# Dressed perturbation theory for nonperturbative QCD

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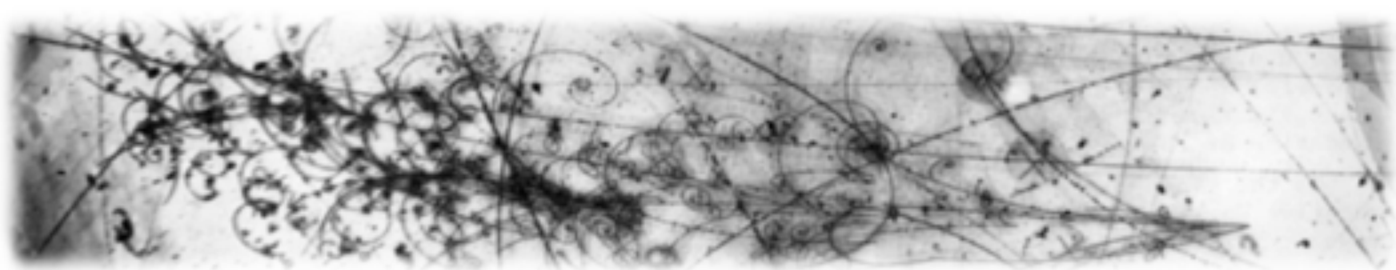

XXV REunião de Trabalho
sobre INterações HAdrônicas

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# Física hadrônica

## Do que se trata?

Estudo da estrutura e interações
de partículas subatômicas
que interagem através
da força forte

# Física hadrônica
## - o desafio maior

Sob a luz da QCD, revelar os graus de liberdade e os mecanismos que governam a formação dos hádrons, suas massas, seus tamanhos e suas interações

# O que é a QCD?!

# QCD é uma teoria, parte do modelo padrão

Modelo padrão está completo com a descoberta do Higgs

Setor forte, descrito pela QCD, único (especial, singular):

- Nunca antes na física uma teoria (ou modelo) é formulada com graus de liberdade que não podem ser observados em instrumentos de medida (confinamento)

- Graus de liberdade fundamentais da matéria visível no universo não pesam quase nada, mas essa matéria é muito pesada (quebra dinâmica da simetria quiral)

Fenômenos emergentes

# QCD - teoria perfeita

Renormalizável não perturbativamente

- uma bem definida teoria quântica de campos

Nenhuma evidência de falhas ou imperfeições

- pelo menos até 8 TeV

Provável modelo para extensões do modelo padrão

- *a la* Technicolor, setor de Higgs é uma descrição efetiva

# Quebra dinâmica da simetria quiral QDSQ

## Simetria aproximada no setor de quarks leves

- quarks de massas quase nulas, dubletos de hádrons de paridade oposta
  MAS: N(940), hádron de paridade oposta mais próximo N(1535)
- simetria é quebrada dinâmicamente (QDSQ)

## QDSQ: teorema de Goldstone

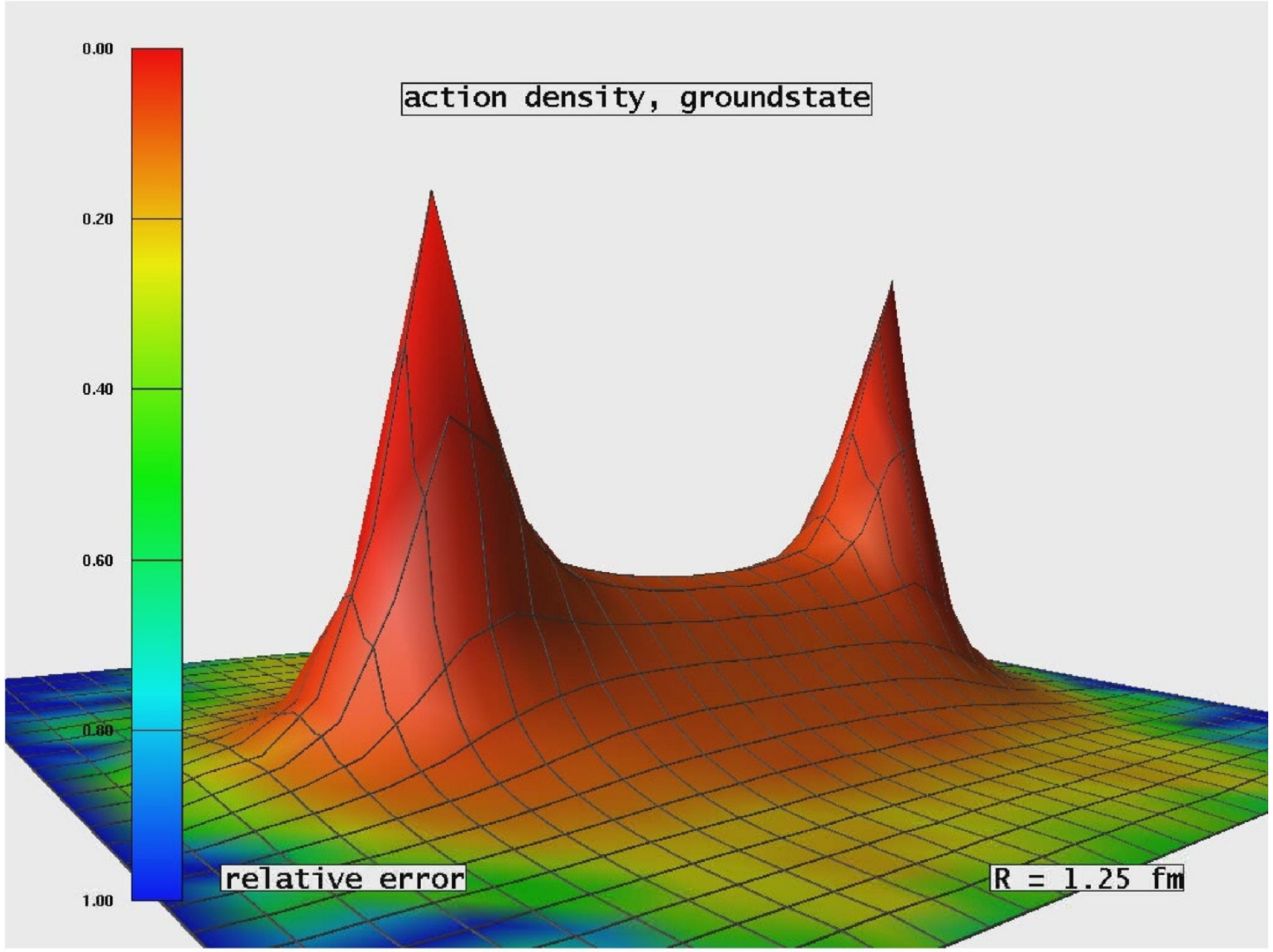
- mésons pi quase sem massa (káons também "sentem" o efeito)
- quarks ficam pesados
- limite quiral: força nuclear de alcance infinito, raios de carga infinitos
- píons leves implodem a noção simplista (convencional) de confinamento

**Mecanismo de Higgs (quase que) completamente irrelevante para a geração da massa visível no universo**

- Folklore … ***Hall-D Conceptual Design Report(5)***

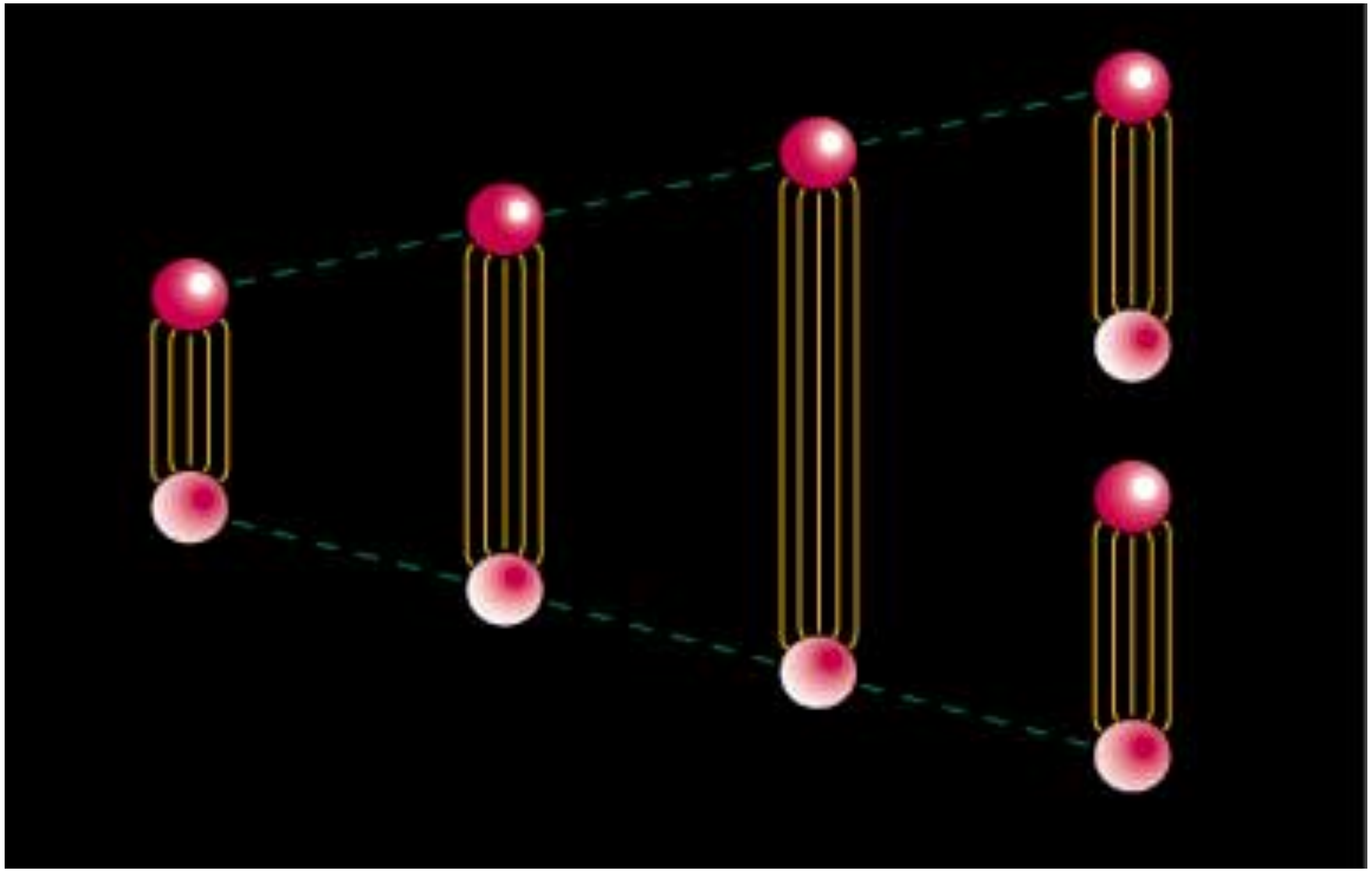
"The color field lines between a quark and an anti-quark form flux tubes. A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons."




Slide de Craig Roberts

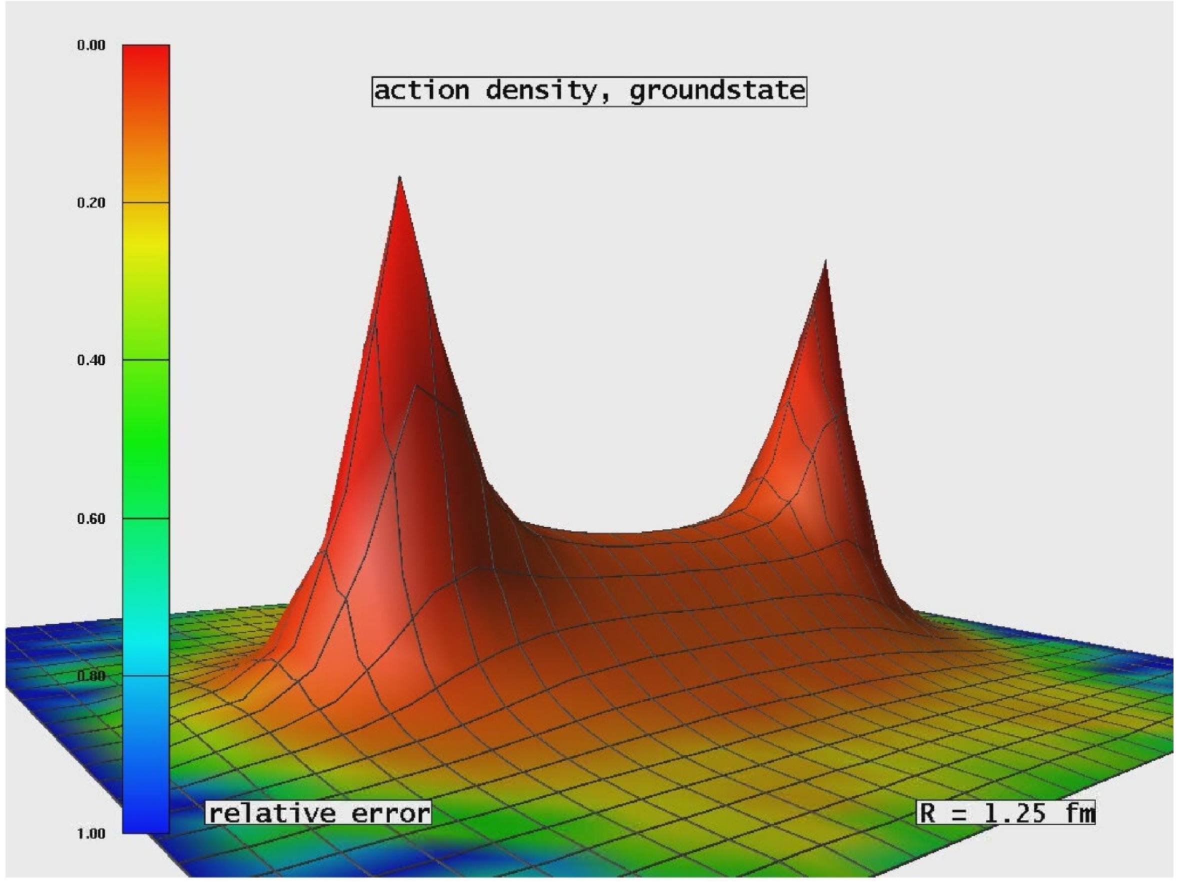
# Tubos de fluxo quebram



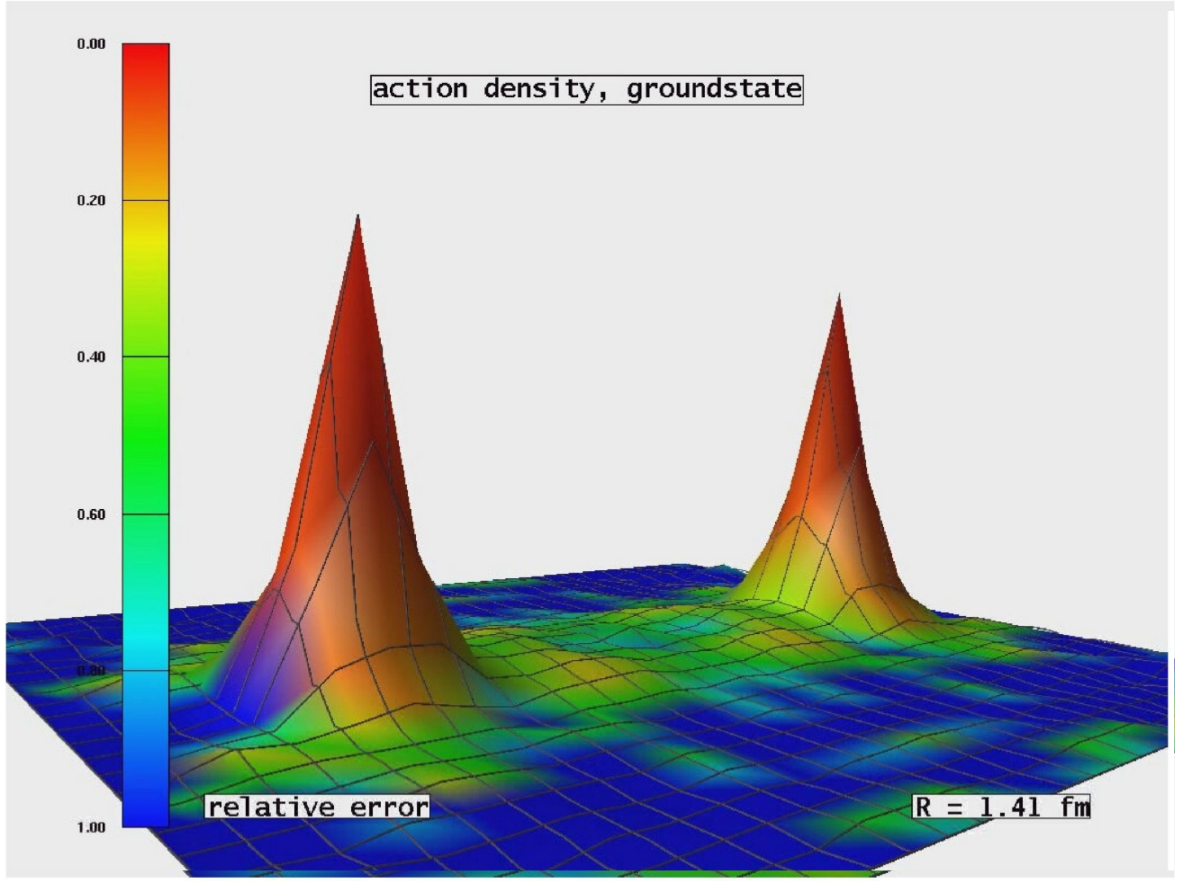
Corda esticada de 1 fm ~ 10 píons

# Presença de quarks leves

# Presença de quarks leves



# Presença de quarks leves



# O que gostaríamos de fazer

A partir da QCD, calcular (e entender) tudo sobre os hádrons

- massas
- tamanhos
- interações

# Próxima década

Uma quantidade enorme de dados experimentais estará disponível, com a qual espera-se:

- Sobre tudo: elucidar o significado do confinamento e possível conexão com a QDSQ

- Calcular PDF, GPD, TMD
  Não basta fitar, não vamos aprender nada fitando

É necessário um esquema de cálculo que respeite invariância de Poincaré

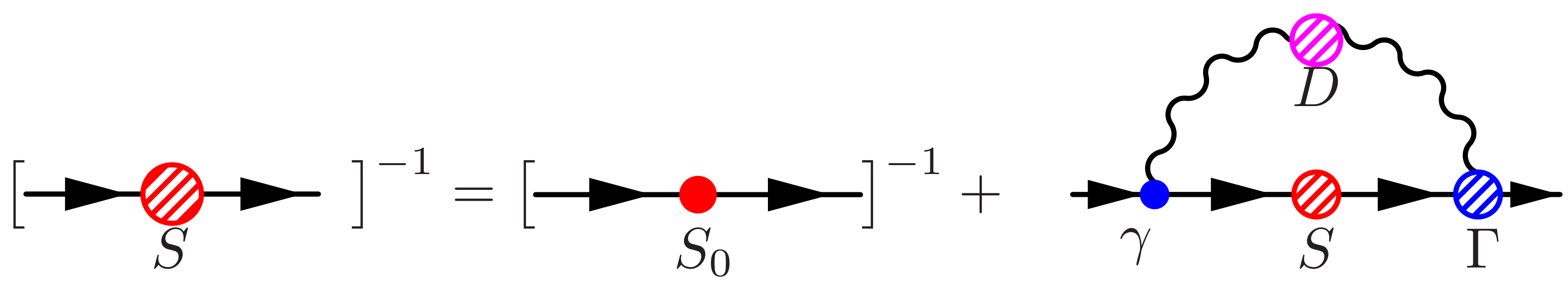
# Acesso a observáveis

## Funções de correlação

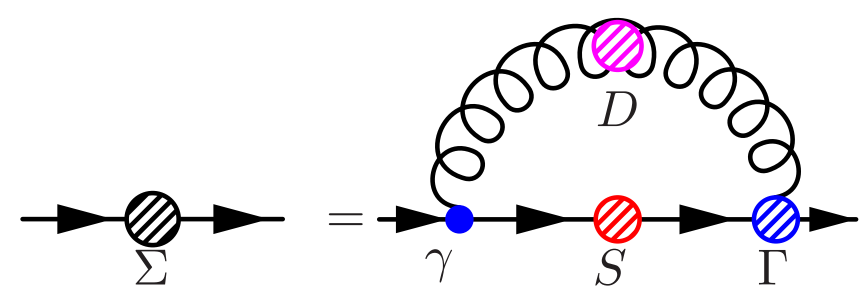
### Equações de Dyson-Schwinger

- propagadores
- funções de vértice
- estados ligados, "funções de onda"
- fatores de forma
- funções de distribuição de partons (PDF, GPD, TMD)
- muitos corpos: coeficientes cinéticos (difusão, condutividade térmica, viscosidade, etc)

# Propagador do quark

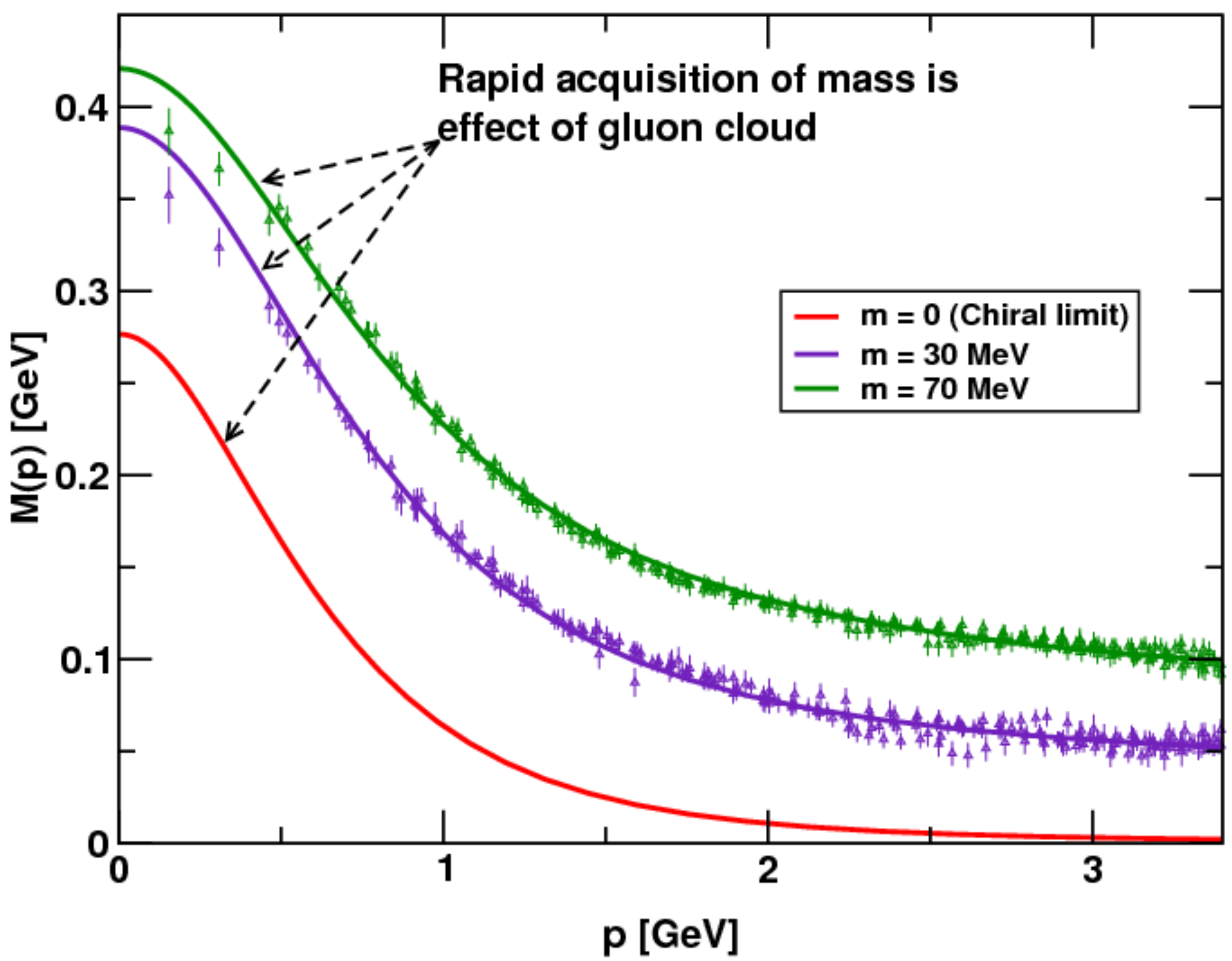



$$S^{-1}(p) = S_0^{-1}(p) - \Sigma(p)$$




$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

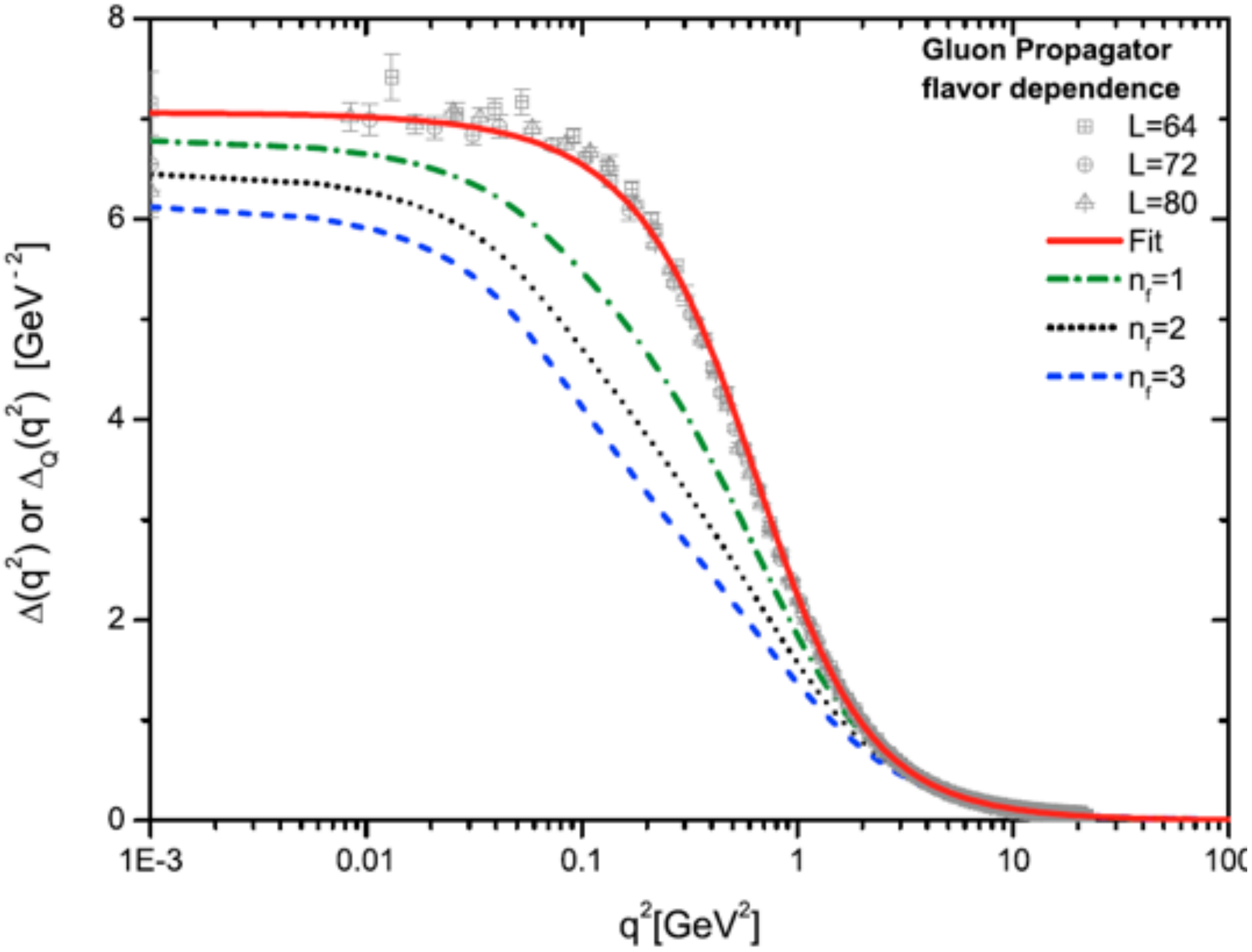
# Propagador do quark




$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$

C.D. Roberts, Prog. Part. Nucl. Phys. 61, 50 (2008)

# Propagador do glúon



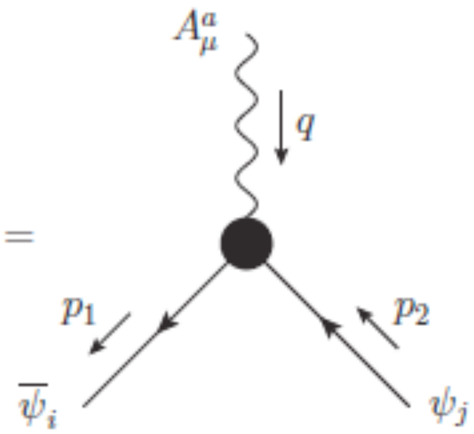

Unquenched gluon propagator:

A.C. Aguilar, D. Binosi, J. Papavassiliou, PRD 86, 14032 (2012)

# Vértice quark-glúon

$$i\Gamma_{A^a_\mu \psi_j \overline{\psi}_i}(q, p_2, -p_1) = i g t^a_{ij} \Gamma_\mu(q, p_2, -p_1); \qquad \Gamma^{(0)}_\mu(q, p_2, -p_1) = \gamma_\mu; \qquad q + p_2 = p_1$$

$$i\Gamma_{A^a_\mu \psi_j \overline{\psi}_i}(q, p_2, -p_1) =$$

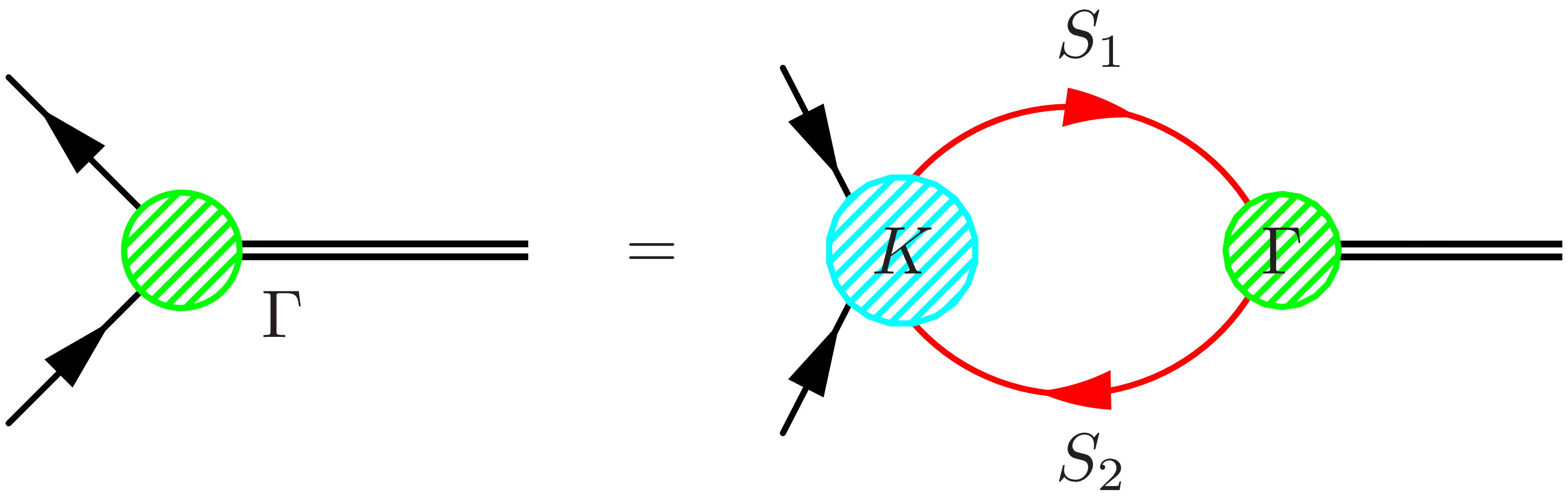



$$q + p_2 = p_1$$

$$\Gamma_\mu(q, p_2, -p_1) = L_1 \gamma_\mu + L_2 (\not{p}_1 + \not{p}_2)(p_1 + p_2)_\mu + L_3 (p_1 + p_2)_\mu + L_4 \widetilde{\sigma}_{\mu\nu} (p_1 + p_2)^\nu$$


A.C. Aguilar, D. Bonosi, J.C. Cardona, J. Papavassiliou, PoS ConfinementX (2012) 103
E. Rojas, J. P. B. C. de Melo, B. El-Bennich, O. Oliveira, T. Frederico, arXiv 1306.3022 [hep-ph]

# Equação de Bethe-Salpeter

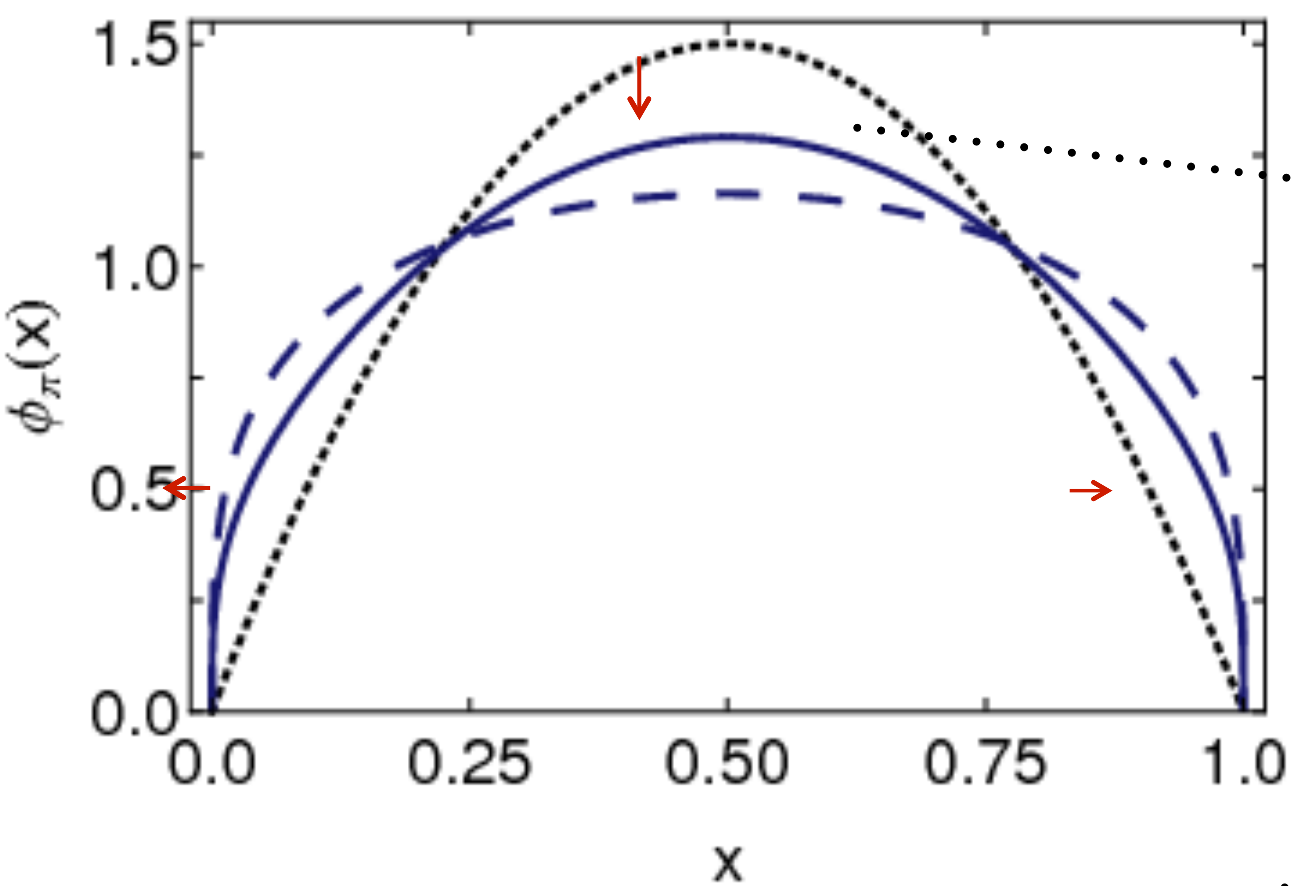



$$\Gamma(k;P) = \int \frac{d^4q}{(2\pi)^4} K(P;k,q)\, S_1(q+\eta_+P)\, \Gamma(q;P)\, S_2(q-\eta_-P)$$

$\eta_+ + \eta_- = 1$ **Estado ligado:** $P^2 = -M^2_{\mathrm{meson}}$

# Distribuição de quarks de valência no píon

$$\varphi_\pi(x) = \int \frac{d^4k}{(2\pi)^4}\,\delta(n\cdot k - x n\cdot P)\,\mathrm{Tr}\left[\gamma_5\,\gamma\cdot n\,S(k)\,\Gamma_\pi(k,P)\,S(k-P)\right]$$




$$\varphi_\pi(x) = x(1-x)$$

*Dilation of pion's wave function is measurable in pion's electromagnetic form factor at JLab12*

L. Chang et al. PRL 110, 132001 (2013)

# Mais ainda:

1) Espectroscopia: estados excitados, glueballs & híbridos, charm X,Y,Z

2) Fatores de forma e.m., papel da QDSQ

3) Produção de hádrons

4) Colisões de íons pesados

5) Estrelas de nêutrons


Thanks to Craig Roberts for the picture

# Dressed perturbation theory (DPT) *

- start with an ansatz (rainbow-ladder) for dressed propagators, improve with perturbation theory using the dressed propagators

Similar to:

- linear δ expansion
  A. Okopinska, Phys. Rev. D 35, 1835 (1987)
  M. Moshe and A. Duncan, Phys. Lett. B 215, 352 (1988).

- screened perturbation theory
  J. O. Andersen, E. Braaten, and M. Strickland, PRD 63, 105008 (2001)

- our work: relativistic nuclear many–body problem & phase transitions
  GK, D. P. Menezes and M. B. Pinto, Phys. Lett. B 370, 5 (1996)
  GK, R. S. M. de Carvalho, D. P. Menezes, M. Nielsen and M. B. Pinto, Eur. Phys. J. A 1, 45 (1998)
  R.L.S. Farias, GK, R.O. Ramos, PRD78, 065046 (2008)
  N. C. Cassol-Seewald, R.L.S. Farias, GK, R.S. Marques de Carvalho, Int. J. Mod. Phys. C 23, 1240016 (2012)

*Fernando Serna & Marco Brito

# QCD generation functional

$$Z = \int \mathcal{D}(\bar{\psi}, \psi, A)\; e^{-\mathcal{S}} = \int \mathcal{D}(\bar{\psi}, \psi, A)\; e^{-\mathcal{S}_0 - \mathcal{S}_I}$$

$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_I$ QCD action

# QCD action

$$\mathcal{S}_0 = \int d^4x\, d^4y\, \bar{\psi}(x)\, S_0^{-1}(x-y)\, \psi(y) + \frac{1}{2}\int d^4x\, d^4y\, A_\mu^a(x) \left(D_0^{-1}\right)_{\mu\nu}^{ab}(x-y)\, A_\nu^b(y)$$

$$\mathcal{S}_I = \int d^4x \Big[ ig\, \bar{\psi}(x)\gamma\cdot A^a T^a \psi(x) - g f^{abc} \partial_\mu A_\nu^a(x) A_\mu^b(x) A_\nu^c(x) + \frac{1}{4} g^2 f^{abc} f^{ade} A_\mu^b(x) A_\nu^c A_\mu^d(x) A_\nu^e(x) \Big]$$

Simplify discussion:

- no ghost fields
- no renormalization factors

# Noniteracting quark and gluon propagators

$$S_0^{-1}(x-y) \quad = \quad (\gamma_\mu \partial_\mu + m)\, \delta^{(4)}(x-y),$$

$$\left(D_0^{-1}\right)^{ab}_{\mu\nu}(x-y) \quad = \quad \delta^{ab}\left[-\delta_{\mu\nu}\partial^2 + \left(1-\frac{1}{\xi}\right)\partial_\mu \partial_\nu\right]\delta^4(x-y)$$

# Idea of DPT

Three steps:

1) Add to and subtract from the QCD action quark and gluon self-energies

$$\mathcal{S}_0 \to \overline{\mathcal{S}}_0 = \mathcal{S}_0 + \langle \bar{\psi}\, \bar{\Sigma}\, \psi \rangle + \langle A_\mu\, \bar{\Pi}_{\mu\nu}\, A_\nu \rangle,$$

$$\mathcal{S}_I \to \overline{\mathcal{S}}_I = \mathcal{S}_I - \langle \bar{\psi}\, \bar{\Sigma}\, \psi \rangle - \langle A_\mu\, \bar{\Pi}_{\mu\nu}\, A_\nu \rangle,$$

2) Correlation functions are calculated perturbatively with

$$\overline{S}_I$$

3) A criterion is defined to fix self-consistently the self-energies

$$\bar{\Sigma} \quad \text{and} \quad \bar{\Pi}_{\mu\nu}$$

# First step

$$
\begin{aligned}
\bar{\mathcal{S}}_0 &= \int d^4x\, d^4y\, \bar{\psi}(x)\, \bar{S}^{\,-1}(x-y)\, \psi(y) \\
&+ \frac{1}{2}\int d^4x d^4y\, A^a_\mu(x) \left(\bar{D}^{-1}\right)^{ab}_{\mu\nu}(x-y)\, A^b_\nu(y)
\end{aligned}
$$

New "noninteracting"quark and gluon propagators:

$$
\begin{aligned}
\bar{S}^{\,-1}(x-y) &= S_0^{-1}(x-y) + \bar{\Sigma}(x-y), \\
\left(\bar{D}^{\,-1}\right)^{ab}_{\mu\nu}(x-y) &= \left(D_0^{-1}\right)^{ab}_{\mu\nu}(x-y) + \bar{\Pi}^{ab}_{\mu\nu}(x-y)
\end{aligned}
$$

# New propagators

$$\overline{S}(p) = \frac{1}{i\gamma \cdot p + m + \overline{\Sigma}(p)},$$

$$\overline{D}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2[1+\overline{\Pi}(p^2)]} + \xi\frac{p_\mu p_\nu}{p^4}$$

New interactions:

$$\begin{aligned}\bar{S}_I &= \delta \int d^4x \Big[ ig\bar{\psi}(x)\gamma\cdot A^a T^a \psi(x) - g f^{abc}\partial_\mu A^a_\nu(x) A^b_\mu(x) A^c_\nu(x) \\ &\quad + \frac{1}{4}\delta g^2 f^{abc} f^{ade} A^b_\mu(x) A^c_\nu A^d_\mu(x) A^e_\nu(x)\Big] \\ &\quad - \delta^2 \int d^4x d^4y \left[\bar{\psi}(x)\,\bar{\Sigma}(x-y)\,\psi(y) - \frac{1}{2} A^a_\mu(x)\,\bar{\Pi}^{ab}_{\mu\nu}(x-y)\,A^b_\nu(y)\right]\end{aligned}$$

$\delta$ : bookeeping of the perturbative expansion

# Quark and gluon self-energies

- obtained as power series in $\delta$

$$\Sigma = \Sigma_0 + \delta^2 \Sigma_2 + \delta^4 \Sigma_4 + \cdots$$

$$\Pi = \Pi_0 + \delta^2 \Pi_2 + \delta^4 \Pi_4 + \cdots$$

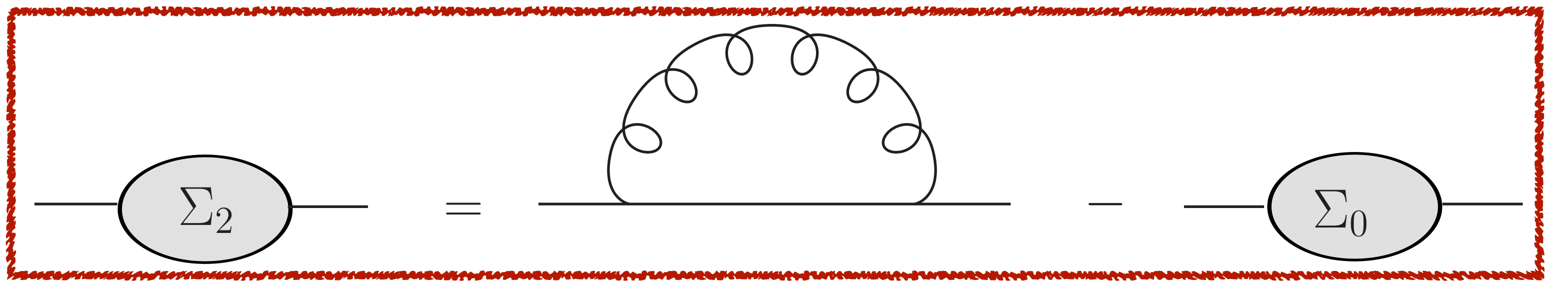
# Criterion to fix the added quark and gluon self-energies

Impose:

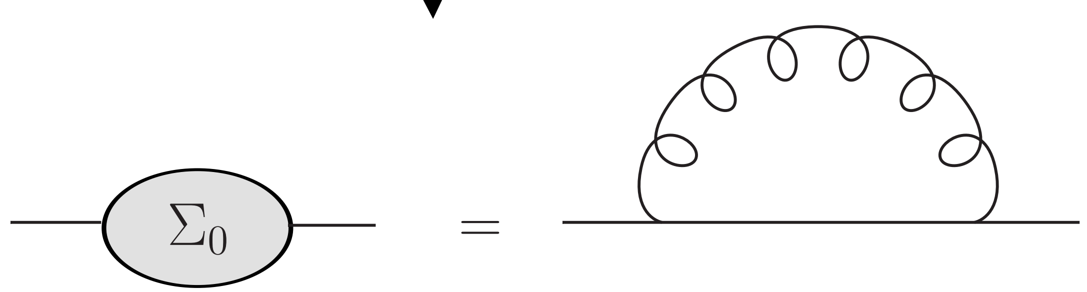
$$\Sigma_2 = 0$$

$$\Pi_2 = 0$$

# Quark DSE - 2nd order

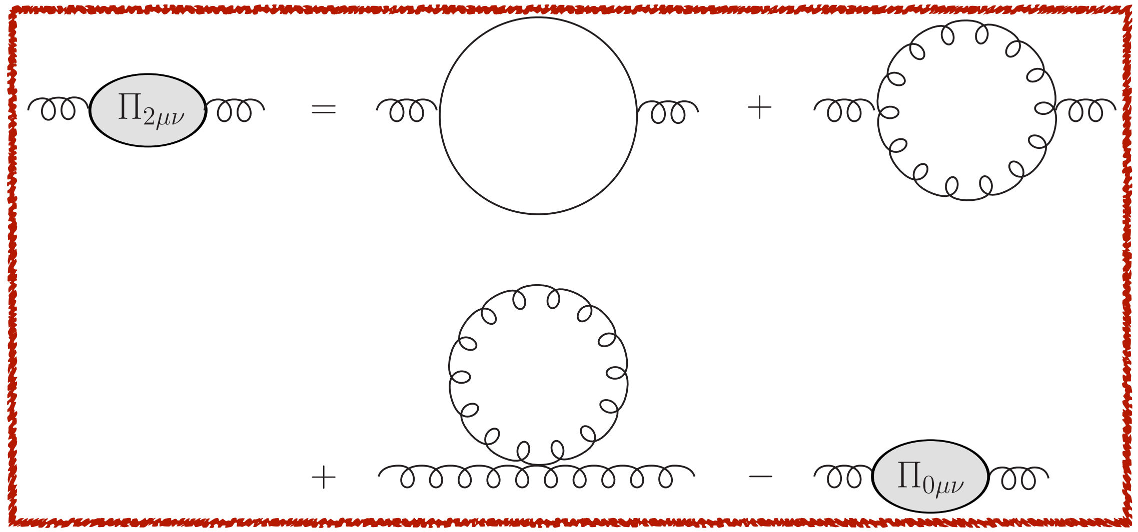



$$\Sigma_2 = 0$$

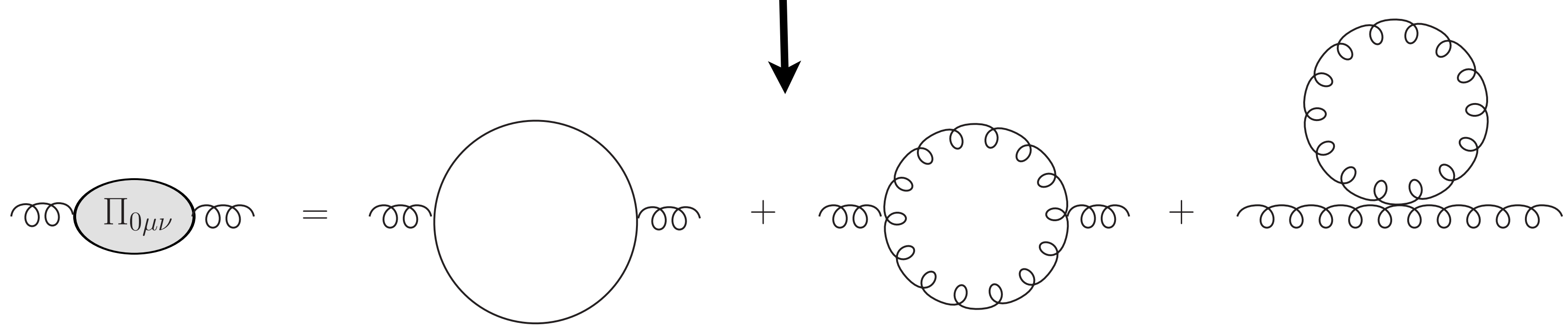



$$\overline{\Sigma}(p) = \frac{4}{3} g^2 \int \frac{d^4q}{(2\pi)^4} \overline{D}_{\mu\nu}(p-q)\, \gamma_\mu\, \overline{S}(q)\, \gamma_\nu$$

# Gluon DSE - 2nd order



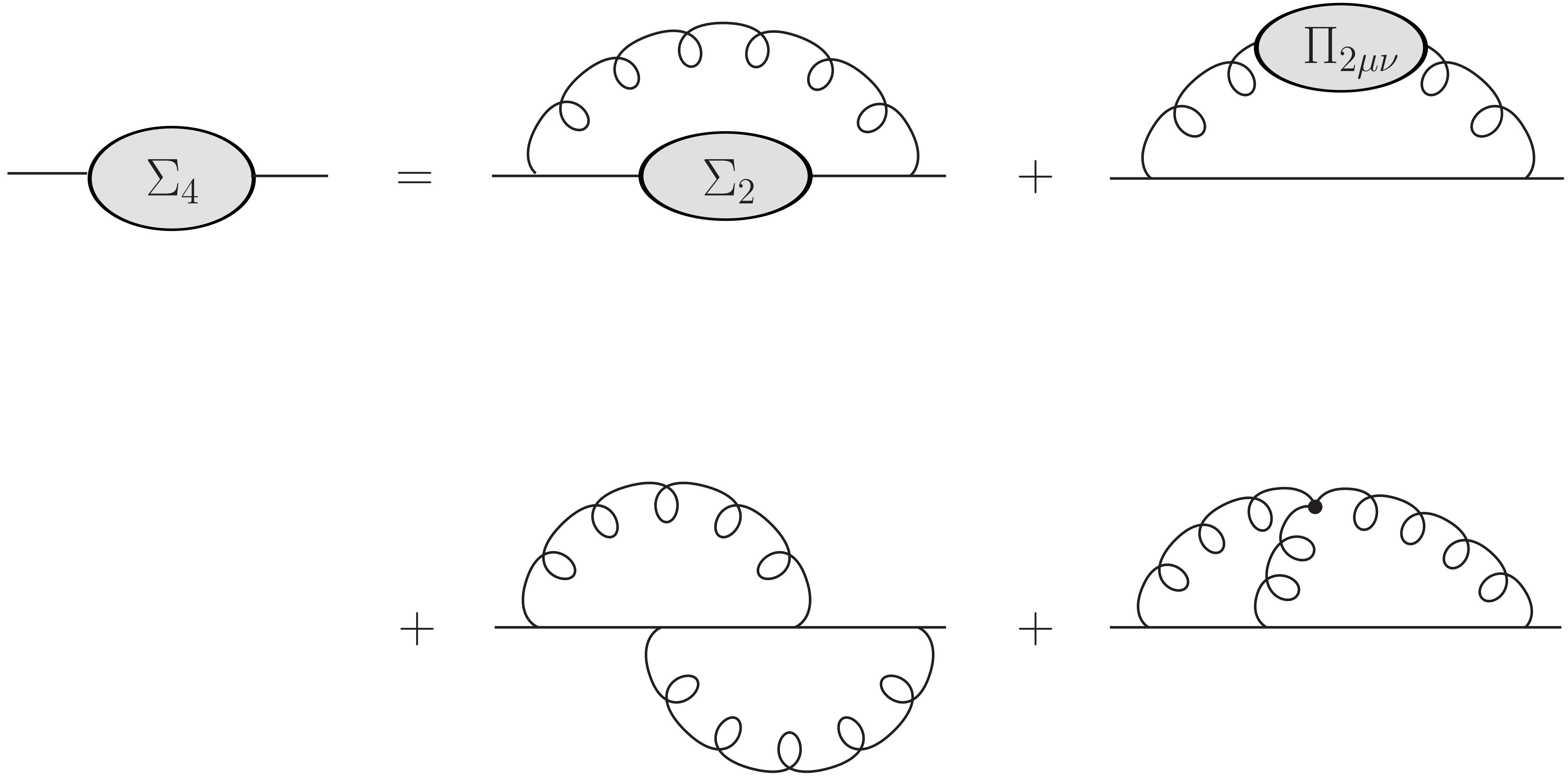

$$\Pi_2 = 0$$



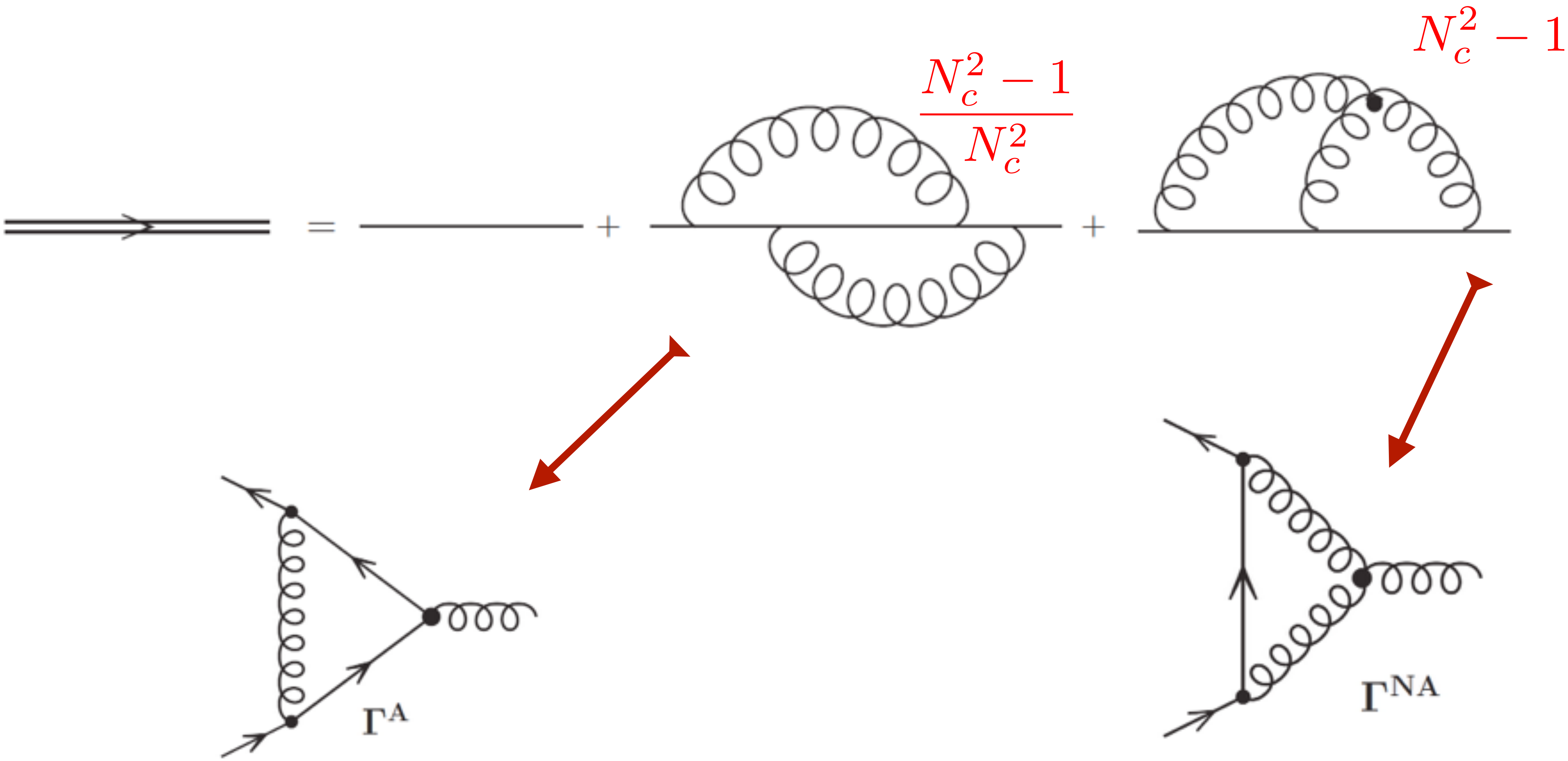

$$\overline{\Pi}_{\mu\nu}(p) = -g^2 \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}_D \left[\gamma_\mu \overline{S}(p+q)\gamma_\nu \overline{S}(q)\right] + \mathrm{CONST}_{tad}$$

$$+\frac{3}{2}g^2 \int \frac{d^4q}{(2\pi)^4} N_{\mu\rho\sigma}(p,-p-q,q)\overline{D}_{\sigma\sigma'}(q)\overline{D}_{\rho\rho'}(p+q)N_{\sigma'\rho'\nu}(q,-(p+q),p)$$

# Quark DSE - 4th order



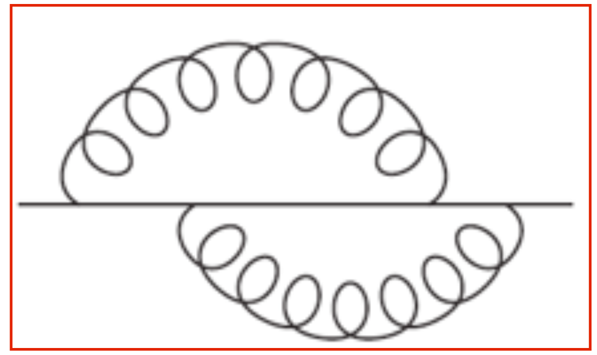
# Nonzero contribution:



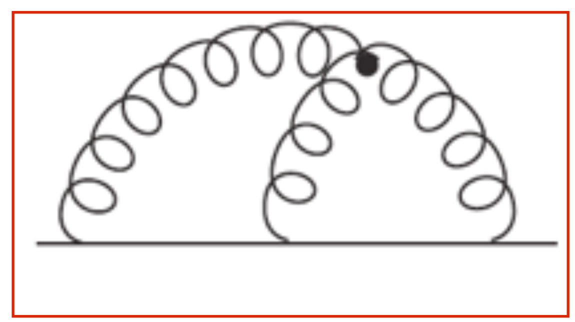


M. S. Bhagwat, A. Höll, A. Krassnigg, C. D. Roberts, P. C. Tandy, PRC 70, 035205 (2004)
C. S. Fischer, R. Williams, PRD 78, 074006 (2008)
G. Eichmann, R. Alkofer, I.C. Kloët, C.D. Roberts, PRC 77, 042202(R) (2008)
R. Alkofer, C. S. Fischer, F. J. Llanes-Estrada, K. Schwenzer, Ann. Phys. 324, 106 (2009)
A. C. Aguilar, J. Papavassiliou, PRD 83, 014013 (2011)
A. Bashir, A. Raya, S. Sanchez-Madrigal, PRD 84, 036013 (2011)
L. von Smekal, M. Q. Huber, JHEP 04, 149 (2013)

# Explicitly:

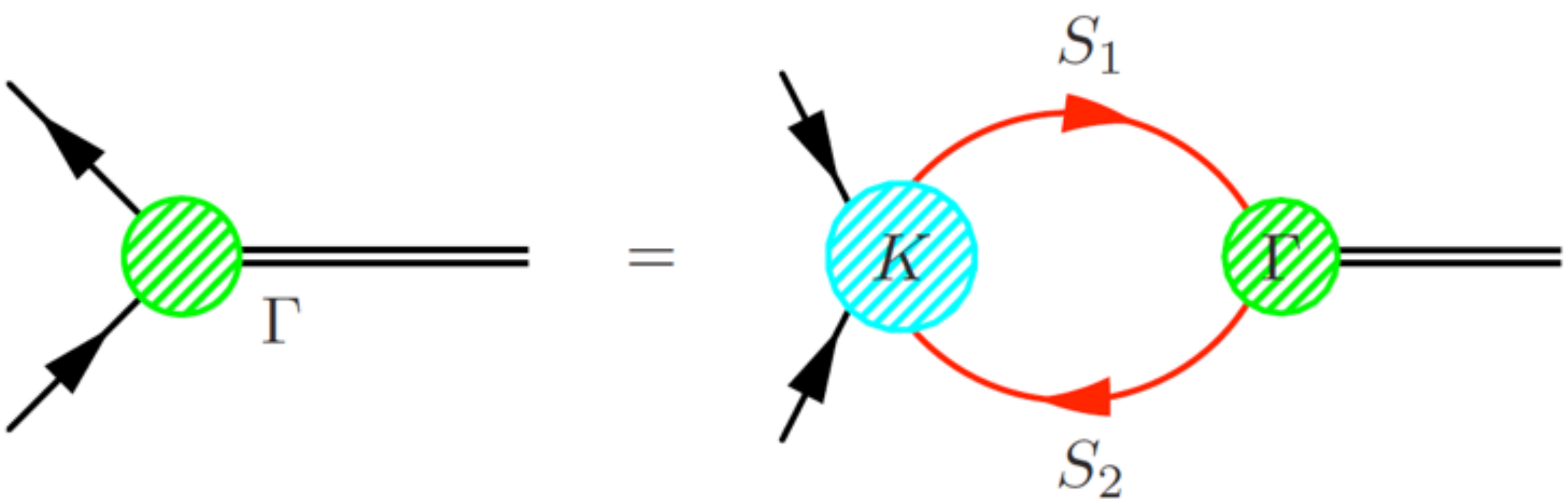


$$\Sigma_4^A(p) = \frac{N_c^2-1}{4N_c^2} g^4 \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \overline{D}_{\mu\nu}(p-q)\,\gamma_\mu\,\overline{S}(q)\,\gamma_\rho\,\overline{S}(q+l)\,\gamma_\nu\,\overline{S}(p+l)\overline{D}_{\rho\sigma}(l)\,\gamma_\sigma$$

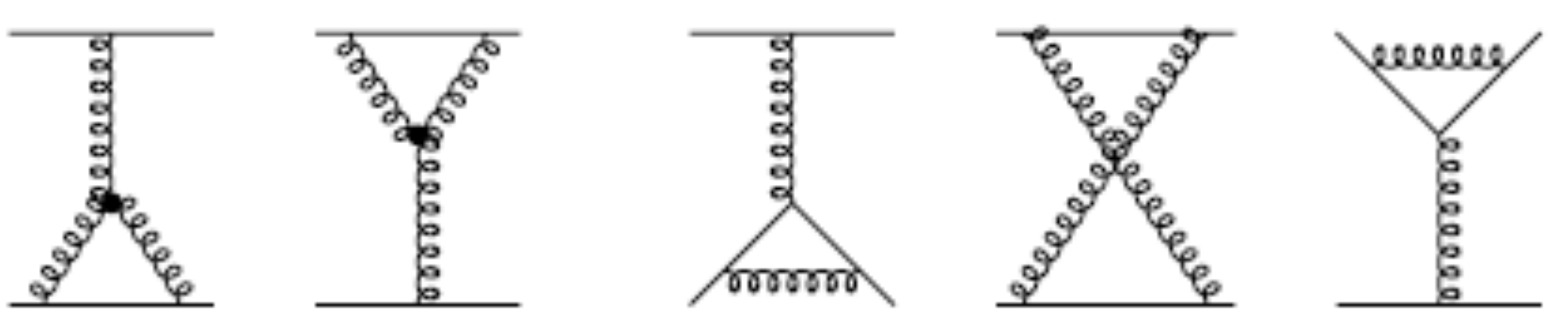


$$\Sigma_4^{nA}(p) = -i\frac{N_c^2-1}{4} g^4 \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \overline{D}_{\mu\nu}(p-q)\,\gamma_\mu\,\overline{S}(q)\,N_{\nu\rho\sigma}(p-q,q-l,l-p)$$
$$\times\,\gamma_{\sigma'}\,\overline{D}_{\sigma'\rho}(q-l)\,\overline{S}(l)\,\gamma_{\rho'}\,\overline{D}_{\sigma'\sigma}(l-p).$$

# Bethe-Salpeter equations




$\mathcal{O}(\delta^4)$ : to satisfy chiral Ward-Takahashi identity



# Complications

- dressed propagators, numerical tables

- loops might/will introduce divergences

- regularization, symmetry preservation

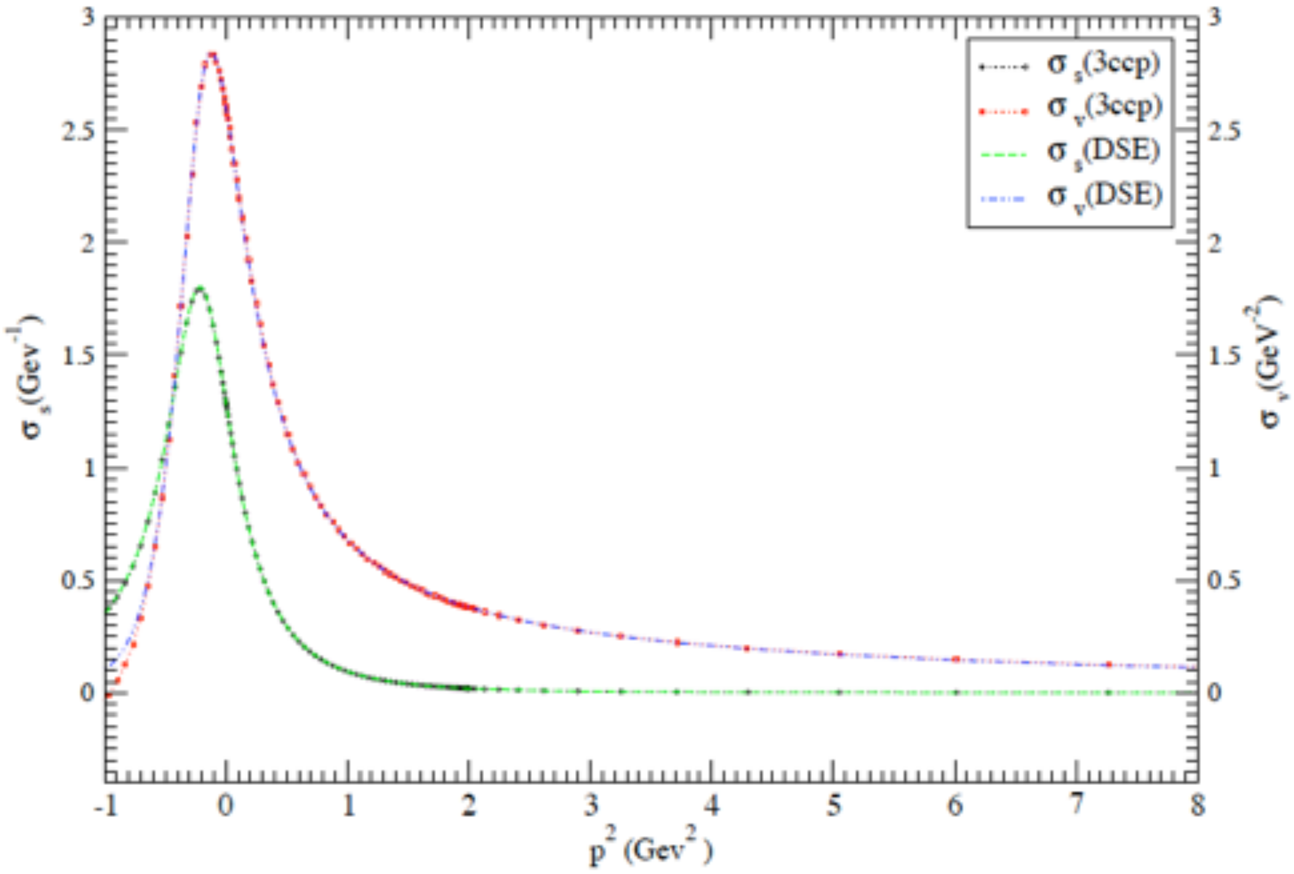
# Tandy

## Feynman Integral Method/Representation

- For triangle diagram, need all momentum integral variables to appear in denominators that are powers of quadratic forms, with necessary finite powers in numerator
- [Could apply to BSE eqn]---see 1960-70s---Perturbation Integral Repn, and Nakanashi Repn of BSE amplitudes
- Here we use it as a convenient fit/representation of existing numerical solns of DSE for q propagator, and of BSE for meson BSE ampls.
- Momentum integrals done analytically, remaining Feyn parameter integrations done numerically
- Only singularities in the resulting physical quantity come from true contour pinches demanded by unitarity and physical thresholds from open hadronic decays
- Accommodates confining propagators via complex conjugate location of spectral properties--non-positive spectral densities
- Trust in essential content of QFT: analyticity, unitarity, principal mass scales, causality.....etc.

# Dressed propagators

- expand in a sum of "free" propagators, with complex-conjugated mass poles*




Quark propagator

$$S(p) = -i \not{p}\, \sigma_v(p) + \sigma_s(p)$$

$$= \sum_{i=k}^{N} \left( \frac{z_k}{i \not{p} + m_k} + \frac{z_k^*}{i \not{p} + m_k^*} \right)$$

Rainbow-ladder, Maris-Tandy model

M. S. Bhagwat, M. A. Pichowsky, P. C. Tandy, PRD 67, 054019 (2003)
S. Souchlas, PhD Thesis, Kent State 2009; J. Phys G 37 , 11501 (2010), PRD 81, 114019 (2010)

*Assumption on the analytical structure of confining propagators

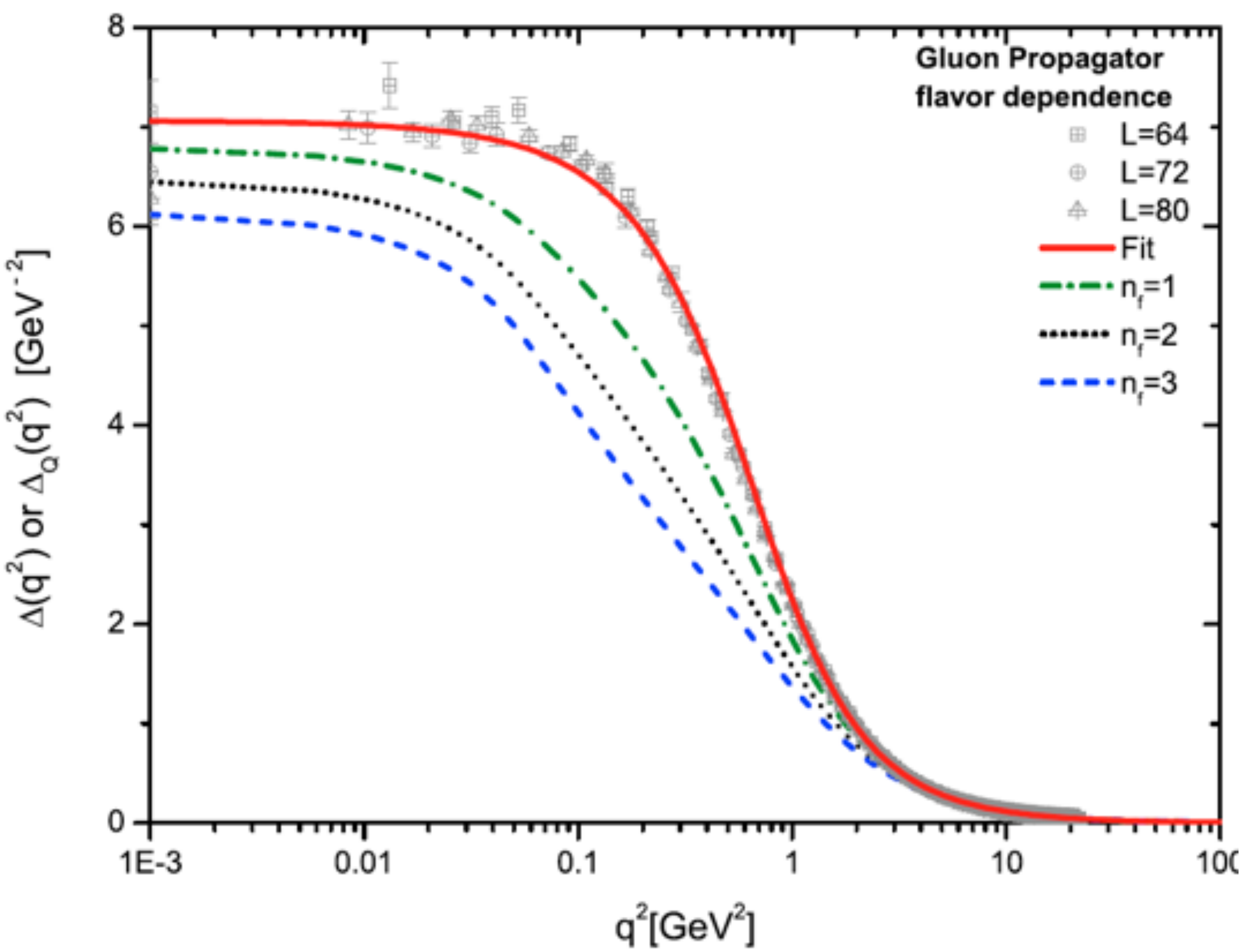
# Same for gluon propagator

$$D(p^2) = \frac{p^2 + M^2}{p^4 + (m^2 + M^2)k^2 + \lambda^4}$$

$$= \frac{R_+}{p^2 + m_+^2} + \frac{R_-}{p^2 + m_-^2}$$

Pure-gauge: refined Gribov-Zwanziger:

D. Dudal, M.S. Guimaraes, S.P. Sorella, PRL 106, 062003 (2011)




Unquenched gluon propagator:

A.C. Aguilar, D. Binosi, J. Papavassiliou, PRD 86, 14032 (2012)

# A simple application

## - contact interaction -

Ansatz gluon propagator

$$g^2 \bar{D}^{(0)}_{\mu\nu} = \delta_{\mu\nu} \frac{4\pi\alpha_{IR}}{m_G^2}$$

Regularization + no quark-antiquark thresholds

$$\frac{1}{k^2 + M^2} \to \frac{e^{-(k^2+M^2)/\Lambda^2} - e^{-(k^2+M^2)/\Lambda^2_{IR}}}{k^2 + M^2}$$

D. Ebert, T. Feldmann, H. Reinhardt, Phys. Lett. B 388, 154 (1996)
L.X. Gutiérrez-Guerrero et al. PRC 81, 065202 (2010)

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- Kazuo Tsushima (IIF - Natal)
- Laura Tólos (Bracelona)
- Lei Chang (Adelaide)
- Leticia Palhares (Heidelberg)
- Marcio Woitek (IFT)
- Marco Brito (IFT)
- Marcus B. Pinto (UFSC)
- Ulf Meissner (Bonn-Juelich)
- Nadiane Cassol Seawald (IFT)
- Rafael Leme (IFT)
- Ricardo L.S. Farias (UFSM)
- Ulf Meissner (Bonn-Juelich)
- Sabyasachi Ghosh (IFT)
- Thiago Peixoto (IFT)
- Ulf Meissner (Bonn-Juelich)