Hadrons in the Light-Front Approach

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Light-Front Motivations

- **Ligh-Front is the Ideal Framework to Describe Hadronic Bound States**
- **Constituent Picture and Unanbiguous Partons Content of the Hadronic System**
- **Light-Front Wavefunctions: Representation of Composite Systems in QFT**
- **Invariant Under Boosts**
- **Light-Front Vacuum is Trivial**
- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian:** $P^2 = P^+P^- - P_\perp^2$

Light-Front Coordinates

Four-Vector $\Longrightarrow$ $x^\mu = (x^0, x^1, x^2, x^3) = (x^+, x^-, x_\perp)$

$x^+ = t + z \qquad x^+ = x^0 + x^3 \qquad \Longrightarrow$ **Time**

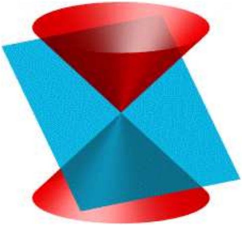
$x^- = t - z \qquad x^- = x^0 - x^3 \qquad \Longrightarrow$ **Position**

Metric Tensor and Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

$$p^+ = p^0 + p^3 \ , \ p^- = p^0 - p^3 \ , \ p^\perp = (p^1, p^2)$$

Wave Function Projection



Dirac Matrix and Electromagnetic Current

$\gamma^{+} = \gamma^{0} + \gamma^{3} \implies$ **Electr. Current** $J^{+} = J^{0} + J^{3}$
$\gamma^{-} = \gamma^{0} - \gamma^{3} \implies$ **Electr. Current** $J^{-} = J^{0} - J^{3}$
$\gamma^{\perp} = (\gamma^{1}, \gamma^{2}) \implies$ **Electr. Current** $J^{\perp} = (J^{1}, J^{2})$

$$p^{\mu} x_{\mu} = \frac{p^{+}x^{-} + p^{-}x^{+}}{2} - \vec{p}_{\perp}\vec{x}_{\perp}$$

$$x^{+}, x^{-}, \vec{x_{\perp}} \implies p^{+}, p^{-}, \vec{p}_{\perp}$$

$p^- \implies$ **Light-Front Energy**

$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$

On-shell

Bosons $\implies \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\implies \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- **Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky**
- **A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.**
- **An Introduction to Light-Front Dynamics for Pedestrians Avaroth Harindranath**

Light-Front book organizer: James Vary and Frank Wolz,(1997)

General Electromagnetic Current: Spin-1

$$J^{\mu}_{\alpha\beta} = [F_1(q^2) g_{\alpha\beta} - F_2(q^2) \frac{q_\alpha q_\beta}{2m_\rho^2}] p^\mu - F_3(q^2)(q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha) \ ,$$

- Polarization Vectors

$\epsilon^\mu_x = (-\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$, $\epsilon^\mu_y = (0,0,1,0)$, $\epsilon^\mu_z = (0,0,0,1)$,
$\epsilon'^\mu_x = (\sqrt{\eta}, \sqrt{1+\eta}, 0, 0)$, $\epsilon'^\mu_y = \epsilon_y$, $\epsilon'^\mu_z = \epsilon_z$,
where $\eta = q^2/4m_\rho^2$

- Breit Frame:

$p^\mu_i = (p^0, -q_x/2, 0, 0)$ (Initial) where $p^0 = m_\rho \sqrt{1+\eta}$.
$p^\mu_f = (p^0, q_x/2, 0, 0)$ (Final)

$$J_{ji}^{+} = \imath \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon_j^{'\beta}\Gamma_\beta(k,k-p_f)(\not{k}-\not{p}_f+m)}{((k-p_i)^2-m^2+\imath\epsilon)(k^2-m^2+\imath\epsilon)}$$
$$\times \frac{\gamma^+(\not{k}-\not{p}_i+m)\epsilon_i^\alpha\Gamma_\alpha(k,k-p_i)(\not{k}+m)]\Lambda(k,p_f)\Lambda(k,p_i)}{((k-p_f)^2-m^2+\imath\epsilon}$$

- Regulator Function

$$\Lambda(k,p_{i(f)}) = N/((p-k)^2-m_R+\imath\epsilon)^2$$

- ρ-Meson Vertex

$$\Gamma^\mu(k,p) = \gamma^\mu - \frac{m_\rho}{2}\frac{2k^\mu-p^\mu}{p.k+m_\rho m-\imath\epsilon}$$

- Mass Squared $\left(x = \frac{k^+}{P^+} \implies 0 < x < 1\right)$

$$M^2(m_a, m_b) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_b^2}{1 - x} - p_\perp^2$$

- Free Mass $M_0^2(m, m)$ and Function $M_R^2(m, m_R)$

The function M_R^2 is given by

$$M_R^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m_R^2}{1 - x} - p_\perp^2$$

$$M_0^2 = \frac{k_\perp^2 + m^2}{x} + \frac{(\vec{p} - \vec{k})_\perp^2 + m^2}{1 - x} - p_\perp^2$$

- Wave Function

$$\Phi_i(x, \vec{k}_\perp) = \frac{N^2}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2} \vec{\epsilon}_i . [\vec{\gamma} - \frac{\vec{k}}{\frac{M_0}{2} + m}]$$

Refs.

- Phy.Rev. **C55** (1997) 2043 J.P.B. C. de Melo and T. Frederico
- Phy.Lett. **B708** (2012) 87 J.P.B. C. de Melo and T. Frederico
- Few.Body.Syst. **52**(2012) 403 J.P.B. C. de Melo and T. Frederico

- Instant-Form Spin Base

$$J^{+}_{ji} = \frac{1}{2}\begin{pmatrix} J^{+}_{xx} + J^{+}_{yy} & \sqrt{2}J^{+}_{zx} & J^{+}_{yy} - J^{+}_{xx} \\ -\sqrt{2}J^{+}_{zx} & 2J^{+}_{zz} & \sqrt{2}J^{+}_{zx} \\ J^{+}_{yy} - J^{+}_{xx} & -\sqrt{2}J^{+}_{zx} & J^{+}_{xx} + J^{+}_{yy} \end{pmatrix}$$

- Light-Front

$$I^{+}_{m'm} = \begin{pmatrix} I^{+}_{11} & I^{+}_{10} & I^{+}_{1-1} \\ -I^{+}_{10} & I^{+}_{00} & I^{+}_{10} \\ I^{+}_{1-1} & -I^{+}_{10} & I^{+}_{11} \end{pmatrix}$$

$\Longrightarrow \quad R^{\dagger}_{M} I^{+} R^{\dagger}_{M} = J^{+} \quad \Longleftrightarrow$ **Melosh**

• Matrix Elements

$$I^+_{11} = \frac{J^+_{xx} + (1+\eta)J^+_{yy} - \eta J^+_{zz} - 2\sqrt{\eta}J^+_{zx}}{2(1+\eta)}$$

$$I^+_{10} = \frac{\sqrt{2\eta}J^+_{xx} + \sqrt{2\eta}J^+_{zz} - \sqrt{2}(\eta-1)J^+_{zx}}{2(1+\eta)}$$

$$I^+_{1-1} = \frac{-J^+_{xx} + (1+\eta)J^+_{yy} + \eta J^+_{zz} + 2\sqrt{\eta}J^+_{zx}}{2(1+\eta)}$$

$$I^+_{00} = \frac{-\eta J^+_{xx} + J^+_{zz} - 2\sqrt{\eta}J^+_{zx}}{(1+\eta)}$$

$$
\begin{aligned}
J^+_{xx} &= \frac{1}{1+\eta}[I^+_{11} + 2\sqrt{2\eta} I^+_{10} - \eta I^+_{00} - I^+_{1-1}] \\
J^+_{zx} &= \frac{\sqrt{2}}{1+\eta}[\frac{\sqrt{2\eta}}{2} I^+_{11} + (\eta - 1) I^+_{10} + \sqrt{\frac{\eta}{2}} I^+_{00} \\
&\quad - \frac{\sqrt{2\eta}}{2} I^+_{1-1}] \\
J^+_{yy} &= I^+_{11} + I^+_{1-1} \\
J^+_{zz} &= \frac{1}{1+\eta}[-\eta I^+_{11} + 2\sqrt{2\eta} I^+_{10} + I^+_{00} + \eta I^+_{1-1}]
\end{aligned}
$$

$$\Delta(q^2) = (1+2\eta)I^+_{11} + I^+_{1-1} - \sqrt{8\eta}I^+_{10} - I^+_{00} = (1+\eta)(J^+_{yy} - J^+_{zz}) = 0$$

- **Angular Condition: Violation !!**

$$q_x \Longrightarrow J^+_{yy} = J^+_{zz} \begin{cases} \textit{Parity} \\ + \\ \textit{Rotations} \end{cases}$$

$$\Delta(q^2) \neq 0$$

- **Ref:**
- **Sov. J. Nucl. Phys. 39 (1984) 198**

I.Grach and L.A. Kondratyku

- **Phy. Rev. Lett. 62 (1989) 387**

L.L. Frankfurt, I.Grach, L.A. Kondratyku and M. Strikman

Prescriptions

$$\begin{cases} FFS \\ GK \\ CCKP \\ BH \end{cases} \quad \textbf{vs} \quad \textbf{COVARIANT}$$

- **Breit Frame** $\Longrightarrow P^+ = P'^+$, $P^- = P'^-$, $\vec{P'}_\perp = -\vec{P}_\perp = \vec{q}/2$
- **B.F:** $q^+ = q^0 + q^3 = 0$
- $J^+_\rho \begin{cases} 4 \; \textit{Current Elements} \\ 3 \; \textit{Form Factors} \;\; G_0 \;, \; G_1 \; \textit{and} \; G_2 \end{cases}$

Inna Grach Prescription: I^+_{00}

$$
\begin{aligned}
G_0^{GK} &= \frac{1}{3}[(3-2\eta)I^+_{11} + 2\sqrt{2\eta}I^+_{10} + I^+_{1-1}] = \\
&\frac{1}{3}[J^+_{xx} + \eta J^+_{zz}(2-\eta)J^+_{yy}] \\
G_1^{GK} &= 2[I^+_{11} - \frac{1}{\sqrt{2\eta}}I^+_{10}] = J^+_{yy} - J^+_{zz} - \frac{J^+_{zx}}{\sqrt{\eta}} \\
G_2^{GK} &= \frac{2\sqrt{2}}{3}[-\eta I^+_{11} + \sqrt{2\eta}I^+_{10} - I^+_{1-1}] = \\
&\frac{\sqrt{2}}{3}[J^+_{xx} + J^+_{yy}(-1-\eta) + \eta J^+_{zz}] \ .
\end{aligned}
$$

CCKP

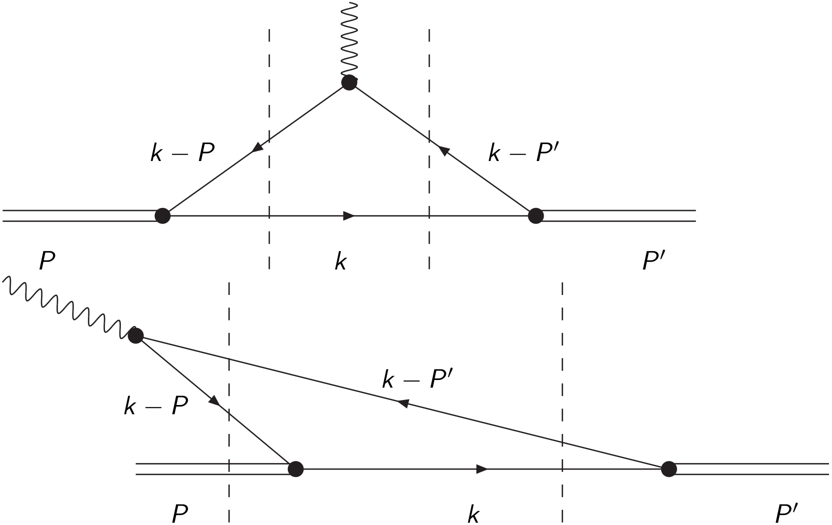
$$
\begin{aligned}
G_0^{CCKP} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}] \\
&= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}] \\
G_1^{CCKP} &= \frac{1}{(1+\eta)}[I_{11}^{+}+I_{00}^{+}-I_{1-1}^{+}-\frac{2(1-\eta)}{\sqrt{2\eta}}I_{10}^{+}] = -\frac{J_{zx}^{+}}{\sqrt{\eta}} \\
G_2^{CCKP} &= \frac{\sqrt{2}}{3(1+\eta)}[-\eta I_{11}^{+}-\eta I_{00}^{+}+2\sqrt{2\eta}I_{10}^{+}-(\eta+2)I_{1-1}^{+}] = \\
&\quad \frac{\sqrt{2}}{3}[J_{xx}^{+}-J_{yy}^{+}]
\end{aligned}
$$

Brodsky-Hiller - (BH) - I_{11}^{+}

$$
\begin{aligned}
G_0^{BH} &= \frac{1}{3(1+2\eta)}[(3-2\eta)I_{00}^{+} + 8\sqrt{2\eta}I_{10}^{+} + 2(2\eta-1)I_{1-1}^{+}] \\
&= \frac{1}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)] \\
G_1^{BH} &= \frac{2}{(1+2\eta)}[I_{00}^{+} - I_{1-1}^{+} + \frac{(2\eta-1)}{\sqrt{2\eta}}I_{10}^{+}] \\
&= \frac{1}{(1+2\eta)}[\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}] \\
G_2^{BH} &= \frac{2\sqrt{2}}{3(1+2\eta)}[\sqrt{2\eta}I_{10}^{+} - \eta I_{00}^{+} - (\eta+1)I_{1-1}^{+}] \\
&= \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}]
\end{aligned}
$$

FFS

$$
\begin{aligned}
G_0^{FFS} &= \frac{1}{3(1+\eta)}[(\frac{3}{2}-\eta)(I_{11}^{+}+I_{00}^{+})+5\sqrt{2\eta}I_{10}^{+}+(2\eta-\frac{1}{2})I_{1-1}^{+}] \\
&= \frac{1}{6}[2J_{xx}^{+}+J_{yy}^{+}+3J_{zz}^{+}] \\
G_1^{FFS} &= G_1^{CCKP} \\
G_2^{FFS} &= G_2^{CCKP}
\end{aligned}
$$



Light-front time-ordered diagrams for the current: Triangle Diagram and Pair Terms

- **Vertex** $\Gamma(\gamma^\mu, \gamma^\nu)$

$$Tr[gg]_{ji} = Tr[\not\epsilon_f^\alpha (\not k - \not p' + m)\gamma^+ (\not k - \not p + m)\not\epsilon_i^\alpha (\not k + m)]$$

- **+Z (Pair Terms)** $\propto (k^-)$

$$Tr[gg]_{ji}^{+Z} = \frac{k^-}{2} \; R_{gg}$$

$$R_{gg} = Tr[\not\epsilon_f^\alpha (\not k - \not p' + m)\gamma^+ (\not k - \not p + m)\not\epsilon_i^\alpha \gamma^+]$$

- **Fact:** $\rightarrow k^{-(m+1)} \; (p^+ - k^+)^n$

  No Pair Terms Contribution if $m < n$

Ref.: Nucl. Phys. A660 (1999) 219, De Melo, Frederico, Naus and Sauer

- **Simplification:** $[\gamma^{\mu}, \gamma^{\nu}]$ **Dirac Trace:**

$$\begin{aligned} Tr[gg]^{+Z}_{xx} &= -\eta \; Tr[gg]^{+Z}_{zz} \\ Tr[gg]^{+Z}_{zx} &= -\sqrt{\eta} \; Tr[gg]^{+Z}_{zz} \\ Tr[gg]^{+Z}_{zz} &= R_{gg} \end{aligned}$$

- **Also:**

$$Tr[gg]^{+Z}_{yy} = 4k^{-}(p^{+} - k^{+})^{2}$$

- **Pair Terms**

$$J^{+Z}_{xx} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{xx}]}{[1][2][4][5][6][7]} \neq 0$$

$$J^{+Z}_{zx} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{zx}]}{[1][2][4][5][6][7]} \neq 0$$

$$J^{+Z}_{zz} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{zz}]}{[1][2][4][5][6][7]} \neq 0$$

$$J^{+Z}_{yy} = \lim_{\delta^+ \to 0} \int d^3K \frac{Tr[J^{+Z}_{zz}]}{[1][2][4][5][6][7]} = 0$$

- **Basis $I^+_{m'm}$:**

$$I^{+Z}_{11} = 0, \; I^{+Z}_{10} = 0$$
$$I^{+Z}_{1-1} = 0, \; I^{+Z}_{00} = (1+\eta)J^+_{zz} \neq 0$$

- **Pair Term Contribution:** **only: I^{+Z}_{00} !!**
- **Inna Grach:** **Elimination** I^+_{00}

$$
\begin{aligned}
G_0^{GK} &= \frac{1}{3}[J^+_{xx} + 2J^+_{yy} - \eta J^+_{yy} + \eta J^+_{zz}] \\
G_1^{GK} &= J^+_{yy} - J^+_{zz} - \frac{J^+_{zx}}{\sqrt{\eta}} \\
G_2^{GK} &= \frac{\sqrt{2}}{3}[J^+_{xx} + J^+_{yy}(-1 - \eta) + \eta J^+_{zz}]
\end{aligned}
$$

$$
\begin{aligned}
G_0^{GK\ (+Z)} &= \frac{1}{3}\left(J_{xx}^{(+Z)}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \\
&\quad \frac{1}{3}\left(-\eta J_{zz}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = 0 \\
G_1^{GK\ (+Z)} &= \left(-J_{zz}^{+Z}[gg] - \frac{J_{zx}^{+Z}[gg]}{\sqrt{\eta}}\right) = \\
&\quad -J_{zz}^{+Z}[gg] + \sqrt{\eta}\frac{J_{zz}^{+Z}[gg]}{\sqrt{\eta}} = 0 \\
G_2^{GK\ (+Z)} &= \frac{\sqrt{2}}{3}\left(J_{xx}^{+Z}[gg] + \eta J_{zz}^{+Z}[gg]\right) = \frac{\sqrt{2}}{3}\left(-\eta J_{zz}^{+} + \eta J_{zz}^{+Z}\right) = 0
\end{aligned}
$$

Cross term with γ^μ and derivatives

$$Tr[dg]_{ji} = \epsilon'_j \cdot (2k - p')\ Tr[(\not{k} - \not{p}' + m)\gamma^+(\not{k} - \not{p} + m)\not{\epsilon}_i(\not{k} + m)]$$

- Terms with $m \geq n$:

$$Tr[dg]^Z_{ji} = \epsilon'^+_j \epsilon^+_i R_{dg} - 4m\ k^- k^+ \epsilon'^+_j\ \vec{\epsilon}_{i\perp} \cdot \vec{q}_\perp$$

•$R_{dg} = 4m\ k^- \left(k^-(k^+ - p^+) + (\vec{k}_\perp - \vec{p}\,'_\perp) \cdot (\vec{k}_\perp - \vec{p}_\perp) + q_\perp \cdot k_\perp + m^2 \right)$

•The **Z-modes to yy is zero** $\rightarrow$ $\epsilon^+_y = 0$ **and** $\epsilon'^+_y = 0$

- **Term with k^- and $k^{-2}(k^+ - p^+)$**
- **Interval $0 < k^+ - p^+ < \delta^+$ with γ^μ and derivative couplings:**

$$J_{ji}^{+Z}[dg] = \lim_{\delta^+ \to 0_+} \int [d^4 k]^Z \frac{Tr[dg]_{ji}^Z}{\{1\}\{2\}\{3\}} \frac{1}{\{4\}^2} \frac{1}{\{5\}^2} \frac{m_v}{2(p' \cdot k + m\, m_v - i\epsilon)}$$

- **Zero of $\{3\}$ is dislocated by using $p'^+ = p^+ + \delta^+$**
- **Cauchy integration in k^- with k^+ in the interval $0 < k^+ - p^+ < \delta^+$**
- **Two poles: one from the dislocated denominator $\{3\} = 0$, and**

$$k^- = \frac{1}{p^+} \left(2\vec{p}\,'_\perp \cdot \vec{k}_\perp - k^+ \, p^- - 2\, m\, m_v + \imath\epsilon \right) \; .$$

- **The residue from the zero of $\{3\}$ is $\mathcal{O}[(\delta^+)^2]$**
- **The residue from the pole gives a contribution $\mathcal{O}[(\delta^+)^0]$.**

After integration in k^-, one has that:

$$J_{ji}^{+Z}[dg] \sim \mathcal{O}[\delta^+] \; .$$

Direct term with derivative couplings

$$Tr[dd]_{ji} = \left[A_{dd}\ \frac{k^-}{2} + B_{dd}\right] \epsilon'_j \cdot (2k - p')\ \epsilon_i \cdot (2k - p)\ .$$

$$A_{dd} \ = \ Tr[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\gamma^+] = 8(p^+ - k^+)^2$$
$$B_{dd} \ = \ Tr\left[(\not k - \not p' + m)\gamma^+(\not k - \not p + m)\left(\frac{\gamma^-}{2}k^+ - \vec{\gamma}_\perp.\vec{k}_\perp + m\right)\right]\ .$$

$$J^{+Z}_{ji}[dd] \ = \ \lim_{\delta^+\to 0_+}\int [d^4k]^Z \frac{Tr[dd]_{ji}}{\{1\}\{2\}\{3\}\{5\}^2\{6\}^2} \times \frac{m_v^2}{4(p\cdot k + m\, m_v - i\epsilon)(p'\cdot k + m\, m_v - i\epsilon)}$$

$\Longrightarrow \ J^{+Z}_{ij}[dd] \sim \ \mathcal{O}[\delta^+]$

$I^{+Z}_{11} = 0\ ,\ I^{+Z}_{10} = 0\ ,\ I^{+Z}_{1-1} = 0$ **and** $I^{+Z}_{00} = (1+\eta)J^{+Z}_{zz}$ **with** $\lim_{\delta^+\to 0_+} J^{+Z}_{zz} \neq 0$

Final Result:

No Zero Modes or Pair Terms Contribution with Inna Grach prescp.!!

REf.:

- **J.P.B.C. de Melo and T. Frederico**, **Phys. Lett. B708, (2012) 87**
- **J.P.B.C. de Melo and T. Frederico, Few Body Syst. 52 (2012) 403**

- **Similar Results are found by Ji, Bakker and Choi: Only** $\Rightarrow \; \Gamma[\gamma^\mu, \gamma^\nu]$
- **Phy.Rev.D65 (2002) 116001**
- **Phy.Rev.D70 (2004) 053015**

Elimination of Zero Modes

- **VIP:**

$$\begin{aligned} J_{xx}^{+z} &= -\eta J_{zz}^{+z} \\ J_{zx}^{+z} &= -\sqrt{\eta} J_{zz}^{+z} \\ J_{yy}^{+z} &= 0 \ . \end{aligned}$$

- **Also:**

$$J_{zz}^{+Z} = J_{yy}^{+V} - J_{zz}^{+V}$$

- **J.P.B.C. de Melo and T. Frederico, Phys. Lett. B708, (2012) 87**

Electromagnetic Form Factors Free of Zero Modes

$$
\begin{aligned}
G_0^{CCKP} &= \frac{1}{6}[2J_{xx}^{+} + J_{yy}^{+} + 3J_{zz}^{+}] = \frac{1}{3}[J_{xx}^{+V} - (-2-\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}] \\
G_1^{CCKP} &= -\frac{J_{zx}^{+}}{\sqrt{\eta}} = [J_{yy}^{+V} - \frac{J_{zx}^{+V}}{\sqrt{\eta}} - J_{zz}^{+V}] \\
G_2^{CCKP} &= \frac{\sqrt{2}}{3}[J_{xx}^{+} - J_{yy}^{+}] = \frac{\sqrt{2}}{3}[J_{xx}^{+V} - (1+\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}]
\end{aligned}
$$

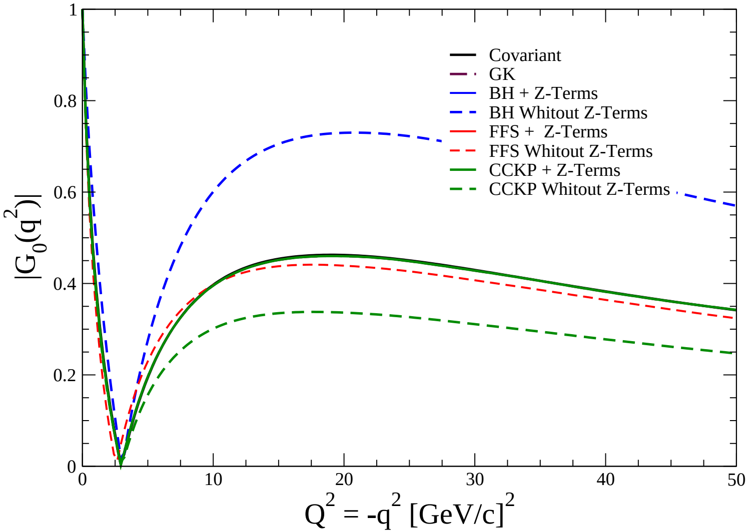
$$
\begin{aligned}
G_0^{FFS} &= \frac{1}{3}[J_{xx}^{+} + 2J_{yy}^{+}] = \frac{1}{3}[J_{xx}^{+V} - (-2-\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}] \\
G_1^{FFS} &= G_1^{CCKP} \\
G_2^{FFS} &= G_2^{CCKP}
\end{aligned}
$$

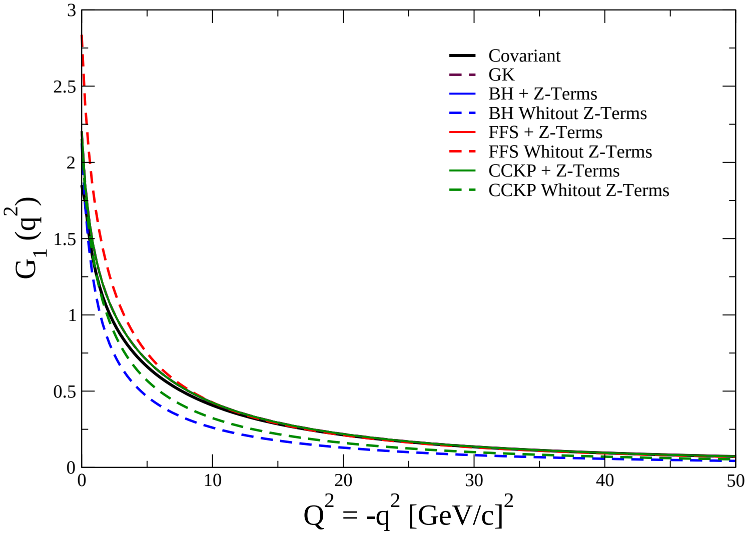
$$G_0^{BH} = \frac{1}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) + J_{yy}^{+}(2\eta-1) + J_{zz}^{+}(3+2\eta)]$$
$$= \frac{1}{3}[J_{xx}^{+V} - (-2-\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}]$$
$$G_1^{BH} = \frac{1}{(1+2\eta)}[\frac{J_{zx}^{+}}{\sqrt{\eta}}(1+2\eta) - J_{yy}^{+} + J_{zz}^{+}]$$
$$= [J_{yy}^{+V} - \frac{J_{zx}^{+V}}{\sqrt{\eta}} - J_{zz}^{+V}]$$
$$G_2^{BH} = \frac{\sqrt{2}}{3(1+2\eta)}[J_{xx}^{+}(1+2\eta) - J_{yy}^{+}(1+\eta) - \eta J_{zz}^{+}]$$
$$= \frac{\sqrt{2}}{3}[J_{xx}^{+V} - (1+\eta)J_{yy}^{+V} + \eta J_{zz}^{+V}]$$

Important: All Prescriptions Given the Same Electr. Form Factors
With the relation in Phys. Lett. B708, (2012) 87, De Melo and Frederico

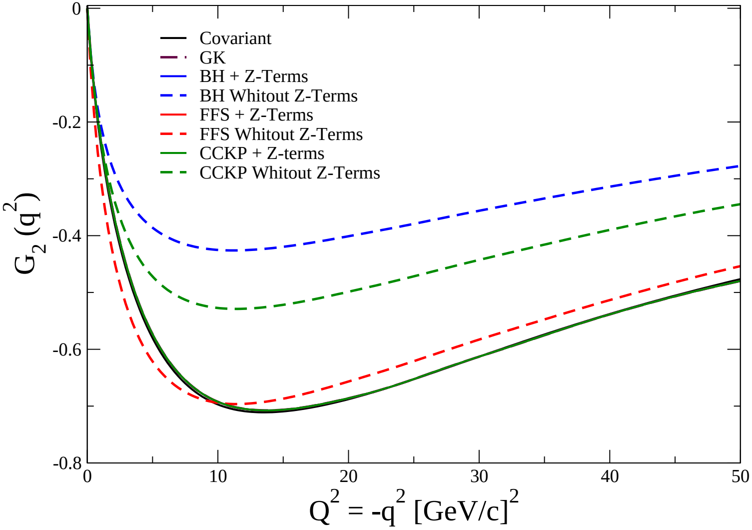
Angular Condition: Free of Zero Modes

$$
\begin{aligned}
\Delta(q^2) &= (1+2\eta)I^{+}_{11} + I^{+}_{1-1} - \sqrt{8\eta}I^{+}_{10} - I^{+}_{00} \\
&= (1+\eta)(J^{+V}_{yy} - J^{+V}_{zz}) = 0
\end{aligned}
$$

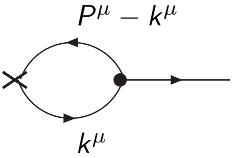

Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-Terms
CCKP Whitout Z-Terms
$|G_0(q^2)|$
$Q^2 = -q^2$ [GeV/c]^2
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50



Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-Terms
CCKP Whitout Z-Terms
$G_1(q^2)$
$Q^2 = -q^2\ [\mathrm{GeV/c}]^2$
0
0.5
1
1.5
2
2.5
3
0
10
20
30
40
50


Covariant
GK
BH + Z-Terms
BH Whitout Z-Terms
FFS + Z-Terms
FFS Whitout Z-Terms
CCKP + Z-terms
CCKP Whitout Z-Terms
$G_2(q^2)$
$Q^2 = -q^2$ [GeV/c]^2
0
-0.2
-0.4
-0.6
-0.8
0
10
20
30
40
50

Decay Constant



$$< 0|J^\mu(0)|p,\lambda >= \imath\sqrt{2} f_V M \epsilon^\mu_\lambda$$

- $\epsilon^\mu_\lambda \Longrightarrow$ **Polarization** $\rightarrow \ \epsilon^+_z \ = \ 1$

Dirac Trace:

$$Tr[O^+] \ = \ [-4k^{+2} + 4k^2_\perp + 4k^+p^+ + 4m^2] - \frac{m_V}{2} \frac{[4m(2k^+ - p^+)(k^- \ - \ k^+)]}{\frac{p^+k^- + p^-k^+}{2} + m_v m}$$

- **Because the denominator:** **the zero mode is cancel out!!**

Observables

m_q / m_R [GeV]	f_ρ [GeV]	$< r^2 >$ [fm^2]	μ [$2/m_v$]	Q_d [e/m_v^2]
0.430 / 3.0	0.154	0.267	2.10	-0.898
[1]	0.134	0.296	2.10	-0.910
[2]	0.130	0.312	2.11	-0.850
[3]	0.207	0.540	2.01	-0.410
Exp. [PDG]	0.153±0.008			

[1] **Phy.Rev.D65 (2002) 116001, B. Bakker, H. M. Choi and C. R. Ji**
[2] Phy.Rev.C83 (2011) 065206, H. L. Roberts, A. Bashir, L.X.G. Guerrero, C. Roberts,
[3] Phy.Rev.C77 (2008) 025203, M. S. Bhagwat and P. Maris

Pion Decay: Motivations

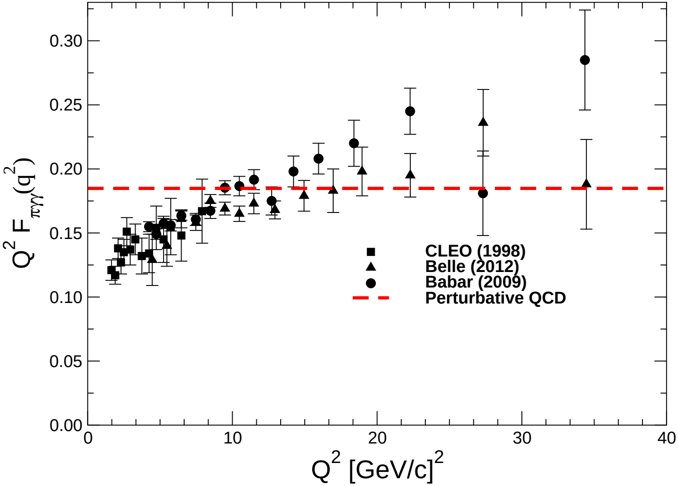
- $\pi^0 \rightarrow \gamma\gamma$ **Most Important Example of the Triangle Anomaly**

- π^0 **Meson is the Lightest Meson cannot Decay to Another Hadronic State**

- $\pi^0 \rightarrow \gamma\gamma$ **Is Conected to the Adler-Bell-Jackiw Anomaly**

- **Babar Experiment (2009)**

- **Belle Experiment (2012)**



- **QCD:** $2f_\pi \implies$ **Brodsky & Lepage (1980)**

Effective Interaction Lagrangian

$$\mathcal{L}^{int}_{\pi q} = -\imath \frac{M}{f_\pi} \vec{\pi} \cdot \vec{q} \gamma^5 \vec{\tau} q \ ;$$

Where:

$\rightarrow$ M**:** **Constituent Quark Mass**

$\rightarrow$ f_π**:** **Weak Decay Constant**

$\rightarrow$ π**:** **Pion Field**

$\rightarrow$ q**:** **Quark Field**

- $\hbar = c = 1$

$T^{\mu\nu}$ Tensor : Amplitude (a) and (b)

$$T^{\mu\nu} = t_{\mu\nu}(k_1, k_2) + t_{\mu\nu}(k_2, k_1)$$

- **Lorentz Invariant, Parity Conservation, Gauge Invariance**

After Calculation of Trace in Spinor and Flavour Basis:

$$t_{\mu\nu} = \frac{4}{3}\frac{M^2}{f_\pi} e_0^2 N_c \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta I(k_1^2) \;;$$

$$I(k_1^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{((k_2-k)^2 - M^2 + \imath\epsilon)} \frac{1}{(k^2 - M^2 + \imath\epsilon)} \frac{1}{((k_\pi - k)^2 - M^2 + \imath\epsilon)} \,.$$

- **VIP** $\Longrightarrow$ $\mathbf{N_C = 3}$

Light-Front Wave Function

- **The Wave Function // Replaced**

$$\frac{1}{-m_\pi^2 + M_0^2} \rightarrow \frac{\pi^{\frac{3}{2}} f_\pi}{M\sqrt{M_0 N_c}} \Phi_\pi(K^2)$$

- **Wave Function $\Phi(k^2)$ Normalization**

$$\int d^3K \Phi_\pi^2(k^2) = 1 \ ,$$

Ref.

• T. Frederico and G. Miller, Phy. Rev. D45 (1992) 071901
ibid. Phy. Rev. D50 (1994) 210
• de Melo, T. Frederico and H.L. Naus
Phy.Rev. C59 (1999) 2278
• de Melo, E. Pace, T. Frederico, G. Salmé,
Nucl. Phys. A 707 (2002) 399-424
• da Silva, de Melo, B. El-Bennich, V. Filho,
Phy.Rev. C86 (2012) 038202

Pion - Wave Function

i) **Gaussian Wave Function**

$$\Phi_\pi = \left(\frac{8r_{NR}^2}{3\pi}\right)^{3/4} exp\left[-\left(\frac{4}{3}\right)(r_{NR}k)^2\right]$$

ii) **Hydrogen-Atom Wave Function**

$$\Phi_\pi = \frac{1}{2\pi}\left(\frac{\sqrt{3}}{r_{NR}}\right)^{5/2}\left[\frac{1}{(\frac{3}{4}r_{NR}^{-2}+k^2)^2}\right]$$

- **Two Independents Parameters**

i) Quark Mass: M

ii) Non-Relativistic Charge Radius: r_{NR}

- **Final Pion Transition Form Factor**

$$F_{\pi^0}(-q^2) = \frac{\sqrt{N_c}M}{6\pi^{\frac{3}{2}}} \int \frac{dxd^2K_\perp}{(1-x)\sqrt{M_0}} \frac{\Phi_\pi(K^2)}{((\vec{K}-x\vec{q})^2_\perp + M^2)}$$

- **Soft Pion Limit** ***:**

$$F_{\gamma\pi^0}(0) = \frac{1}{4\pi^2 f_\pi} \ , \qquad (\textbf{SAB})$$

$\star$ **J. S. Schwinger, Phy. Rev.82, (1951) 664.**
S. L. Adler, Phys. Rev. Phy. Rev. 177, (1969) 2426.
J. S. Bell, R. Jackiw, Nuovo Cimento, 60 , (1969) 47.

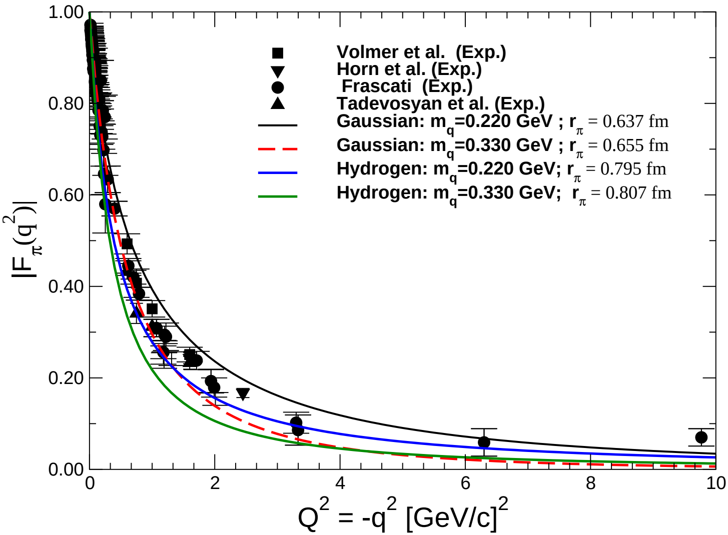
Some Results

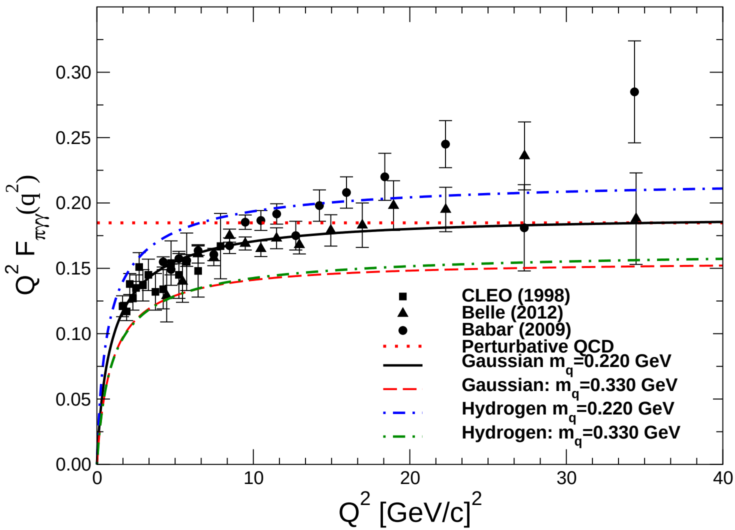
Table-I: f_π : 92.4 MeV **Fixed**

model	$m_{u,d}$ $[GeV]$	r_{nr} $[fm]$	$< r^2 >$ $[fm^2]$	$< r^2_{\pi^0} >$ $[fm^2]$
Gaussian	**0.220**	**0.345**	**0.637**	**0.683**
	0.330	**0.472**	**0.655**	**0.552**
Hydrogen	**0.220**	**0.593**	**0.795**	**0.782**
	0.330	**0.708**	**0.807**	**0.582**
Exp.[PDG]			**0.672±0.008**	

Table-II: **Quark Mass fixed :** $m_{u,d} = 0.220 \;\; GeV$

model	f_π [MeV]	r_{nr} [fm]	$< r^2 >$ [fm^2]	$< r^2_{\pi^0} >$ [fm^2]
Gaussian	92.4	0.345	0.637	0.683
	97.0	0.303	0.589	0.657
	110.0	0.172	0.406	0.664
Hydrogen	92.4	0.593	0.795	0.782
	97.0	0.543	0.750	0.767
	110.0	0.410	0.626	0.720
Exp.[PDG]	92.2 ± 0.021		0.672 ± 0.008	

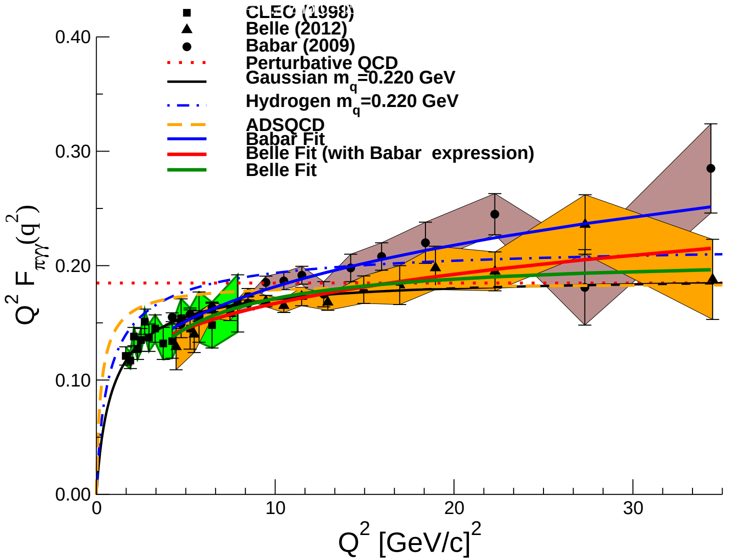

1.00
0.80
0.60
0.40
0.20
0.00
0
2
4
6
8
10
$|F_\pi(q^2)|$
$Q^2 = -q^2$ [GeV/c]^2
Volmer et al. (Exp.)
Horn et al. (Exp.)
Frascati (Exp.)
Tadevosyan et al. (Exp.)
Gaussian: m_q=0.220 GeV ; r_π = 0.637 fm
Gaussian: m_q=0.330 GeV ; r_π = 0.655 fm
Hydrogen: m_q=0.220 GeV; r_π = 0.795 fm
Hydrogen: m_q=0.330 GeV; r_π = 0.807 fm



$Q^2 F_{\pi\gamma\gamma}(q^2)$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
10
20
30
40
Q^2 [GeV/c]^2
CLEO (1998)
Belle (2012)
Babar (2009)
Perturbative QCD
Gaussian m_q=0.220 GeV
Gaussian: m_q=0.330 GeV
Hydrogen m_q=0.220 GeV
Hydrogen: m_q=0.330 GeV

Fit Curves:

- **Babar:** $Q^2|F(Q^2)| \; = \; A \; (\frac{Q^2}{10 \;\; GeV^2})^{\beta} \;\Longrightarrow\; \begin{cases} A = 0.182 \;\; GeV \\ \beta = 0.250 \end{cases}$
- **Belle:** $Q^2|F(Q^2)| \; = \; A_1 \; (\frac{Q^2}{10 \;\; GeV^2})^{\beta_1} \;\Longrightarrow\; \begin{cases} A_1 = 0.167 \;\; GeV \\ \beta_1 = 0.204 \end{cases}$
- **Belle:** $Q^2|F(Q^2)| \; = \; \frac{B \;\, Q^2}{Q^2+C} \;\Longrightarrow\; \begin{cases} B = 0.209 \;\; GeV \\ C = 2.2 \;\; GeV^2 \end{cases}$


CLEO (1998)
Belle (2012)
Babar (2009)
Perturbative QCD
Gaussian m_q=0.220 GeV
Hydrogen m_q=0.220 GeV
ADSQCD
Babar Fit
Belle Fit (with Babar expression)
Belle Fit
0.40
0.30
0.20
0.10
0.00
0
10
20
30
$Q^2 F_{\pi\gamma\gamma}(q^2)$
Q^2 [GeV/c]^2

Some Coments

- **Theoretical Analyses:** **Explain Babar Data** **and** **Not Explain**

Models try reproduce Babar:

- **Alteration of the asymptotic pion wave function or distribuition amplitude**
- **Dressing $\gamma - q\bar{q}$-vertex with Phenomelogical Interactions, ie., like VMD**

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- Rotational Invariance Broken $\Longrightarrow k^-$ Problematic
- Terms $\begin{cases} - \textit{Good / Covariant} \\ - \textit{Z Terms / Pair Terms} \end{cases}$
- Electromagnetic Current:
- $\begin{cases} - \textbf{Present Work}: \mathbf{J}^+ \textbf{ Component} \\ - \textbf{Future Works}: \mathbf{J}^- \textbf{ and } \mathbf{J}_\perp \end{cases}$
- Pair Terms Contribution / Zero Modes: $\Longrightarrow J^+, \; J^-$ and $J_\perp$
- J^+ is not free of the Pair Terms Contribution !!!
- Take New Informations about Bound States

$\Longrightarrow$ • Correlation $q\bar{q}$ (Pion)

$\Longrightarrow$ • Meson Decays

$\Longrightarrow$ • Heavy Mesons Physics

$\Longrightarrow$ • N-N Interaction

$\Longrightarrow$ • Pion: Space-like and Time-like

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