Symmetry-preserving contact interaction model for heavy-light mesons

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Introduction

Contact fermonic interactions find widespread applications in hadron physics with Nambu–Jona-Lasinio (NJL) type of models. The lack of nonrenormalizabilty of such models can lead to gross violations of global symmetries due to the regularization procedure - because of ambiguities arising from momentum shifts in divergent integrals. Despite well known, practitioners at large inexplicably ignore these problems.

In this work we employ a subtraction scheme [1, 2, 3] to separate symmetry-offending parts in amplitudes in a way independent of choices of momentum routing in divergent integrals. We apply it to the contact interaction model of Ref. [4] to heavy-light mesons. After solving the gap equation – Dyson-Schwinger (DSE) equation – for the u, s, and c quarks, we obtain the Bethe-Salpeter equations (BSE) for the bound states of pseudoscalar π, K and D mesons in a way that they satisfy the Ward-Green-Takahashi (WGT) identities for arbitrary routing of the momenta running in loop integrals.

DSE, BSE and WGT Identities

The DSE for the quark propagator of flavour f is

$$S_f(k)^{-1} = i\gamma\cdot k + m_f + \int_q g^2 D_{\mu\nu}(k-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,k) \quad (1)$$

where m_f is the current-quark mass $D_{\mu\nu}$ the gluon propagator, and Γ^f_ν the quark-gluon vertex . The meson mass, m_{PS}, is the eigenvalue $P^2 = -m^2_{PS}$ that solves the homogeneous BSE for the pseudoscalar vertex $\Gamma^{lh}_{PS}(k;P)$

$$\left[\Gamma^{lh}_{PS}(k;P)\right]_{AB} = \int_q [K(k,q;P)]_{AC,DB} \times \left[S_l(q_+)\Gamma^{lh}_{PS}(q;P)S_h(q_-)\right]_{CD} \quad (2)$$

where $K(q,k;P)$ is the fully amputated quark-antiquark scattering kernel; $A, B, \cdots$ denote collectively color, flavor, and spinor indices; $q_\pm = q \pm \eta_\pm P$, with $\eta_+ + \eta_- = 1$. The Ward-Green-Takahashi (WGT) identity is

$$P_\mu \Gamma^{lh}_{5\mu}(k;P) = S_l^{-1}(k_+)i\gamma_5 + i\gamma_5 S_h^{-1}(k_-) - i\,(m_l + m_h)\,\Gamma^{lh}_{PS}(k;P) \quad (3)$$

where the pseudovector vertex $\Gamma^{lh}_{5\mu}(k;P)$ satisfies an inhomogeneous BSE. The contact-interaction scheme of Ref. [4] amounts to make the replacements in the DSE

$$g^2 D_{\mu\nu}(p-q) \to G_S\,\delta_{\mu\nu} \text{ and } \Gamma^a_\mu \to \frac{\lambda^a}{2}\gamma_\mu \quad (4)$$

together with the replacement in the BSE

$$[K(k,q;P)]_{AC,DB} = -G_S\left(\frac{\lambda^a}{2}\gamma_\mu\right)_{AC}\left(\frac{\lambda^a}{2}\gamma_\mu\right)_{DB} \quad (5)$$

to preserve the WGT identity [5]. Here, G_s is a (flavor-independent) coupling constant. In this case, the pseudoscalar vertex is $\Gamma^{lh}_{PS}(k,P) = \gamma_5\left[iE^{lh}_{PS} + \gamma\cdot P\,F^{lh}_{PS}\right]$

Symmetry-preserving subtraction scheme

Physical observables cannot depend on the choice of $\eta_\pm$. The subtraction scheme makes repeated use of the identity in the integrals in the BSE:

$$\frac{1}{q^2_\pm + M^2} = \left(\frac{1}{q^2_\pm + M^2} - \frac{1}{q^2 + M^2}\right) + \frac{1}{q^2 + M^2} = \frac{1}{q^2 + M^2} - \frac{q^2_\pm - q^2}{(q^2 + M^2)}\frac{1}{(q^2_\pm + M^2)} \quad (6)$$

Performing two of such subtractions in Eq. (2), the BSE can be written in a matrix form

$$\begin{bmatrix} E^{lh}_{PS}(P) \\ F^{lh}_{PS}(P)\end{bmatrix} = \frac{8G_s}{3}\begin{bmatrix}\mathcal{K}^{EE}_{lh} & \mathcal{K}^{EF}_{lh} \\ \mathcal{K}^{FE}_{lh} & \mathcal{K}^{FF}_{lh}\end{bmatrix}\begin{bmatrix} E^{lh}_{PS}(P) \\ F^{lh}_{PS}(P)\end{bmatrix} \quad (7)$$

where

$$\mathcal{K}^{EF}_{PS} = 8\Big\{\left[\eta^2_+ A_{\mu\nu}(M^2_l) + \eta^2_- A_{\mu\nu}(M^2_h)\right]P_\mu P_\nu + I_{quad}(M^2_l) + I_{quad}(M^2_h) - \left[P^2 + (M_h - M_l)^2\right] \times \left[I_{log}(M^2_h) - Z_0(M^2_h, M^2_l, P^2; M^2_h)\right]\Big\} \quad (8)$$

where $Z_0(M^2_h, M^2_l, P^2; M^2_h)$ is the finite integral

$$Z_0(M^2_h, M^2_l, P^2; M^2_h) = \int_0^1 dz\,[H(z) - M^2_h] \times \int_q \frac{2q^2 + M^2_h + H(z)}{(q^2 + M^2_h)^2[q^2 + H(z)]^2} \quad (9)$$

with $H(z) = z(1-z)P^2 - (M^2_l - M^2_h)z + M^2_l$ and $A_{\mu\nu}(M^2)$, $I_{quad}(M^2)$ and $I_{log}(M^2)$ are the divergent integrals (when $\Lambda \to \infty$)

$$A_{\mu\nu}(M^2) = \int_q^\Lambda \frac{4q_\mu q_\nu - (q^2 + M^2)\delta_{\mu\nu}}{(q^2 + M^2)^3} \quad (10)$$

$$I_{quad}(M^2) = \int_q^\Lambda \frac{1}{q^2 + M^2}$$

$$I_{log}(M^2) = \int_q^\Lambda \frac{1}{(q^2 + M^2)^2} \quad (11)$$

It is important to note that to arrive at these results, no momentum shifts were made in divergent integrals. The other amplitudes, $\mathcal{K}^{EF}_{lh}$, $\mathcal{K}^{FE}_{lh}$, and $\mathcal{K}^{FF}_{lh}$ are given by similar integrals [6].

Note that whatever choice made for $\eta_\pm$, unavoidably implies translation symmetry breaking; *unless* the regularization scheme leads to $A_{\mu\nu}(M^2) = 0$.

Let us next examine the WGT identity in Eq. (3). One needs to deal with integrals of the form

$$\int_q^\Lambda \frac{q_\mu}{q^2_\pm + M^2} = \mp\eta_\pm P_\mu\,I_{quad}(M^2) \pm \frac{1}{3}\eta^3_\pm\left[P^2 A_{\mu\rho}(M^2)P_\rho - P_\mu A_{\rho\sigma}(M^2)P_\rho P_\sigma\right] \mp \eta_\pm B_{\mu\rho}(M^2)P_\rho \mp \frac{1}{3}\eta^3_\pm C_{\mu\rho\sigma\lambda}(M^2)P_\sigma P_\rho P_\lambda \quad (12)$$

where $B_{\mu\nu}(M^2)$ and $C_{\mu\nu\rho\sigma}(M^2)$ are the new structures

$$B_{\mu\nu}(M^2) = \int_q^\Lambda \frac{2q_\mu q_\nu - (q^2 + M^2)\delta_{\mu\nu}}{(q^2 + M^2)^2} \quad (13)$$

$$C_{\mu\nu\rho\sigma}(M^2) = \int_q^\Lambda \frac{c_{\mu\nu\rho\sigma}(q, M^2)}{(q^2 + M^2)^4} \quad (14)$$

with

$$c_{\mu\nu\rho\sigma}(q^2, M^2) = 24 q_\mu q_\nu q_\rho q_\sigma - 4(q^2 + M^2) \times (\delta_{\mu\nu}q_\rho q_\sigma + \text{perm. } \nu\sigma\rho) \quad (15)$$

Using such integrals in the WGT identity, one obtains

$$0 = (M_l - m_l) + (M_h - m_h) - 48G_S\left[I_{quad}(M^2_l) + I_{quad}(M^2_h)\right] + \left[\eta^2_+ A_{\mu\nu}(M^2_l) + \eta^2_- A_{\mu\nu}(M^2_h)\right]P_\mu P_\nu \quad (16)$$

and

$$0 = \int_q^\Lambda\left(\frac{q_+\cdot P}{q^2_+ + M^2} - \frac{q.P}{q^2 + M^2}\right) \sim \text{terms prop. to } \eta_\pm\,(A_{\mu\nu}, B_{\mu\nu}C_{\mu\nu\rho\sigma}) \quad (17)$$

We see that for arbitrary momentum routing in the loop integrals, the subtraction scheme allows to identify in a systematic way symmetry offending terms; they are the integrals $A_{\mu\nu}$, $B_{\mu\nu}$ and $C_{\mu\nu\rho\sigma}$ in Eqs. (10), (13) and (14). A consistent regularization scheme should make the integrals vanish automatically. Otherwise, the vanishing of the integrals must be imposed; in doing so, the regularization scheme becomes part of the model. It is worth mentioning that dimensional regularization and Pauli-Villars regularization are examples of schemes that lead to $A_{\mu\nu} = 0$, $B_{\mu\nu} = 0$, and $C_{\mu\nu\rho\sigma} = 0$.

Numerical results

Masses and electroweak decay constants of π, K, and D mesons have been obtained in the NJL model in Ref. [7] using the random phase approximation with cutoff regularization - results of this reference are shown in the first two columns of Table 1.

The last two columns in the table are the results employing the subtraction scheme. They were obtained calculating $I^\Lambda_{quad}(M_f)$ and $I^\Lambda_{log}(M_f)$ with proper time regularization. We used the set parameters

$$G_s = \frac{16\pi\alpha_{IR}}{36m^2_G}, \quad \alpha_{IR} = 0.93\pi, \quad m_G = 0.8\,\text{GeV},$$

$$m_u = 0.0071\text{GeV}, \; m_s = 0.176\text{GeV}, \; m_c = 1.356\text{GeV}.$$

$$\Lambda_{UV} = 0.905, \quad \Lambda_{IR} = 0.240.$$

and finite integrals like $Z_0(M^2_h, M^2_l, P^2; M^2_h)$ are integrated *without* a cutoff.

Table 1 : Masses and e.w. decay constants (in GeV).

Meson	m_{PS}(Ref. [7])	f_{PS}(Ref. [7])	m_{PS}	f_{PS}
π	0.135	0.092	0.140	0.100
K	0.498	0.095	0.494	0.110
D	1.869	0.079	1.730	0.132

The experimental values of the decay constants are in the direction of $f_\pi < f_K < f_D$. This pattern is nicely reproduced by the subtraction scheme, while the traditional cutoff regularization gives $f_D < f_\pi$.

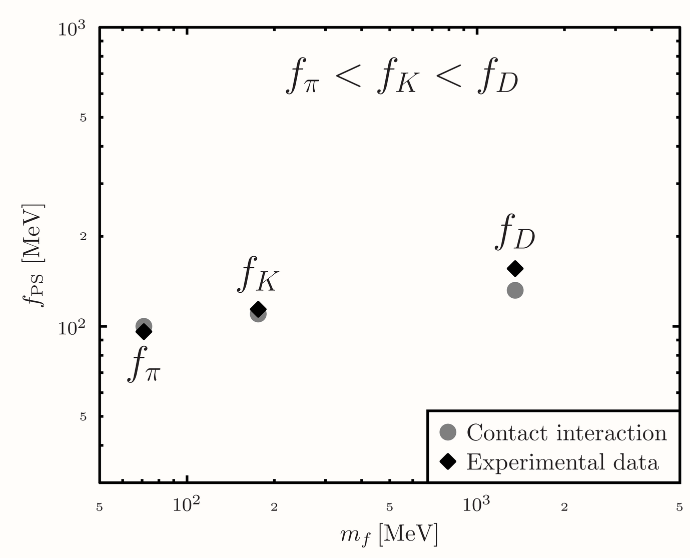

Figure 1 : Electroweak decay constants.

Better values can be obtained using a smaller value for the effective coupling G_S for the heavy quarks.

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