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HYDRODYNAMIC BEHAVIORS INDUCED BY MICROSCOPIC DYNAMICS IN RELATIVISTIC HEAVY-ION COLLISIONS

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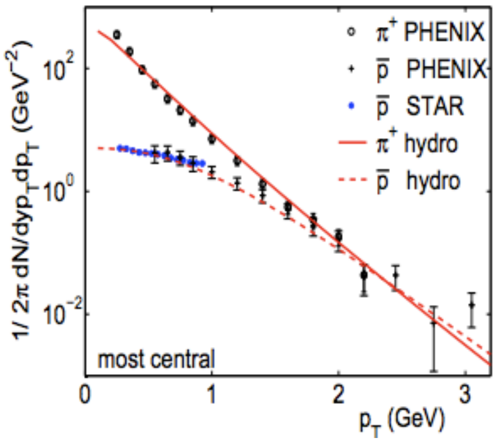
Outline

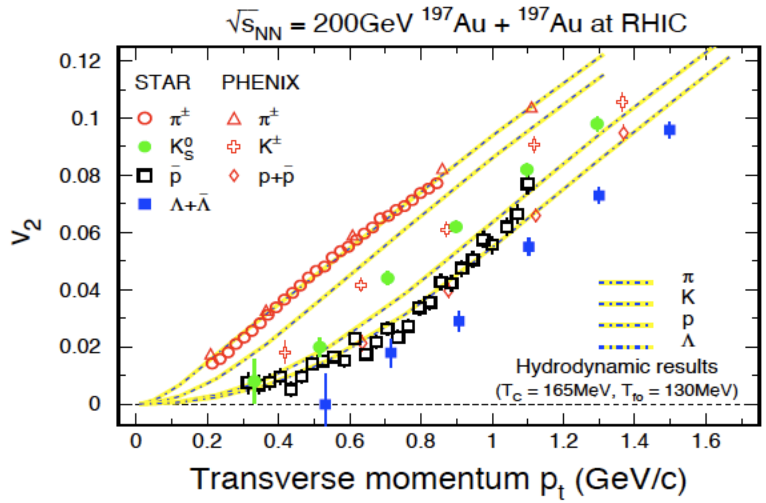
- Introduction
- Coarse graining scale of hydrodynamic modeling
- Non-equilibrium dynamics – PHSD model
- Evaluating the system evolution
- Preliminary results
- Summary

INTRODUCTION

- Success of hydrodynamics in describing the experimental observations in heavy-ion collisions



P. Kolb and U. Heinz, arXiv:nucl-th/0305084



Oldenburg M.D. (STAR Collab.), J. Phys. G 31, S437

Ideal hydro:

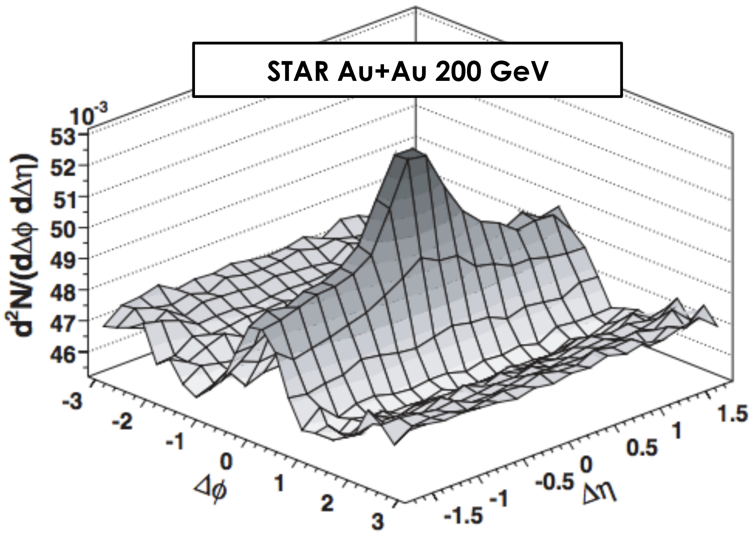
- local thermal equilibrium
- conservation laws + equation of state

collective flow:

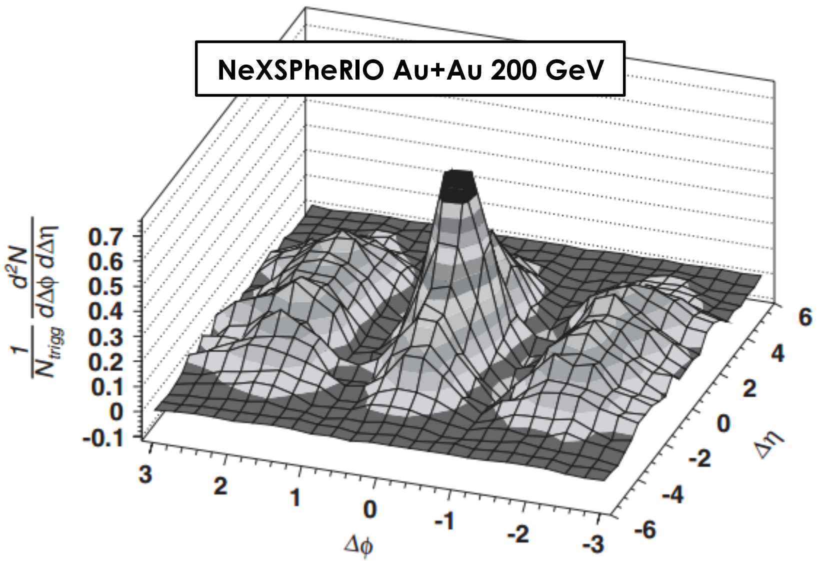
- hydro models can reproduce the anisotropic momentum distribution of the final particles
- the system behaves collectively (like a strongly interacting liquid)

INTRODUCTION

- Success of hydrodynamics in describing the experimental observations in heavy-ion collisions



STAR Collab., Phys. Rev. C, 80, 064912



Takahashi J., *et al.*, Phys. Rev. Lett. 103, 242301

Event-by-event hydrodynamics:

- 2-particle correlation analysis considering inhomogeneous initial condition + hydro evolution reproduces the ridge structure observed experimentally

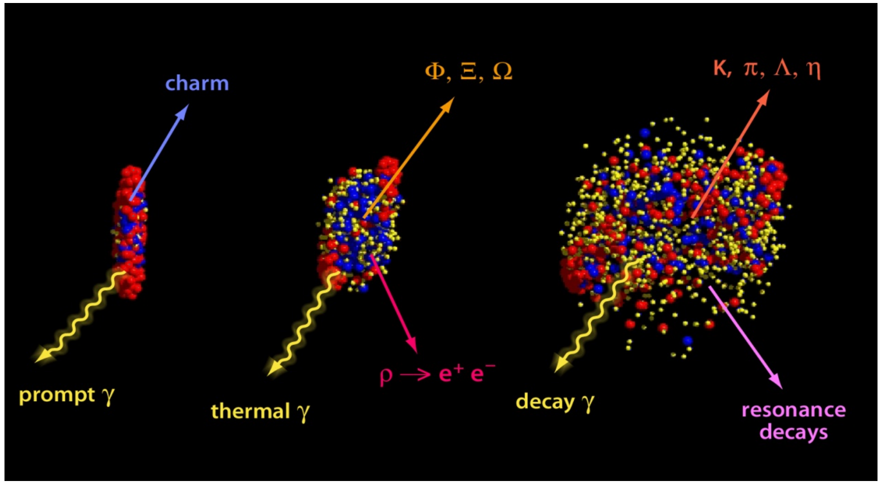
Coarse Graining Scale of Hydrodynamic Modeling

> *"… why at all the hydrodynamic approach works so well for such a violent and almost microscopic collisional process?"*
>
> (Ph. Mota, *et al*, Eur. Phys. J A, 48, 165)

- Basic ideas:
  - The larger the coarse graining scale, larger the probability of success of hydrodynamic modeling
  - On the other hand, in order to probe small inhomogeneities of a given initial condition, a small coarse graining scale is required
  - The most important issue is that we do not know *a priori* the coarse graining scale for the hydrodynamic modeling

NON-EQUILIBRIUM DYNAMICS – PHSD MODEL

- Parton-Hadron-String Dynamics



W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919;
NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3
A. Peshier, W. Cassing, PRL 94 (2005) 172301;
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons

NON-EQUILIBRIUM DYNAMICS – PHSD MODEL

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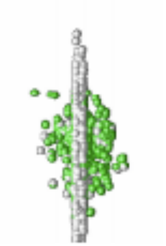
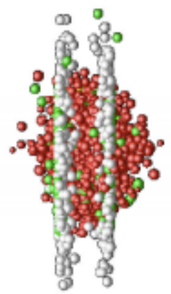


- Initial A+A collisions:

LUND string model

  - ✓ *String formation in primary NN collisions*
  - ✓ *String decay to pre-hadrons (B – baryons; m – mesons)*



- Formation of QGP phase: $\varepsilon > \varepsilon_{critical}$

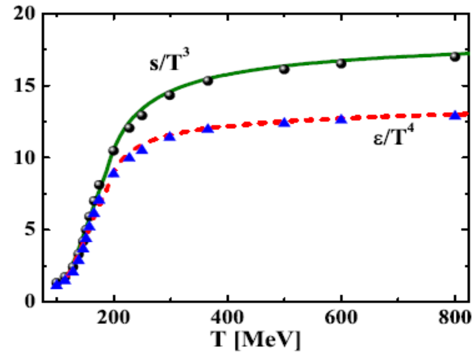
  - ✓ *dissolution of pre-hadrons into massive colored quarks + mean field energy*

$$B \rightarrow qqq, \;\; m \rightarrow q\bar{q} \quad \forall \quad U_q$$

  - ✓ ***Dynamical QuasiParticle Model (DQPM)***

(defines quark spectral functions, i.e. masses $M_q(\varepsilon)$ and widths $\Gamma_q(\varepsilon)$ + mean field potential at a given ε (local energy density))

(ε related by lQCD EoS to *T* in the local cell)

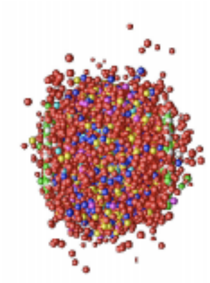


W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; EPJ ST 168 (2009) 3; NPA856 (2011) 162.

NON-EQUILIBRIUM DYNAMICS – PHSD MODEL

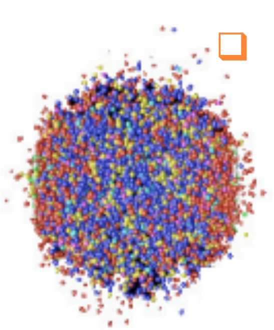
- Parton-Hadron-String Dynamics

  - Partonic phase – QGP:



    - ✓ *quarks and gluons = dynamical quasi-particles with off-shell spectral functions (width, mass) defined by DQPM*
    - ✓ *self generated mean field potential for quarks and gluons U_q, U_g from DQPM*
    - ✓ ***EoS of partonic phase**: crossover from lattice QCD (fitted by DQPM)*
    - ✓ *(quasi-) elastic and inelastic parton-parton interactions using effective cross-section from the DQPM*

  - Hadronization:

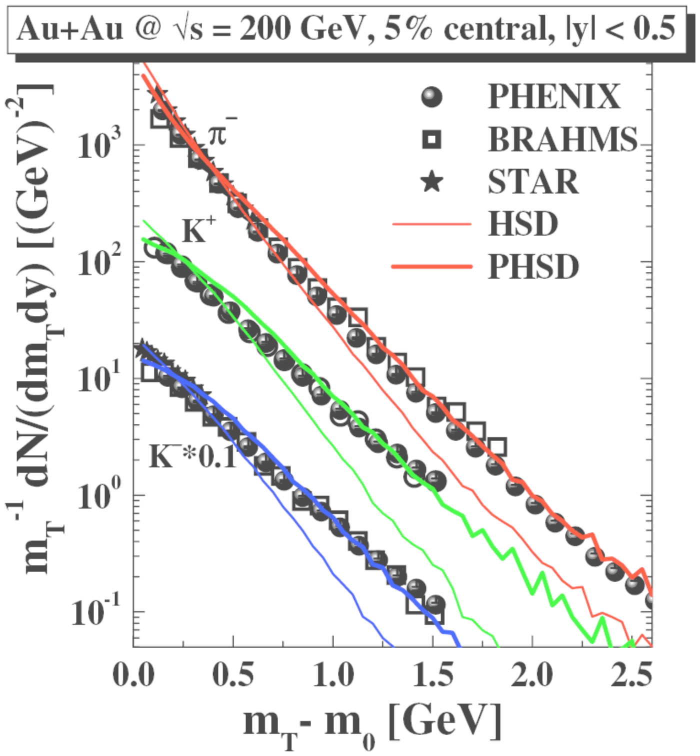


    - ✓ *massive, off-shell (anti-)quarks with broad spectral functions hadronize to off-shell mesons and baryons or color neutral excited states – 'strings'*

$$g \to q + \bar{q}, \qquad q + \bar{q} \leftrightarrow meson \ ('string')$$
$$q + q + q \leftrightarrow baryon \ ('string')$$

Non-equilibrium dynamics – PHSD model

- Parton-Hadron-String Dynamics



W. Cassing & E. Bratkovskaya, NPA 831 (2009) 215

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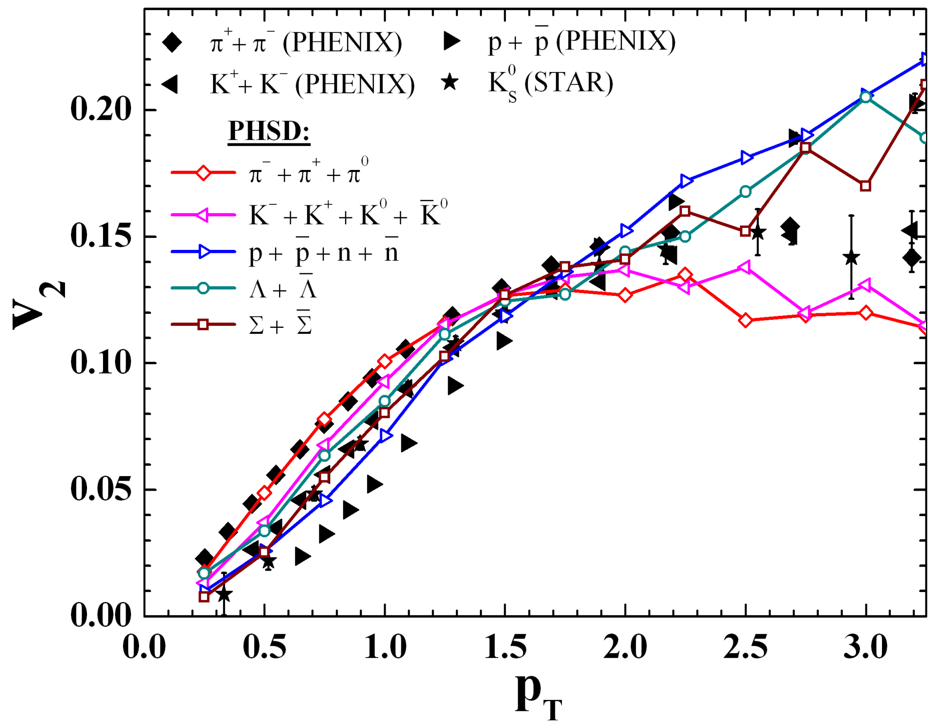

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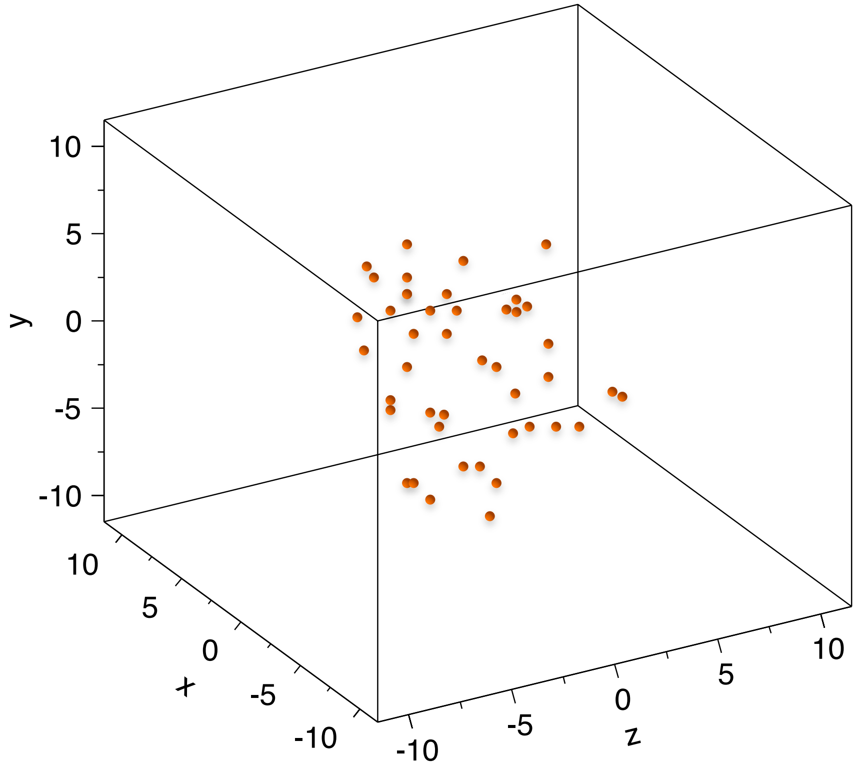
Evaluating the system evolution in PHSD

$$T^{\mu\nu} = \sum_i \left(\frac{P_i^\mu P_i^\nu}{P_i^0} \right) \delta^3(\vec{r} - \vec{r}_i)$$



EVALUATING THE SYSTEM EVOLUTION IN PHSD

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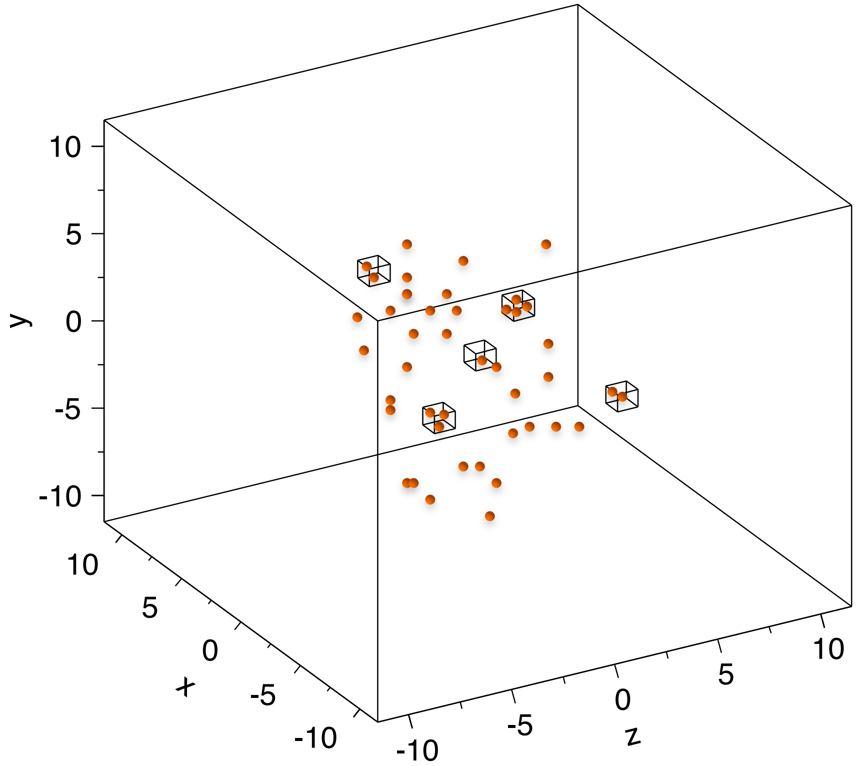
$$\delta^3(\vec{r} - \vec{r}_i) \to W(|\vec{r} - \vec{r}_i|; \Delta_x)$$



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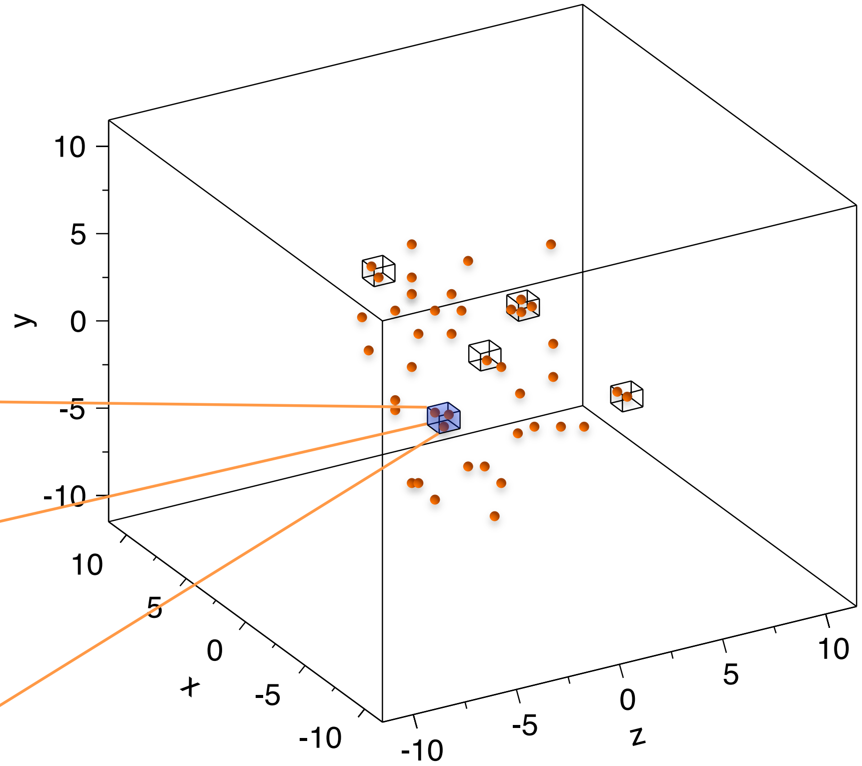
$$T^{\mu\nu} \to \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$



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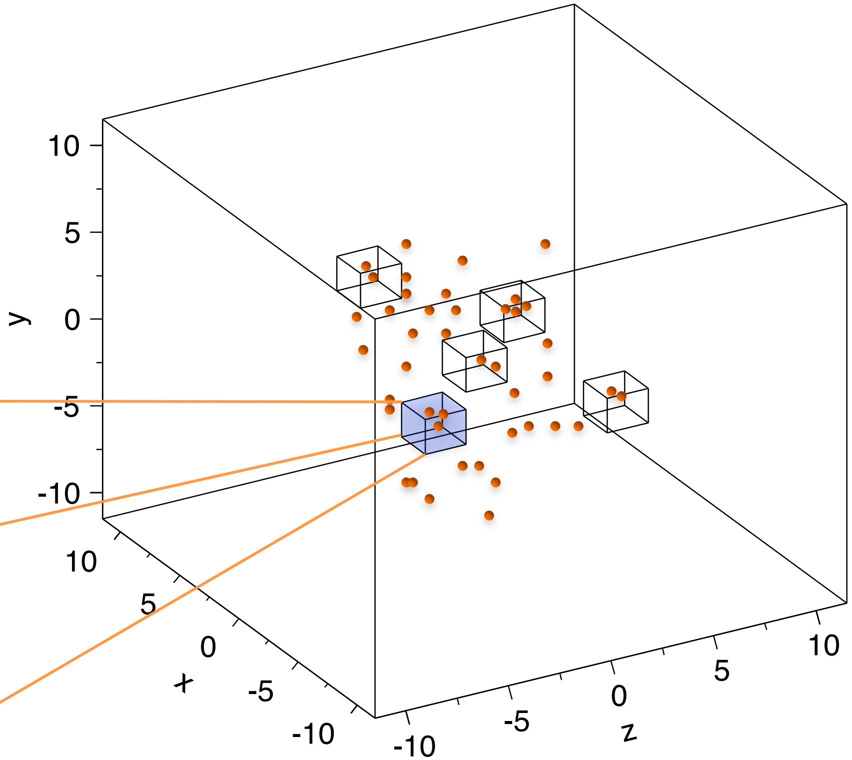

Evaluating the coarse graining scale:

✧ *Varying Δx and Δy (currently Δz is fixed)*

✧ *Varying Δt (over 'n' time steps of the evolution)*

✧ *Varying number of parallel events used to calculate the mean field (parameter NUM in the PHSD code)*

Evaluating the system evolution in PHSD

- Diagonalizing the energy-momentum tensor

(solve the eigenvalue/eigenvector problem)

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix} \longrightarrow \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P_1 & 0 & 0 \\ 0 & 0 & P_2 & 0 \\ 0 & 0 & 0 & P_3 \end{pmatrix}$$

  - The four velocity can be identified with the eigenvector associated with the eigenvalue ε

    $T^{\mu\nu} u_\nu = \varepsilon u^\mu$ *(time-like eigenvector)*

  - If the system is in local thermal equilibrium:

    $P_1 = P_2 = P_3$ *(isotropic pressure)*

Extracting the hydrodynamic properties of the system evolution

- Once the four-velocity is known, one can extract the hydrodynamic quantities from the energy-momentum tensor:

$$T^{\mu\nu} = \varepsilon \Delta_{\parallel}^{\mu\nu} - (P + \zeta)\, \Delta_{\perp}^{\mu\nu} + \Pi^{\mu\nu}$$

with $\begin{cases} \Delta_{\parallel}^{\mu\nu} = u^\mu u^\nu \\ \Delta_{\perp}^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \end{cases}$

$$\Rightarrow \begin{cases} \varepsilon = \Delta_{\parallel}^{\alpha\beta} T_{\alpha\beta} \\ (P + \zeta) = -\frac{1}{3} \Delta_{\perp}^{\alpha\beta} T_{\alpha\beta} \\ \Pi^{\mu\nu} = \Delta_{\perp}^{\mu\alpha} \Delta_{\perp}^{\nu\beta} T_{\alpha\beta} - \frac{1}{3} \Delta_{\perp}^{\alpha\beta} T_{\alpha\beta} \Delta_{\perp}^{\mu\nu} \end{cases}$$

EXTRACTING THE HYDRODYNAMIC PROPERTIES OF THE SYSTEM EVOLUTION

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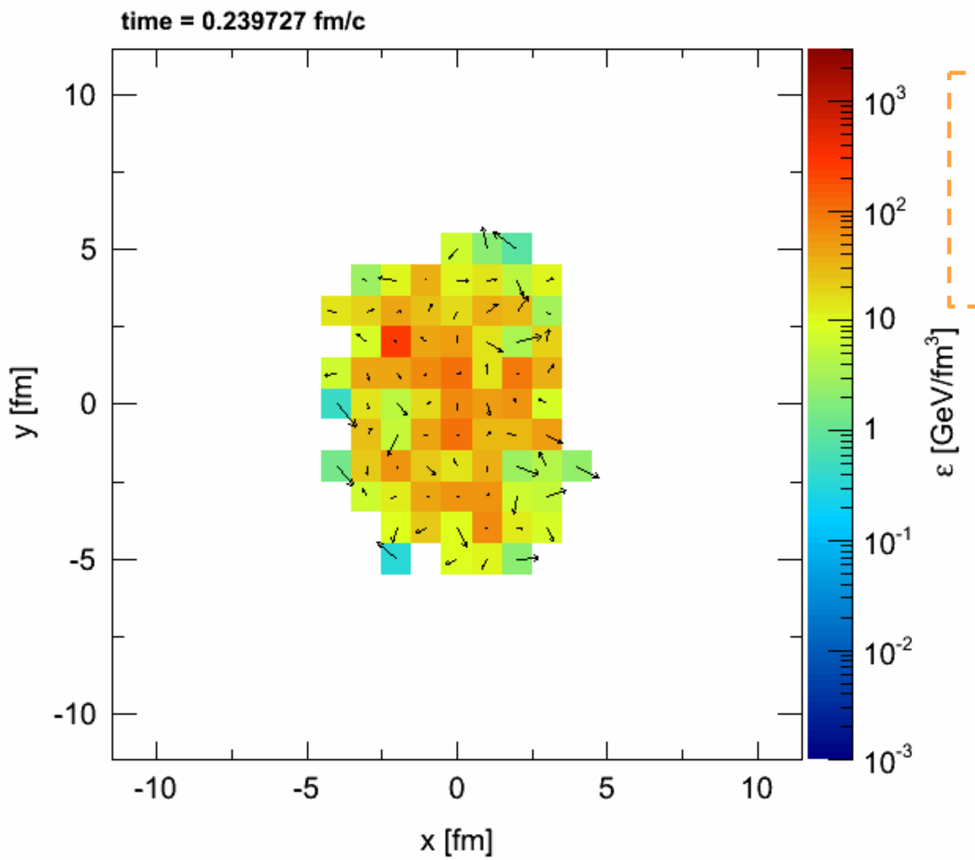
?

EXTRACTING THE HYDRODYNAMIC PROPERTIES OF THE SYSTEM EVOLUTION

- **Preliminary results:** (Au+Au collision at 200 GeV)
  - *Impact parameter: b = 7 fm*
  - *parallel events to calculate partonic mean field potential: NUM = 10*



Transverse profile (z=0) of the local energy density distribution of each cell in the grid

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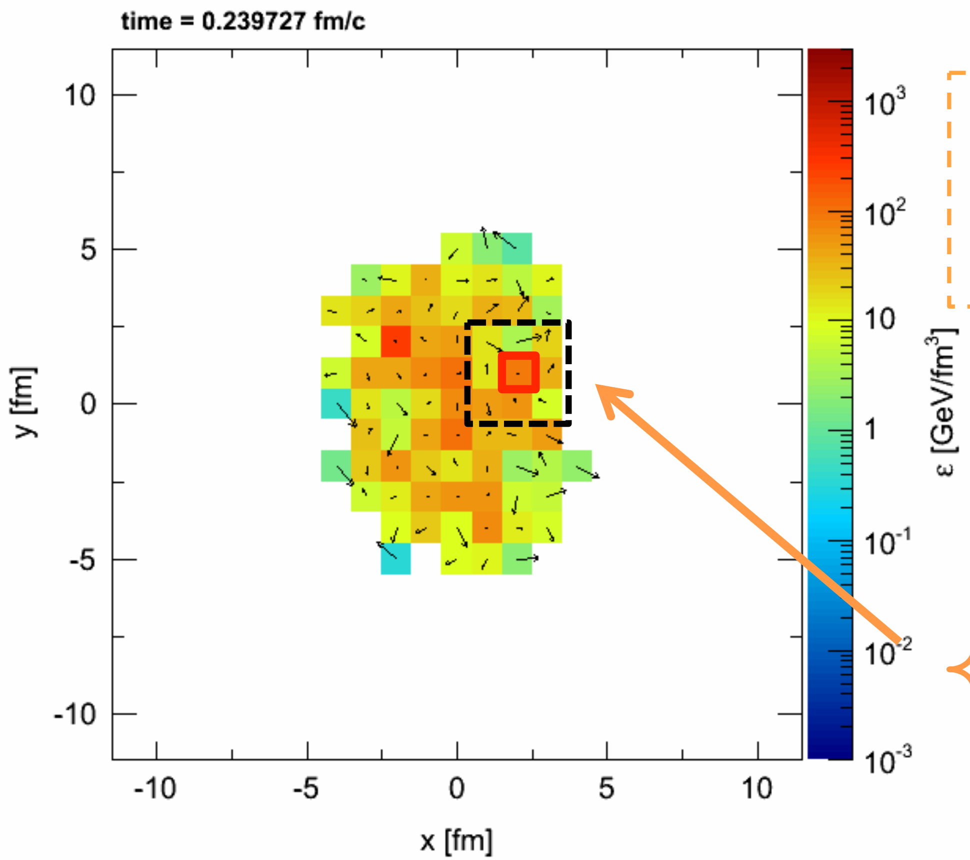

Transverse profile (z=0) of the local energy density distribution of each cell in the grid

Initial idea to evaluate the coarse graining scale:

- *choose one point (x,y) in the transverse plane*
- *change the "size" of the neighborhood used to calculate the energy momentum tensor*

EXTRACTING THE HYDRODYNAMIC PROPERTIES OF THE SYSTEM EVOLUTION

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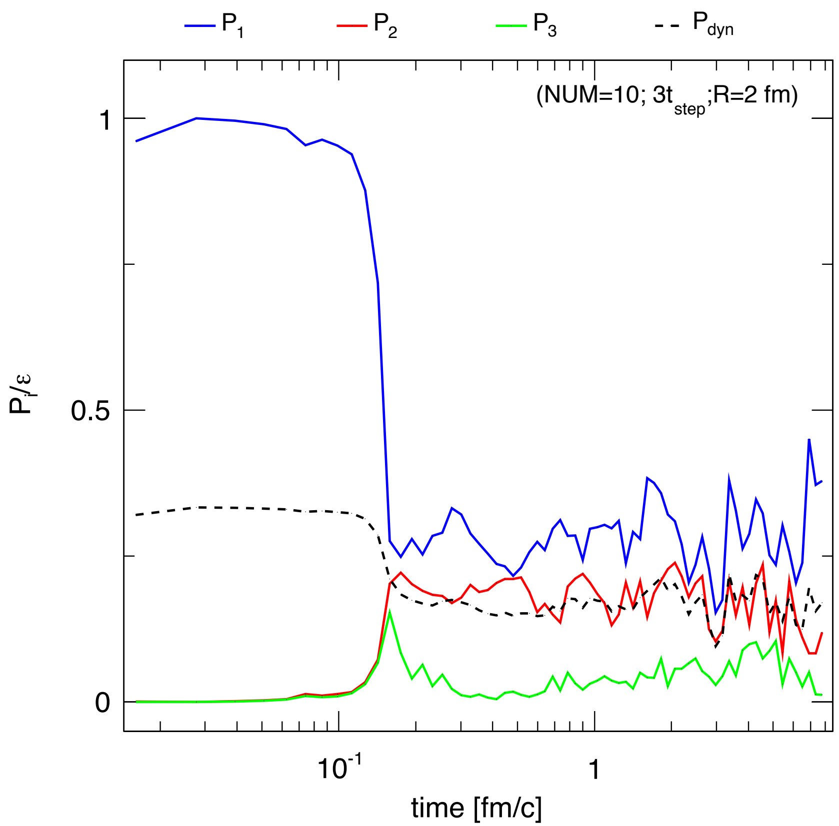

$$P_1 \neq P_2 \neq P_3$$

system evolution does not seem to reach equilibrium!

Extracting the hydrodynamic properties of the system evolution

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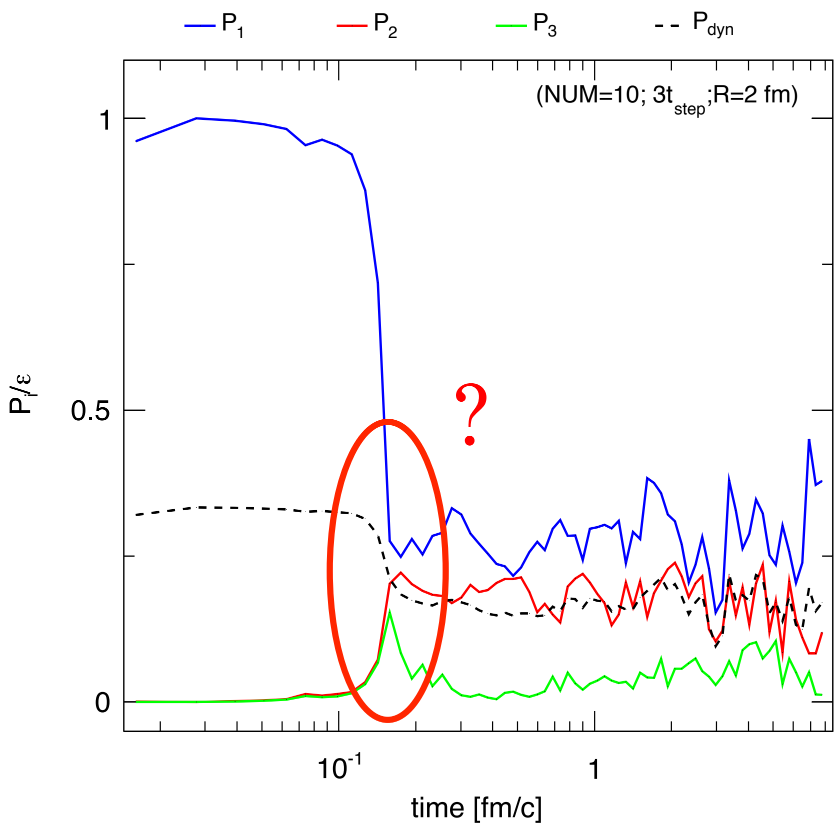

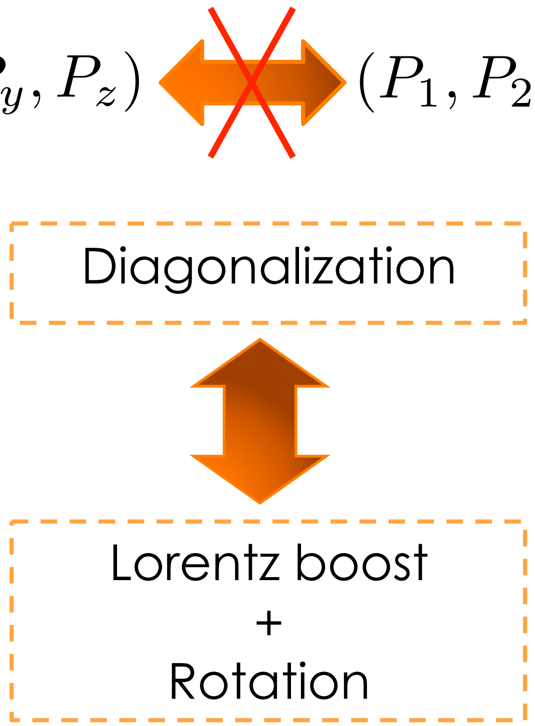

(P_x, P_y, P_z) (P_1, P_2, P_3)

Extracting the hydrodynamic properties of the system evolution

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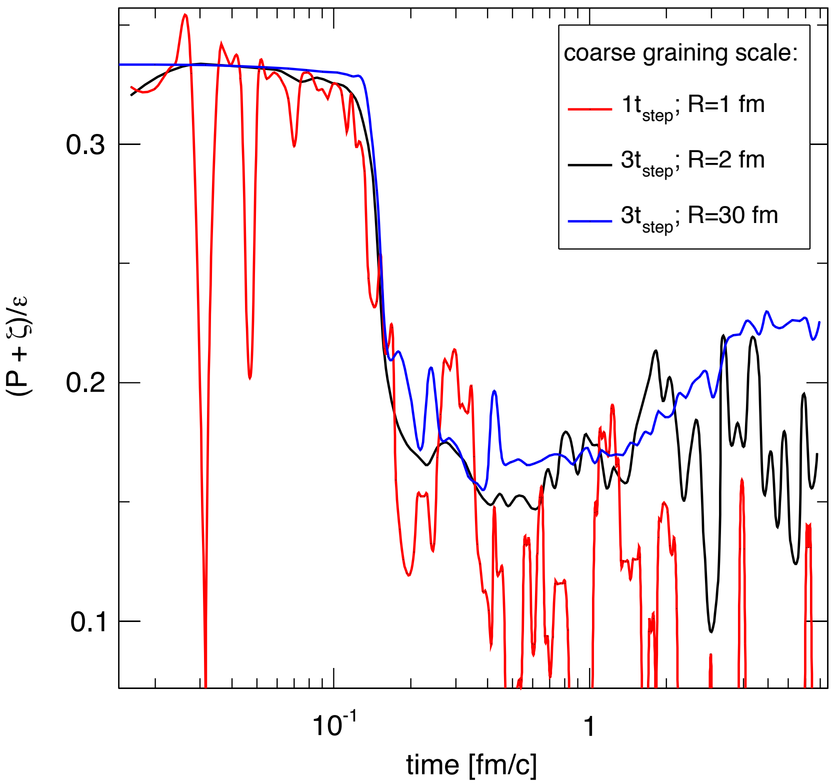

$$\frac{(P+\zeta)}{\varepsilon}\ vs\ \mathrm{time}$$

Small scale – large fluctuations

Averaged over all cells in the xy-plane – smaller fluctuations

Intermediate scale – fluctuations already close to the averaged situation

EXTRACTING THE HYDRODYNAMIC PROPERTIES OF THE SYSTEM EVOLUTION

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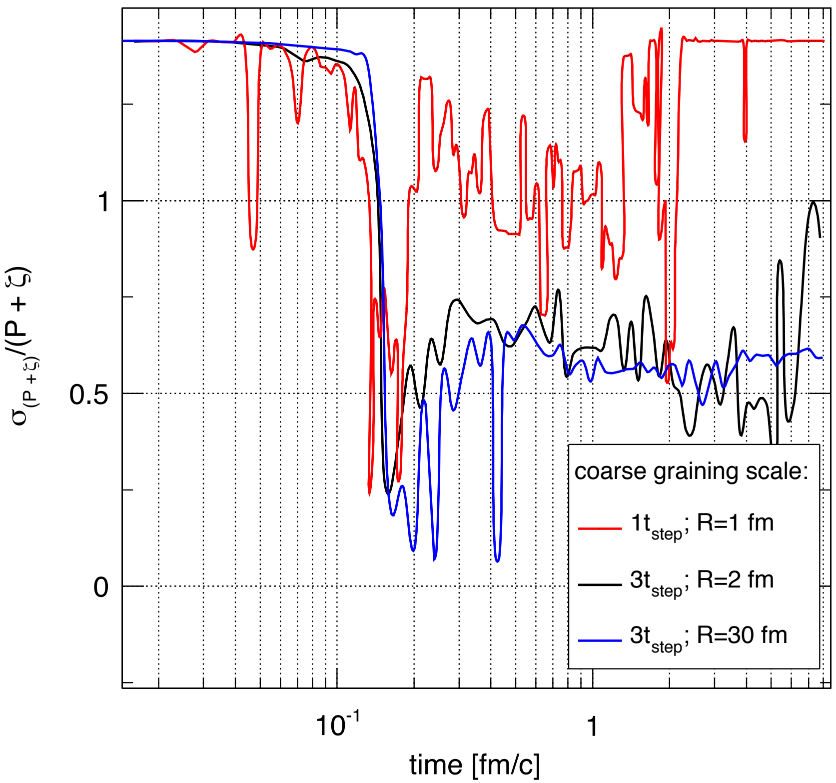

$$\frac{\sigma}{(P+\zeta)}\, vs\, \mathrm{time}$$

Small scale – large fluctuations

Averaged over all cells in the xy-plane – smaller fluctuations

Intermediate scale – fluctuations already close to the averaged situation

SUMMARY

- ✓ The coarse graining scale is intimately related to the validity of the hydrodynamic approach
- ✓ PHSD model provides a convenient way to test the coarse graining scale of hydrodynamics within a scenario of microscopic dynamics
- ✓ So far, we have not yet succeeded in showing hydrodynamic behavior emerging from microscopic dynamics but the preliminary results seems promising

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To do:

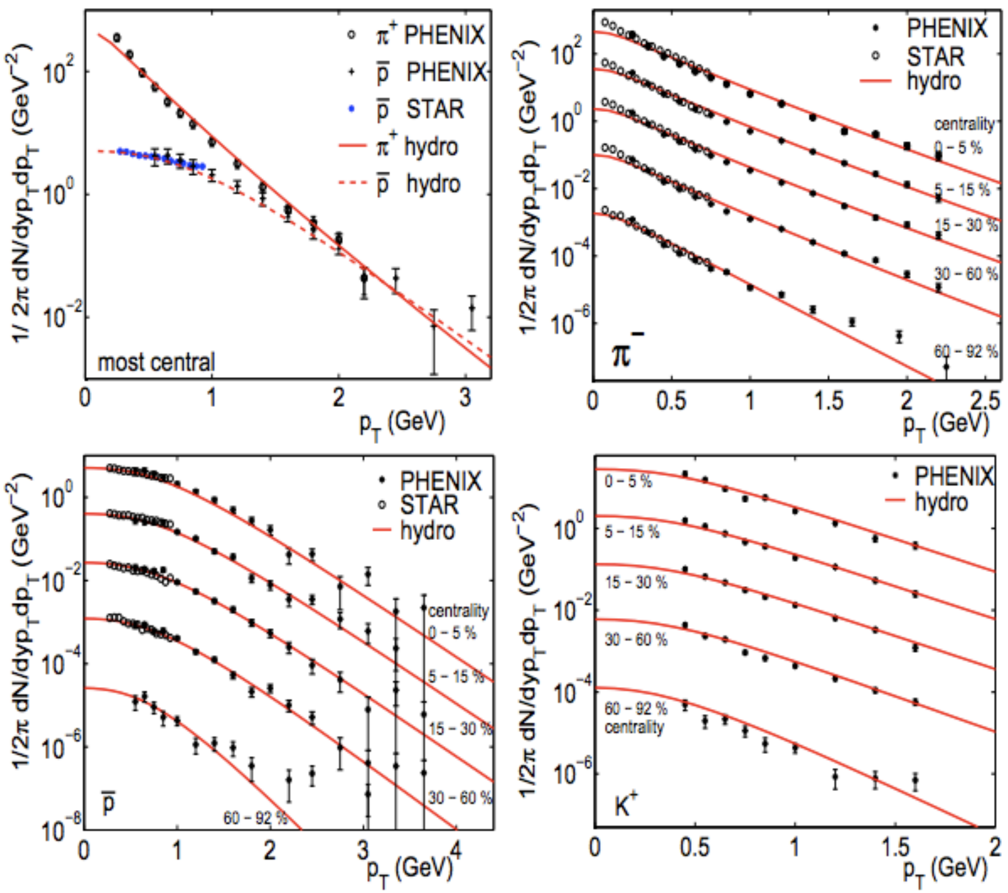
- *We need to use common 'labels' related to the spatial components of the pressure*
- *Instead of diagonalizing $T^{\mu\nu}$, only apply the Lorentz boost, therefore, keeping the information about x,y,z directions, which may allow us to obtain information such as 'local elliptic flow' – $(P_{xx} - P_{yy})/(P_{xx} + P_{yy})$*

Thank you

Backup

INTRODUCTION

- Success of hydrodynamics in describing the experimental observations in heavy-ion collisions



P. Kolb and U. Heinz, arXiv:nucl-th/0305084

Ideal hydro:

- local thermal equilibrium
- conservation laws + EoS

good description of identified hadrons p_t spectra

INTRODUCTION

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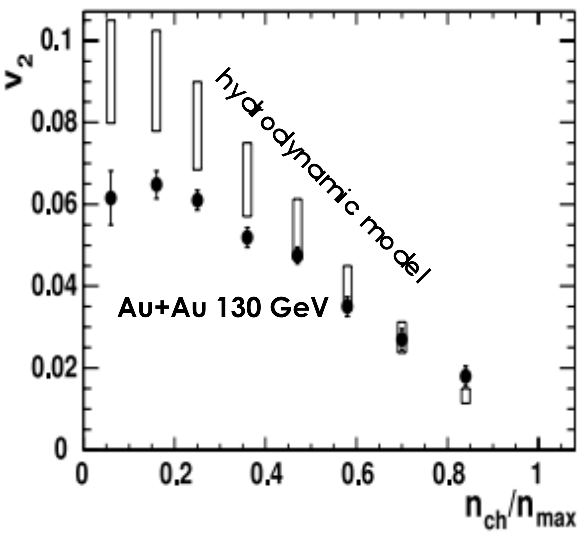
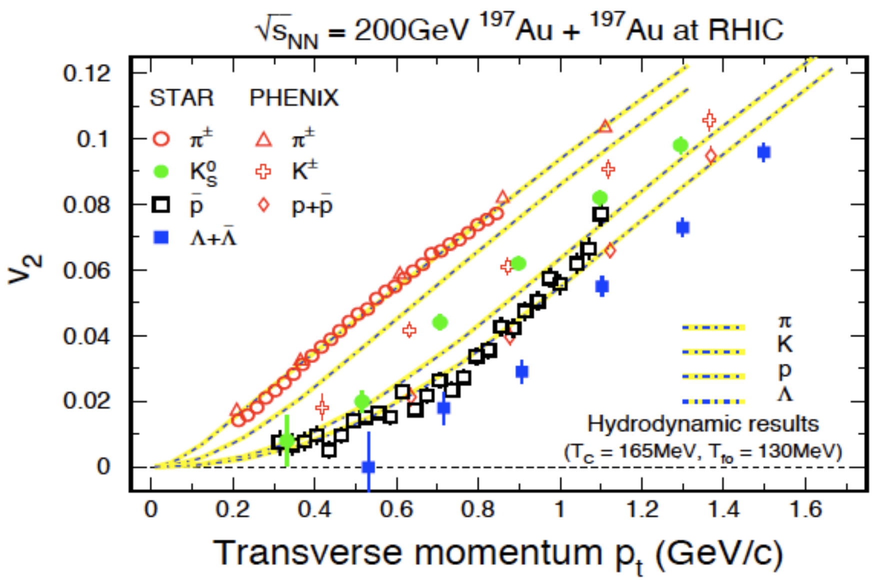


Ackermann K.H., *et al* (STAR Collab.), Phys. Rev. Lett. 86, 402



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collective flow:

- hydro models can reproduce the anisotropic momentum distribution of the final particles
- the system behaves collectively (like a strongly interacting liquid)

Evaluating the system evolution in PHSD

- Diagonalizing the energy-momentum tensor

  (finding the appropriate Lorentz transformation)

$$T^{\mu\nu}_{\text{rest}} = \Lambda^{\mu}{}_{\alpha} \Lambda_{\beta}{}^{\nu} T^{\alpha\beta}$$

proper energy density

$$T^{\mu\nu}_{\text{rest}} \rightarrow \begin{pmatrix} \varepsilon & \vec{0}^t \\ \vec{0} & \hat{\mathbb{T}} \end{pmatrix}$$

3x3 matrix

Lorentz transformation

$$\Lambda^{\mu}{}_{\nu} \rightarrow \begin{pmatrix} \gamma & -\gamma\vec{\beta}^t \\ -\gamma\vec{\beta} & \gamma\mathbb{P} + \mathbb{Q} \end{pmatrix}$$

where $\mathbb{P}$ and $\mathbb{Q}$ are projection operators

EVALUATING THE SYSTEM EVOLUTION IN PHSD

- Diagonalizing the energy-momentum tensor

  (finding the appropriate Lorentz transformation)

  - Solve the system for **ε** and **β**:

$$\Lambda^{-1} T_{\text{rest}} = T\Lambda$$

$$\begin{pmatrix} \gamma & \gamma\vec{\beta}^t \\ \gamma\vec{\beta} & \gamma\mathbb{P}+\mathbb{Q} \end{pmatrix} \begin{pmatrix} \varepsilon & \vec{0}^t \\ \vec{0} & \hat{\mathbb{T}} \end{pmatrix} = \begin{pmatrix} T_{00} & \vec{T}^t \\ \vec{T} & \mathbb{T} \end{pmatrix} \begin{pmatrix} \gamma & -\gamma\vec{\beta}^t \\ -\gamma\vec{\beta} & \gamma\mathbb{P}+\mathbb{Q} \end{pmatrix}$$

$$\ldots \quad \begin{cases} \varepsilon = T_{00} - \vec{T}^t\vec{\beta} \\ \varepsilon\vec{\beta} = \vec{T} - \mathbb{T}\vec{\beta} \end{cases} \quad \Rightarrow \quad \begin{cases} \varepsilon^{(k)} = T_{00} - \vec{T}^t\vec{\beta}^{(k-1)} \\ \vec{\beta}^{(k)} = \frac{\vec{T}-\mathbb{T}\vec{\beta}^{(k-1)}}{\varepsilon^{(k)}} \end{cases}$$

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$$u^\mu \to (\gamma, \gamma\beta_x, \gamma\beta_y, \gamma\beta_z)$$

it may present some convergence problem for β ~ 1

EXTRACTING THE HYDRODYNAMIC PROPERTIES OF THE SYSTEM EVOLUTION

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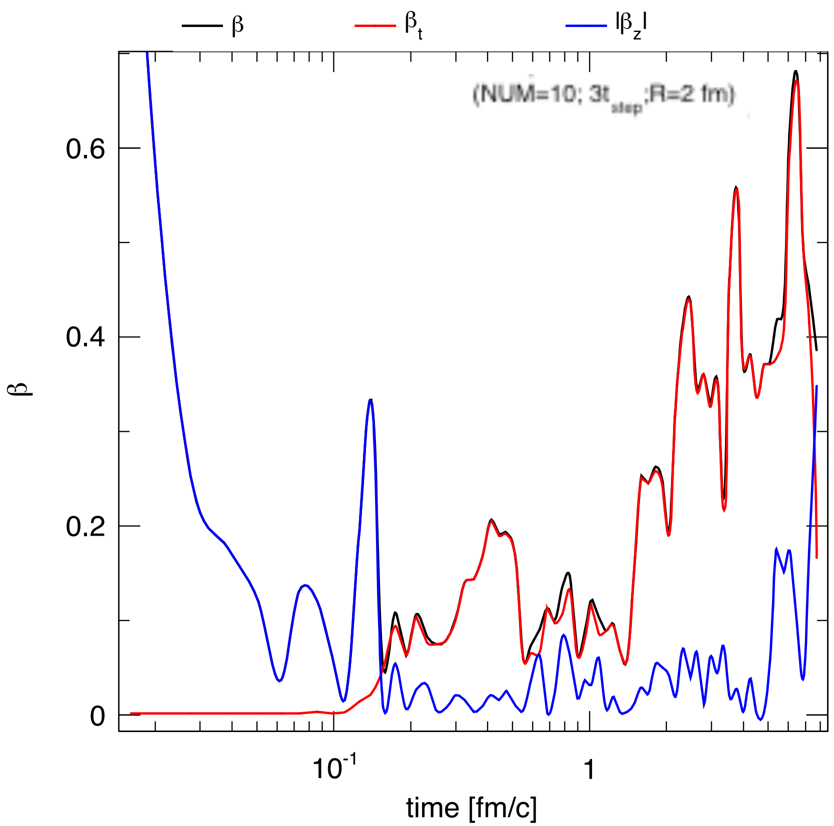

As the system expands the longitudinal component of beta becomes very small while the transverse components increase