A holographic calculation of the electric conductivity of the strongly coupled quark-gluon plasma near the deconfinement transition

arXiv:1311.6675

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Outline

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a) Electric conductivity of a plasma
b) Gauge/gravity duality

2 – Electric conductivity from holography

3 – Einstein + Scalar backgrounds

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1- Introduction

a) Electric conductivity of a plasma

Heavy ion collisions generate strong electric and magnetic fields due to the current created by the spectating nuclei.

Example: magnetic fields in heavy ion collisions

Origin	Magnetic field (G)
EM shock wave (strongest natural magnetic field on Earth)	10^7
Neutron star	10^{10}-10^{13}
Magnetar (strongest magnetic field of astrophysical origin)	10^{15}
Heavy ion collisions at RHIC	10^{18}
Heavy ion collisions at LHC	10^{19}

Heavy Ion Collisions produce the strongest magnetic fields ever known.

(see V. Skovok, arXiv:0907.1396; K. Tuchin, arXiv:1301.0099)

What is the role of such strong external EM fields on the dynamics of the quark-gluon plasma created in heavy ion collisions?

What is the role of such strong external EM fields on the dynamics of the quark-gluon plasma created in heavy ion collisions?

One way of studying this question is looking at the response of the plasma to external electromagnetic fields.

The electric response of the medium to a (weak) external EM field is given by the electric conductivity tensor σ_{ij}:

$$\langle J^i(\omega)\rangle = \sigma^{ij}(\omega)F_{jt}(\omega)$$

By linear response theory,

$$\sigma^{ij}(\omega) = -\frac{G_R^{ij}(\omega, \mathbf{k}=0)}{i\omega}$$

Where G_R is the retarted correlation fuction of the current J^i:

$$G_R^{ij}(k) = -i\int d^4x\, e^{-ik\cdot x}\theta(t)\left\langle\left[\hat{J}^i(t,\mathbf{x}), \hat{J}^j(0,\mathbf{0})\right]\right\rangle_T$$

For isotropic media, $\sigma^{ij}(\omega) = \sigma(\omega)\delta^{ij}$

The QGP is a strongly coupled non-Abelian plasma. How to compute G_R non-pertubatively in these conditions?

Lattice QCD

G_R is a real time Green's function.

Lattice QCD is naturally suited to compute the Euclidean Green's function G_E - real time quantities can be extracted using MEM (maximum entropy method), but this requires high statistics.

One alternative approach, which is naturally suited to the computation of real time quantities of strongly coupled non-Abelian plasmas, is the gauge/gravity duality.

Objective: to compute holographically the electric conductivity in a bottom-up model that matches the thermodynamics of the lattice near the crossover.

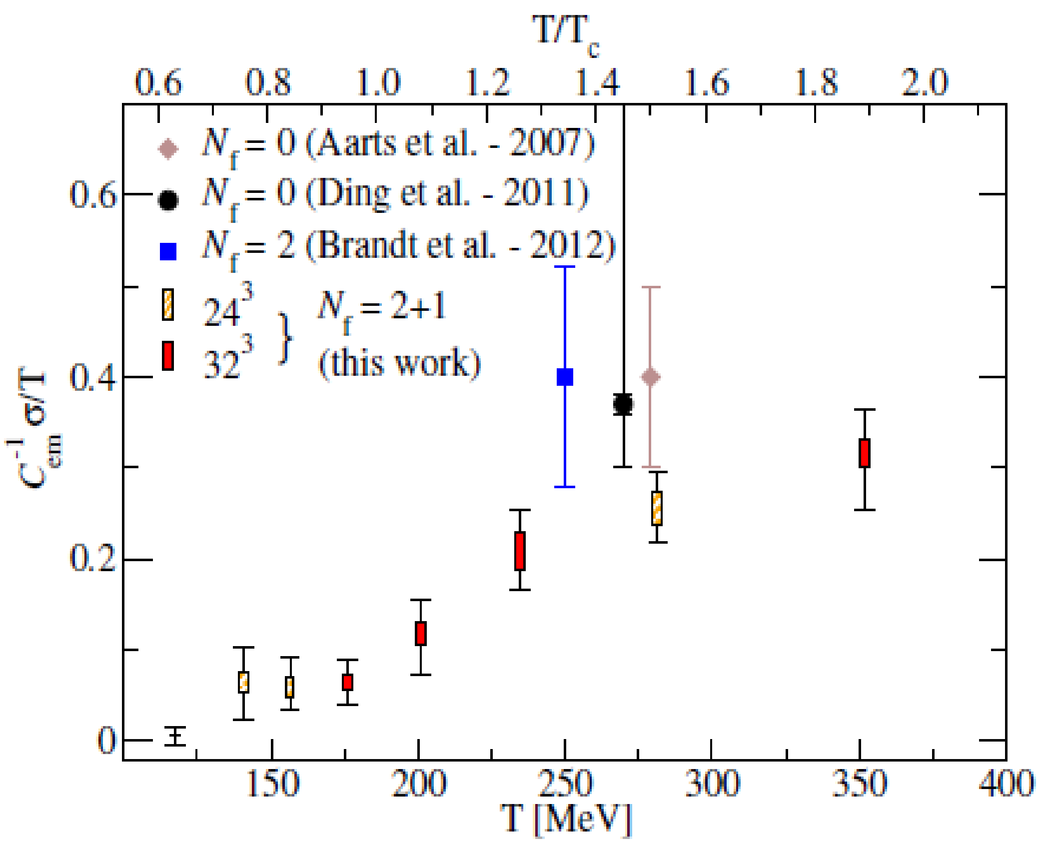
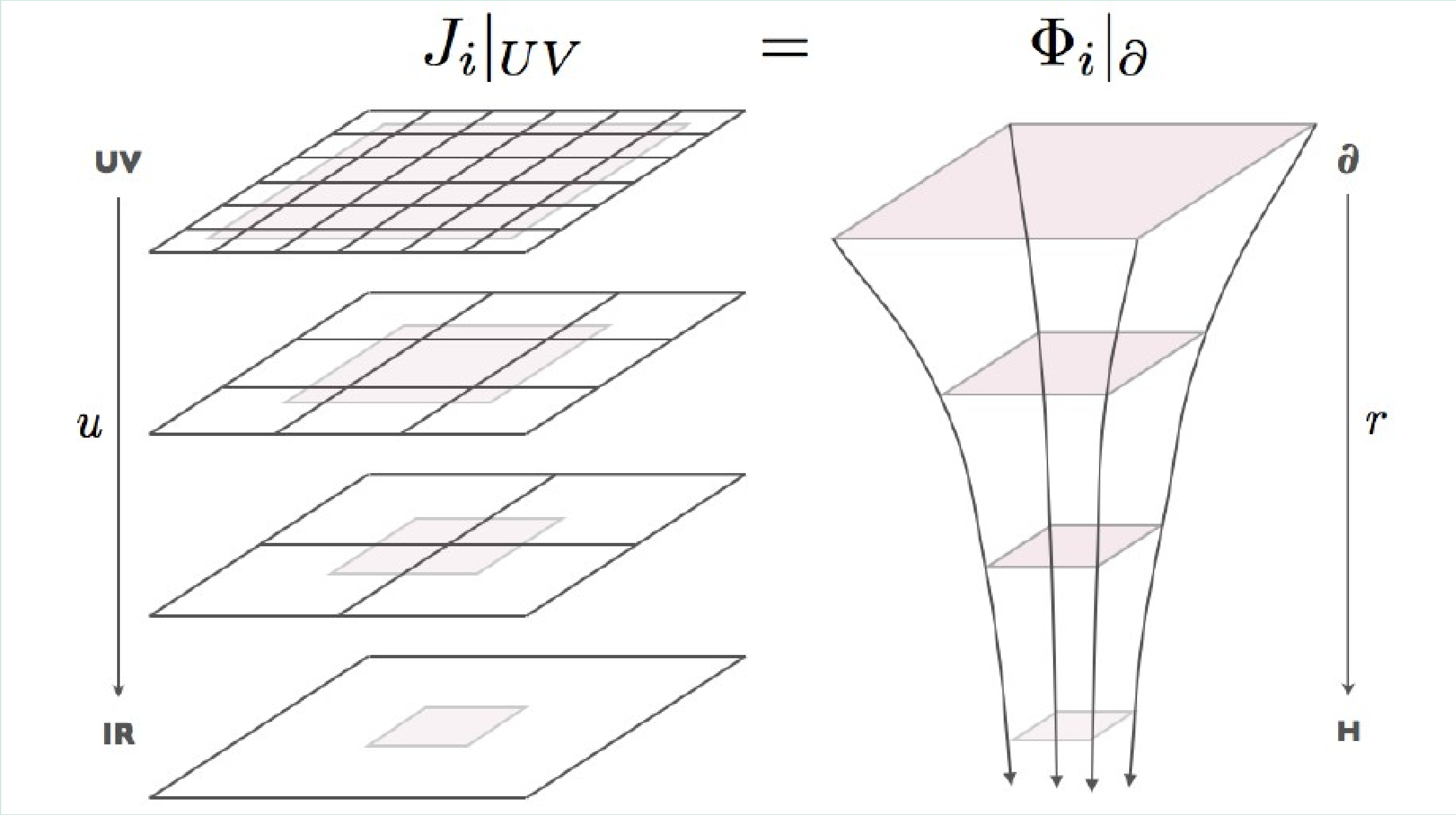


FIG. 3. Temperature dependence of $C_{em}^{-1}\sigma/T$, where $C_{em} = 5/9e^2$ for two light flavours. The vertical size of the rectangles reflects the systematic uncertainty due to changes in the default model, by varying $0.4 < b < 1$. The error bars indicate the statistical jackknife error, combining all b values between 0.4 and 1. Previously obtained results [6, 9, 10] are indicated as well: the $N_f = 0$ results are inserted matching the values of T/T_c. Note that the black circle has two error bars [9].

Lattice results (from 1307.6763)

b) Gauge/gravity duality

Geometrizing the renormalization group.



(Adams et al., arXiv:1205.5180)

The energy scale *u* becomes a coordinate for the 5th dimension.
We can compute observables in the strongly coupled IR fixed point of the gauge theory by studying classical Einstein gravity in the bulk.

2 – Electrical conductivity from holography

One must consider fluctuations, in the bulk gravity theory, of a U(1) Abelian field – which is assumed to be dual to the electromagnetic field of the boundary gauge theory.

Metric ansatz:

$$ds^2 = e^{2\tilde{A}(z)}\left(-\tilde{h}(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{\tilde{h}(z)}\right)$$

Boundary at *z->0*
Horizon at $z=z_h$

Action for the bulk U(1) field fluctuations:

$$S_M = -\frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\,\frac{f(\phi)}{4}F_{MN}F^{MN}$$

$f(\phi)$ is a dilaton-depedent coupling function

Equations of motion:

$$\partial_z\bar{\sigma}(\omega,z) = i\,\omega\,\frac{\Sigma(z)}{\tilde{h}(z)}\left[\frac{\bar{\sigma}(\omega,z)^2}{\Sigma(z)^2} - 1\right]$$

(horizon regularity is used to reduce the EOM from order 2 to order 1)

$$\Sigma(z) = f(\phi(z))\,e^{\tilde{A}(z)}.$$

From the real-time gauge/gravity duality prescription for G_R,

$$\sigma(\omega) = -\frac{G_R(\omega)}{i\omega} = \bar{\sigma}(\omega, z \to 0)$$

DC conductivity:

Taking $\omega \to 0$ implies that $\bar{\sigma}$ is a constant; thus

$$\sigma_{DC} = f(\phi(z_h))e^{\tilde{A}(z_h)}$$

This is exactly the result from the membrane paradigm (see Iqbal, Liu, arXiv:0809.3808).

At zero frequency, the membrane conductivity is the same as the as boundary conductivity.

AC conductivity:

One solves (numerically, if the case) the eom for $\bar{\sigma}$.

The boundary condition is $\bar{\sigma}(\omega, z_h) = \sigma_{DC}$

3 – Einstein + Scalar backgrounds

Our holographic models will be bottom-up gravity theories described by Einstein gravity coupled to a scalar field.

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\left[R - \frac{4}{3}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)\right]$$

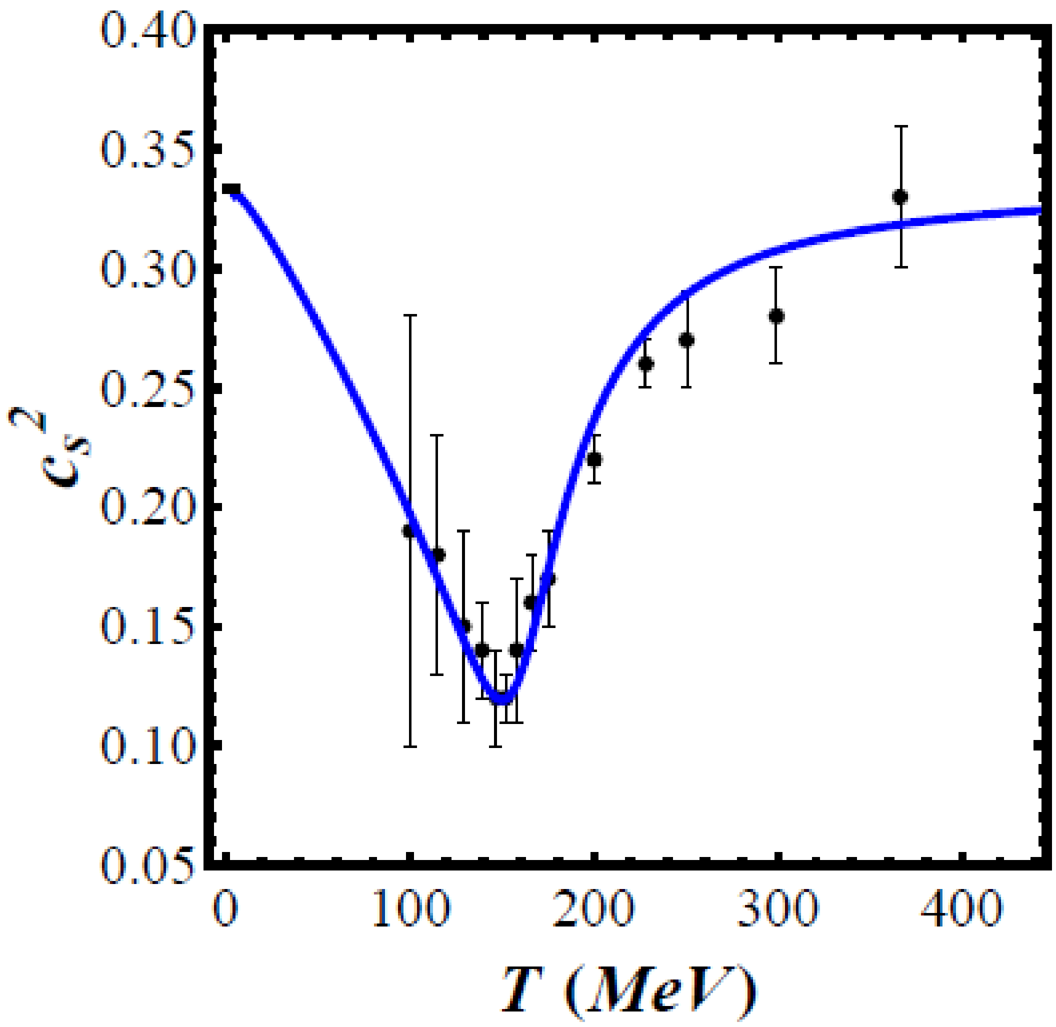
The potential V encodes, in the geometry, the IR properties of the gauge theory.

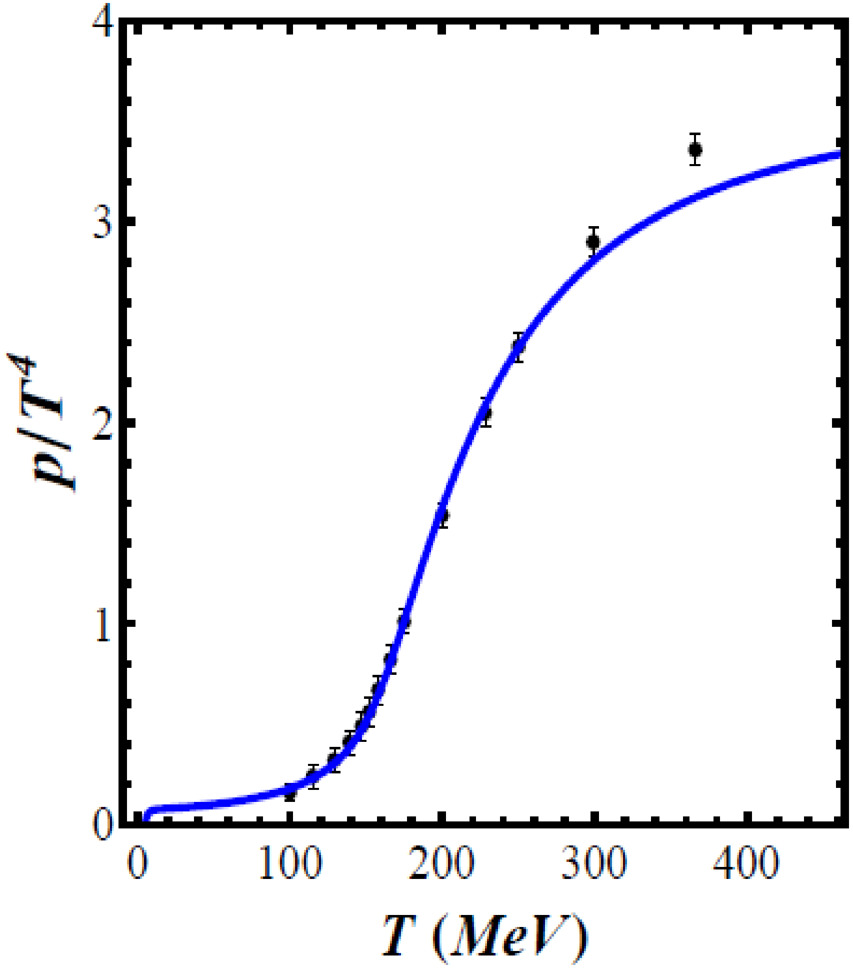
For the dilaton dependent couplingof the bulk U(1) field, we have chosen three simple parametrizations to check the robustness of our results:

$$f_1(\phi) = \frac{\mathrm{sech}(a_1\,\phi)}{g_{5,1}^2},$$
$$f_2(\phi) = \frac{1}{g_{5,2}^2}\,\frac{1}{(\phi^2 + a_2^2)}$$
$$f_3(\phi) = \frac{e^{-a_3^2\phi^2}}{g_{5,3}^2},$$

Potential choice and matching to lattice QCD thermodynamics

$$V(\phi) = -12\cosh\gamma\phi + b_2\phi^2 + b_4\phi^4 + b_6\phi^6$$



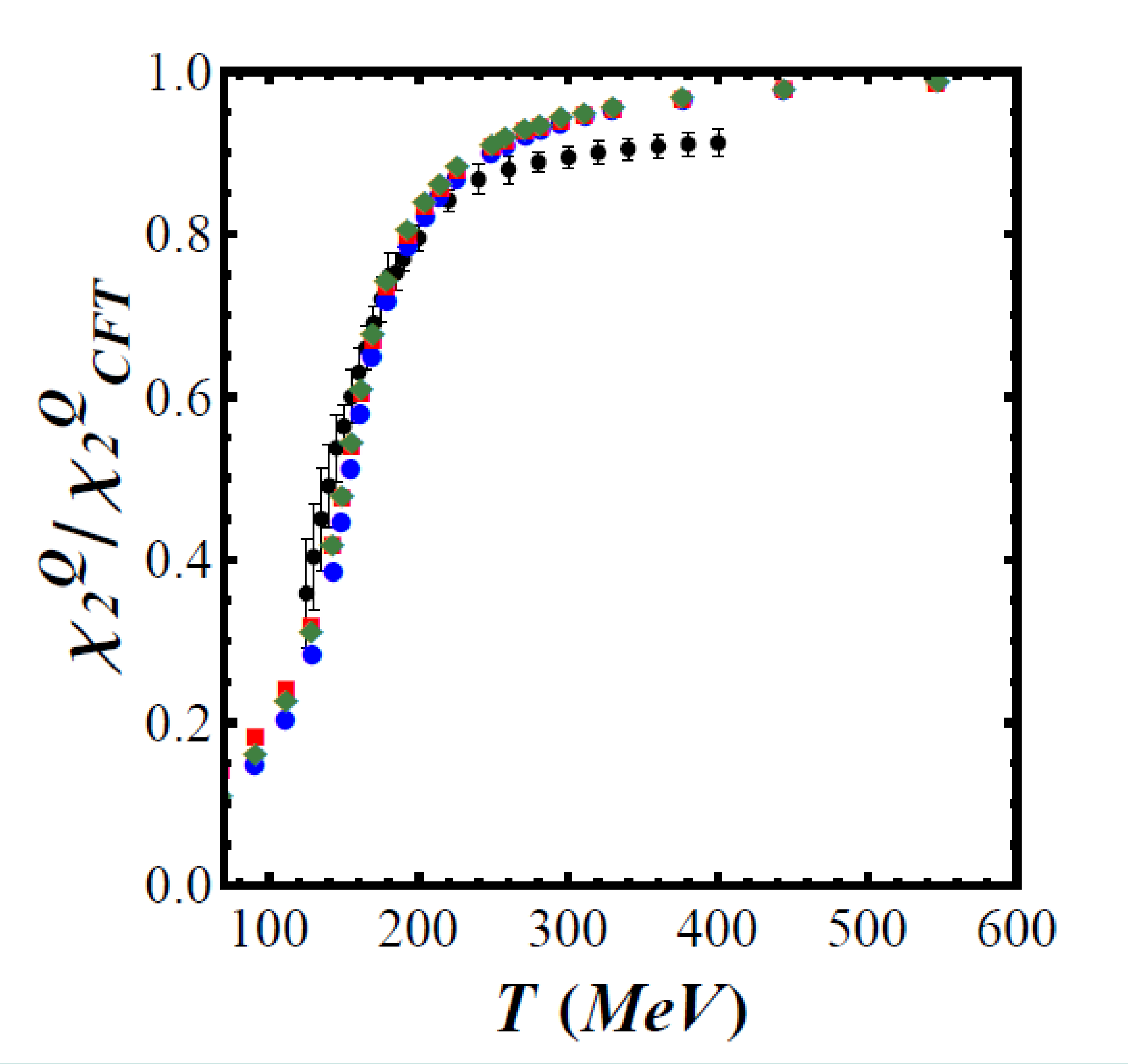


Points with error bars: Lattice data for (2+1) flavors (arXiv:1007.2580)
Curve: our holographic model

U(1) coupling choice and matching to charge susceptibility from the lattice

From holography,

$$\chi_2^Q = \frac{1}{\int_0^{z_h} dz \, [e^{\tilde{A}(z)} f(\phi(z))]^{-1}}$$



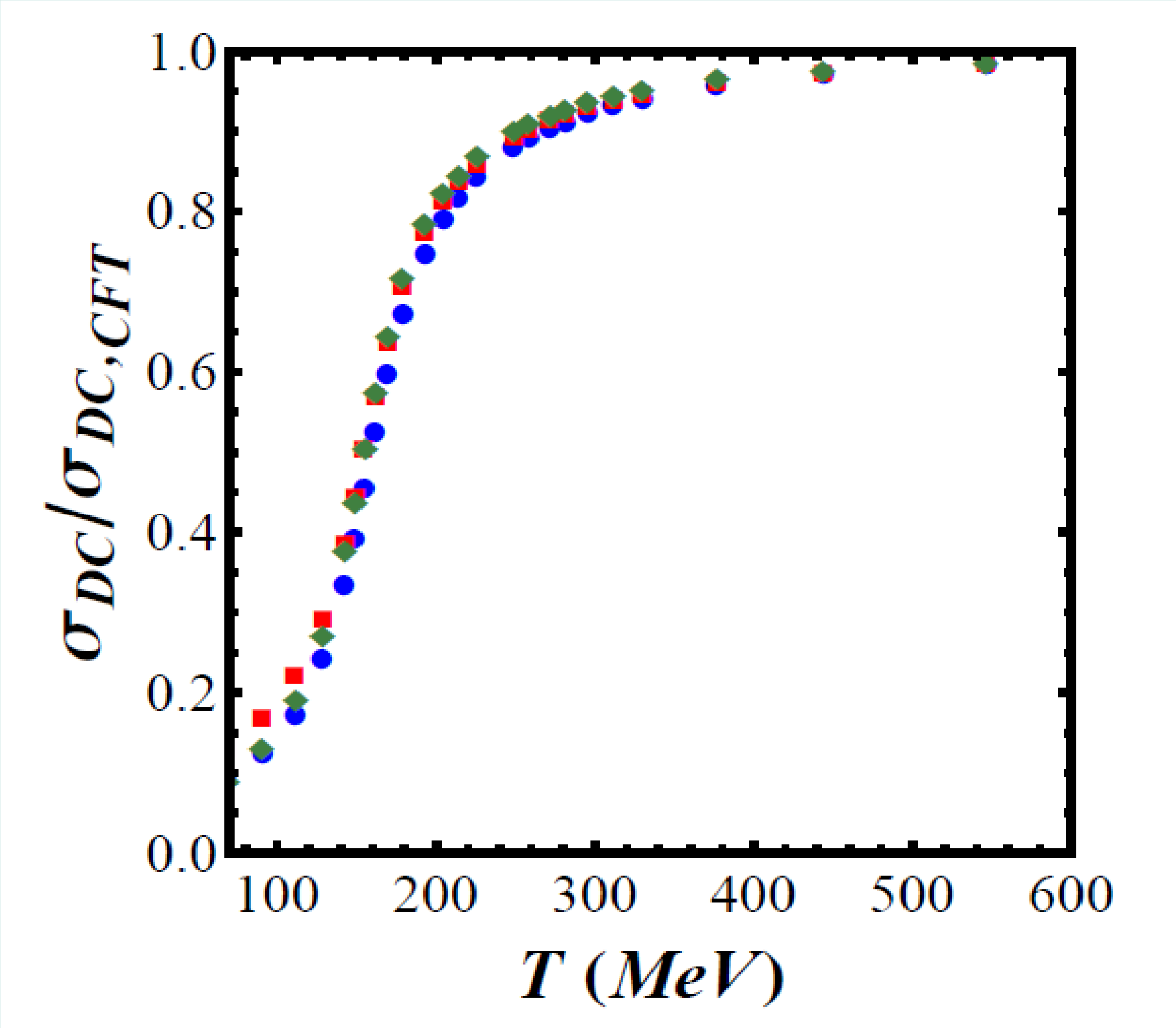
Lattice data (points with error bars) from arXiv:1112.4416

Circles – Model 1
Squares – Model 2
Diamonds – Model 3

4 - Results

With every parameter in the holographic model fixed, we can proceed to make predictions for the electrical conductivity for a strongly coupled non-Abelian plasma.

DC conductivity:

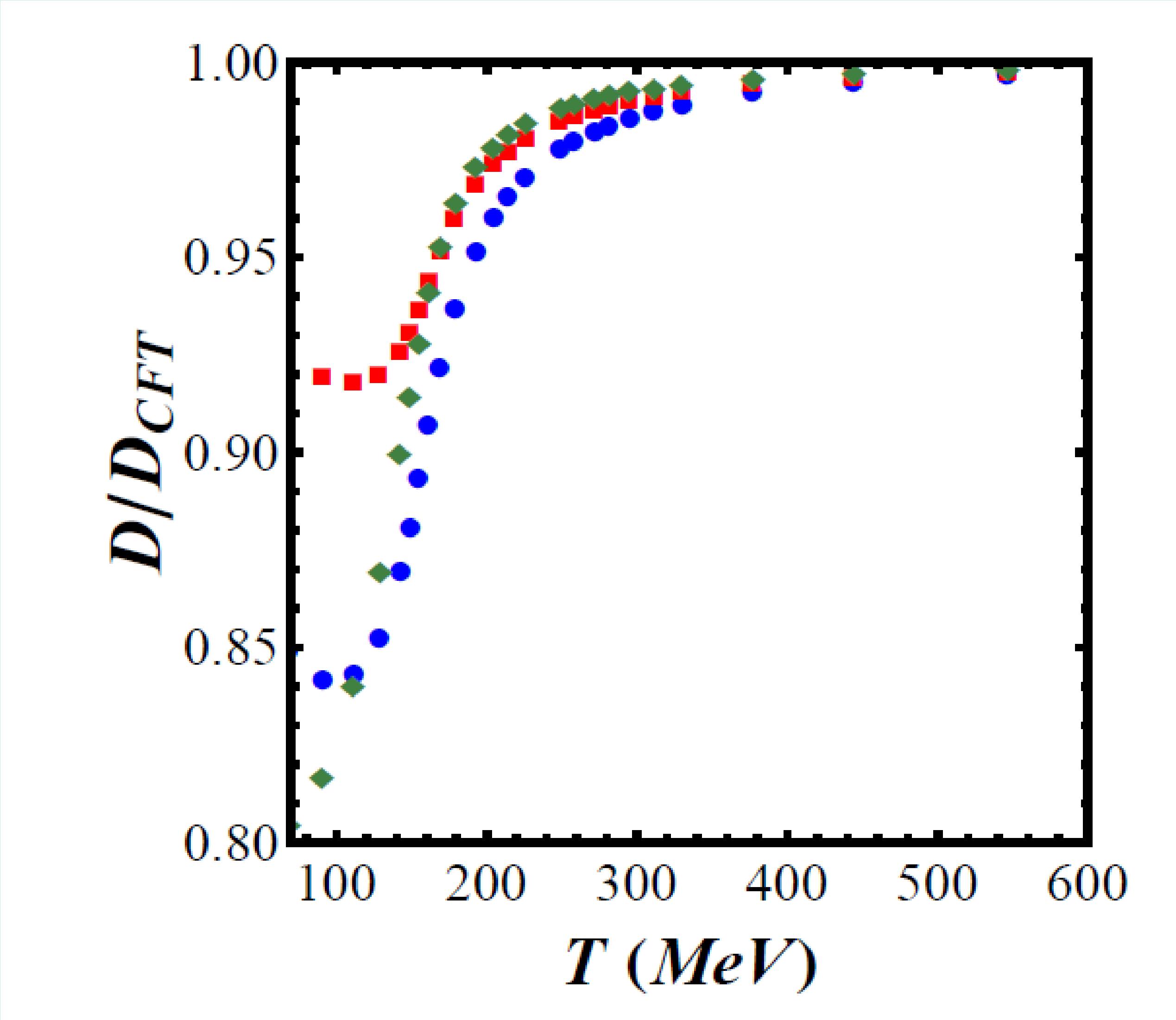


We get the enhancement near the phase transition seen on the lattice.

Bonus: diffusion constant

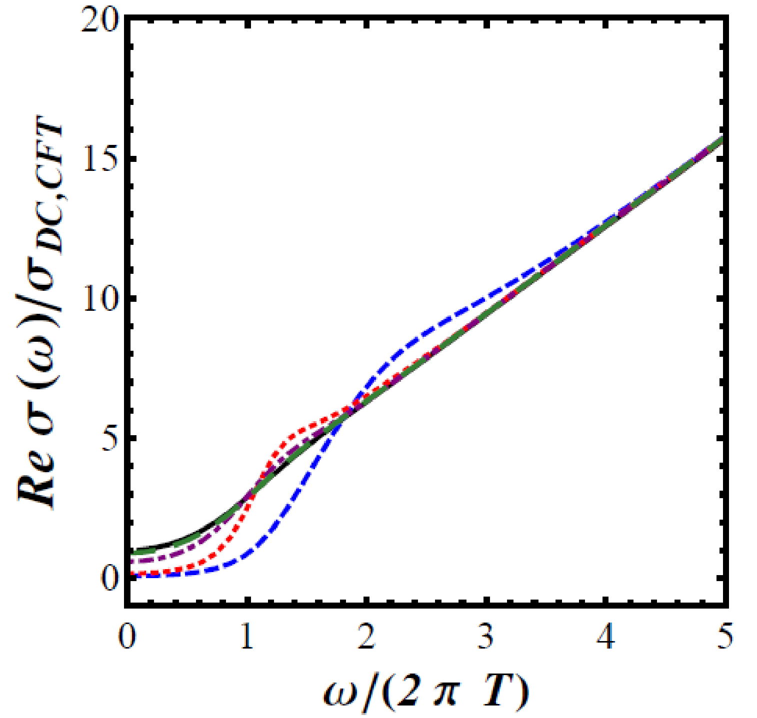
Einstein's relation can be shown to hold in this holographic model:

$$D = \frac{\sigma_{DC}}{\chi_2^Q}$$

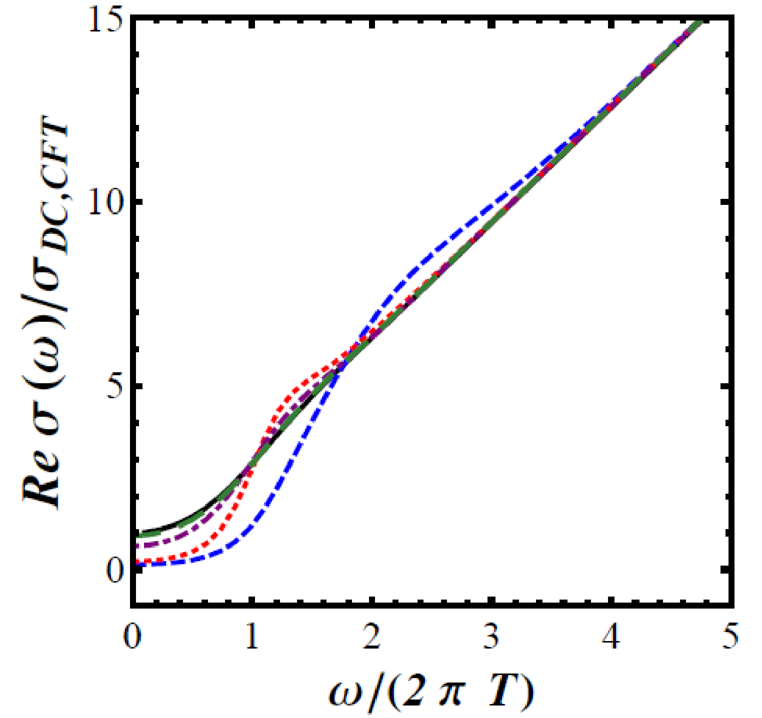


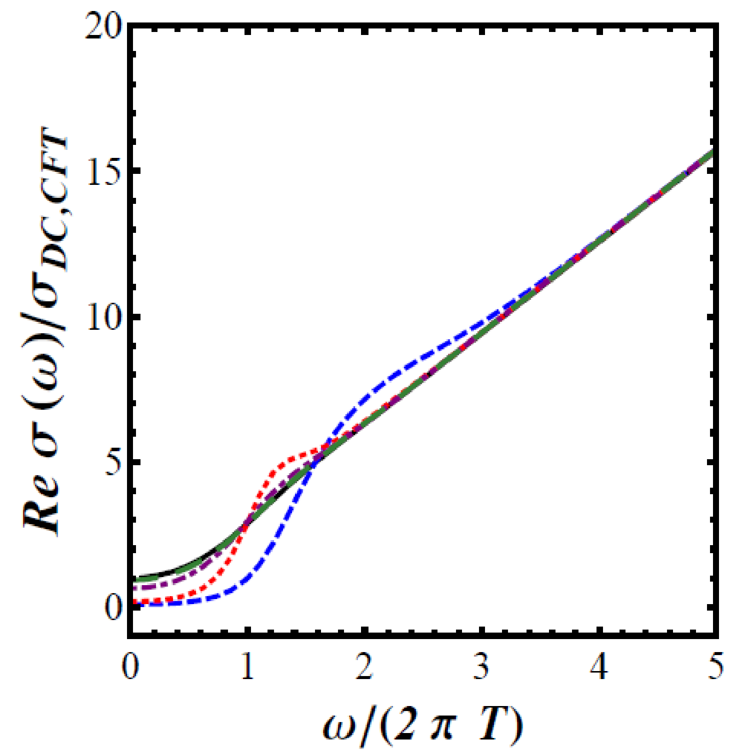
AC results:

Model 1



Model 2





Model 3

Temperatures:

Solid black – CFT
Blue dashed – $T/T_c = 0.45$
Red dotted – $T/T_c = 0.74$
Dash-dotted magenta – $T/T_c = 1.12$
Green long dashed – $T/T_c = 1.81$

5 - Outlook

a) Once the potential and coupling are fixed by lattice QCD thermodynamics and charge susceptibility, the electric conductivity and the diffusion rate are predictions of the model.

b) Enhancement of the DC conductivity near the phase transition coherent with lattice results

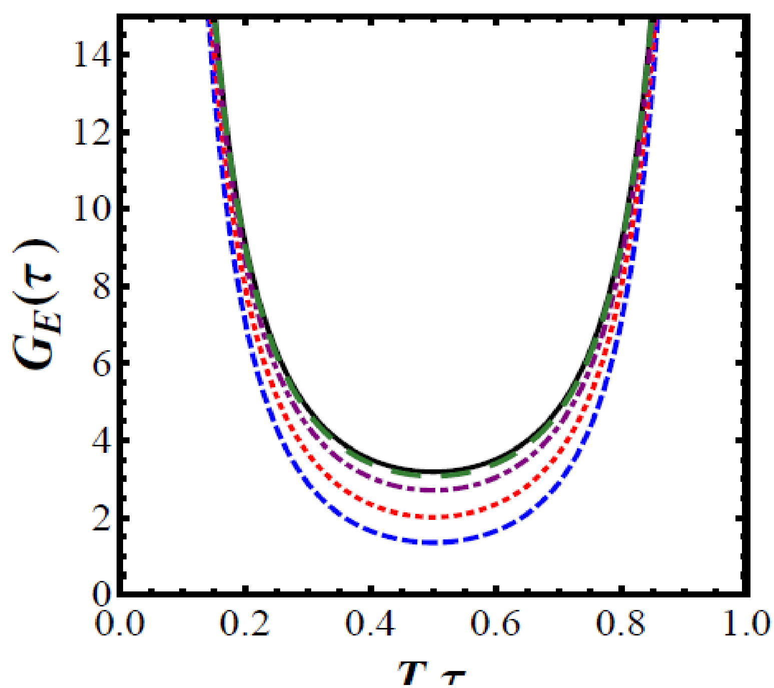
c) Oscillations of the AC conductivity are quickly damped at temperatures near or above the phase transition.

d) The results are fairly independent of the choice of the bulk U(1) gauge field coupling.

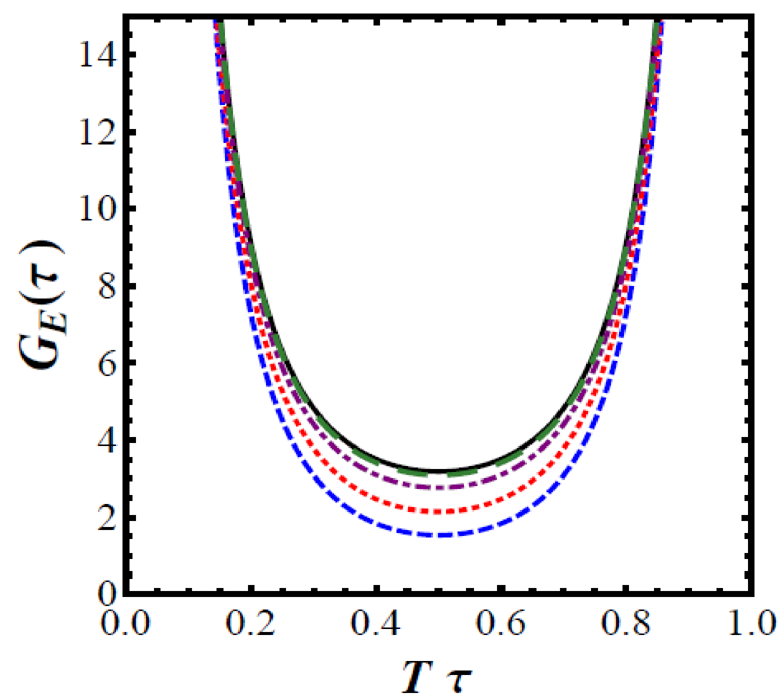
Thanks!

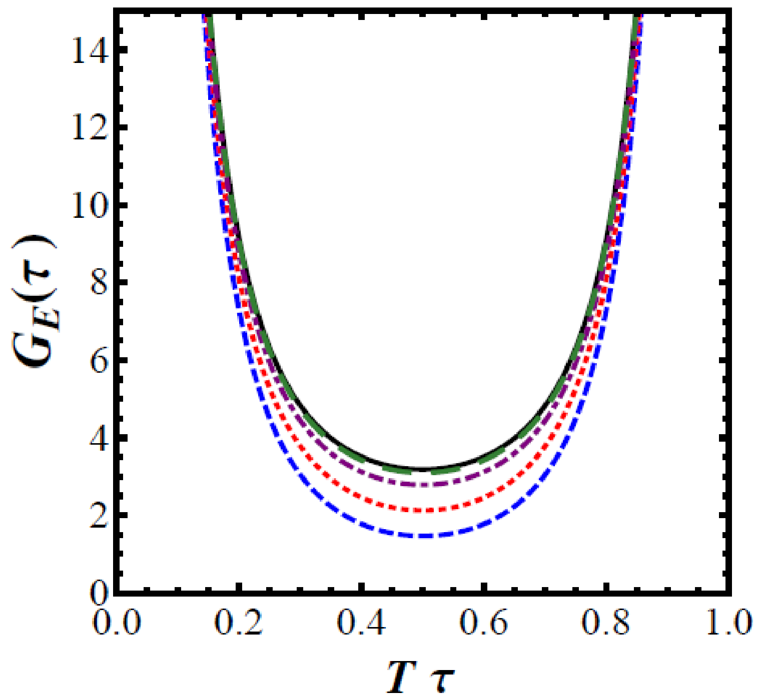
Euclidean Correlator:

Model 1



Model 2





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