QCD vacuum effects for Effective quark interactions

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With
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First results: - Barros Jr- F.L.B. , Phys. Rev. D 88, 034011 (2013)
- Paulo Jr – F.L.B. , In preparation
- Mod. Phys. Lett A (2013) = Fierz transformation

MOTIVATIONS

· Derivation of higher order effective interactions for quarks

· (NJL and similar : limitations)

* stability of QCD effective theories / vacuum / expansions on currents

* hadron structure Multi-quark interaction

* Role of gluon condensate $\langle AA \rangle$,
(including IR effects) to quark interactions,

* eventually incorporating confinement / chiral symmetry breaking

[one well known example: ‘t Hooft interaction – instanton mediation]

From QCD to effective quark interactions

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \mathcal{D}^\mu - M)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{\lambda}(\partial_\mu A^\mu)^2 + \mathcal{L}_{FP}$$

Covariant gauge, Landau gauge

However: first analysis will neglect Gribov horizon problem

(Results capture some of the leading order terms

of a more complete calculation: to continue on this point…)

$$Z = N \int \mathcal{D}[\psi, \overline{\psi}]\mathcal{D}[A_\mu]\mathcal{D}[\chi, \chi^*] e^{<\overline{\psi}(i\gamma_\mu \mathcal{D}^\mu - m_i)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{FP} + \mathcal{L}_{gf}>}$$

$\mathcal{L}_{FP}, \mathcal{L}_{gf}$ stand for the Faddeev Popov ghost term gauge fixing.

Seemingly Impossible task:

to integrate out gluons exactly to obtain effective model for quarks

One of the needs for improvement: Gluon propagator

"MASSIVE" TRANSVERSE GLUON

* It may reduce to One- gluon exchange
* Transverse mass : endowed by auxiliary field $\phi(x)$
* Eventually k-dependent mass

$$(R^{\mu\nu}_{bc})^{-1}(k,-k) = \delta_{bc}\left[(k^2 + c_1\phi(k))\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \lambda k_\mu k_\nu\right]^{-1}$$

Integrating out gluons approximatedly

The gap equation for ϕ_0 is found in the stationary point:

$$\left.\frac{\partial S_{eff}}{\partial \phi}\right|_{\phi(x)\rightarrow\phi_0} = 0$$

For usual homogeneous condensate:
(Euclidean k-space)

$$\alpha_s = 8.92/N_c$$

(Lerche et al
, Cucchieri et al

$$\phi_0 = \frac{3}{2}(N_c^2 - 1)\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + M_G^2}.$$

$\Lambda \simeq 435$ MeV. > Λ_{QCD}

$M_G = 600$ MeV

It allows for defining a momentum dependent (non-homogeneous) gluon mass or ground state....

under work

At very low energies,

an expansion can be performed around ϕ_0

By considering scalar gluon condensate $\langle 0|AA|0\rangle$

$$\phi(x) \rightarrow \phi_0 + \xi(x)$$

$$J = j^a_\mu z^{\mu\nu} j^a_\nu,$$

$$z^{\mu\nu} = \left(g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2}\right)(\partial^2 + \phi_0)^{-1}$$

$$z^{\mu\nu} \to g^{\mu\nu}$$

IT FOLLOWS – AFTER INTEGRATING OUT FLUCTUATIONS OF SCALAR FIELD ξ

Non derivative couplings (below) and derivative couplings (under work)

$$S_{\text{eff}} = \int_x \left[\frac{1}{2} G j^\mu_a(x) j^a_\mu(x) + \bar{\psi}(i\gamma_\mu \partial^\mu - M)\psi\right]$$
$$- \frac{1}{2} \underline{\text{Tr} \ln[(t_1[\phi_0] + c)\delta^4(x-y) \quad + t_2(x,y) j^a_\mu(x) j^\mu_a(x)]} + C_{\phi_0}.$$

8^{th} order effective quark interaction : current – current type

$$\overline{\psi}\gamma_\mu\lambda_1\psi\overline{\psi}\gamma^\mu\lambda_2\psi\overline{\psi}\gamma_\nu\lambda_3\psi\overline{\psi}\gamma^\nu\lambda_4\psi$$

Performing a Fierz transformation it yields usual

(eighth order chiral) scalar /pseudo scalar quark-quark interaction

Scalar /pseudo scalar interactions
(eg. Alkofer / Zahed, Osipov / Hiller, et al) = stability of ground state

$$\bar{\psi}_a\Gamma_{ad}\psi_d\cdot\bar{\psi}_c\Gamma_{cf}\psi_f\bar{\psi}_e\Gamma_{eh}\psi_h\cdot\bar{\psi}_g\Gamma_{gb}\psi_b$$

$$\rightarrow\sum(\bar{\psi}_a\Gamma_{ab}\psi_b\cdot\bar{\psi}_c\Gamma_{cd}\psi_d\bar{\psi}_e\Gamma_{ef}\psi_f\cdot\bar{\psi}_g\Gamma_{gh}\psi_h)\,,$$

SU(2), SU(3) F.L. Braghin, Mod. Phys. Lett. A 28, (2013) 1350023 :

Fierz transformation

$$F_I^{(3)} = \frac{1}{27} + \frac{1}{12}\lambda_1 \cdot \lambda_2 \lambda_3 \cdot \lambda_4 + \frac{1}{18}(\lambda_1 \cdot \lambda_2 + \lambda_2 \cdot \lambda_3 + \lambda_3 \cdot \lambda_4 + \lambda_2 \cdot \lambda_4 + \lambda_1 \cdot \lambda_3 + \lambda_1 \cdot \lambda_4) + \frac{1}{8} h_{mnk} h_{ijk} \lambda_1^m \lambda_2^n \lambda_3^i \lambda_4^j + \frac{1}{12} h_{ijk} (\lambda_1^i \lambda_2^j \lambda_3^k + \lambda_2^i \lambda_3^j \lambda_4^k + \lambda_1^i \lambda_2^j \lambda_4^k + \lambda_1^i \lambda_3^j \lambda_4^k) .$$

SU(3)

$$F_8^{(3)} = \frac{8.32}{3.81} - \frac{16}{81}(\lambda_1 \cdot \lambda_2 + \lambda_3 \cdot \lambda_4 + \lambda_2 \cdot \lambda_3 + \lambda_2 \cdot \lambda_4 + \lambda_1 \cdot \lambda_3 + \lambda_1 \cdot \lambda_4) + \frac{1}{27} d_{ijk} (\lambda_1^i \lambda_2^j \lambda_3^k + \lambda_2^i \lambda_3^j \lambda_4^k + \lambda_1^i \lambda_2^j \lambda_4^k + \lambda_1^i \lambda_3^j \lambda_4^k) + \frac{1}{18}(d_{ijm} d_{klm} + f_{ijn} f_{kln}) \lambda_1^i \lambda_2^j \lambda_3^k \lambda_4^l + i\frac{1}{6}(d_{ijm} f_{klm} + f_{ijm} d_{klm}) \lambda_1^i \lambda_2^j \lambda_3^k \lambda_4^l .$$

8^{th} order effective quark interaction (SCALAR + PSEUDOSCALAR)

Final shape of the quark interaction (quark field expansion), SU(3) invariant

$$\mathcal{L}_8 = g_8(\bar{\psi} P_R \psi \, \bar{\psi} P_L \psi \, \bar{\psi} P_R \psi \, \bar{\psi} P_L \psi),$$

Inclusion of quark-antiquark condensate

$$\bar{q}\lambda^i q \rightarrow (\bar{q}\lambda^i q)_c + \bar{q}\lambda^i q.$$

Those that condense
S, P usual variables

Those that form hadron states

As the entire quark field will not be integrated out, it is enough to apply

Auxiliary variable method (scalar and pseudo scalar) for $(\Psi\Psi)_c$, not for $(\Psi\Psi)_q$

$$S^{\phi}_{eff}[\bar{\psi}, \psi, \phi] = tr \int_{x,y} \left[\bar{\psi}\left(i\gamma_\mu \partial^\mu - M\right)\psi\delta(x-y) - j^b_\mu (R^{\mu\nu}_{bc})^{-1} j^c_\nu\right]$$
$$-tr_C \int_x \left[c_2\phi^2 - \frac{1}{2}\int_p \ln((R^0_{\nu\mu,cb})^{-1} R^{\mu\nu}_{bc})\right]$$

Usual auxiliary variables for scalar/pseudoscalar quark currents S P

$$S_{eff} = -\frac{i}{2} Trln\left(D^{\mu\nu}(x-y)\right) - \frac{c}{2}\phi^2 + iTrln\left(S^{-1}(x-y)\right) - \frac{1}{g_4}\left(S_i^2 + P_i^2\right)$$

$$- g_4 \int_x \left[(\overline{\psi}\lambda_a\psi)^2 + (\overline{\psi}i\gamma_5\lambda_a\psi)^2\right]$$

Where

$$S^{-1}(x-y) = (i\partial\!\!\!/ - M_i^* - 2(\overline{\psi}\lambda_a\psi + \overline{\psi}i\gamma_5\lambda_a\psi))\delta(x-y)$$

$$M^* = M + S_i\lambda_i,$$

SU(3) GROUND STATE
WITH gluon and qq condensates

$$\left.\frac{\partial V_{eff}}{\partial \phi}\right|_{\phi=\phi_0, S^i=S_0^i} = 0,$$

$$\left.\frac{\partial V_{eff}}{\partial S_i}\right|_{S^i=S_0^i, \phi=\phi_0} = 0,$$

$$\left.\frac{\partial V_{eff}}{\partial P_i}\right|_{S^i=S_0^i, \phi=\phi_0} = 0,$$

Most general expansion of the determinants,

mixing scalar, pseudo-scalar and vector interactions (Fierz)-

yield chiral invariant structures of the types

$$S_{eff} \to Tr \sum_n \left[c_n (\overline{\psi}\psi)^n + d_n (j_\mu j^\mu)^n + g_n \sum_m b_m (\overline{\psi}\psi)^m \, (j_\mu j^\mu)^{n-m} \right]$$

coefficients being calculated

8^{th} order effective quark interaction

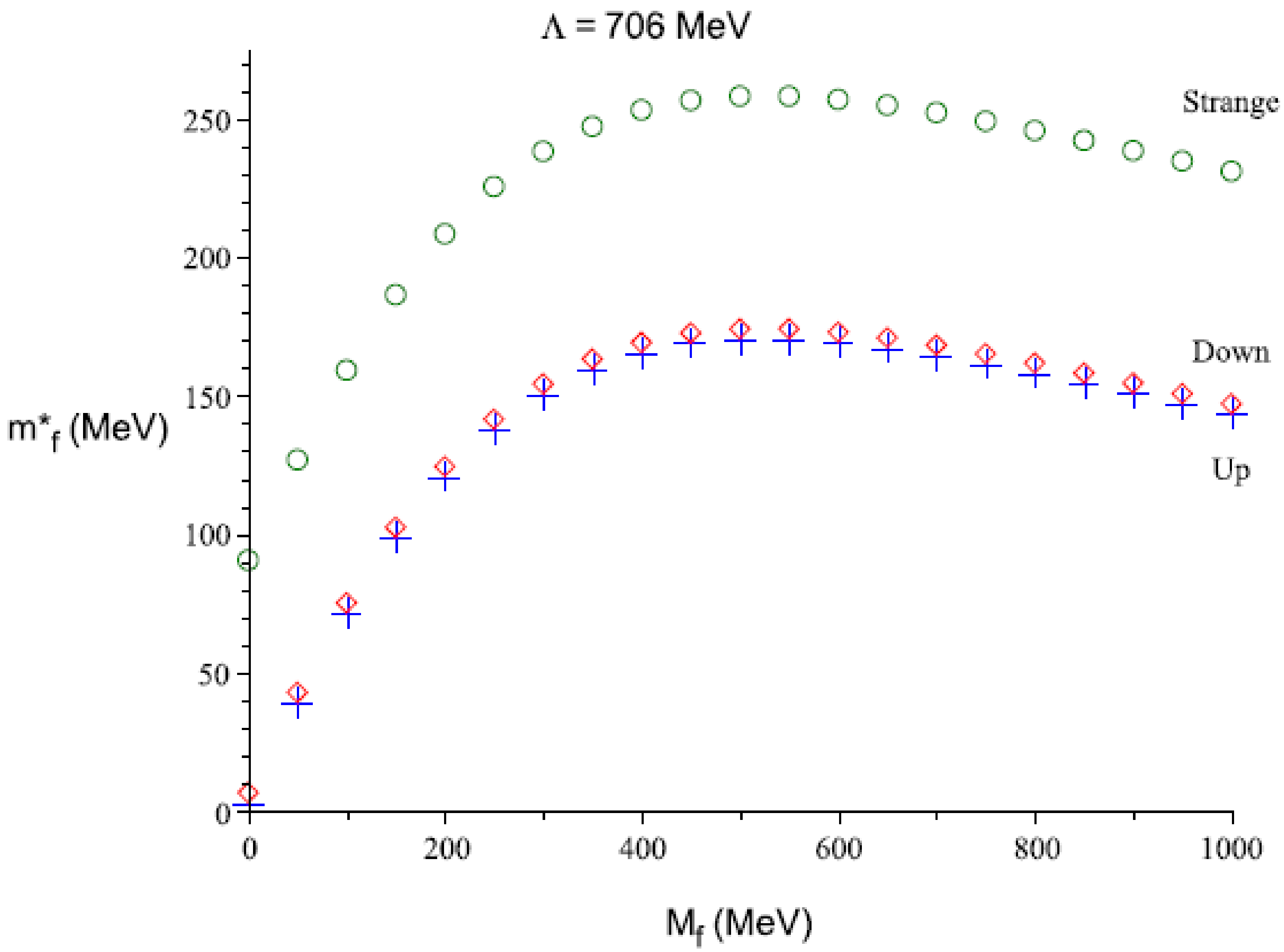
Final shape of the quark interaction (quark field expansion)

$$\mathcal{L}_{8,2} = g_{8,2} \left(\bar{\psi} P_R \psi \; \bar{\psi} P_L \psi \; \bar{\psi} P_R \psi \; \bar{\psi} P_L \psi \right) ,$$

Most general chiral invariant
eighth order interactions

m_g	100	200	300	400	500	600	650	700	800
Λ	109	217	325	435	542	651	706	760	870
M_u	50	94	139	183	229	274	297	319	365
s_u	47	91	136	180	226	271	294	316	362
M_d	56	100	145	189	235	281	303	326	372
s_d	49	93	138	182	228	274	296	319	365
M_s	137	192	245	295	345	395	419	443	492
s_s	46	101	154	204	254	304	328	352	401

Resultados obtidos por Ademar Paulo Jr.



Λ = 706 MeV
Strange
Down
Up
m*f (MeV)
Mf (MeV)
0
50
100
150
200
250
0
200
400
600
800
1000

Corrections to 4th and 6th order couplings

M (MeV)	Δg_4 (GeV^{-2})	$\frac{\Delta g_4}{g_4}$
100	2,2	0,23
200	1,9	0,20
300	1,6	0,16
400	1,3	0,14
500	1,1	0,11
600	0,9	0,09
700	0,8	0,08
800	0,6	0,06

M (MeV)	Δg_6 (GeV^{-5})	$\frac{\Delta g_6}{g_6}$
100	273	1,14
200	288	1,21
300	246	1,03
400	194	0,81
500	147	0,61
600	110	0,46
700	82	0,34
800	47	0,26

Summary

* We obtained fourth, sixth, eighth (and higher) order quark interaction by considering the contribution of scalar (auxiliary) fields

* Expansion of determinant (integration of gluons) compared to Det (constant field)
Same structure from auxiliary field ϕ

Some of the calculations on the way …

· To improve integration of gluons

·* Computation of contribution of scalar quark-antiquark condensate and fluctuations for the eighth order interactions

* Improve calculation of the determinants

·* Enlarge gluon configurations considered: more complete account of gluon interactions

·* Momentum dependent gluon mass

·* More general solutions for the GAP equations

·* diquark interactions