Tomography of Elastic pp Scattering

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IF/UFRJ

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Outline

- Elastic differential cross sections
- t - space amplitudes
- Regular behaviour with the energy
- b - space analytical forms and eikonals
- Unitarity conditions
- High energy extrapolations
- Conclusions

Differential cross section and t space amplitudes

$$\frac{d\sigma}{dt} = (\hbar c)^2 \; |T_R(s,t) + iT_I(s,t)|^2$$

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$$F^C(s,t)e^{i\alpha\,\Phi(s,t)} = (-/+)\,\frac{2\alpha}{|t|}e^{i\alpha\,\Phi(s,t)}\; F^2_{\text{proton}}(t)$$

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Coulomb Phase

$$F_{\text{proton}}(t) = [0.71/(0.71+|t|)]^2$$

$(\hbar c)^2 = 0.3894$ mb GeV^2

$$T_K^N(s,t) = \alpha_K(s)\mathrm{e}^{-\beta_K(s)|t|} + \lambda_K(s)\Psi_K(\gamma_K(s),t)$$

Nuclear amplitudes

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Nuclear amplitudes

$$\Psi_K(\gamma_K(s),t) = 2\ \mathrm{e}^{\gamma_K}\left[\frac{\mathrm{e}^{-\gamma_K\sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - \mathrm{e}^{\gamma_K}\ \frac{e^{-\gamma_K\sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}}\right]$$

Shape functions

where $K = R$, $K = I$ for real and imaginary amplitudes

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where $K = R$, $K = I$ for real and imaginary amplitudes

For |t|=0 we have the forward quantities...

Quantities in forward scattering

$$\sigma(s) = 4\sqrt{\pi}\,(\hbar c)^2\ \left(\alpha_I(s) + \lambda_I(s)\right)$$

Total cross section

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Real/Imaginary

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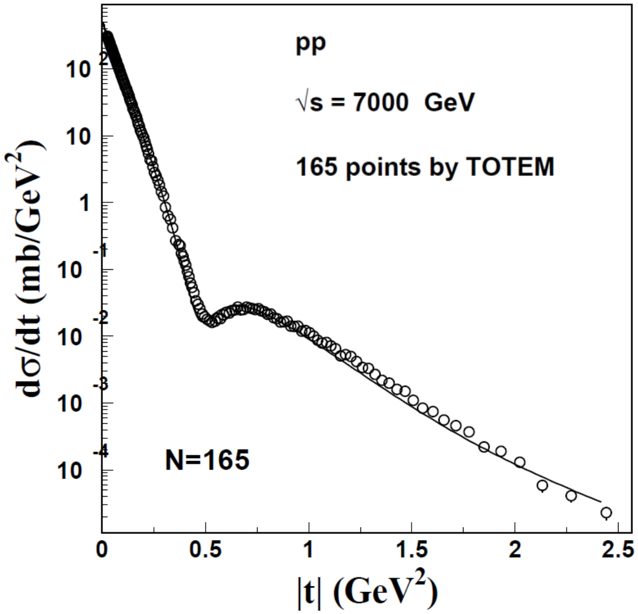
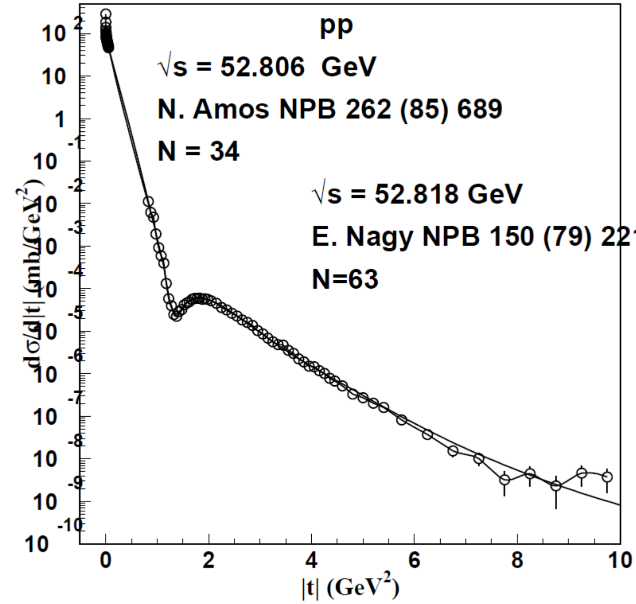
Real/Imaginary

$$B_K(s) = \frac{2}{T_K^N(s,t)} \frac{dT_K^N(s,t)}{dt}\Big|_{t=0}$$
$$= \frac{1}{\alpha_K(s) + \lambda_K(s)}\left[\alpha_K(s)\beta_K(s) + \frac{1}{8}\lambda_K(s)a_0\Big(6\gamma_K(s) + 7\Big)\right]$$

Real and Imaginary slopes

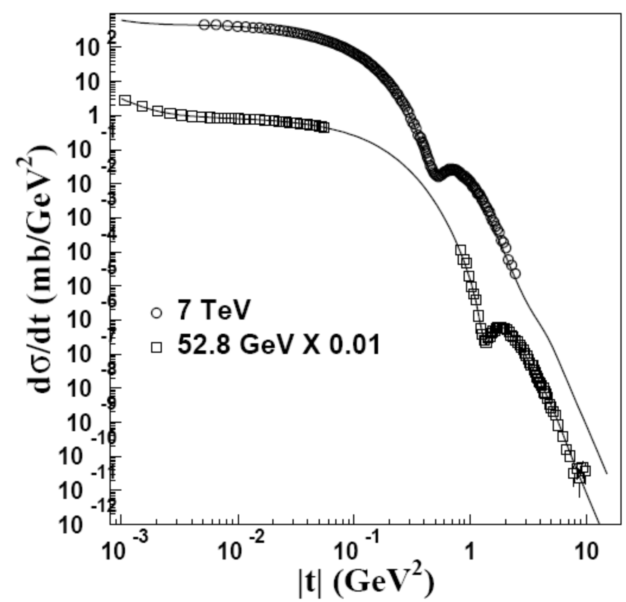
Differential cross sections



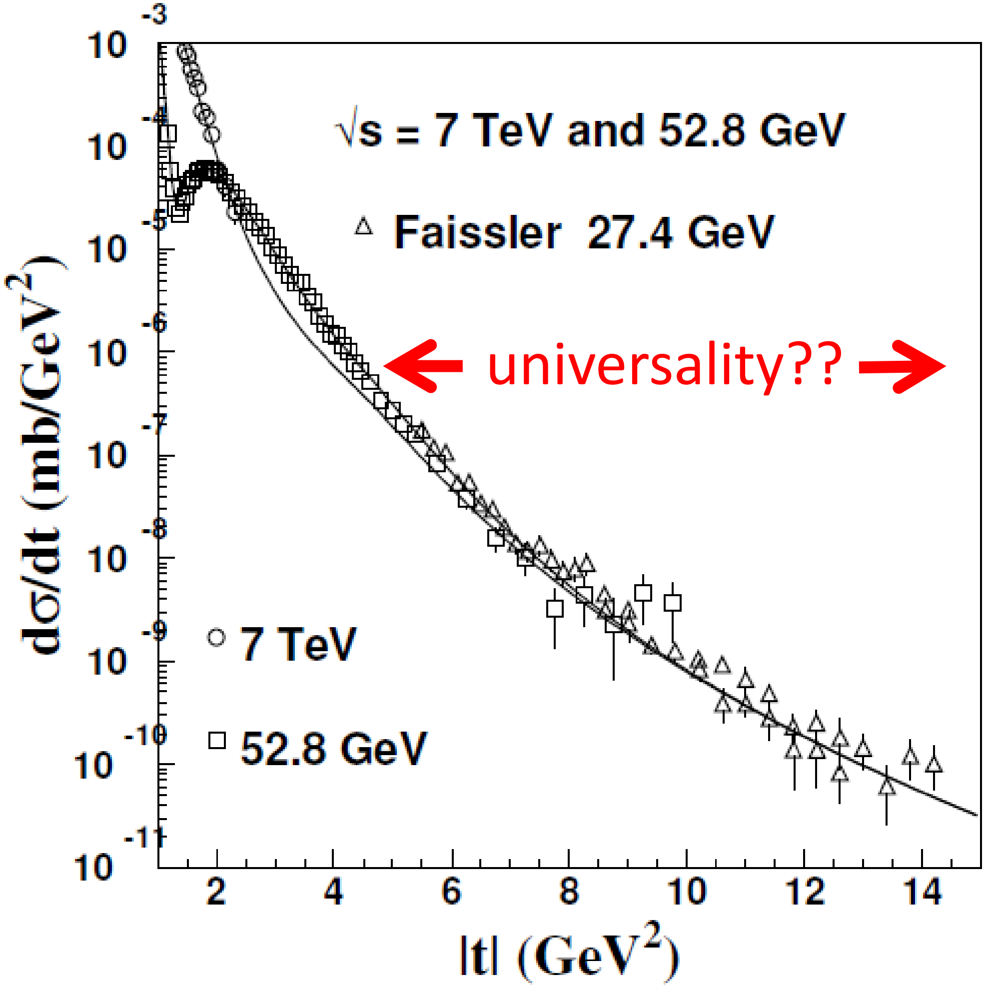


A. K. Kohara, E. Ferreira, T. Kodama *Eur. Phys. J. C* **73**, 2326 (2013)

Totem Experiment *et al.*, *Europhys. Lett.* **95**, 41001 (2011)

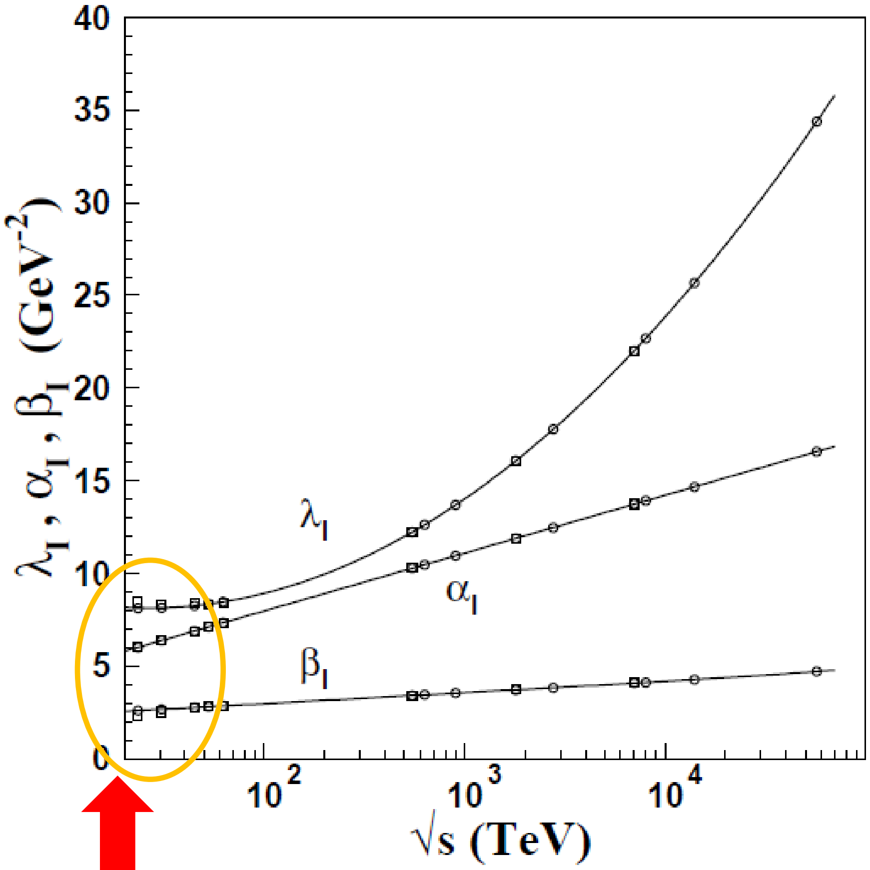


Universality at large |t|?

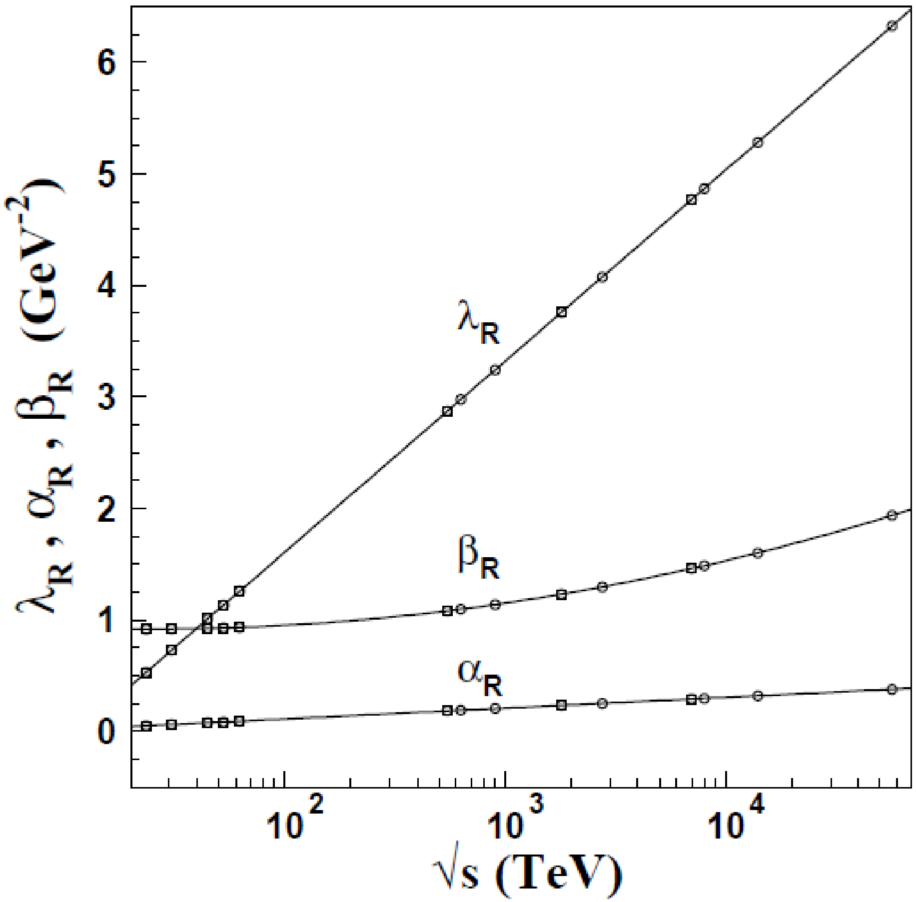


W. Fassler et al. , *Phys. Rev. D* **23**, 33 (1981)

Energy dependence of the parameters



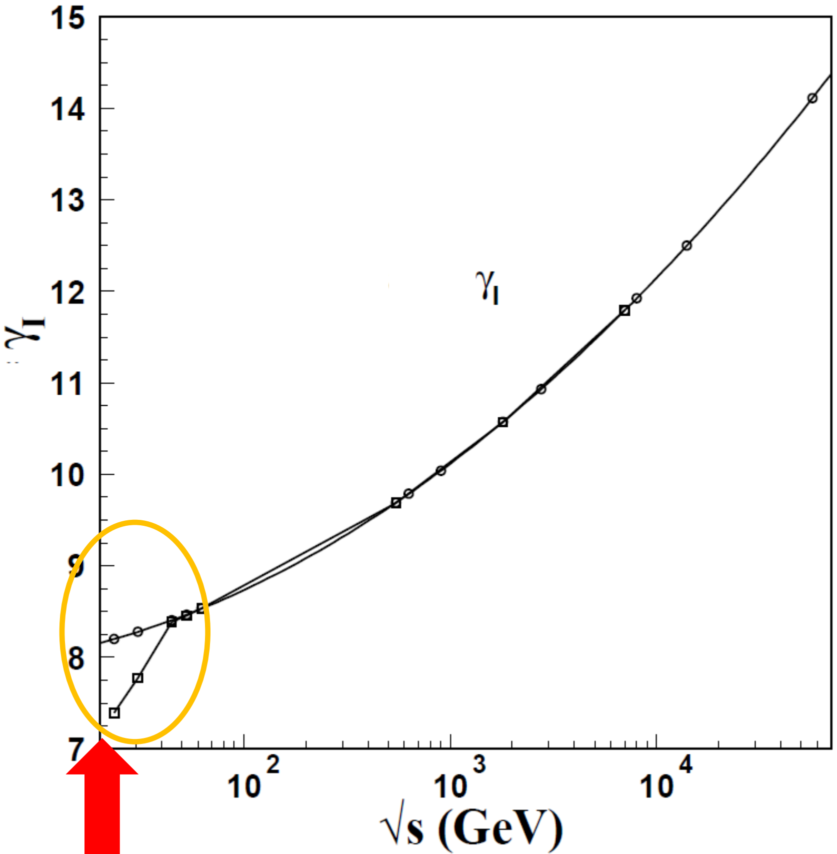
Real amplitudes



Imaginary amplitudes

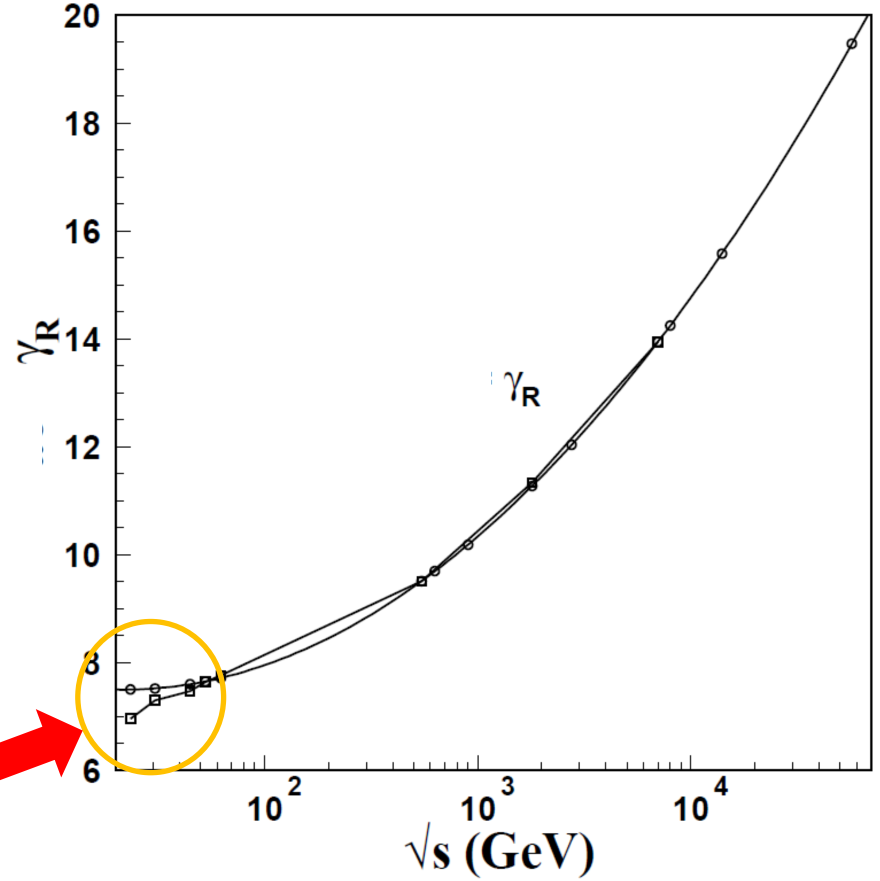
Influence of p structure in the low energies

Energy dependence of the parameters



Imaginary amplitudes

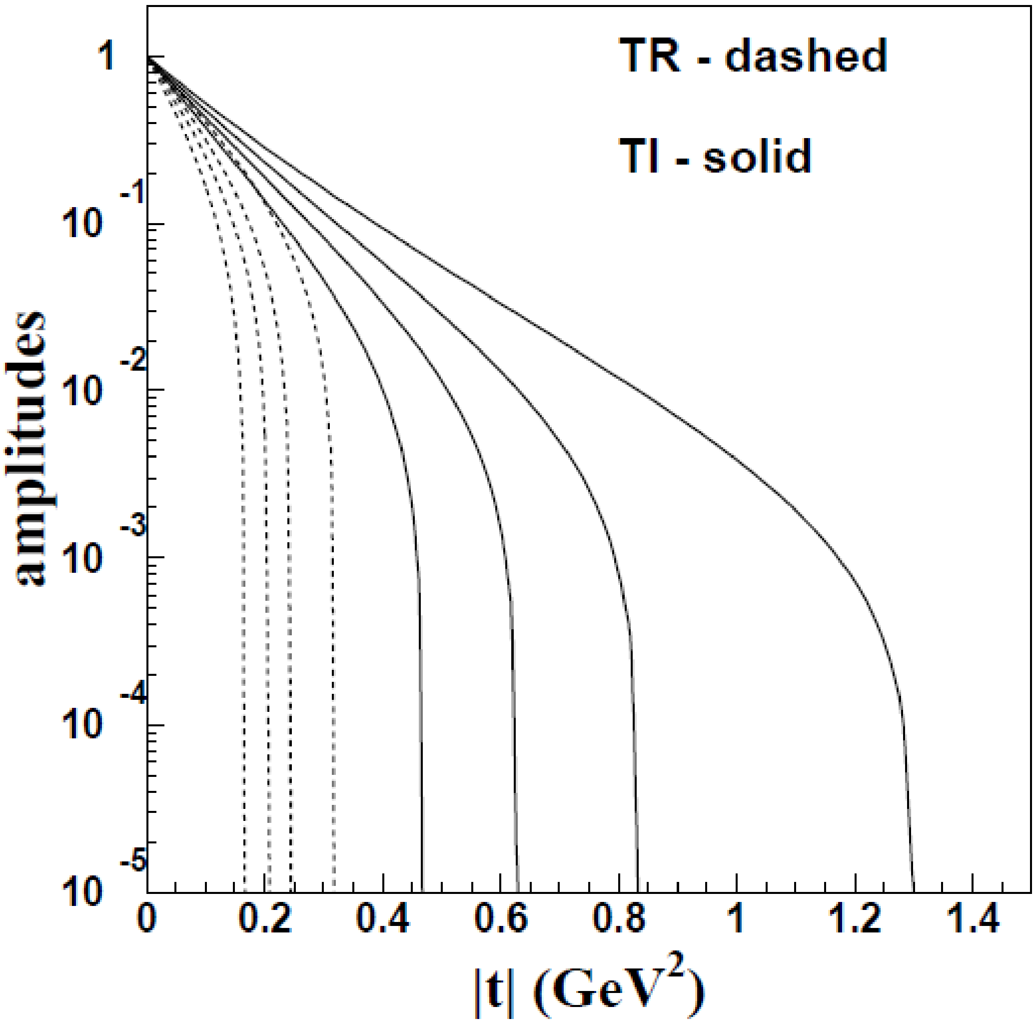
Real amplitudes



Influence of p structure in the low energies

t - space amplitudes

Forward region



t - space amplitudes

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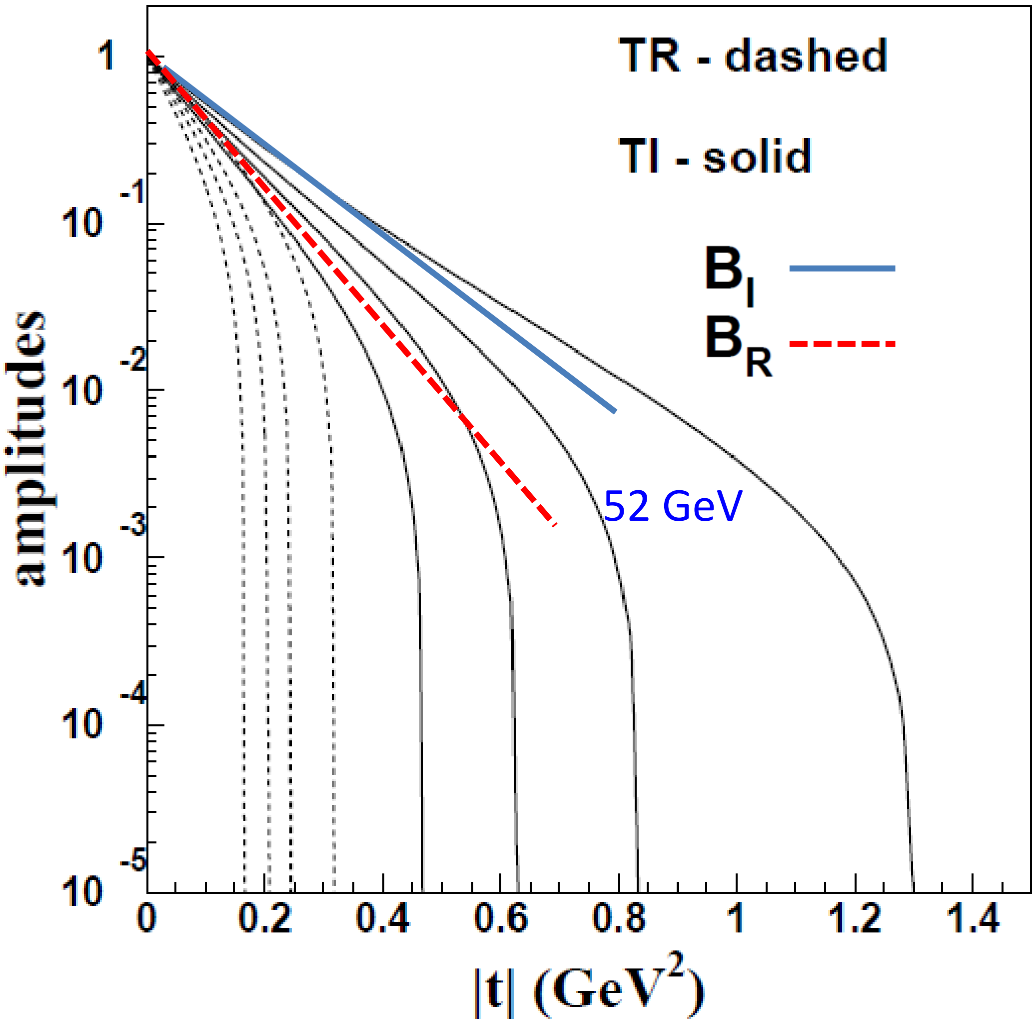

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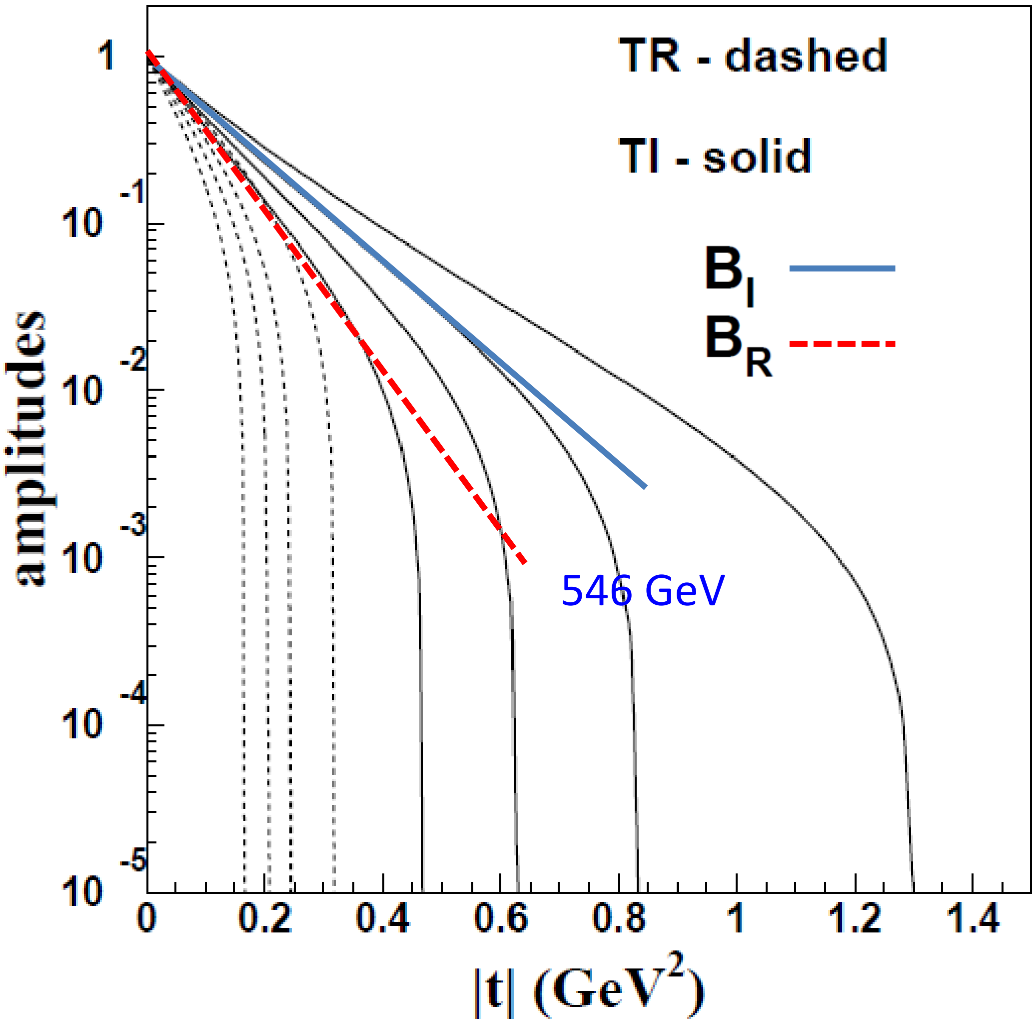

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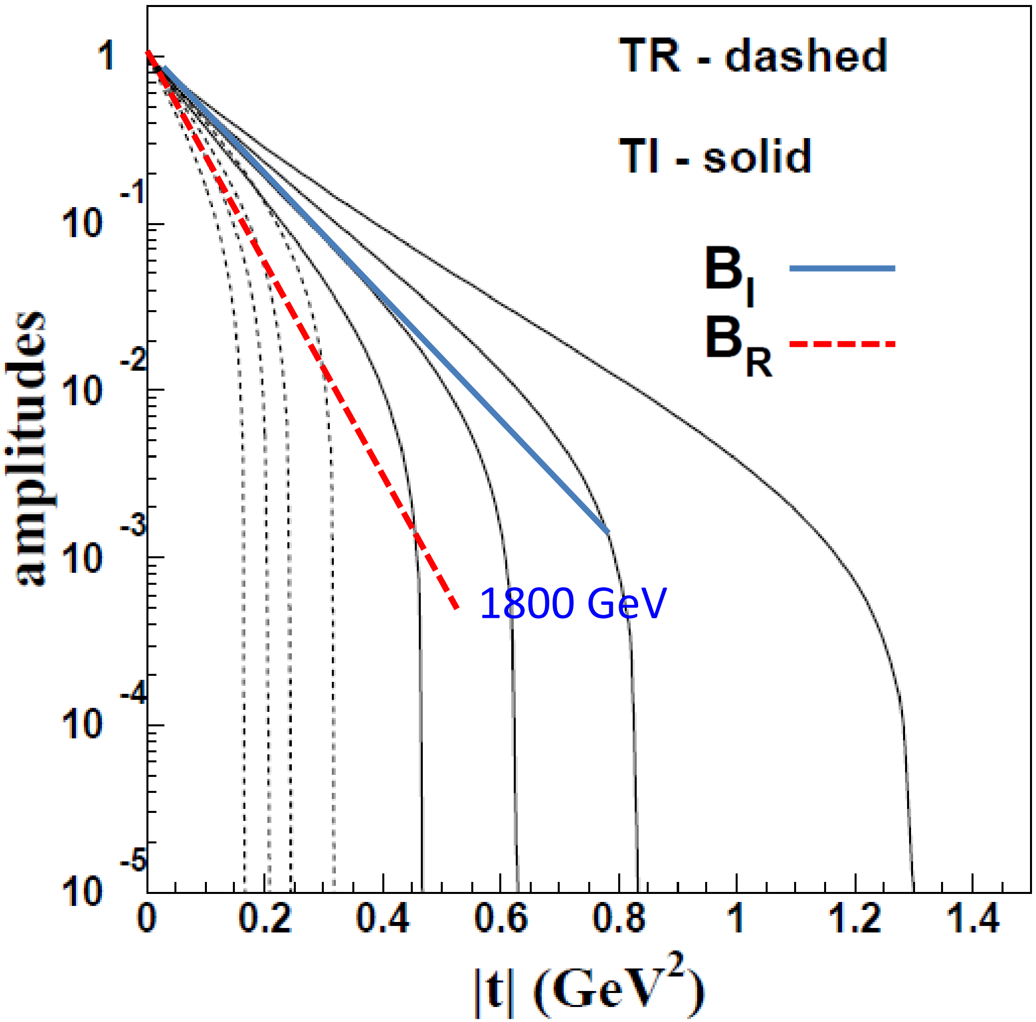

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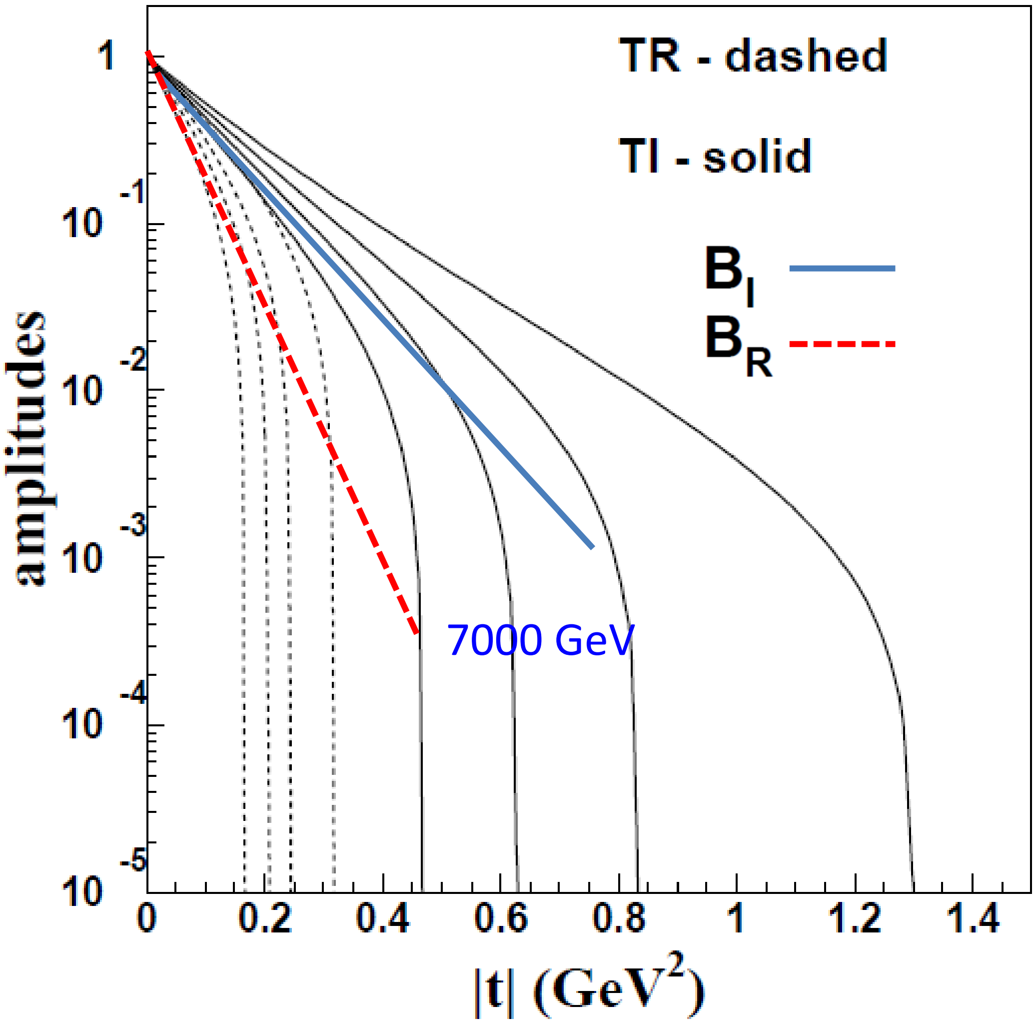

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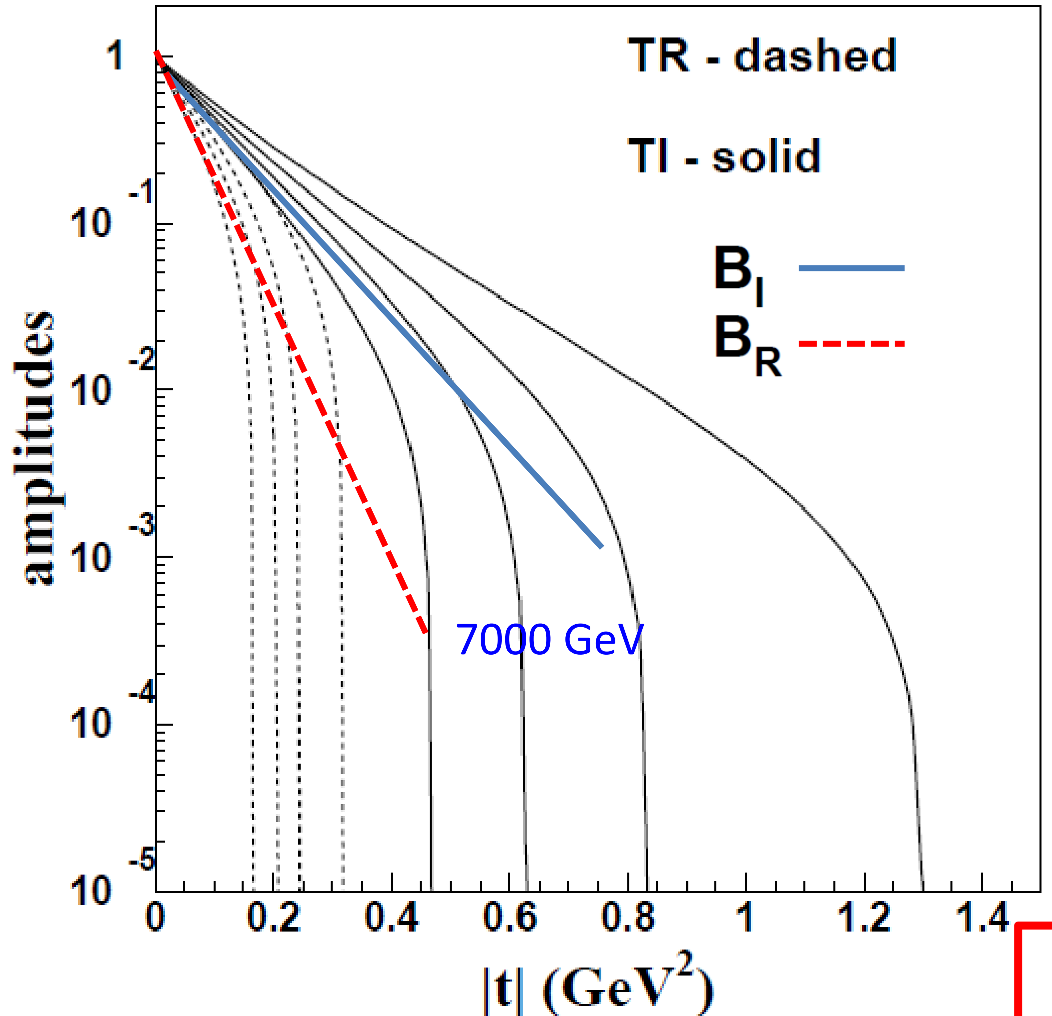
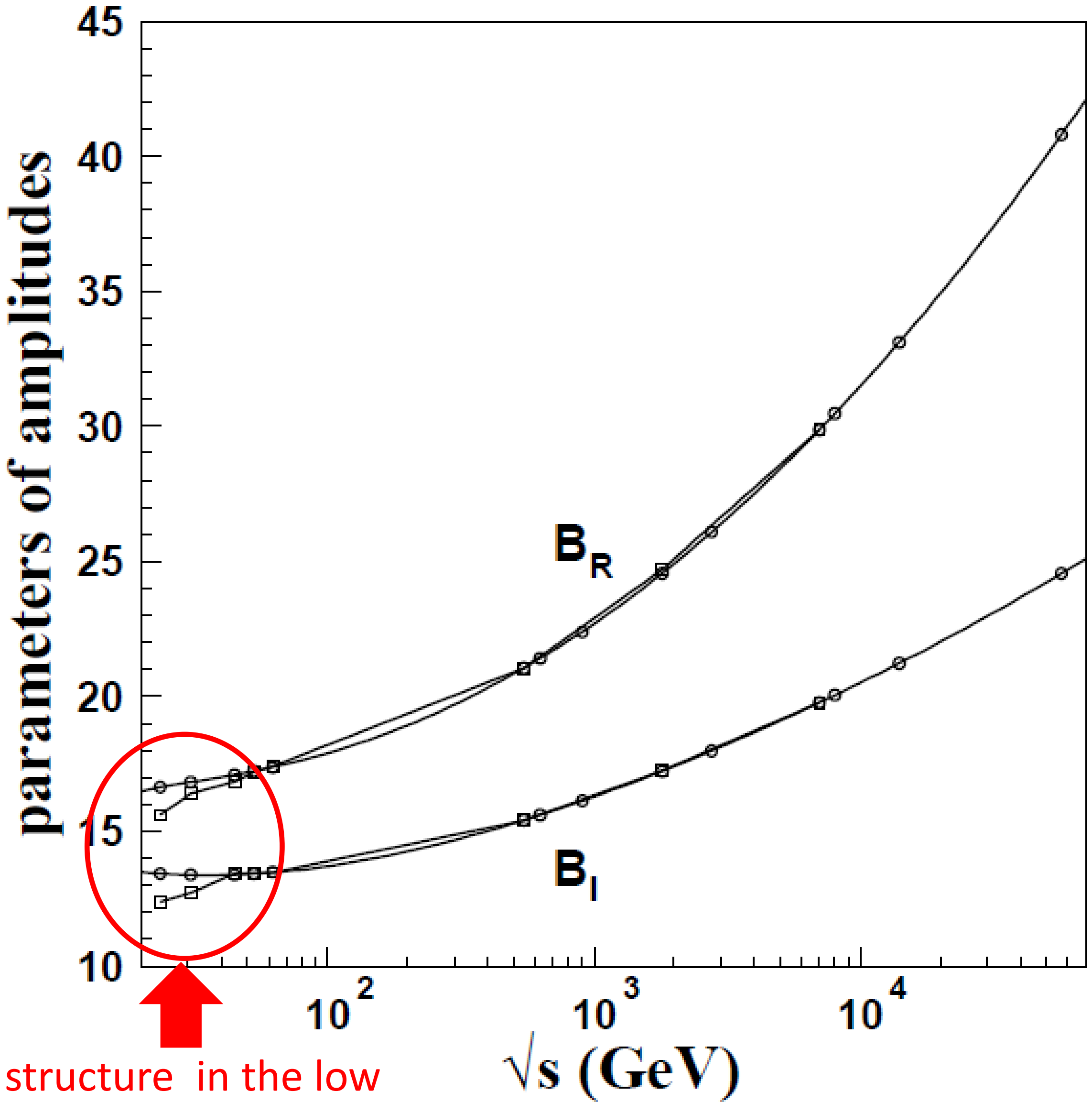


Obviously

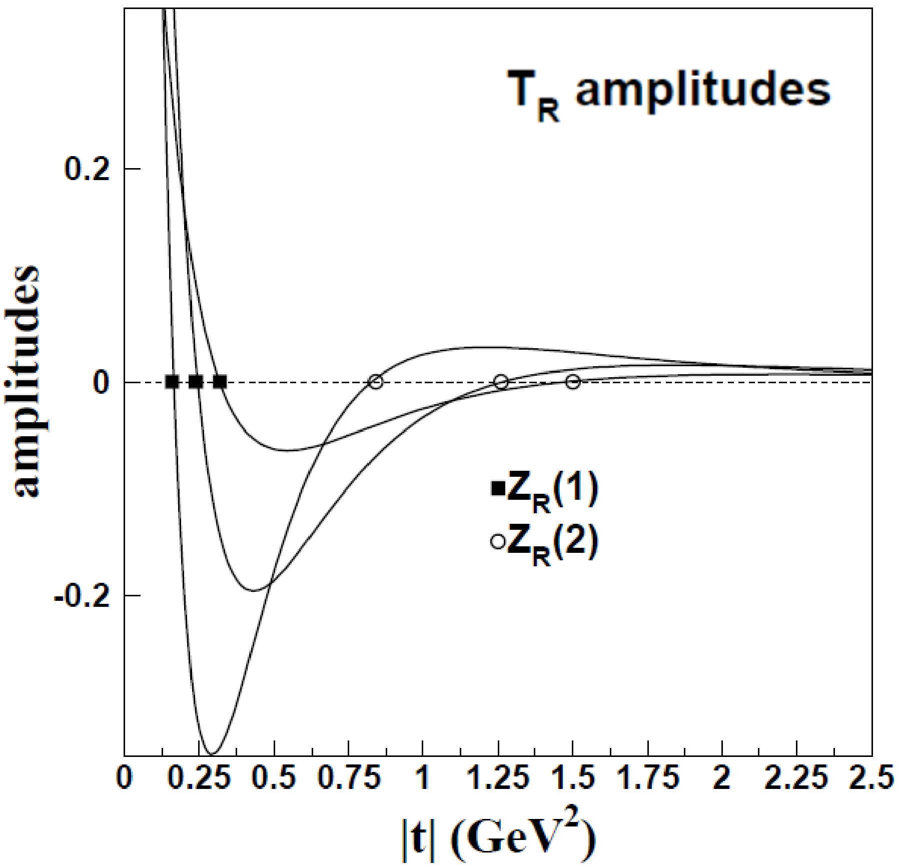
$$B_R \neq B_I$$

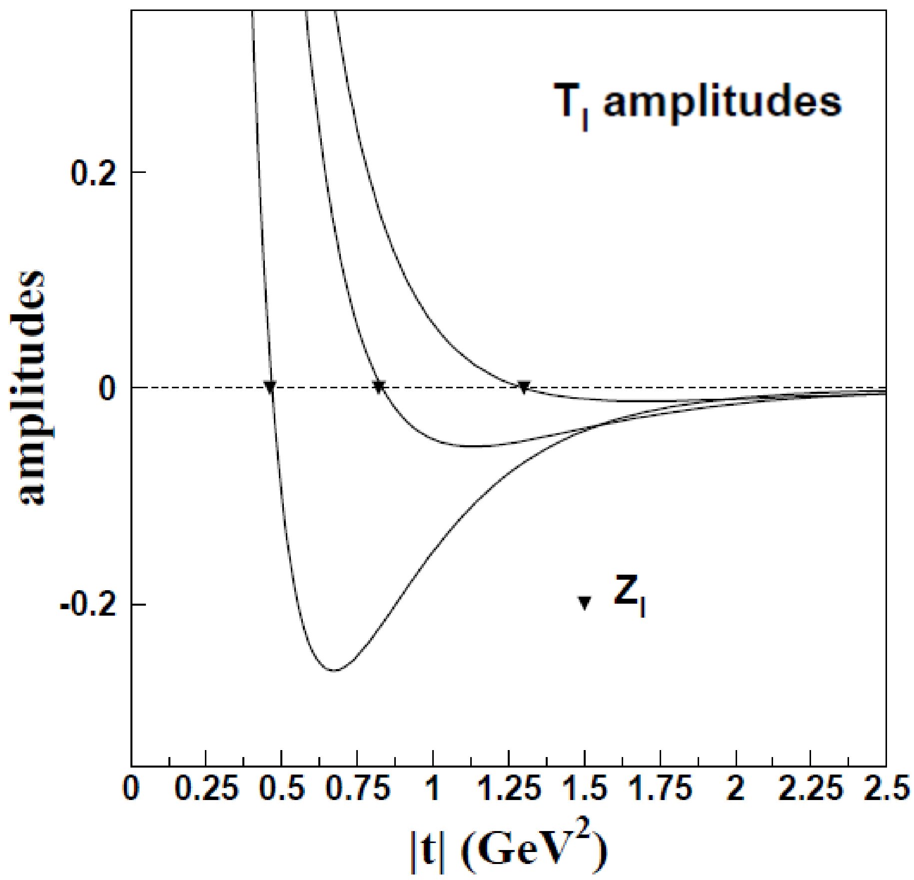
Evolution of the slopes



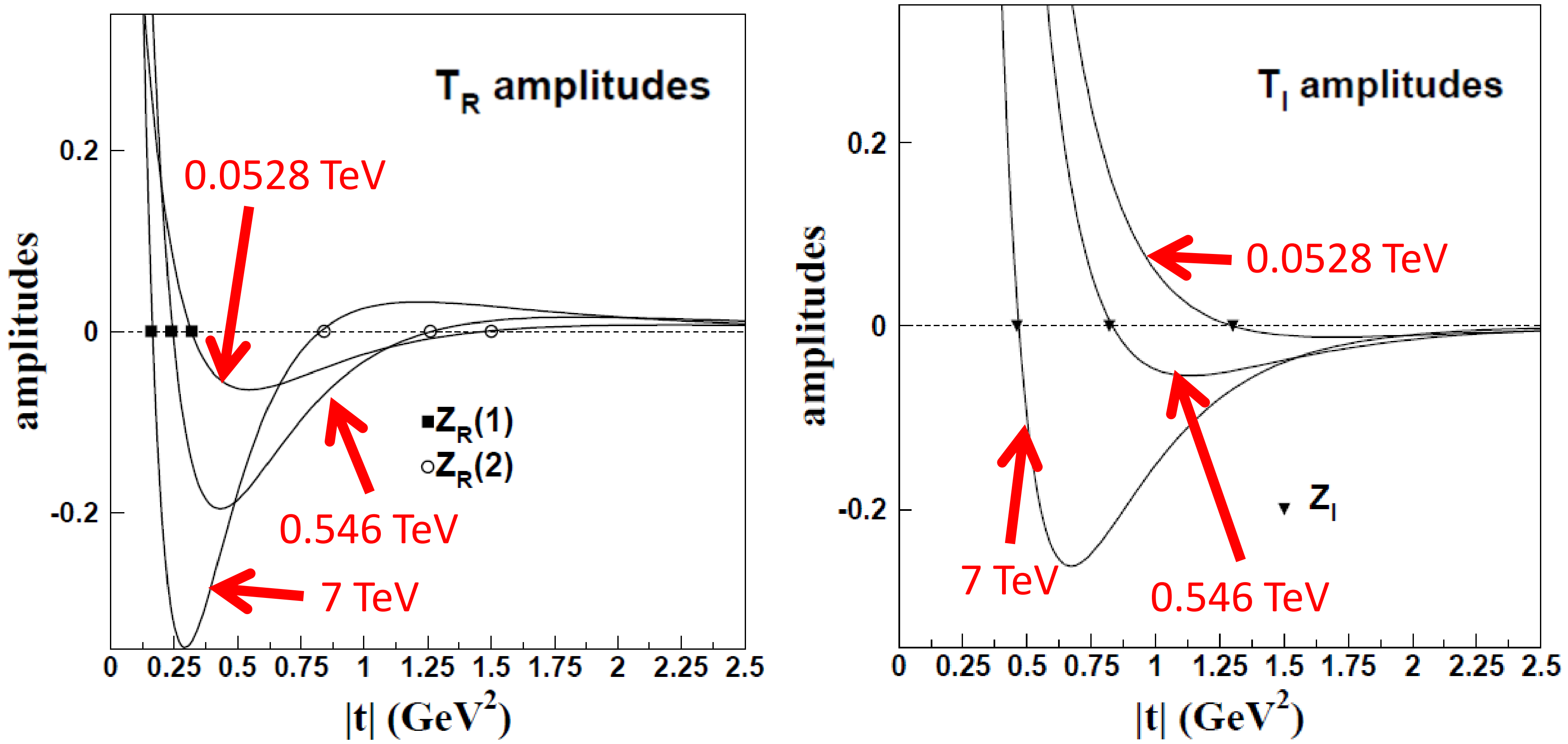
Influence of p structure in the low energies

Structure behind the data

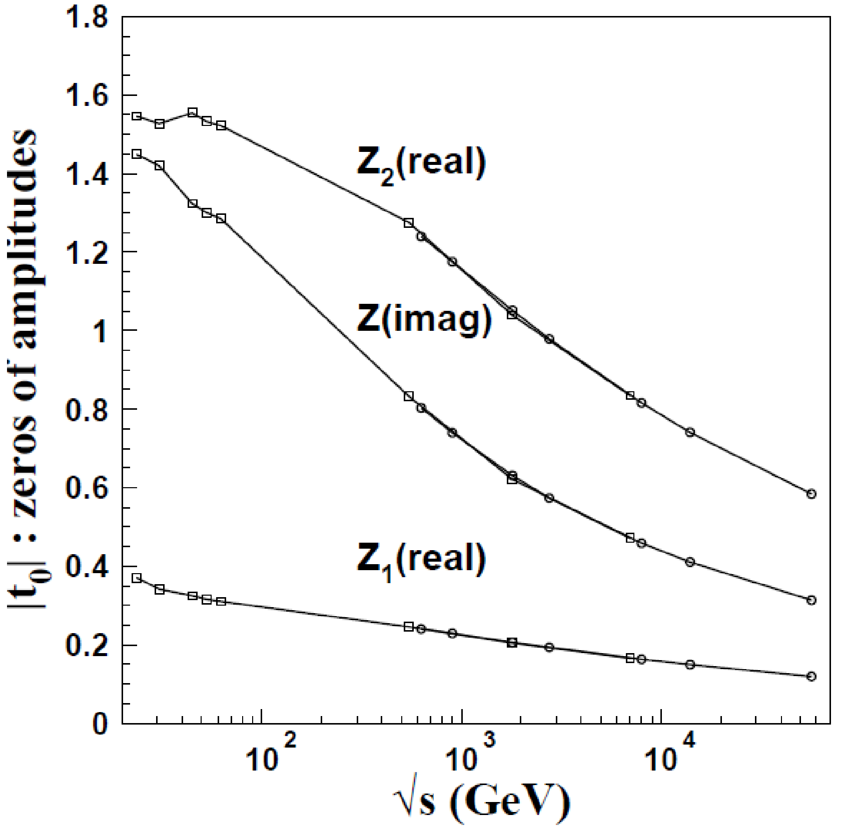




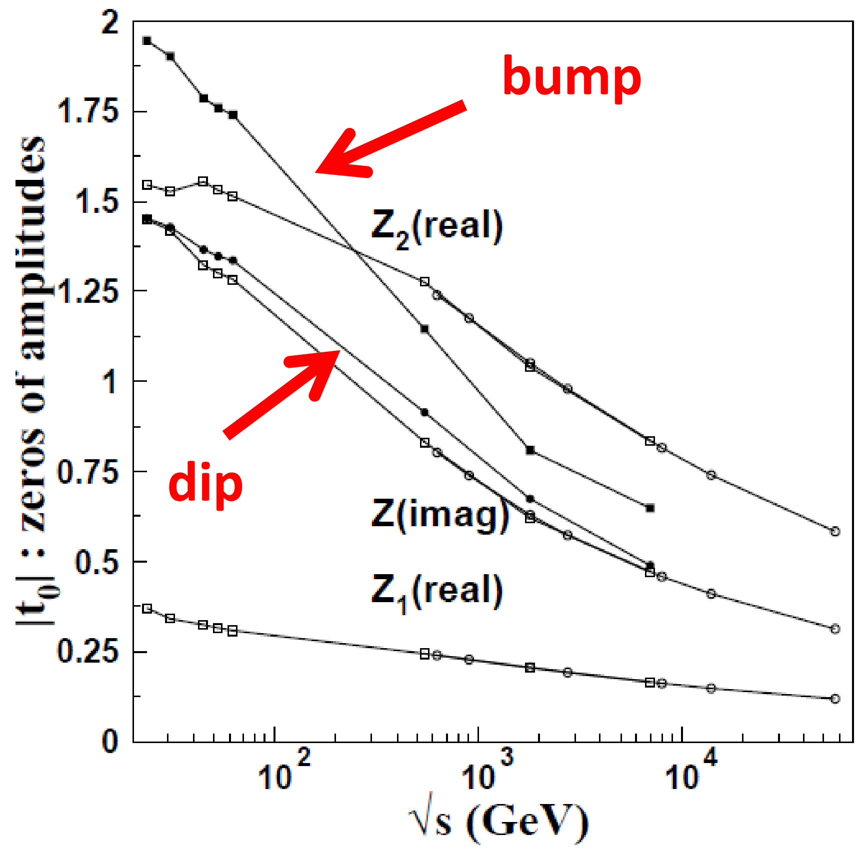
Structure behind the data



Zeros dips and bumps

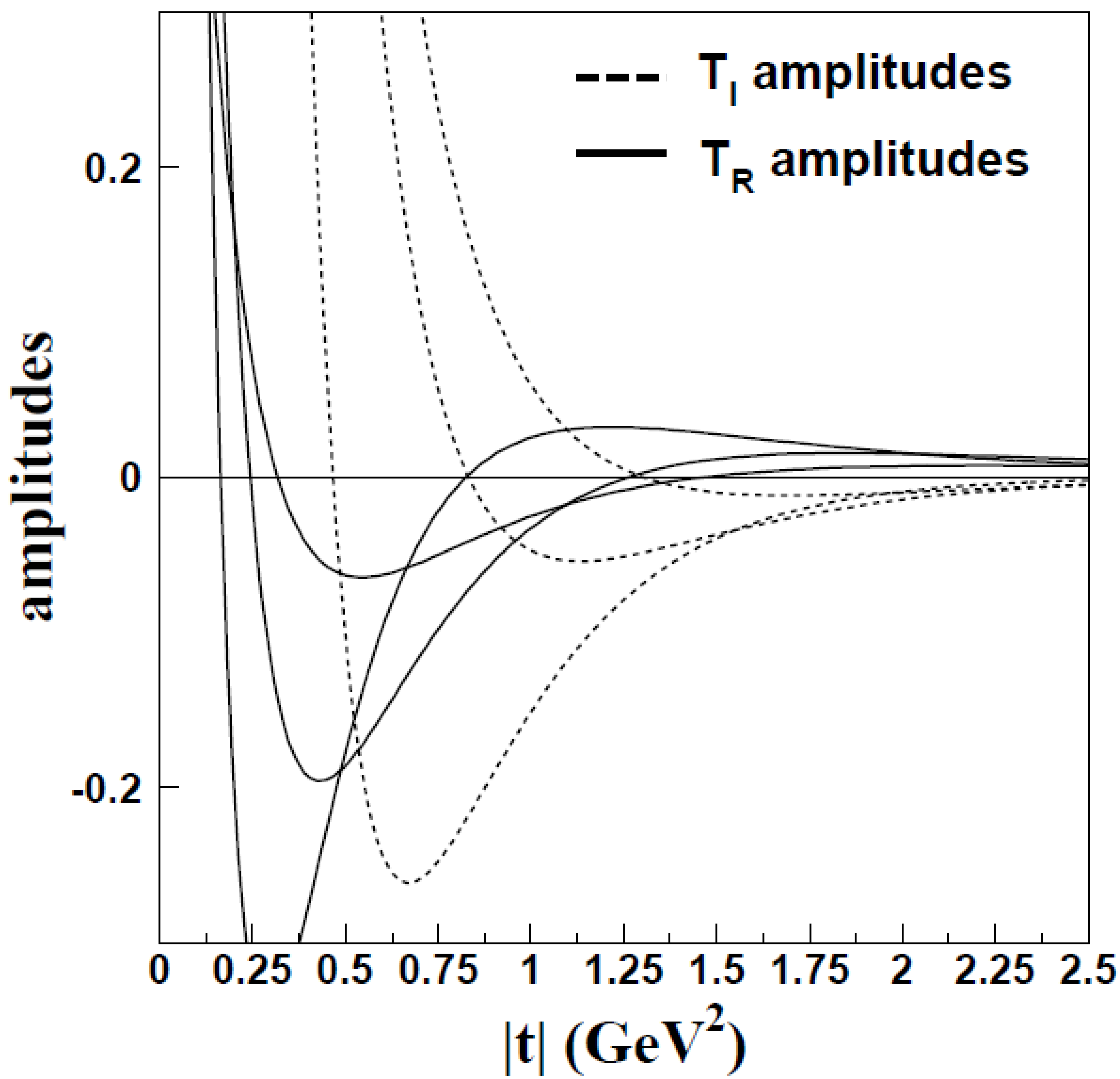


Zeros of amplitudes

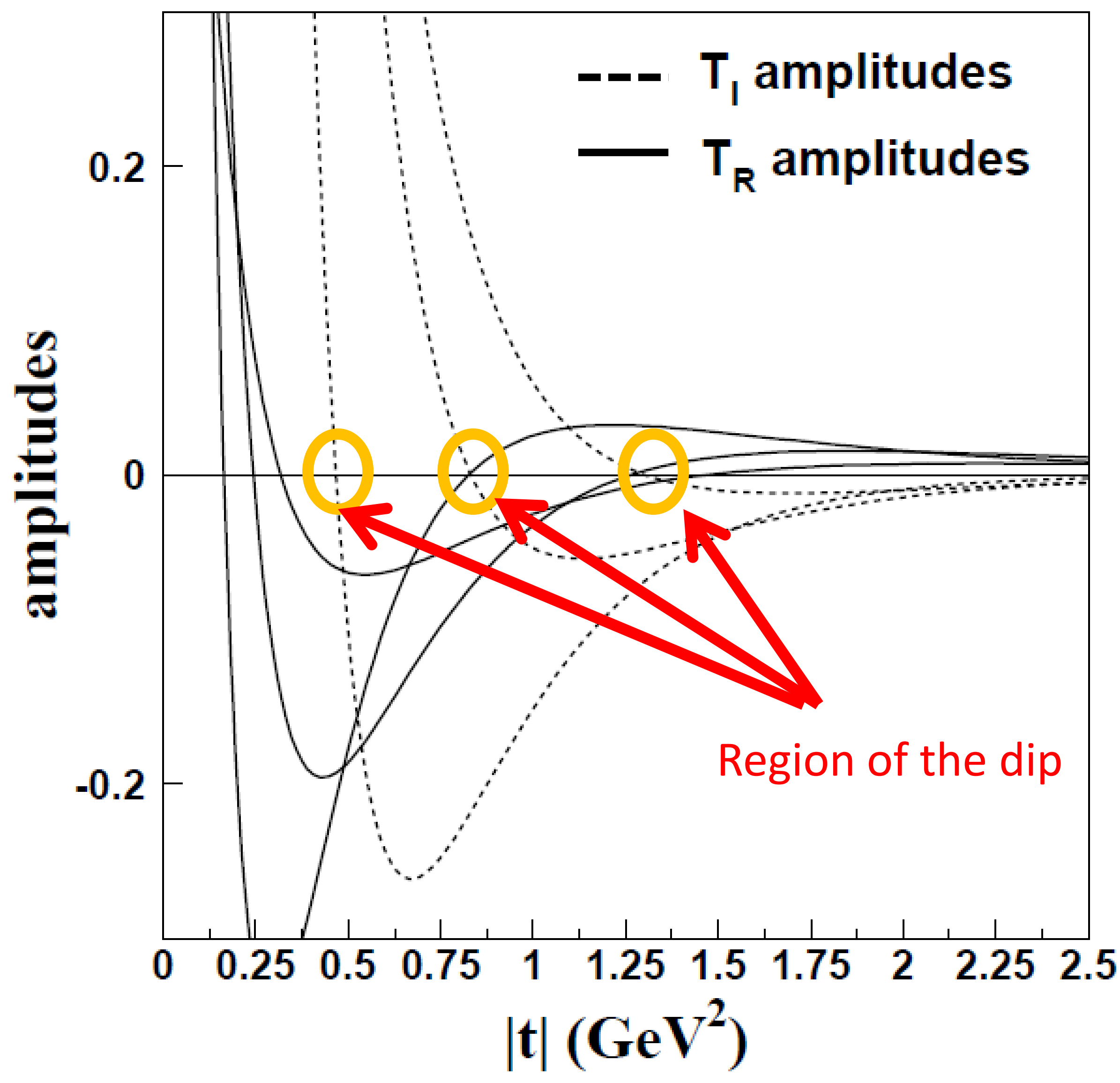


Zeros, dips and bumps

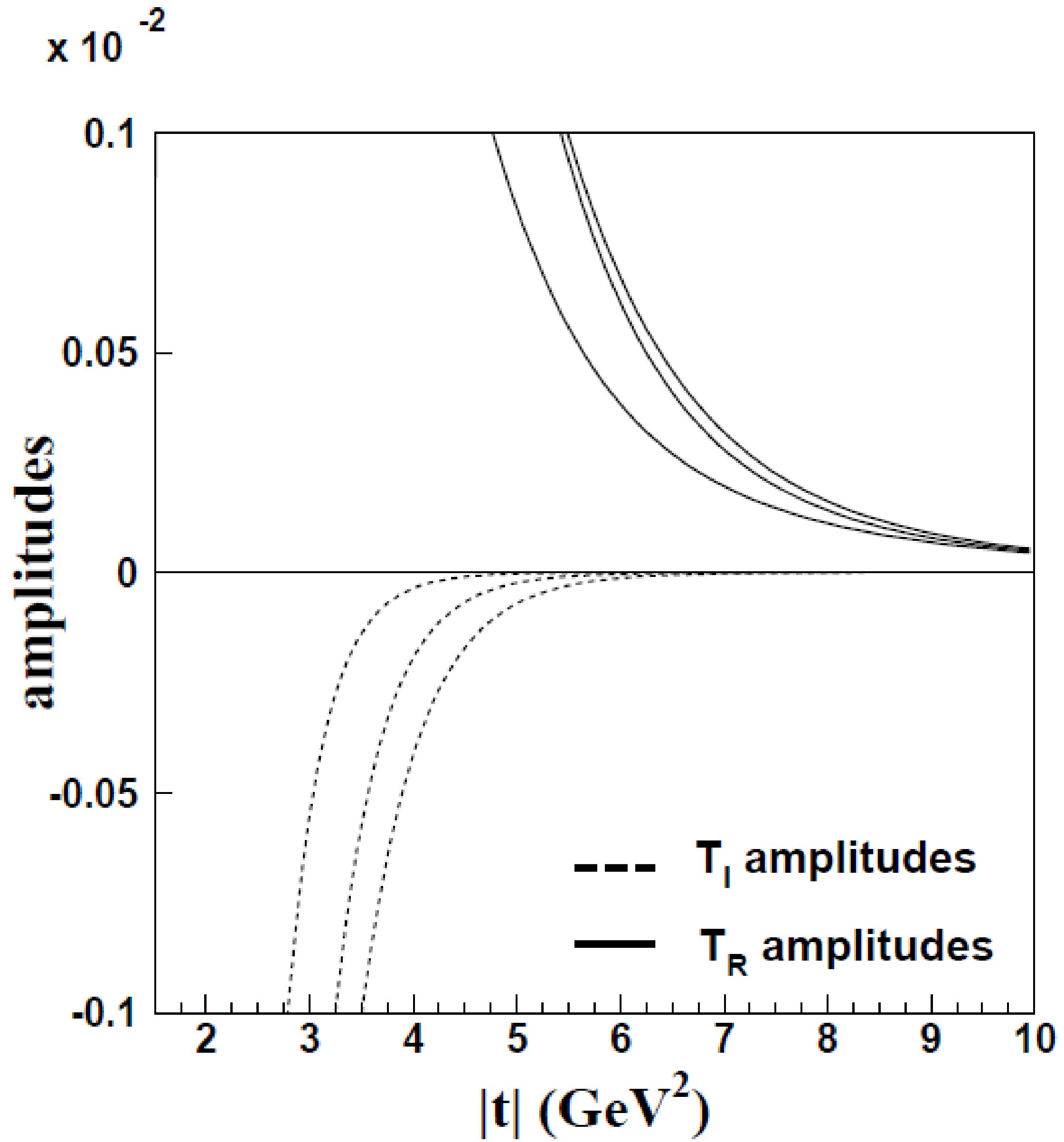
Real and imaginary amplitudes



Real and imaginary amplitudes



Dominance of the real part at large t



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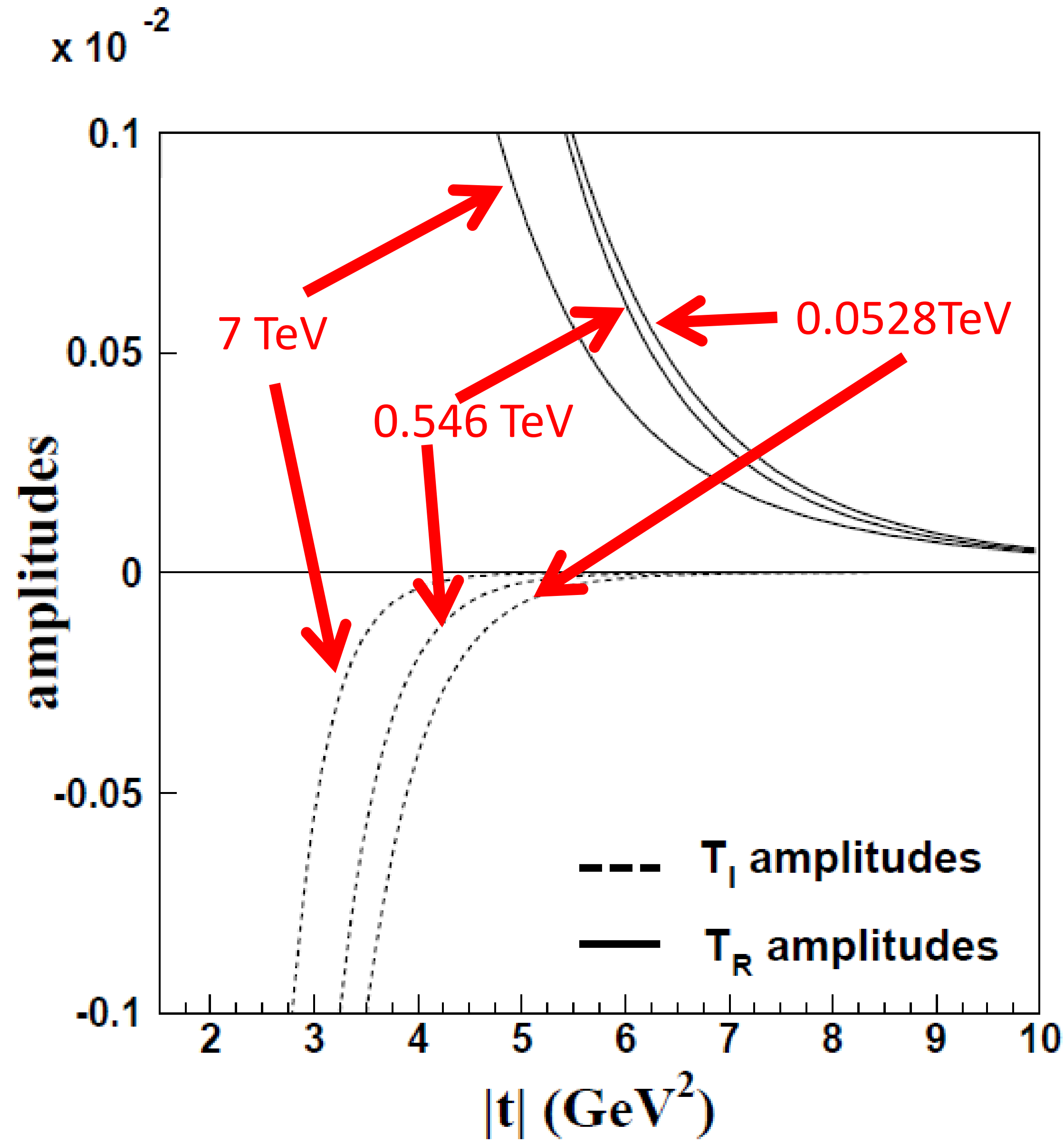
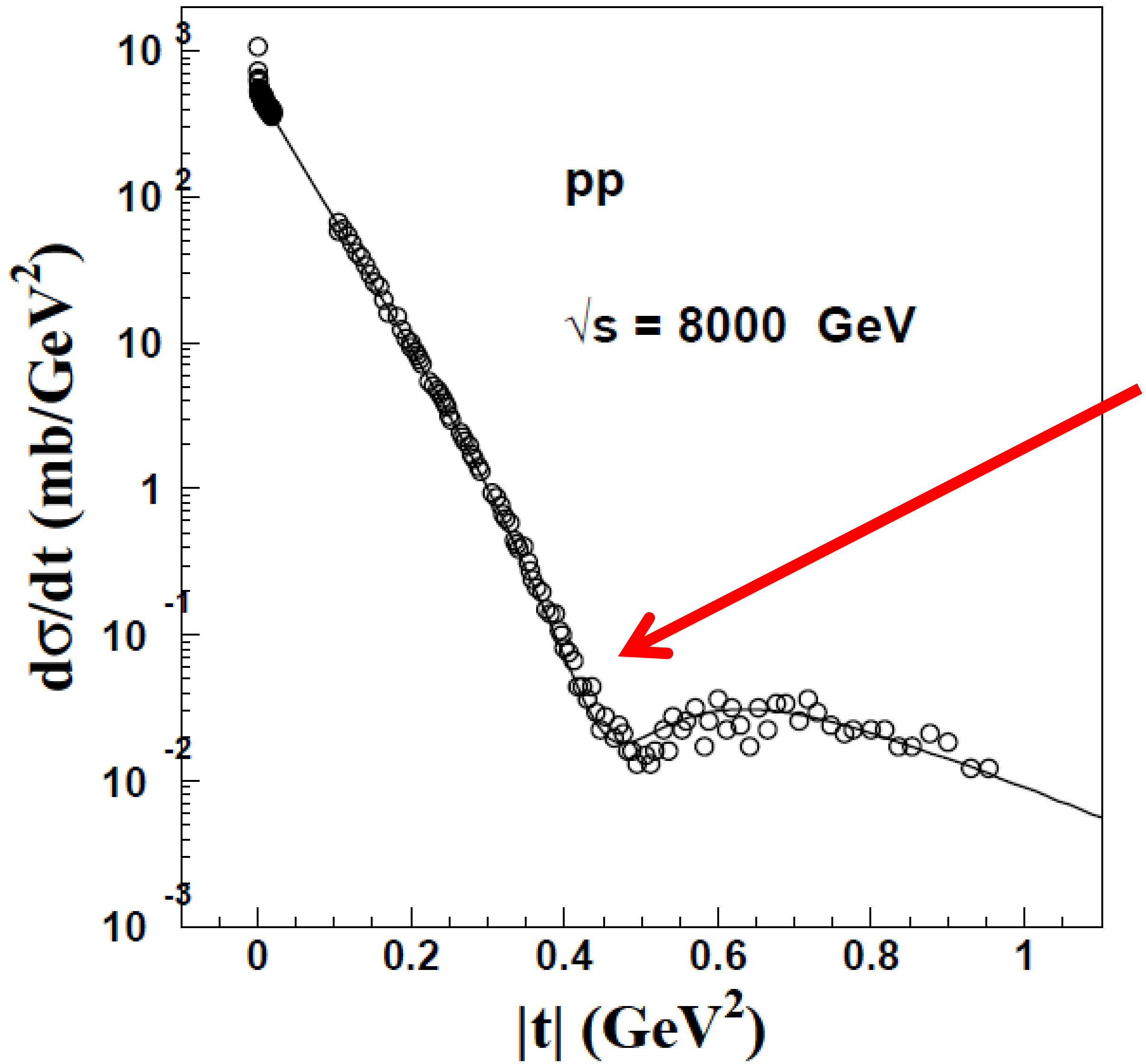
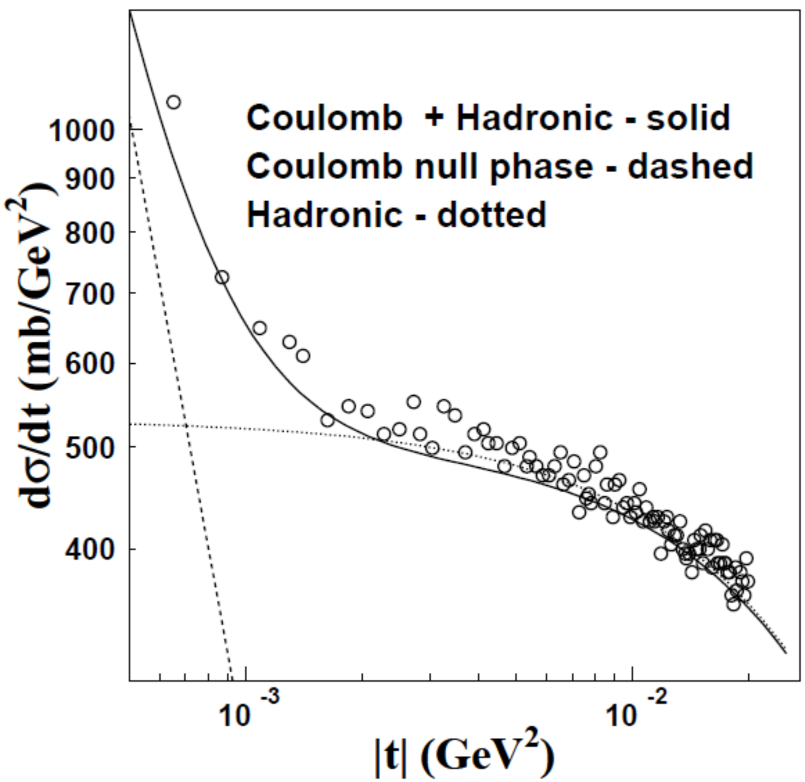


Predictions to pp at 8 TeV



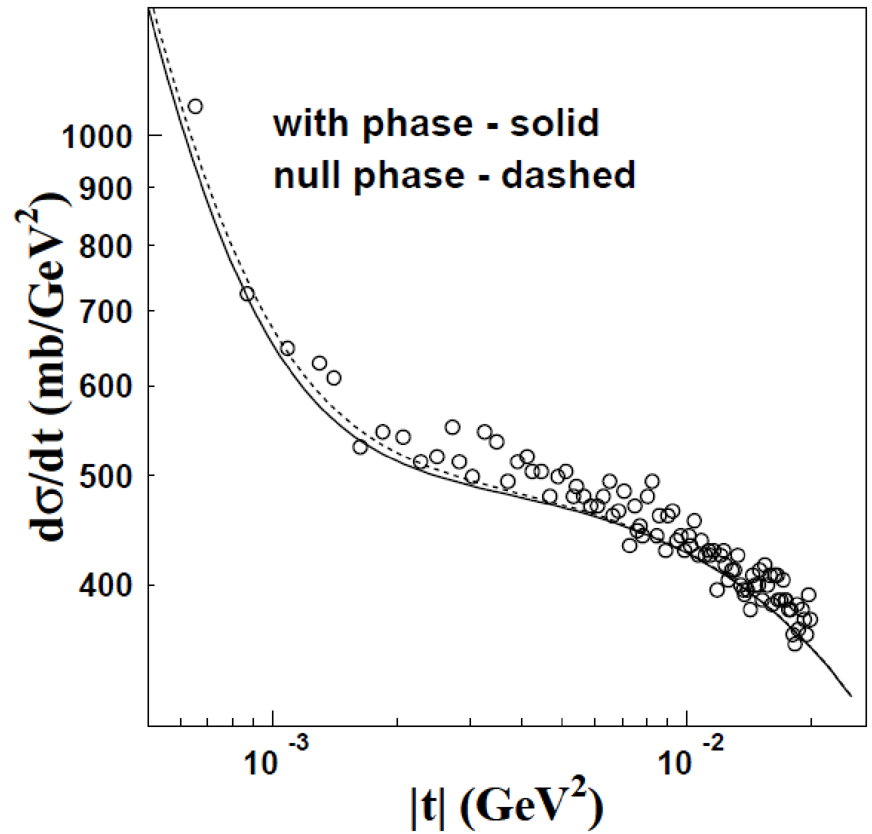
Data extracted by eye from preliminar plot.

Coulomb inteference at 8 TeV



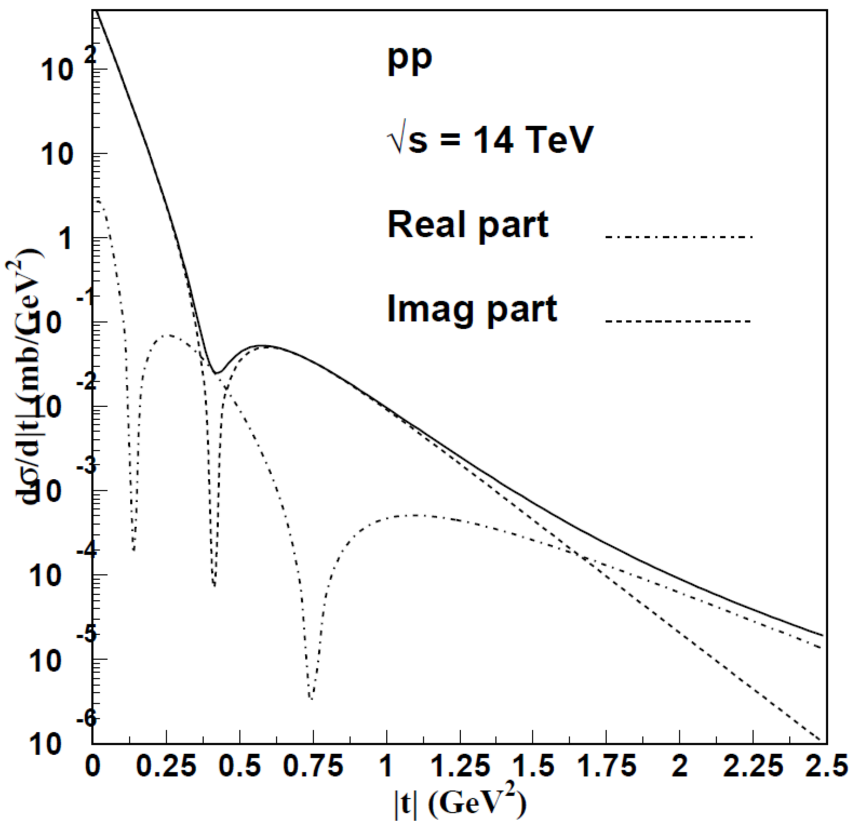
Region of Coulomb interference

Testing the Coulomb phase effect

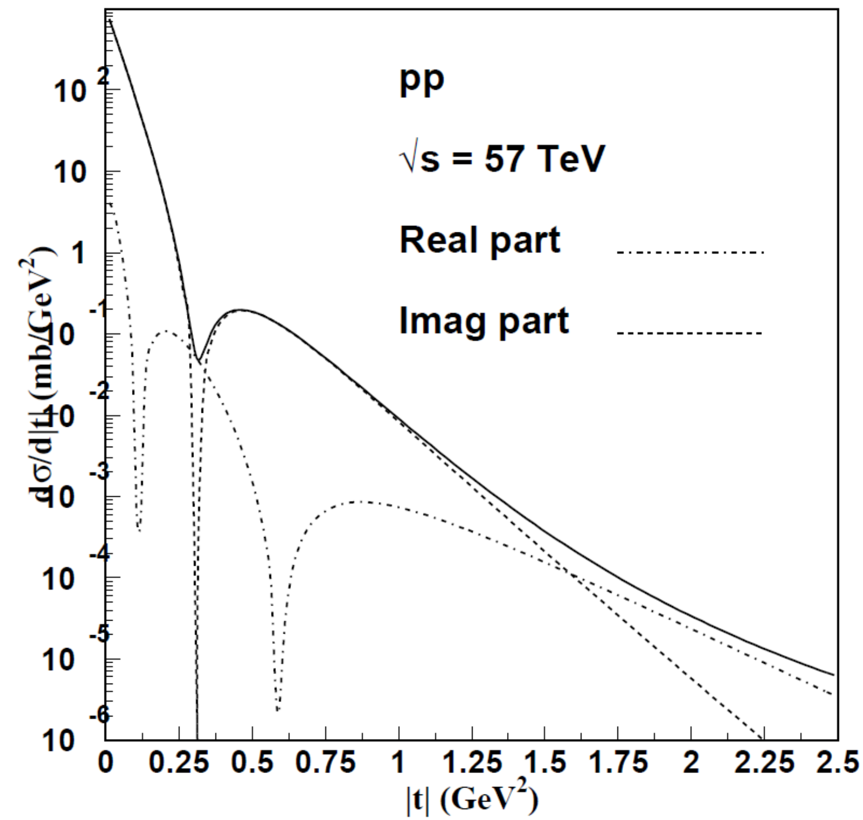


LHC 14 TeV and cosmic ray 57 TeV extrapolations

LHC at 14 TeV



Cosmic ray at 57 TeV



Forward quantities:

$\sigma = 111.34$ mb $\quad \rho = 0.139$

$B_I = 21.23\ \mathrm{GeV}^{-2} \quad B_R = 33.11\ \mathrm{GeV}^{-2}$

$\sigma = 140.66$ mb $\quad \rho = 0.132$

$B_I = 24.56\ \mathrm{GeV}^{-2} \quad B_R = 40.82\ \mathrm{GeV}^{-2}$

b space amplitudes

Fourier Transform

$$\tilde{T}_K(s,b) = \frac{1}{2\pi}\int d^2\vec{q}\, e^{-i\vec{q}.\vec{b}}\, T_K^N(s,t=-q^2)$$

Analytical forms

$$T_K(s,b) = \frac{\alpha_K}{2\beta_K} e^{-\frac{b^2}{4\beta_K}} + \lambda_K \tilde{\psi}(\gamma_k(s),b)$$

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with the shape functions

$$\tilde{\psi}(\gamma_K(s),b) = \frac{2e^{\gamma_K}}{a_0} \frac{e^{\sqrt{\gamma_K^2+\frac{b^2}{a_0}}}}{\sqrt{\gamma_K^2+\frac{b^2}{a_0}}} \left[1 - e^{\gamma_K} e^{-\sqrt{\gamma_K^2+\frac{b^2}{a_0}}}\right]$$

b space amplitudes

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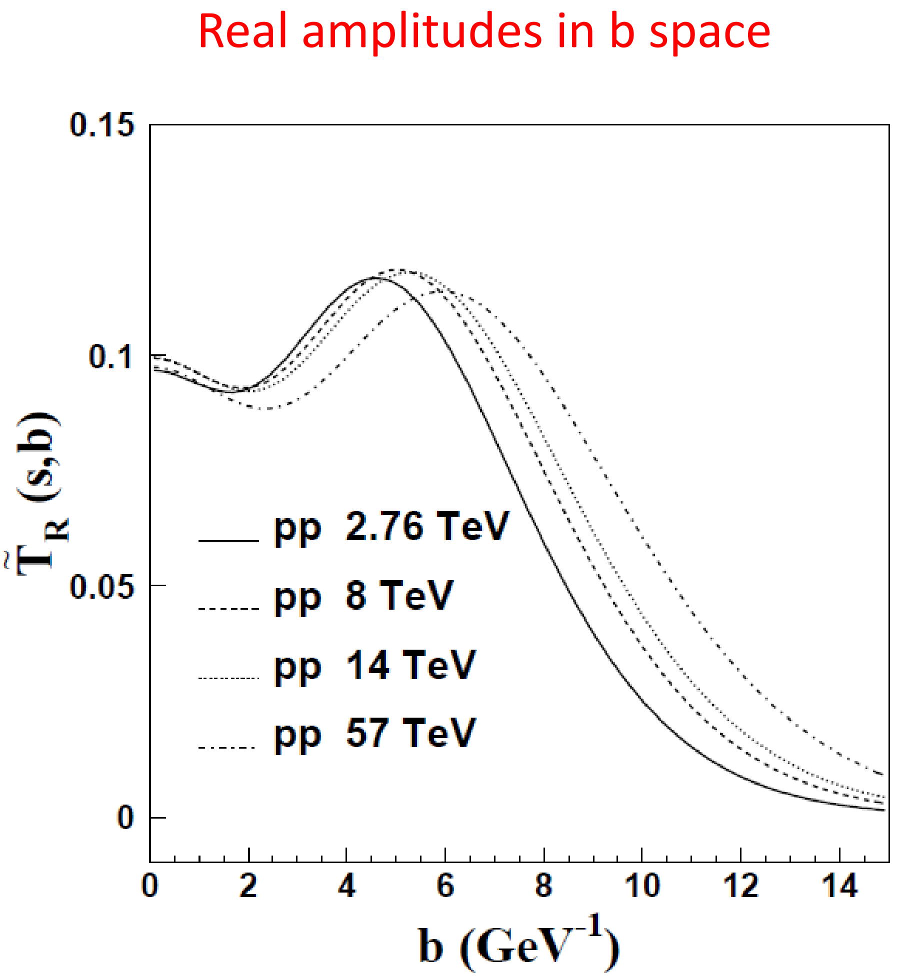
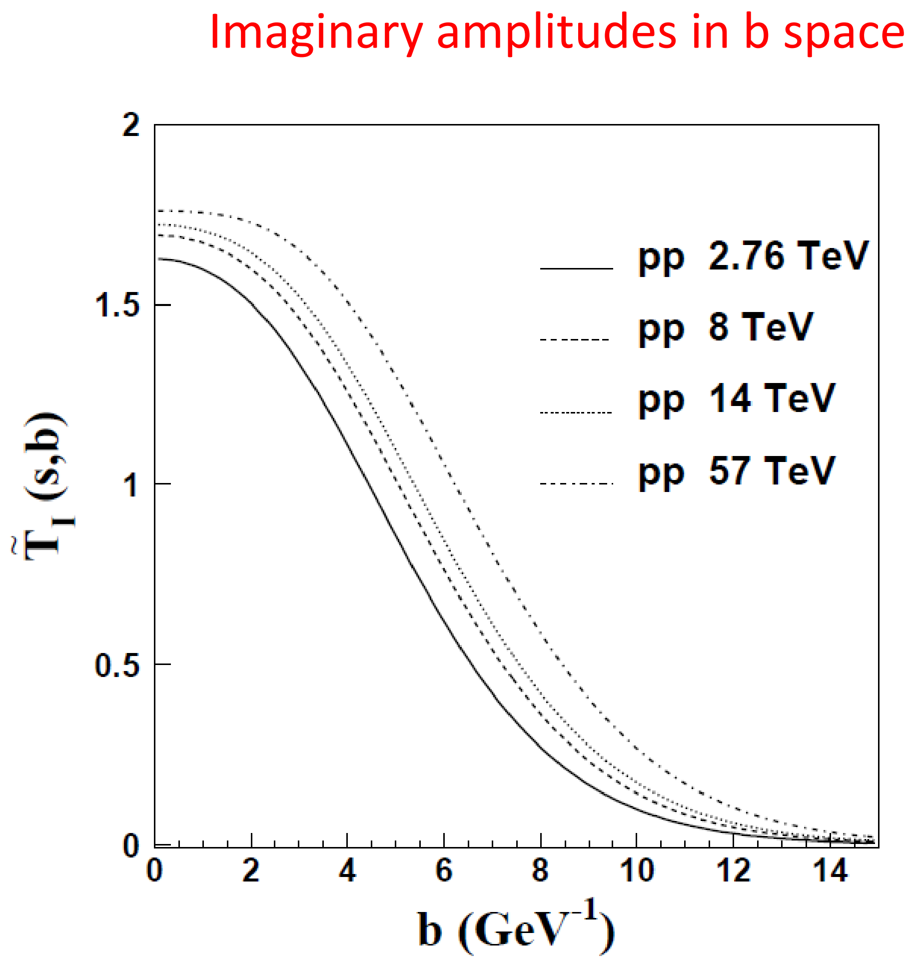
Large b

$$e^{-\gamma b}/b$$

Yukawa like



Real amplitudes in b space
0.15
0.1
0.05
0
$\tilde{T}_R$ (s,b)
pp 2.76 TeV
pp 8 TeV
pp 14 TeV
pp 57 TeV
0 2 4 6 8 10 12 14
b (GeV^{-1})



Imaginary amplitudes in b space
2
1.5
1
0.5
0
$\tilde{T}_I$ (s,b)
pp 2.76 TeV
pp 8 TeV
pp 14 TeV
pp 57 TeV
0 2 4 6 8 10 12 14
b (GeV^{-1})

Eikonal representation

Introducing eikonal formalism

$$\tilde{T}_R\left(s,\vec{b}\right)=\tilde{T}_R\left(s,\vec{b}\right)+i\tilde{T}_I\left(s,\vec{b}\right)\equiv i\sqrt{\pi}\left(1-e^{i\chi(s,b)}\right)$$

with the complex eikonal function $\chi=\chi_R+i\chi_I$

Eikonal representation

Introducing eikonal formalism

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with the complex eikonal function $\chi=\chi_R+i\chi_I$

The real and imaginary parts are

$$\chi_R=\tan^{-1}\left(\frac{\tilde{T}_R}{\sqrt{\pi}-\tilde{T}_I}\right)$$

$$\chi_I=-\ln\sqrt{\left(\frac{1}{\sqrt{\pi}}\tilde{T}_R\right)^2+\left(1-\frac{1}{\sqrt{\pi}}\tilde{T}_I\right)^2}$$

Eikonal representation

Introducing eikonal formalism

$$\tilde{T}_R\left(s,\vec{b}\right) = \tilde{T}_R\left(s,\vec{b}\right) + i\tilde{T}_I\left(s,\vec{b}\right) \equiv i\sqrt{\pi}\left(1 - e^{i\chi(s,b)}\right)$$

with the complex eikonal function $\chi = \chi_R + i\chi_I$

The real and imaginary parts are

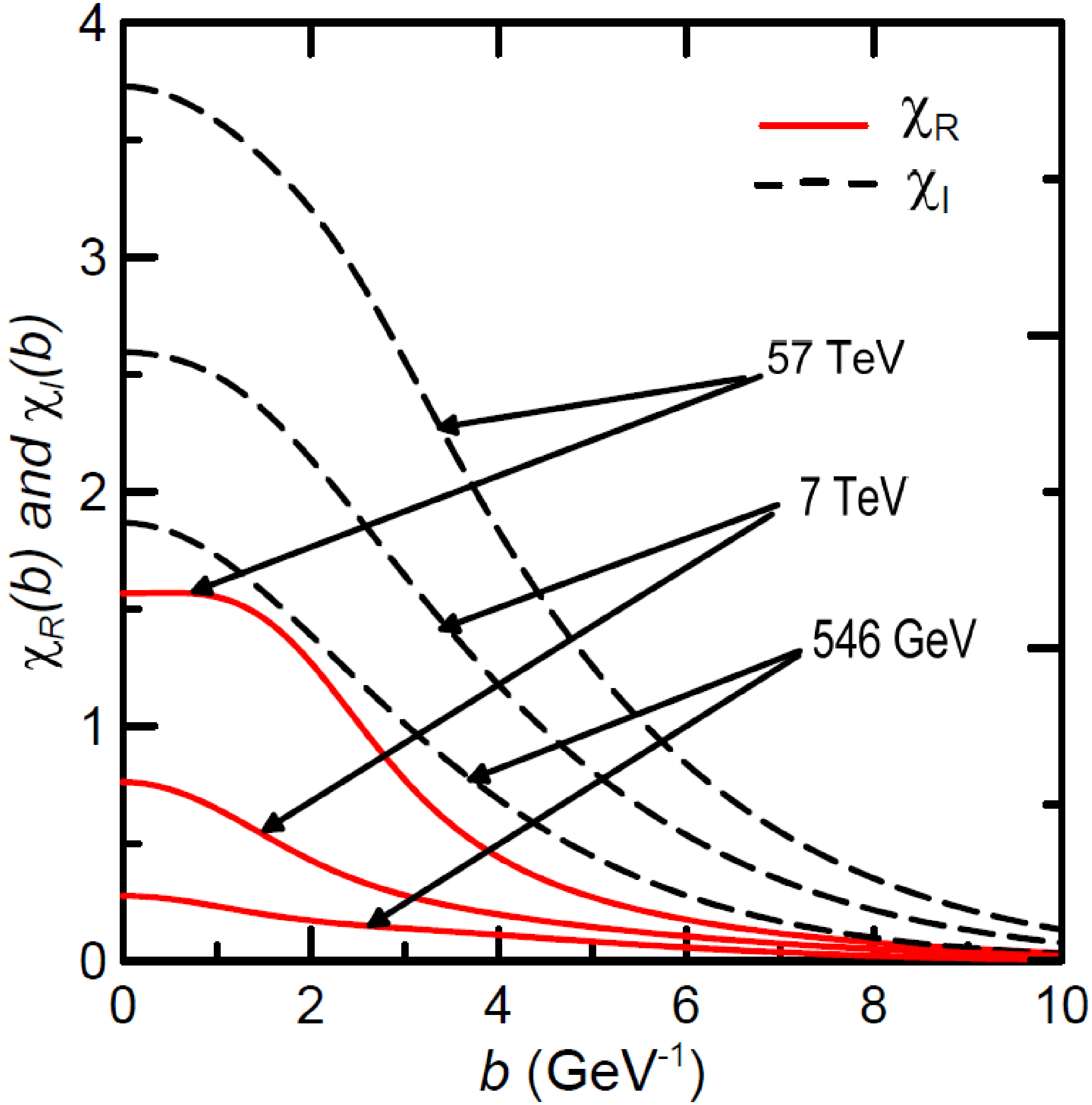
$$\chi_R = \tan^{-1}\left(\frac{\tilde{T}_R}{\sqrt{\pi} - \tilde{T}_I}\right)$$

$$\chi_I = -\ln\sqrt{\left(\frac{1}{\sqrt{\pi}}\tilde{T}_R\right)^2 + \left(1 - \frac{1}{\sqrt{\pi}}\tilde{T}_I\right)^2}$$

A pole is avoided if

$$\tilde{T}_I(s,b) < \sqrt{\pi}$$

Eikonal representation



We write elastic, total and inelastic differential cross sections in b space

$$\sigma_{el} = \int d^2\vec{b}\, \frac{d\bar{\sigma}_{el}}{d^2\vec{b}}\ , \qquad \sigma_{Tot} = \int d^2\vec{b}\, \frac{d\bar{\sigma}_{Tot}}{d^2\vec{b}}\ , \quad \sigma_{inel} = \int d^2\vec{b}\, \frac{d\bar{\sigma}_{inel}}{d^2\vec{b}}$$

and identify adimensional differential cross sections

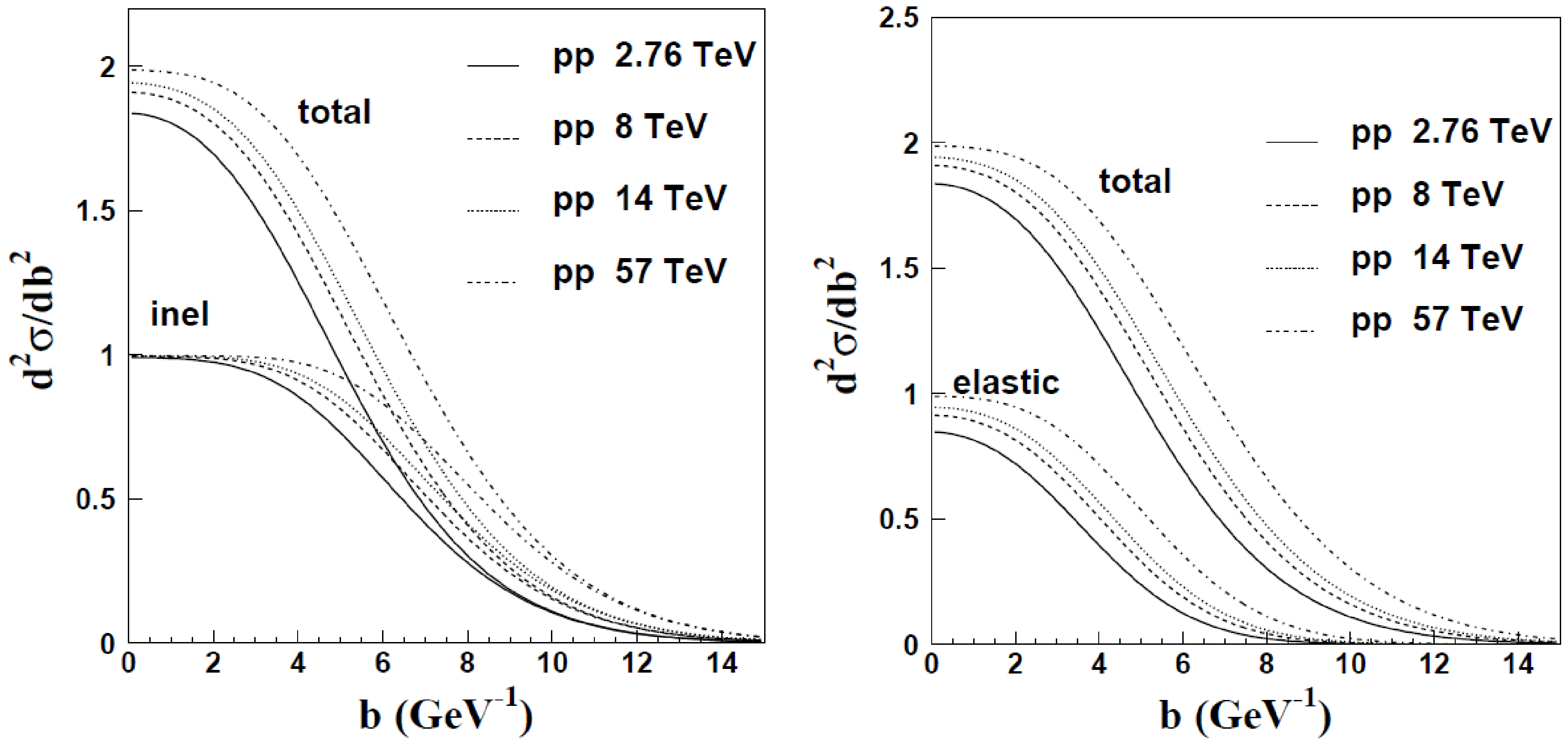
$$\frac{d\bar{\sigma}_{el}}{d^2\vec{b}} = 1 - 2\cos(\chi_R)\, e^{-\chi_I} + e^{-2\chi_I} = \frac{1}{\pi}|\tilde{T}(s,b)|^2$$

$$\frac{d\bar{\sigma}_{Tot}}{d^2\vec{b}} = 2\left\{1 - \cos(\chi_R)\, e^{-\chi_I}\right\} = \frac{2}{\sqrt{\pi}}\tilde{T}_I(s,b)$$

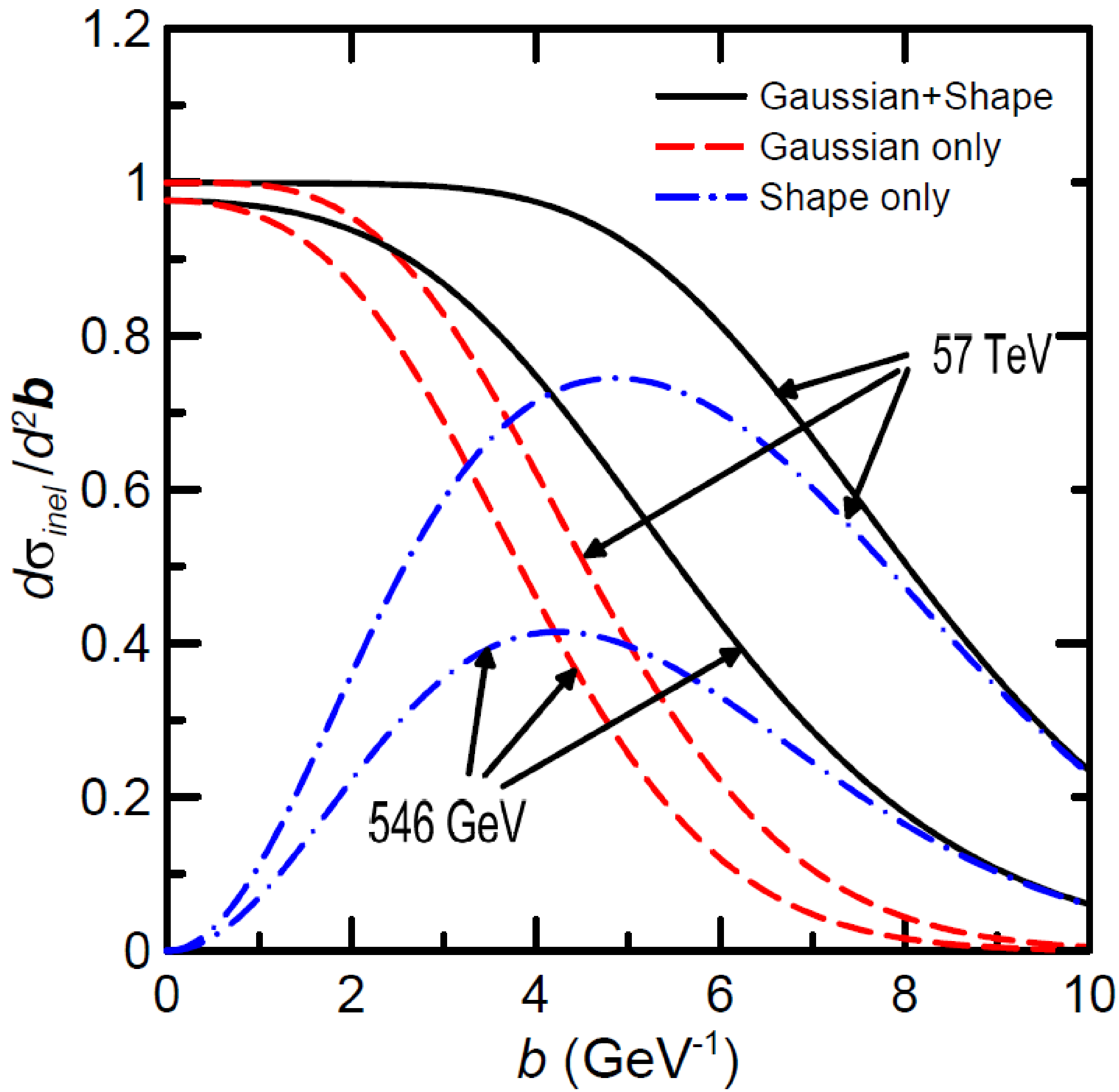
Inelasticity

$$\frac{d\bar{\sigma}_{inel}}{d^2\vec{b}} = 1 - e^{-2\chi_I} = \frac{2}{\sqrt{\pi}}\tilde{T}_I(s,\vec{b}) - \frac{1}{\pi}|\tilde{T}(\vec{b},s)|^2 \equiv G(s,\vec{b})$$

Differential cross sections in b space



Inelastic cross sections



Unitarity

Unitarity condition $$\left| e^{i\chi(s,b)} \right| \leq 1$$

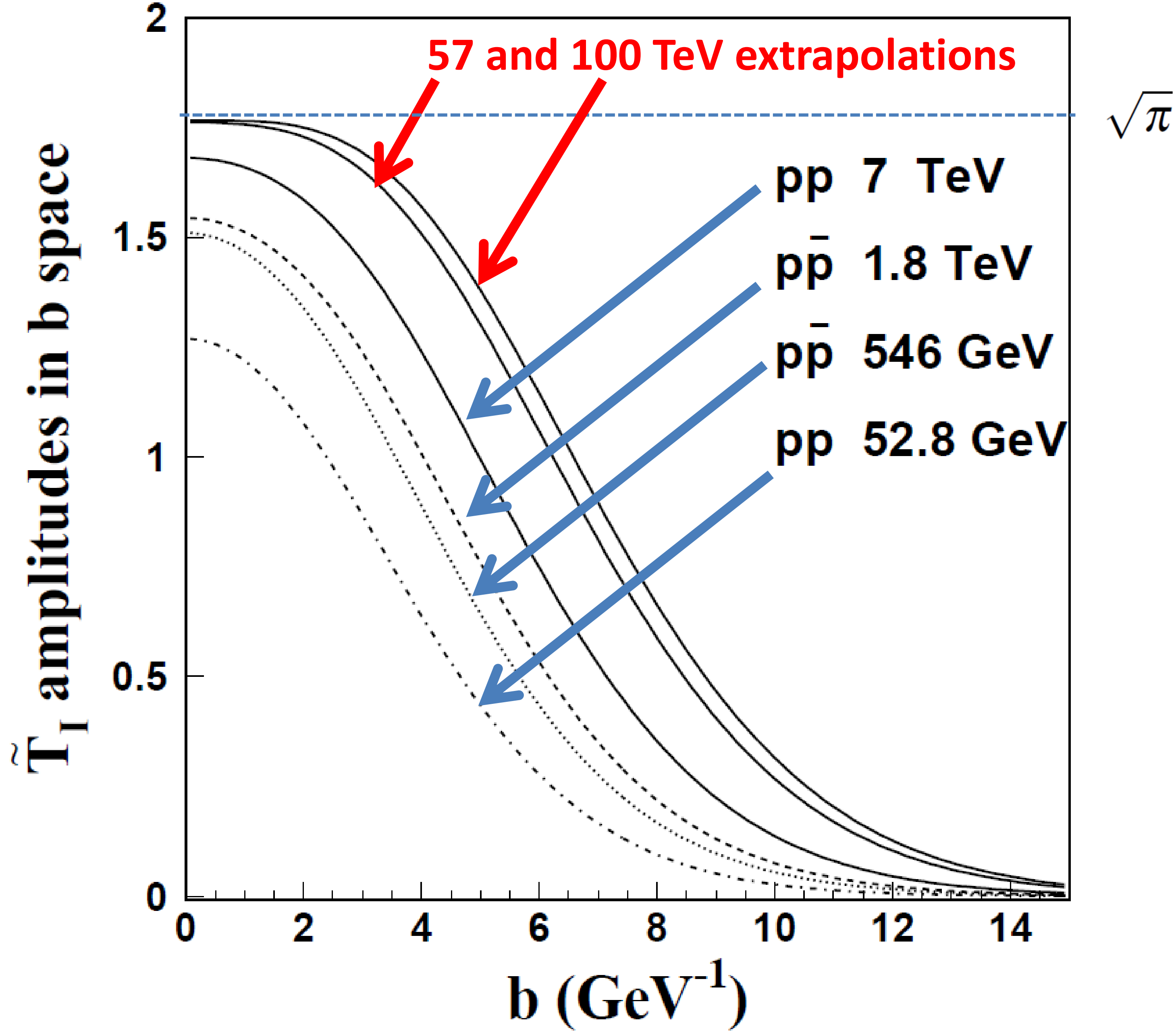
is equivalente to $$\chi_I(s,b) \geq 0$$

In terms of imaginary amplitude

$$\frac{1}{\sqrt{\pi}} T_I(s,b) = 1 - \cos(\chi_R) e^{-\chi_I} \leq 1$$

for all s and b.

These condition are satisfied in our representation.



57 and 100 TeV extrapolations
$\sqrt{\pi}$
pp 7 TeV
p$\bar{p}$ 1.8 TeV
p$\bar{p}$ 546 GeV
pp 52.8 GeV
$\tilde{T}_I$ amplitudes in b space
b (GeV^{-1})
0
0.5
1
1.5
2
0
2
4
6
8
10
12
14

Slopes and total cross section

We define the average radius of interaction

$$\langle b^2 \rangle_{\rm tot} = \left(\int b^2 \frac{1}{(\hbar c)^2} \frac{d\sigma_{tot}}{d^2\vec{b}} d^2\vec{b} \right) \Bigg/ \left(\int \frac{1}{(\hbar c)^2} \frac{d\sigma_{tot}}{d^2\vec{b}} d^2\vec{b} \right)$$

From the definition of the imaginary slope

$$B_I(s) = \frac{1}{2} \langle b^2 \rangle_{\rm tot}$$

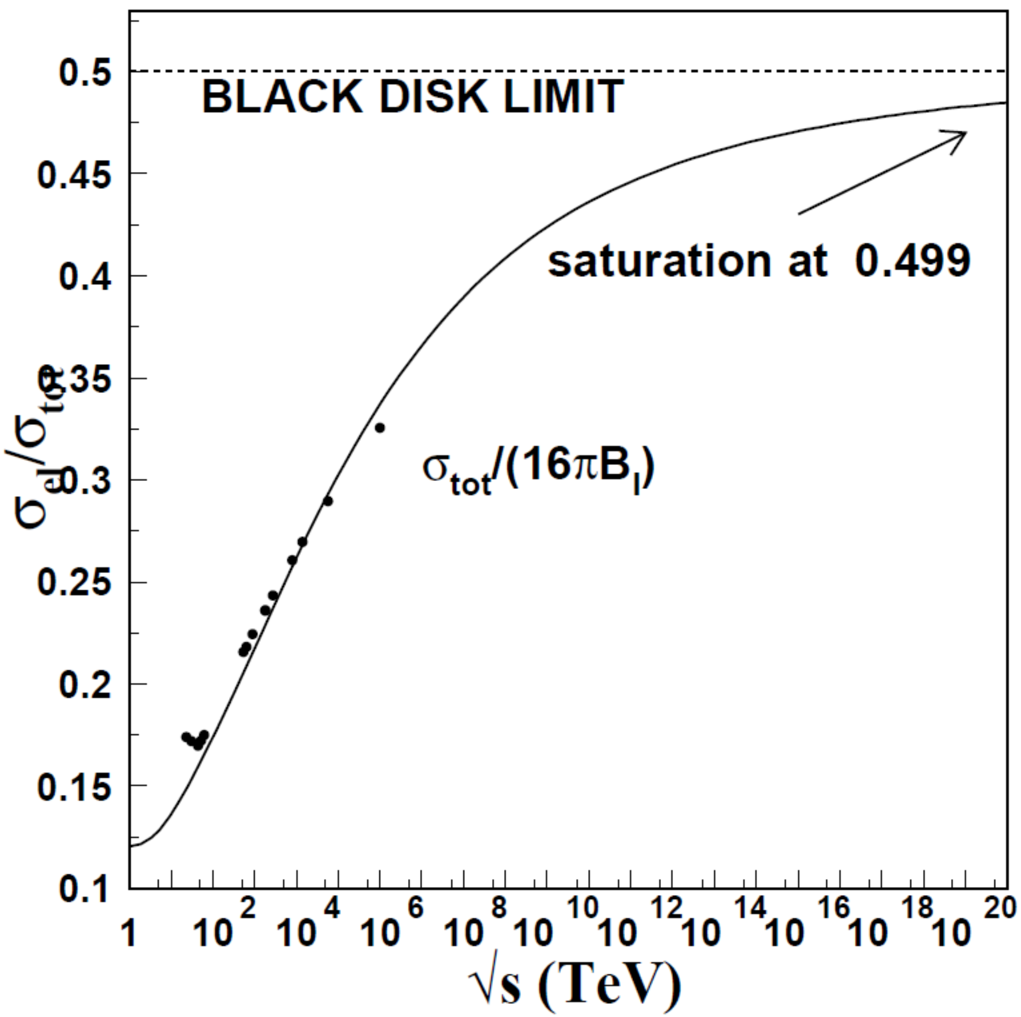
As $\sigma_{\rm tot}(\sqrt{s}) \sim \langle b^2 \rangle_{\rm tot}$

It follows that $B_I(s) \sim \log^2 \sqrt{s}$

Relations between cross sections and slope

$$\frac{1}{16\pi}\frac{\sigma_{\mathrm{tot}}}{B_I(s)} \approx \frac{\sigma_{elas}}{\sigma_{tot}}$$

D. A. Fagundes, M. J. Menon and P. V. R. G. Silva
J. Phys. G: Nucl. Part. Phys. **40**, 065005 (2013)

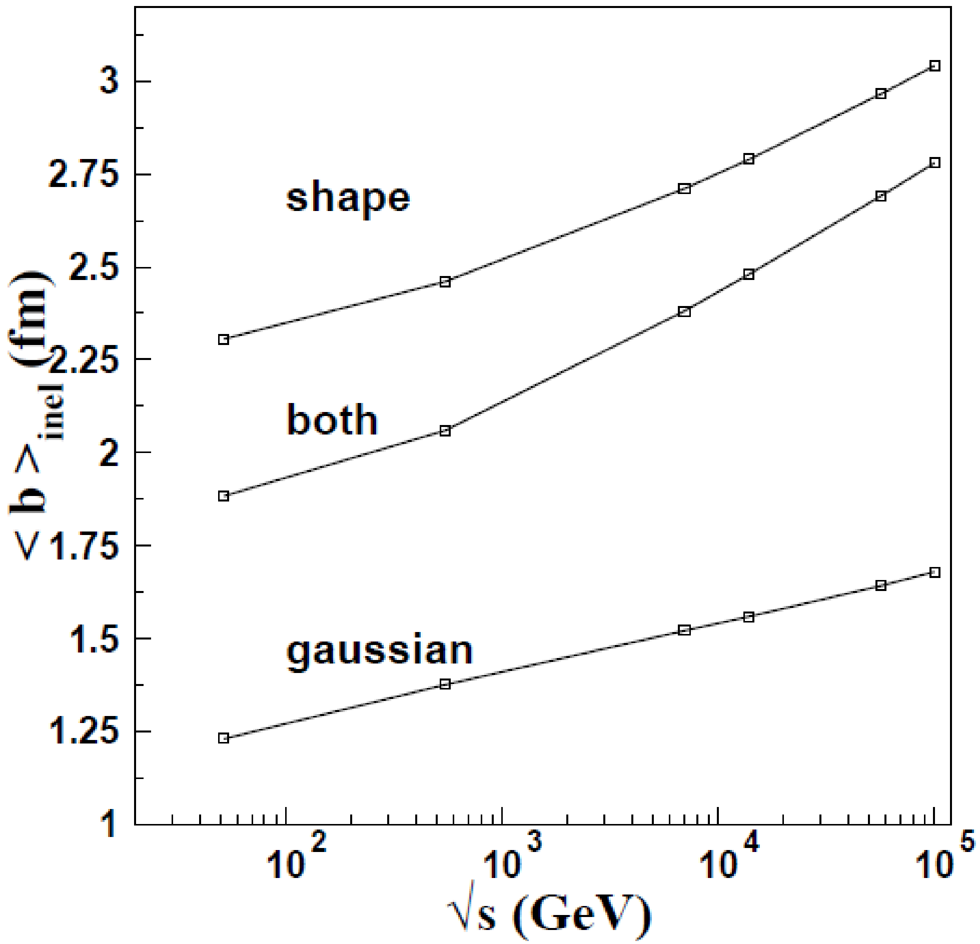


Thanks

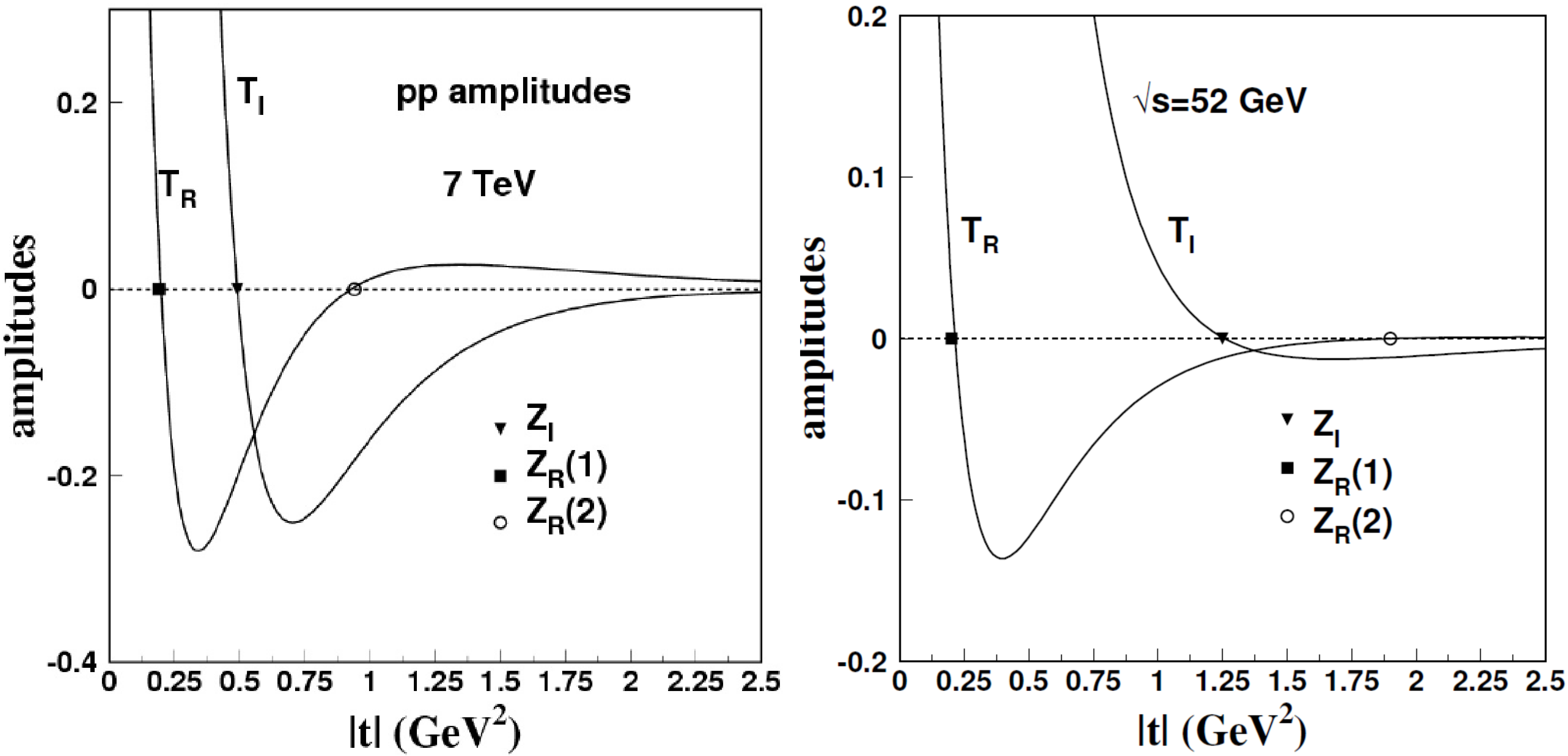
Interaction Range

Average radius of interaction for the inelastic channels

$$\langle b \rangle = \frac{1}{N}\int b \frac{1}{(\hbar c)^2}\frac{d\sigma_{inel}}{d^2\vec{b}} d^2\vec{b} = \frac{1}{N}\int b\; G(s,b) d^2\vec{b}$$



comparison of pp 7 TeV with 0.0528 TeV amplitudes



Prediction for the large |t| region

The amplitudes for the data up to |t| = 2.5 GeV² are

$$T_K^N(s,t) = \alpha_K(s)\mathrm{e}^{-\beta_K(s)|t|} + \lambda_K(s)\Psi_K(\gamma_K(s),t)$$ Nuclear amplitudes

$$\Psi_K(\gamma_K(s),t) = 2\ \mathrm{e}^{\gamma_K}\left[\frac{\mathrm{e}^{-\gamma_K\sqrt{1+a_0|t|}}}{\sqrt{1+a_0|t|}} - \mathrm{e}^{\gamma_K}\ \frac{e^{-\gamma_K\sqrt{4+a_0|t|}}}{\sqrt{4+a_0|t|}}\right]$$ Shape functions

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For large |t| we add a perturbative tri-gluon exchange Rggg

A. Donnachie, P. V. Landshoff, *Zeit. Phys. C* **2**, 55 (1979)

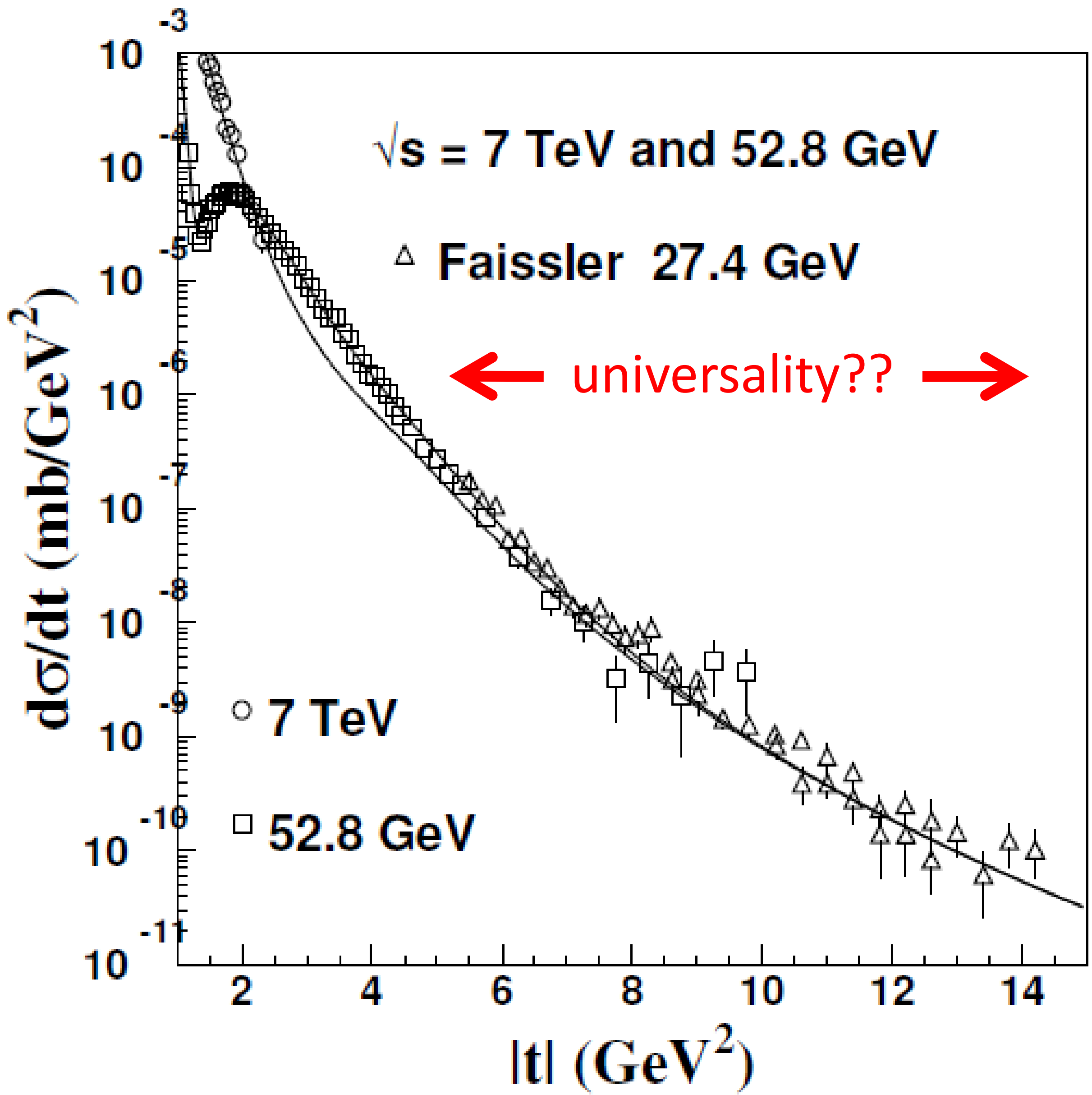
$$T_{R(\mathrm{tail})}(s,t) = T_K^N(s,t) + R_{ggg}(t)$$

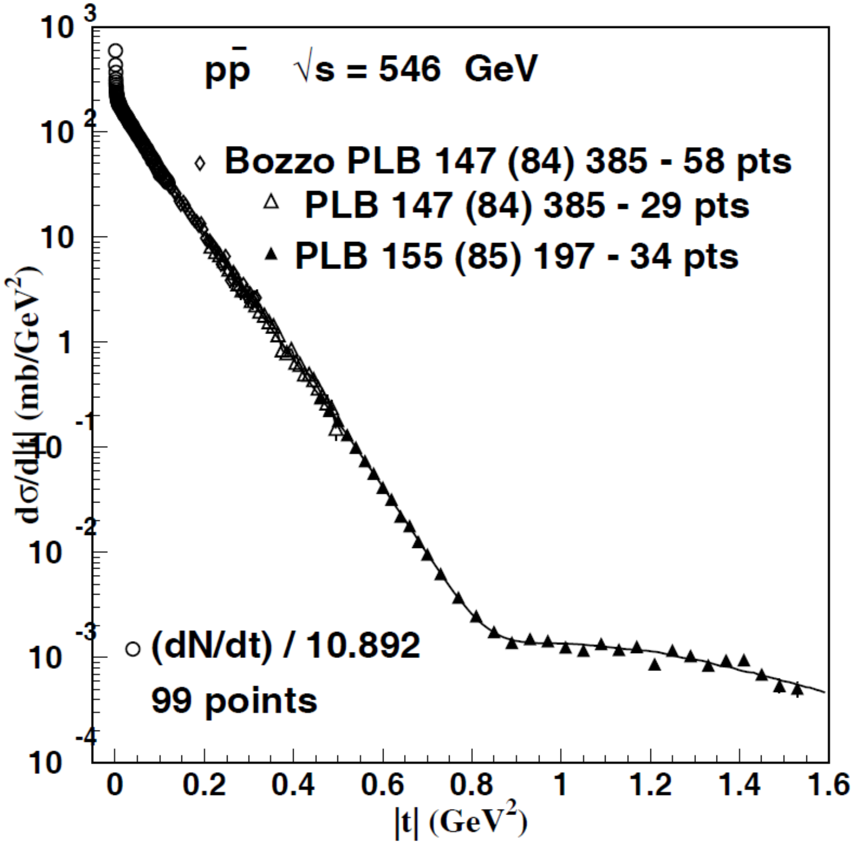
$$R_{ggg}(t) \equiv \pm 0.45\ t^{-4}(1-e^{-0.005|t|^4})(1-e^{-0.1|t|^2})$$

For large |t|

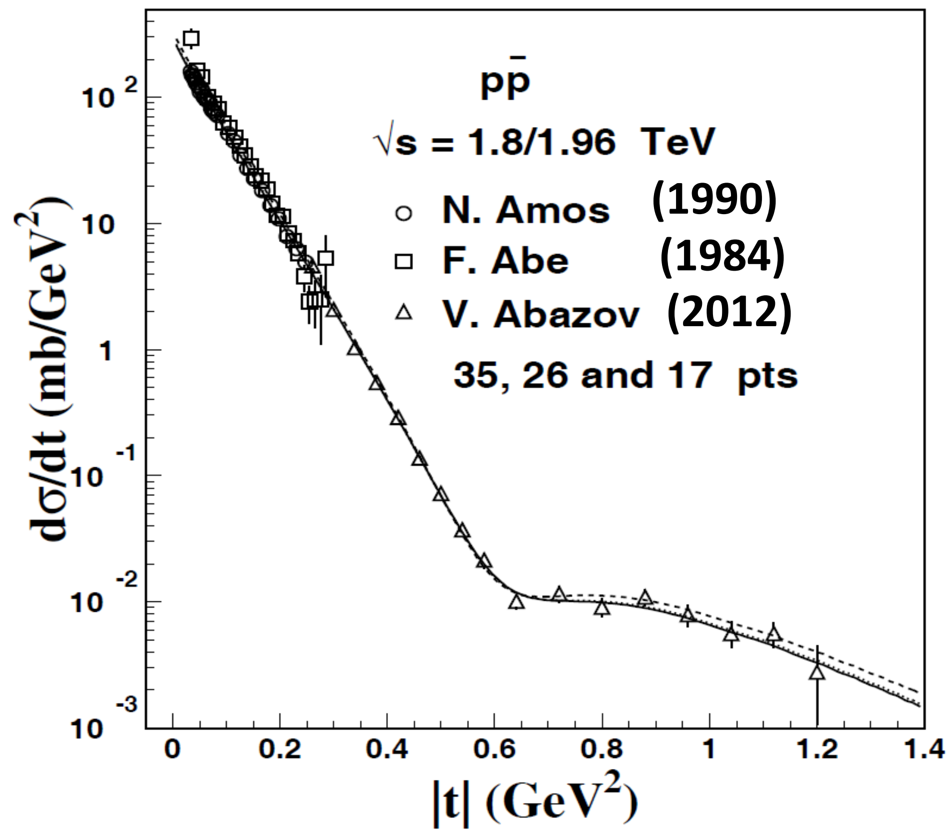
$$d\sigma/dt \approx (\hbar c)^2\ [T_{R(\mathrm{tail})}(t)]^2 \approx 0.08\ t^{-8}\quad (\mathrm{mb/GeV^2})$$ energy independent !!!!

Large t region (compare with 52.8 GeV)



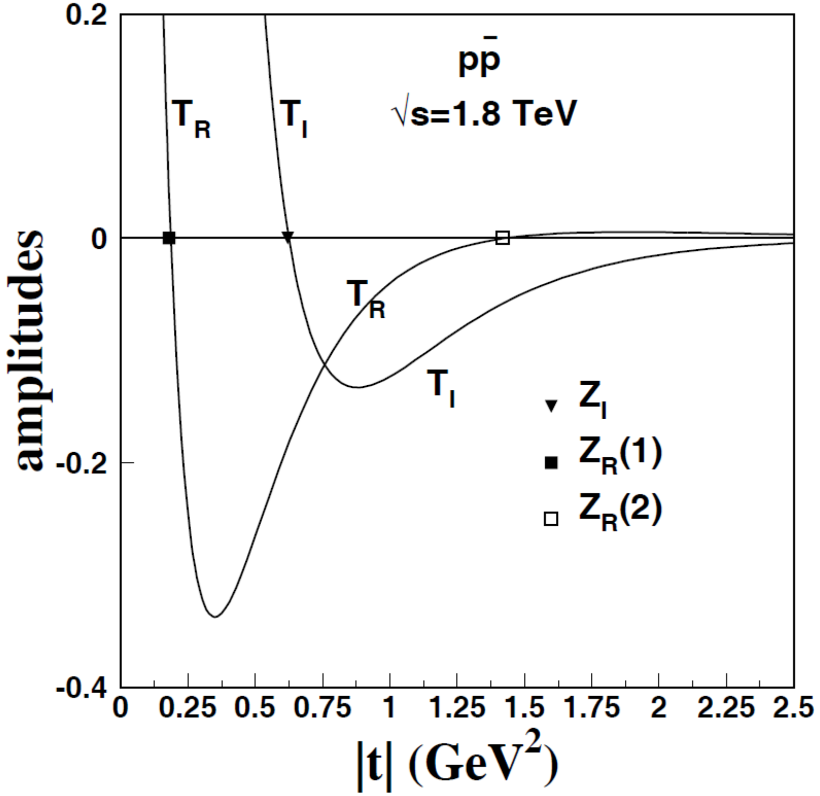


pp̄ √s = 546 GeV
Bozzo PLB 147 (84) 385 - 58 pts
PLB 147 (84) 385 - 29 pts
PLB 155 (85) 197 - 34 pts
(dN/dt) / 10.892
99 points
dσ/d|t| (mb/GeV²)
|t| (GeV²)

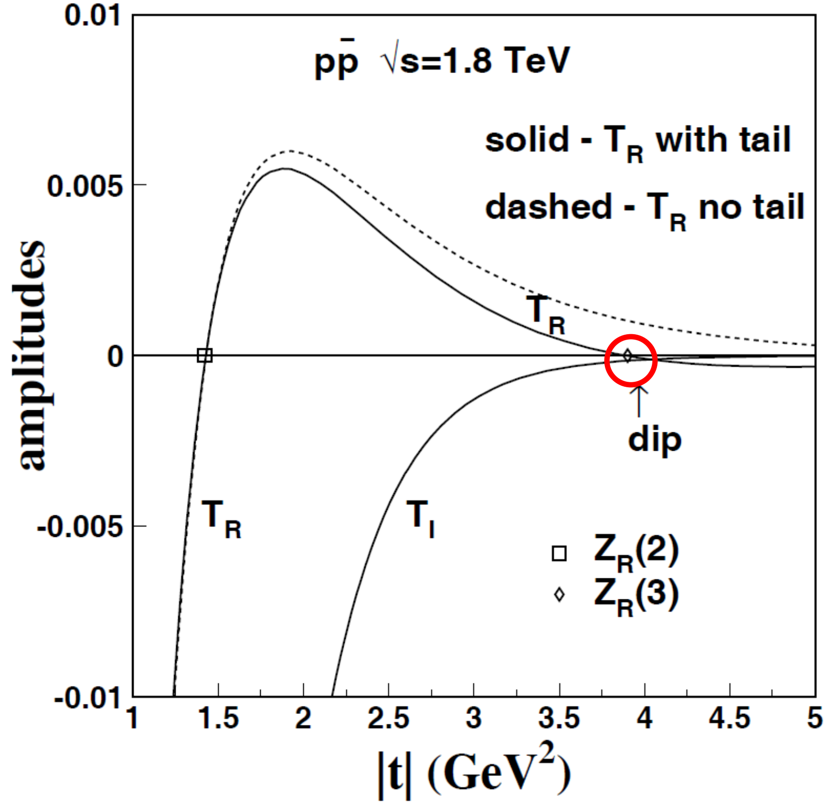


pp̄
√s = 1.8/1.96 TeV
N. Amos (1990)
F. Abe (1984)
V. Abazov (2012)
35, 26 and 17 pts
dσ/dt (mb/GeV²)
|t| (GeV²)

Amplitudes



0.2
0
-0.2
-0.4
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
amplitudes
|t| (GeV²)
pp̄
√s=1.8 TeV
T_R
T_I
T_R
T_I
Z_I
Z_R(1)
Z_R(2)



0.01
0.005
0
-0.005
-0.01
1
1.5
2
2.5
3
3.5
4
4.5
5
amplitudes
|t| (GeV²)
pp̄ √s=1.8 TeV
solid - T_R with tail
dashed - T_R no tail
T_R
T_R
T_I
dip
Z_R(2)
Z_R(3)