Gribov horizon, BRST symmetry and non-perturbative formulation of linear covariant and maximal Abelian gauges

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Short summary of the Gribov-Zwanziger framework in the Landau gauge

Linear covariant gauges

Maximal Abelian gauge

A look at 2d: decoupling versus scaling solution for the gluon propagator.

# Short summary of the Gribov-Zwanziger framework in the Landau gauge

The Gribov-Zwanziger framework implements the restriction of the domain of integration in the functional integral to the Gribov region  $\Omega$ . This restriction is needed to account for the Gribov copies.

$$\Omega = \{ A^{a}_{\mu}, \partial A^{a} = 0, \ \mathcal{M}^{ab} > 0 \}$$
$$\mathcal{M}^{ab} = -\partial_{\mu} \left( \delta^{ab} \partial_{\mu} + f^{acb} A^{c}_{\mu} \right)$$
$$\int [d\mu]_{FP} e^{-S_{YM}} \to \int_{\Omega} [d\mu]_{FP} e^{-S_{YM}}$$

 $[d\mu]_{FP} = [DA]\delta(\partial A)det(\mathcal{M})$ 

The region  $\Omega$  has important properties: 1) it is bounded in all direction in field space, 2) it is convex, 3) all gauge orbits crosses the region  $\Omega$  at least once.

As we have learned from the previous talk by D. Dudal

$$\int_{\Omega} [d\mu]_{FP} \ e^{-S_{YM}} = \int [d\mu]_{FP} \ e^{-(S_{YM} + \gamma^4 H(A) - \mathcal{C})}$$
$$\mathcal{C} = 4V\gamma^4 (N^2 - 1))$$
$$H = \int d^4x d^4y f^{abc} A^b_{\mu}(x) \left[\frac{1}{\mathcal{M}(x, y)}\right]^{ad} f^{dec} A^e_{\mu}(y)$$

<u>Remark</u>: similarly to the case of the Faddeev-Popov determinant, the horizon function can be cast in local form by introducing a suitable set of localizing fields. Once cast in local form, the resulting action turns out to be renormalizable: GZ action.

$$\mathcal{Z}_{GZ} = \int [D\Phi] \ e^{-(S_{GZ} - \mathcal{C})}$$
$$S_{GZ} = S_{FP} + \int d^4x \ \left(\bar{\phi}\mathcal{M}(A)\phi - \bar{\omega}\mathcal{M}(A)\omega + \gamma^2 A(\phi + \bar{\phi})\right)$$
$$S_{FP} = S_{YM} + \int (b\partial A + \bar{c}\partial D(A)c)$$

The massive Gribov parameter  $\gamma$  is determined in a selfconsistent dynamical way through the gap equation

$$\frac{\partial \mathcal{E}_{v}}{\partial \gamma^{2}} = 0 \qquad \mathcal{Z}_{GZ} = e^{-V\mathcal{E}_{v}}$$

$$\gamma^{2} \sim e^{-\frac{1}{g^{2}}} \qquad \begin{array}{l} \text{See L. Palhares's talk for} \\ \text{more about } \end{array}$$

The GZ action breaks in a soft way the standard BRST transf., see previous talks by D. Dudal and A. Cucchieri

$$sA^{a}_{\mu} = -D^{ab}_{\mu}c^{b} \qquad sc^{a} = \frac{g}{2}f^{abc}c^{b}c^{c} \qquad s\bar{c}^{a} = b^{a} \qquad sb^{a} = 0$$
$$s\bar{\omega} = \bar{\phi} \qquad s\bar{\phi} = 0 \qquad s\phi = \omega \qquad s\omega = 0$$

$$sS_{GZ} = \gamma^2 \int d^4x \left( -Dc(\phi + \bar{\phi}) + A\omega \right)$$

Recently, we have been able to find out a nilpotent nonperturbative extension of the standard BRST operator which is an exact symmetry of the GZ action

Capri et.al, arXiv:1506.06995, Phys. Rev. D 92, 045039 (2015)

$$s \to s_{\gamma} \qquad s_{\gamma} s_{\gamma} = 0$$
  
 $s_{\gamma} S_{GZ} = 0$ 

An important step in the construction of the non-perturbative BRST symmetry is the introduction of the gauge invariant transverse field

$$A^{h}_{\mu} = \mathcal{P}_{\mu\nu} \left( A_{\nu} - ig \left[ \frac{\partial A}{\partial^{2}}, A_{\nu} \right] + \frac{ig}{2} \left[ \frac{\partial A}{\partial^{2}}, \partial_{\nu} \frac{\partial A}{\partial^{2}} \right] \right) + O(A^{3})$$
$$\mathcal{P}_{\mu\nu} = \left( \delta_{\mu\nu} - \frac{\partial_{\mu} \partial_{\nu}}{\partial^{2}} \right)$$

The field A<sup>h</sup> is transverse and invariant under infinitesimal gauge transformations order by order.

The GZ action can be rewritten in the following way

$$H(A) = H(A^{h}) - \mathcal{R}(A)\partial A \qquad b^{h} = b - \gamma^{4}\mathcal{R}(A)$$
$$S_{GZ} = S_{YM} + \int (b^{h}\partial A + \bar{c}\partial Dc)$$
$$+ \int (\bar{\phi}\mathcal{M}(A^{h})\phi - \bar{\omega}\mathcal{M}(A^{h})\omega + \gamma^{2}A^{h}(\bar{\phi} + \phi))$$

$$s_{\gamma}A_{\mu} = -D_{\mu}c \qquad s_{\gamma}c = gc^{2} \qquad s_{\gamma}\bar{c} = b^{h} \qquad s_{\gamma}b^{h} = 0$$
$$s_{\gamma}\bar{\omega} = \bar{\phi} + \gamma^{2}A^{h}\frac{1}{\mathcal{M}(A^{h})} \qquad s_{\gamma}\bar{\phi} = 0 \qquad s_{\gamma}\phi = \omega \qquad s_{\gamma}\omega = 0$$

$$s_{\gamma}S_{GZ} = 0$$

the operator  $s_{y}$  exhibits the following properties

1) it is nilpotent

$$s_{\gamma}s_{\gamma}=0$$

2) it depends explicitly on the non-perturbative Gribov parameter *§*. In this sense, it represents a non-perturbative extension of the standard BRST operator

*3) it reduces to the standard operator when*  $\gamma$ =0

$$s_{\gamma}\big|_{\gamma=0} = s$$

4) the operator  $s_{\gamma}$  generalizes to the case of the Refined-Gribov-Zwanziger action

$$S_{RGZ} = S_{GZ} + \int d^4x \left(\frac{m^2}{2} (A^h)^2 + \mu^2 (\bar{\phi}\phi - \bar{\omega}\omega)\right)$$

where the dynamical parameters  $(m,\mu)$  are related to dimension two condensates

$$m^2 \sim \langle A^h A^h 
angle \qquad \mu^2 \sim \langle \bar{\phi}\phi - \bar{\omega}\omega 
angle$$
  
 $ilde{s}_{\gamma} S_{RGZ} = 0 \qquad ilde{s}_{\gamma} ilde{s}_{\gamma} = 0$ 

$$\tilde{s}_{\gamma}A_{\mu} = -D_{\mu}c \qquad \tilde{s}_{\gamma}c = gc^{2} \qquad \tilde{s}_{\gamma}\bar{c} = b^{h} \qquad \tilde{s}_{\gamma}b^{h} = 0$$
$$\tilde{s}_{\gamma}\bar{\omega} = \bar{\phi} + \gamma^{2}A^{h}\frac{1}{\mathcal{M}(A^{h}) + \mu^{2}} \qquad \tilde{s}_{\gamma}\bar{\phi} = 0 \qquad \tilde{s}_{\gamma}\phi = \omega \qquad \tilde{s}_{\gamma}\omega = 0$$

# The linear covariant gauges

Following the general BRST set up, we employ the operator  $s_{\gamma}$  to address the issue of the Gribov copies in other gauges as, for example, the linear covariant gauges.

The idea is that of defining the gauge-fixing as an exact  $s_{\gamma}$ -variation, while providing a geometrical understanding in terms of elimination of Gribov copies.

$$S_{gf} = s_{\gamma} \ \tilde{S}$$

The linear covariant gauges are defined by the gauge condition

$$\partial A = \alpha b$$

Some references on linear covariant gauges

### Lattice formulation:

- 1) L. Giusti, hep-lat/9605032, Nucl. Phys. B498 (1997) 331
- 2) A. Cucchieri et al., arXiv:0907.4138, PRL 103 (2009) 141602
- 3) P. Bicudo et al., arXiv:1505.05897

#### Variational methods

F. Siringo, arXiv:1408.5313, PRD 90 (2014) 9094021

#### Schwinger-Dyson framework

A.C. Aguilar et al., arXiv:1501.07150, PRD 91 (2015) 8, 085014 M. Huber, arXiv: 1502.0457, PRD 91 (2015) 8, 085018

### Gribov's point of view

R.F. Sobreiro et al., JHEP 0506, 054 (2005) M. A. L. Capri et al., arXiv:1505.05467 M. A. L. Capri et al., arXiv:1506.06995, PRD For the analogue of the GZ action in linear covariant gauges, we have found

$$S_{GZ}^{LCG} = S_{YM} + \int \left(\bar{\phi}\mathcal{M}(A^{h})\phi - \bar{\omega}\mathcal{M}(A^{h})\omega + \gamma^{2}A^{h}(\bar{\phi} + \phi)\right) \\ + s_{\gamma}\int \left(\bar{c}\partial A - \frac{\alpha}{2}\bar{c}b^{h}\right) \\ = S_{YM} + \int \left(\bar{\phi}\mathcal{M}(A^{h})\phi - \bar{\omega}\mathcal{M}(A^{h})\omega + \gamma^{2}A^{h}(\bar{\phi} + \phi)\right) \\ + \int \left(b^{h}\partial A - \frac{\alpha}{2}b^{h}b^{h} + \bar{c}\partial D(A)c\right)$$

$$s_{\gamma}S_{GZ}^{LCG} = 0$$

## The action $S_{GZ}^{LCG}$ enjoys the following properties:

1) It enables us to keep control of the gauge parameter  $\alpha$ . From the explicit one-loop computation of the vacuum energy (see arXiv:1505.05467), it follows that both vacuum energy and Gribov parameter  $\gamma$  are independent from the gauge parameter  $\alpha$ . A twoloop check of this statement is being worked out.

2) Nilpotency of the operator  $s_{Y}$  enables us to speak about the cohomolgy of  $s_{Y}$ . In particular, local gauge invariant operators as  $F^{2}(x)$ , etc., belong to the cohomology of  $s_{Y}$ . As a consequence, the correlation functions of gauge invariant quantities,  $< F^{2}(x)$   $F^{2}(y)$ , are independent from  $\alpha$ , a result easily checked already at the first order.

3) As we have seen from Dudal's talk, both  $s_{\gamma}$  and  $S_{GZ}^{LCG}$  can be cast in local form. Local action, local Ward identities.

4) <u>Geometrical picture</u>. The action  $S_{GZ}^{LCG}$  implements the restriction of the domain of integration in the functional integral to the following region

$$\Omega^h = \{\partial A = \alpha b, \ \partial A^h = 0, \ \mathcal{M}(A^h) > 0\}$$

$$\mathcal{M}^{ab}(A^h) = -\partial_{\mu} \left( \delta^{ab} \partial_{\mu} + g f^{abc} A^{h,c}_{\mu} \right)$$

The region  $\Omega^h$  reduces to the Gribov region of the Landau gauge when  $\alpha=0$ .

For a given field b and for a given  $\alpha$ , the region  $\Omega^h$  is convex and bounded in field space. This is a consequence of the linearity and hermiticity of the operator  $M(A^h)$ 

For sufficiently enough small values of the parameter  $\alpha$ , it can be proven that the restriction to the region  $\Omega^h$  implies that the Faddeev-Popov operator of the linear covariant gauge M(A)

 $\mathcal{M}^{ab}(A) = -\delta^{ab}\partial^2 - g\alpha f^{abc}b^c - gf^{abc}A^c_{\mu}\partial_{\mu}$ 

has no zero modes, showing that the restriction to the region  $\Omega^h$  enables us to eliminate infinitesimal Gribov copies.

<u>Observation</u>. Unlike the operator  $M(A^h)$ , the Faddeev-Popov operator M(A) of the linear covariant gauge is not hermitian.

<u>Observation.</u> Within the GZ formulation of the linear covariant gauges, we have two ghosts: the Faddeev-Popov ghosts c, related to the non-hermitian operator M(A), and the auxiliary ghost  $\omega$  related to the hermitian operator  $M(A^h)$ . The ghost  $\omega$  carries information about the region  $\Omega^h$ . The study of the ghost sector in linear covariant gauges requires a careful analysis. Under investigation.

# **Gluon propagator in linear covariant gauges**

As in the case of the Landau gauge, before looking at the gluon propagator, we need to take into account the existence of the dimension two condensates

 $\langle A^h A^h \rangle \qquad \langle \bar{\phi}\phi - \bar{\omega}\omega \rangle$ 

A direct calculation shows in fact that these condensates are nonvanishing for non-zero Gribov parameter  $\gamma$ .

Suppose we want to evaluate the condensate <O> of a certain operator O. We introduce the operator O coupled to a source J and we compute the functional W(J) defined as

$$e^{-VW(J)} = \int [D\Phi] e^{-(S+J\int d^d x O)}$$
$$\langle O \rangle = \frac{\partial W(J)}{\partial J} \bigg|_{J=0}$$

Employing dimensional regularization, a first-order calculation shows that

$$\langle \bar{\phi}\phi - \bar{\omega}\omega \rangle_{first-order} \sim \gamma^4 \int \frac{d^a k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^4 + 2g^2 N \gamma^4} \\ \langle A^h A^h \rangle_{first-order} \sim \gamma^4 \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^4 + 2g^2 N \gamma^4}$$

Observe also that the integral

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^4 + 2g^2 N \gamma^4}$$

is perfectly convergent in both UV and IR in 4d as well as in 3d

$$S_{Refined}^{LCG} = S_{GZ}^{LCG} + \int d^4x \left(\frac{m^2}{2}(A^h)^2 + \mu^2(\bar{\phi}\phi - \bar{\omega}\omega)\right)$$

For the gluon propagator we get

$$\langle A^{a}_{\mu}(k)A^{b}_{\nu}(-k)\rangle = \delta^{ab} \left( (\delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^{2}})\mathcal{A} + \frac{k_{\mu}k_{\nu}}{k^{2}}\mathcal{B} \right)$$

$$\mathcal{A} = \frac{k^{2} + \mu^{2}}{k^{4} + (m^{2} + \mu^{2})k^{2} + m^{2}\mu^{2} + 2g^{2}N\gamma^{4}} \qquad \mathcal{B} = \frac{\alpha}{k^{2}}$$

The transverse part of the gluon propagator shares great similarity with the gluon propagator in Landau gauge. It is suppressed in the IR, attaining a non-vanishing value at k=0. This behavior is of the decoupling type.

The longitudinal component is proportional to the gauge parameter  $\alpha$ , being identical to its usual perturbative expression.

This behavior is in agreement with the recent lattice data (see Cucchieri and Bicudo papers) as well as with the results found from the Schwinger-Dyson equations, see Aguilar's talk. <u>A similar result holds also in 3d</u>. The transverse part is of the decoupling type, while the longitudinal part is equal to  $\alpha$ .

The situation seems to be different in 2d. Notice in fact that the integral

$$\int \frac{d^{a}k}{(2\pi)^{d}} \frac{1}{k^{2}} \frac{1}{k^{4} + 2g^{2}N\gamma^{4}}$$

is plagued by IR singularities. This is a signal that in 2d, the condensates cannot be safely introduced. The argument can be made more precise by looking at the Gribov's no pole condition for the auxiliary ghost  $\omega$ , as done in the case of the Landau gauge, see D. Dudal et al., PLB 680 (2009) 377.

Similarly to the case of the Landau gauge, we expect a scaling type behavior in 2d for the transverse component of the gluon propagator

$$\mathcal{A}\Big|_{2d} = \frac{k^2}{k^4 + 2g^2 N \gamma^4}$$

# A few words on the MAG

The maximal Abelian gauge is widely employed within the context of the dual superconductivity mechanism for confinement. Useful to check the Abelian dominance hypothesis.

$$A_{\mu} = A^{\alpha}_{\mu}T^{\alpha} + A_{\mu}T^3 \qquad \alpha = 1,2 \qquad SU(2)$$

The maximal Abelian gauge is a non-linear covariant gauge defined by the conditions

$$D^{\alpha\beta}_{\mu}A^{\beta}_{\mu} = \partial_{\mu}A^{\alpha}_{\mu} - g\varepsilon^{\alpha\beta}A_{\mu}A^{\beta}_{\mu} = 0 \qquad \varepsilon^{\alpha\beta} = \varepsilon^{\alpha\beta3}$$
$$\partial_{\mu}A_{\mu} = 0$$

Some references on the MAG

For lattice formulation and gluon propagator in momentum space see, for example, A. Cucchieri et al., hept-lat/0611002, S. Gongyo, arXiv:1411.2211

<u>Schwinger-Dyson framework</u> R. Alkofer et al., arXiv:1112.6173

#### Gribov's point of view

M. A. L. Capri et al., arXiv:1507.05481 and refs. therein

The Faddeev-Popov of the maximal Abelian gauge reads

$$\mathcal{M}^{\alpha\beta}(A) = -D^{\alpha\delta}_{\mu}D^{\delta\beta}_{\mu} + g^2\varepsilon^{\alpha\sigma}\varepsilon^{\beta\delta}A^{\sigma}_{\mu}A^{\delta}_{\mu}$$

<u>This operator is hermitian</u>. Similarly to the case of the Landau gauge, the issue of the Gribov copies can be addressed by restricting the path integral to the region  $\Omega_{MAG}$ 

$$\Omega_{MAG} = \{ \partial A = 0, \ D^{\alpha\beta} A^{\beta} = 0, \ \mathcal{M}^{\alpha\beta}(A) > 0 \}$$
$$\downarrow$$
$$S_{GZ}^{MAG}$$

As in the Landau and linear covariant gauges, with the help of the gauge invariant field A<sup>h</sup> we can rewrite the GZ action of the MAG in such a way that it exhibits an exact nilpotent BRST symmetry

$$S_{GZ}^{MAG} = S_{YM} + S_{FP} + S_{\gamma}$$
$$S_{FP} = \int d^4x \left( b^{\alpha} D^{\alpha\beta}_{\mu} A^{\beta}_{\mu} - \bar{c}^{\alpha} \mathcal{M}^{\alpha\beta} c^{\beta} + b \partial_{\mu} A_{\mu} + \bar{c} \partial_{\mu} (\partial_{\mu} c + g \varepsilon^{\alpha\beta} A^{\alpha}_{\mu} c^{\beta}) \right)$$
$$S_{\gamma} = \int d^4x \left( \bar{\phi}^{\alpha\beta}_{\mu} \mathcal{M}^{\alpha\delta} (A^h) \phi^{\delta\beta}_{\mu} - \bar{\omega}^{\alpha\beta}_{\mu} \mathcal{M}^{\alpha\delta} (A^h) \omega^{\delta\beta}_{\mu} + g \gamma^2 \varepsilon^{\alpha\beta} A^{h,3}_{\mu} (\phi^{\alpha\beta}_{\mu} - \bar{\phi}^{\alpha\beta}_{\mu}) \right)$$

$$\begin{split} s_{\gamma}A^{\alpha}_{\mu} &= -(D^{\alpha\beta}_{\mu}c^{\beta} + g\varepsilon^{\alpha\beta}A^{\beta}_{\mu}c) \qquad s_{\gamma}A_{\mu} = -(\partial_{\mu}c + g\varepsilon^{\alpha\beta}A^{\alpha}_{\mu}c^{\beta}) \\ s_{\gamma}c^{\alpha} &= g\varepsilon^{\alpha\beta}c^{\beta}c \qquad s_{\gamma}c = \frac{g}{2}\varepsilon^{\alpha\beta}c^{\alpha}c^{\beta} \\ s_{\gamma}\bar{c}^{\alpha} &= b^{\alpha} \qquad s_{\gamma}b^{\alpha} = 0 \qquad s_{\gamma}\bar{c} = b \qquad s_{\gamma}b = 0 \\ s_{\gamma}\phi^{\alpha\beta}_{\mu} &= \omega^{\alpha\beta}_{\mu} \qquad s_{\gamma}\omega^{\alpha\beta}_{\mu} = 0 \\ s_{\gamma}\bar{\omega}^{\alpha\beta} &= \bar{\phi}^{\alpha\beta} + g\gamma^{2}\varepsilon^{\delta\beta}A^{h,3}_{\mu}\left(\mathcal{M}^{-1}(A^{h})\right)^{\delta\alpha} \qquad s_{\gamma}\bar{\phi}^{\alpha\beta}_{\mu} = 0 \end{split}$$

$$s_{\gamma}S_{GZ}^{MAG} = 0 \qquad s_{\gamma}s_{\gamma} = 0$$

# Gluon propagator in the MAG

As in the Landau and linear covariant gauges, also here we have to take into account dimension two-condensates, see *D*, *Dudal et al*, *PRD 70 (2004) 114038*, *M*. *A*. *L*. *Capri et al.*, *arXiv:1507.05481 and refs. therein* 

$$\begin{split} m_{off}^2 &\sim \langle A_{\mu}^{h,\alpha} A_{\mu}^{h,\alpha} \rangle \qquad m_{diag}^2 \sim \langle A_{\mu}^{h,3} A_{\mu}^{h,3} \rangle \qquad \mu^2 \sim \langle \bar{\phi}\phi - \bar{\omega}\omega \rangle \\ \text{A first order direct calculation with the GZ action in the MAG yields} \\ &\langle A_{\mu}^{h,3} A_{\mu}^{h,3} \rangle \qquad \sim \gamma^4 \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^4 + 4g^2 \gamma^4} \\ &\langle \bar{\phi}\phi - \bar{\omega}\omega \rangle \qquad \sim \gamma^4 \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2} \frac{1}{k^4 + 4g^2 \gamma^4} \end{split}$$

Once again, the condensates can be safely introduced in both 4d and 3d. In 2d, IR divergences show up.

#### For the gluon propagator, we have

$$\langle A^{\alpha}_{\mu}(k)A^{\beta}_{\nu}(-k)\rangle_{4d} = \delta^{\alpha\beta}\mathcal{P}_{\mu\nu}(k) \frac{1}{k^{2}+m_{off}^{2}} \qquad \mathcal{P}_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^{2}}\right)$$

$$\langle A_{\mu}(k)A_{\nu}(-k)\rangle_{4d} = \mathcal{P}_{\mu\nu}(k) \frac{k^{2}+\mu^{2}}{k^{4}+(m_{diag}^{2}+\mu^{2})k^{2}+\mu^{2}m_{diag}^{2}+4g^{2}\gamma^{4}}$$

Good agreement with available lattice data, see A. Cucchieri et al., hept-lat/0611002

In 3d we expect a similar behavior.

In 2d, we have indication that something different might occur, as also advocated in a recent lattice study, see *S. Gongyo, arXiv:* 1411.2211.

## Conclusion

We have presented an extension of the Landau Gribov-Zwanziger framework to the case of the linear covariant and maximal Abelian gauges.

The resolution of the Gribov issue through the Refined Gribov-Zwanziger action looks quite promising. The resulting gluon propagators in 4d look in nice agreement with the available lattice data.

Our analysis shows that, in both 4d and 3d, a decoupling type gluon propagator emerges. Nevertheless, in 2d, the situation seems to be different.

Is there a general pattern in 2d? Coleman theorem about the absence of Goldstone type modes? Worth for further lattice investigation.

The issue of the BRST symmetry in presence of the Gribov horizon is a key feature. The output of our analysis shows that the standard BRST symmetry is plagued by a soft breaking.

Though, recently, we have been able to construct a nonperturbative extension of the BRST operator which is an exact symmetry of the (R)GZ action in Landau, linear covariant and MAG gauges. The Coulomb gauge can be added as well to this list.

## **Obrigado!**