Introduction to Quantum Walks – part 3

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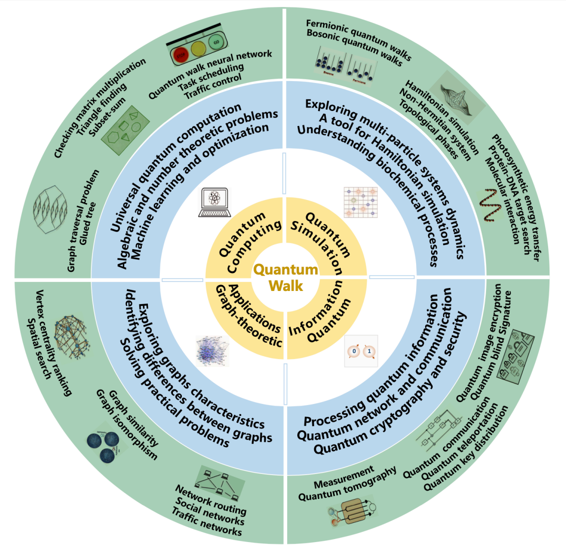
4-18 June 2024

QW applications



Xiaogang Qiang, Shixin Ma, and Haijing Song arXiv:2404.04178 (2024)

QW search algorithms

- QW spread faster than RW – ballistic $O(t)$ vs diffusive $O(\sqrt{t})$
- In some graphs spread is even faster
- This can be used to speed up search

Neil Shenvi, Julia Kempe, and K. Birgitta Whaley Phys. Rev. A 67, 052307 (2003)

- Structured database – graph

Miklos Santha, TAMC, Lecture Notes in Computer Science, vol 4978. Springer, Berlin, Heidelberg (2008)

$$\vdots$$

- Idea: Start QW in a uniform state

$$|\psi(0)\rangle = \frac{1}{\sqrt{N}} \sum_{x=1}^{N} |x\rangle \otimes |anc\rangle$$

Find evolution to localise walk in the marked vertex $x = m$

$$|\psi(t)\rangle \approx |m\rangle \otimes |anc'\rangle$$

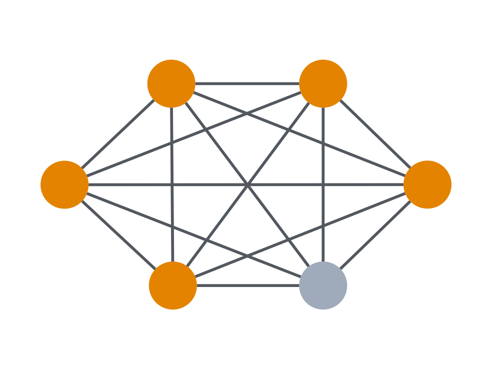
QW search algorithms

Grover algorithm as QW

Lov K. Grover, Am. J. Phys. 69, 769–777 (2001)

$$U = U_1 U_0, \qquad U_0|x\rangle = e^{i\pi\delta_{x,m}}|x\rangle, \qquad U_1 = 2|\psi\rangle\langle\psi| - I,$$

$$|\psi\rangle = \frac{1}{\sqrt{n}}\sum_{x=1}^{N}|x\rangle, \qquad U_0|x'\rangle = \frac{2}{N}\sum_{x=1}^{N}|x\rangle - |x'\rangle$$



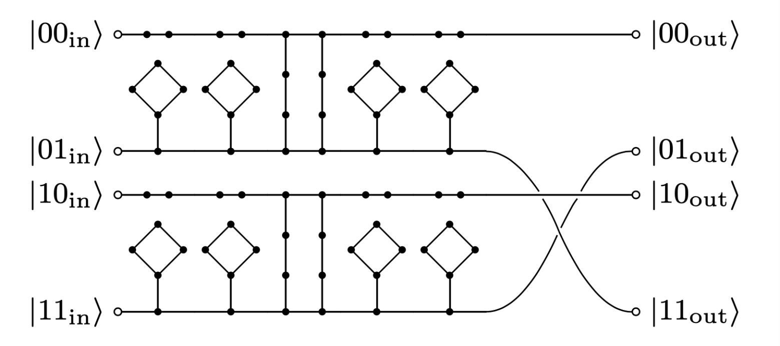
QW search algorithms

- In a connected graph the search time is $O(\sqrt{N})$
- In a structured database (not a connected graph) the walker cannot get straight to the marked vertex
- Structured graph search time is typically $\sqrt{N} \leq t \leq O(N)$
- Example: search in 1D (chain) does not exhibit quadratic speedup $t = O(N)$

QW as Universal Quantum Computers

CTQW as Universal Quantum Computer

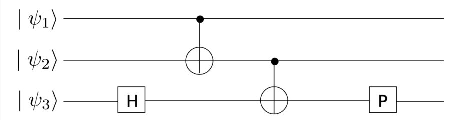
$$H \to CNOT$$

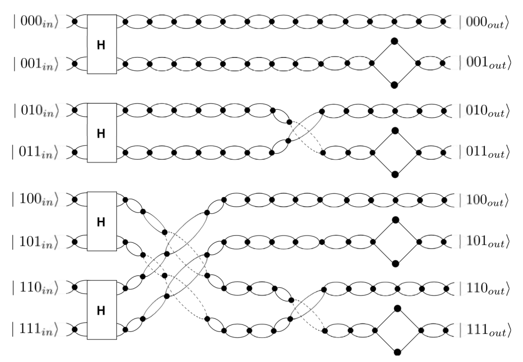


Andrew M Childs, Phys. Rev. Lett. 102, 180501 (2009)

QW as Universal Quantum Computers

DTQW as Universal Quantum Computer





Neil B Lovett, et. al., Phys. Rev. A 81, 042330 (2010)

QW as simulators of physical dynamics

QW simulation of solid state systems

- Graph – tight binding and crystal structure
- How to introduce electric potentials $\phi(x)$?
- Dirac analogy

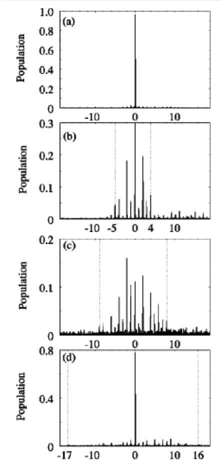
$$U = S(I \otimes C) = e^{ip\sigma_z} e^{i\theta\sigma_y} \;\rightarrow\; e^{-i(cp\sigma_z + mc^2\sigma_y)t}$$

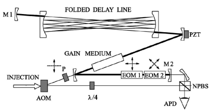
$$e^{-i(cp\sigma_z + mc^2\sigma_y + \phi(x))t} \rightarrow e^{ip\sigma_z} e^{i\theta\sigma_y} e^{i\phi(x)} = S(I \otimes C)P = U'$$

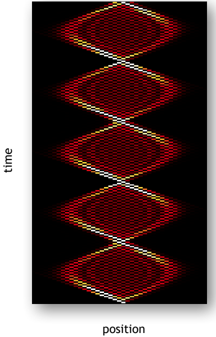
$$P = \left(\sum_{x=-\infty}^{\infty} e^{i\phi(x)} |x\rangle\langle x| \right) \otimes I$$

- Constant force: $\phi(x) = \varphi x$

QW as simulators of physical dynamics





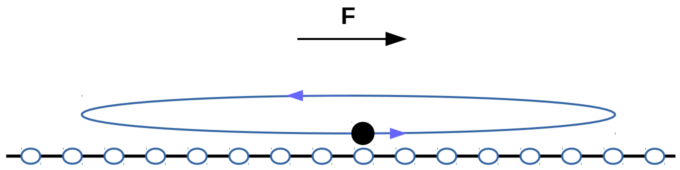


D. Bouwmeester, et. al., Phys. Rev. A 61, 013410 (1999)

A. Wojcik, et. al., Phys. Rev. Lett. 93, 180601 (2004)

QW as simulators of physical dynamics

Bloch oscillations

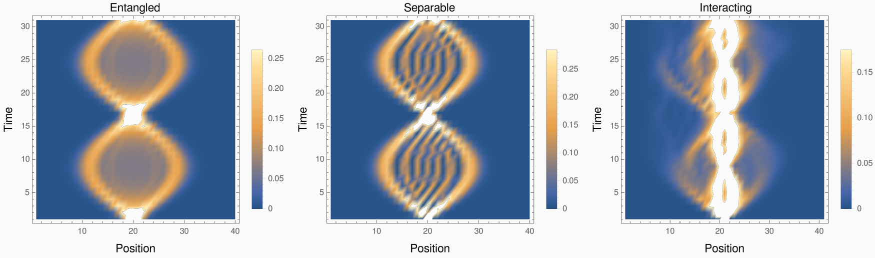


- Constant force $e^{i\eta x}|k\rangle = |k+\eta\rangle$
- Oscillations due to momentum in the first Brillouin zone $k \in [-\pi, \pi]$
- Period of Bloch oscillations (unit mass) $T_{BO} = \frac{2\pi}{\eta}$
- Period of Bloch oscillations ($N\times$ unit mass) $T_{BO} = \frac{2\pi}{N\eta}$

A. Wojcik, et. al., Phys. Rev. Lett. 93, 180601 (2004)

QW as simulators of physical dynamics

Bloch oscillations of two quantum walkers (center of mass)



Z. Lasmar, et. al., Phys. Rev. A 98, 062105 (2018)

QW as simulators of physical dynamics

Magnetic quantum walks (2D)

I. Yalcınkaya and Z. Gedik, Phys. Rev. A 92, 042324 (2015)

- How to introduce vector potentials $\vec{A}(x, z)$?
- $\vec{p} \quad \rightarrow \vec{p} - q\vec{A}$
- Dirac analogy

$$U = S_x(I \otimes C)S_z(I \otimes C) = e^{ip_x\sigma_x} e^{i\theta\sigma_y} e^{ip_z\sigma_z} e^{i\theta\sigma_y} \quad \rightarrow$$

$$e^{-i(cp_x\sigma_x + cp_z\sigma_z + mc^2\sigma_y)t}$$

$$e^{-i(c(p_x - qA_x)\sigma_x + c(p_z - qA_z)\sigma_z + mc^2\sigma_y)t} \quad \rightarrow$$

$$e^{ip_x\sigma_x} e^{iA_x\sigma_x} e^{i\theta\sigma_y} e^{ip_z\sigma_z} e^{iA_x\sigma_x} e^{i\theta\sigma_y} = S_x M_x (I \otimes C) S_z M_z (I \otimes C) = U'$$

QW as simulators of physical dynamics

- Position dependent coin ($A_x = A_x(x,z), \quad A_z = A_z(x,z)$):

$$M_x = I \otimes C_x(x,z), \qquad M_z = I \otimes C_z(x,z)$$

$$C_x(x,z) = e^{iA_x\sigma_x} = \begin{pmatrix} \cos A_x(x,z) & i\sin A_x(x,z) \\ i\sin A_x(x,z) & \cos A_x(x,z) \end{pmatrix}$$

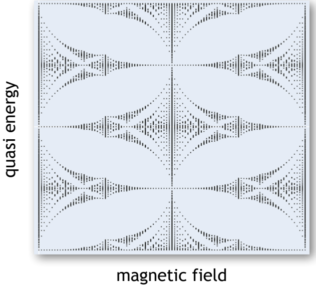
$$C_z(x,z) = e^{iA_z\sigma_z} = \begin{pmatrix} e^{iA_z(x,z)} & 0 \\ 0 & e^{-iA_z(x,z)} \end{pmatrix}$$

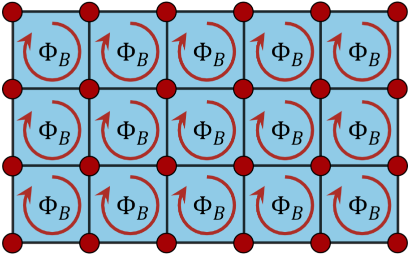
- Constant magnetic field perpendicular to the plane (y-direction):

$$A_x = -\varphi z, \qquad A_z = \varphi x$$

QW as simulators of physical dynamics

Cyclotron orbit (see video) and Hofstadter butterfly

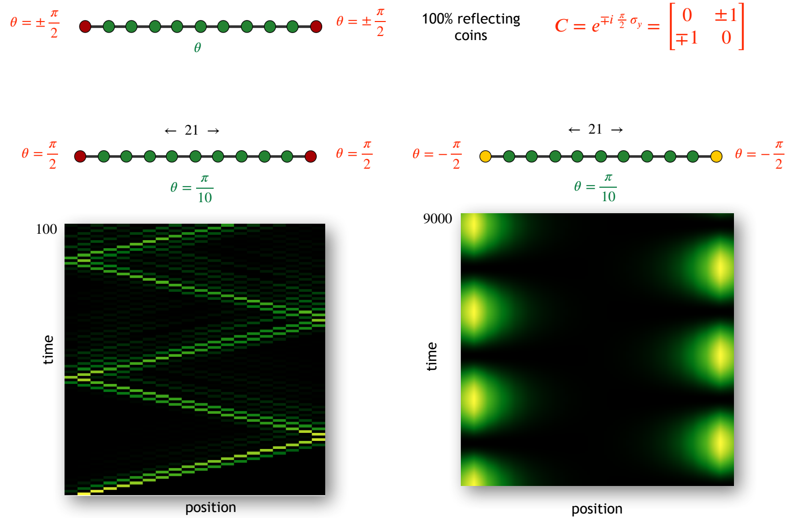




QW as simulators of physical dynamics

Topological effects

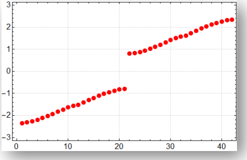
T. Kitagawa, Quant. Inf. Proc. 11, 1107-1148 (2012)

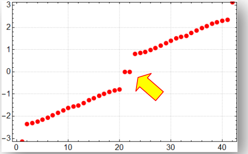


Sorted energies

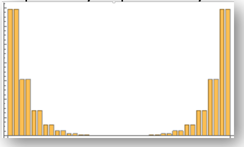
$\theta = \frac{\pi}{2}$ $\theta = \frac{\pi}{10}$ $\theta = \frac{\pi}{2}$

$\theta = -\frac{\pi}{2}$ $\theta = \frac{\pi}{10}$ $\theta = -\frac{\pi}{2}$





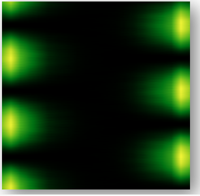
$|LEFT\rangle \pm |RIGHT\rangle$



Gap states

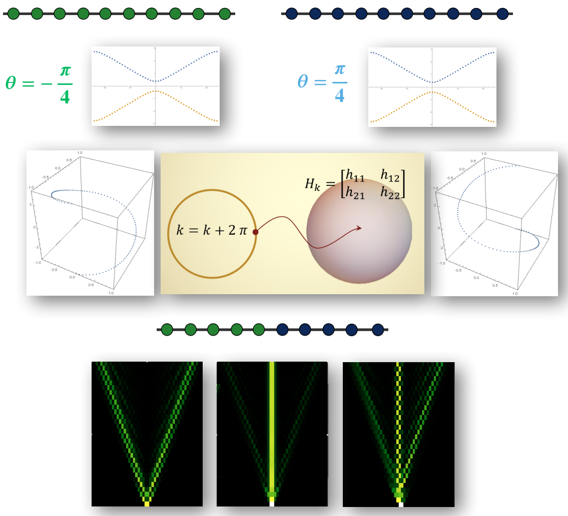
Symmetry protection of topological states

$\theta = -\frac{\pi}{2}$ $\theta = random$ $\theta = -\frac{\pi}{2}$



QW as simulators of physical dynamics

Topological phases and edge states



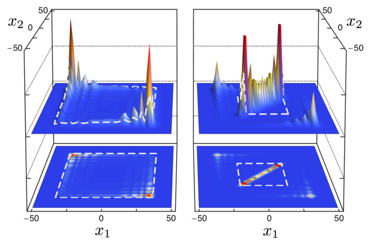
QW as simulators of physical dynamics

Multipartite quantum walks

- State: $|x_1, c_1\rangle \otimes |x_2, c_2\rangle$
- Evolution: $S_1(I \otimes C_1)S_2(I \otimes C_2)I(x_1, x_2)$
- Example (hard core interaction)

$$I(x_1, x_2) = \left(\sum_{x_1, x_2} e^{i\varphi\delta_{x_1,x_2}} |x_1, x_2\rangle\langle x_1, x_2| \right) \otimes I$$

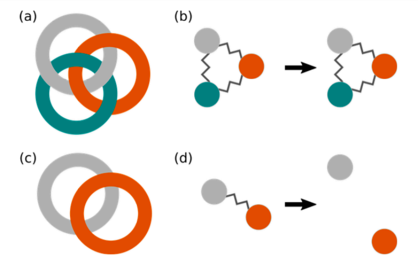
- Creation of bound states



A. Ahlbrecht et al. New J. Phys. 14, 073050 (2012)

QW as simulators of physical dynamics

Borromean states



- State: $|x_1, c_1\rangle \otimes |x_2, c_2\rangle \otimes |x_3, c_3\rangle$
- Evolution: $S_1 S_2 S_3 (I \otimes C(x_1, x_2, x_3))$
- Bound state: $\sum_x |x, x, x\rangle \otimes |GHZ\rangle$

Marcin Markiewicz, Marcin Karczewski, Pawel Kurzynski, Quantum 5, 523 (2021)

QW as simulators of physical dynamics

- One-particle quantum walks: can be simulated classically (computers, coherent light)
- Classical: multipartite walks $\rightarrow$ cellular automata (Lattice Boltzmann Automata, Conway's Life, etc.)
- Quantum: multipartite walks $\rightarrow$ quantum cellular automata

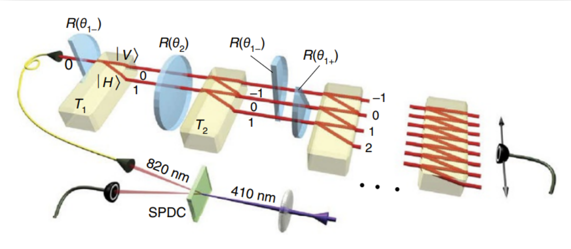
  Pablo Arrighi, arXiv:1904.12956 (2019)
- Cellular automata require powerful classical computers
- Quantum cellular automata require powerful quantum computers!

QW – experimental implementations

Photon

position

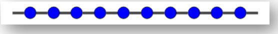
polarization
H, V



Nature Communications 3, 882, 2012

QW – experimental implementations

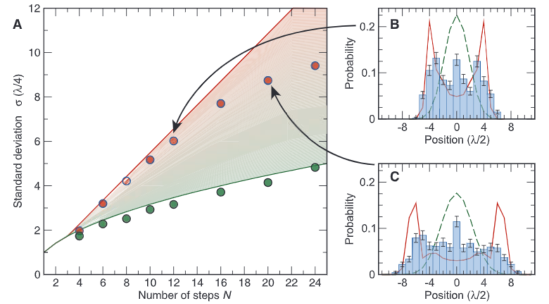
Atom in optical lattice



position

Cesium ground states
hyperfine sublevels

$F = 3, 4$



Science 325 (2009) 174

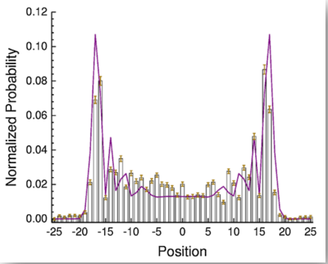
QW – experimental implementations

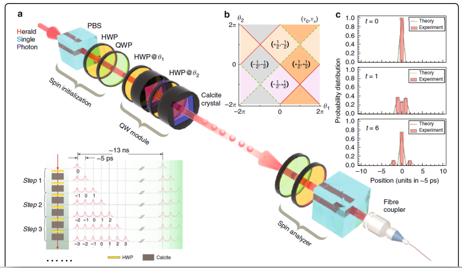
photon

Time bins

polarization

H, V

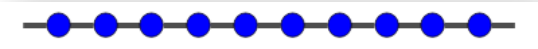




PRL 120 (2018) 260501; Science & Applications (2020) 9:7

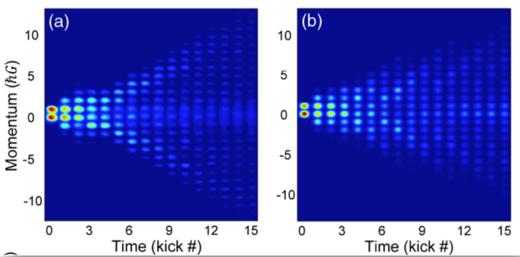
QW – experimental implementations

BEC



momentum
$k \pm \Delta k$

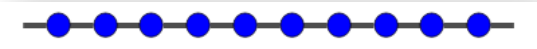
Rubidium ground states hyperfine sublevels
$F = 1,2$



PRL 121 (2018) 070402

QW – experimental implementations

Ion

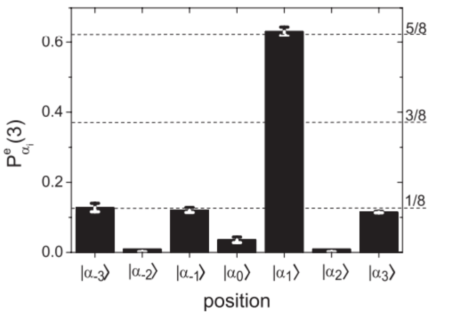


Oscillator phase space position

$|\alpha\rangle$

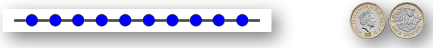
Magnesium ground states hyperfine sublevels

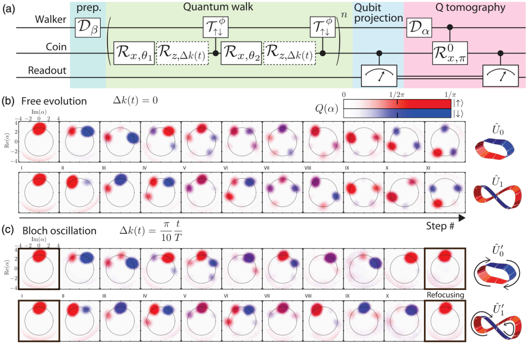
$F = 2, 3$



PRL 103 (2009) 090504

Cavity QED + Superconducting Qubit





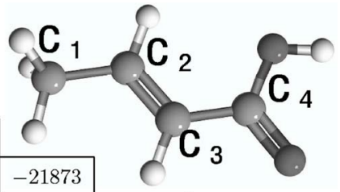
PRX 7 (2017) 031023

QW – experimental implementations

NMR

Two spins

One spin



C_1	-2546			
C_2	40.5	-21873		
C_3	1.5	69.9	-18498	
C_4	7.1	1.4	72.4	-25322
	C_1	C_2	C_3	C_4

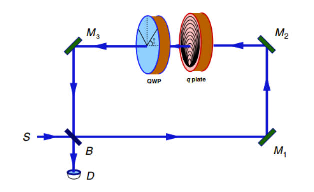
PRA 72 (2005) 062317

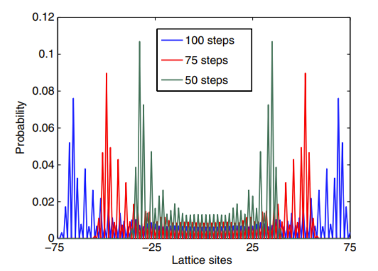
QW – experimental implementations

Light pulse

Orbital momentum

Polarization





PRL 110 (2013) 263602