Asymptotic Scenarios in Proton-Proton Scattering

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Outline

- Motivation
- Asymptotic Scenarios
- Goals and Dataset
- Results
- Conclusions and Perspectives

Motivation

- Cosmic ray experiments $\longrightarrow$ extensive air shower studies:

  extrapolation from accelerator experiments $\Rightarrow \dfrac{\sigma_{\rm tot}}{B}(s)$

- Problems with $B \longrightarrow$ different intervals in momentum transfeer

  large uncertainties

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[1] D.A. Fagundes, M.J. Menon, Nucl. Phys. A **880**, 1 (2012)

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- Problems with $B \longrightarrow$ different intervals in momentum transfeer

large uncertainties

- Aproximate relation: $\boxed{\dfrac{\sigma_{\rm tot}}{B} = 16\pi \dfrac{\sigma_{\rm el}}{\sigma_{\rm tot}}}$

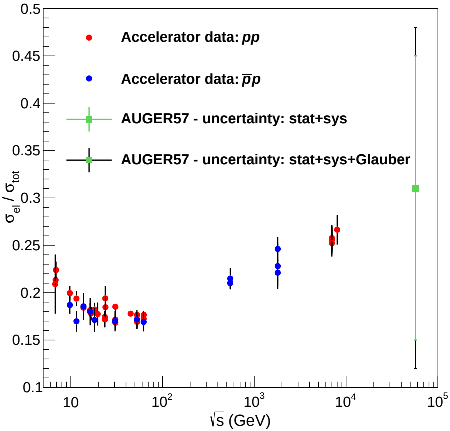
- Strategy [Fagundes and Menon[1] (FM)]

empirical fit $\dfrac{\sigma_{\rm el}}{\sigma_{\rm tot}}$ data $\longrightarrow$ *prediction* $\dfrac{\sigma_{\rm tot}}{B}(s)$

[1]D.A. Fagundes, M.J. Menon, Nucl. Phys. A **880**, 1 (2012)

Motivation

$\dfrac{\sigma_{\rm el}}{\sigma_{\rm tot}}(s)$ data $\longrightarrow$ rise with energy for $\sqrt{s} \gtrsim 100$ GeV



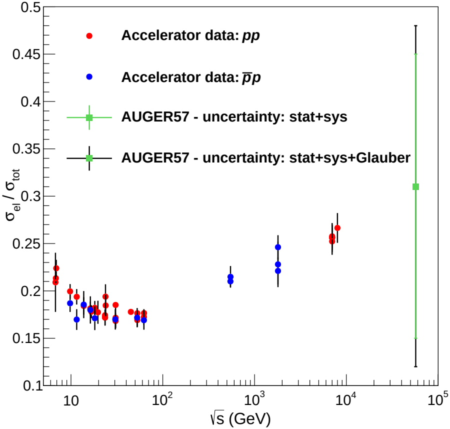
Motivation

$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}(s)$ data $\longrightarrow$ rise with energy for $\sqrt{s} \gtrsim 100$ GeV

Asymptotic limit $\longrightarrow$ Expected (all contexts)

$$s \to \infty \;\Rightarrow\; \frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \to \text{cte}$$

- 1/2 (black-disk)
- 1 (maximum/unitarity)



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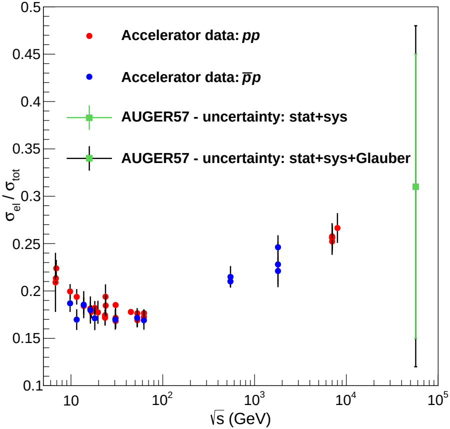

Rise and saturation (cte value) $\longrightarrow$ **change of curvature**

Motivation

Fagundes and Menon[2] (FM): empirical description with

$$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}(s) = A \tanh\left[\gamma_0 + \gamma_1 \ln s + \gamma_2 \ln^2 s\right]$$

[2] D.A. Fagundes, M.J. Menon, Nucl. Phys. A **880**, 1 (2012)

Motivation

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$$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}(s) = A \tanh\left[\gamma_0 + \gamma_1 \ln s + \gamma_2 \ln^2 s\right]$$

A → asymptotic value

tanh → change of curvature

- $A = 1/2$ and $A = 1$ (fixed parameters)
- pp accelerator data only

  $\sqrt{s}_{\text{min}} = 10$ GeV

  $\sqrt{s}_{\text{max}} = 7$ TeV (1 TOTEM point)
- Extension to $\frac{\sigma_{\rm tot}}{B}(s) = 16\pi \frac{\sigma_{\rm el}}{\sigma_{\rm tot}}(s)$ (cosmic-rays; uncertainties)

[2] D.A. Fagundes, M.J. Menon, Nucl. Phys. A **880**, 1 (2012)

This presentation

- Inclusion of all TOTEM data on $\sigma_{\rm el}$ and $\sigma_{\rm tot}$ (7 and 8 TeV)
- $\sqrt{s}_{\sf min} = 5$ GeV, pp and $\bar{p}p$ dataset
- Study on 3 scenarios: **black-disk**, below and above

Empirical results $\longrightarrow$ favour below black-disk

Asymptotic Scenarios I: The Black-Disk Limit

- Naive model (Gray-Disk):

  Profile function $\Gamma(s,b) = \Gamma_0(s)$ for $b \leq R(s)$ (0 otherwise)

$$\sigma_{\text{el}}(s) = \pi R^2 \Gamma_0^2$$

$$\sigma_{\text{tot}}(s) = 2\pi R^2 \Gamma_0$$

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} = \frac{\Gamma_0}{2}}$$

- **Black-Disk Model**: $\Gamma_0 = 1$

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} = \frac{1}{2}}$$

Asymptotic Scenarios I: The Black-Disk Limit

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$$\sigma_{\rm tot}(s) = 2\pi R^2 \Gamma_0$$

$$\boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} = \frac{\Gamma_0}{2}}$$

- **Black-Disk Model**: $\Gamma_0 = 1$

$$\boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} = \frac{1}{2}}$$

- Typical of eikonal models (unitarized by construction): Chou-Yang, Bourrely-Soffer-Wu, Block-Halzen, etc

Asymptotic Scenarios II: Below the Black-Disk

(1) FMS[3] and MS[4]

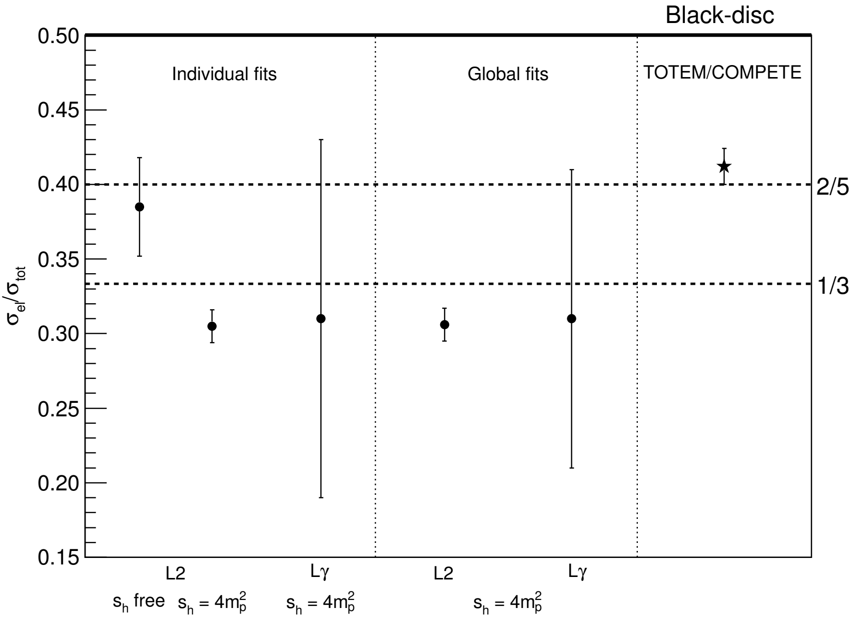
$$\sigma_{\text{tot}}(s) = a_1 \left(\frac{s}{s_l}\right)^{-b_1} + \tau a_2 \left(\frac{s}{s_l}\right)^{-b_2} + \alpha + \beta \ln^{\gamma}(s/s_h)$$

- $\gamma = 2$ fixed and γ as a free fit parameter
- Extension to σ_{el} data ($\gamma = 2$ and $\gamma > 2$)
- Lowest value:

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \to 0.3}$$

[3] D.A. Fagundes, M.J. Menon, P.V.R.G. Silva, J. Phys. G **40**, 065005 (2013)
[4] M.J. Menon, P.V.R.G. Silva, Int. J. Mod. Phys. A **28**, 1350099 (2013)
M.J. Menon, P.V.R.G. Silva, J. Phys. G **40**, 125001 (2013).

Asymptotic Scenarios II: Below the Black-Disk



Asymptotic Scenarios II: Below the Black-Disk

(2) COMPETE and TOTEM results

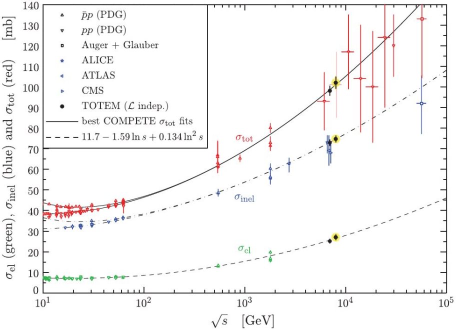
COMPETE[5] highest-rank result:

$$\sigma_{\rm tot}(s) = \text{Regge} + 35.5 + 0.307\ln^2(s/29.1\ \text{GeV}^2)$$

TOTEM[6] empirical fit to $\sigma_{\rm el}$ data:

$$\sigma_{\rm el}(s) = 11.7 - 1.59\ln s + 0.134\ln^2 s$$

$$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} = 0.436$$



[5]J.R. Cudell *et al* (COMPETE Collab.), Phys. Rev. Lett. **89**, 201801 (2002)
[6]G. Antchev *et al* (TOTEM Collab.), Phys. Rev. Lett. **111**, 012001 (2013)

Asymptotic Scenarios III: Above the Black-Disk

(1) Obvious bound from Unitarity: $\dfrac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \leq 1$

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \to 1} \quad (s \to \infty)$$

(2) U-matrix unitarization[7] $\longrightarrow$ Predicts $\dfrac{\sigma_{\text{el}}}{\sigma_{\text{tot}}}$ beyond black-disk limit

[7] S.M. Troshin, N.E. Tyurin, Phys. Lett. B **316**, 175 (1993)
S.M. Troshin, N.E. Tyurin, Int. J. Mod. Phys. A **22**, 4437 (2007)

Asymptotic Scenarios III: Above the Black-Disk

(3) Two formal results[8,9] ($s \to \infty$):

$$\boxed{\sigma_{\rm tot}(s) \leq \frac{\pi}{m_\pi^2} \ln^2 s} \quad \text{and} \quad \boxed{\sigma_{\rm in}(s) \leq \frac{\pi}{4 m_\pi^2} \ln^2 s}$$

If **both** limits saturate:

$$\frac{\sigma_{\rm in}}{\sigma_{\rm tot}} \to \frac{1}{4} \xrightarrow{\text{Unitarity}} \boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \to \frac{3}{4} = 0.75}$$

[8] M. Froissart, Phys.Rev. **123**, 1053 (1961)
A. Martin, Il Nuovo Cimento **42**, 930 (1966)
L. Lukaszuk, A. Martin, Il Nuovo Cimento **52**, 122 (1967)
[9] A. Martin, Phys. Rev. D **80**, 065013 (2009)

Goals in this work

- Studies with 5 different asymptotic scenarios:

$$A = 0.3,\ 0.436,\ \mathbf{0.5},\ 0.75,\ 1$$

- Include new data by TOTEM (7 and 8 TeV)
- Include of data from $\bar{p}p$ scattering
- Empirical parametrization improved

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Dataset (accelerator data)

- $\sqrt{s}_{\text{min}} = 5$ GeV
- $\sqrt{s}_{\text{max}} = 8$ TeV
- $pp + \bar{p}p$ data

Parametrization

$$\boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}}(s) = f(s) = A \tanh g(s)}$$

Improved empirical parametrization (trial and error)

$$g(s) = \alpha + \beta \ln^{1/2}(s/s_0) + \gamma \ln(s/s_0)$$

- α, β and γ are free dimensionless parameters
- $s_0 = 25$ GeV^2 fixed (energy cutoff)

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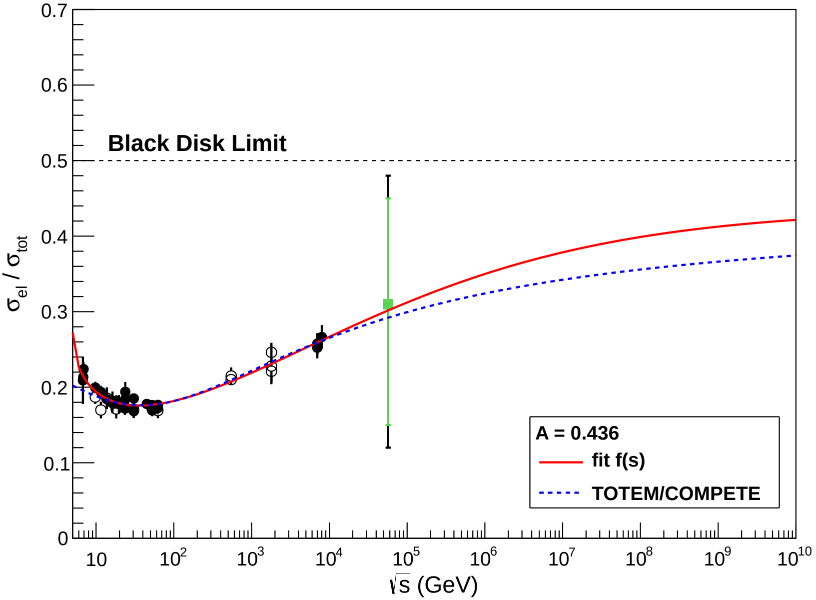
- α, β and γ are free dimensionless parameters
- $s_0 = 25$ GeV^2 fixed (energy cutoff)

- Importance of $f(s) \longrightarrow$ only 3 free dimensionless parameters
  (A and s_0 fixed)

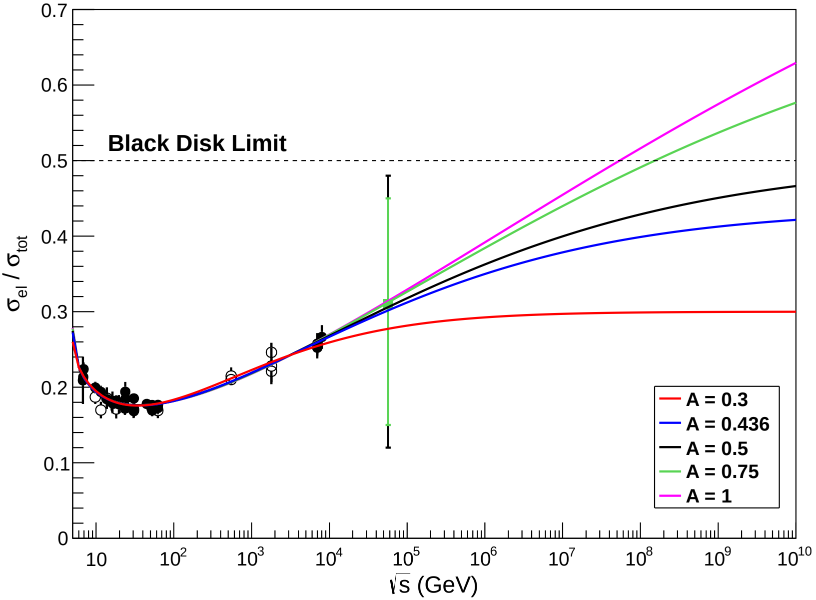
Example $\longrightarrow$ fit $\sigma_{\rm tot}$ and $\sigma_{\rm el}$ (FMS, MS, TOTEM/COMPETE):

12 - 14 free parameters!

TOTEM/COMPETE and fit with $f(s)$



Results with A fixed



A as a free parameter

With the previous results as Initial Values, consider A as a free parameter

A fixed	Results with A free
0.3	
0.436	
0.5	
0.75	
1	

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A fixed		Results with A free
0.3	$\longrightarrow$	0.360 ± 0.078
0.436		
0.5		
0.75		
1		

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0.3	$\longrightarrow$	0.360 ± 0.078
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0.5		
0.75		
1		

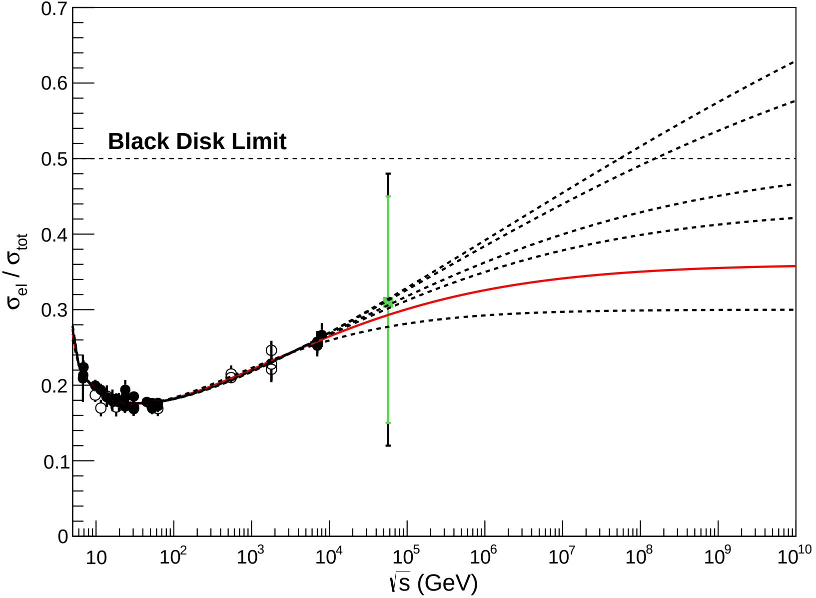
A as a free parameter

With the previous results as Initial Values, consider A as a free parameter

A fixed	Results with A free
0.3	0.360 $\pm$ 0.078
0.436	0.361 $\pm$ 0.078
0.5	0.361 $\pm$ 0.077
0.75	0.361 $\pm$ 0.077
1	0.361 $\pm$ 0.077

All cases: $\boxed{A \simeq 0.36 \pm 0.08}$ $\Rightarrow$ below black-disk limit.

$A = 0.36 \pm 0.08$



0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Black Disk Limit
σel / σtot
10
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^10
√s (GeV)

A as a free parameter

From above results:

$$\boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \to 0.36 \pm 0.08}$$

Agreement with Pumplin bound[10]

$$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} + \frac{\sigma_{\rm diff}}{\sigma_{\rm tot}} \leq \frac{1}{2}$$

Estimations[11] at 7 TeV: $\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} = 0.256 \pm 0.013$, $\frac{\sigma_{\rm diff}}{\sigma_{\rm tot}} \simeq 0.24^{+0.05}_{-0.06}$

$$\boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} + \frac{\sigma_{\rm diff}}{\sigma_{\rm tot}} = 0.496^{+0.05}_{-0.06}}$$

[10] J. Pumplin, Phys. Rev. D **8**, 2899 (1973)
[11] P. Lipari, M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013)

Conclusions and Perspectives

- **Conclusions**
  - Asymptotic $\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} < 1/2$ is a possibility
  - Asymptotic scenario for $\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} \longrightarrow$ Still an *open problem*

Conclusions and Perspectives

- **Conclusions**
    - Asymptotic $\dfrac{\sigma_{\rm el}}{\sigma_{\rm tot}} < 1/2$ is a possibility
    - Asymptotic scenario for $\dfrac{\sigma_{\rm el}}{\sigma_{\rm tot}} \longrightarrow$ Still an *open problem*
- **Perspectives**
    - Uncertainty region studies (due different scenarios) $\sigma_{\rm el}/\sigma_{\rm tot}$
    - Extension to $\dfrac{\sigma_{\rm tot}}{B}(s)$

Sponsors

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

FAPESP

THANK YOU!!

Backup Slides

Relation between $\sigma_{\rm tot}/B$ and $\sigma_{\rm el}/\sigma_{\rm tot}$

- Differential cross section (forward peak): $\frac{d\sigma}{dt} = \left.\frac{d\sigma}{dt}\right|_{t=0} e^{Bt}$

- Optical point: $\left.\frac{d\sigma}{dt}\right|_{t=0} = \frac{(1+\rho^2)}{16\pi}\sigma^2_{\rm tot}$

- Integrated elastic cross section: $\sigma_{\rm el} = \int_{t_0}^{0} \frac{d\sigma}{dt} dt$

- With assumption $1+\rho^2 \approx 1$, taking limit $t_0 \to -\infty$ and using the optical point:

$$\sigma_{\rm el}(s) = \frac{1}{B(s)}\frac{\sigma^2_{\rm tot}(s)}{16\pi} \Rightarrow \boxed{\frac{\sigma_{\rm tot}(s)}{B(s)} = 16\pi\frac{\sigma_{\rm el}(s)}{\sigma_{\rm tot}(s)}}$$

Black-disk model

- Impact parameter formalism (azimuthal symmetry)

$$F(s,q) = ik\int_0^\infty bdbJ_0(qb)\Gamma(s,b)$$

Gray-disk (Profile function):

$$\Gamma(s,b) = \begin{cases} \Gamma_0(s), & b \le R(s) \\ 0, & b > R(s) \end{cases}$$

$$\sigma_{\rm tot} = 2\pi R^2 |\Gamma_0|^2$$

$$\sigma_{\rm el} = \pi R^2 \,\mathrm{Re}\,\Gamma_0$$

$$\frac{d\sigma}{dq^2} = \frac{|\Gamma_0|^2 k^2 R^4}{16\pi s^2}\left|\frac{J_1(qR)}{qR}\right|^2$$

- **Black-disk**: $\Gamma_0 \to 1$

$$\boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} = \frac{1}{2}}$$

(black-disk limit)

Estimation[12] of $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ at 7 TeV

- TOTEM (indep. lum.):

  $\sigma_{\text{tot}} = 98.0 \pm 2.5$ mb, $\sigma_{\text{el}} = 25.1 \pm 1.1$ mb, $\sigma_{\text{in}} = 72.9 \pm 1.5$ mb,

  $\sigma_{\text{el}}/\sigma_{\text{tot}} = 0.256 \pm 0.013$

- ALICE: Fraction of single (SD) and double (DD) diffraction in inelastic collisions:

$$\frac{\sigma_{\text{SD}}}{\sigma_{\text{in}}} = 0.20^{+0.04}_{-0.07} \quad \text{and} \quad \frac{\sigma_{\text{DD}}}{\sigma_{\text{in}}} = 0.12^{+0.05}_{-0.04}$$

  With $\sigma_{\text{diff}} = \sigma_{\text{SD}} + \sigma_{\text{DD}}$:

$$\frac{\sigma_{\text{diff}}}{\sigma_{\text{in}}} = 0.32^{+0.06}_{-0.08}$$

[12] P. Lipari, M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013)

Estimation[12] of $\sigma_{\rm diff}/\sigma_{\rm tot}$ at 7 TeV

- Combining TOTEM and ALICE results:

$$\sigma_{\rm diff} = 23.3^{+4.4}_{-5.9}\ \text{mb} \quad \text{and} \quad \frac{\sigma_{\rm diff}}{\sigma_{\rm tot}} \simeq 0.24^{+0.05}_{-0.06}$$

$$\boxed{\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} + \frac{\sigma_{\rm diff}}{\sigma_{\rm tot}} = 0.496^{+0.05}_{-0.06}}$$

- Indicates saturation of Pumplin bound at LHC energy

[12] P. Lipari, M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013)

σ_{SD}

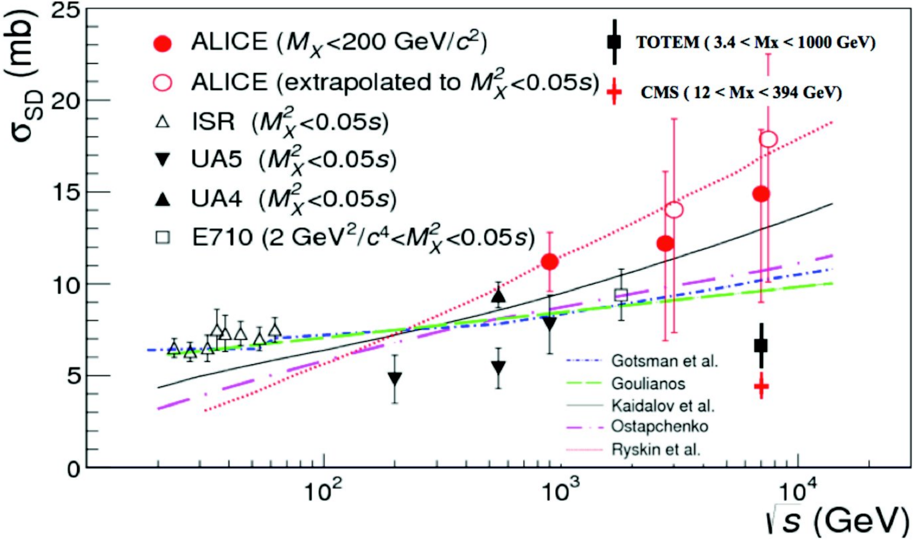


Figure: N. Cartiglia, arXiv:1305.6131v3 [hep-ex]

Variants for $g(s)$

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}}(s) = A \tanh g(s)}$$

- $g_2(s) = \gamma_0 + \gamma_1 \ln(s/s_0) + \gamma_2 \ln^2(s/s_0)$ [Fagundes and Menon]

- $g_\gamma(s) = \gamma_0 + \gamma_1 \ln(s/s_0) + \gamma_2 \ln^{\gamma_3}(s/s_0)$

- γ_i $(i = 0, 1, 2, 3)$ are free parameters
- $s_0 = 25$ GeV^2 fixed (energy cutoff)

Variants for $g(s)$

- Fits with $pp + \bar{p}p$ dataset and for all A values [$g_\gamma(s)$ variant]:

$$\gamma_3 \in [0.31, 0.60] \quad \xrightarrow[\text{Decreasing } A]{}$$

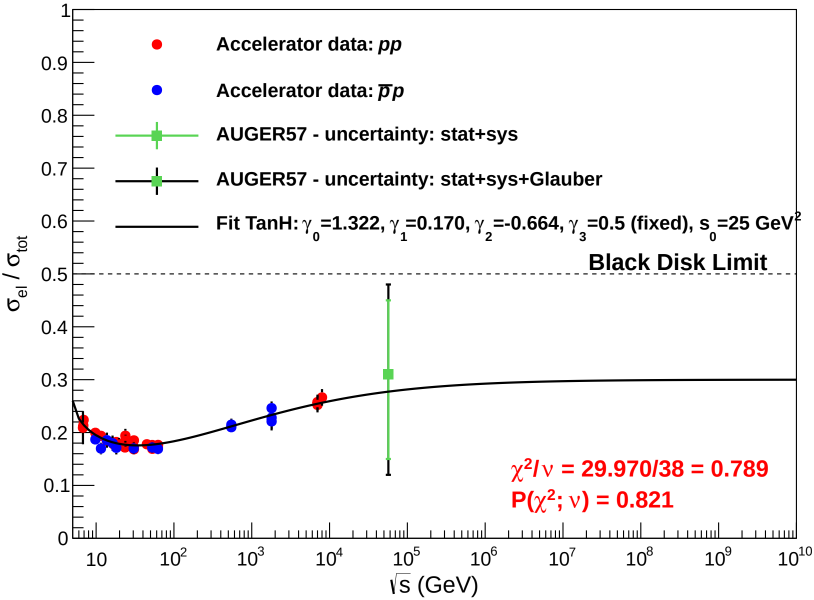
- New variant: $\gamma_3 = 0.5$ fixed

$$g_{1/2}(s) = \gamma_0 + \gamma_1 \ln(s/s_0) + \gamma_2 \ln^{1/2}(s/s_0)$$

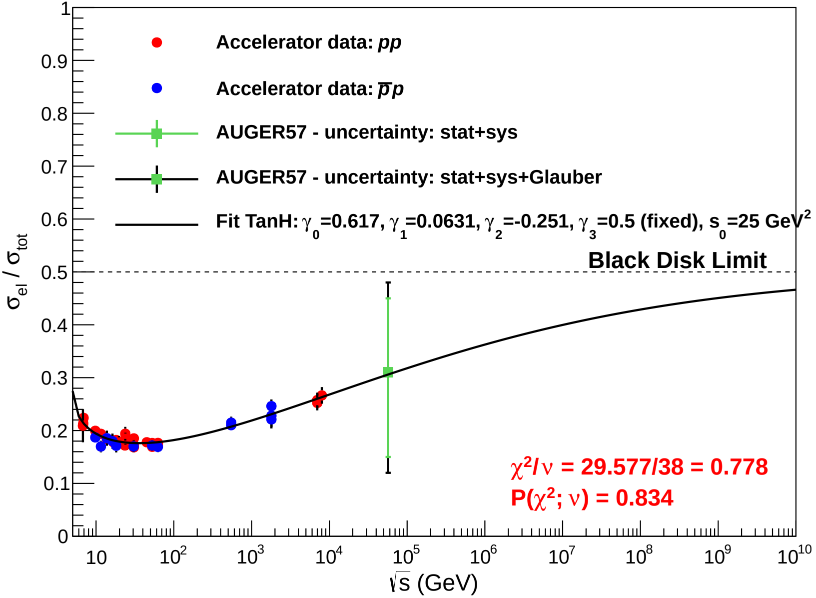
Parameters

A	α	β	γ	$P(\chi^2, \nu)$
0.3 (fixed)	1.36	-0.66	0.17	0.82
0.361	0.96	-0.43	0.11	0.81
0.436 (fixed)	0.73	-0.31	0.078	0.84
0.361	0.96	-0.43	0.11	0.81
0.5 (fixed)	0.62	-0.25	0.063	0.83
0.361	0.96	-0.43	0.11	0.81
0.75 (fixed)	0.39	-0.15	0.038	0.82
0.361	0.96	-0.43	0.11	0.81

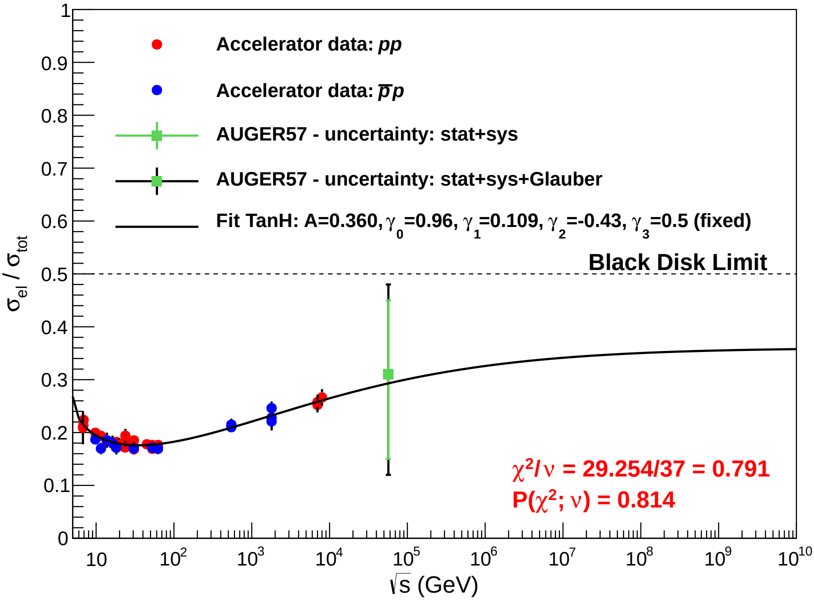
Result: $A = 0.3$ (fixed), $g_{1/2}(s)$



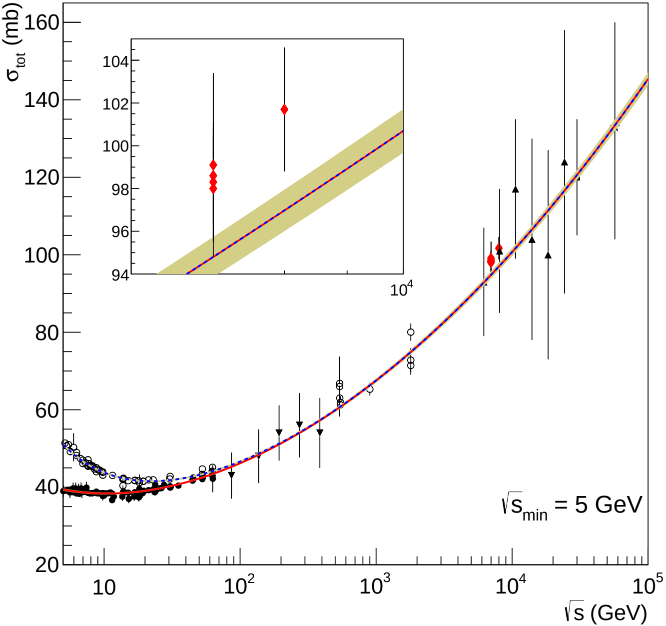
Result: $A = 0.5$ (fixed), $g_{1/2}(s)$



Fit with A free $\longrightarrow$ Initial Value: $A = 0.3$

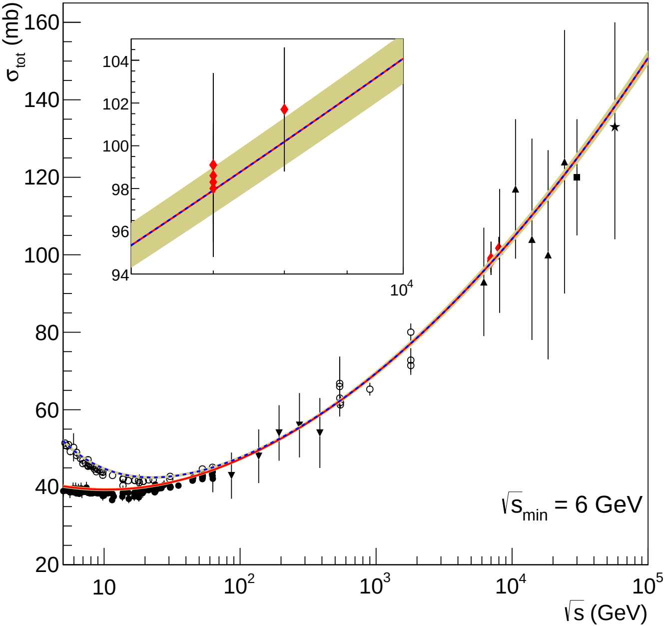


PDG



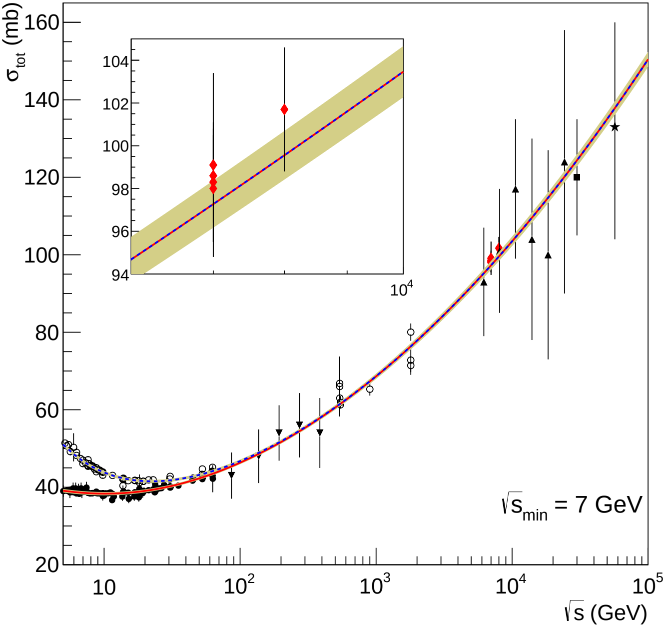
σ_tot (mb)
160
140
120
100
80
60
40
20
104
102
100
98
96
94
10^4
√s_min = 5 GeV
10
10^2
10^3
10^4
10^5
√s (GeV)

PDG



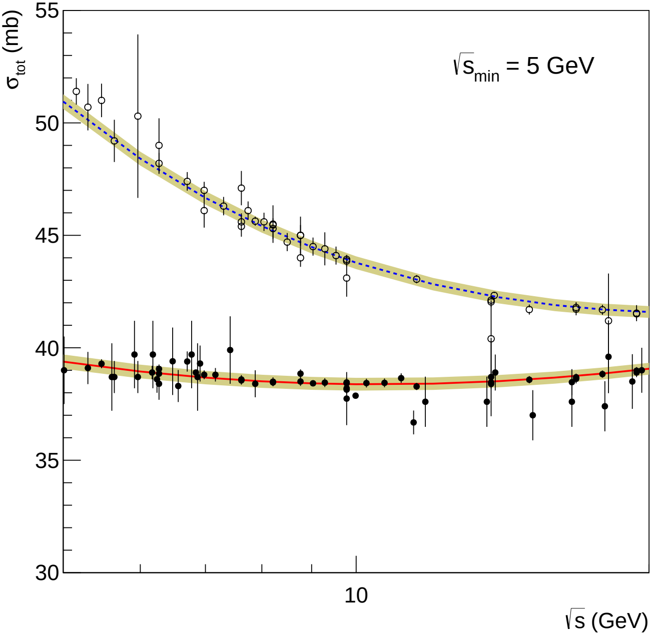
σ_tot (mb)
160
140
120
100
80
60
40
20
104
102
100
98
96
94
10^4
√s_min = 6 GeV
10
10^2
10^3
10^4
10^5
√s (GeV)

PDG

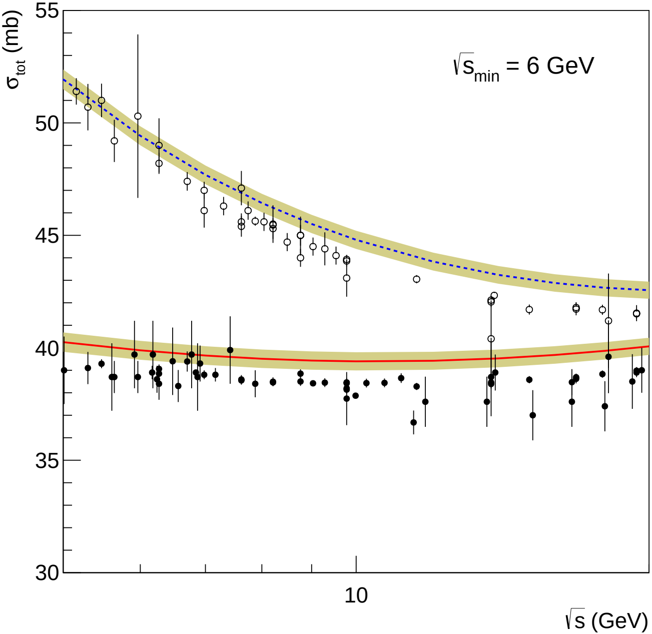


σ_{tot} (mb)
160
140
120
100
80
60
40
20
10
10^2
10^3
10^4
10^5
$\sqrt{s}$ (GeV)
104
102
100
98
96
94
10^4
$\sqrt{s_{min}}$ = 7 GeV

PDG (5 GeV - 20 GeV)



PDG (5 GeV - 20 GeV)



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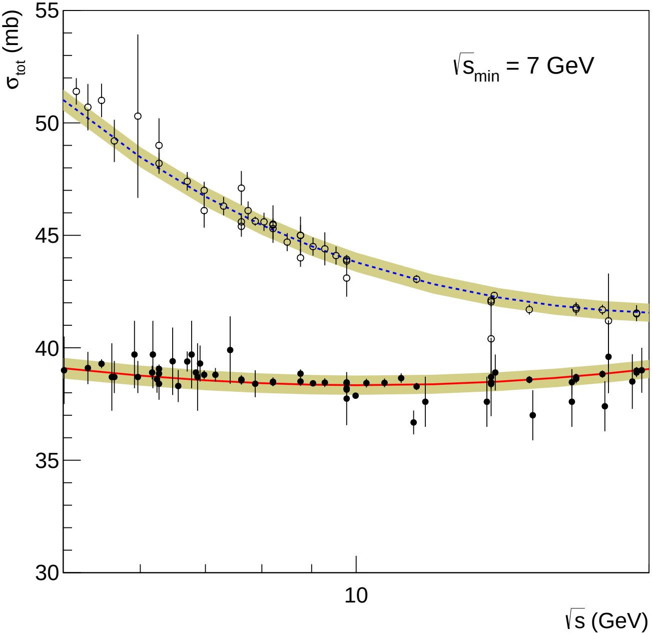

Martin

Ref.: A. Martin, Phys. Rev. D **80**, 065013 (2009)

Depois de obter o limite $\sigma_{\rm in} < \dfrac{\pi}{4m_\pi^2}\ln^2 s$ (pag. 3):

> **This ends the rigorous part of this paper.** *Now comes the fact that most theoreticians believe that the worse that can happen at high energies is that the elastic cross section reaches half of the total cross section, which corresponds to an expanding black disk.*

Martin

No final do artigo (pag. 3), a respeito do limite $\sigma_{\text{el}}/\sigma_{\text{tot}} > 1/2, \quad s \to \infty$:

> *To say the least, this seems to me extremely unlikely and, therefore,* **I tend to believe** *that we have*
>
> $$\sigma_{\text{tot}} < \frac{\pi}{2m_\pi^2} \ln^2 s.$$

Entretanto, alguns artigos passaram a utilizar este “resultado” como o Limite de Froissart-Martin, por exemplo

- M.M. Block, F. Halzen, Phys. Rev. Lett. **107**, 212002 (2011)
- N. Cartiglia, *Measurement of the proton-proton total, elastic, inelastic and diffractive cross sections at 2, 7, 8 and 57 TeV*, arXiv:1305.6131v3 [hep-ex]
- I.M. Dremin, *Hadron structure and elastic scattering*, arXiv:1311.4159 [hep-ph]