Ondas não-lineares em fluidos dissipativos e causais

David A. Fogaça ,   Hugo Marrochio ,
Fernando Navarra e   Jorge Noronha

GRHAFITE
IFUSP
GRupo de HÁdrons e FÍsica TEórica

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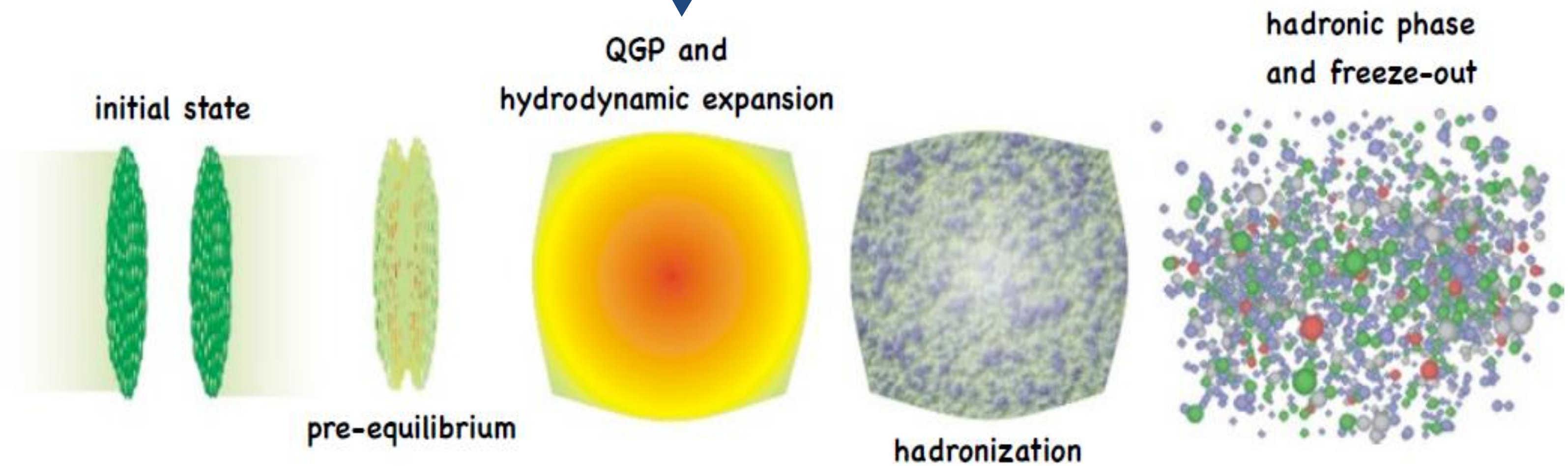
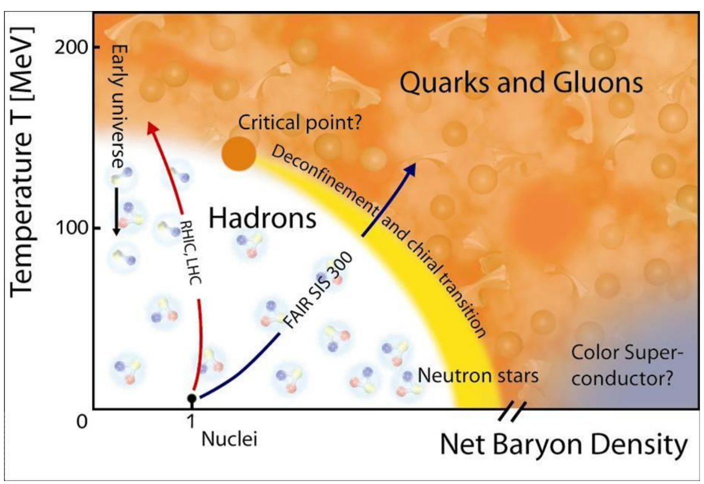
aprox. fluido ideal
viscosidade pequena

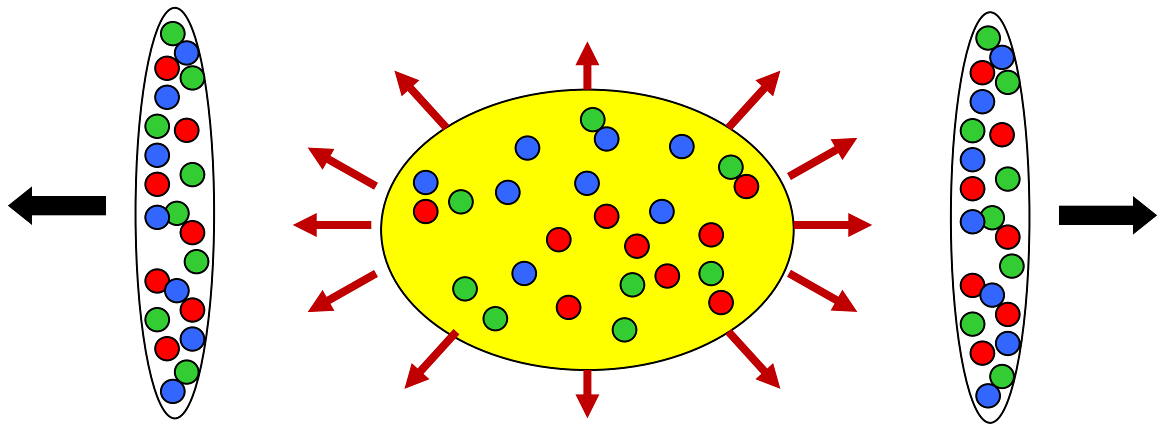


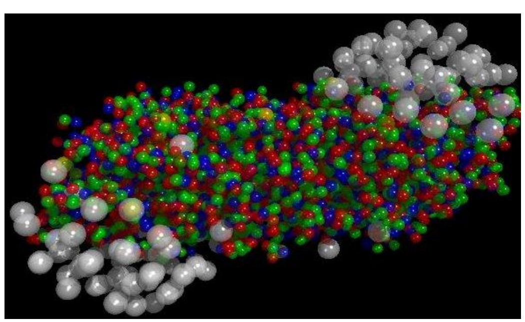


gás de hadrons
viscosidade grande

2 atos da hidrodinâmica !

Primeiro ato da Hidrodinâmica : ***o fluido***





2004: Descoberta do “fluido perfeito” no RHIC

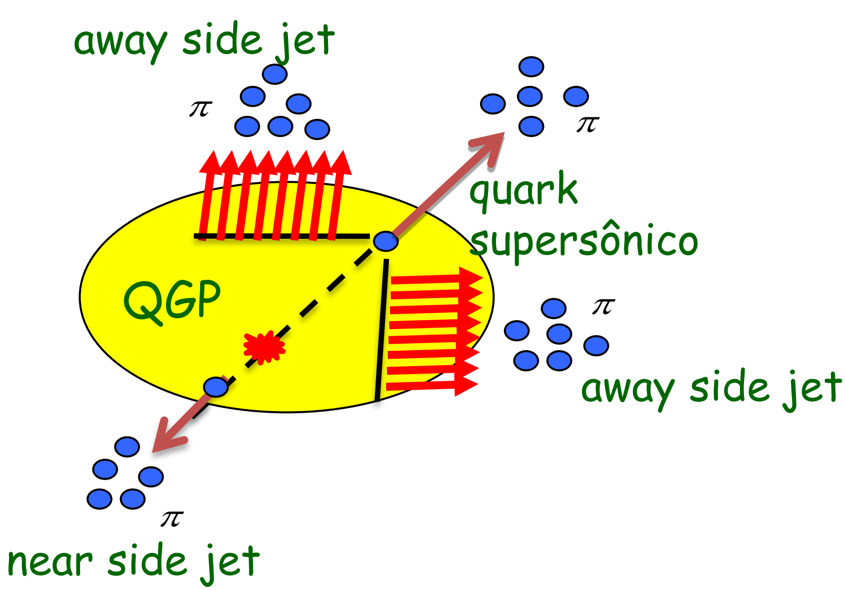
Segundo ato da hidrodinâmica : ***ondas no fluido***

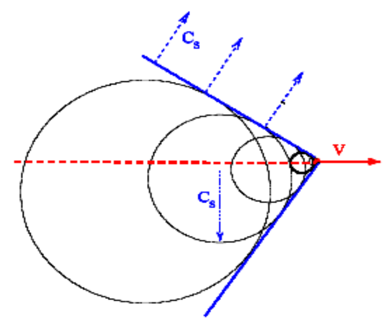
- A passagem de partons energéticos pelo QGP pode formar ondas de choque e "cones de Mach":

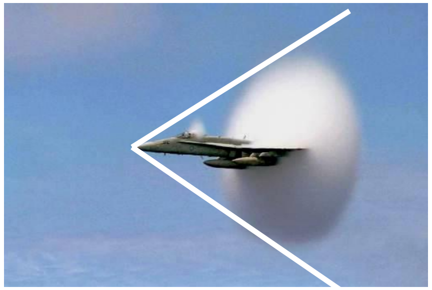
L. M. Satarov, H. Stoecker and I. N. Mishustin, Phys. Lett. **B627**, 64 (2005)

- Evidências experimentais disso no RHIC:

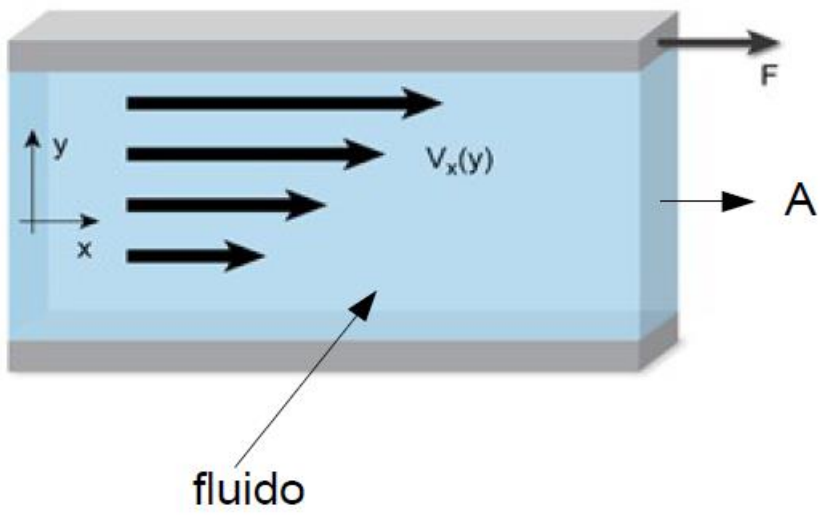
S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **97**, 052301(2006).
J. Adams et al., STAR Collab. *Phys. Rev. Lett.* **95**, 152301 (2005).







Mas e a viscosidade?



Pressão: $\frac{F}{A} \sim \boxed{\eta} \frac{dV_x(y)}{dy}$

η → viscosidade de cisalhamento

P. Danielewicz and M. Gyulassy, *Phys. Rev. D* **31**, 53 (1985)

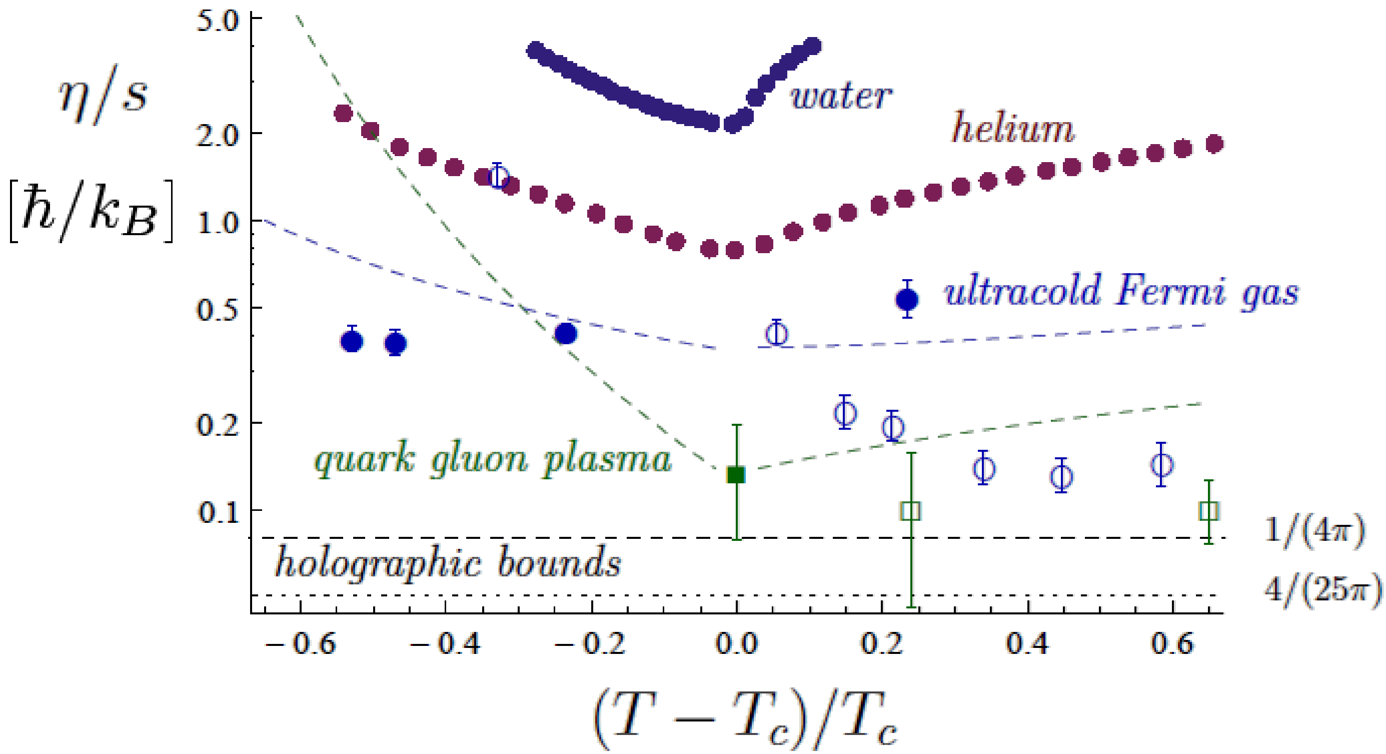
Dissipative phenomena in quark-gluon plasmas

Teoria Cinética dos gases
e princípio da incerteza

$$\left.\begin{array}{r} \eta \sim n < p > l \\ < p > l \sim \hbar \\ s \sim k_B\, n \end{array}\right\} \longrightarrow \frac{\eta}{s} \sim \frac{\hbar}{k_B}$$

n : densidade
$< p >$: momento médio
l : livre caminho médio
s : densidade de entropia

A. Adams, L. D. Carr, T. Schaefer, P. Steinberg, J. E. Thomas, arXiv:1205.5180v1 [hep-th] :



Para o QGP: $T_c \cong 150 MeV/k_B \sim 2 \times 10^{12} K$

" duração " $\sim 15\, fm/c \sim 10^{-23}\, s$

" tamanho " $\sim 15\, fm \sim 10^{-14}\, m$

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

Hidrodinâmica relativística com viscosidade

P. Romatschke, *Int. J. Mod. Phys. E* **19**, 1 (2010)

$$\hbar = c = k_B = 1$$

$$D\varepsilon + (\varepsilon + p)\partial_\mu u^\mu - \Pi^{\mu\nu}\nabla_{(\mu}u_{\nu)} = 0$$

$$(\varepsilon + p)Du^\alpha - \nabla^\alpha p + \Delta^\alpha_\nu \partial_\mu \Pi^{\mu\nu} = 0$$

$$T^{\mu\nu}_{(0)} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu}$$

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}$$

Navier-Stokes *(NS)* ← $\Pi^{\mu\nu} = \eta \nabla^{\langle\mu}u^{\nu\rangle}$

$$u^\mu = (\gamma, \gamma\vec{v}) \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$\nabla_{\langle\mu}u_{\nu\rangle} \equiv 2\nabla_{(\mu}u_{\nu)} - \frac{2}{3}\Delta_{\mu\nu}\nabla_\alpha u^\alpha$$

$$A_{(\mu}B_{\nu)} = \frac{1}{2}(A_\mu B_\nu + A_\nu B_\mu)$$

$$D \equiv u^\mu \partial_\mu$$

$$\nabla^\alpha \equiv \Delta^{\mu\alpha}\partial_\mu$$

A *NS* viola causalidade !

Fazendo a linearização :

$$\left.\begin{aligned} \varepsilon &= \varepsilon_0 + \delta\varepsilon(t,\vec{x}) \\ p &= p_0 + \delta p(t,\vec{x}) \\ u^\mu &= (1,\vec{0}) + \delta u^\mu(t,\vec{x}) \end{aligned}\right\} \longrightarrow NS \longrightarrow \left\{ \frac{\partial}{\partial t}\delta u^y - \frac{\eta}{(\varepsilon_0+p_0)}\frac{\partial^2}{\partial x^2}\delta u^y = 0 \right.$$

$$v_g = 2\frac{\eta}{(\varepsilon_0+p_0)}k \qquad \lim_{k\to\infty} v_g(k) \to \infty$$

Lei de "Maxwell-Cattaneo" :

J.C. Maxwell, Phil. Trans. R. Soc. **157** (1867) 49
C. Cattaneo, Atti Sem. Mat. Fis. Univ. Modena **3** (1948) 3

Incluir uma escala de tempo de tal forma que:

$$\tau_\pi\frac{\partial^2}{\partial t^2}\delta u^y + \frac{\partial}{\partial t}\delta u^y - \frac{\eta}{(\varepsilon_0+p_0)}\frac{\partial^2}{\partial x^2}\delta u^y = 0$$

$$v_g = \sqrt{\frac{\eta}{\tau_\pi(\varepsilon_0+p_0)}}$$

τ_π → tempo de relaxação

Hidrodinâmica relativística dissipativa e causal: teoria de *Israel-Stewart* (IS)

W. Israel, Ann. Phys. (N.Y.) **100**, 310 (1976)
J. M. Stewart, Proc. Roy. Soc. A **357**, 59 (1977)
W. Israel and J. M. Stewart, Ann. Phys. (N.Y.) **118**, 341 (1979)

S. Pu, T. Koide and D. H. Rischke, *Phys. Rev. D* **81**, 114039 (2010).

G. S. Denicol, T. Kodama, T. Koide and Ph. Mota, *J. Phys. G* **35**, 115102 (2008).

H. Marrochio, J. Noronha, G.S. Denicol, M. Luzum, S. Jeon and C. Gale, arXiv:1307.6130 [nucl-th]

$$D\varepsilon + (\varepsilon + p)\theta - \pi^{\mu\nu} \nabla_{\perp(\mu} u_{\nu)} = 0 \longrightarrow \text{" energia "}$$

$$(\varepsilon + p) D u^{\alpha} - \nabla_{\perp}^{\alpha} p + \Delta_{\nu}^{\alpha} \partial_{\mu} \pi^{\mu\nu} = 0 \longrightarrow \text{" momento "}$$

$$\underline{\tau_{\pi}} \left(\Delta_{\alpha}^{\mu} \Delta_{\beta}^{\nu} D \pi^{\alpha\beta} + \frac{4}{3} \pi^{\mu\nu} \theta \right) + \pi^{\mu\nu} = 2\eta \sigma^{\mu\nu} \longrightarrow \text{" dissipação "}$$

τ_{π} → tempo de relaxação

$$D \equiv u^{\mu} \partial_{\mu}$$

$$\theta \equiv \partial^{\mu} u_{\mu}$$

$$\nabla_{\perp}^{\alpha} \equiv \Delta^{\alpha\mu} \partial_{\mu}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu} u^{\nu}$$

$$\sigma^{\mu\nu} \equiv \Delta^{\mu\nu\alpha\beta} \partial_{\alpha} u_{\beta}$$

$$\Delta^{\mu\nu\alpha\beta} \equiv (\Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\mu\beta} \Delta^{\nu\alpha})/2 - \Delta^{\mu\nu} \Delta^{\alpha\beta}/3$$

linearização:

$$\varepsilon(x,t) = \varepsilon_0 + \delta\varepsilon(x,t) \qquad p(x,t) = p_0 + \delta p(x,t)$$

$$\pi^{\mu\nu} = \delta\pi^{\mu\nu}(x,t) \qquad u^\mu(x,t) = (1,0,0,0) + (0,\delta u^x(x,t),0,0)$$

IS

$$\frac{\partial^2}{\partial x^2}\delta\varepsilon - \frac{1}{{c_s}^2}\frac{\partial^2}{\partial t^2}\delta\varepsilon - \frac{\tau_\pi}{{c_s}^2}\frac{\partial^3}{\partial t^3}\delta\varepsilon = -\left(\frac{\chi}{T_0\,{c_s}^2} + \tau_\pi\right)\frac{\partial}{\partial t}\frac{\partial^2}{\partial x^2}\delta\varepsilon$$

<u>Casos particulares :</u>

$$\chi = \frac{4}{3}\frac{\eta}{s_0}$$

Navier-Stokes : $\tau_\pi = 0 \longrightarrow \frac{\partial^2}{\partial x^2}\delta\varepsilon - \frac{1}{{c_s}^2}\frac{\partial^2}{\partial t^2}\delta\varepsilon = -\left(\frac{\chi}{T_0\,{c_s}^2}\right)\frac{\partial}{\partial t}\frac{\partial^2}{\partial x^2}\delta\varepsilon$

Fluido ideal : $\tau_\pi = 0$ e $\chi = 0 \longrightarrow \frac{\partial^2}{\partial x^2}\delta\varepsilon - \frac{1}{{c_s}^2}\frac{\partial^2}{\partial t^2}\delta\varepsilon = 0$

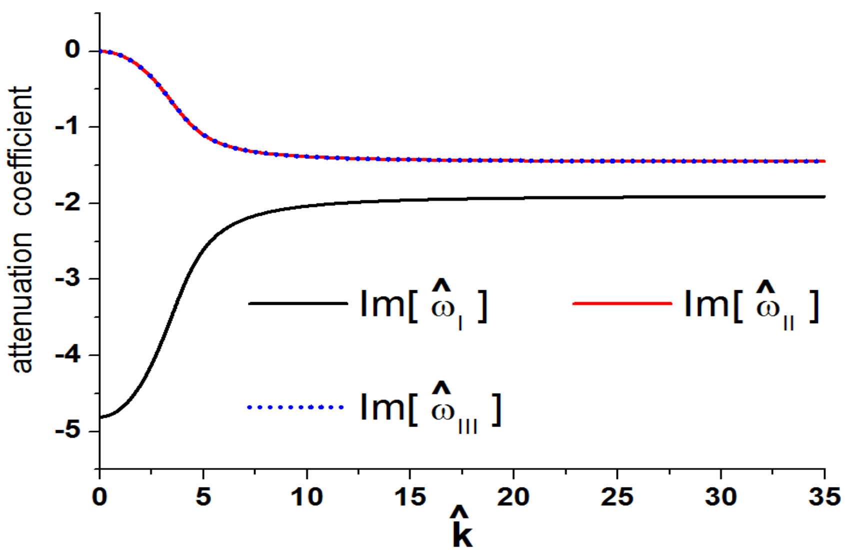
A equação :

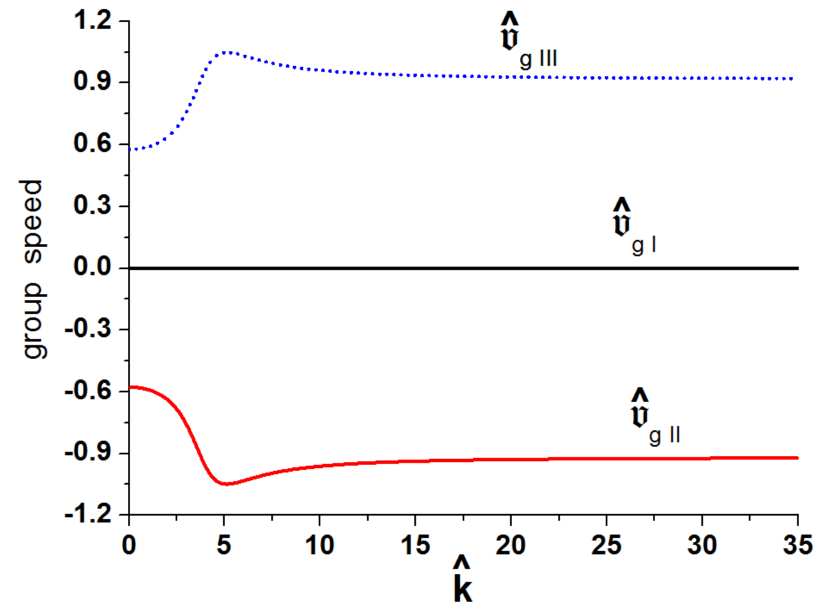
$$\frac{\partial^2}{\partial x^2}\delta\varepsilon - \frac{1}{{c_s}^2}\frac{\partial^2}{\partial t^2}\delta\varepsilon - \frac{\tau_\pi}{{c_s}^2}\frac{\partial^3}{\partial t^3}\delta\varepsilon = -\left(\frac{\chi}{T_0\,{c_s}^2} + \tau_\pi\right)\frac{\partial}{\partial t}\frac{\partial^2}{\partial x^2}\delta\varepsilon$$

com solução onda plana tem relação de dispersão:

$$-i\,\hat{\tau}_\pi\,\hat{\omega}^3 + \hat{\omega}^2 + \left(i\,\hat{k}^2\,\frac{4}{3}\frac{\eta}{s_0} + i\,\hat{k}^2\,{c_s}^2\,\hat{\tau}_\pi\right)\hat{\omega} - \hat{k}^2\,{c_s}^2 = 0$$

$$\hat{\omega} = Re[\hat{\omega}] + i\,Im[\hat{\omega}] \quad \Rightarrow \quad \delta\varepsilon(\hat{x},\hat{t}) = \mathcal{A}\,e^{Im[\hat{\omega}]\hat{t}}\,e^{iRe[\hat{\omega}]\left(\hat{k}\hat{x}/Re[\hat{\omega}]-\hat{t}\right)}$$





estabilidade : $Im[\hat{\omega}] < 0$

causalidade : $\hat{v}_g = dRe[\hat{\omega}]/d\hat{k}$ não diverge !

$\hat{t} = T_0\,t \quad \hat{k} = k/T_0 \quad \hat{\omega} = \omega/T_0$

$\hat{x} = T_0\,x \quad \hat{\tau}_\pi = T_0\,\tau_\pi$

${c_s}^2 = 1/3 \qquad \eta/s_0 = 1/(4\pi) \qquad \hat{\tau}_\pi = [2 - ln(2)]/(2\pi)$

Como observar melhor os efeitos de dissipação e relaxação ?

$$\frac{\partial^2}{\partial x^2}\delta\varepsilon - \frac{1}{{c_s}^2}\frac{\partial^2}{\partial t^2}\delta\varepsilon - \frac{\tau_\pi}{{c_s}^2}\frac{\partial^3}{\partial t^3}\delta\varepsilon = -\left(\frac{\chi}{T_0\,{c_s}^2} + \tau_\pi\right)\frac{\partial}{\partial t}\frac{\partial^2}{\partial x^2}\delta\varepsilon$$

$$\hat{\omega}_I \cong \hat{\omega}_1 = c_s\,\hat{k} - i\,\frac{2}{3}\frac{\eta}{s_0}\hat{k}^2 - \frac{2}{9\,c_s}\frac{\eta}{s_0}\left(\frac{\eta}{s_0} - 3{c_s}^2\,\hat{\tau}_\pi\right)\hat{k}^3 - i\frac{2}{9}\frac{\eta}{s_0}\hat{\tau}_\pi\left(4\frac{\eta}{s_0} - 3{c_s}^2\,\hat{\tau}_\pi\right)\hat{k}^4 + \mathcal{O}(\hat{k}^5)$$

$$\hat{\omega}_{II} \cong \hat{\omega}_2 = -c_s\,\hat{k} - i\,\frac{2}{3}\frac{\eta}{s_0}\hat{k}^2 + \frac{2}{9\,c_s}\frac{\eta}{s_0}\left(\frac{\eta}{s_0} - 3{c_s}^2\,\hat{\tau}_\pi\right)\hat{k}^3 - i\frac{2}{9}\frac{\eta}{s_0}\hat{\tau}_\pi\left(4\frac{\eta}{s_0} - 3{c_s}^2\,\hat{\tau}_\pi\right)\hat{k}^4 + \mathcal{O}(\hat{k}^5)$$

$$\hat{\omega}_{III} \cong \hat{\omega}_3 = -\frac{i}{\hat{\tau}_\pi} + i\,\frac{4}{3}\frac{\eta}{s_0}\hat{k}^2 + i\frac{4}{9}\frac{\eta}{s_0}\hat{\tau}_\pi\left(4\frac{\eta}{s_0} - 3{c_s}^2\,\hat{\tau}_\pi\right)\hat{k}^4 + \mathcal{O}(\hat{k}^5)$$

Como o tempo de relaxação se manifesta em ondas não-lineares ?

O que podemos aprender com isso ?

Ondas não-lineares

Reescrever as equações de IS nas variáveis adimensionais:

$$\hat{\varepsilon}(x,t) = \frac{\varepsilon(x,t)}{\varepsilon_0} \quad , \quad \hat{v}_x(x,t) = \frac{v_x(x,t)}{c_s} \quad \text{e} \quad \hat{\pi}^{xx}(x,t) = \frac{\pi^{xx}(x,t)}{\pi_0^{xx}}$$

e aplicar um método de tratamento de perturbações além da linearização:

Reductive Perturbation Method (RPM)

H. Washimi and T. Taniuti, Phys. Rev. Lett. 17, 996 (1966)

H. Leblond, J. Phys. B: At. Mol. Opt. Phys. 41, 043001 (2008)

→

Preservar a não-linearidade nas equações originais da hidrodinâmica.

Deduzir equações diferenciais para as perturbações.

então:

$$D\varepsilon + (\varepsilon + p)\theta - \pi^{\mu\nu}\nabla_{\perp(\mu}u_{\nu)} = 0$$

$$(\varepsilon + p)Du^{\alpha} - \nabla_{\perp}^{\alpha} p + \Delta_{\nu}^{\alpha}\partial_{\mu}\pi^{\mu\nu} = 0$$

$$\tau_{\pi}\left(\Delta_{\alpha}^{\mu}\Delta_{\beta}^{\nu}D\pi^{\alpha\beta} + \frac{4}{3}\pi^{\mu\nu}\theta\right) + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

→ RPM

$$\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_0} = 1 + \hat{\varepsilon}_1 + \hat{\varepsilon}_2 + \hat{\varepsilon}_3 + \dots$$

$$\frac{\partial}{\partial \hat{t}}\hat{\varepsilon}_1 + c_s\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 + \left[(1 - {c_s}^2)\left(\frac{\varepsilon_0}{\kappa {T_0}^4}\right)\right] c_s\hat{\varepsilon}_1\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 = \left(\frac{2}{3}\frac{\eta}{s_0}\right)\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_1$$

Burgers

$$\frac{\partial}{\partial \hat{t}}\hat{\varepsilon}_2 + c_s\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_2 + (1 - {c_s}^2)c_s\left(\frac{\varepsilon_0}{\kappa {T_0}^4}\right)\hat{\varepsilon}_1\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_2 - \left(\frac{2}{3}\frac{\eta}{s_0}\right)\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_2 + (1 - {c_s}^2)c_s\left(\frac{\varepsilon_0}{\kappa {T_0}^4}\right)\hat{\varepsilon}_2\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1$$

$$+\left(\frac{1}{3}\frac{\eta}{s_0}\right)\left(\frac{\varepsilon_0}{\kappa {T_0}^4}\right)(1 + {c_s}^2)\left(\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1\right)^2 + \left(\frac{2}{3}\frac{\eta}{s_0}\right)\left(\frac{\varepsilon_0}{\kappa {T_0}^4}\right)(1 + {c_s}^2)\hat{\varepsilon}_1\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_1 + {c_s}^2\left(\frac{\varepsilon_0}{\kappa {T_0}^4}\right)\hat{\varepsilon}_1\frac{\partial}{\partial \hat{t}}\hat{\varepsilon}_1$$

$$+{c_s}^3\left(\frac{\varepsilon_0}{\kappa {T_0}^4}\right)\hat{\varepsilon}_1\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 + \left(\frac{2}{3}\frac{\eta}{s_0}\right)\left[\left(\frac{1}{3c_s}\right)\left(\frac{\eta}{s_0}\right) - c_s\hat{\tau}_{\pi}\right]\frac{\partial^3}{\partial \hat{x}^3}\hat{\varepsilon}_1 = 0$$

$$\hat{\tau}_{\pi} = T_0\,\tau_{\pi} \qquad \varepsilon_0/(\kappa {T_0}^4) = 3/4 \qquad {c_s}^2 = 1/3$$

$$\hat{t} = T_0\,t \qquad \hat{x} = T_0\,x$$

$$\frac{\partial}{\partial \hat{t}}\hat{\varepsilon}_1 + c_s \frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 + \left[(1 - c_s{}^2)\left(\frac{\varepsilon_0}{\kappa\, T_0{}^4}\right)\right] c_s \hat{\varepsilon}_1 \frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 = \left(\frac{2}{3}\frac{\eta}{s_0}\right)\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_1$$

(igual em NS)

$$\frac{\partial}{\partial \hat{t}}\hat{\varepsilon}_2 + c_s \frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_2 + (1 - c_s{}^2) c_s \left(\frac{\varepsilon_0}{\kappa\, T_0{}^4}\right)\hat{\varepsilon}_1 \frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_2 - \left(\frac{2}{3}\frac{\eta}{s_0}\right)\frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_2 + (1 - c_s{}^2) c_s \left(\frac{\varepsilon_0}{\kappa\, T_0{}^4}\right)\hat{\varepsilon}_2 \frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1$$

$$+\left(\frac{1}{3}\frac{\eta}{s_0}\right)\left(\frac{\varepsilon_0}{\kappa\, T_0{}^4}\right)(1 + c_s{}^2)\left(\frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1\right)^2 + \left(\frac{2}{3}\frac{\eta}{s_0}\right)\left(\frac{\varepsilon_0}{\kappa\, T_0{}^4}\right)(1 + c_s{}^2)\hat{\varepsilon}_1 \frac{\partial^2}{\partial \hat{x}^2}\hat{\varepsilon}_1 + c_s{}^2\left(\frac{\varepsilon_0}{\kappa\, T_0{}^4}\right)\hat{\varepsilon}_1 \frac{\partial}{\partial \hat{t}}\hat{\varepsilon}_1$$

$$+c_s{}^3\left(\frac{\varepsilon_0}{\kappa\, T_0{}^4}\right)\hat{\varepsilon}_1 \frac{\partial}{\partial \hat{x}}\hat{\varepsilon}_1 + \left(\frac{2}{3}\frac{\eta}{s_0}\right)\left[\left(\frac{1}{3c_s}\right)\left(\frac{\eta}{s_0}\right) - c_s \hat{\tau}_\pi\right]\frac{\partial^3}{\partial \hat{x}^3}\hat{\varepsilon}_1 = 0$$

termo dispersivo

interessante ...

$$\hat{\omega}_1 = \cdots - \frac{2}{9\, c_s}\frac{\eta}{s_0}\left(\frac{\eta}{s_0} - 3 c_s{}^2\, \hat{\tau}_\pi\right)\hat{k}^3 - i\frac{2}{9}\frac{\eta}{s_0}\hat{\tau}_\pi\left(4\frac{\eta}{s_0} - 3 c_s{}^2\, \hat{\tau}_\pi\right)\hat{k}^4 + \cdots$$

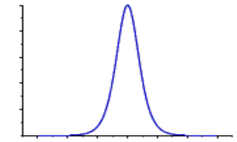
$$\hat{\omega}_2 = \cdots + \frac{2}{9\, c_s}\frac{\eta}{s_0}\left(\frac{\eta}{s_0} - 3 c_s{}^2\, \hat{\tau}_\pi\right)\hat{k}^3 - i\frac{2}{9}\frac{\eta}{s_0}\hat{\tau}_\pi\left(4\frac{\eta}{s_0} - 3 c_s{}^2\, \hat{\tau}_\pi\right)\hat{k}^4 + \cdots$$

$$\hat{\omega}_3 = \cdots + i\frac{4}{9}\frac{\eta}{s_0}\hat{\tau}_\pi\left(4\frac{\eta}{s_0} - 3 c_s{}^2\, \hat{\tau}_\pi\right)\hat{k}^4 + \cdots$$

Soluções numéricas

$$\hat{\varepsilon}_1(\hat{x},0) = A_1\ sech^2\left(\frac{\hat{x}}{B_1}\right)$$

$sech^2 \rightarrow$



$$\frac{\partial}{\partial\hat{t}}\hat{\varepsilon}_1 + c_s\frac{\partial}{\partial\hat{x}}\hat{\varepsilon}_1 + \left[(1-{c_s}^2)\left(\frac{\varepsilon_0}{\kappa\,{T_0}^4}\right)\right]c_s\hat{\varepsilon}_1\frac{\partial}{\partial\hat{x}}\hat{\varepsilon}_1 = \left(\frac{2}{3}\frac{\eta}{s_0}\right)\frac{\partial^2}{\partial\hat{x}^2}\hat{\varepsilon}_1$$

$\hat{\varepsilon}_1$ solução numérica

$$\hat{\varepsilon}_2(\hat{x},0) = A_2\ sech^2\left(\frac{\hat{x}}{B_2}\right)$$

$$\frac{\partial}{\partial\hat{t}}\hat{\varepsilon}_2 + c_s\frac{\partial}{\partial\hat{x}}\hat{\varepsilon}_2 + (1-{c_s}^2)c_s\left(\frac{\varepsilon_0}{\kappa\,{T_0}^4}\right)\hat{\varepsilon}_1\frac{\partial}{\partial\hat{x}}\hat{\varepsilon}_2 - \left(\frac{2}{3}\frac{\eta}{s_0}\right)\frac{\partial^2}{\partial\hat{x}^2}\hat{\varepsilon}_2 + (1-{c_s}^2)c_s\left(\frac{\varepsilon_0}{\kappa\,{T_0}^4}\right)\hat{\varepsilon}_2\frac{\partial}{\partial\hat{x}}\hat{\varepsilon}_1$$

$$+\left(\frac{1}{3}\frac{\eta}{s_0}\right)\left(\frac{\varepsilon_0}{\kappa\,{T_0}^4}\right)(1+{c_s}^2)\left(\frac{\partial}{\partial\hat{x}}\hat{\varepsilon}_1\right)^2 + \left(\frac{2}{3}\frac{\eta}{s_0}\right)\left(\frac{\varepsilon_0}{\kappa\,{T_0}^4}\right)(1+{c_s}^2)\hat{\varepsilon}_1\frac{\partial^2}{\partial\hat{x}^2}\hat{\varepsilon}_1 + {c_s}^2\left(\frac{\varepsilon_0}{\kappa\,{T_0}^4}\right)\hat{\varepsilon}_1\frac{\partial}{\partial\hat{t}}\hat{\varepsilon}_1$$

$$+{c_s}^3\left(\frac{\varepsilon_0}{\kappa\,{T_0}^4}\right)\hat{\varepsilon}_1\frac{\partial}{\partial\hat{x}}\hat{\varepsilon}_1 + \left(\frac{2}{3}\frac{\eta}{s_0}\right)\left[\left(\frac{1}{3c_s}\right)\left(\frac{\eta}{s_0}\right) - c_s\hat{\tau}_\pi\right]\frac{\partial^3}{\partial\hat{x}^3}\hat{\varepsilon}_1 = 0$$

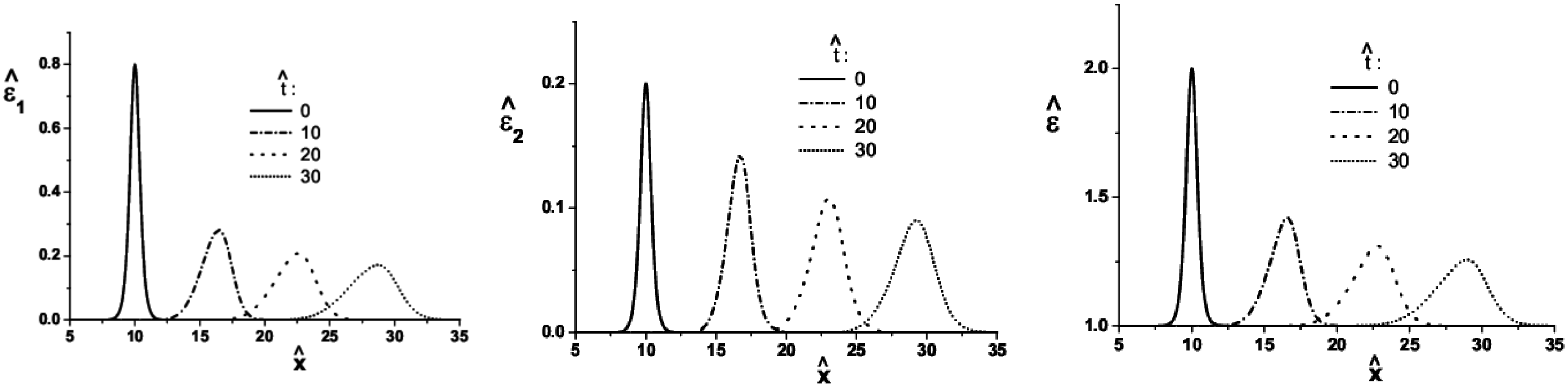
$\hat{\varepsilon}_2$ solução numérica

$$\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_0} = 1 + \hat{\varepsilon}_1 + \hat{\varepsilon}_2 + \ldots$$

$B_1 = B_2 = 0.5$

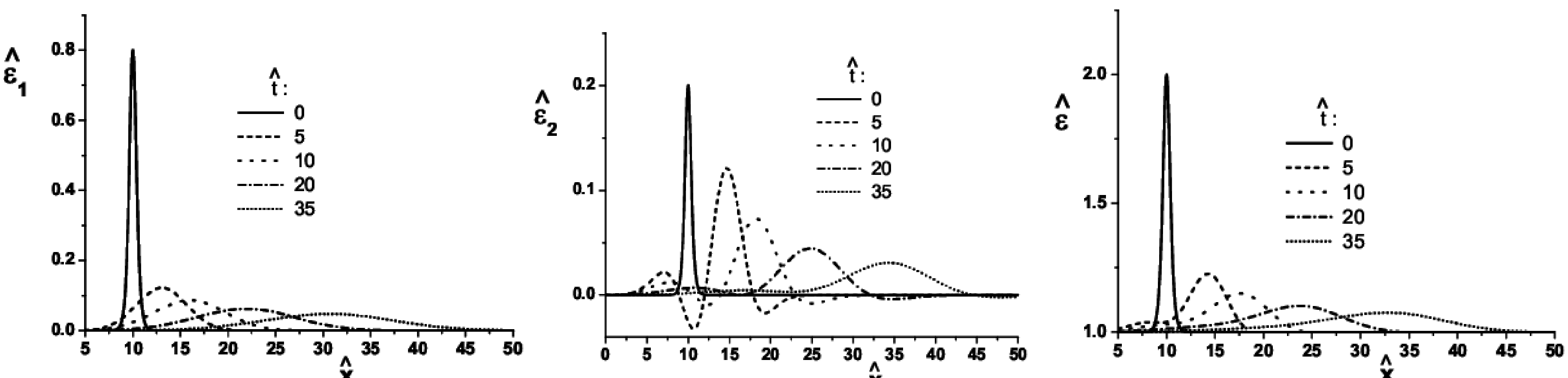
Resultados esperados:
Pulso inicial sobrevive apesar de sofrer perdas.
Aumento da viscosidade e relaxação: dissipação acentuada.

$\eta / s_0 = 1/(4\pi) \qquad \hat{\tau}_\pi = [2 - ln(2)]/(2\pi)$:



$$\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_0} = 1 + \hat{\varepsilon}_1 + \hat{\varepsilon}_2 + \ldots$$

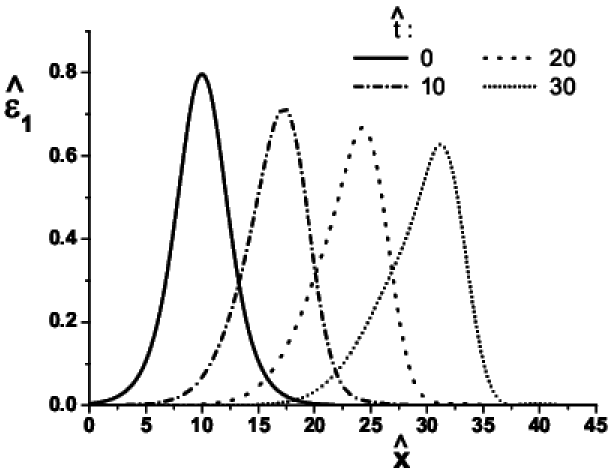
$\eta / s_0 = 1 \qquad \hat{\tau}_\pi = 5$:

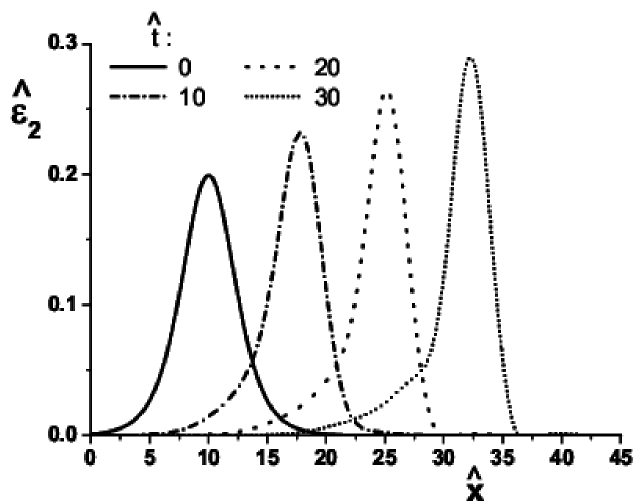


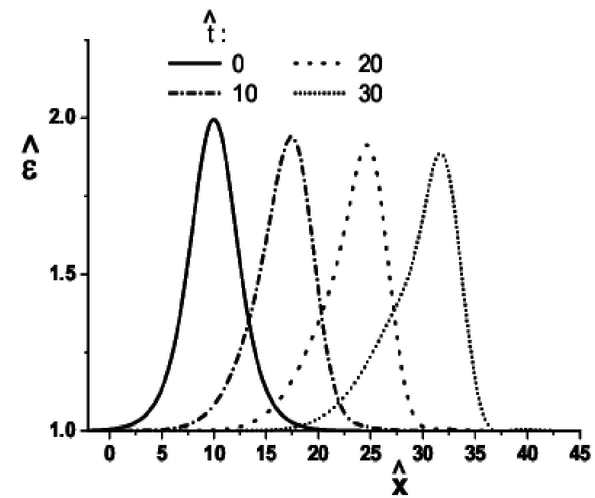
<u>Efeitos de aumento de largura :</u> comportamento " solitônico " com formação de parede:

$\eta/s_0 = 1/(4\pi)$ $\quad \hat{\tau}_\pi = [2 - ln(2)]/(2\pi)$:

$B_1 = B_2 = 3$

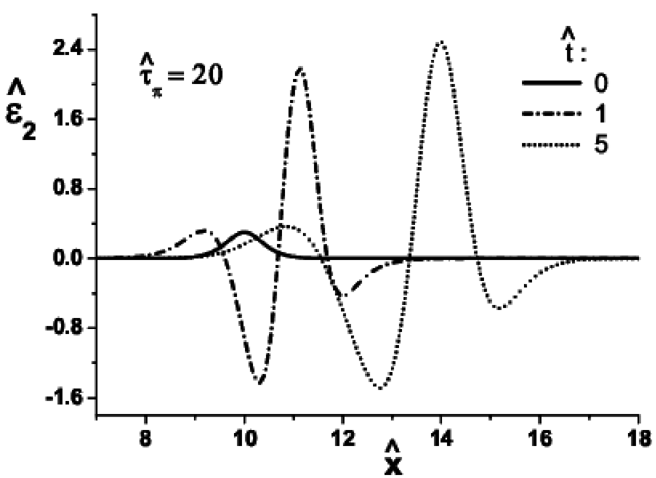


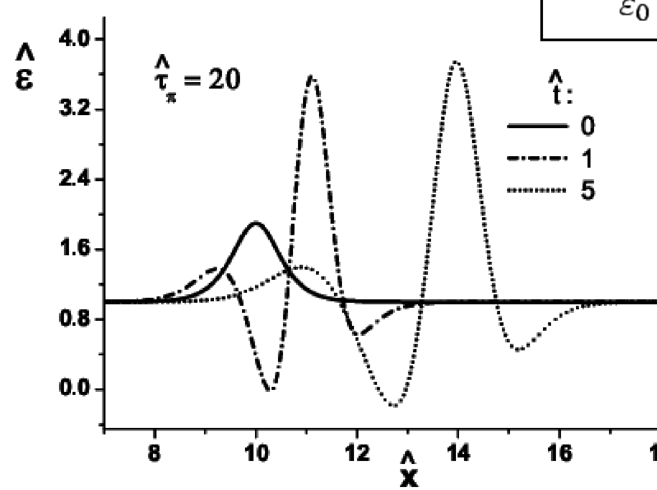




<u>Extrapolação de Israel-Stewart</u>: aumento exagerado de relaxação causa problemas !

$$\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_0} = 1 + \hat{\varepsilon}_1 + \hat{\varepsilon}_2 + \dots$$





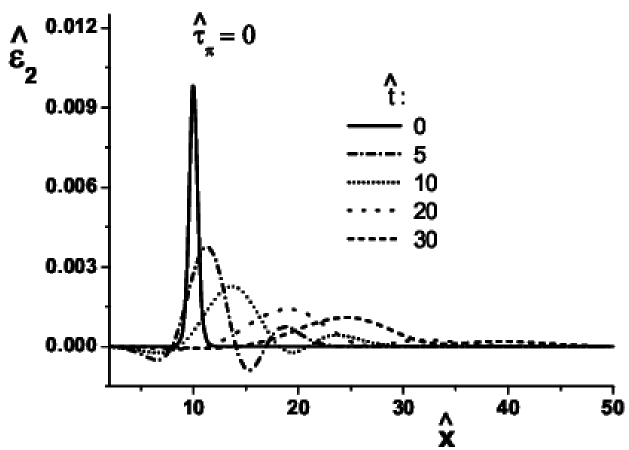
$\eta/s_0 = 1/(4\pi)$

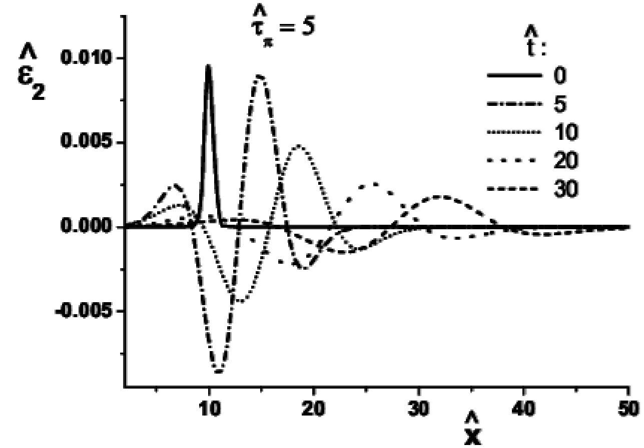
$A_1 = 0.6 \quad A_2 = 0.3$

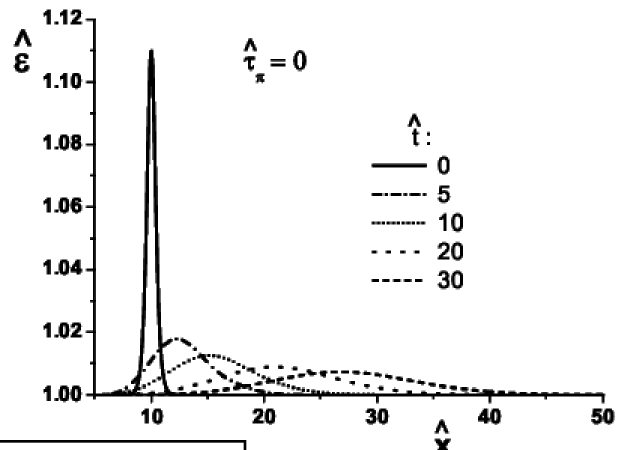
$B_1 = 0.7 \quad B_2 = 0.5$

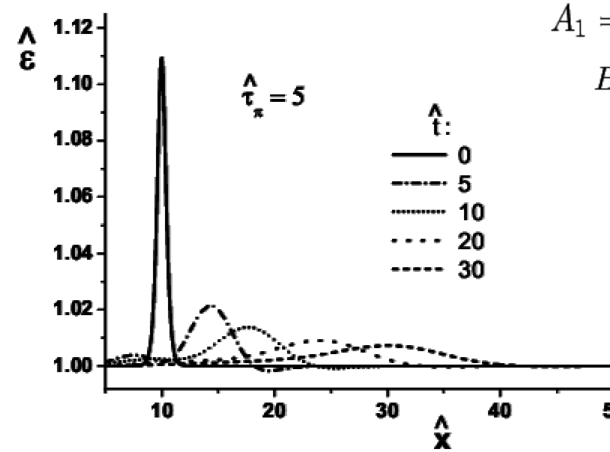
Navier-Stokes versus Israel-Stewart :

$\eta / s_0 = 1$









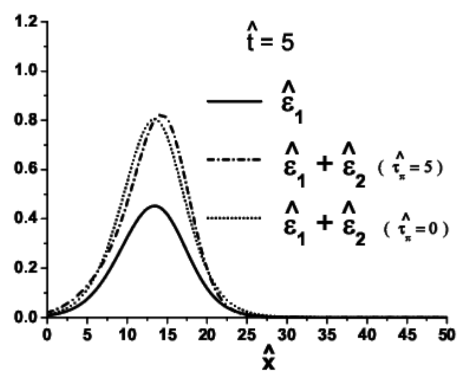
$A_1 = 0.1 \;\; A_2 = 0.01$

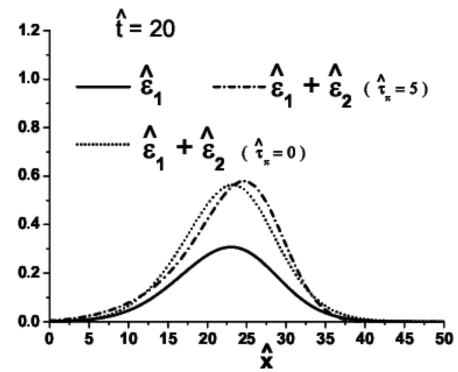
$B_1 = B_2 = 0.5$

$$\hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_0} = 1 + \hat{\varepsilon}_1 + \hat{\varepsilon}_2 + \ldots$$

$A_1 = 0.6 \;\; A_2 = 0.4$

$B_1 = B_2 = 4$





Considerações finais

Perturbações não-lineares em Israel-Stewart: sistema de equações diferenciais acopladas para perturbações na densidade de energia

Tratamento semi-analítico: possibilita investigar efeitos de relaxação sem utilizar simulações hidrodinâmicas mais complexas

Equação de Burgers : obtida para a perturbação em primeira ordem, equivale ao resultado não-linear de Navier-Stokes

Largura inicial : pulsos mais largos " vivem mais " , estabilidade tipo " sóliton "

Possível limite superior da escala de relaxação :
relaxação muito grande aumenta amplitude dos pulsos, contradizendo a teoria de perturbação.

Israel-Stewart : favorece a formação de " parede ", onda de choque.

Obrigado !