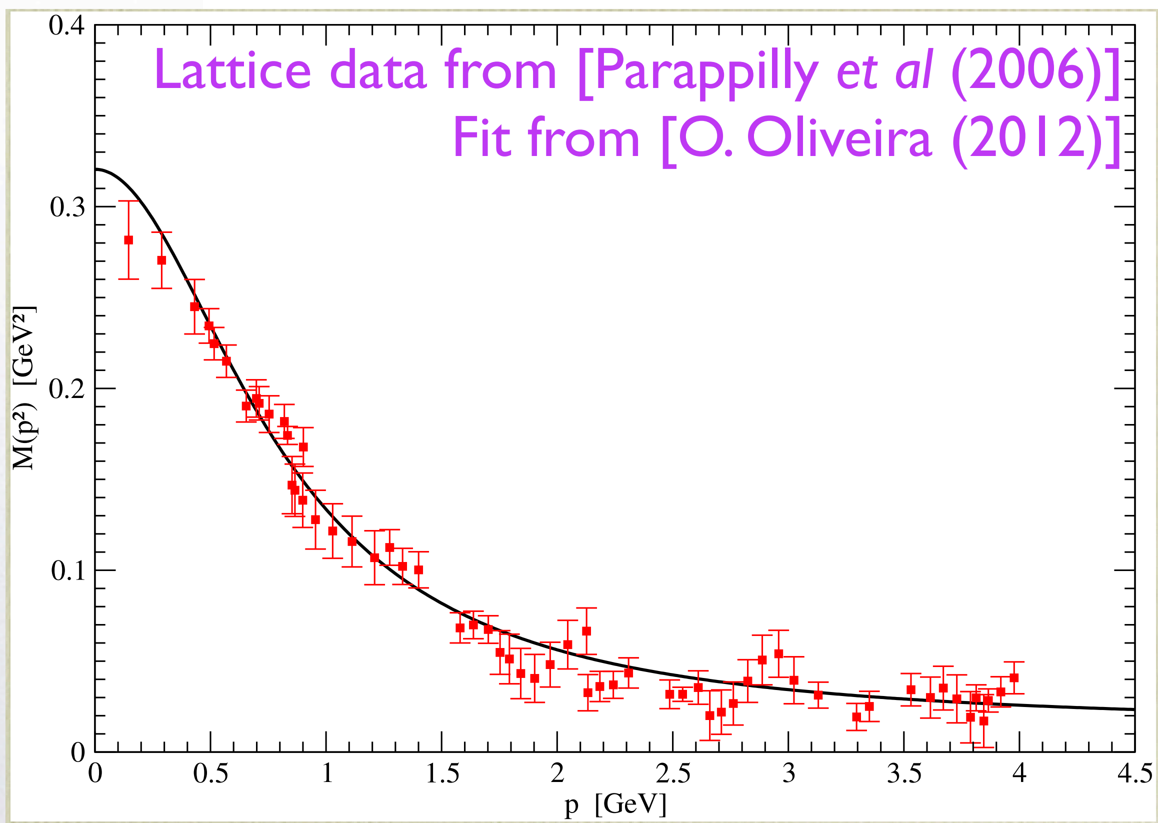
Quark (fundamental fermion) case:

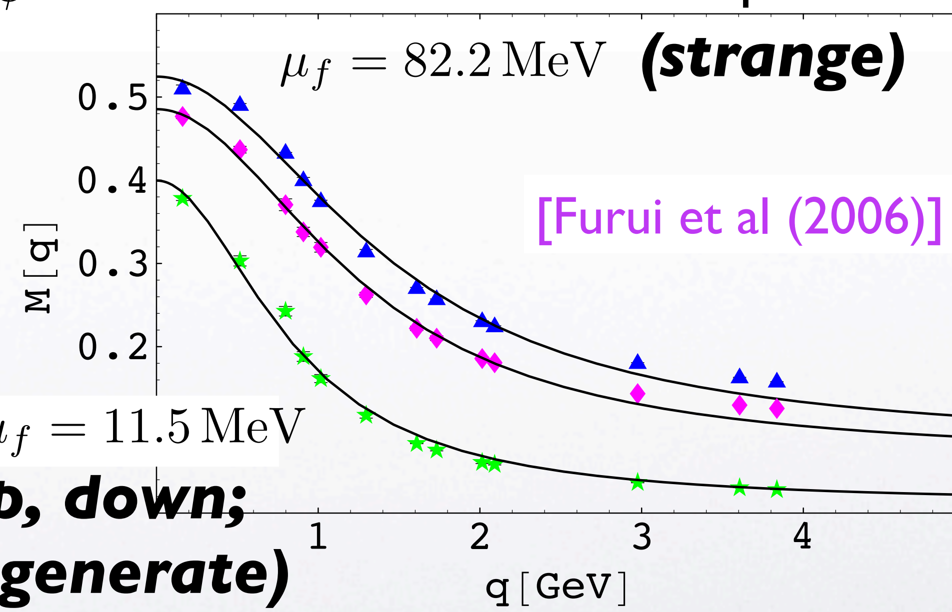
$$\langle \psi^i(k)\bar{\psi}^j(-k)\rangle = \delta^{ij}\ \frac{-ik_\mu\gamma_\mu + \mathcal{A}(k^2)}{k^2 + \mathcal{A}^2(k^2)}\ ,$$

where

$$\mathcal{A}(k^2) = m_\psi + \frac{g^2\sigma^3 C_F}{k^2+\mu_\psi^2}\ ,$$

where m_ψ is the fermion mass and μ_ψ is the condensate mass parameter.





➡ With the obtained fit values, the propagator displays: **one real pole and two complex-conjugated ones**

Violation of positivity

Osterwalder-Schrader axiom of reflection positivity

$$\int d^4x d^4y f^\dagger(-x_0,\vec{x})\Delta(x-y)f(y_0,\vec{y}) \geq 0$$

where f is a test function with support for positive times. Making a Fourier transformation, one gets the condition

$$\int dt dt' d^4y f^\dagger(t',\vec{p})\Delta((t'-t),\vec{p})f(t,\vec{p}) \geq 0 \;\; \forall \vec{p}$$

If Δ(t'-t, p) is negative within any domain, it is easy to find a test function which will pinpoint this negative region, thus violating the O-S positivity. In practice, one looks at p=0

$$\Delta(t) = \int dp\, e^{itp}\tilde{\Delta}(p^2 = p_4^2)$$

Confined quarks: violation of positivity!

For the quark propagator, we have two form factors:

$$S(p) = -i\gamma_\mu p_\mu \, \sigma_v(p^2) + \sigma_s(p^2)$$

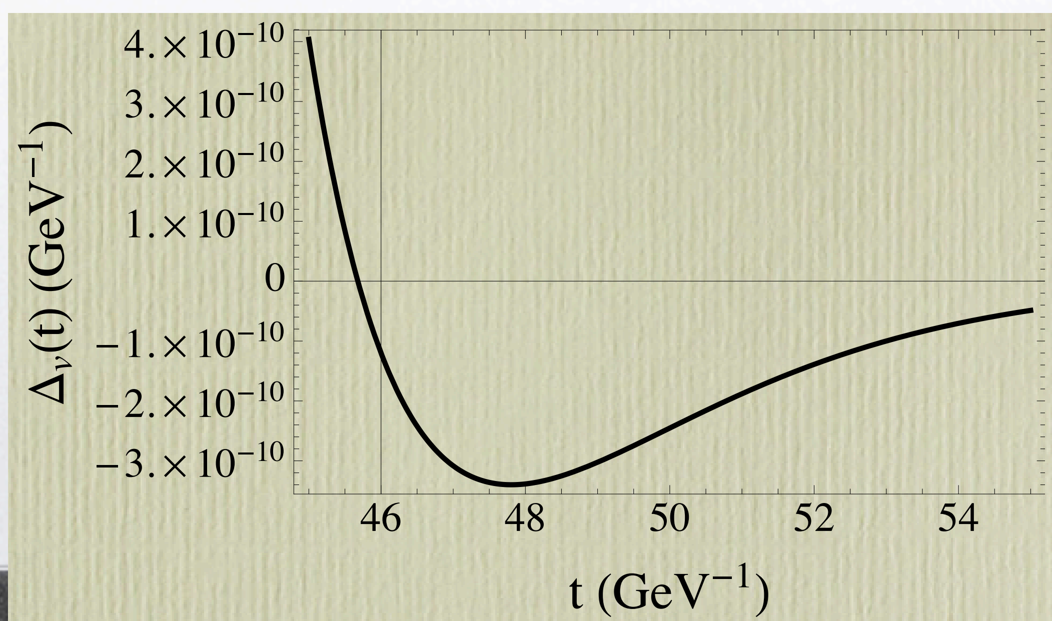
We need to study two functions:

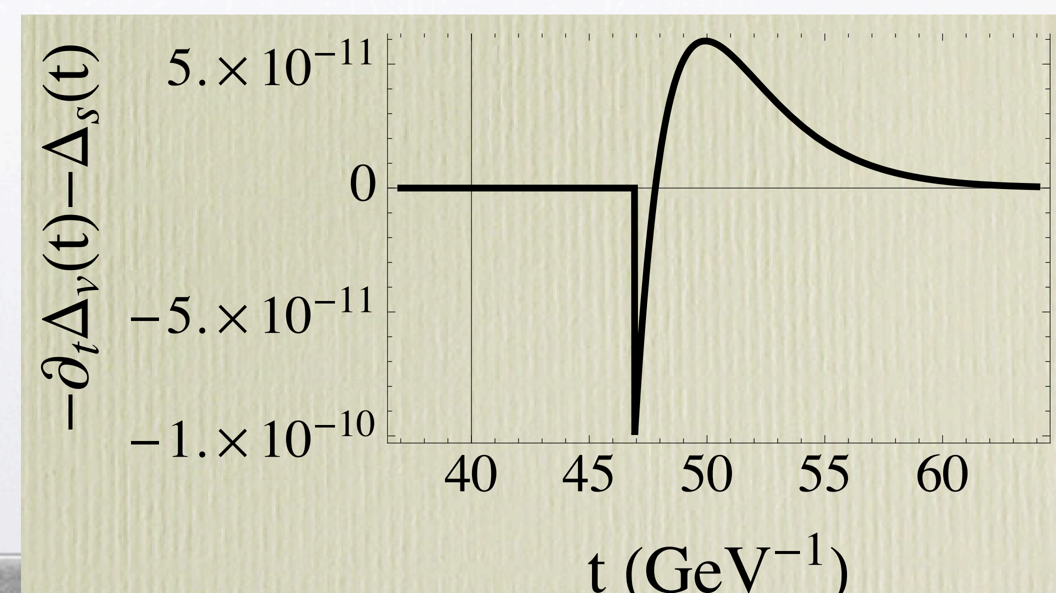
$$\Delta_v(t) = \int dp \, e^{ipt} \sigma_v(p^2) \qquad \Delta_v(t) = \int dp \, e^{ipt} \sigma_s(p^2)$$

In the fermionic case the O-S positivity implies that

$$\Delta_v(t) \geq 0 \qquad -\partial_t \Delta_v(t) \geq \Delta_s(t)$$

[Dudal,Guimaraes,LFP,Sorella (2012)]

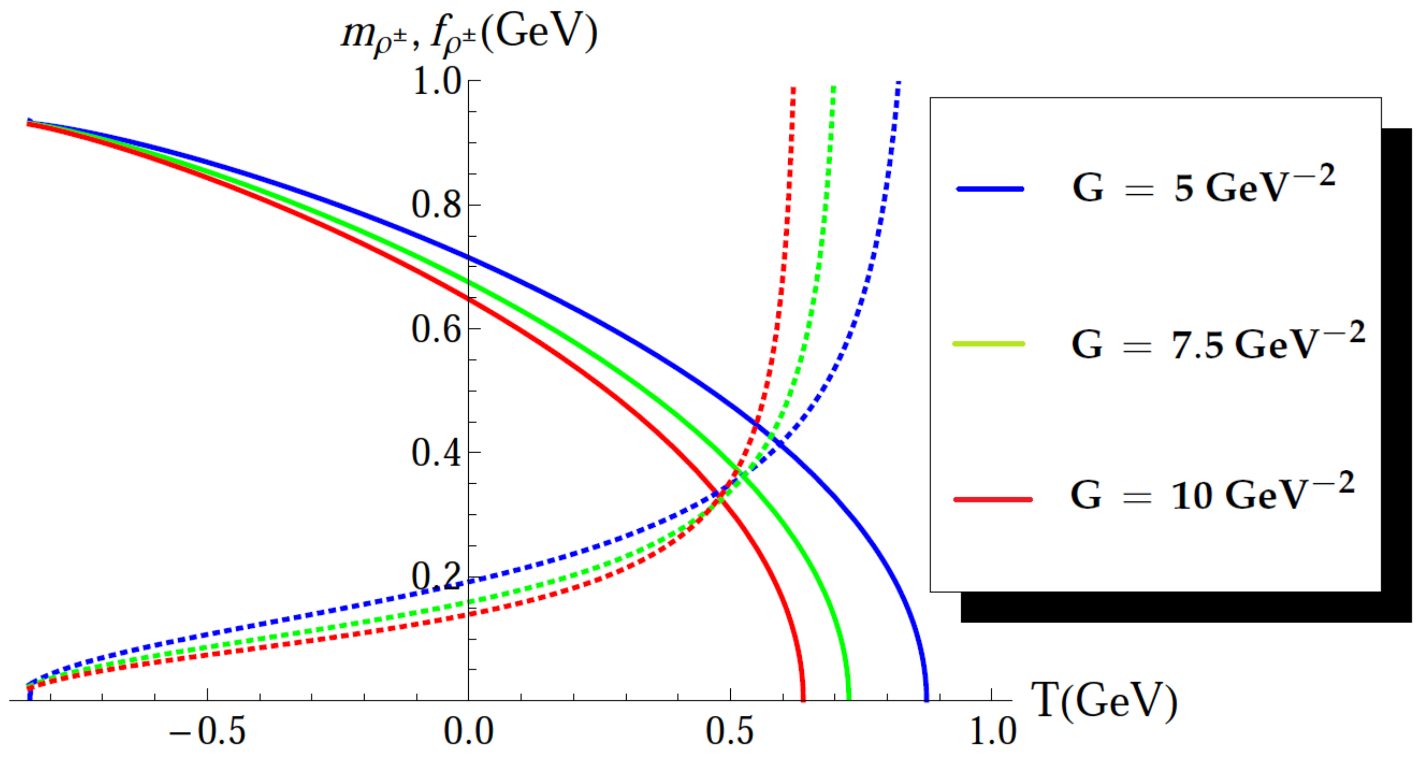


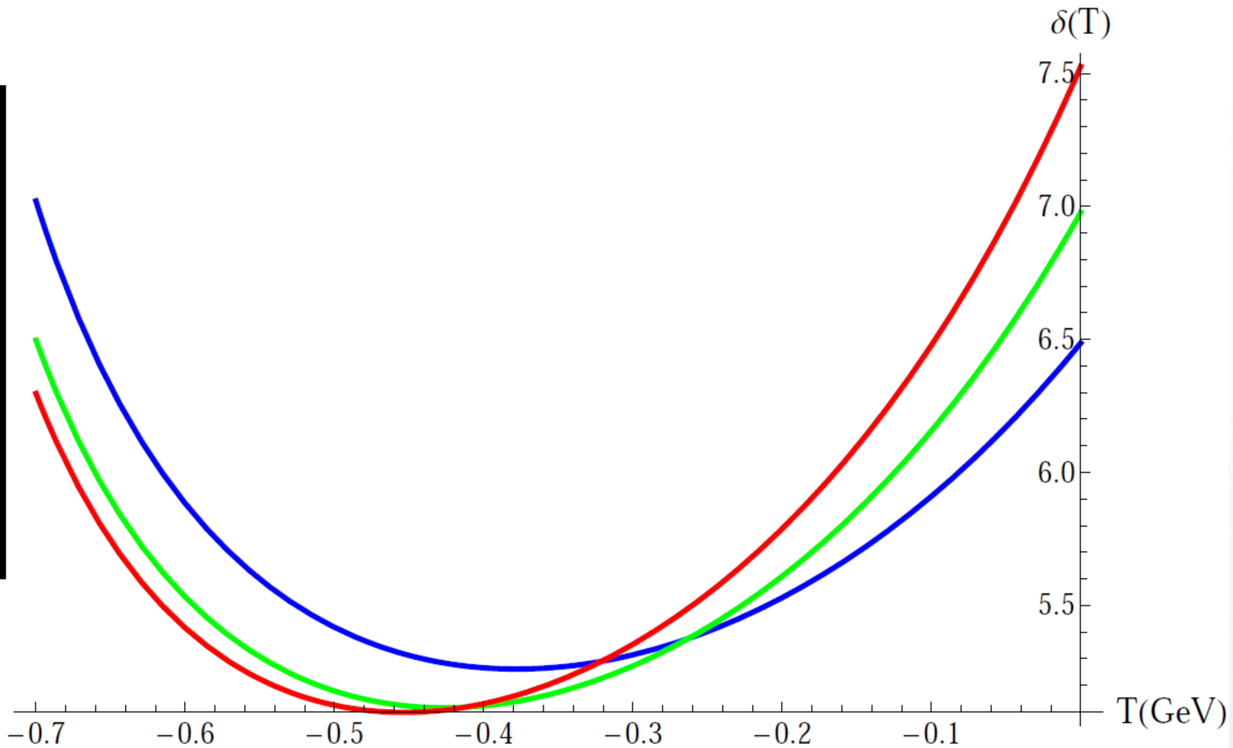


From confined quarks to mesons...

[Dudal,Guimaraes,LFP,Sorella (2012)]

- Pion as a pseudo-Goldstone boson
- Estimate of the rho meson mass and decay constant:





We obtain for the masses
$m_{\rho^\pm}^{G=5,\ 7.5,\ 10} \approx 0.84, 0.83, 0.83$ GeV ($m_{\rho^\pm}^{exp} = 775.49 \pm 0.34$ MeV)
and decay constants
$f_{\rho^\pm}^{G=5,\ 7.5,\ 10} \approx 0.13, 0.10, 0.09$ GeV. ($f_{\rho^\pm}^{exp} \approx 0.208$ GeV,
$f_{\rho^\pm}^{latt} \approx 0.25$ GeV)

Thermodynamics of confined quarks

Motivation:

- Physical observables (gauge-invariant!) computed directly, and should satisfy the thermodynamical constraints
  ➠ Direct test of the confinement scenario

- Indications of thermodynamic instabilities in quark systems with complex conjugated poles **[Benic, Blaschke, Buballa (2012)]**

- New models of low-energy QCD: true ***constituent*** quarks should be absent from spectrum!

Pure gauge related studies:

[Reinosa, Serreau, Tissier, Wschebor (2015)] (see talk by M. Tissier)
[Canfora *et al* (2015)] (see poster by I. Justo)

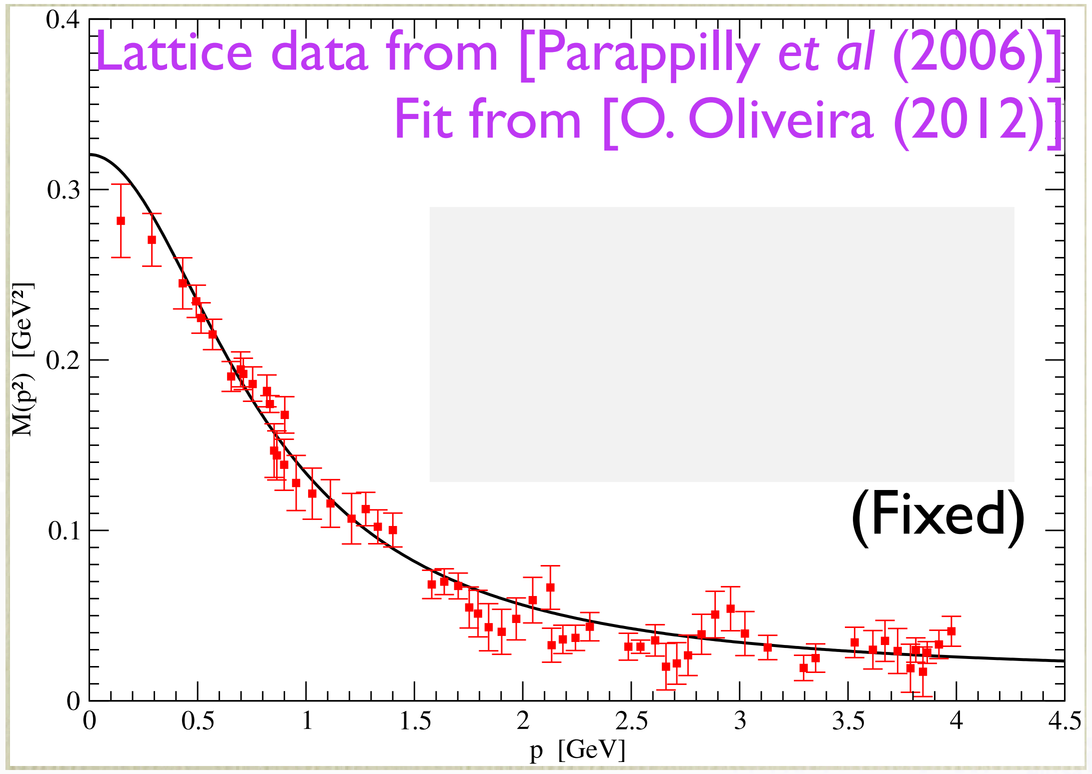
A model of confined quarks

Lowest order approximation: ***dressed quarks***

$$\boxed{\mathcal{L}_{IRq} = \bar{\psi}\left[i\partial\!\!\!/ - \left(\frac{M_3}{-\partial^2 + m^2} + m_0\right)\right]\psi}$$

- Positivity violation: sign of confinement
- Dynamically generated mass, compatible with lattice and DSE.

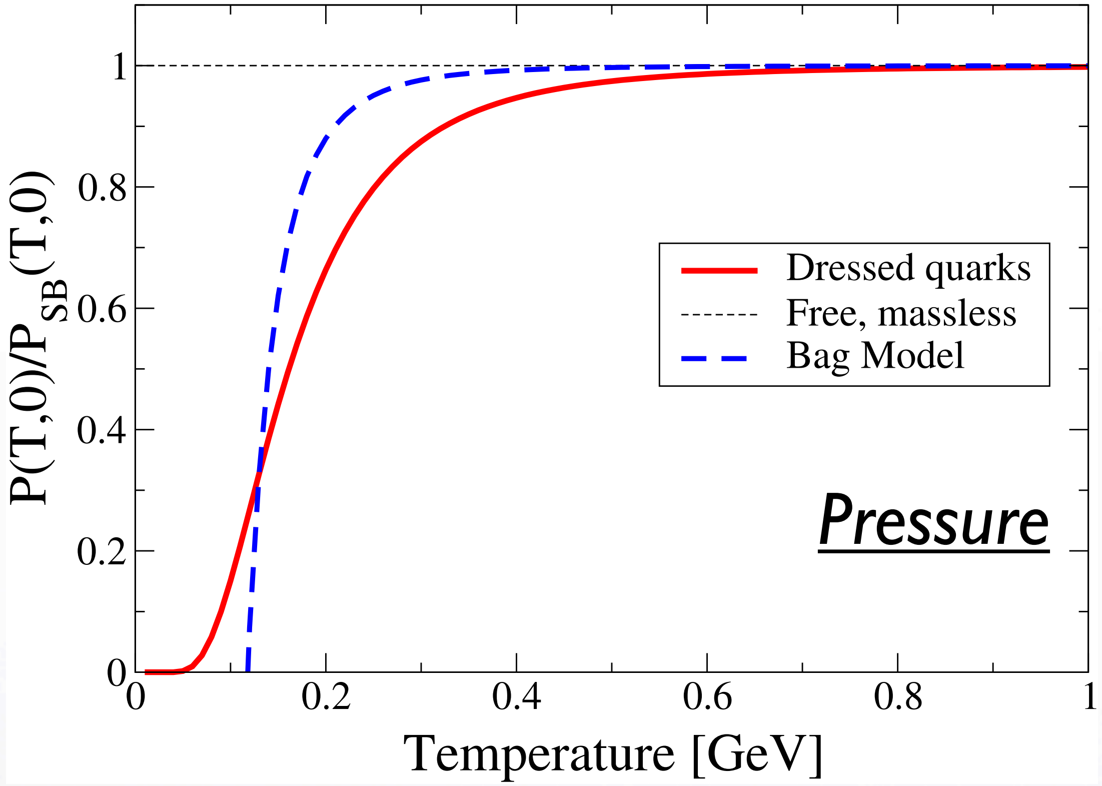
[Guimaraes, Mintz, LFP (2015)]

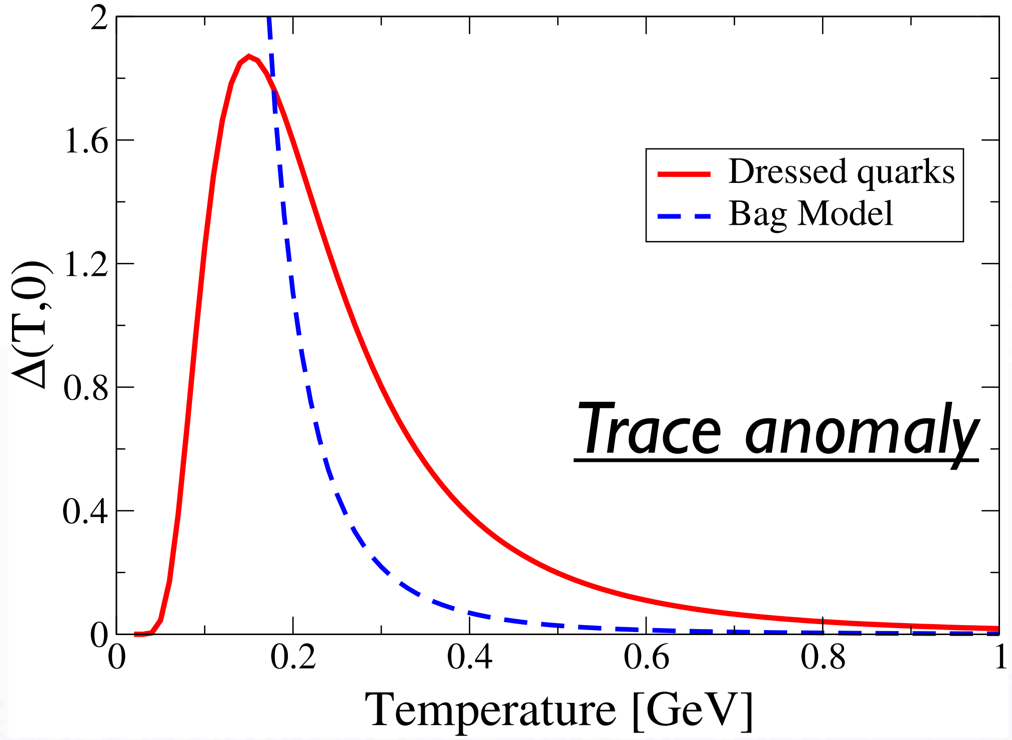


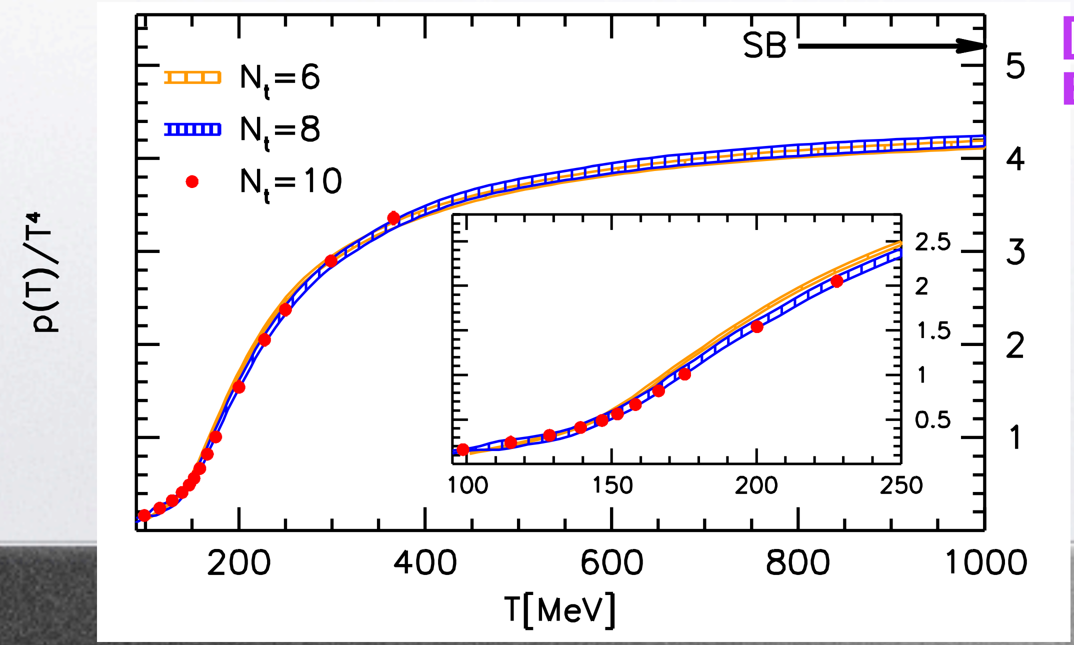
- The partition function of this model is quadratic and can be computed exactly for any T and μ in the standard imaginary-time formalism

Thermodynamics of hot confined quarks

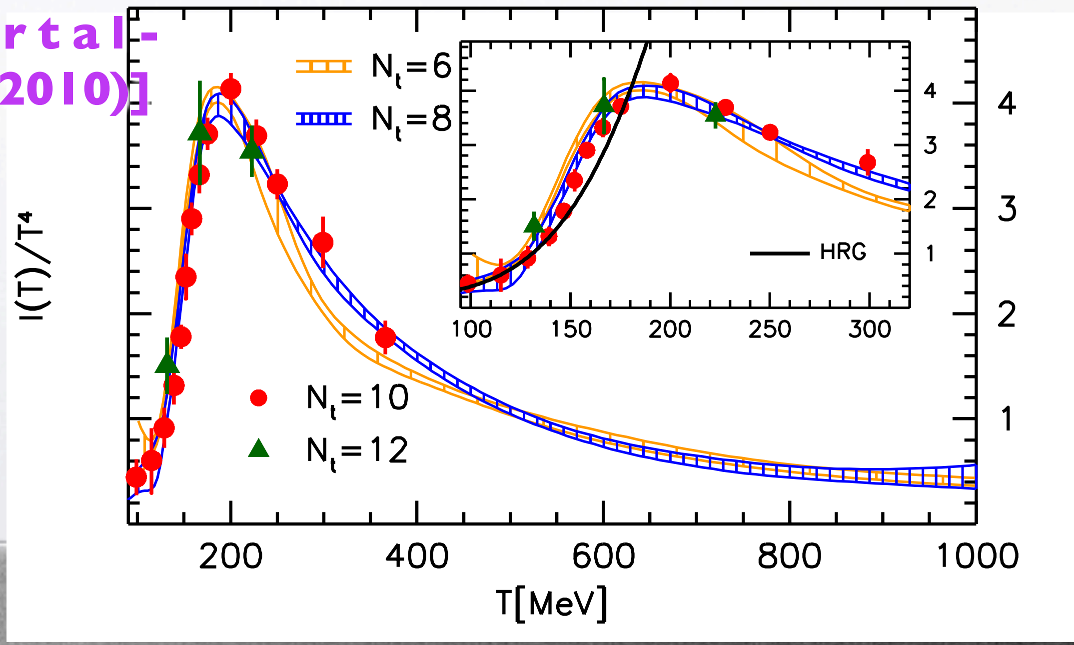
- The ***leading-order contribution*** to the thermodynamics of these confined quarks is already highly non-trivial (and stable)!
  Qualitative behavior compatible with lattice (thermal crossover)







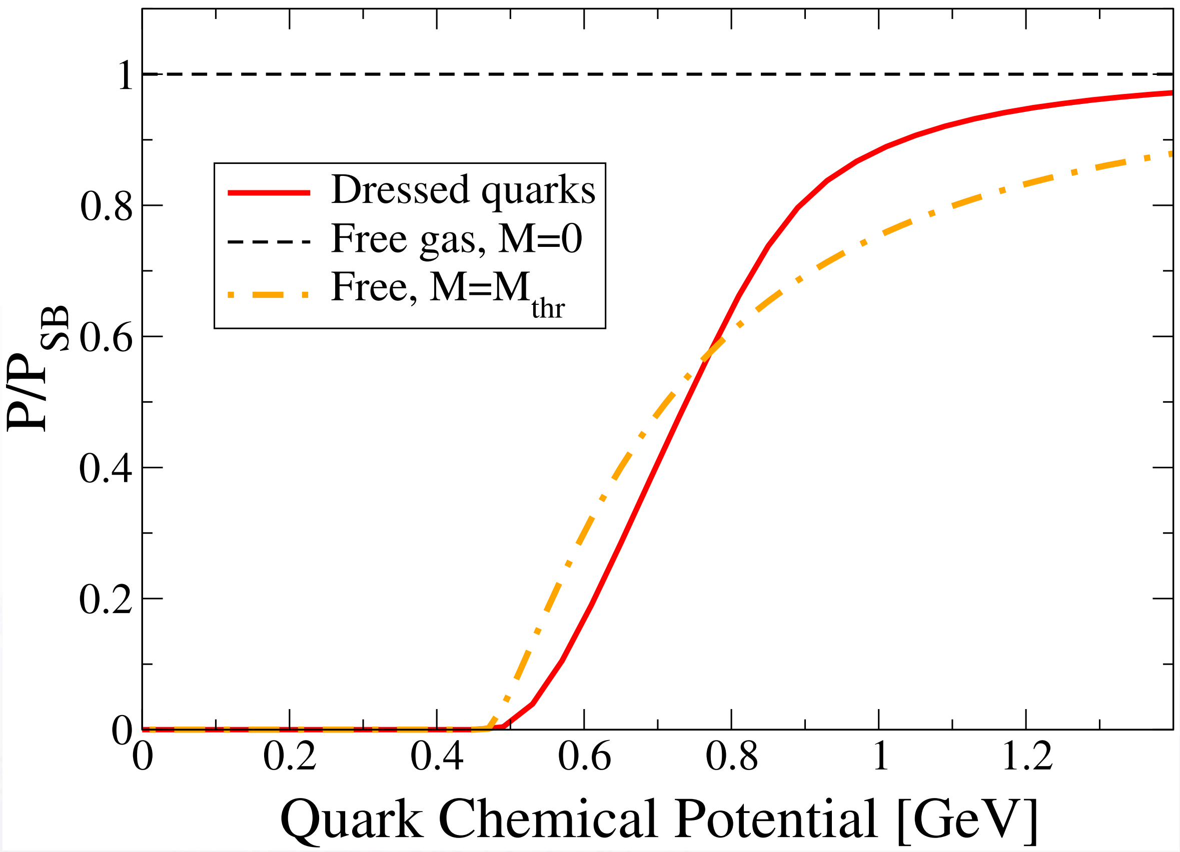
[Wuppertal-Budapest (2010)]

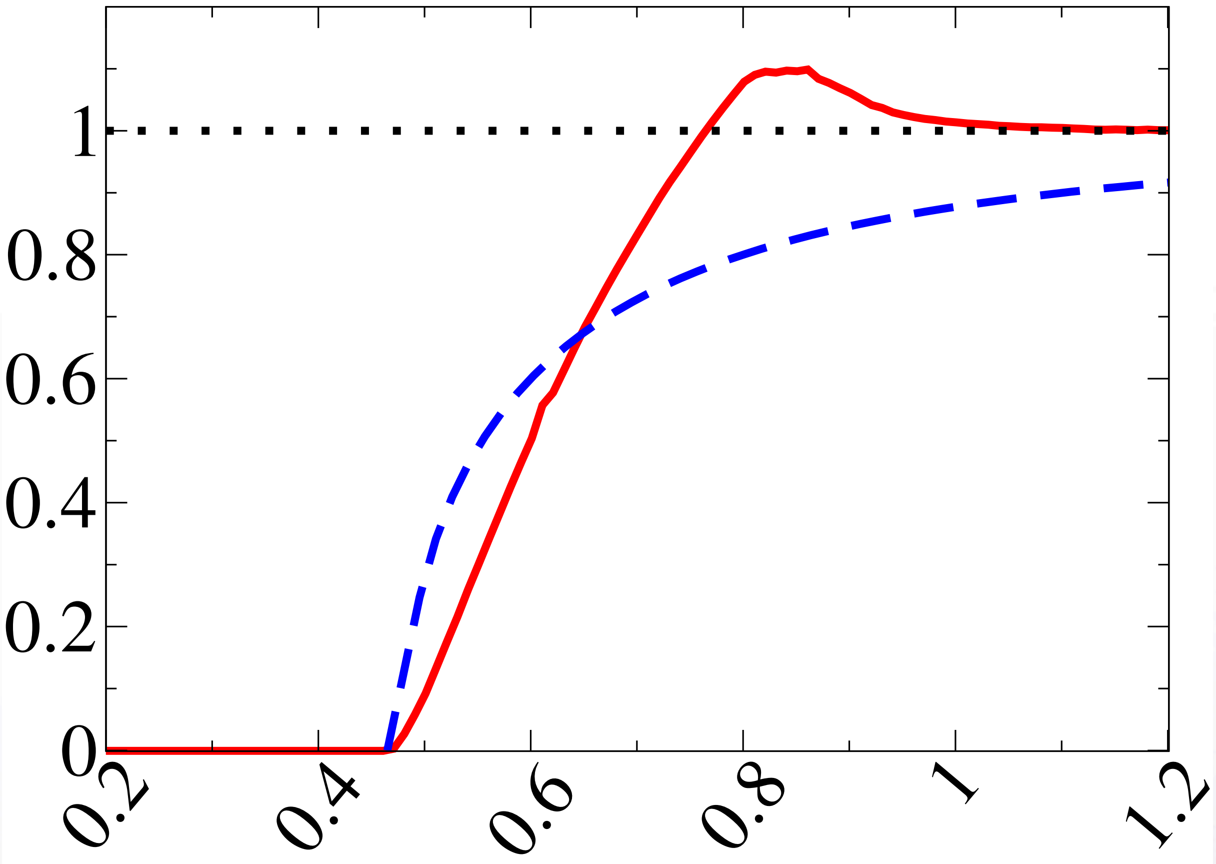


Thermodynamics of cold and dense confined quarks

[Guimaraes,Mintz,LFP (2015)]

Pressure at finite density (Sign problem region)



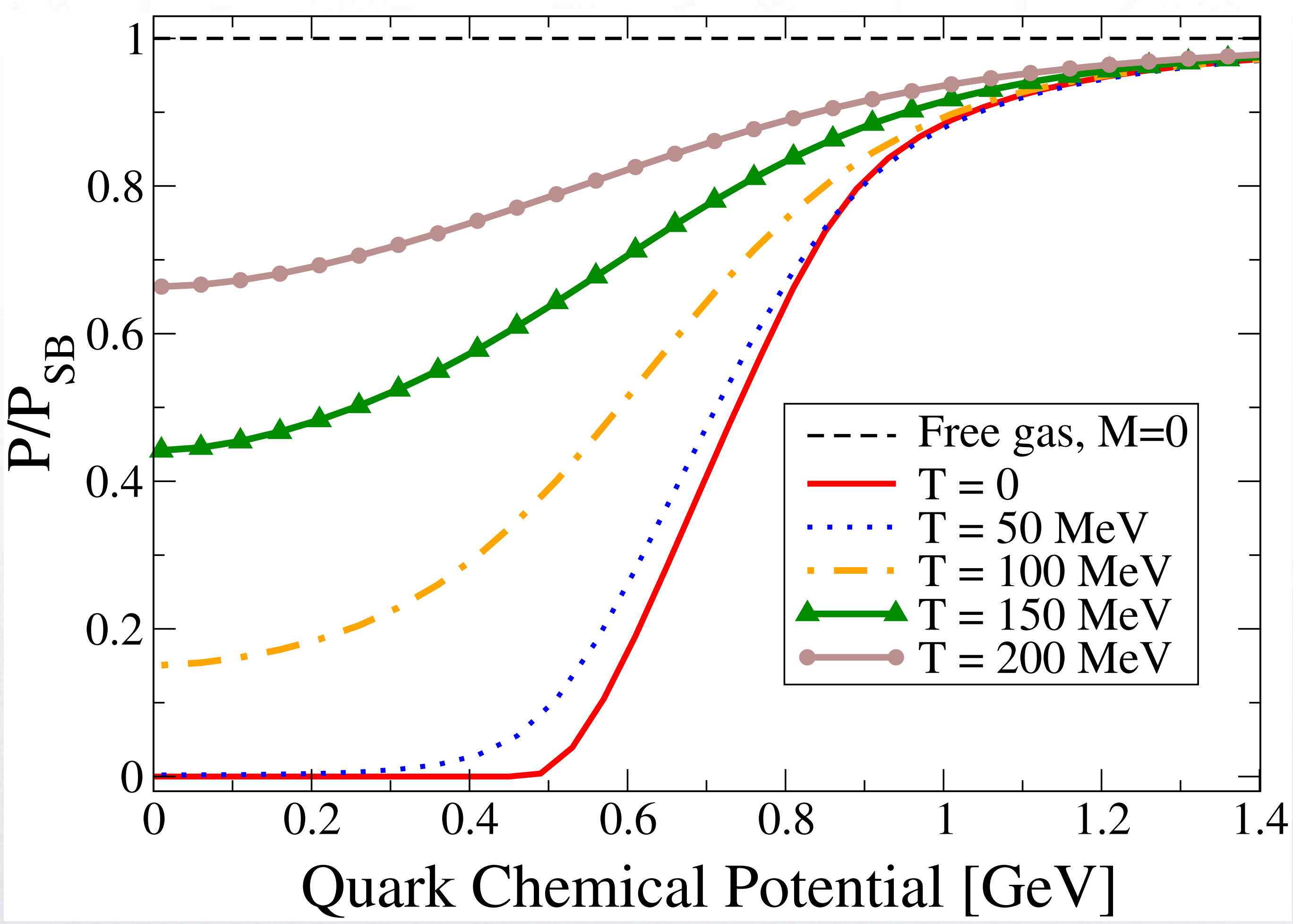


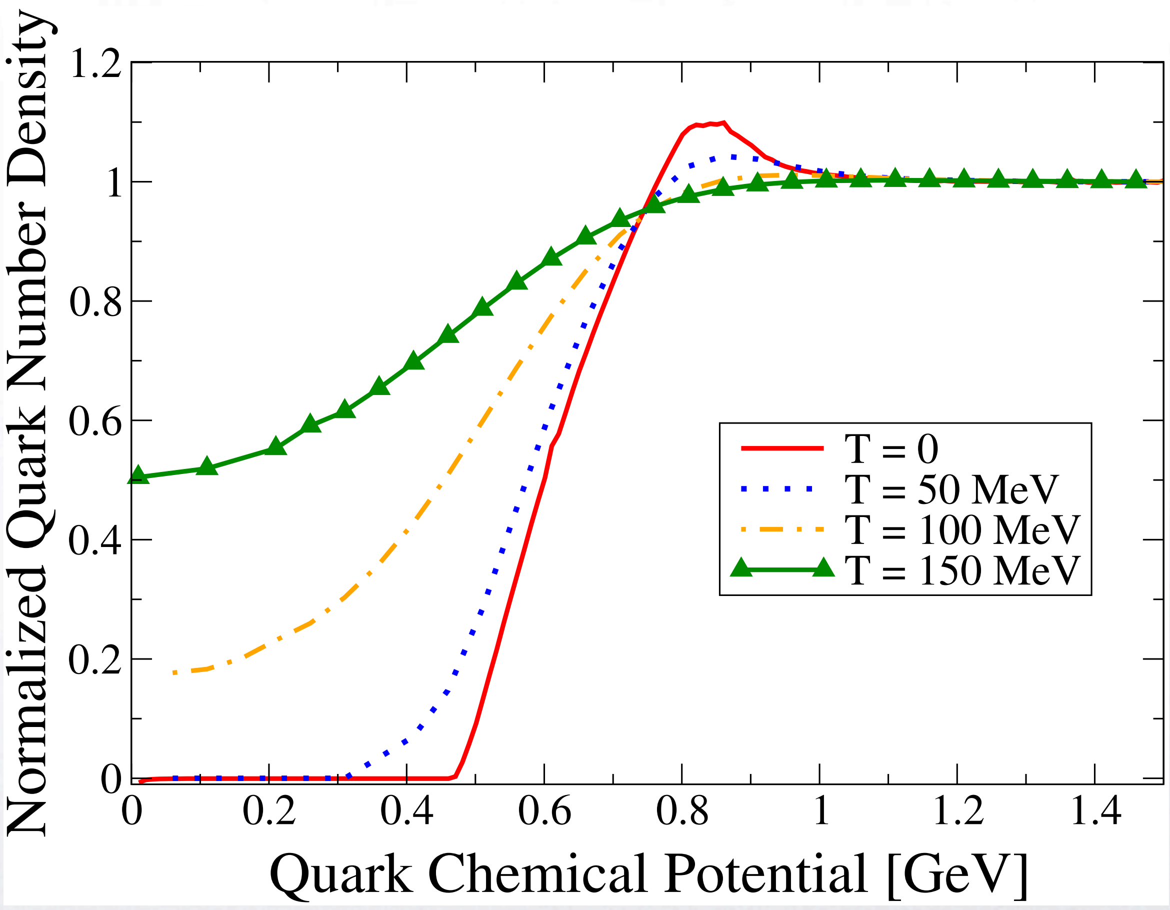
- gluons suppressed: good approximation, but no comparison with lattice possible
- Silver Blaze problem: ok!
- No phase transition in this approximation (T- and μ-independent parameters)

Thermodynamics of confined quarks

[Guimaraes,Mintz,LFP (2015)]

- Turning on the temperature, ***consistent physical results***:
  excitations for all values of μ,
  smoothening of the transition,
  shift of inflection point to lower μ's.





Final comments

- ***The mechanism of confinement is still not understood.***

- The ***Gribov problem*** is present in non-Abelian gauge theories and should profoundly affect the IR regime. Its connection to ***confinement*** seems thus natural.

- The ***RGZ framework*** represents a consistent scenario to study the non-perturbative IR physics and has provided many ***interesting results for the gluon sector***.

- ***BRST breaking*** turns out to play an important role in the non-perturbative regime of non-Abelian gauge theories (in the Landau gauge).
  Its possible connection with confinement and the possibility of extending the non-perturbative model to the matter sector seems fruitful.

- ***Thermodynamics*** of a confining quark model shows no instabilities and nontrivial results

- **Perspectives:** including gluons and Polyakov l.; phase structure; other non-perturbative observables (such as Casimir energy, Debye masses, ...); more spectrum predictions; transmission of BRST breaking from the gluon sector; other gauges...

- **Many caveats, of course:** approximations, physical operators, unitarity, ...

Thank you for your attention!