Hamiltonian approach to QCD in Coulomb gauge: from the vacuum to finite temperatures

H. Reinhardt

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

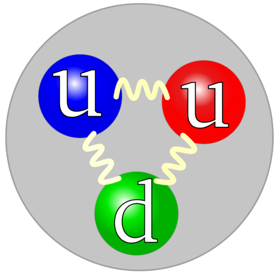
collaborators:

J. Heffner, P. Vastag, H. Vogt, E. Ebadati

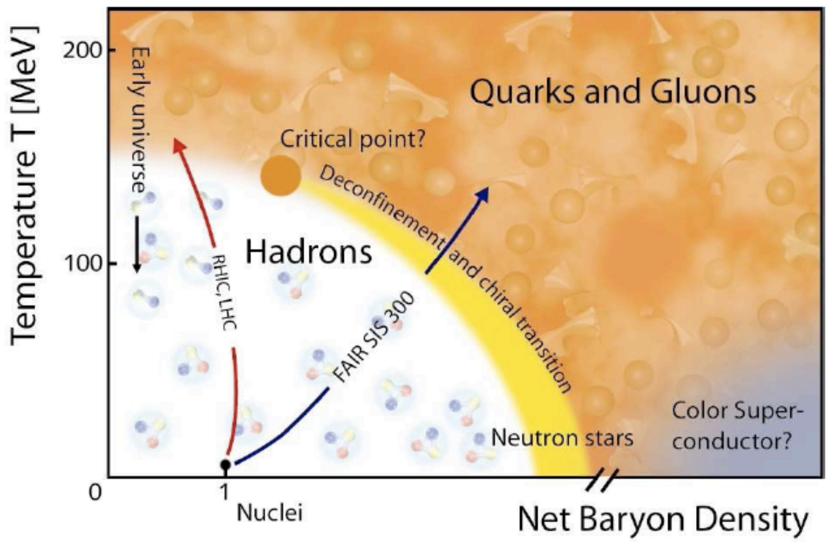
G. Burgio, D. Campagnari, M. Quandt, P.Watson

QCD

- *vacuum*
  - confinement
  - SB chiral symmetry



- *phase diagram*
  - deconfinement
  - rest. chiral symm.



- latticeMC-fail at large chemical potential

  continuum approaches required

  Hamiltonian approach in Coulomb gauge

Outline

- introduction
- Hamiltonian approach to QCD in Coulomb gauge
  - quark sector
- Hamiltonian approach to finite temperature QFT by compactification of a spatial dimension
- QCD at finite T
  - effective potential for the Polyakov loop
  - dual quark condensate
- conclusions

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J\Pi + B^2) + H_C \qquad \Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density $\rho^a = -f^{abc}A^b\Pi^c + \rho_m^a$

$$\langle\phi|\ldots|\psi\rangle = \int DA J(A)\phi^*(A)\ldots\psi(A)$$

$$H\psi[A] = E\psi[A]$$

Variational approach to YMT

- Gaussian ansatz,

$$\Psi(A) = \exp\left[-\tfrac{1}{2}\int dx\, dy A(x)\omega(x,y)A(y)\right]$$

D. Schütte 1984

....

A.Szczepaniak & E. Swanson 2002

C. Feuchter & H. R. 2004

-ansatz
-FP determinant
-renormalization

- Greensite, Matevosyan,Olejnik,Quandt, Reinhardt, Szczepaniak,PRD83

Variational approach to YMT

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\,\omega(x,y)\,A(y)\right]$$

gluon propagator $\quad \langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

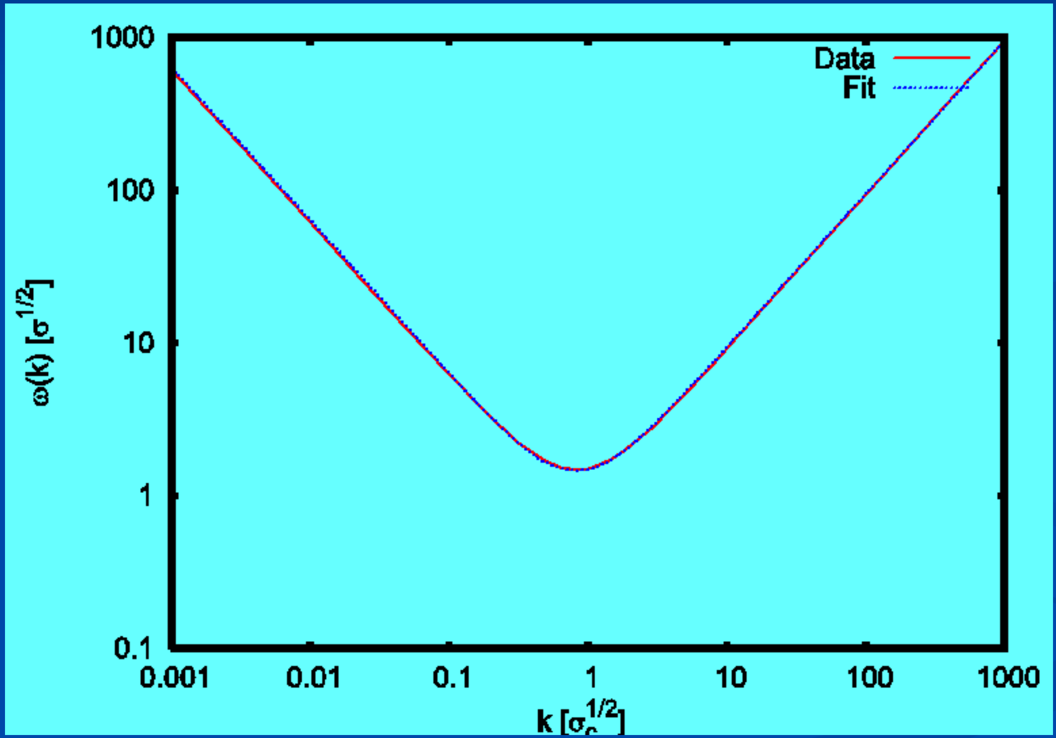
variational kernel $\quad \omega(x,x') \quad$ determined from

$$\boxed{\langle\Psi|H|\Psi\rangle \to \min}$$

Numerical results

gluon energy

D. Epple, H. R., W.Schleifenbaum, PRD 75 (2007)



$$IR: \quad \omega(k) \sim 1/k \qquad UV: \quad \omega(k) \sim k$$

Static gluon propagator in D=3+1

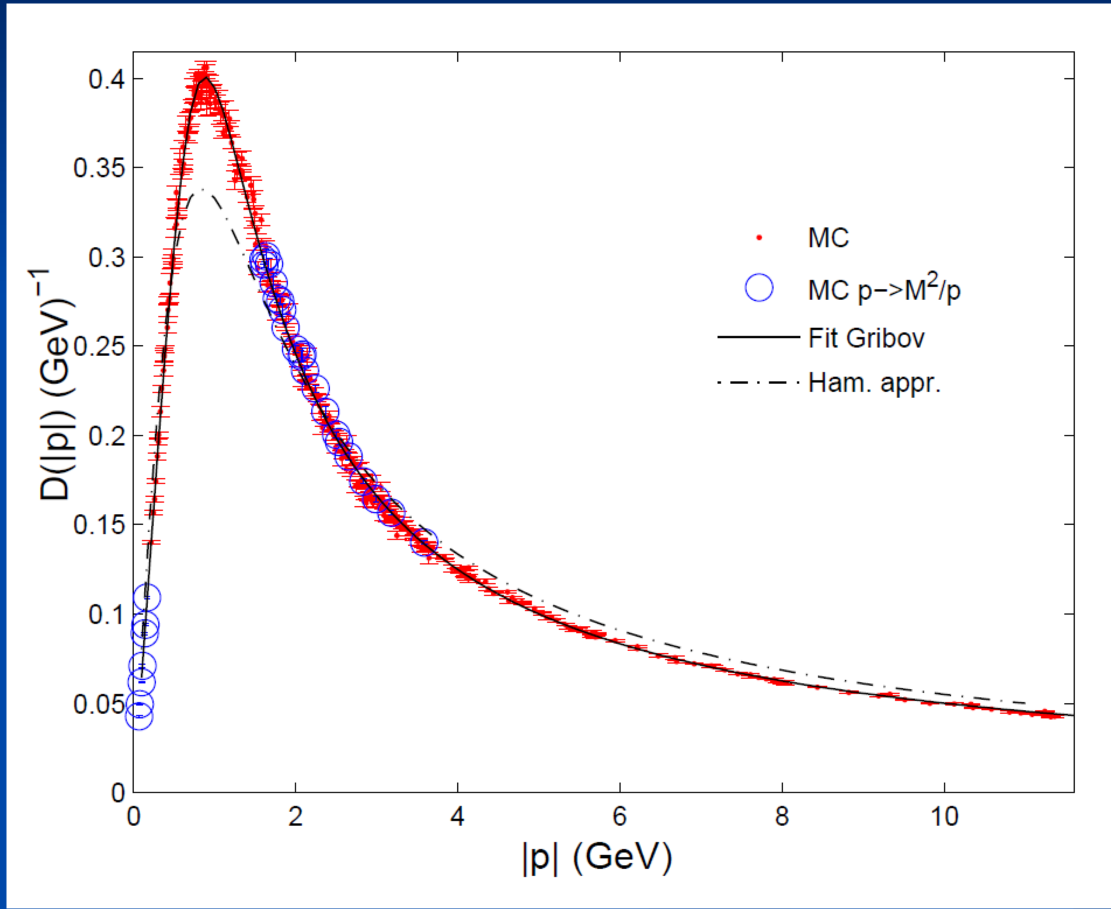
$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 GeV$$

missing strength in
mid momentum regime:
missing gluon loop



G. Burgio, M.Quandt , H.R., **PRL102(2009)**

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,
Phys.Rev.D82(2010)

wave functional

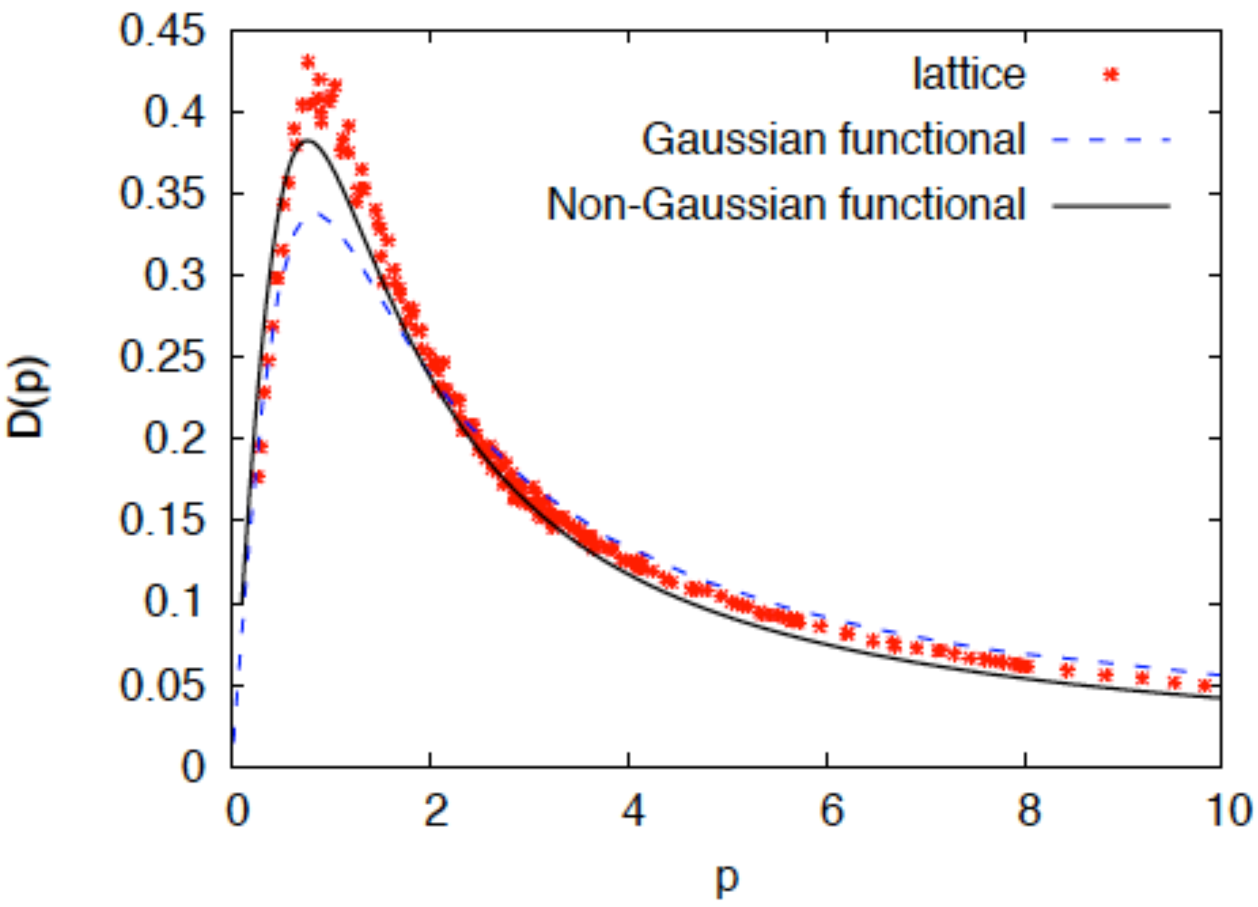
$$|\psi[A]|^2 = \exp(-S[A])$$

ansatz

$$S[A] = \int \omega A^2 + \tfrac{1}{3!}\int \gamma^{(3)} A^3 + \tfrac{1}{4!}\int \gamma^{(4)} A^4$$

exploit DSE

Corrections to the gluon propagator



D. Campagnari & H.R, Phys.Rev.D82(2010)

variational Hamiltonian approach with non-Gaussian wave functionals CRDSE

YMT D. Campagnari & H. R. , PRD82(2010)

applications: ghost loop dominance

> 3-gluon vertex

> ghost-gluon vertex

D. Campagnari & H. R.,PLB707(2012)

> 3-gluon vertex+ghost-gluon vertex

M. Huber, D. Campagnari & H. R. , PRD91(2015)

QCD D. Campagnari & H. R., arXiv:1507.01414
PRD, in press

Hamiltonian approach to YMT in Coulomb gauge $\partial A = 0$

$$H = \tfrac{1}{2}\int (J^{-1}\Pi J \Pi + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D^{ab} = \delta^{ab}\partial + gf^{abc}A^c$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

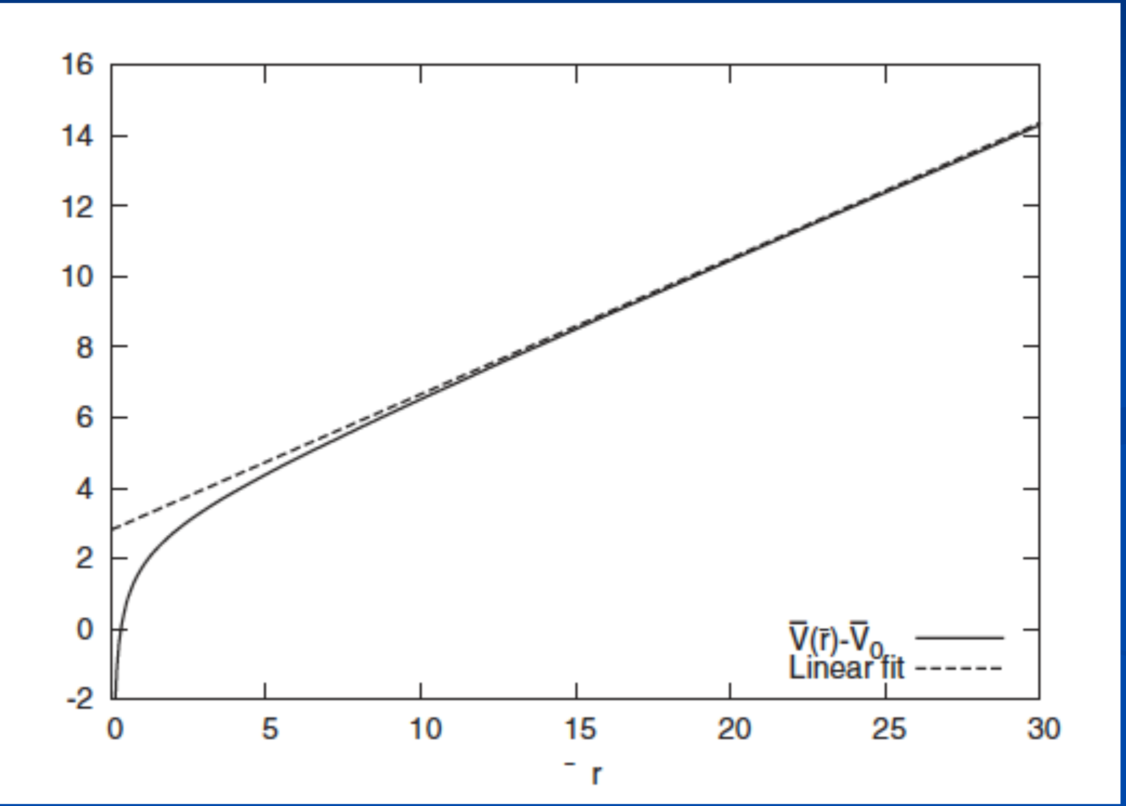
color charge density $\qquad \rho^a = -f^{abc}A^b\Pi^c + \rho_m^a$

$$\langle\phi|\ldots|\psi\rangle = \int DA J(A)\phi^*(A)\ldots\psi(A)$$

$$H\psi[A] = E\psi[A]$$

Static Coulomb potential

$$V(|x-y|) = g^2 \left\langle \langle x | (-D\partial)^{-1} (-\partial^2)(-D\partial)^{-1} | y \rangle \right\rangle$$



D. Epple, H. Reinhardt
W.Schleifenbaum,
PRD 75 (2007)

$$V(r) = \xrightarrow[r\to\infty]{} \sigma_C r, \qquad \sigma_C \geq \sigma_W \qquad \text{lattice}: \ \sigma_C = 2...3\sigma_W$$

$$V(r) = \xrightarrow[r\to 0]{} \sim 1/r$$

The QCD Hamiltonian in Coulomb gauge

$$\boxed{H_{QCD} = H_{YM} + H_C + H_q}$$

gluon part

$$H_{YM} = \tfrac{1}{2}\int (J^{-1}\Pi J \Pi + B^2) \qquad \Pi = -i\delta / \delta A \qquad J(A^{\perp}) = Det(-D\partial)$$

quark part

$$H_q = \int \Psi^{\dagger}(x)[\vec{\alpha}(\vec{p} + g\vec{A}) + \beta m_0]\Psi(x) \qquad \vec{\alpha}, \beta - \textit{Dirac matrices}$$

Coulomb term

$$H_C = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

color charge density

$$\rho^a = -f^{abc}A^b\Pi^c + \Psi^{\dagger}(x)t^a\Psi(x)$$

quark wave functional

P. Vastag & H. R.
to be published

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A}) \Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\quad \vec{\alpha}, \beta$ – *Dirac matrices*

quark wave functional

P. Vastag & H. R.
to be published

$$\langle A|\Phi\rangle_q = \exp\Big[\int \Psi^\dagger_+ (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi_-\Big]|0\rangle$$

$s, v, w -$ variational kernels $\quad \vec{\alpha}, \beta - \textit{Dirac matrices}$

$\mathrm{v} = \mathrm{w} = 0:\ \ BCS - \textit{wave function}$

Finger & Mandula
Adler & Davis, Alkofer

quark wave functional

P. Vastag & H. R.
to be published

$$\langle A | \Phi \rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A}) \Psi_- \right] | 0 \rangle$$

s, v, w – variational kernels $\vec{\alpha}, \beta$ – *Dirac matrices*

$\mathrm{v}=\mathrm{w}=0$: *BCS – wave function*

Finger & Mandula
Adler & Davis, Alkofer

$\mathrm{v}\neq 0, \mathrm{w}=0$: *quark - gluon - coupling*

Pak & Reinhardt,

quark wave functional

P. Vastag & H. R.
to be published

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s, v, w – variational kernels $\vec{\alpha}, \beta$ – Dirac matrices

$v = w = 0$: *BCS – wave function* — Finger & Mandula; Adler & Davis, Alkofer

$v \neq 0, w = 0$: *quark - gluon - coupling* — Pak & Reinhardt,

alternatives to include the coupling to the spatial gluons

- *exponential- S-method* — Kümmel etal.

$$|\Phi\rangle_q = \exp\left[-S^{(3)}\right]|BCS\rangle$$ *linarized eq.* $S^{(3)}$ — Szczepaniak & Krupinski

- *effective interaction*

$|BCS\rangle$ $\quad \int d\boldsymbol{x}\, d\boldsymbol{y}\, J_i^a(\boldsymbol{x})\, D^{ij}(|\boldsymbol{x}-\boldsymbol{y}|)\, J_j^a(\boldsymbol{y})$ — Fontoura, Krein, Vizcarra

quark wave functional

P. Vastag & H. R.
to be published

$$\langle A | \Phi \rangle_q = \exp\Big[\int \Psi_+^\dagger (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi_- \Big] |0\rangle$$

s, v, w – variational kernels $\quad \vec{\alpha}, \beta$ – *Dirac matrices*

> calculate $\langle H_{QCD} \rangle$ up to 2 loops

> variation w.r.t. **s, v, w**

$$v(p,q) = f_v[s,\omega]$$

$$w(p,q) = f_w[s,\omega]$$

$$s(p) = f_s[s,v,w;p]$$

gap equation

Renormalization

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi\right]|0\rangle$$

gap equation: $s(p) = f_s[s,v,w;p]$

strict cancelation of linear divergencies

Renormalization

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi^\dagger(\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi\right]|0\rangle$$

gap equation: $s(p) = f_s[s, v, w; p]$

strict cancelation of linear divergencies

logarithmic divergencies $g^2 \ln(\Lambda/\mu) = \tilde{g}^2(\mu)$

Renormalization

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi\right]|0\rangle$$

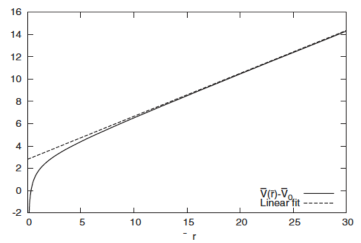
gap equation: $s(p) = f_s[s, v, w; p]$

strict cancelation of linear divergencies

logarithmic divergencies $\boxed{g^2 \ln(\Lambda/\mu) = \tilde{g}^2(\mu)}$

input: *non-abelian Coulomb potential*

$\Rightarrow$ *scale* $\mu = \sqrt{\sigma_C}$



lattice: $\sigma_C = 2\sigma$

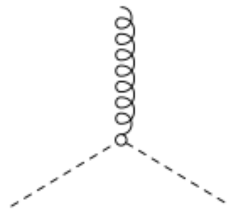
choose $\tilde{g}(\sqrt{\sigma_C})$ *to reproduce* $\langle\bar{q}q\rangle = (-235\,MeV)^3$

$\Rightarrow \tilde{g}(\sqrt{\sigma_C}) \approx 3.57$

running coupling constant

$$\alpha(p) = \frac{\tilde{g}^2(p)}{4\pi}$$

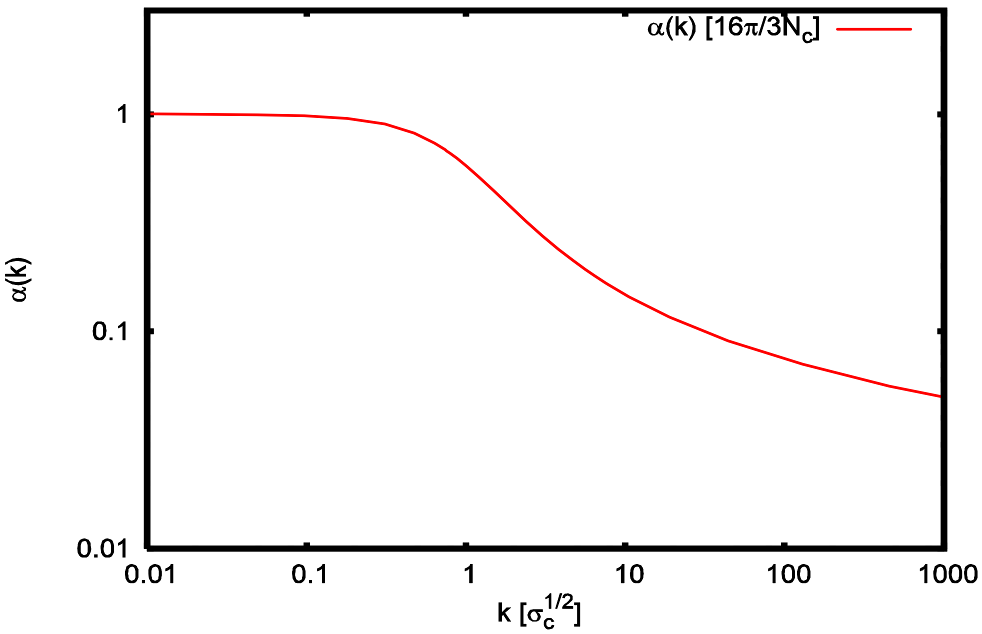
from ghost-gluon vertex



$$\tilde{g}(\sqrt{\sigma_C}) \approx 3.73$$

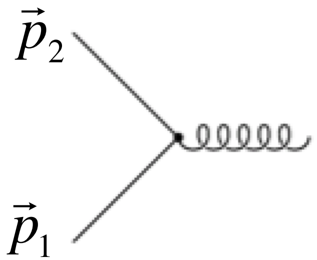
$$\langle \bar{q}q \rangle = (-235\, MeV)^3$$

$$\Rightarrow \tilde{g}(\sqrt{\sigma_C}) \approx 3.57$$

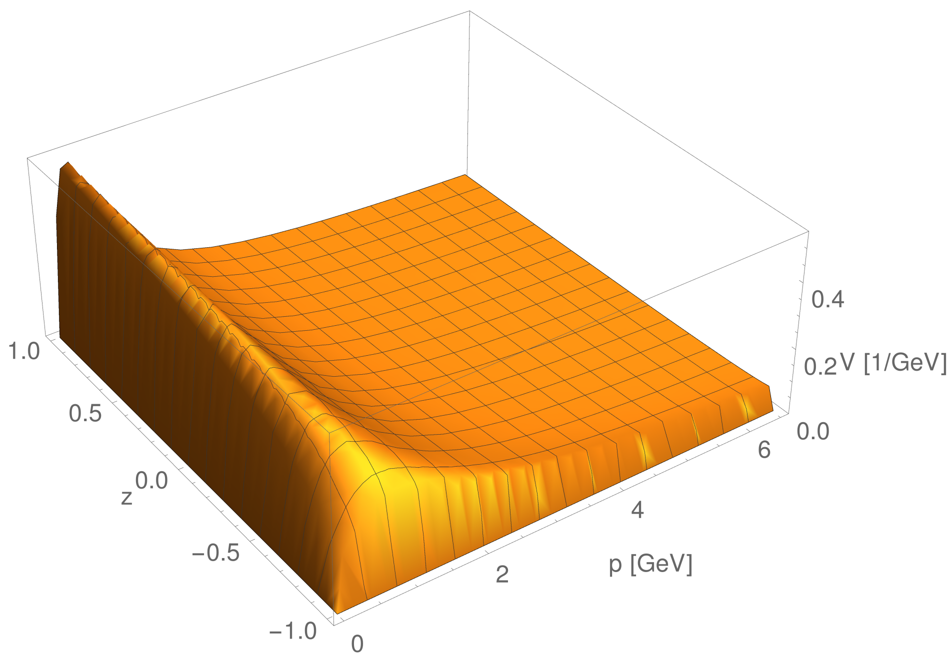


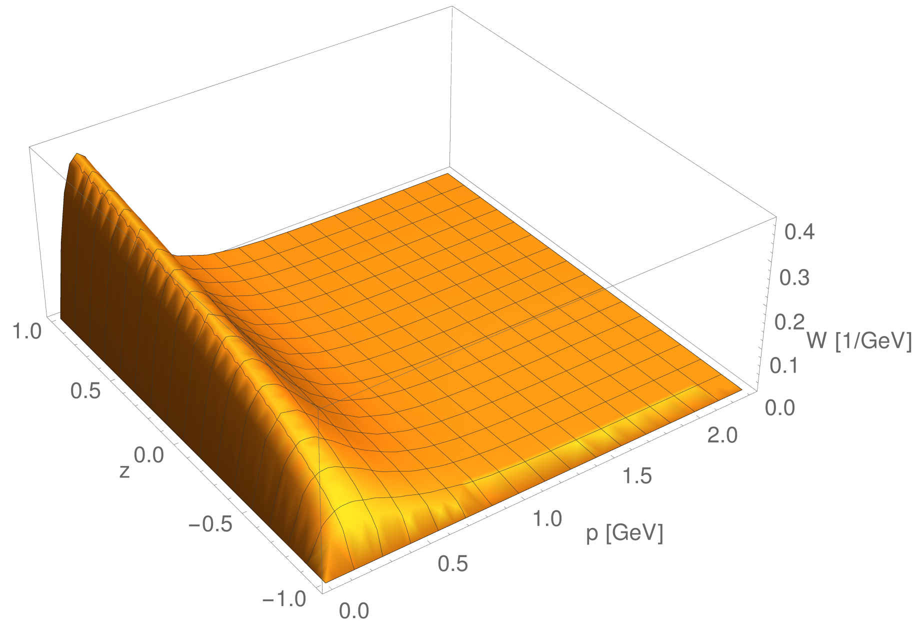
D. Epple, H. Reinhardt and W. Schleifenbaum, Phys. Rev. D75(2007)045011

vector form factors v , w

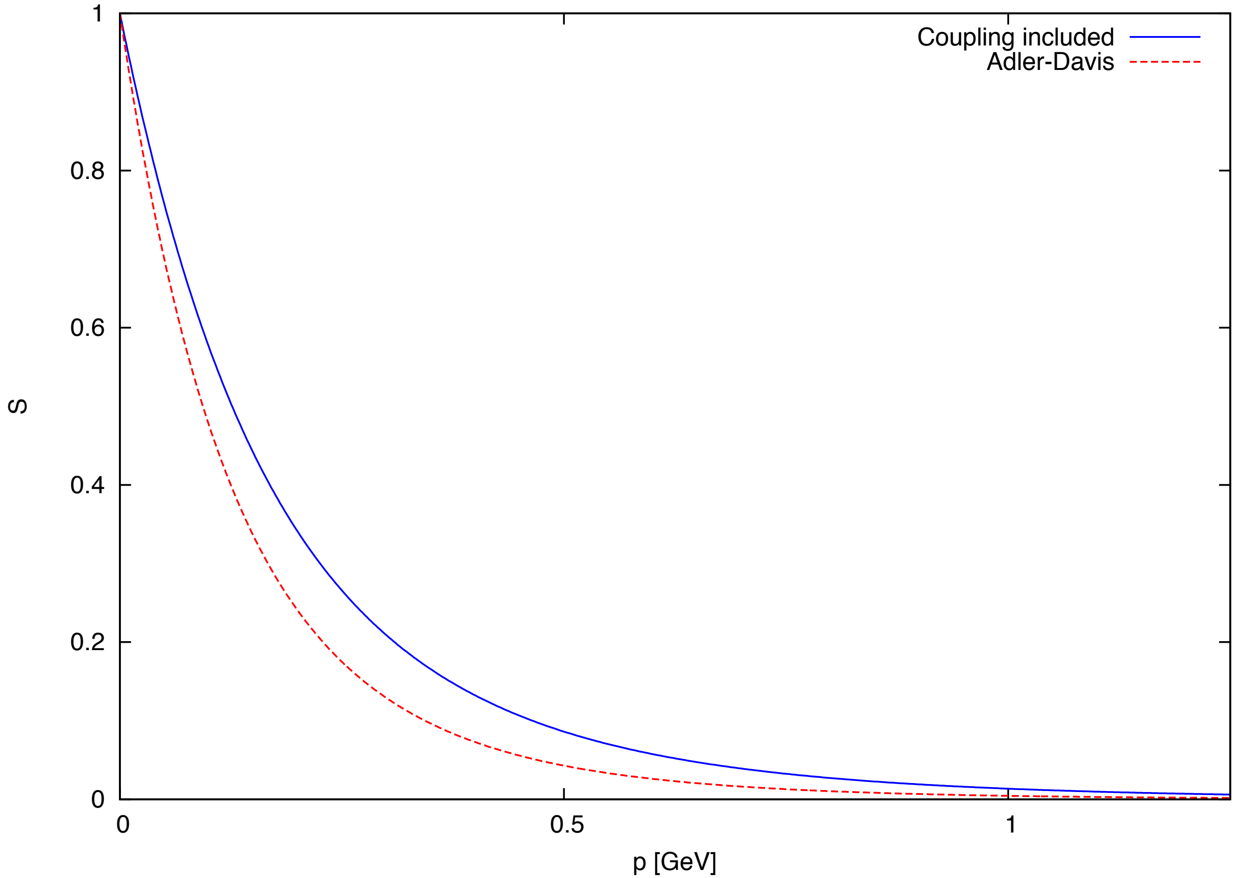


$$v,w(\vec{p}_1,\vec{p}_2): \quad p := |\vec{p}_1| = |\vec{p}_2|, \;\; z = \cos\sphericalangle(\vec{p}_1,\vec{p}_2)$$





scalar form factor

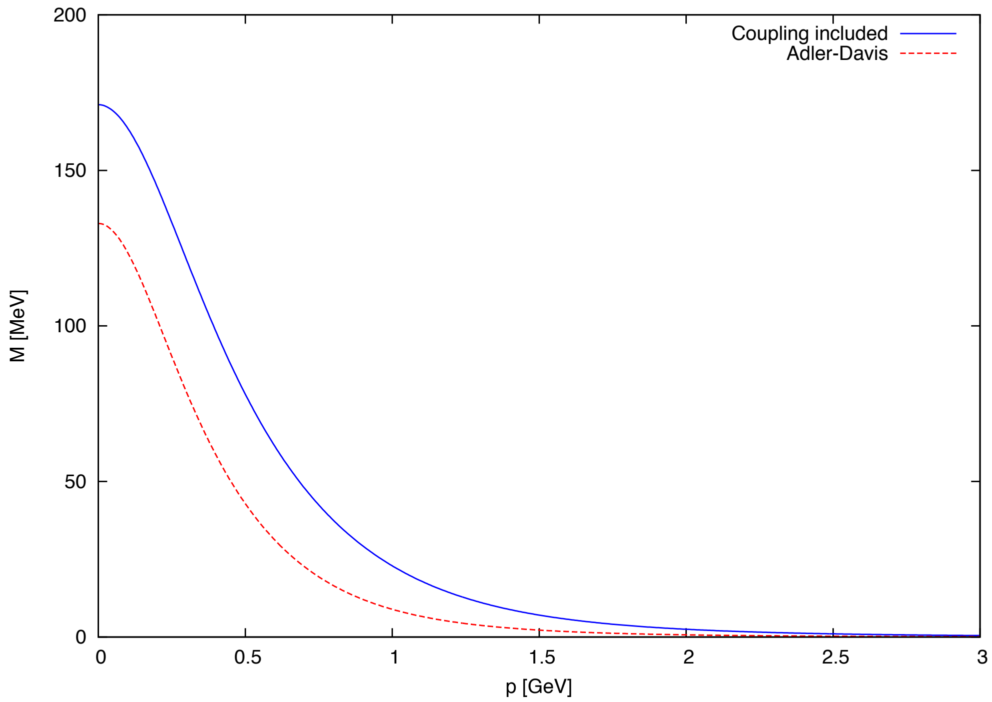


effective quark mass

$$M(p) = \frac{2ps(p)}{1 - s^2(p)}$$

$$\langle \bar{q}q \rangle_{phen} = (-235\,MeV)^3$$

$$\langle \bar{q}q \rangle_{AD} = (-179\,MeV)^3$$



> coupling to tranversal gluons substantially increases chiral symmetry breaking

unquenching the gluon propagator

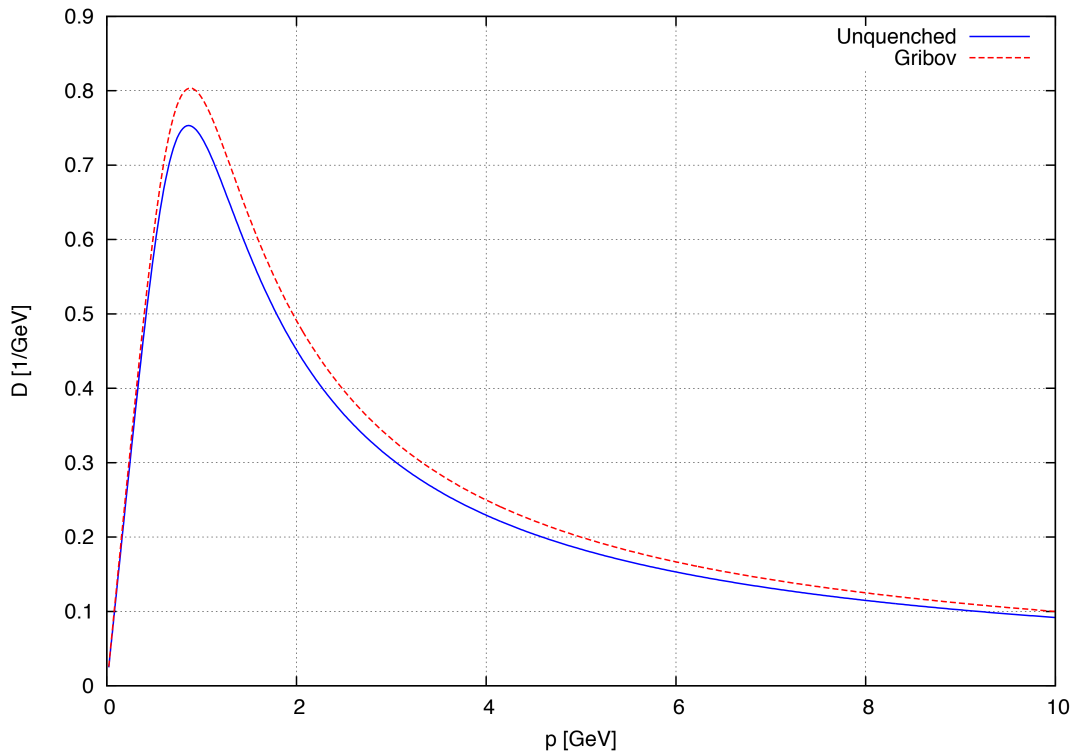
$\omega^2(p) = \omega_{YM}^2(p) +$ several „quark loop“ terms

quenched propagator:

Gribov formula

$$\omega_{YM}(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88GeV$$

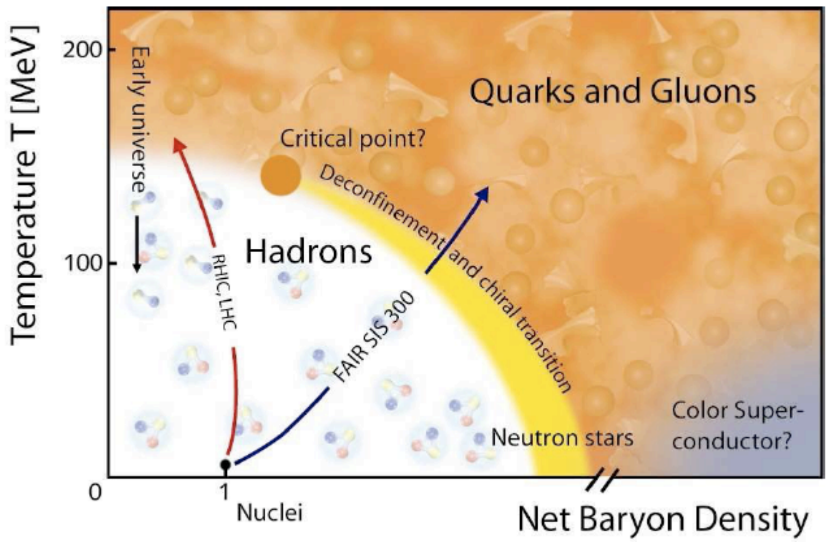


unquenching reduces gluon propagator somewhat

QCD at finite temperature: grand canonical ensemble

- quasi-particle ansatz for the density operator

- minimization of the thermodynamic potential



YM sector:

H.Reinhardt, D.Campagnari & A. Szczepaniak, Phys.Rev.D84(2011)

J.Heffner, H.Reinhardt & D.Campagnari, Phys.Rev.D85(2012)

Alternative Hamiltonian approach to finite temperature QFT

H. Reinhardt & J. Heffner,
Phys.Rev.D88(2013)
Phys.Rev.D91(2015)
and to be published

- no ansatz for the density matrix required

- motivation: Polyakov loop

$$P[A_0](\vec{x}) = \frac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

  - $\langle P[A_0] \rangle$ order parameter of confinement

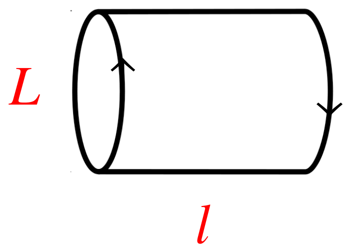
- Hamiltonian approach

  - Weyl gauge $A_0 = 0$

- How to calculate the Polyakov loop in the Hamiltonian approach?

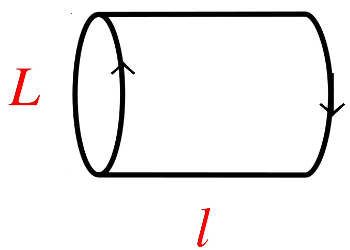
Finite temperature QFT

- compactification of (Euclidean) time
- bc: $A(x^0 = L/2) = \mathrm{A}(x^0 = -L/2)$ Bose fields
  $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields



$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\qquad l \to \infty$
- exploit the O(4)-invariance of the Euclidean Lagrangian
  - O(4)-rotation $x^0 \to x^3 \quad \mathrm{A}^0 \to A^3 \quad \gamma^0 \to \gamma^3$
    $x^1 \to x^0 \quad \mathrm{A}^1 \to A^0 \quad \gamma^1 \to \gamma^0$
  - one compactified spatial dimension
  - bc: $A(x^3 == L/2) = \mathrm{A}(x^3 = -L/2)$ Bose fields
    $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields
- spatial manifold: $\mathbb{R}^2 \times S^1(L)$

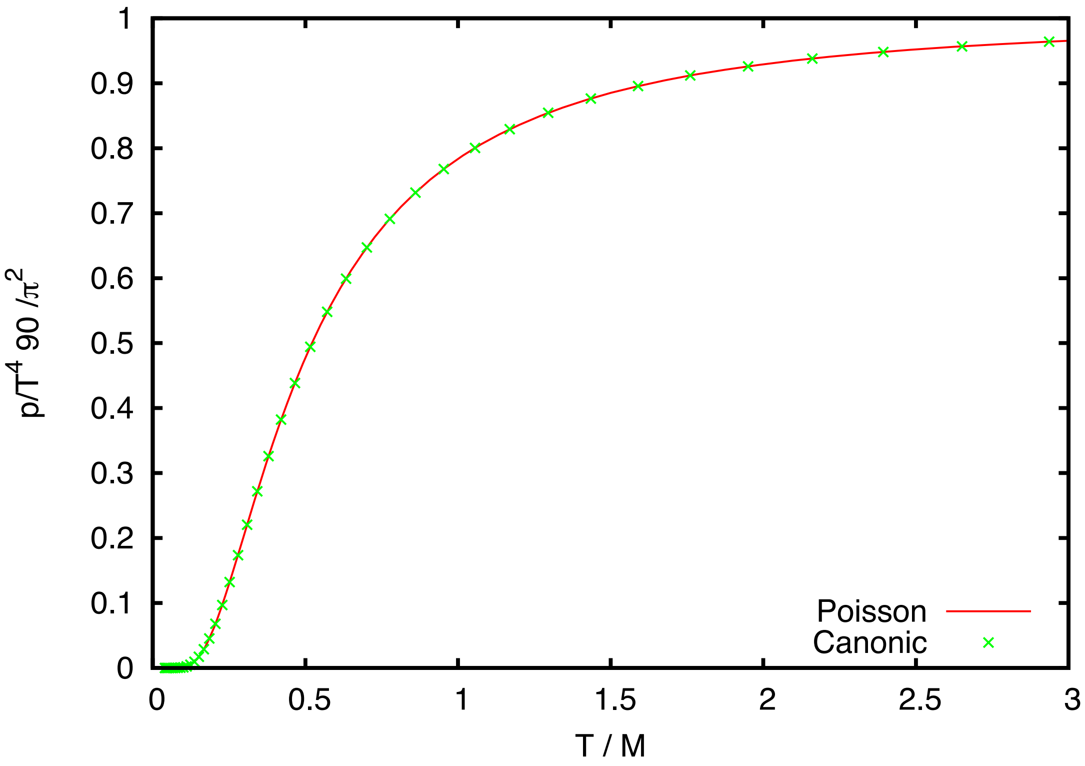


massive bosons

$$\omega(p) = \sqrt{p^2 + m^2}$$

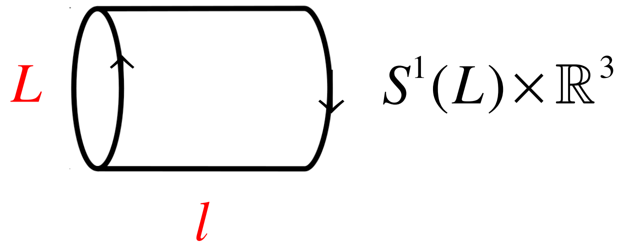
$$P = \tfrac{2}{3}\int d^3p\, p \frac{p^2}{\omega(p)} n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1}$$

$$P = -e(L) = -\frac{1}{2\pi^2} \sum_{n=-\infty}^{\infty} \left(\frac{m}{n\beta}\right)^2 K_{-2}(n\beta m)$$

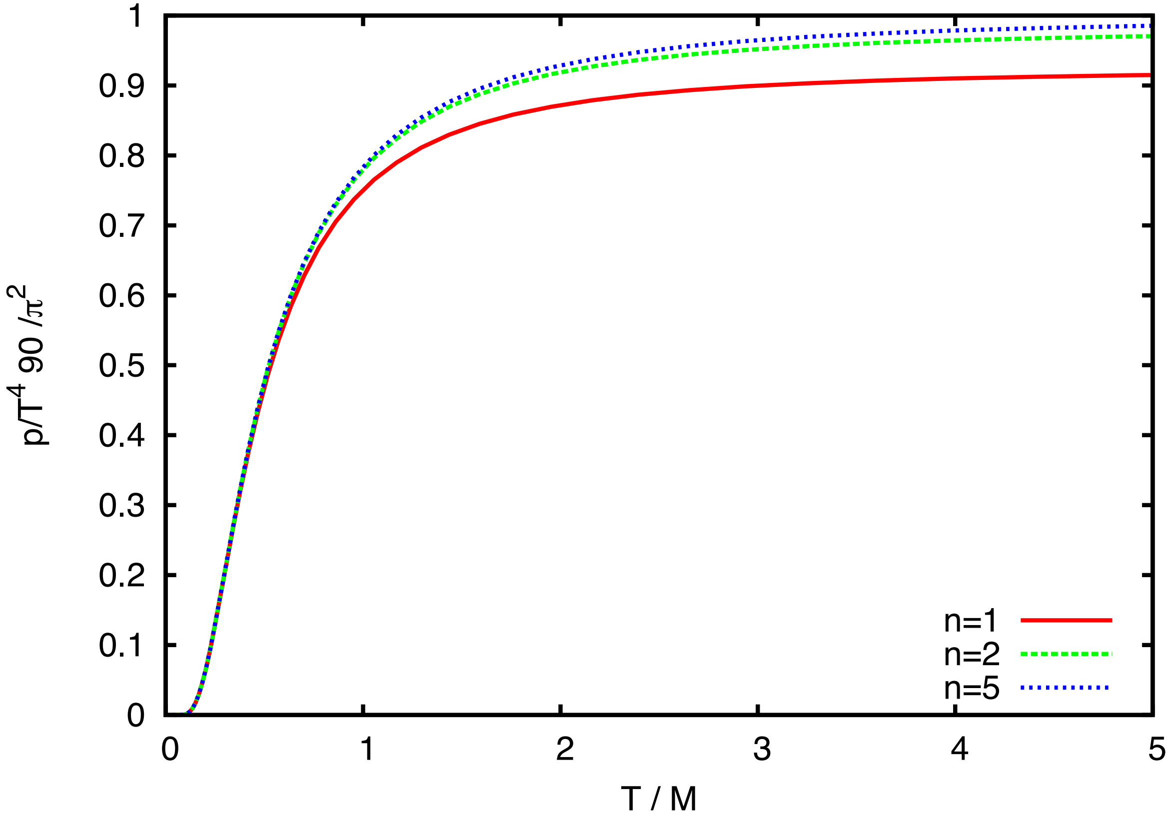


pressure of a massive relativistic Bose gas

$$e(L) = \frac{1}{2}\int d^2 p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2 + p_\perp^2 + \omega_n^2} \qquad \omega_n = \frac{2\pi n}{L}$$



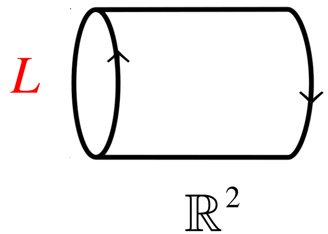
$$P = \frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{m}{n\beta}\right)^2 K_{-2}(nLm)$$



a few terms are sufficient to reproduce the result of the usual grand canonical ensemble

QCD at finite T

- Hamiltonian approach in Coulomb gauge on the partially compactified spatial manifold $\mathbb{R}^2 \times S^1(L)$



- variational solution of the Schrödinger equation for the vacuum
- finite temperature QCD is fully encoded in its vacuum

YMT at finite T

J. Heffner & H. R.
Phys.Rev.D91(2015)

Polyakov loop

H. R. & J. Heffner,
Phys.Rev.D88(2013)

dual quark condensate

H. R. & P. Vastag
to be published

recent work on the Polyakov-loop

- FRG
- *F.Marhauser and J. M. Pawlowski, arXiv:0812.11144*
- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*
- *J. Braun,T.K. Herbst, arXiv1205.0779*
- *J.Braun etal., Eur.Phys.J C70(2010)1007*
- ....
- DSE
- *C. Fischer, L. Fister, J. Lücker, J.M. Pawlowski, arXiv1306.6022*
- *U.Reinosa, J. Serreau, M. Tissier, N. Wschebor, Phys. Lett. B742(2015)61*

- Hamiltonian approach
- *H. R. & J. Heffner, Phys. Lett.B718(2012)672*
- *PRD88(2013)045024*

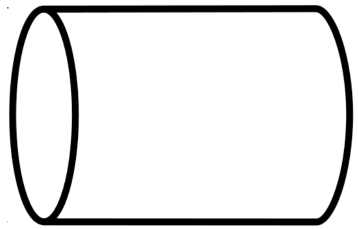
- strong coupling lattice
- *M. Fromm etal. JHEP1201(2012)042.*
- lattice
- *J. Greensite, Phys. Rev. D86(2012)114507*
- *J. Greensite and K. Langfeld, D87(2013)094501*
- *D. Smith etal arXiv1307.6339*
- .....

Polyakov loop

- YMT at finite temperature T :compact Euclidean time

$$P[A_0](\vec{x}) = \frac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$$T^{-1} = L$$



- order parameter for confinement: $\langle P[A_0](\vec{x}) \rangle \sim \exp\left[-F_\infty(\vec{x}) L \right]$
  - conf. phase: center symmetry $\langle P[A_0](\vec{x}) \rangle = 0$
  - deconf. phase: center symmetry-broken $\langle P[A_0](\vec{x}) \rangle \neq 0$

- Polyakov gauge $\partial_0 A_0 = 0, \quad A_0 = diagonal$ $\quad SU(2): \; P[A_0](\vec{x}) = \cos(\frac{A_0(\vec{x})L}{2})$
  - fundamental modular region $0 < A_0 L/2 < \pi$ $\quad P[A_0] - unique\ function\ of\ A_0$
  - **Jensen´s inequality:** $\langle P[A_0](\vec{x}) \rangle \;\leq\; P[\langle A_0(\vec{x}) \rangle]$
  - alternative order parameters: $\langle P[A_0](\vec{x}) \rangle \quad P[\langle A_0(\vec{x}) \rangle] \quad \langle A_0(\vec{x}) \rangle$

- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*
- *F.Marhauser and J. M. Pawlowski, arXiv:0812.11144*
- *J. Braun,T.K. Herbst,arXiv1205.0779*

Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle$ – const, diagonal (Polyakov gauge)
- effective potential $e[a_0] \to \min \qquad \Rightarrow a_0 = \bar{a}_0$
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$
- 1-loop perturbation theory

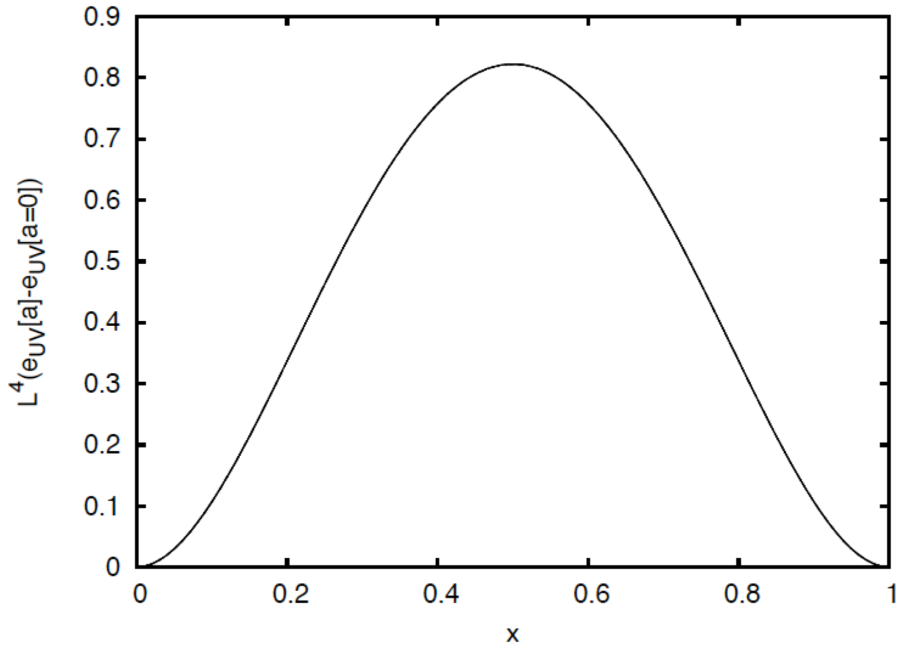
$$e_{PT}[a_0 = x 2\pi / L]$$

Gross, Pisarski, Yaffe, Rev.Mod.Pys.53(1981)

N.Weiss, Phys.Rev.D24(1981)

$$P[\bar{a}_0 = 0] = 1$$

deconfined phase



non-perturbative evaluation of $e[a_0]$ in the Hamiltonian approach

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013)

Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle$ – const, diagonal (Polyakov gauge)
- effective potential $e[a_0] \to \min \qquad \Rightarrow a_0 = \bar{a}_0$

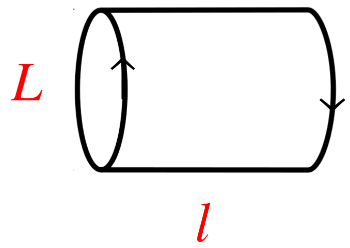
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$

- ordinary Hamiltonian approach assumes Weyl gauge $A_0 = 0$

  - O(4)-invariance

  - compactify (instead of time) one spatial axis to a circle of circumference L and interprete L^{-1} as temperature

  - Hamiltonian approach on $\mathbb{R}^2 \times S^1(L)$



  - compactify x_3 – axis $\vec{a} = a\vec{e}_3$

  - calculate the effective potential $\boxed{e[a]}$

The effective potential in the Hamiltonian approach

- effective potential $e(\vec{a})$ of a spatial background field $\vec{a}$

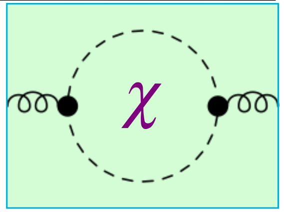
$$\langle H \rangle_{\vec{a}} = \min \langle H \rangle \qquad \langle \vec{A} \rangle = \vec{a}$$

$$\langle H \rangle_{\vec{a}} = (spatial\ volume) \times e(\vec{a})$$

$$e(\vec{a}) - effective\ potential$$

The gluon effective potential

- energy density
$$e(a,L)=\sum_{\sigma}\tfrac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\chi(p^\sigma))$$



- background field $\vec{p}^{\,\sigma}=\vec{p}_\perp+(p_n-\sigma a)\vec{e}_3 \quad p_n=2\pi n/L \quad \sigma - roots$
- roots

$SU(2)$: $H_1=T_3$ $\quad \sigma_1=0,\ \pm 1$ $\quad$ *positive roots*

$SU(3)$: $H_1=T_3 \quad H_2=T_8$ $\quad \sigma=(1,0),\ \ (\tfrac{1}{2},\tfrac{1}{2}\sqrt{3}),\ \ (\tfrac{1}{2},-\tfrac{1}{2}\sqrt{3})$

- periodicity $e(a,L)=e(a+\mu_k/L,L) \qquad \exp\left(i\mu_k\right)=z_k\in Z(N)$

$\mu_k - coweights$

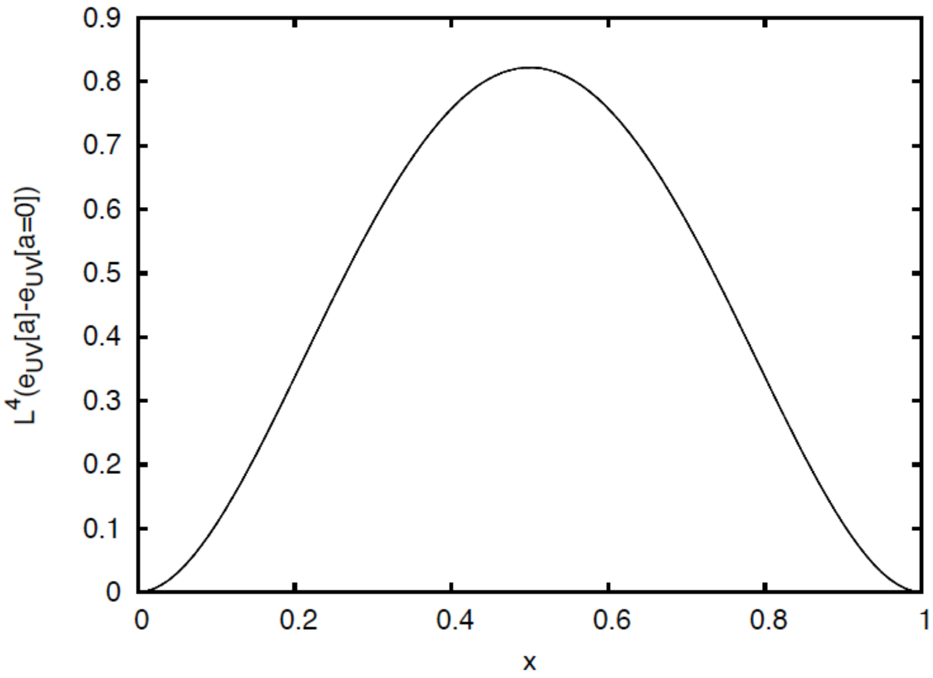
- input: $\omega(p),\chi(p)$ from the variational calculation in Coulomb gauge at T=0

C. Feuchter & H. Reinhardt,Phys. Rev.D71(2005)
D. Epple, H. Reinhardt, W. Schleifenbaum, Phys. Rev.D75(2007)

The gluon UV-effective potential

$$\chi(p)=0 \qquad \omega(p)=p \qquad e(a,L)=\sum_{\sigma}\tfrac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$

$$e(a,L)=\frac{8}{\pi^2L^4}\sum_{n=1}^{\infty}\frac{\sin^2(naL/2)}{n^4}$$
$$=\frac{4\pi^2}{3L^4}\underbrace{\left(\frac{aL}{2\pi}\right)}_{x}^2\left[\frac{aL}{2\pi}-1\right]^2$$



N.Weiss 1-loop PT

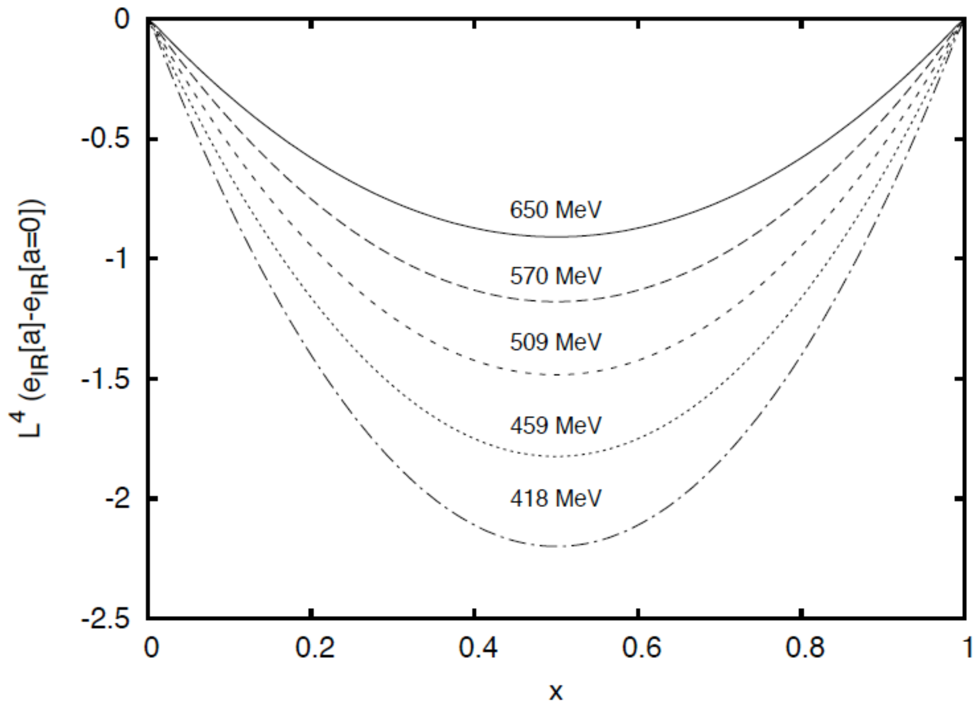
$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=0]=1 \qquad deconfining \quad phase$$

The gluon IR-effective potential

$$\chi(p)=0 \qquad \omega(p)=M^2/p \qquad e(a,L)=\sum_\sigma \frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$

$$e_{IR}(a,L)=-\frac{4M^2}{\pi^2L^2}\sum_{n=1}^{\infty}\frac{\sin^2(naL/2)}{n^2}$$

$$=\frac{2M^2}{L^2}\underbrace{\left(\frac{aL}{2\pi}\right)}_{x}\left[\frac{aL}{2\pi}-1\right]$$



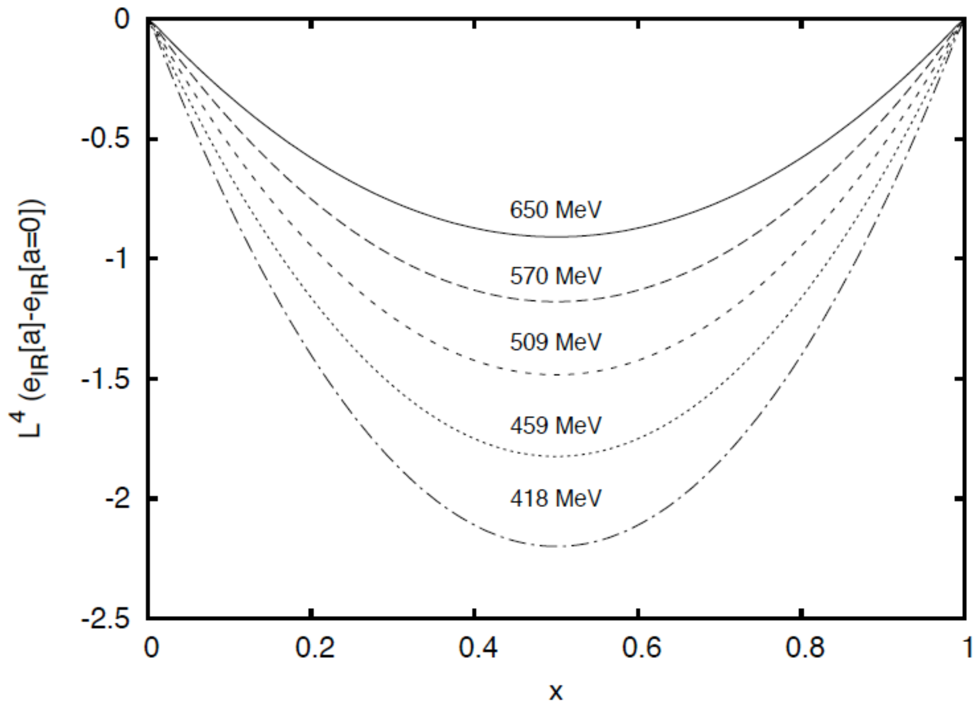
$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=\pi/L]=0 \qquad confining \quad phase$$

The gluon IR-effective potential

$$\chi(p) = 0 \qquad \omega(p) = M^2 / p \qquad e(a,L) = \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \cancel{\chi(p^\sigma)})$$

$$e_{IR}(a,L) = -\frac{4M^2}{\pi^2 L^2} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^2}$$

$$= \frac{2M^2}{L^2} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x} \left[\frac{aL}{2\pi} - 1\right]$$



$$Polyakov-loop \qquad \langle P \rangle \simeq P[a_{\min} = \pi / L] = 0 \qquad confining \quad phase$$

deconfinement phase transition results from the interplay between the confining IR-potential and deconfining UV-potential

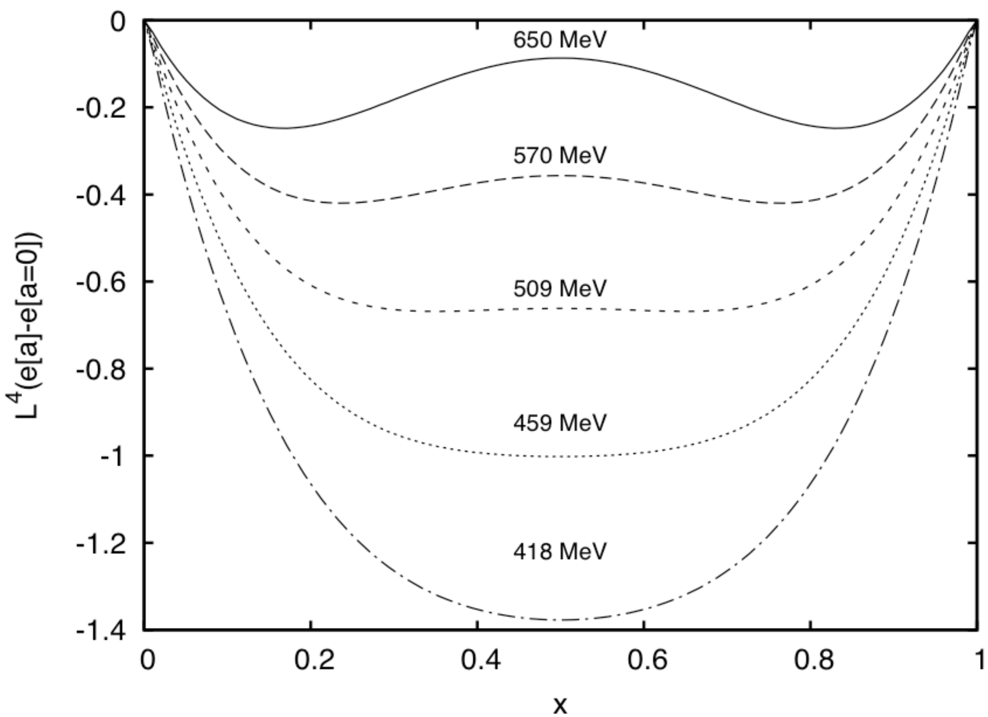
The gluon IR+UV effective potential:

$\chi(p) = 0$ $\omega(p) = p + M^2 / p$ $e(a, L) = e_{UV}(a, L) + e_{IR}(a, L)$

phase transition

critical temperature:

$T_C = \sqrt{3} M / \pi$

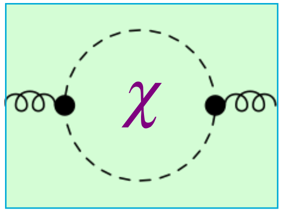


$lattice : M \simeq 880\, MeV \quad \Rightarrow \quad T_C \simeq 485\, MeV$

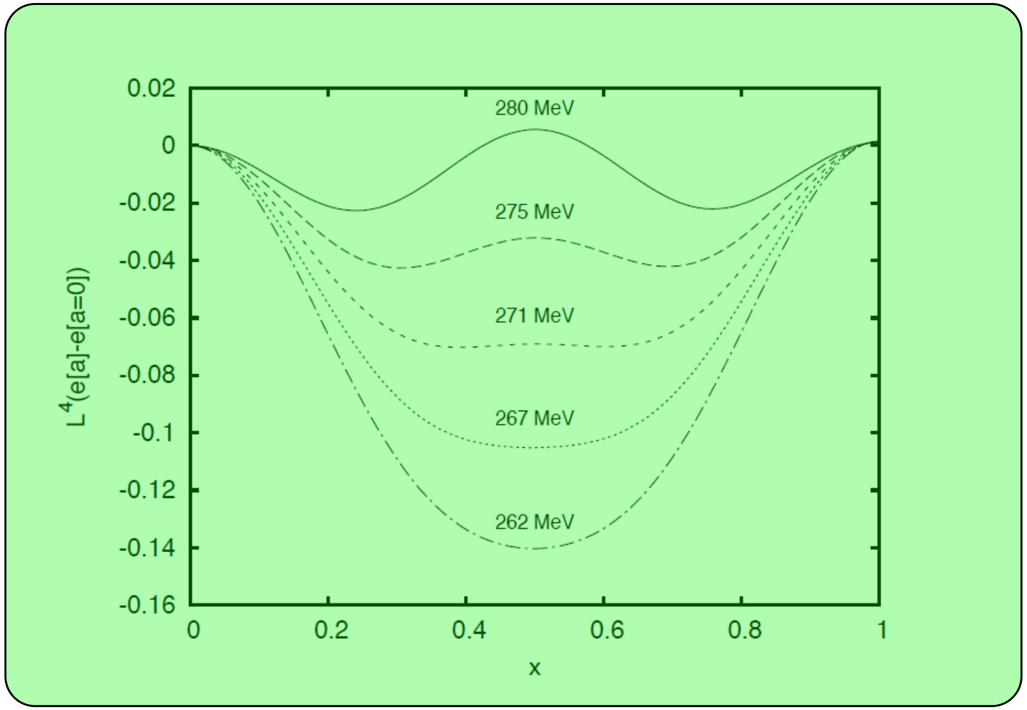
$\chi(p) = 0$ $\omega(p) = \sqrt{p^2 + M^4 / p^2}$ $T_C \simeq 432\, MeV$

The full effective potential

$$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_{\perp} (\omega(p^{\sigma}) - \chi(p^{\sigma}))$$



variational calculation in Coulomb gauge



SU(2)

critical temperature:

$$T_C \simeq 270\, MeV$$

The effective potential for SU(3)

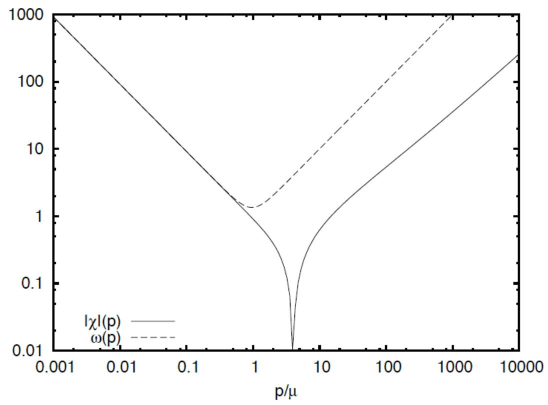
SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive rooots

$$\boxed{\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)}$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

The full effective potential for SU(3)

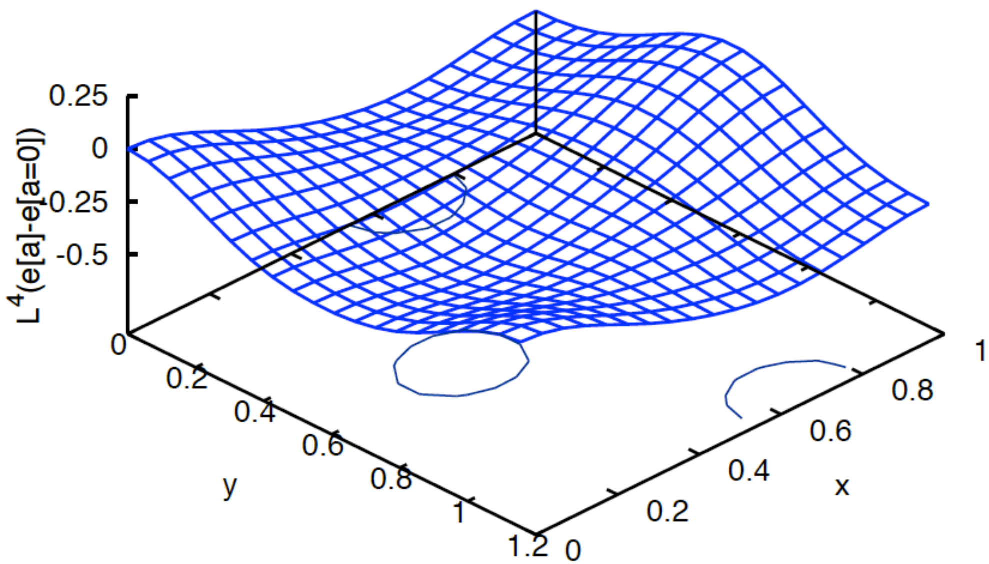
$$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_{\perp} (\omega(p^{\sigma}) - \chi(p^{\sigma}))$$

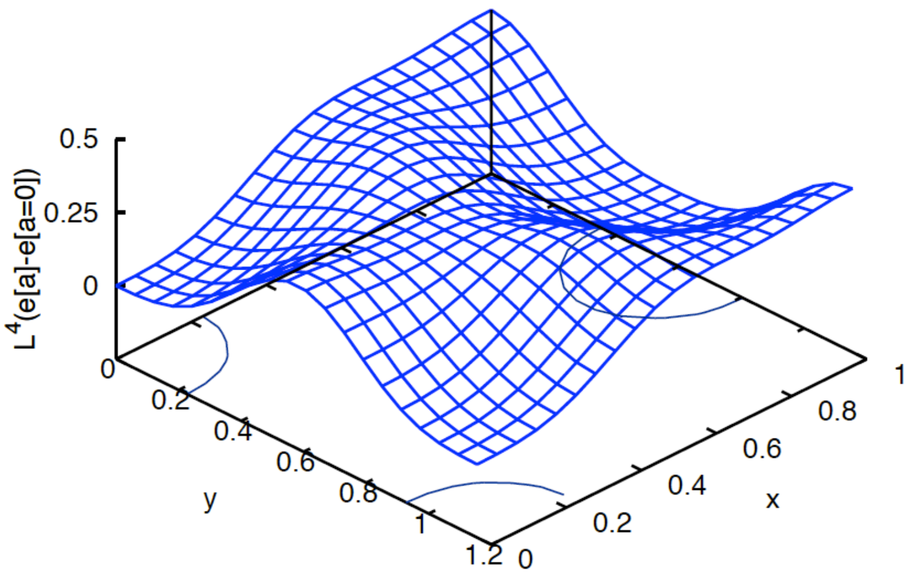


variational calculation in Coulomb gauge

$T < T_C$

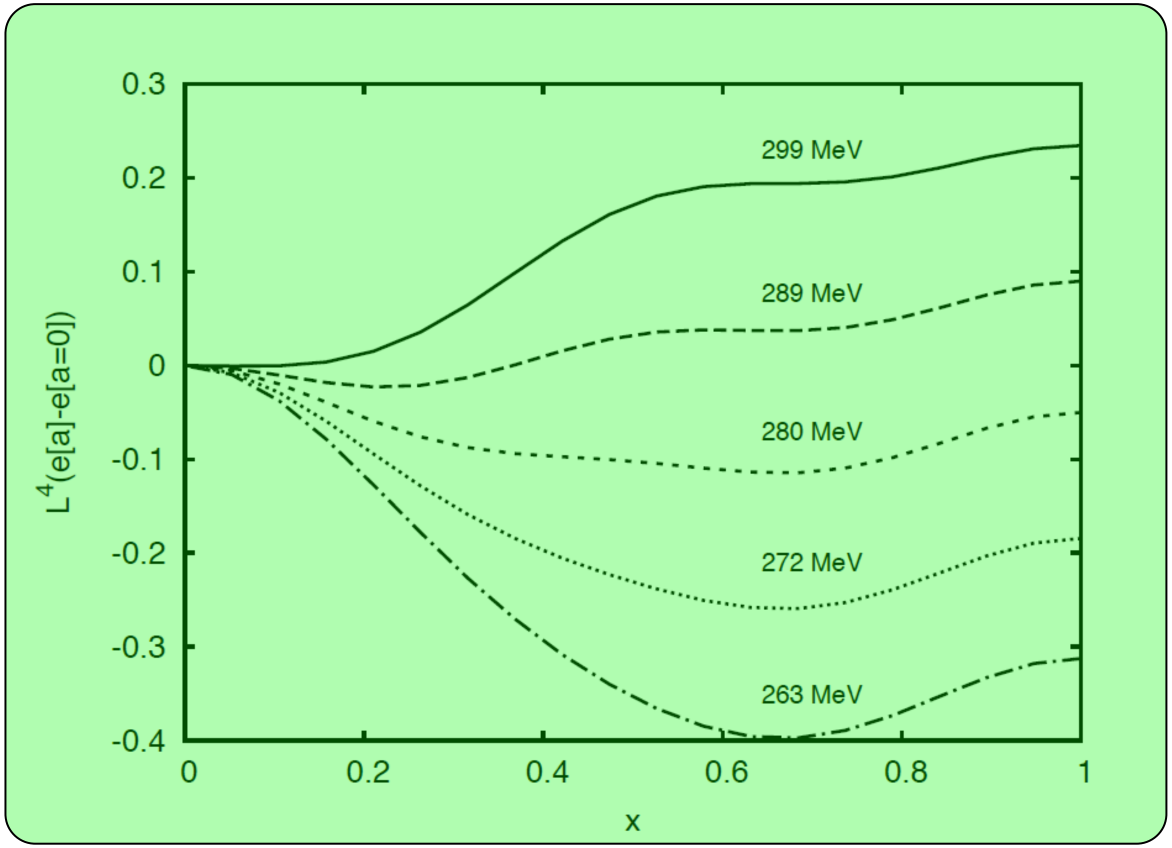
$T > T_C$





$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi}$

Polyakov loop potential for SU(3)



$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi} = 0$$

$$input: \ SU(2) - data:$$
$$M = 880\,MeV$$

$$T_C = 283\,MeV$$

critical temperature

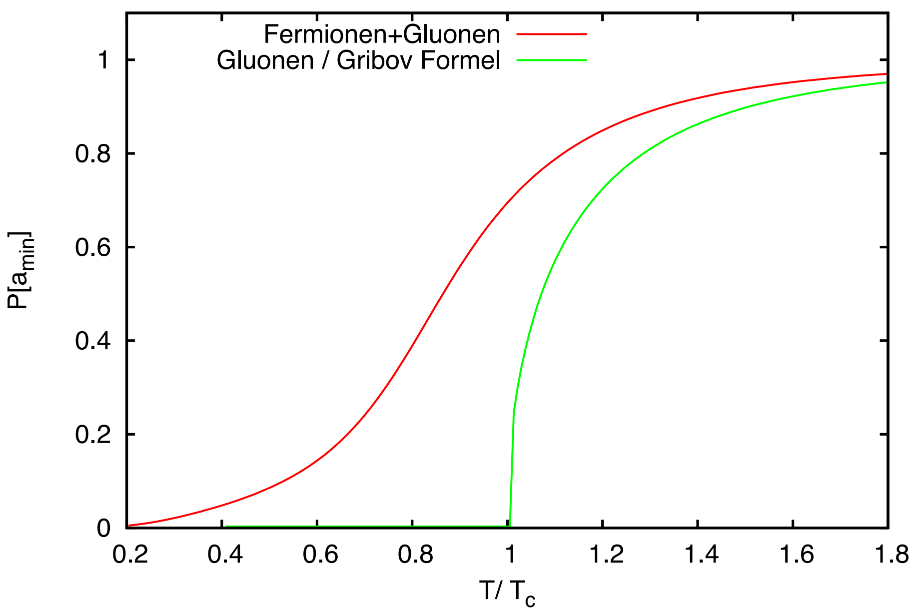
$lattice:$ $T_C^{SU(2)} = 312\,MeV$ $T_C^{SU(3)} = 284\,MeV$

$this\ work:$ $T_C^{SU(2)} = 269\,MeV$ $T_C^{SU(3)} = 283\,MeV$

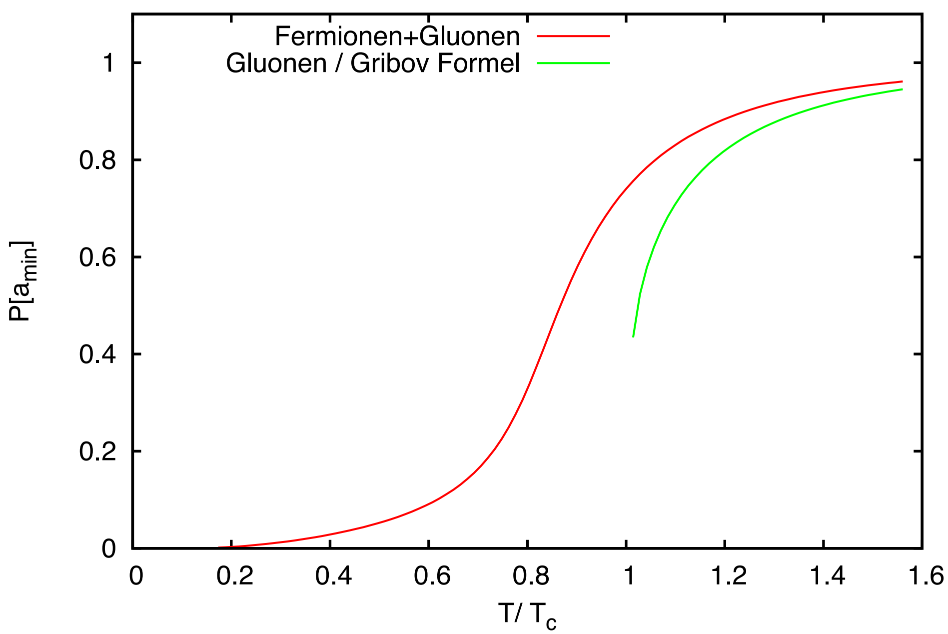
$FRG(Fister\ \&\ Pawlowski):$ $T_C^{SU(2)} = 230\,MeV$ $T_C^{SU(3)} = 275\,MeV$

lattice: *B. Lucini, M. Teper, U. Wenger, JHEP01(2004)061*

The Polyakov loop



SU(2)



SU(3)

center symmetry

DECONFINEMENT PHASE TRANSION:

confined phase: center symmetry
deconfined phase: center symmetry broken

any observable transforming non-trivially under the center may serve as order parameter for confinement

prototype: Polyakov loop

$$P[A_0](\vec{x}) = \frac{1}{d_r} trP\exp\left[i\int_0^L dx_0 A_0(x_0,\vec{x}) \right]$$

dual quark condensate -dressed Polyakov loop

Gattringer, PRL. 97(2006)
Synatschke, Wipf, Wozar, Phys. Rev. D75(2007)
....

lattice: Bilgici, Bruckmann, Gattringer, Hagen, PR D77(2008)...
FRG: Braun, Haas, Marhauser, Pawlowski, PRL 106(2011)
DSE: Fischer, Maas, Müller, Eur. Phys. J. 68(2010) ...

Hamiltonian approach: H. R. & P. Vastag, to be published

dual quark condensate -dressed Polyakov loop

$$\Sigma_n = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-in\varphi} \left\langle (\bar{q}q)_\varphi \right\rangle \qquad q(\beta) = e^{i\varphi} q(0)$$

Σ_n *loops winding n-times around the compact time axis*

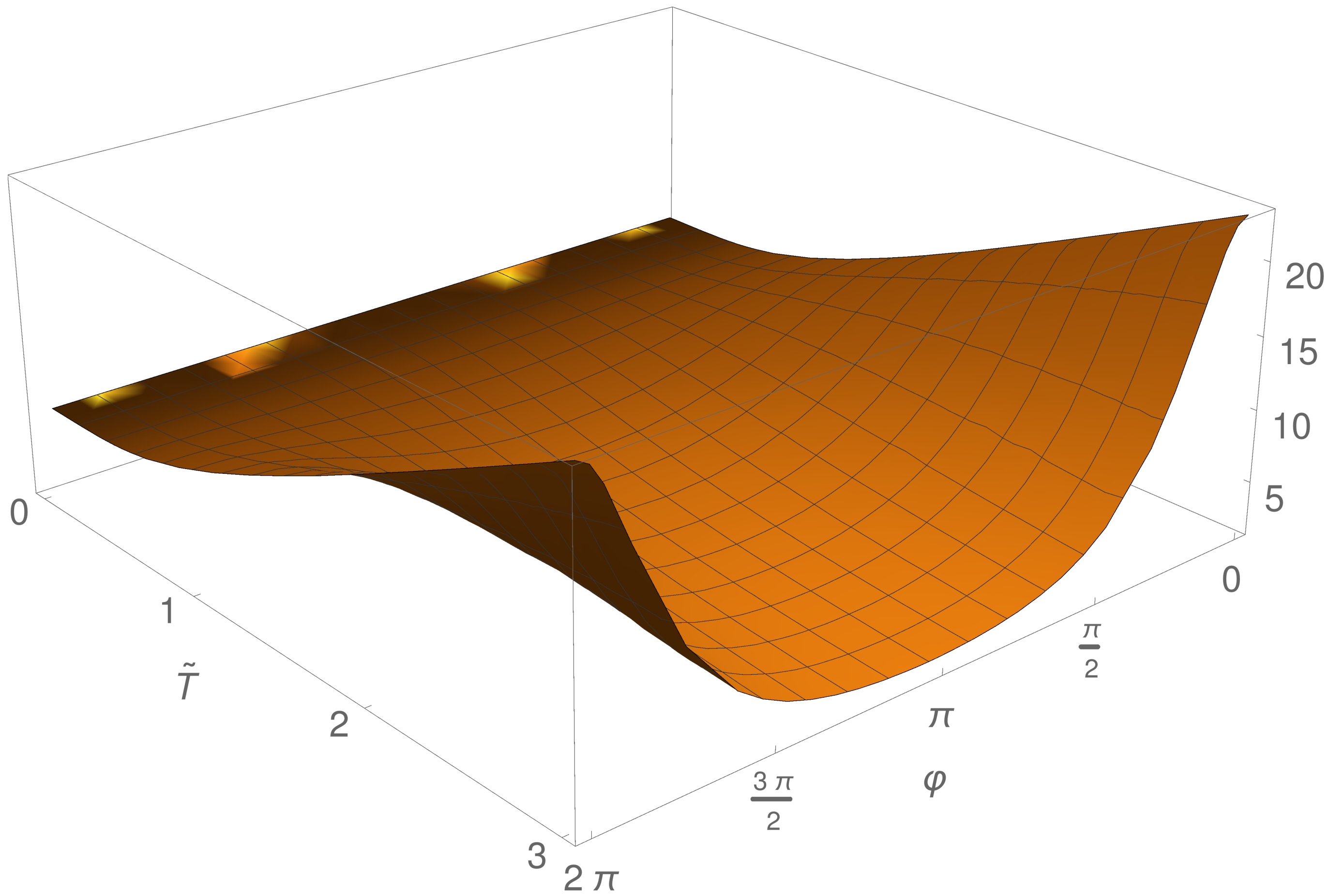
Σ_1 *dressed Polyakov loop*

Gattringer
PRL. 97(2006)

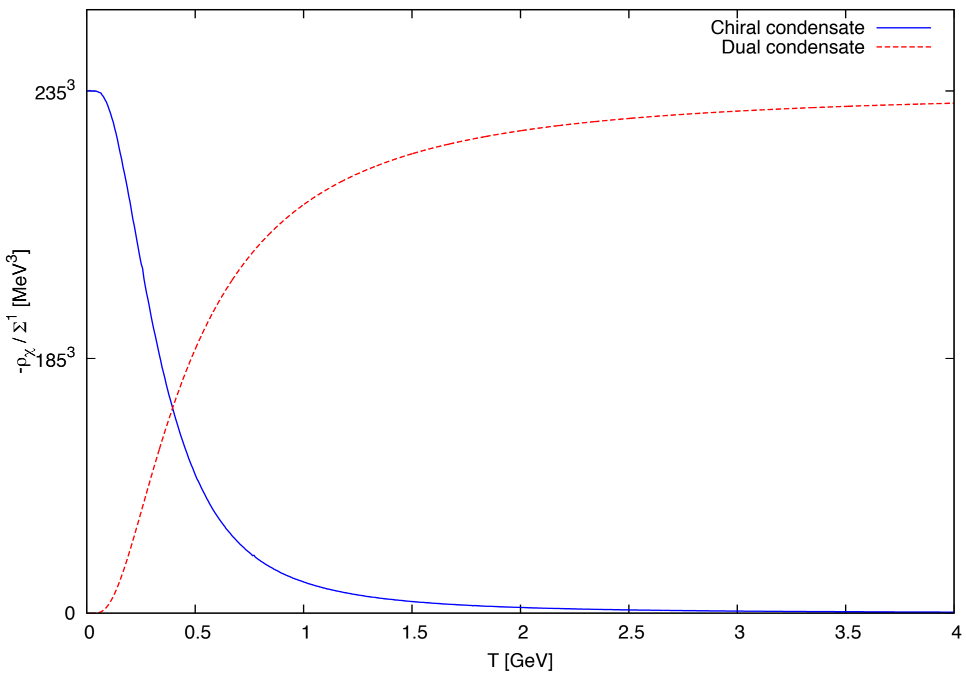
$$imaginary\ chemical\ potential: \quad \mu = i\frac{\pi - \varphi}{\beta}$$

$$compactified\ \ 3\text{-}axis\ potential: \quad p_3 = \Omega_n + i\mu = \frac{2\pi n + \varphi}{\beta}$$

quark condensate $\left\langle (\bar{q}q)_{\varphi} \right\rangle$



chiral & dual condensate



$$\sigma_C = 2\sigma \qquad T_{PC} \simeq 256\,MeV$$

Conclusions

- variational approach to the Hamiltonian formulation of QCD in Coulomb gauge
  - quark sector
- novel Hamiltonian approach to finite temperature QFT
  - compactification of a spatial dimension
  - the finite QFT is fully encoded in the ground state of the spatial manifold $R^2 \times S^1$
- effective potential of the Polyakov loop gluonic part of the eff. potential:
  - deconfinement phase transition
    - SU(2): 2.order
    - SU(3): 1.order
- inclusion of quarks:
  - the deconfinement phase transition is turned into a crossover
- dual quark condensate