Pseudoscalar meson spectroscopy in the Bethe-Salpter approach

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QCD forthcoming facilities

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- Beijing's electron-positron collider
- COSY (Julich Cooler Synchrotron)
- ELSA (Bonn Electron Strectcher and Accelerator)
- MAMI (Mainz Microtron)
- FAIR (Facility for Antiproton and Ion Research,Darmstadt)

- J-PARC (Japan Proton Accelerator Research Complex,Japan)
- AL-ICE, COMPASS (CERN)
- JLab (USA)
- RHIC (USA)

quark gluon vertex

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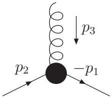

Figure: BSE

$$\Gamma_\nu = \sum_i L_i^B \lambda_i + \sum_i T_i \tau_i \quad (1)$$

Ball-Chiu vertex, longitudinal base elements

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$$L^1_\mu(p_1, p_2) = \gamma_\mu \ , \quad (2)$$

$$L^2_\mu(p_1, p_2) = (\not p_1 - \not p_2)\,(p_1 - p_2)_\mu \ , \quad (3)$$

$$L^3_\mu(p_1, p_2) = i\,(p_1 - p_2)_\mu\,\mathbb{I}_D \ , \quad (4)$$

$$L^4_\mu(p_1, p_2) = \sigma_{\mu\nu}\,(p_1 - p_2)^\nu \quad (5)$$

$$, \quad (6)$$

Ball-Chiu vertex, transversal base elements

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$$T^1_\mu(p_1,p_2) = i\,\left[p_{1\mu}\,(p_2\cdot p_3) - p_{2\mu}\,(p_1\cdot p_3)\right]\mathbb{I}_D\ , \tag{7}$$

$$T^2_\mu(p_1,p_2) = i\,T^1_\mu\left(\not{p}_1 - \not{p}_2\right)\ , \tag{8}$$

$$T^3_\mu(p_1,p_2) = p_3^2\,\gamma_\mu - p_{3\mu}\,\not{p}_3 := p_3^2\,\gamma^{\mathrm{T}}_\mu\ , \tag{9}$$

$$T^4_\mu(p_1,p_2) = -\,i\,T^1_\mu(p_1,p_2)\,\sigma_{\alpha\beta}\,p_2^\alpha\,p_1^\beta\ , \tag{10}$$

$$T^5_\mu(p_1,p_2) = \sigma_{\mu\nu}\,p_3^\nu\ , \tag{11}$$

$$T^6_\mu(p_1,p_2) = -\,\gamma_\mu\left(p_1^2 - p_2^2\right) - (p_1 - p_2)_\mu\,\not{p}_3\ , \tag{12}$$

$$T^7_\mu(p_1,p_2) = \frac{i}{2}\left(p_2^2 - p_1^2\right)\left[\gamma_\mu\left(\not{p}_1 - \not{p}_2\right) - (p_1 - p_2)_\mu\,\mathbb{I}_D\right] + \tag{13}$$

$$(p_1 - p_2)_\mu\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta\ , \tag{14}$$

$$T^8_\mu(p_1,p_2) = i\,\gamma_\mu\,\sigma_{\alpha\beta}\,p_1^\alpha\,p_2^\beta - \left(p_{1\mu}\,\not{p}_2 - p_{2\mu}\,\not{p}_1\right)\ , \tag{15}$$

Ball-Chiu Vertex

$$\lambda_1(p_1, p_2, p_3) = \frac{X_0(p_3^3)\, F(p_3^2)}{2} \left[A(p_1^2) + A(p_2^2) \right] \ , \tag{16}$$

$$\lambda_2(p_1, p_2, p_3) = \frac{X_0(p_3^3)\, F(p_3^2)}{2\ (p_2^2 - p_1^2)} \left[A(p_2^2) - A(p_1^2) \right] \ , \tag{17}$$

$$\lambda_3(p_1, p_2, p_3) = \frac{X_0(p_3^3)\, F(p_3^2)}{p_1^2 - p_2^2} \left[B(p_1^2) - B(p_2^2) \right] \ , \tag{18}$$

$$\lambda_4(p_1, p_2, p_3) = 0 \ . \tag{19}$$

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what says lattice about the vertex?

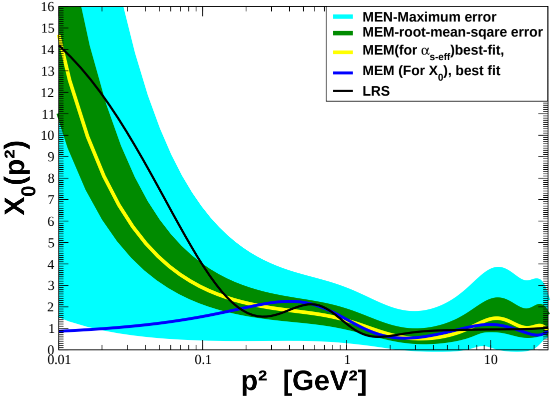

Figure: E.R,J.P.B. de Melo,B. El-Bennich,O.Oliveira,T.Federico 2013

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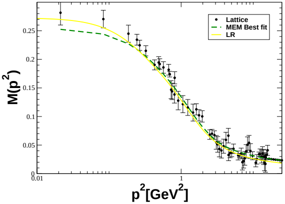

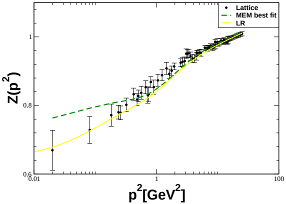

Figure: E.R,J.P.B. de Melo,B. El-Bennich,O.Oliveira,T.Federico 2013

Rainbow Ladder approximation (RLA)

$$\Gamma_\mu = \gamma_\mu \tag{20}$$

Maris Tandy model (1998)

$$\mathcal{G}(s) = \frac{4\pi^2}{\omega^6} D s e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + (1 + s/\Lambda_{QCD}^2)\right]} \mathcal{F}(s) \tag{21}$$

Qin, Chang, Liu, Roberts and Wilson 2011

$$\mathcal{G}(s) = \frac{8\pi^2}{\omega^4} D e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + (1 + s/\Lambda_{QCD}^2)\right]} \mathcal{F}(s) \tag{22}$$

Ec. Bethe-Salpeter (BSE)

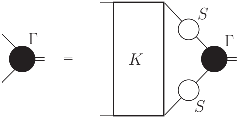

Figure: BSE

$$\lambda(P^2)\Gamma^{ab}(P,p) = \int d^4p'\,\bar{K}^{ab}(p,p';P)S(p'_+)\Gamma^{ab}(P,p')S(p'_-), \qquad (23)$$

Bethe-Salpeter equation

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$$\Gamma = T^{H}\Big[iE_{\pi}(q:P) + \gamma\cdot PF_{\pi}(q:P) + \gamma\cdot q q\cdot PG_{\pi}(q:P) \tag{24}$$

$$+\sigma_{\mu\nu}q_{\mu}P_{\nu}H_{\pi}(q:P)\Big] \equiv F^{\alpha}A^{\alpha} \tag{25}$$

with

$$A^{\beta} = \gamma_5\{i1_{4\times 4}, \gamma_{\mu}P^{\mu}, (q\cdot P)\gamma_{\mu}q^{\mu}, \sigma_{\mu 4}q^{\mu}P^{4}\}$$

$$F^{\beta} = E_{\pi}, F_{\pi}, G_{\pi}, H_{\pi}, \tag{26}$$

$$T^{H} = \lambda^{3}/2 \qquad \text{for Neutral mesons}, \tag{27}$$

$$T^{H} = (\lambda^{4} \pm i\lambda^{5})/2 \qquad \text{for } K^{+/-} \tag{28}$$

Projecting in the tesonrial basis P^α

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$$F^{\alpha}(p;P) = \frac{1}{4} Tr_D[P^{\alpha}(p;P)\Gamma(p:P)] \quad (29)$$

$$\text{para} \quad \alpha = 1, \cdots 4 \quad (30)$$

BS equation as a eigenvalue problem

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$$\lambda(P^2)F_H^{\alpha}(p;P) = \int \frac{d^4q}{(2\pi)^4} K^{\alpha\beta}(p,q;P)F_H^{\beta}(q;P) \tag{31}$$

where

$$K^{\alpha\beta}(p,q;P) \equiv$$
$$= D_{\mu\nu}^{\text{phen}}(k)\frac{1}{4}\text{Tr}_{\text{D}}[P^{\alpha}(p;P)\gamma_{\mu}S^{a}(q_{+})A^{\beta}(q;P)S^{a}(q_{-})\gamma_{\nu}] \tag{32}$$

Chebyshev moments expansion

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$$F_H^{\beta}(q;P) = \sum_{n=0}^{\infty} F_{H,n}^{\beta}(q;P)U_n(z_q). \tag{33}$$

$$\lambda(P^2)F_{H,m}^{\alpha}(p^2;P^2) = \sum_{n=0}^{\infty}\int q^2dq^2K_{mn}^{\alpha\beta}(p^2,q^2,P^2)F_{H,n}^{\beta}(q^2,P^2) \tag{34}$$

$$K_{mn}^{\alpha\beta}(p^2,q^2,P^2) \tag{35}$$

$$=\frac{2}{(2\pi)^4}\int dz_p dz_q dz_{qp} W(z_p)U_m(z_p)K^{\alpha\beta}(p,q;P)W(z_q)U_n(z_q) \tag{36}$$

Normalization

$$2P_\mu = \int \frac{d^4q}{(2\pi)^4}\left\{\mathrm{Tr}\left[\bar{\Gamma}_H(q;-P)\frac{\partial S^a(q_+)}{\partial P_\mu}\Gamma_H(q;P)S^b(q_-)+\right.\right.$$
$$\left.\left.\bar{\Gamma}_H(q;-P)S^a(q_+)\Gamma_H(q;P)\frac{\partial S^b(q_-)}{\partial P_\mu}\right]\right\} \quad (37)$$

$$P_\mu f_H = Z_2^{\mathrm{ch}} \int \frac{d^4q}{(2\pi)^4}\left\{\mathrm{Tr}\left[\frac{T^H}{2}S^a(q_+)\Gamma_H(q;P)S^b(q_-)\right]\right\} \quad (38)$$

Soluciónes a las ecuaciones de Schwinger-Dyson en el plano complejo in the RLA

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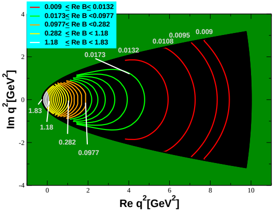

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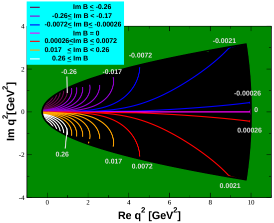

Figure: E.R,J.P.B. de Melo,B. El-Bennich, work in preparation

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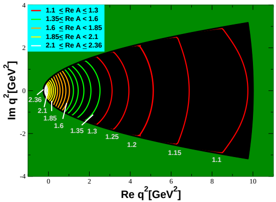


Figure:

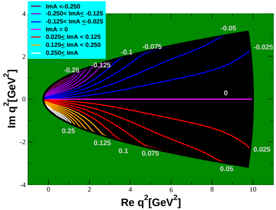
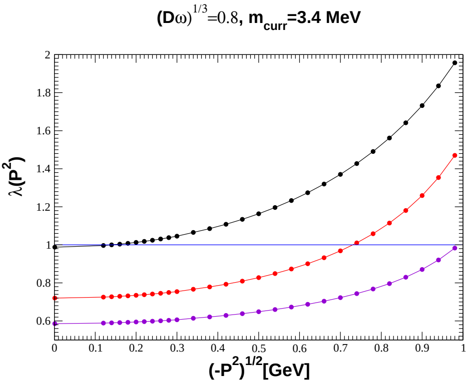


Figure: E.R,J.P.B. de Melo,B. El-Bennich, work in preparation

Bethe-Salpeter spectrum para las resonancias del pion



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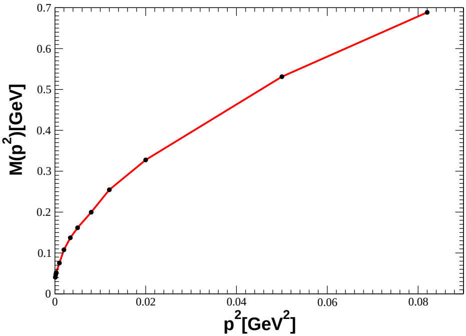
masa del pion como una función de la masa del fermion en el regimen perturbativo (current mass)



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	this work	Roberts *et al.* 2011	exp.
$A(0)$	2.07	2.07	
$M(0)$	0.62	0.62	
m_π	0.1377	0.138	0.135
f_π	0.0955	0.094	0.9221
$m_{\pi 1}$	1.	—	
m_K	0.493	0.496	0.4976
f_K	0.112	0.11	0.1104
m_{D_s}	2.13	—	1.869
f_{D_s}	0.251	—	0.257
$m_{c\bar{c}}^{J^{PC}=0^{-+}}$	3.04	—	
$f_{c\bar{c}}$	0.377		

Table: Dimensioned quantities are reported in GeV. We divide the light quarks decay constants by $\sqrt{2}$ in order to compare with Roberts *et al.* 2011. We use the standard values $\omega = 0.4 GeV$ and $(\omega D) = (0.8 GeV)^3$ in the Roberts *et al.* 2011 model.

Conclusions

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- there exist a great variety of forthcoming experiments and facilities
- implementation of the libraries based in the Arnoldi factorization method allows to calcuate in a easy way calculate high meson resonances and their corresponding Bethe-Salpeter amplitudes

Obrigado