Deconfinement transition from "perturbation theory"

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Introduction

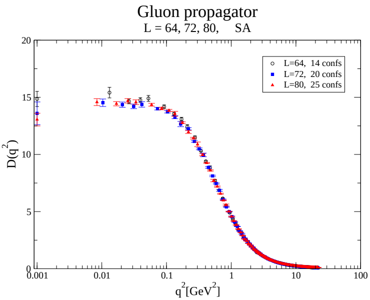
Yang-Mills theory, gauge-fixed in the Landau gauge with the Faddeev-Popov procedure, is described by a set of massless fields: Gluons (A_μ^a), ghosts (c^a and $\bar{c}^a$) and a Lagrange multiplyer (h^a)

$$\mathcal{L} = \frac{1}{4}(F_{\mu\nu}^a)^2 + \partial_\mu \bar{c}^a (D_\mu c)^a + h^a \partial_\mu A_\mu^a$$

However, lattice simulations see unambiguously a gluon propagator that saturates at low momentum.

However...

Gluon propagator is massive! (Sternbeck et al '07)!



The origin of the mass could:

- result from solving Dyson-Schwinger equations;
- be a consequence of a gluon condensate;
- be related to Gribov ambiguity;
- none of those...

Phenomenological model

The mass generation is a difficult issue. Once we are convinced it exists, how much physics can we understand?
Introduce a mass for the gluon by hand in the (gauge-fixed) Lagrangian:

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \partial_\mu \bar{c}^a (D_\mu c)^a + h^a \partial_\mu A^a_\mu + \frac{1}{2} m^2 \left(A^a_\mu\right)^2$$

(Here, we make the assumption that no extra field is needed)
This is one particular representative of the Curci-Ferrari lagrangian.

Overview

- Motivate the interest of this phenomenological model.
  - General arguments;
  - Systematic comparison with Lattice correlation functions.
- Applications to finite temperature physics
  - in the quenched approximation;
  - with dynamic quarks, with and without chemical potential.

Nice properties of the model I

- UV ($p \gg m$) properties are unaffected by the gluon mass.
- In particular, the theory is renormalizable to all orders (De Boer et al). (gluon mass softly breaks the BRST symmetry)
- the (running) gluon mass tends to zero in the ultraviolet ($m(\mu) \propto g^{\alpha}(\mu)$ with $\alpha > 0$).
- Feynman rules are identical to usual ones, except for the massive gluon propagator:

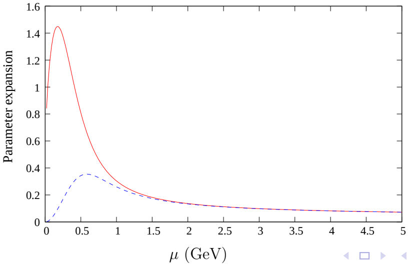
$$\langle A_\mu A_\nu \rangle_0(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{1}{p^2 + m^2}$$

  perturbation calculations are easy to perform.
- Low momentum physics regularized by the gluon mass.

Infrared behavior

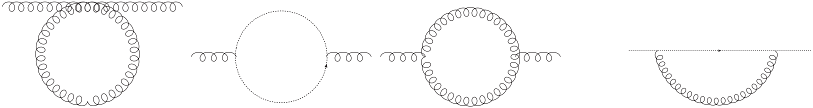
At very low momenta, gluons are frozen. Ghost loop dominates.

- $\Gamma_{A^a_\mu A^b_\nu} \sim \delta^{ab}(\delta_{\mu\nu})p^{d-2}$
- $\Gamma_{A^a_\mu A^b_\nu A^c_\rho} \sim -f^{abc}(ip_\mu\delta_{\nu\rho} + \cdots)p^{d-4}$
- in $d = 4$, leads to log divergences, hard to see...
- in $d = 3$, gluon propag $cte + |p|$, 3-gluon vertex changes sign, consistent with lattice data.
- Interaction between ghosts is mediated by heavy gluons (see also Weber). Effective interaction is suppressed by some positive power of p at low momentum.



Ghost and gluon propagators

Need to compute 4 Feynman diagrams



Define $\langle A_\mu A_\nu \rangle(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) G(p)$ $\qquad \langle c\bar{c}\rangle(p) = \frac{1}{p^2} F(p)$.

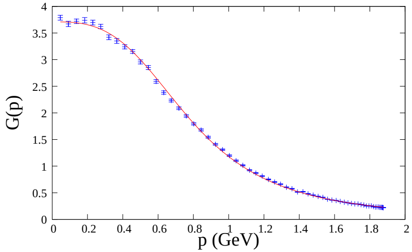
Introduce 4 renormalization parameters and you get ($s = p^2/m^2$.):

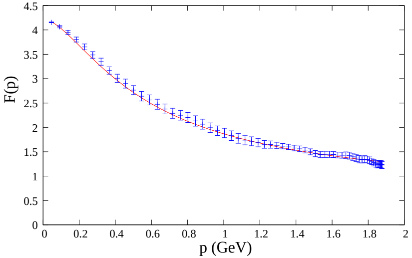
$$G^{-1}(p)/m^2 = s + 1 + \frac{g^2 N}{384\pi^2} s \Big\{ 111 s^{-1} - 2 s^{-2} + (2 - s^2)\log s$$
$$+ (4s^{-1} + 1)^{3/2} \left(s^2 - 20s + 12\right) \log\left(\frac{\sqrt{4+s} - \sqrt{s}}{\sqrt{4+s} + \sqrt{s}}\right)$$
$$+ 2(s^{-1} + 1)^3 \left(s^2 - 10s + 1\right) \log(1+s) - (s \to \mu^2/m^2) \Big\},$$

$$F^{-1}(p) = 1 + \frac{g^2 N}{64\pi^2} \Big\{ - s \log s + (s+1)^3 s^{-2} \log(s+1) - s^{-1} - (s \to \mu^2/m^2) \Big\}.$$

Comparison with lattice data

For SU(2) (Cucchieri, Mendes '08)





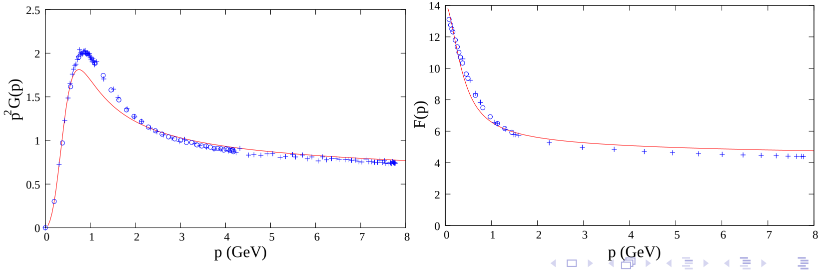
Renormalization-group flow

From renormalization factors, deduce a set of coupled β functions for g and m:

In the UV ($\mu \gg m$) $\beta_g \simeq -\frac{g^3 N}{16\pi^2}\frac{11}{3}$

In the IR ($\mu \ll m$) $\beta_g \simeq +\frac{g^3 N}{16\pi^2}\frac{1}{6}$

For SU(3) (Bogolubsky '09, Dudal '10)

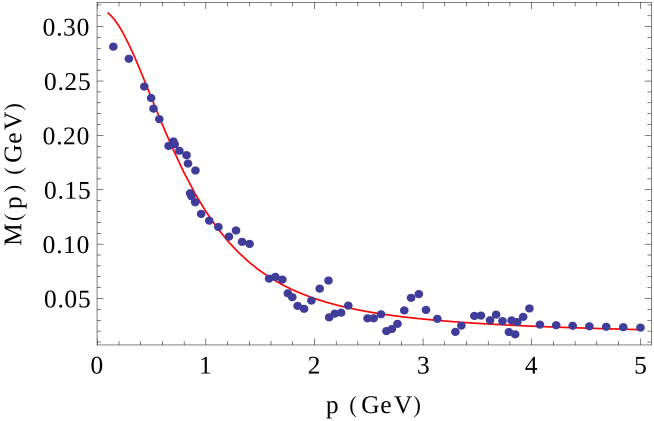


Other correlation functions

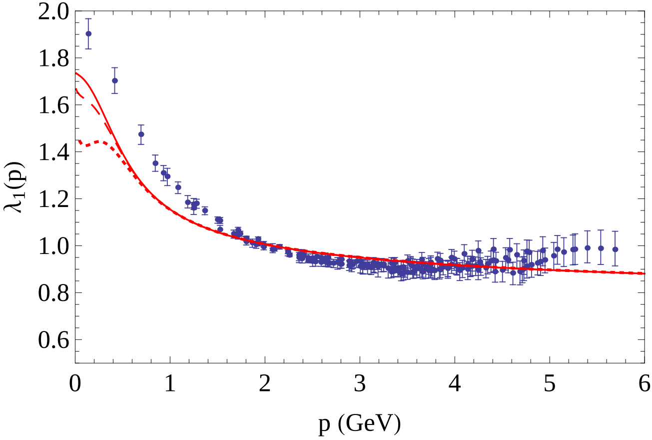
- By the same technique, we have computed (all tensorial components) and compared with lattice data, when available:
  - 3 gluon vertex and ghost-gluon vertex;
  - quark propagator;
  - quark-gluon vertex;
- Agreement (Maximal error of 15-20%) in the quenched approximation.
- In unquenched calculations (Skullerud et al), still ok, but less precise, because the quark-gluon vertex is larger (typically the double of the ghost-gluon vertex).
- 1-loop compares badly to lattice for the quark renormalization factor and for one of the structure tensors of the quark-gluon vertex (λ_2).

Quark mass

The quark mass is enhanced in the infrared. But no chiral symmetry breaking.



Quark-gluon vertex



The phase diagram of QCD I

- In heavy ion collisions, and core of neutron stars, matter reaches extreme conditions, with temperatures of the order of $\sim 10^{12}$ K, densities of $\sim 10^{18}$ kg/m^3.
- Typical values for strong interactions. In strong interactions units: $T \sim 1$ GeV, $\rho \simeq 1$ GeV/fm^3.
- In the quenched approximation (no dynamic quarks), lattice simulations clearly show a phase transition at a temperature ~ 250 MeV, which is in the nonperturbative regime.
- Extension to finite chemical potential is intricate because the action is not real anymore.

Phase diagram of Yang-Mills II

- To study the theory at finite temperature, compactify the time direction, with periodicity $\beta = 1/T$.
- There exist gauge transformations $A \to A^U$ such that A^U is periodic, although U itself is not: $U(\beta, x) = z\, U(0, x)$ with z an element of the centre (for SU(N), $z = e^{im\pi/N}\mathbb{I}$ with $m \in \{0, \cdots, N-1\}$).
- Some quantities are invariant under the centre (such as $F_{\mu\nu}^2$). Other vary, such as the Polyakov loop

$$\ell \propto \text{Tr}\ P e^{ig\int_0^\beta A_0(\tau) d\tau}$$

- $\langle \ell \rangle = \exp(-\beta F_q)$ where F_q is the free energy of a quark.
  - If the centre symmetry is realized, $\langle \ell \rangle = 0$, $F_q = \infty$, confined phase.
  - If the centre is spontaneously broken, $\langle \ell \rangle \neq 0$, F_q finite, deconfined phase.

To encode the centre symmetry, convenient to

- decompose the gluon field in a background $\bar{A}$ and a fluctuating a field: $A = \bar{A} + a$ (we choose $\bar{A}$ constant, temporal, in the Cartan subalgebra);
- introduce the Landau-de Witt gauge $\partial_\mu a_\mu + gf^{abc}\bar{A}^b_\mu a^c_\mu = 0$;
- choose $\bar{A}$ such that $\langle a \rangle = 0$.

This last condition is fulfilled by minimizing some potential $V(\bar{A})$.
Center transformations act on $\bar{A}$!
At leading order in g, the SU(2) potential reads ($r_3 = \beta g \bar{A}_3$)

$$V = T^4 \left(\frac{3}{2} F_{m\beta}(r_3) + \frac{1}{2} F_0(r_3) - 1 F_0(r_3) \right) + \mathcal{O}(g^2)$$

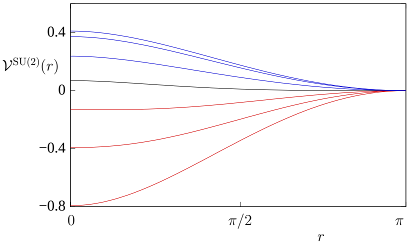
$$F_{\tilde{m}}(r) = \int_q \log(1 + e^{-2\sqrt{\tilde{m}^2+q^2}} - 2e^{-\sqrt{\tilde{m}^2+q^2}} \cos(r))$$

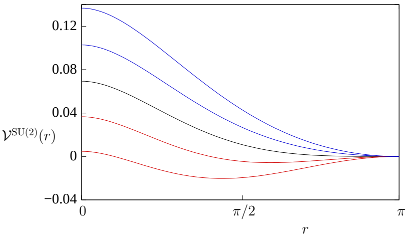
The Polyakov loop:

$$\langle \ell \rangle = \cos(r_3/2) + \mathcal{O}(g^2)$$

Phase diagram of Yang-Mills IV

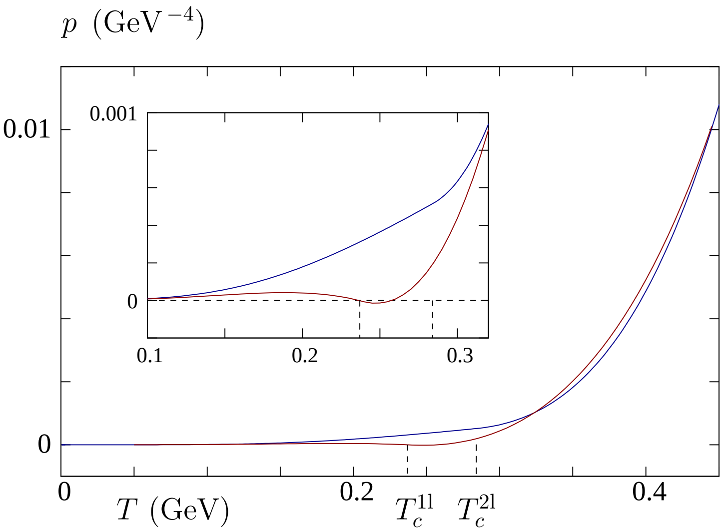
At high temperatures (red), $V \to +1F_0(r_3)$. (Weiss potential)
At low temperatures (blue), $V \to -\frac{1}{2}F_0(\beta g\bar{A})$.



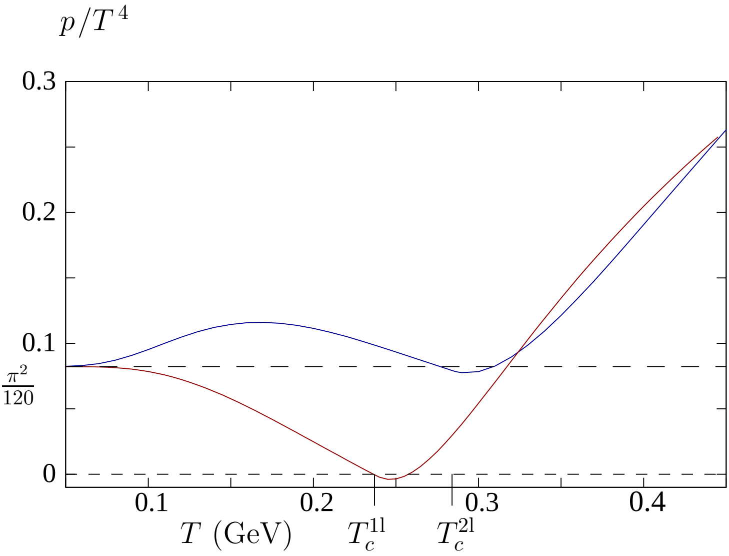


$r = \pi$ is invariant under center transformations and corresponds to $\langle \ell \rangle = 0$.
The leading order approximation captures the good physics!

Pressure I

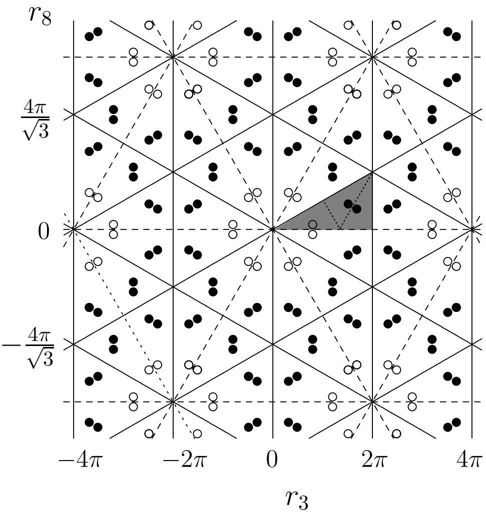


Pressure II



Phase diagram for SU(3) I

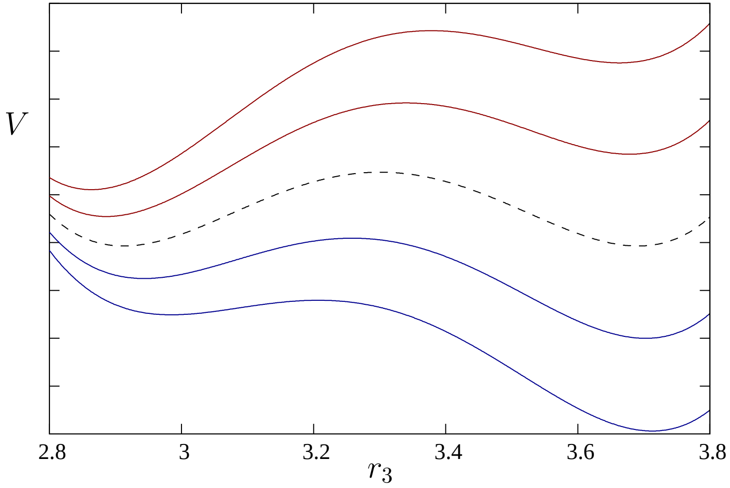
Two fields in the Cartan subalgebra: r_3 and r_8.



Calculation of the potential and Polyakov loop generalizes easily.

Phase diagram for SU(3) II

Leads to first order transition



Finite chemical potential

- Add the quarks lagrangien density, with chemical potential:

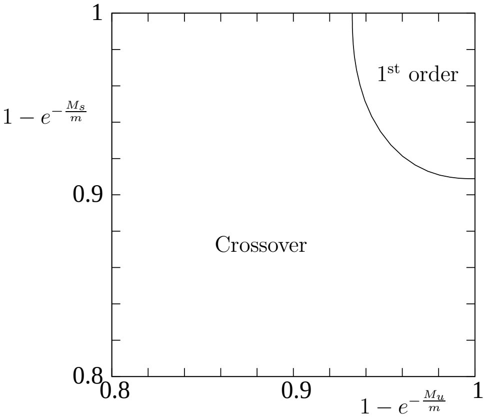
$$\sum_f \bar{\psi}_f (\not{D} + M_f + \mu\gamma_0)\psi_f$$

- Explicitely break centre symmetry.
- Contributes to the potential as:

$$V_f(\mu) = -\frac{T}{\pi^2}\int_0^\infty dq\, q^2 \Big\{ \Big[\log[1 + e^{-\beta(\sqrt{q^2+M_f^2}+\mu)}\Big] + (\mu \to -\mu)\Big\} + \mathcal{O}(g^2)$$

Vanishing chemical potential, SU(3)

Consider 2 +1 quarks (only for large mass because we don't control chiral limit). Columbia plot:

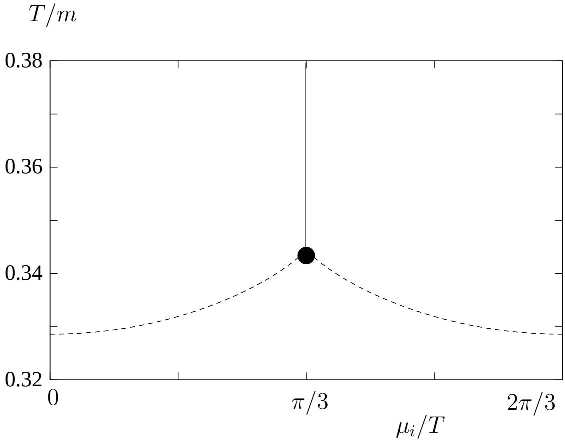


Imaginary chemical potential, SU(3)

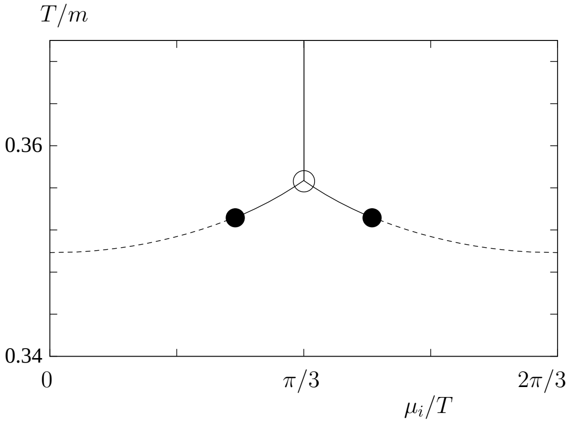
- The action and potential V are real, so lattice simulations can be performed.
- It is symmetric under a simultaneous transformation of the background field and of the chemical potential (Roberge, Weiss). $\mu/T = i\pi/3$ plays a particular role:

We retrieve the properties obtained in Lattice approaches (de Forcrand, Philipsen), in particular at the tricritical point.

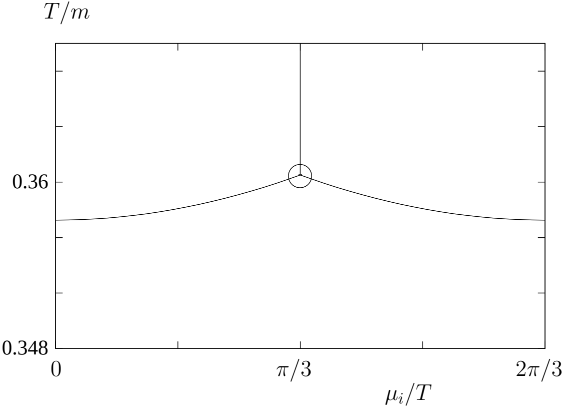
small quark mass



intermediate quark mass

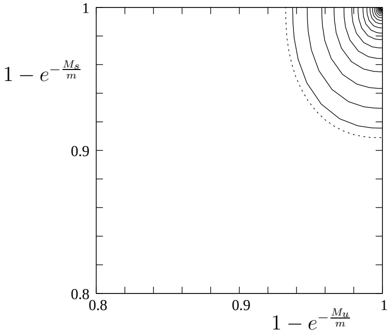


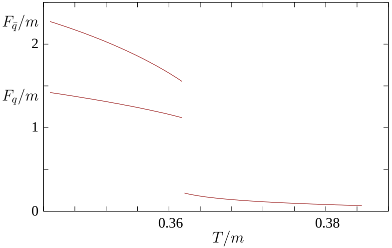
large quark mass



Real chemical potential

- For real background fields, the potential is complex!
- To be consistent, need to choose r_3 real and r_8 imaginary (See also Nishimura 2014). Then the potential is then real.





Conclusions

- Curci-Ferrari seems to capture many "nonperturbative" properties of QCD within "perturbation theory".
- This would mean that the major nonperturbative ingredient is the gluon mass.
- We have a nice model to study low-energy properties of QCD. Tested in several situations.
  - Propagators in the Landau-de Witt gauge.
  - Chiral symmetry breaking?
  - Polyakov loop in other representations?
  - Analytic structure of the correlation functions?
  - Wilson loop?
  - Two-loop calculations for the propagators?
  - Transport coefficients?
  - ...
- Can we generate the mass from first principles (relation with problems with disorder in stat. phys.)?
- Can we build a physical subspace?