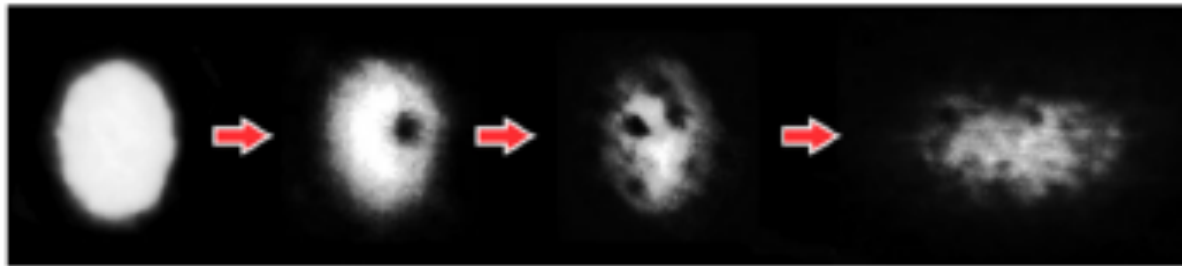


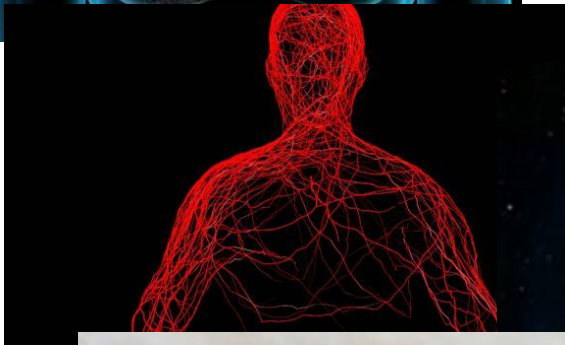
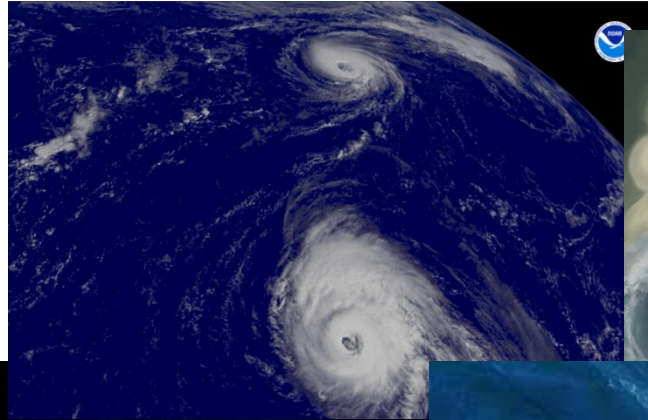
# Quantum Turbulence in BEC overview and new perspectives

**Vanderlei S. Bagnato**

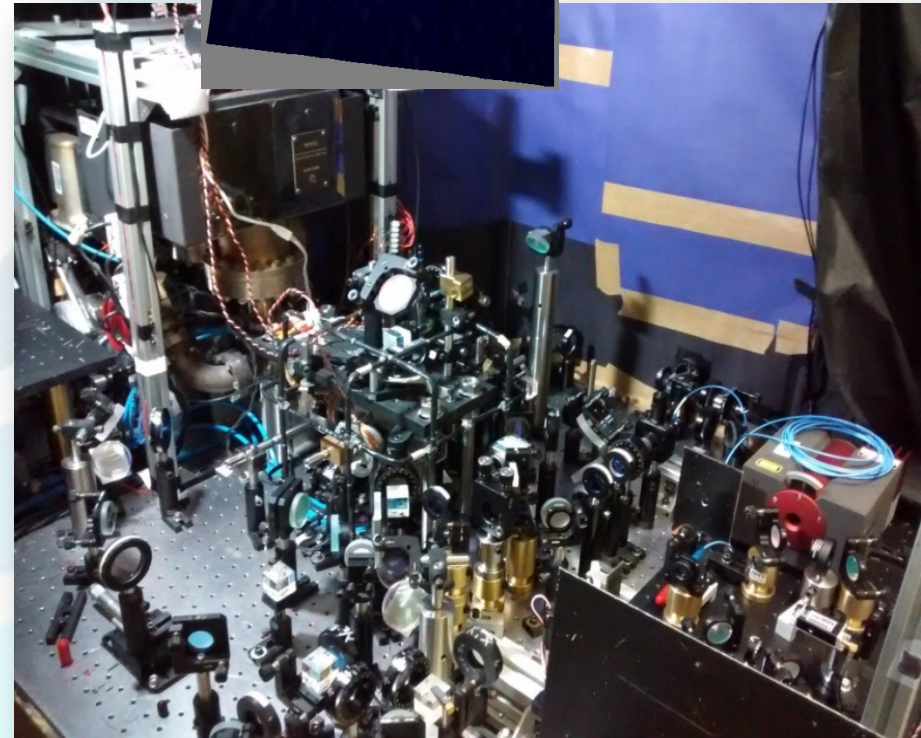
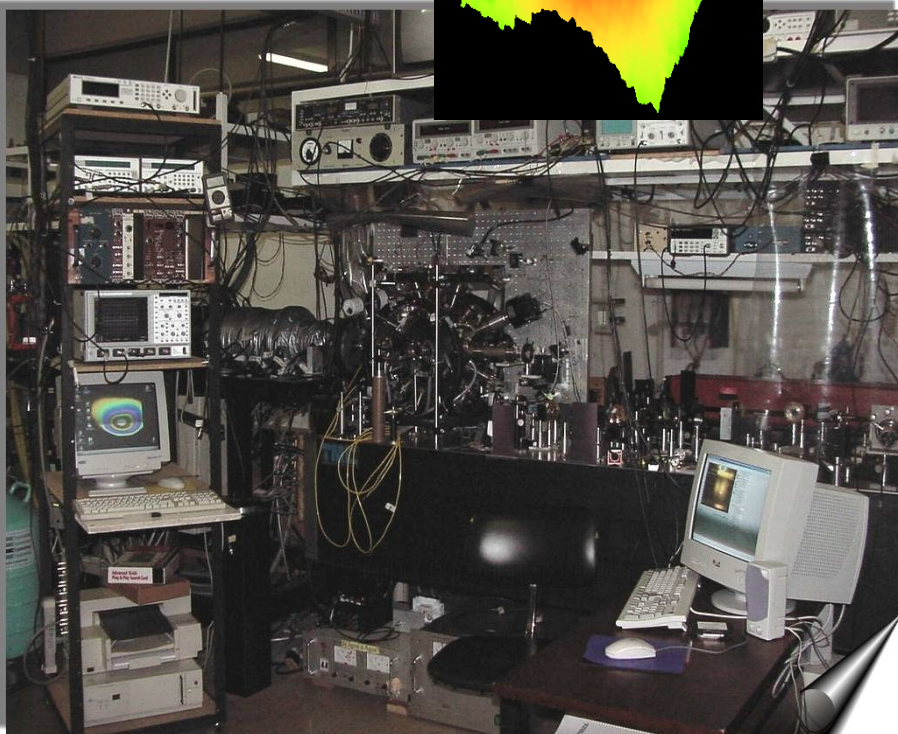
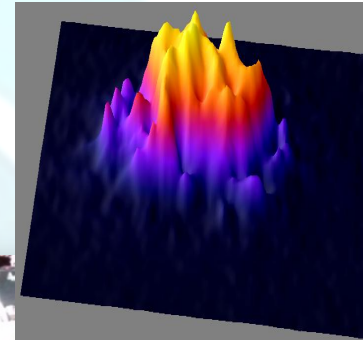
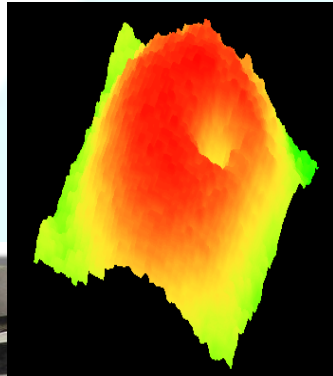
Instituto de Física de São Carlos - University of São Paulo - Brazil



# Turbulence is all around us

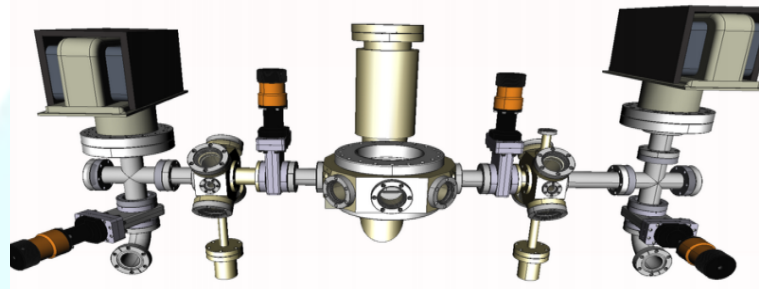


# Quantum turbulence in superfluids



## Na/K BEC Mixture

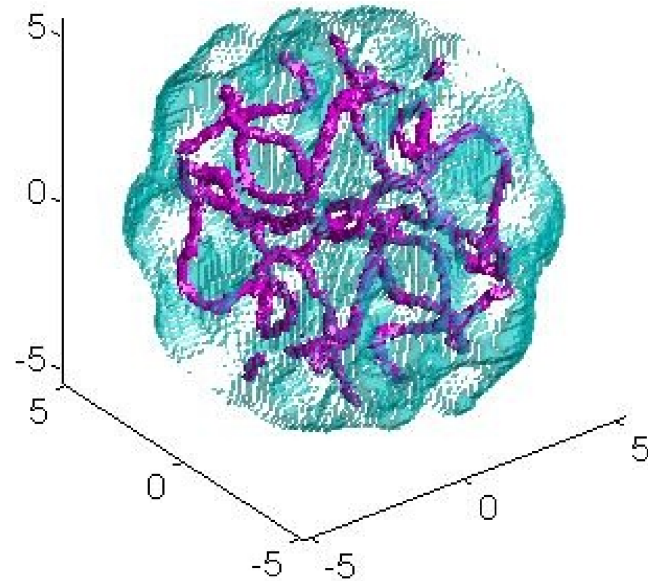
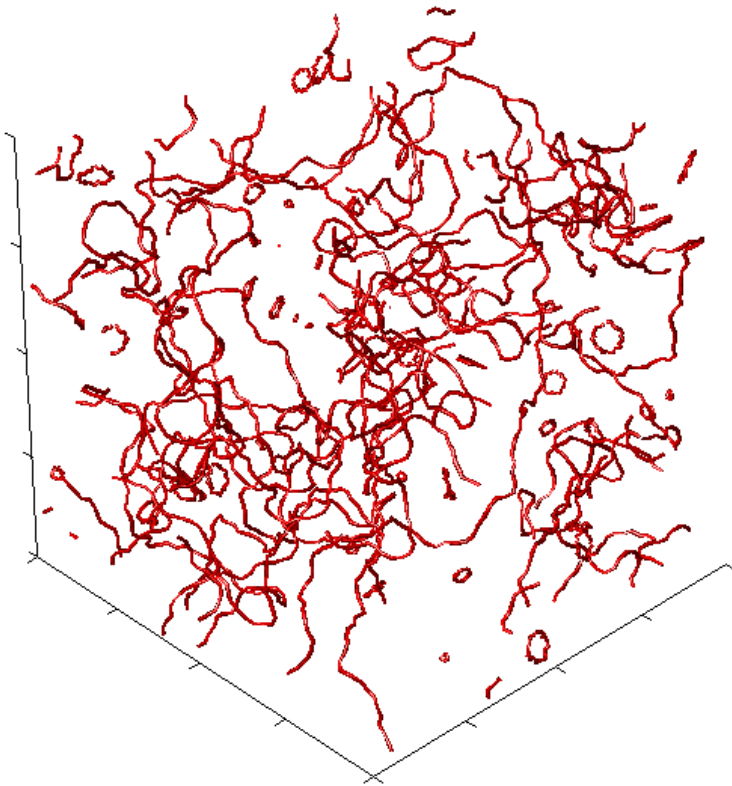
# Mixture of two superfluids



K source



## Idea of turbulent regime in superfluids



1955: **Feynman** proposed that “superfluid turbulence” consists of a tangle of quantized vortices.

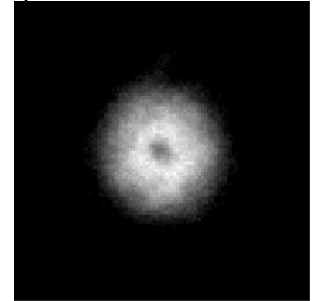
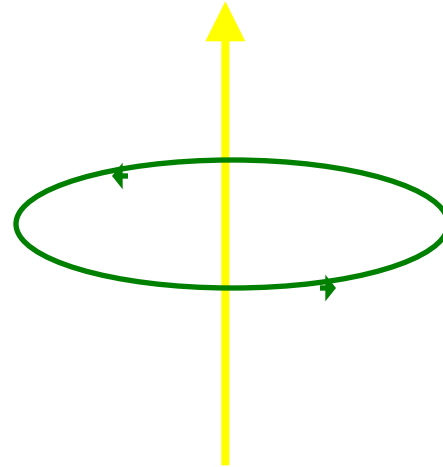
# (1) Circulation

$$\oint \mathbf{v}_s \cdot d\mathbf{s} = \kappa n$$

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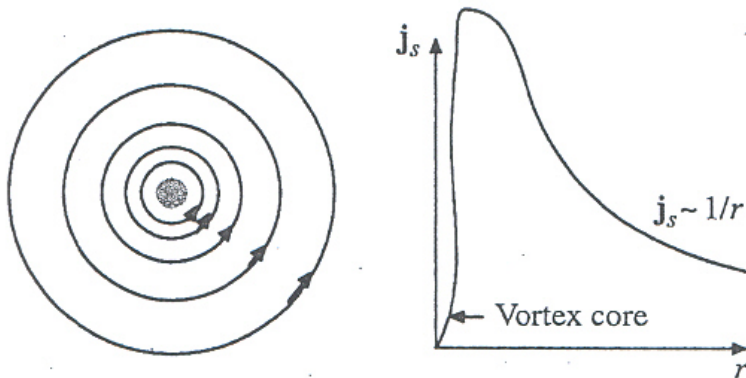
$$\kappa = h / m$$

Stability  $\Rightarrow n = 1$

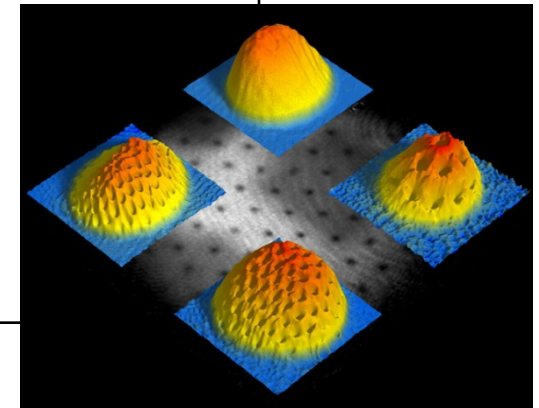


ENS

(2) core size is very small.

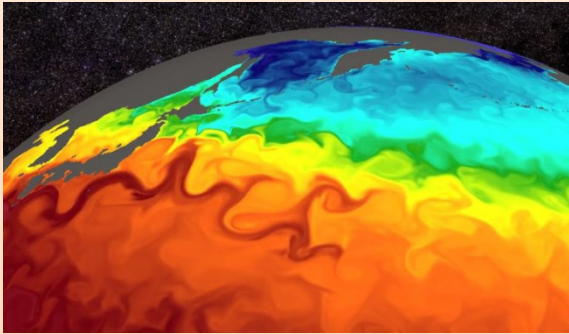


$$\text{Healing length} = (8\pi \rho a)^{-1/2}$$



MIT

## Classical Turbulence



Eddies produced by flow

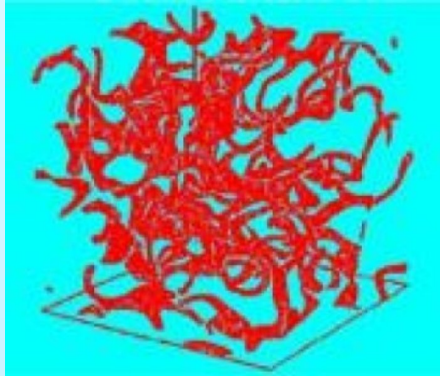
Many values of vorticities

Flux lines crossing

Cascade of energy

Viscous dissipation at small scales

## Quantum turbulence



Quantum vortices – single vorticity

Tangle configurations

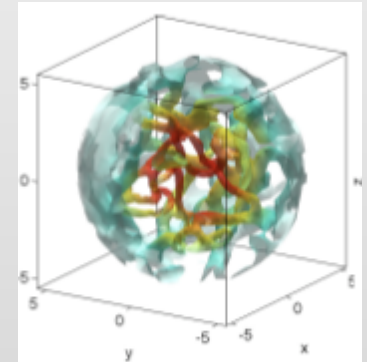
Reconnection

Large number of vortices

Small core

Many decades of scale

## QT in BEC



Fine size

Finite number of vortices

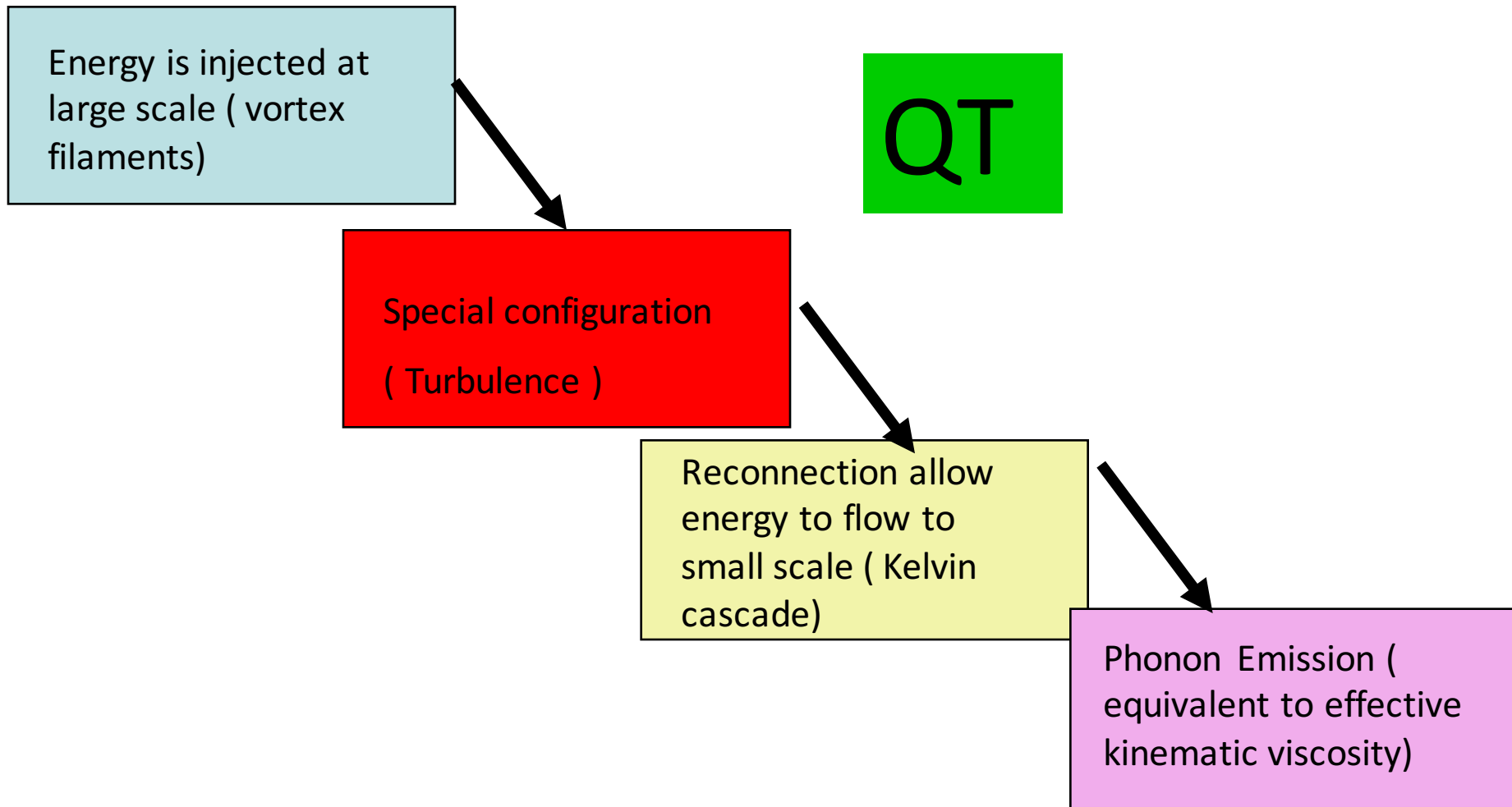
Reconnection

Large Core

Few decades of scale

-In comparison with many other areas, our knowledge and understanding of Turbulence ( classical and quantum ) is primitive

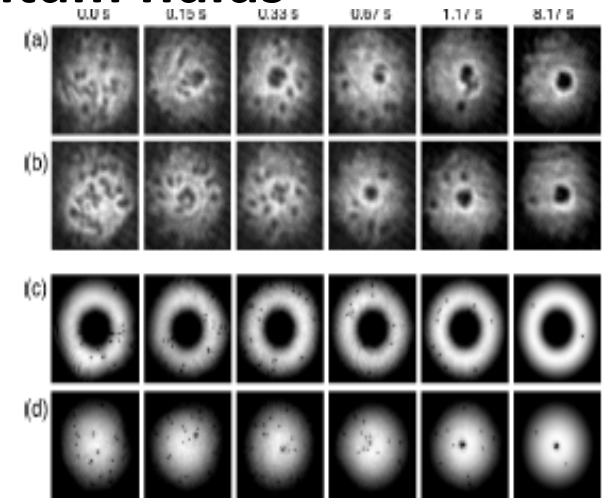
-The topic is placed as a challenges for present decade



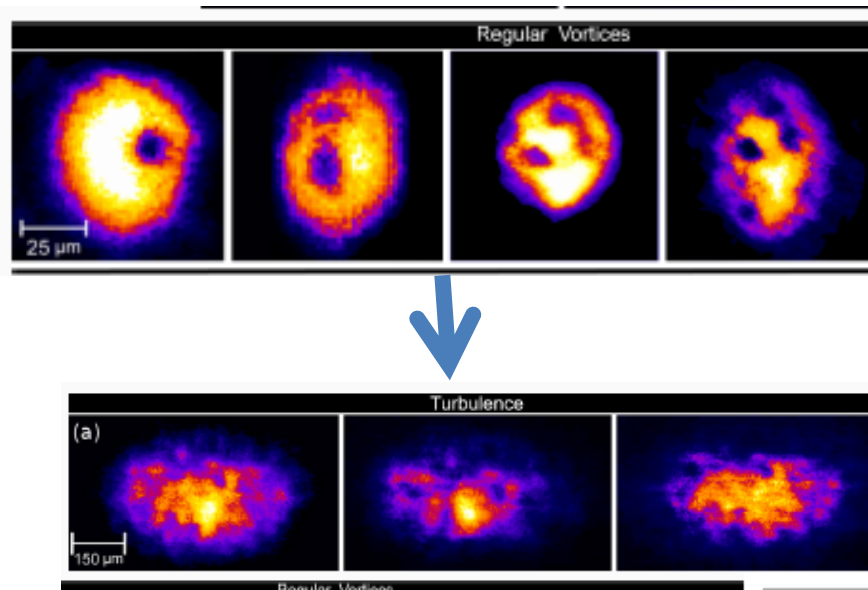
# Investigations involving 2D quantum fluids

2D: Anderson's group, Arizona:

- Highly oblate condensate: aspect ratio  $\omega_z/\omega_\perp \sim 11 \rightarrow$  2D vortex dynamics.
- Gaussian laser beam directed through the trap centre  $\rightarrow$  annular trap.
- Centre of the harmonic trap moves in a circle  $\rightarrow$  small scale forcing induced, vortices nucleated.



## 3D Turbulence



## In terms of QT, BEC of trapped atoms goes beyond superfluid Helium

- Small number of vortices
- Smaller range of intervortex spacing
- Possibility to control vortex lines
- 1D,2D,3D
- Homogeneous and non-homogeneous
- Adjustment of intrinsic atomic properties
- Large range of densities
- Mixtures
- MORE.....

Sequence of works

2009

**GENERATION OF VORTICES**

**FORMATIONS OF VORTICES CLUSTERS**

**EMERGENCE OF TURBULENCE**

**SELF-SIMILAR EXPANSION**

**DIAGRAM OF EXCITATIONS**

**FINITE SIZE EFFECT**

**GRANULATION**

**MODEL FOR SELF-SIMILAR EXPANSION**

**SECOND SOUND EXCITATION (COUNTER FLOW )**

**KINETIC ENERGY SPECTRUM**

**HIGHLY CHARGED VORTEX**

**ESPONTANEOUS GENERATION OF QT**

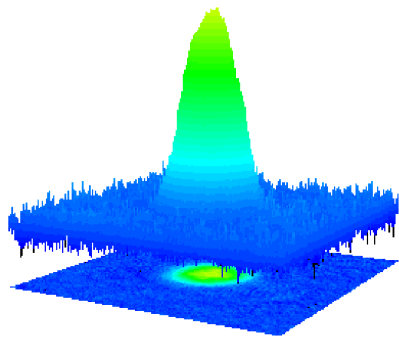
**DYNAMICAL STABILITY OF HIGLY CHARGED VORTEX**

**MATTER WAVE ASPECT - SPECKLES**

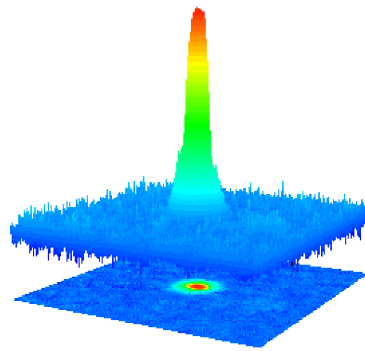
2016

<http://cepof.ifsc.usp.br>

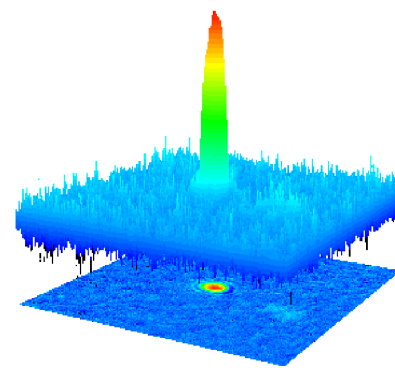
1



$$T > T_c$$

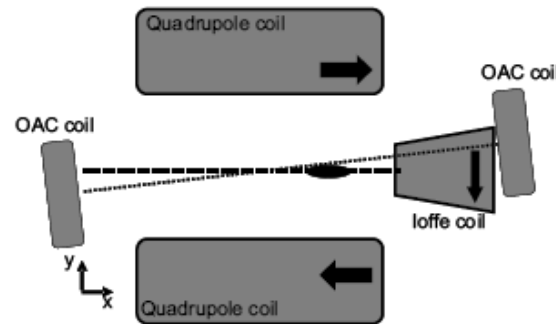


$$T < T_c$$

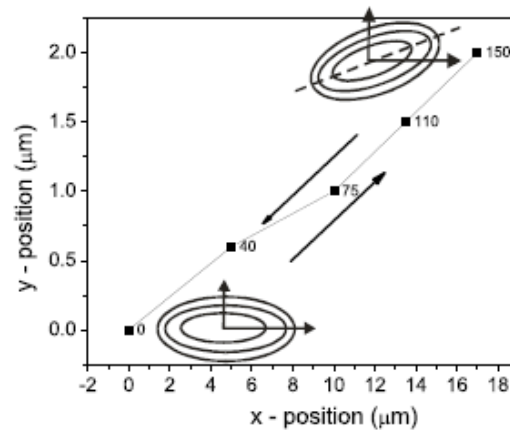


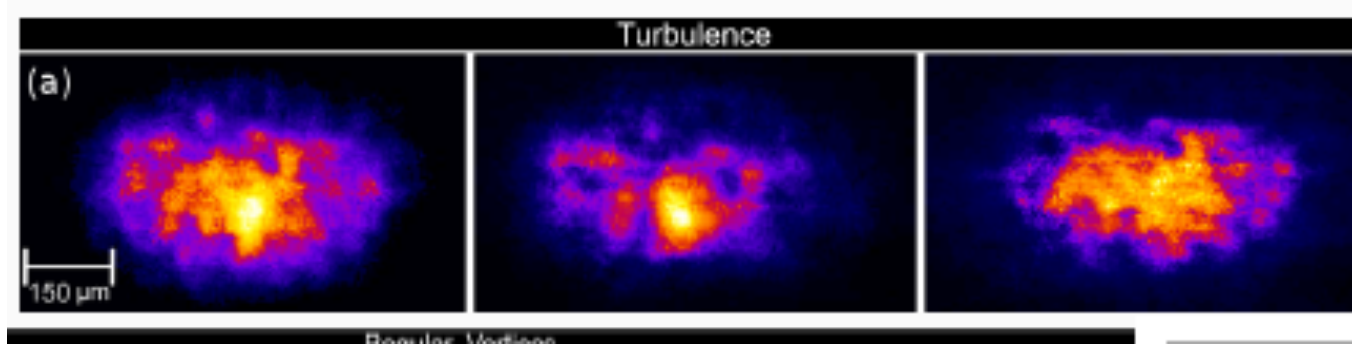
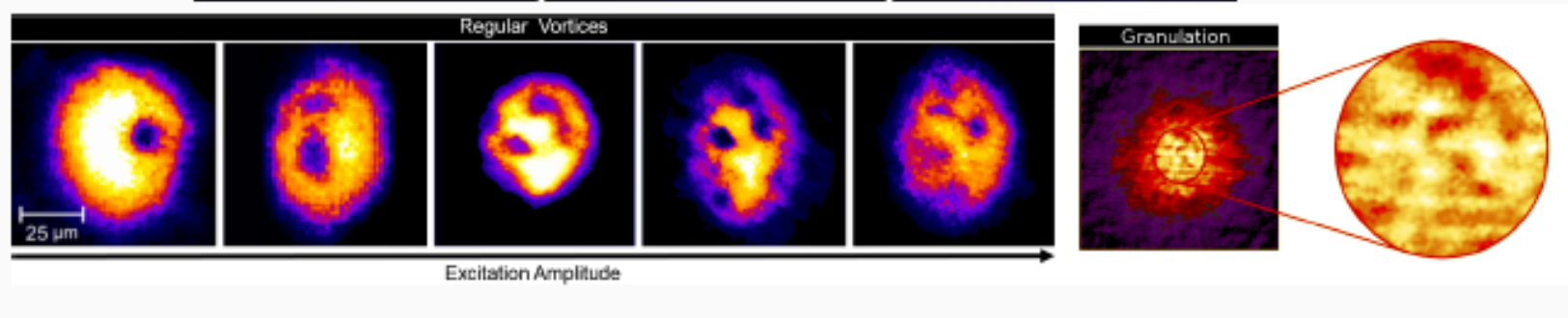
$$T \ll T_c$$

2

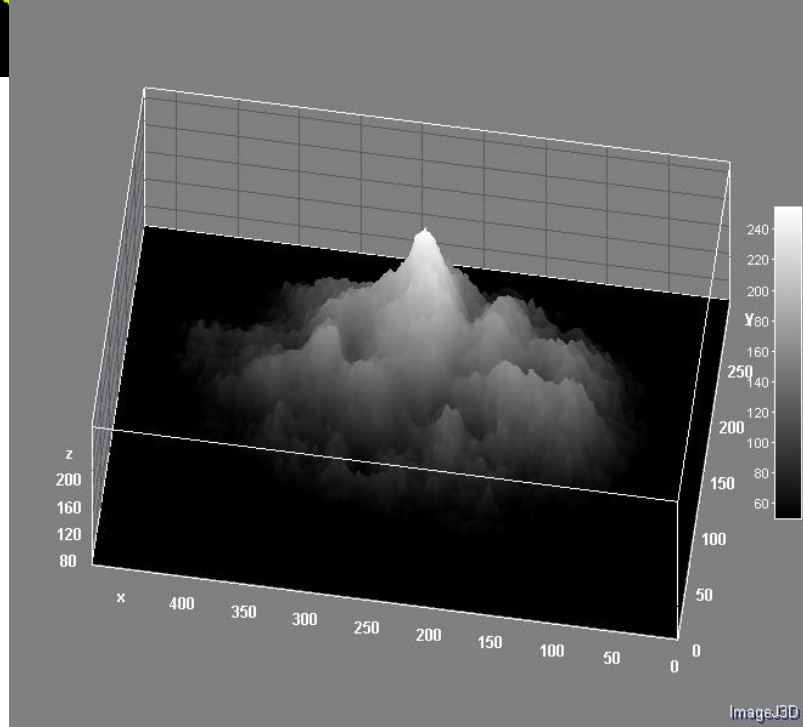
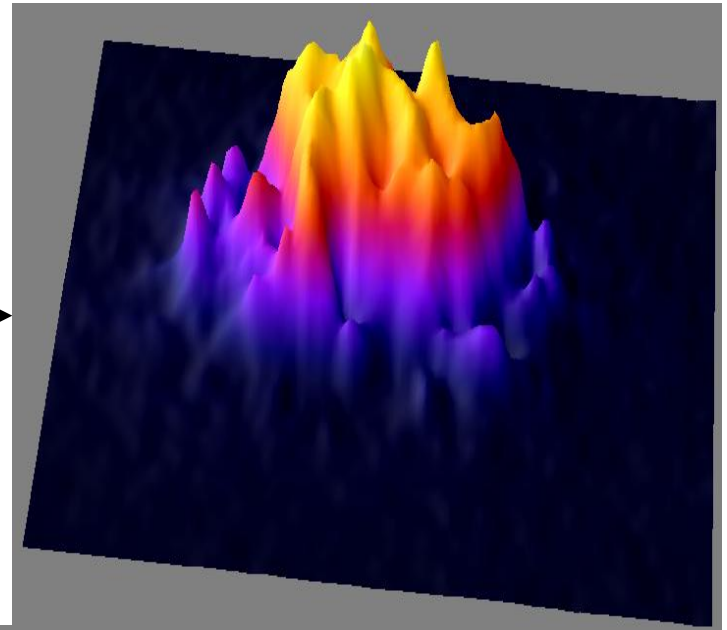
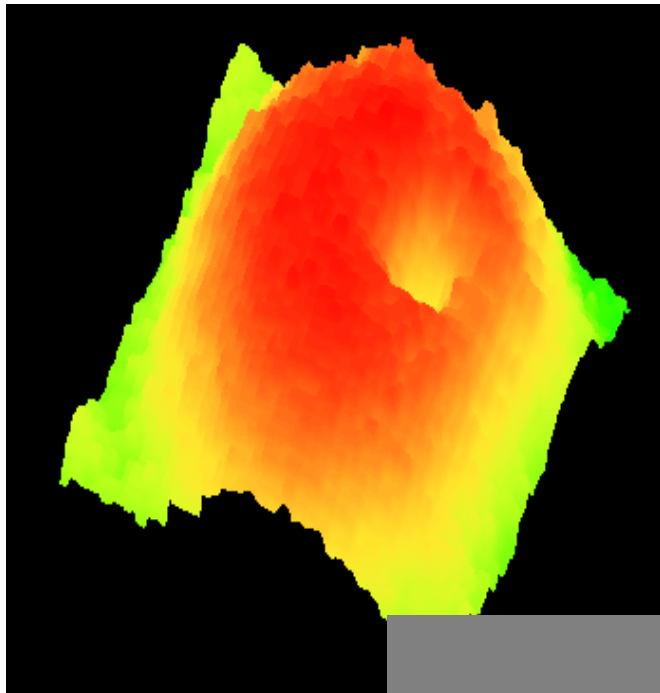


**Atomic washing machine  
( excitation by shaking )**





# Turbulent cloud



# Complete list of publications:

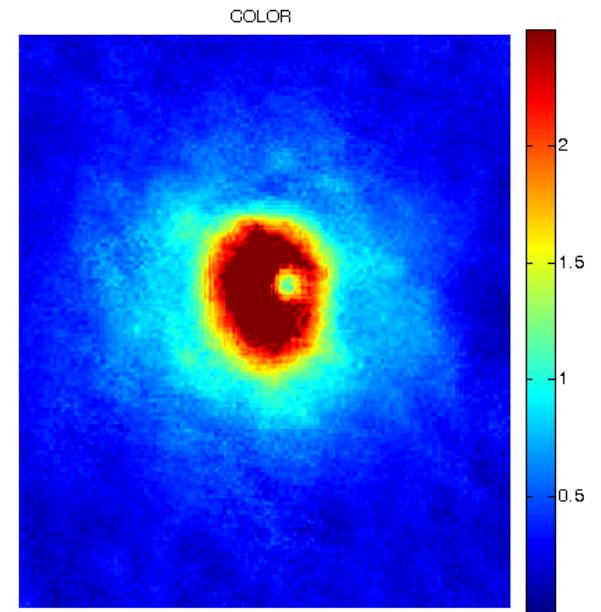
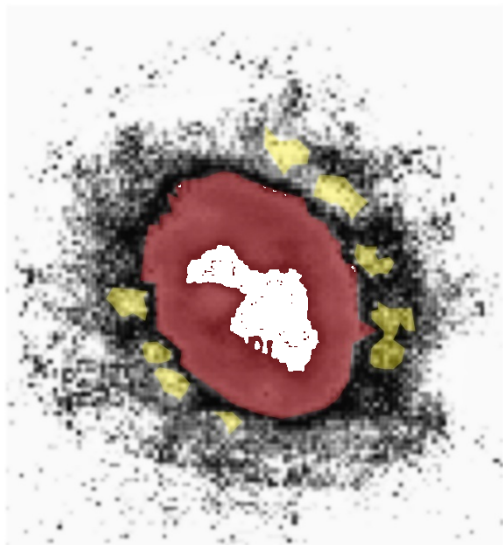
[http:// cepof.ifsc.usp.br](http://cepof.ifsc.usp.br)

( publications – atomic physics)



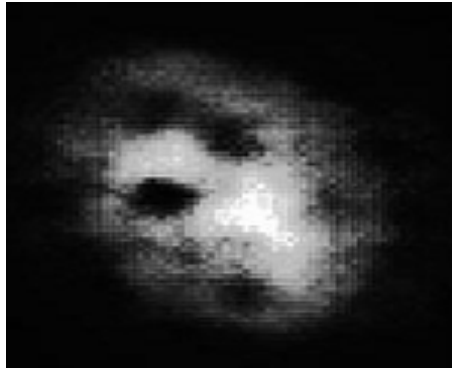
# Vortex formation

## COLLECTIVE MODES



**Increasing amplitude or time of excitation:**

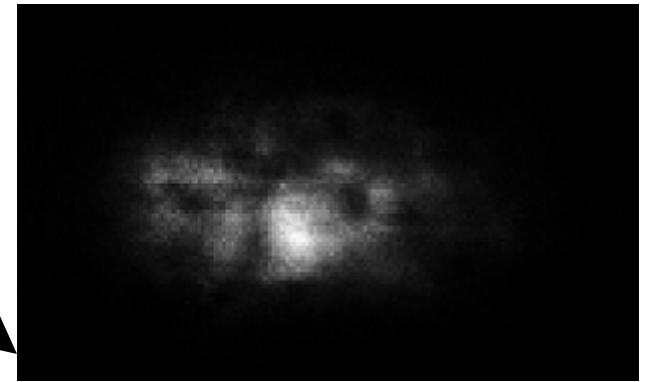
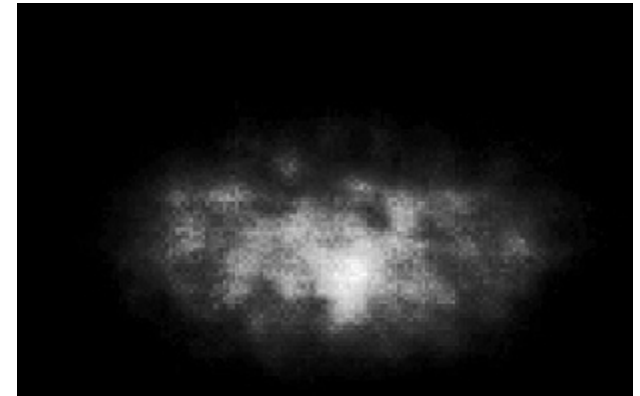
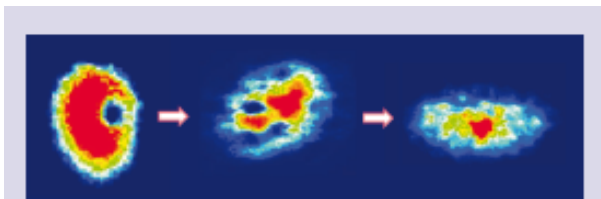
Explosion and proliferation of many vortices  
but no regular pattern and hard to count

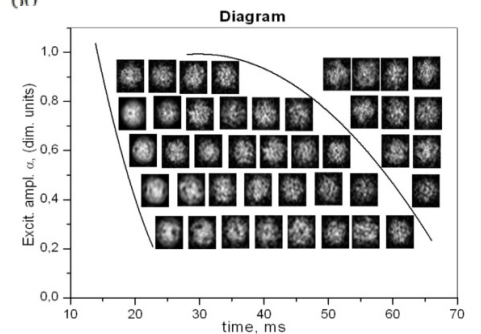
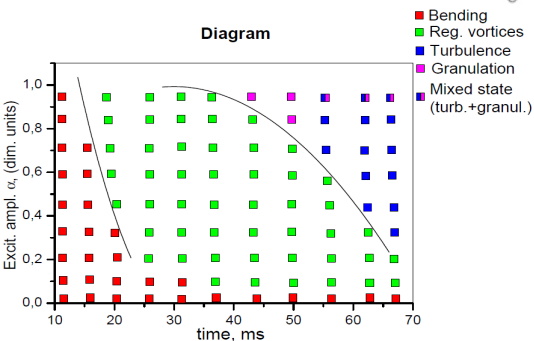
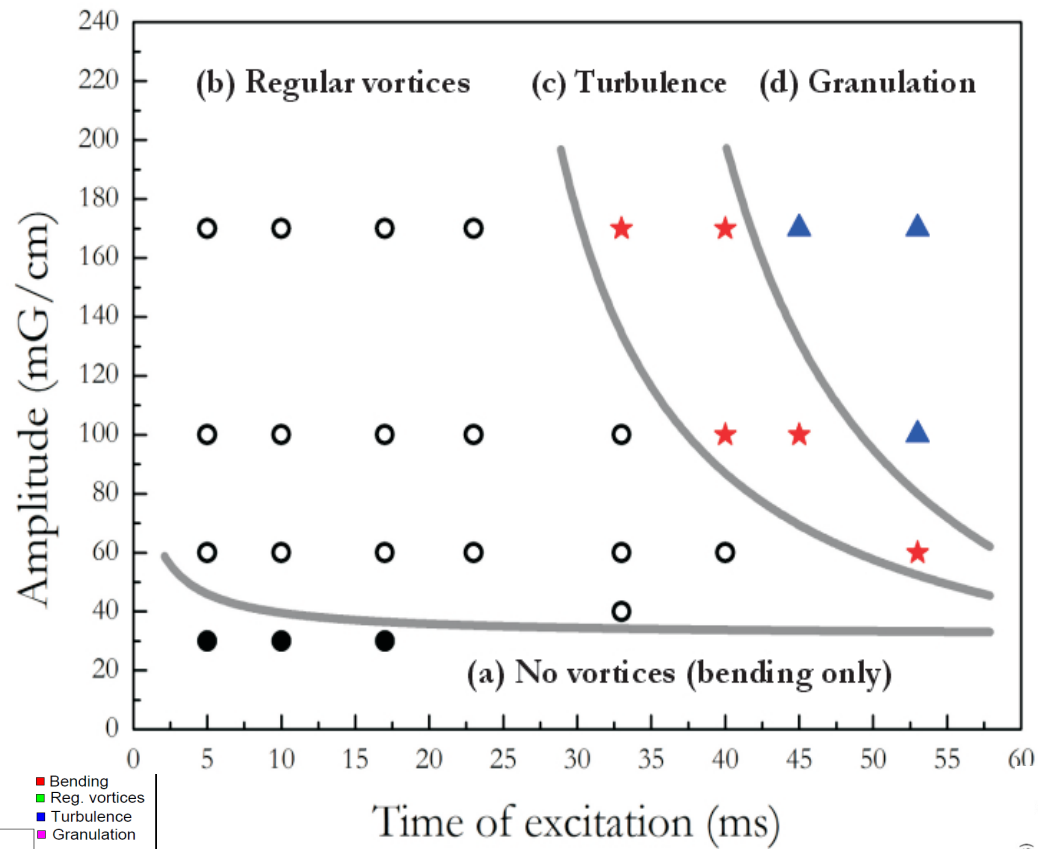
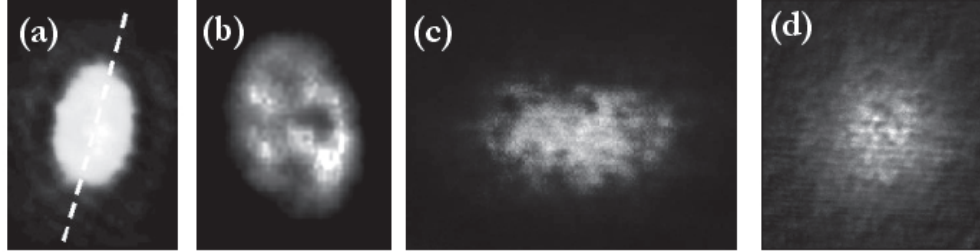


Vortices to tangle vortices  
“TURBULENCE”

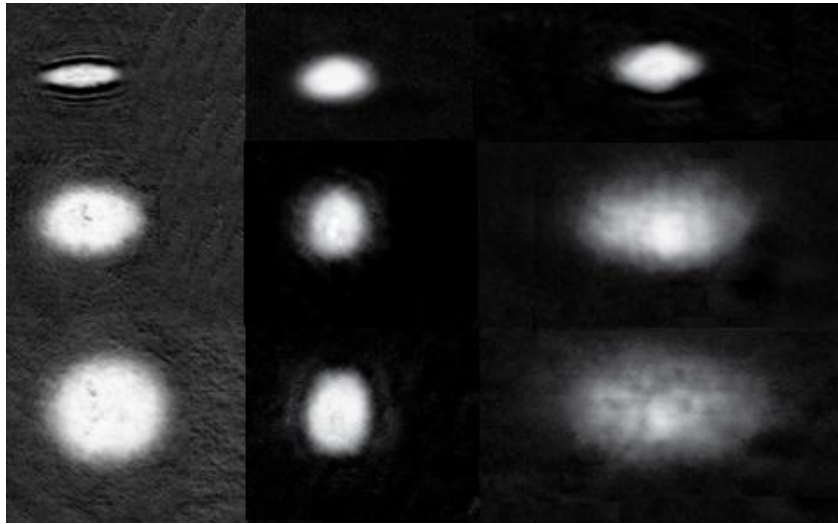
NON REGULAR – MANY POSITIONS

ORIENTATIONS AND LENGTH





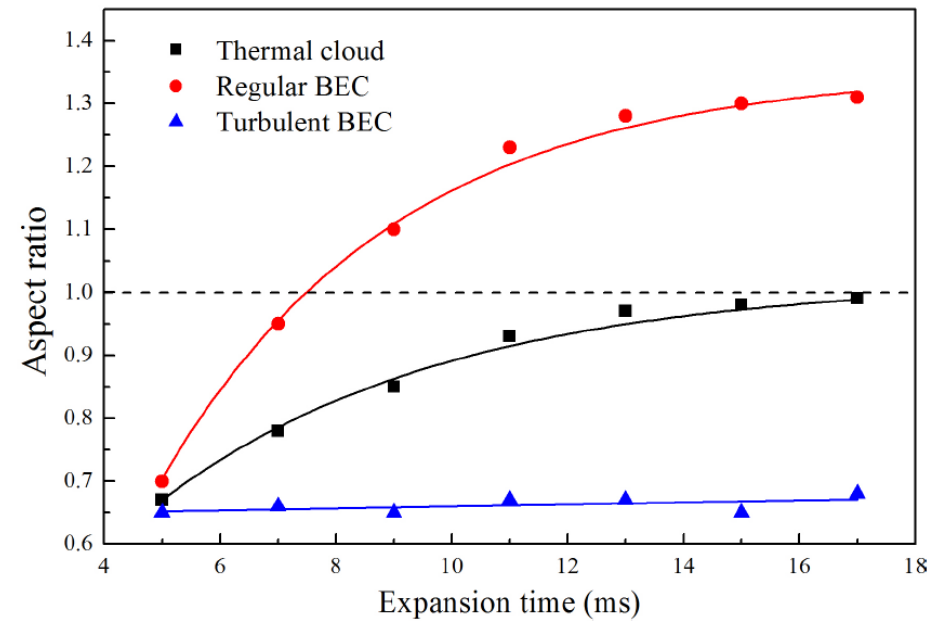
# Cloud expansion



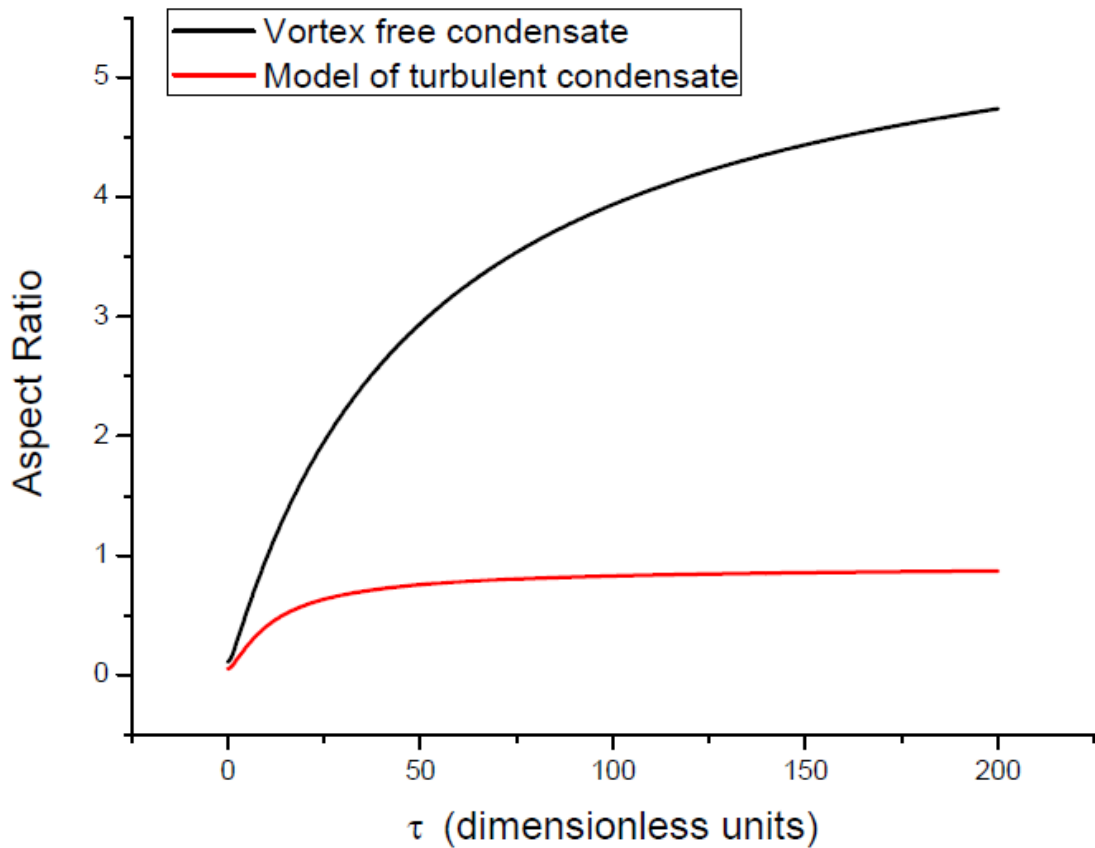
Thermal

BEC

Turbulent



# Self-similar expansion of a turbulent Bose-Einstein condensate: a generalized hydrodynamic model



# Simulation by Tsubota, Kasamatsu and Kobayashi - Japan

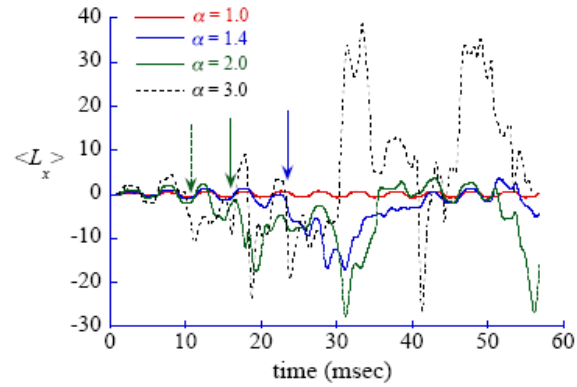
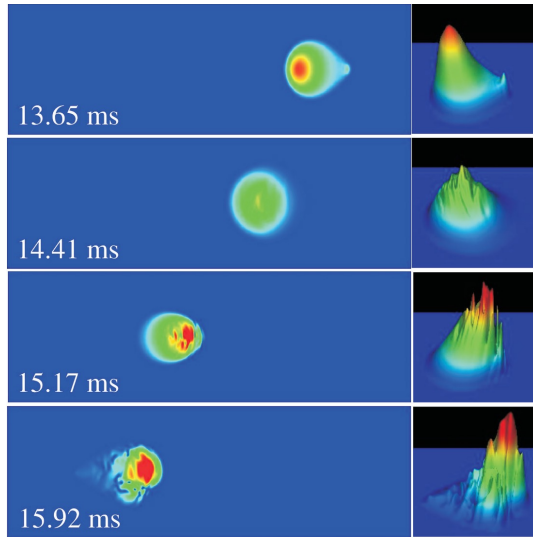


FIG. 5: The time development of the mean angular momentum per atom for  $\gamma = 0.02$  and  $\alpha = 1.0, 1.4, 2.0, 3.0$ . The onset time of vortex nucleation is indicated by arrows.

$$(i - \gamma) \frac{\Omega}{\omega_r} \frac{\partial \Psi}{\partial t} = \left[ -\frac{\nabla^2}{2} - \mu + \frac{1}{2} \left( \frac{\omega_x^2}{\omega_r^2} x^2 + y^2 \right) + u_{2D} |\Psi|^2 - \Omega_z \sin t \cdot L_z \right] \Psi$$

# Kibble–Zurek mechanism (KZM)

**KZM** characterizes the spontaneous formation of defects in the process of system equilibration from an initial strongly nonequilibrium symmetric state to an equilibrium state with broken symmetry

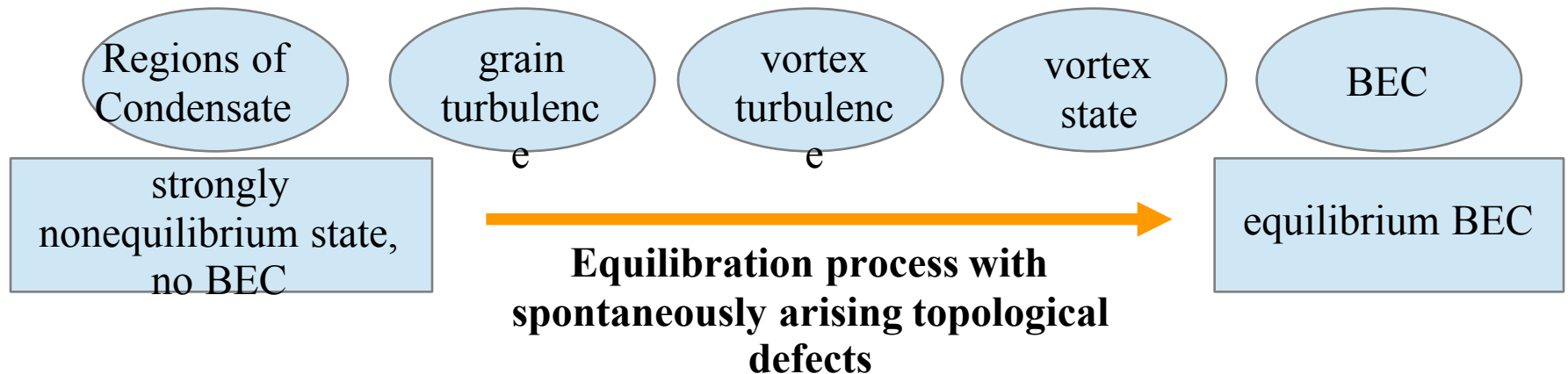
T.W.B. Kibble, J. Phys. A 9 (1976)  
1387,

W.H. Zurek, Nature 317 (1985)  
505

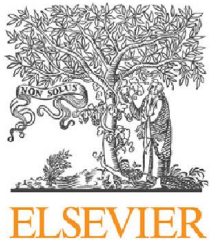
## Experimentally observed in:

- ion crystals
- superconducting films
- superfluid He-3 and He-4,
- trapped Bose–Einstein condensate

## KZM in Bose-Einstein condensate



E. Levich, V. Yakhot, Phys. Rev. B 15 (1977) 243



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# Realization of inverse Kibble–Zurek scenario with trapped Bose gases



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Vortex turbulence

Grain turbulence

Wave turbulence

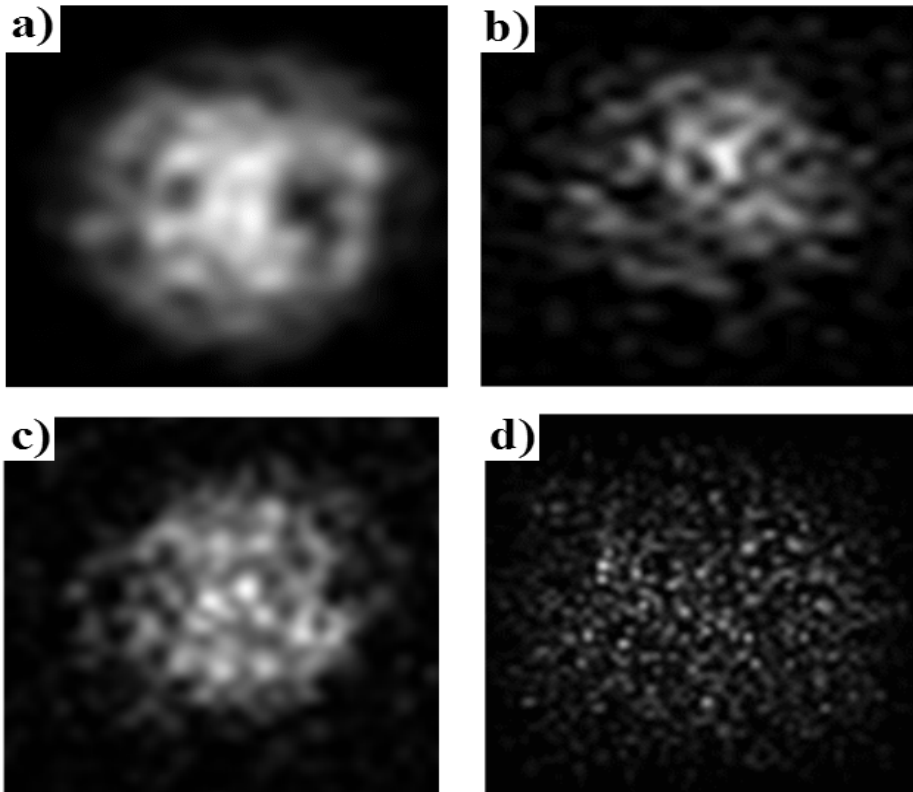
Inverse Kibble–Zurek scenario

### ABSTRACT

We show that there exists the *inverse Kibble–Zurek scenario*, when we start with an equilibrium system with broken symmetry and, by imposing perturbations, transform it to a strongly nonequilibrium symmetric state through the sequence of states with spontaneously arising topological defects. We demonstrate the inverse Kibble–Zurek scenario both experimentally, by perturbing the Bose–Einstein condensate of trapped  $^{87}\text{Rb}$  atoms, and also by accomplishing numerical simulations for the same setup as in the experiment, the experimental and numerical results being in good agreement with each other.

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## Could we realise a **inverse KZ scenario** in trapped BEC?



**The sequence of nonequilibrium states (the density cross-sections) realized in modeling of BEC under external perturbation:**

- a) vortex state**
- b) vortex turbulence**
- c) grain turbulence**
- d) wave turbulence**

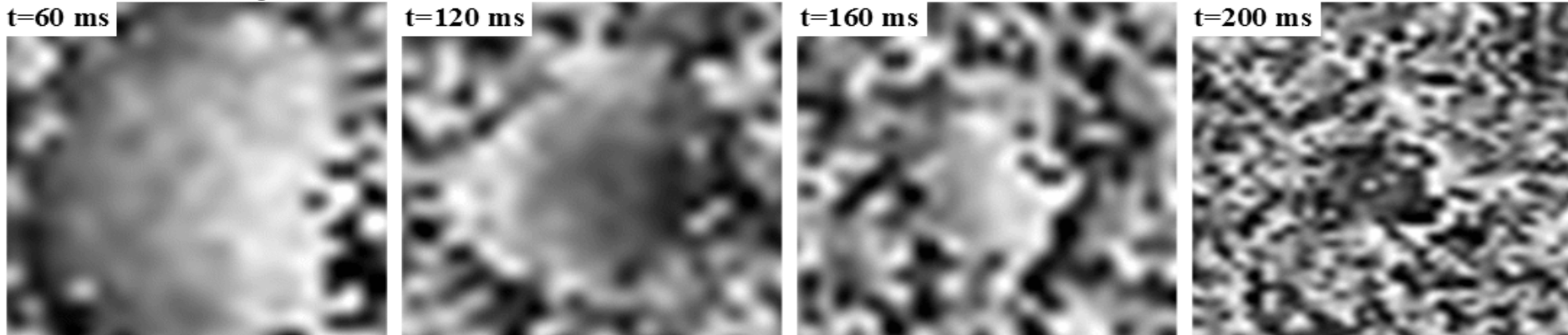
**The results of simulation are in a perfect agreement with the experiment!**

...and some more info

The sequence of snapshots of system phase at consecutive moments of time:

grain turbulence

wave turbulence

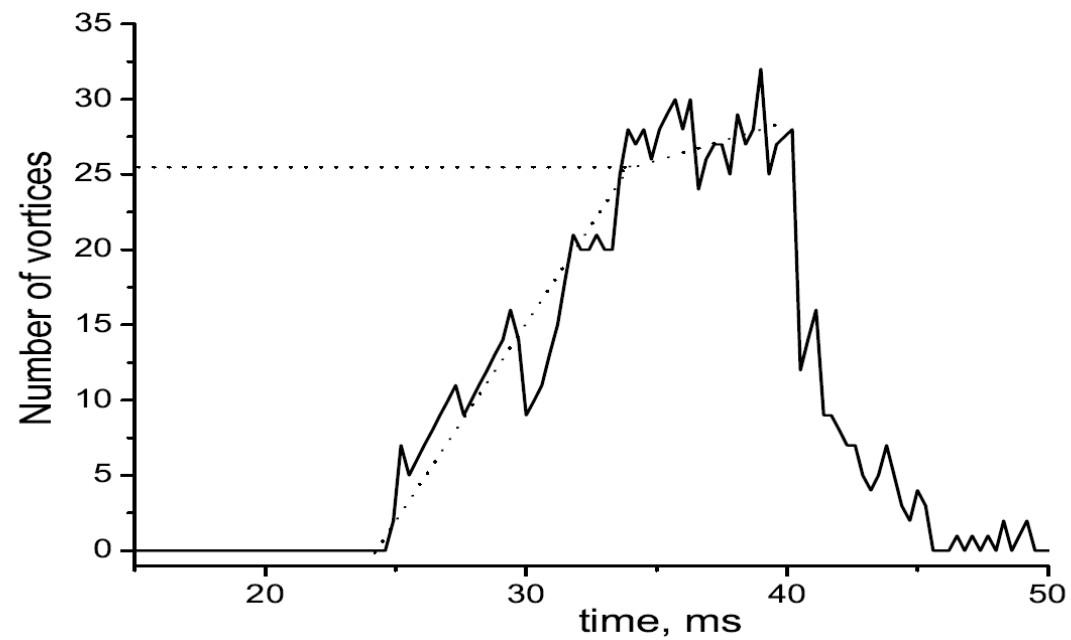
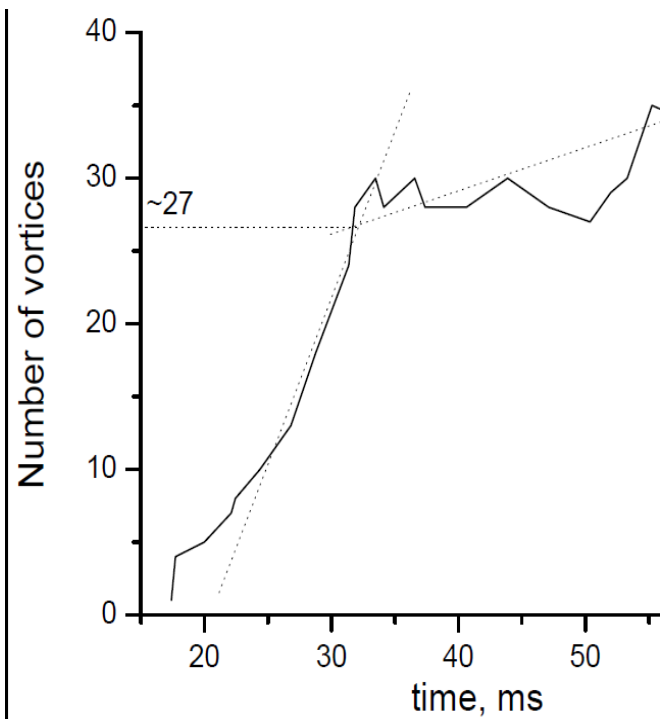


equilibrium Bose-  
condensed state

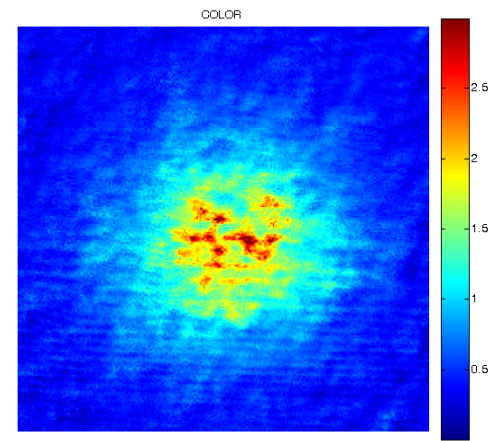
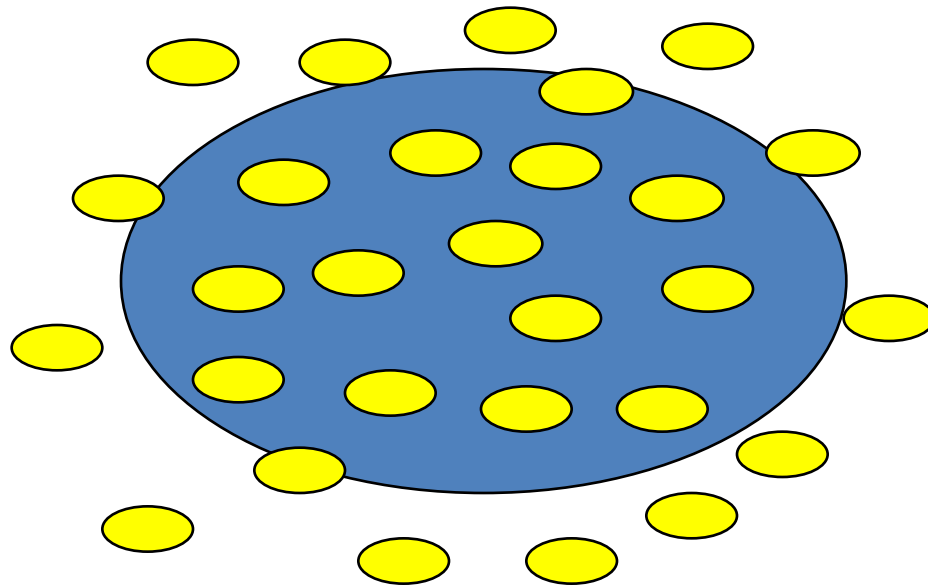


a state with almost  
destroyed coherence

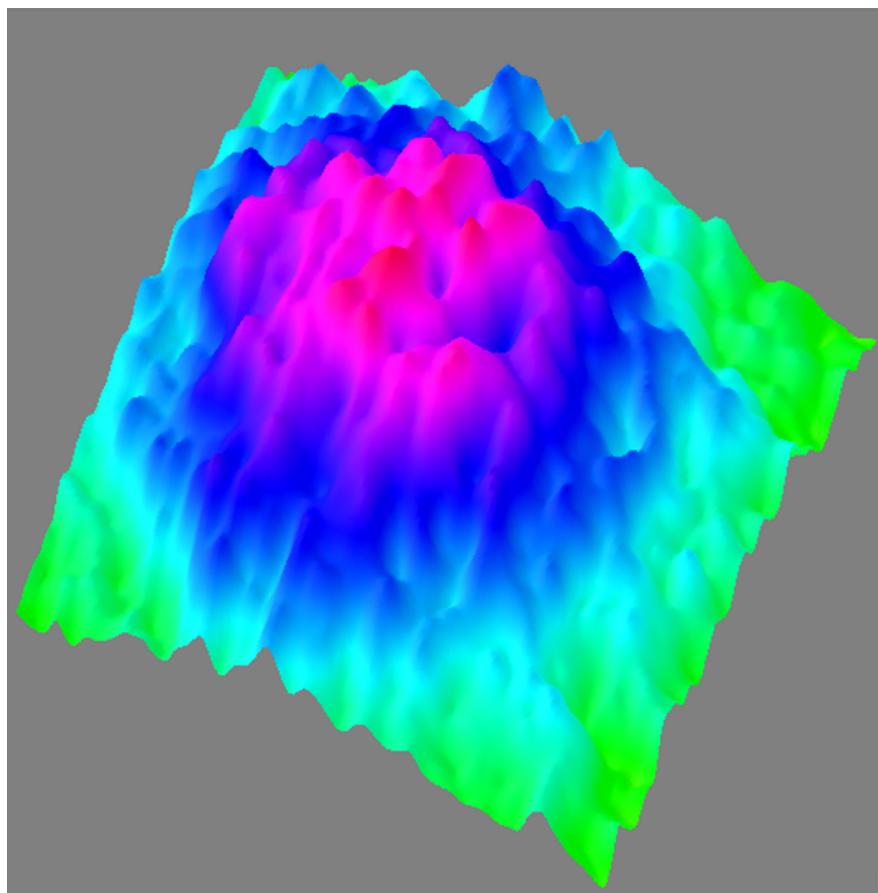
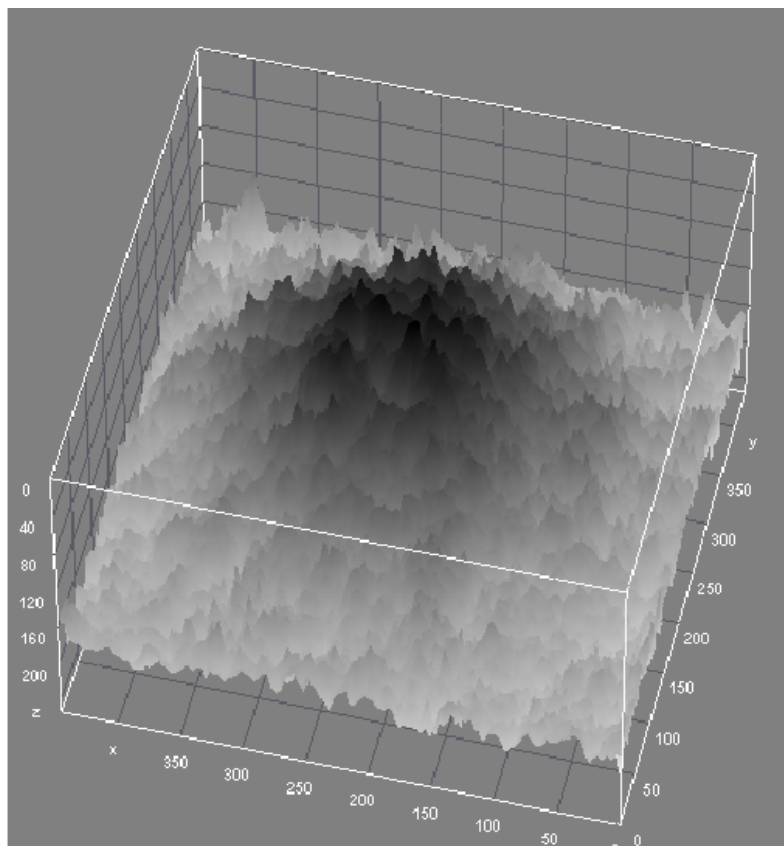
**Opposite Kibble-Zurek mechanism order!**

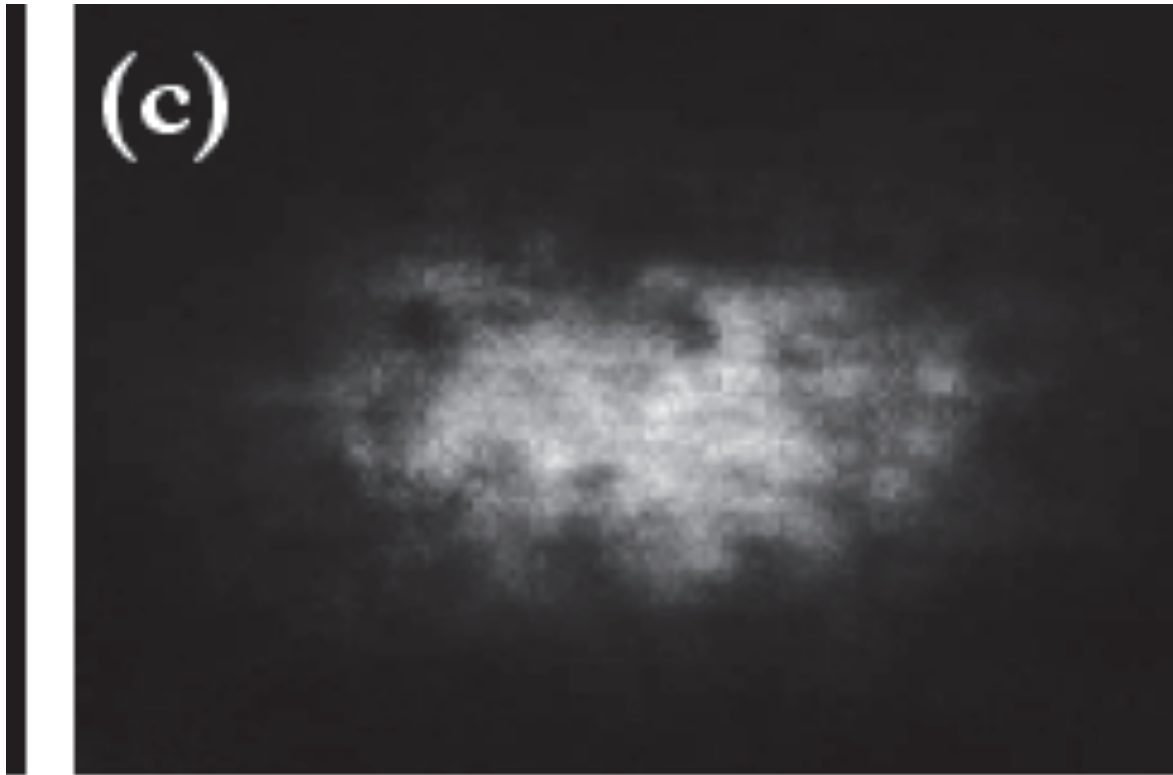


# Granulation

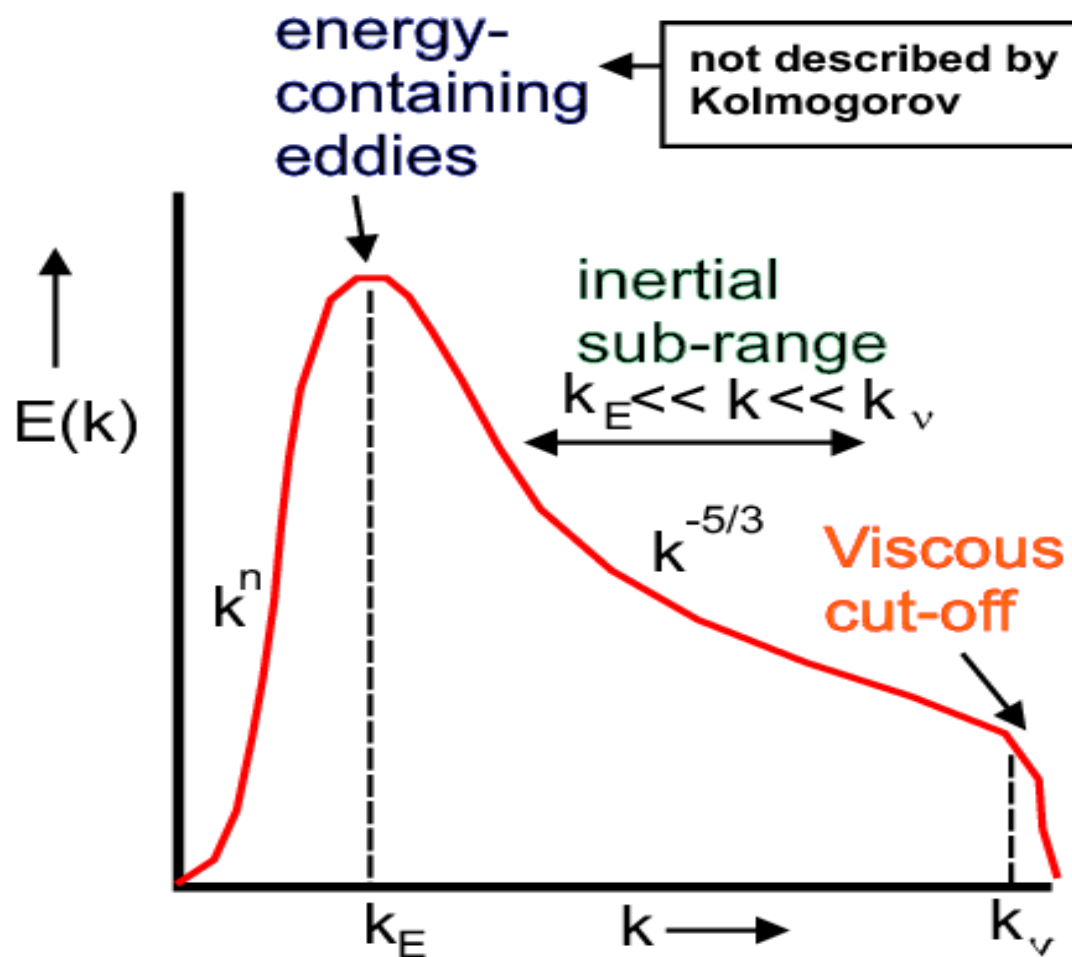


**HIGH DENSITY FLUCTUATIONS**

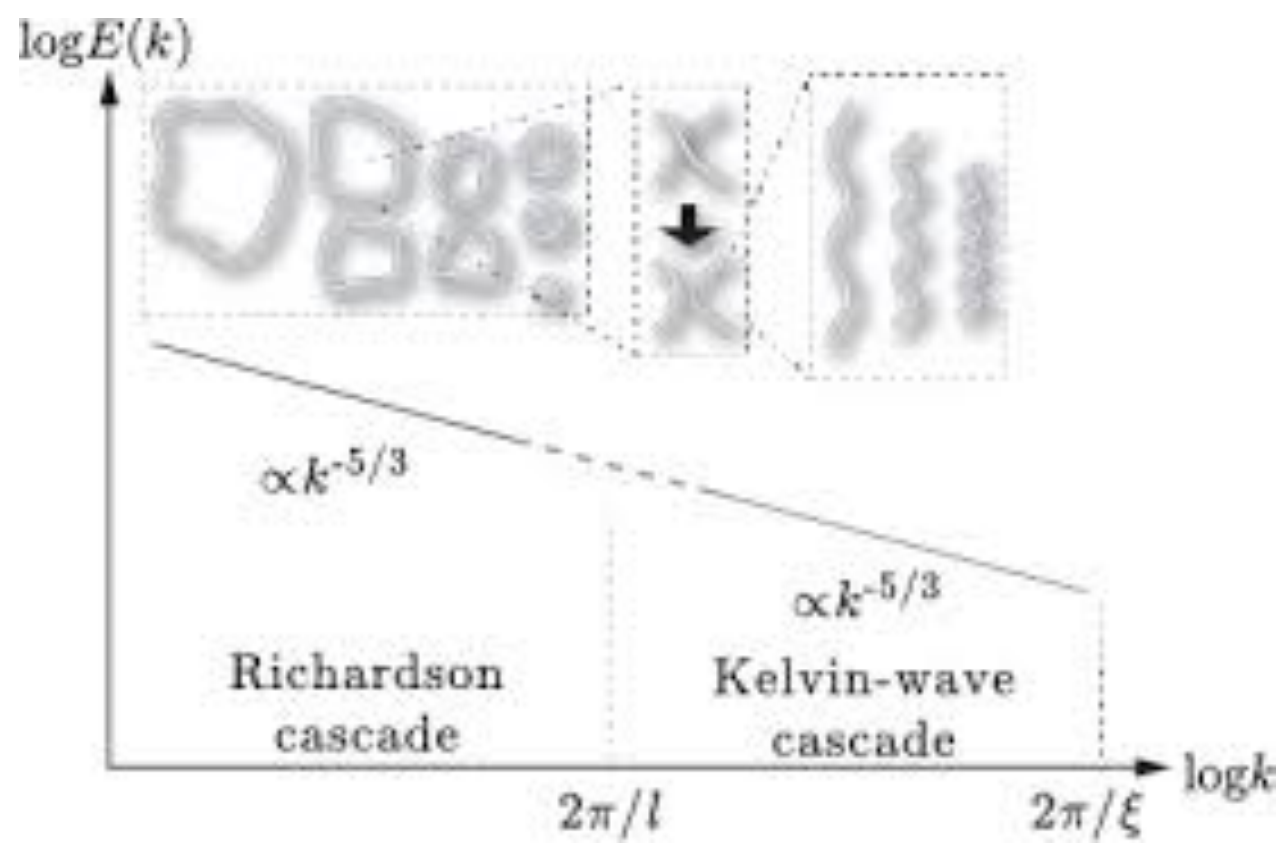




Collective modes are frozen



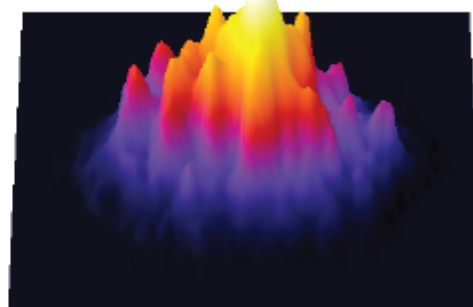
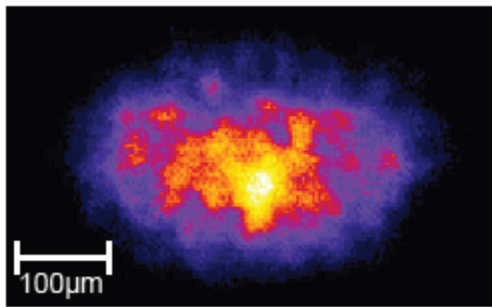
**Kolmogorov:** In the inertial sub-range, no dissipation and local interactions



# HOW TO OBTAIN $n(k)$ in a trapped superfluid?



Time dependence on the free expansion



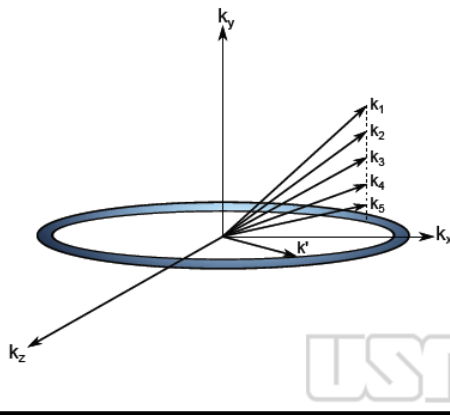
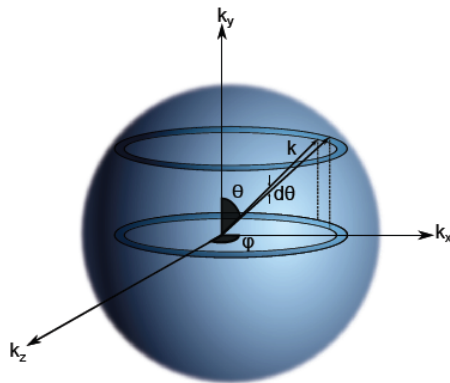
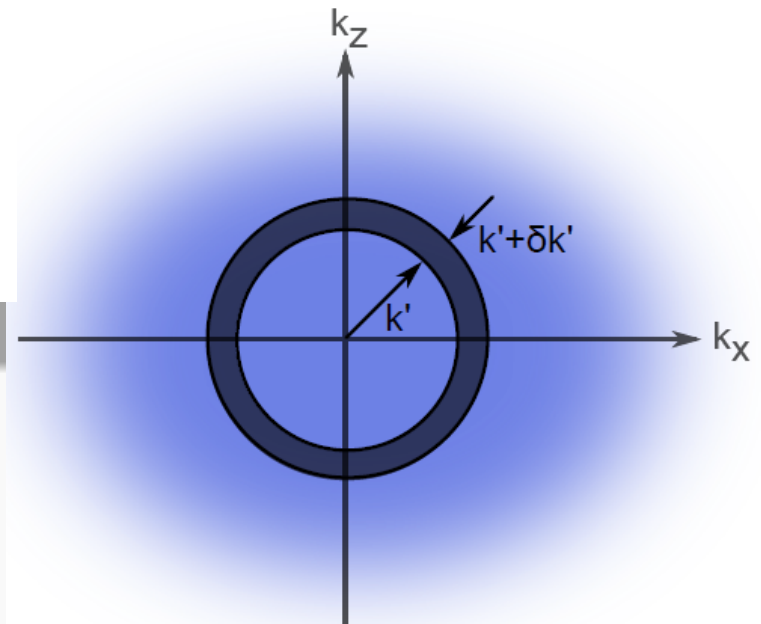
os BEC turbulento

$$R_\rho(0) = 2,65 \mu\text{m}$$

$$R_x(0) = 51,33 \mu\text{m}$$

$$R_\rho(15) = 60 \mu\text{m}$$

$$R_x(15) = 105 \mu\text{m}$$



$$g(k') = 2\pi \int_{k'}^{k'+\delta k'} n'(k') k' dk'$$

$$\int n'(k') k' dk' = N$$

$$n'(k') = 2 \int_{k'}^{\infty} \frac{n(k) dk}{\sqrt{1 - \left(\frac{k'}{k}\right)^2}}.$$

USP

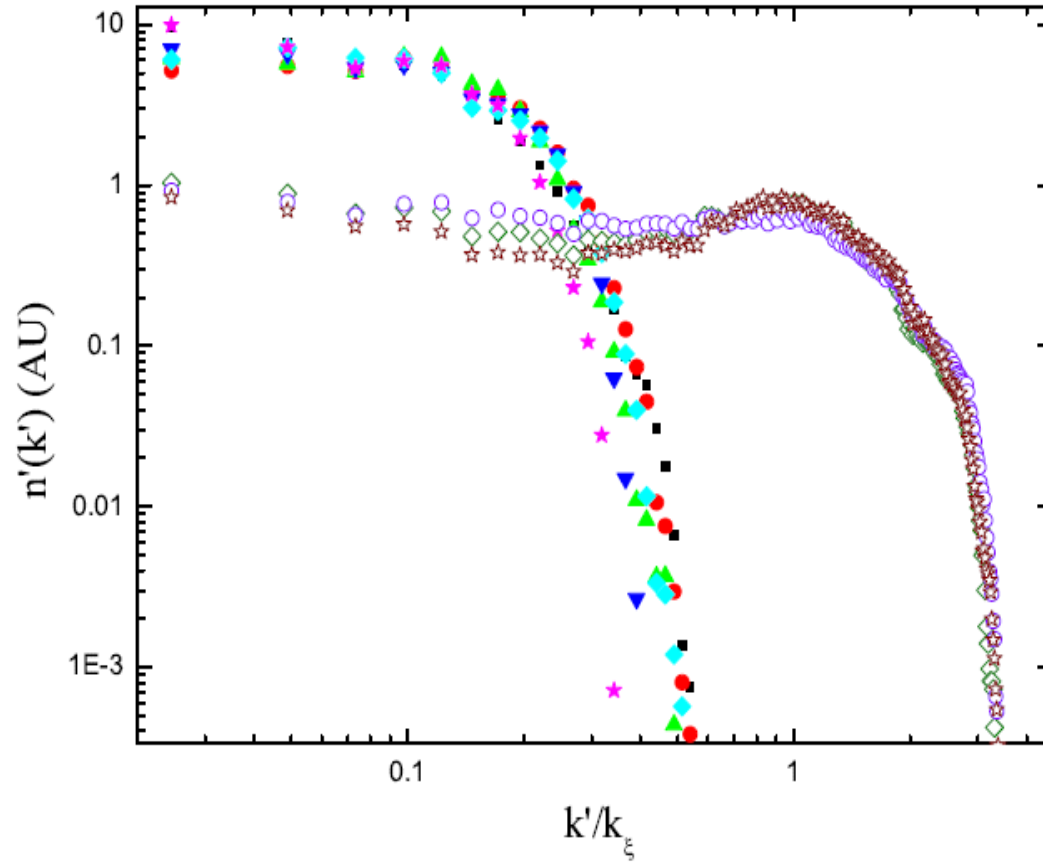
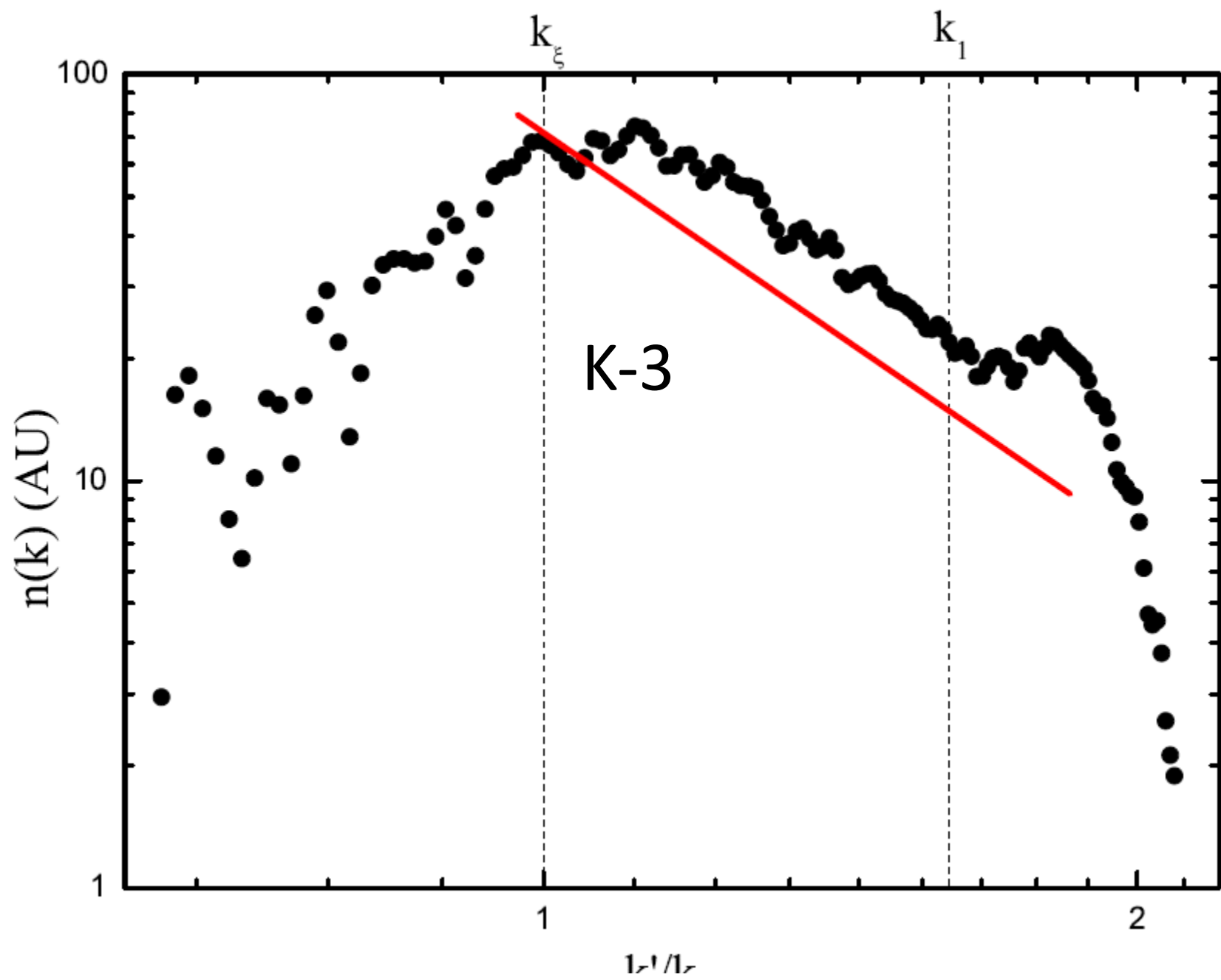
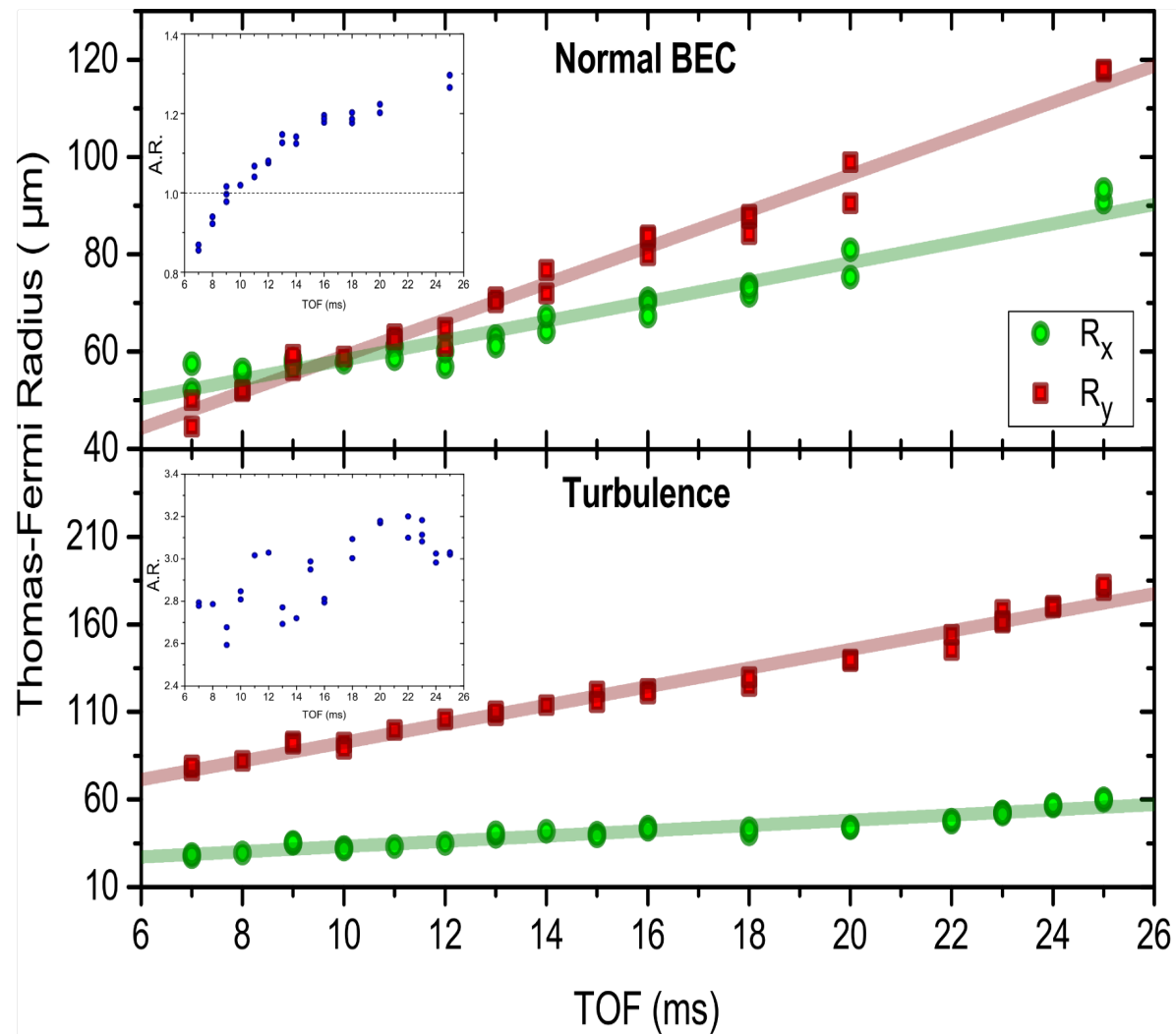
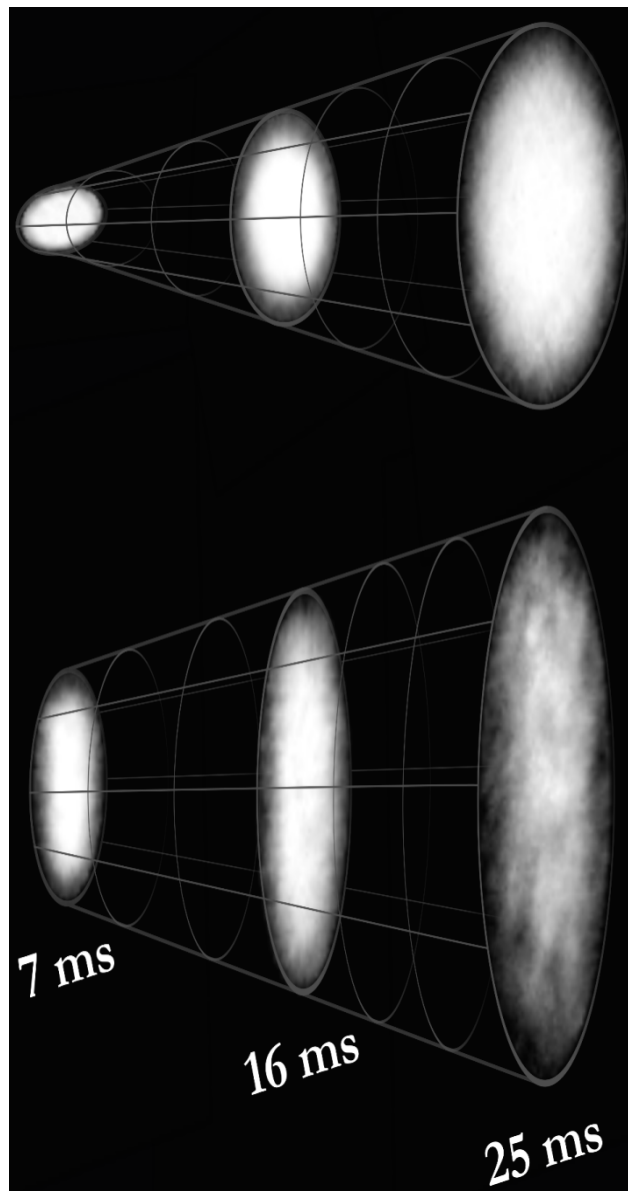
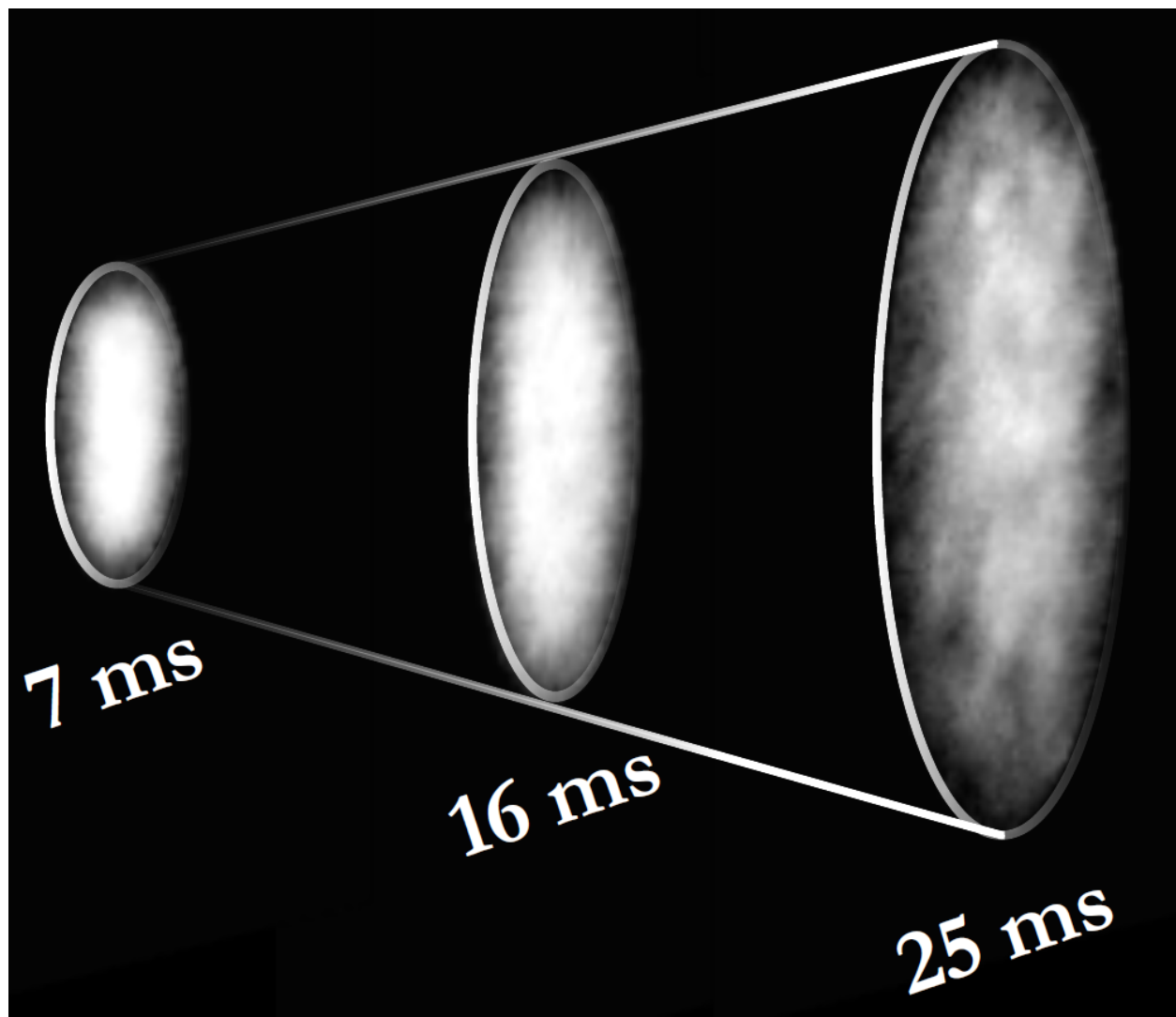


Figure 6: This figure shows the two dimensional projected momentum density,  $n'(k')$ , on a log-log plot. The Thomas-Fermi and condensates with a low number of vortices are shown in closed symbols. The  $\blacksquare$ ,  $\bullet$ ,  $\blacktriangle$ ,  $\blacktriangledown$ ,  $\blacklozenge$ , and  $\star$  symbols represent condensates with 0, 1, 2, 3, 4, and 5 vortices respectively and the open symbols are data from three different realizations of a turbulence. The distinction between the behavior of condensates with energy dominated by internal and kinetic energy are clear from the different behavior.

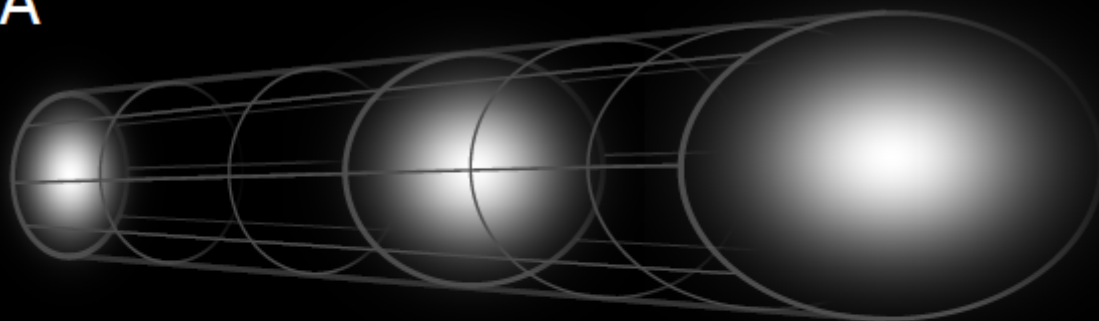


# Matter wave expansion:

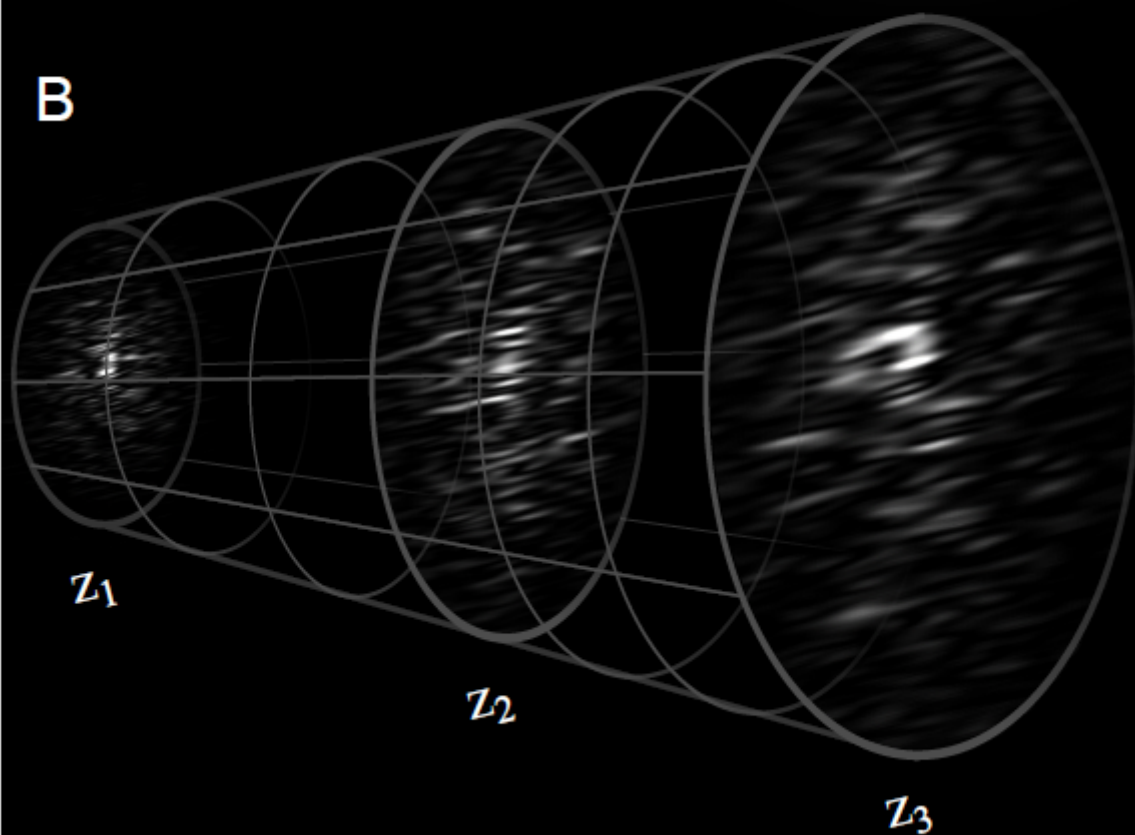


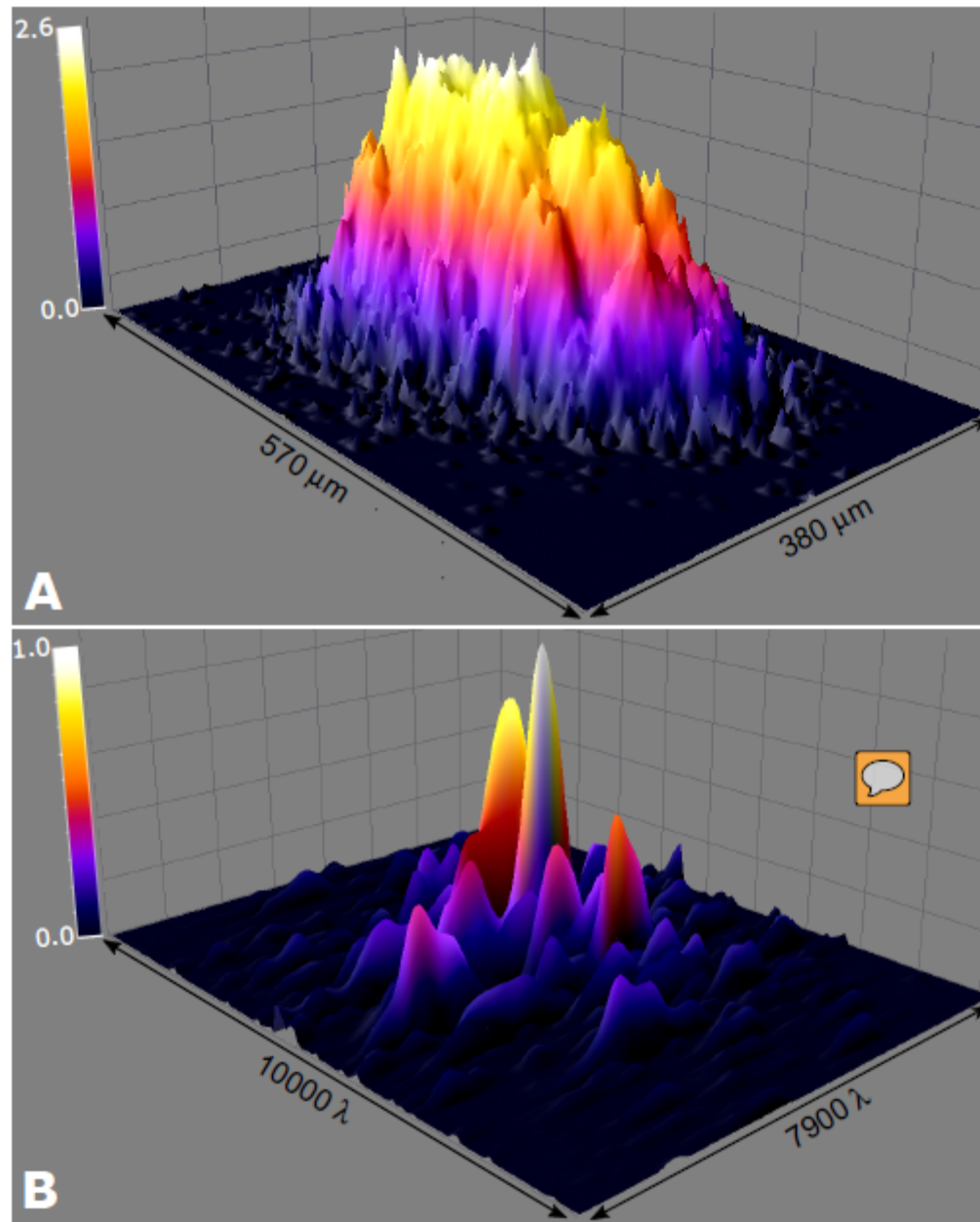


A



B

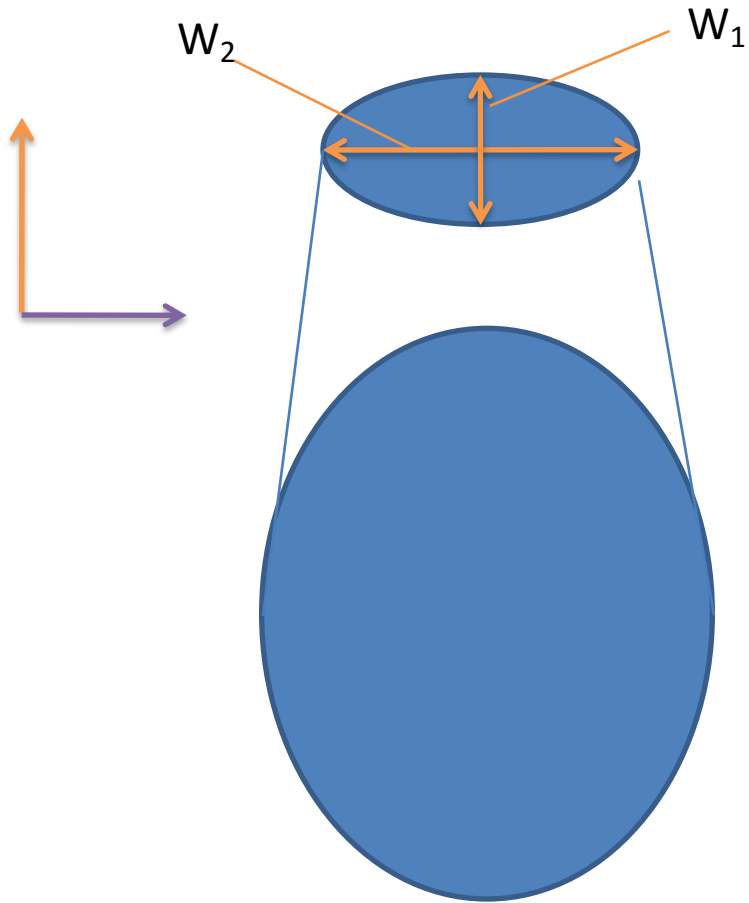




Comparison between turbulent cloud and speckle beam. 3D column :

## COHERENT LASER FIELD

$$\tan \Theta = \frac{\lambda}{w_i}$$

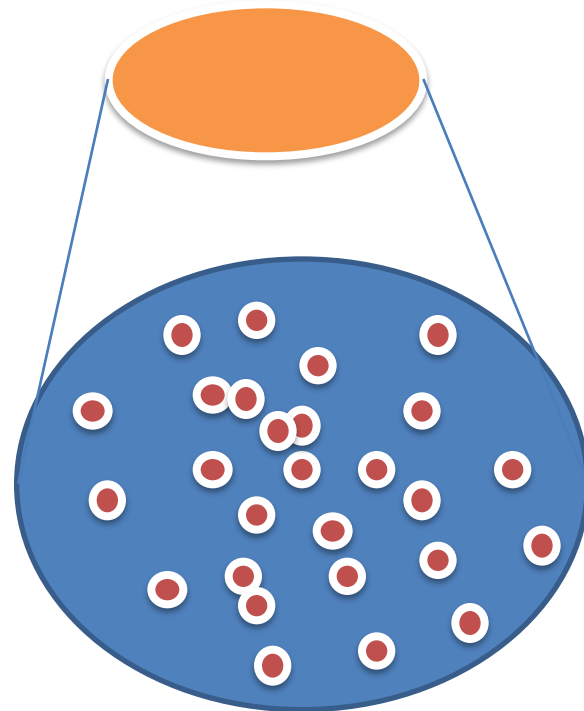


$w_i$  is well determined in both directions

## SPEKLE FIELD

$$\tan \Theta = \frac{\lambda}{l_i}$$

$$\tan \Theta = \frac{\lambda}{l_i}$$



$l_i$  is the **correlation length**, equal in all directions

# Equivalences in Expansion and Propagation

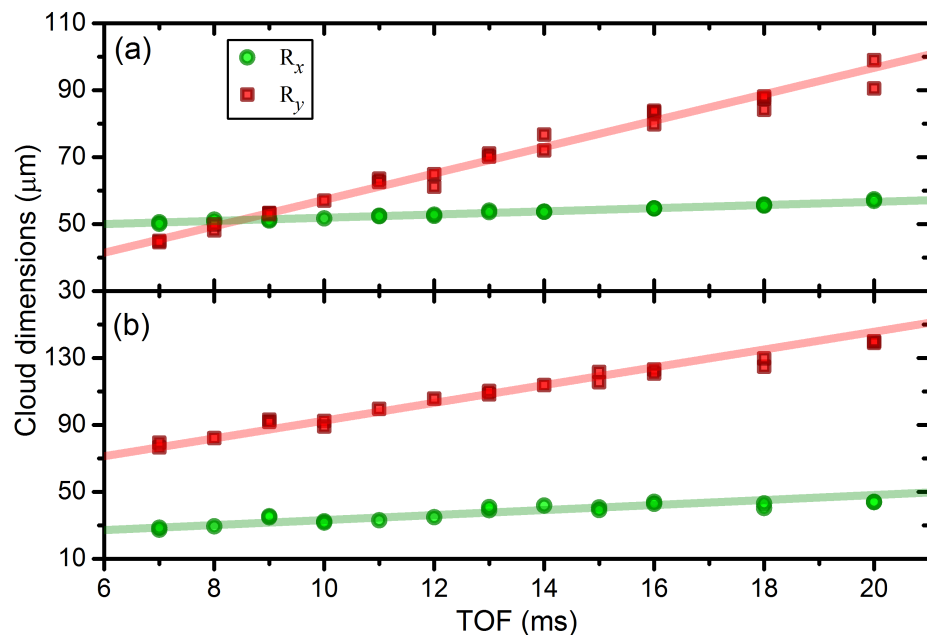
➤ Divergence angles:

$$\tan \theta_i^c = \frac{\lambda}{w_{0i}}$$

$$\tan \theta_i^d = \frac{\lambda}{\ell_i}$$

$$\ell_i < w_{0i}$$

$$\tan \theta_i^d > \tan \theta_i^c$$

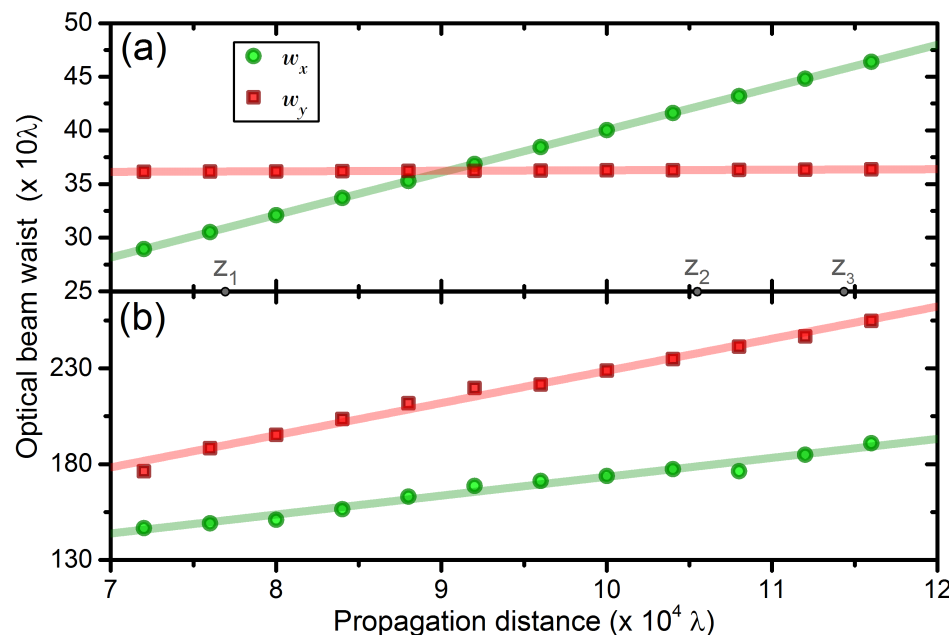


$$\dot{R}_x^{\text{BEC}} = 0.47(1) \mu\text{m/ms}$$

$$\dot{R}_y^{\text{BEC}} = 3.9(1) \mu\text{m/ms}$$

$$\dot{R}_x^{\text{Turb}} = 1.5(1) \mu\text{m/ms}$$

$$\dot{R}_y^{\text{Turb}} = 5.3(2) \mu\text{m/ms}$$



$$\dot{w}_x^{\text{Gauss}} = 39.64(2) \times 10^{-3}$$

$$\dot{w}_y^{\text{Gauss}} = 0.48(1) \times 10^{-3}$$

$$\dot{w}_x^{\text{Speckle}} = 0.098(5)$$

$$\dot{w}_y^{\text{Speckle}} = 0.167(6)$$

## DIVERGENCE DURING PROPAGATION

For the coherent light field,  $\Theta_1 > \Theta_2 \rightarrow$  **inversion ratio**

For the speckle field,  $I_1 \sim I_2 \rightarrow$   $\Theta_1 \sim \Theta_2 \rightarrow$  **self-similar expansion**

$$g_2(\Delta \mathbf{r}) = \frac{\langle I(\mathbf{r}) I(\mathbf{r} + \Delta \mathbf{r}) \rangle}{\langle I(\mathbf{r}) \rangle \langle I(\mathbf{r} + \Delta \mathbf{r}) \rangle}.$$

Optical field  $\rightarrow$  intensity –intensity correlation

Matter field  $\rightarrow$  density –density correlation

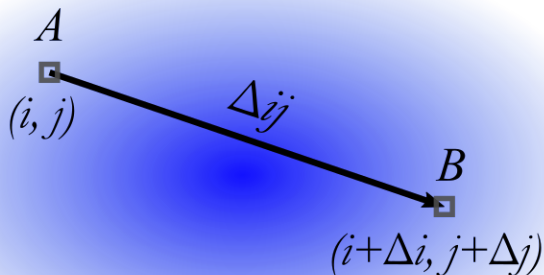
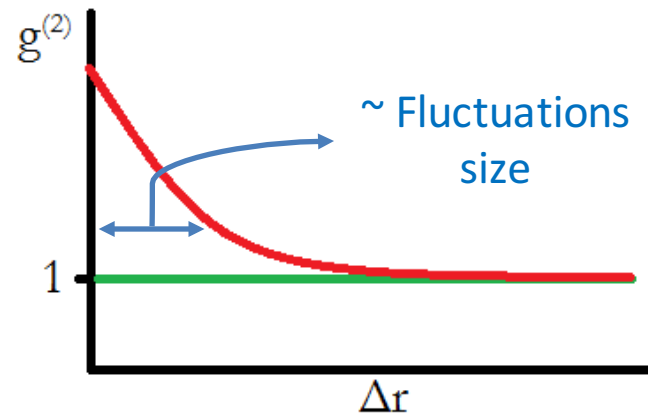
$$g^{(2)}(\Delta r) = \frac{\langle n(r) n(r + \Delta r) \rangle}{\langle n(r) \rangle \langle n(r + \Delta r) \rangle}$$

# Second-order Correlation Function



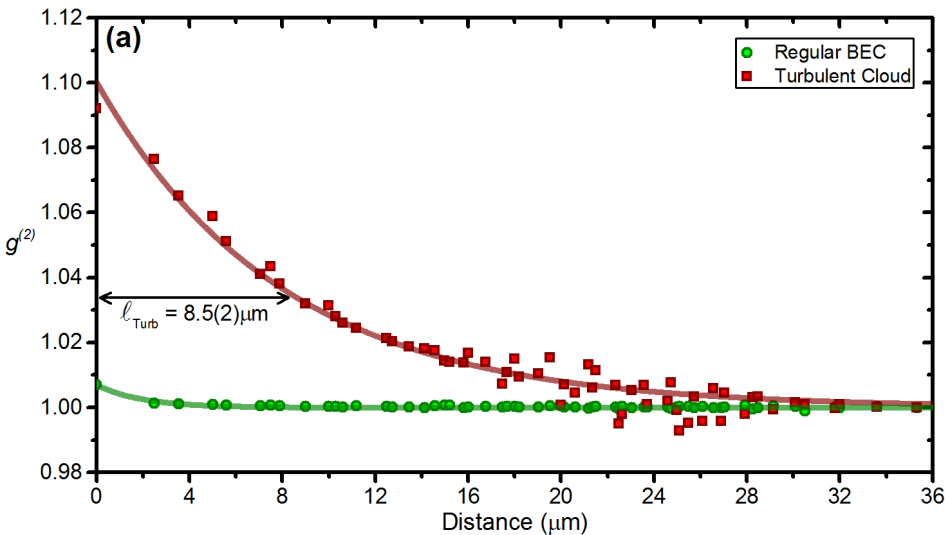
$$g^{(2)}(\Delta r) = \frac{\langle n(r) n(r + \Delta r) \rangle}{\langle n(r) \rangle \langle n(r + \Delta r) \rangle}$$

- For coherent state  $\rightarrow g^{(2)}(\Delta r) = 1$
- For disordered state  $\rightarrow g^{(2)}(0) > 1$   
 $\rightarrow$  decay to 1



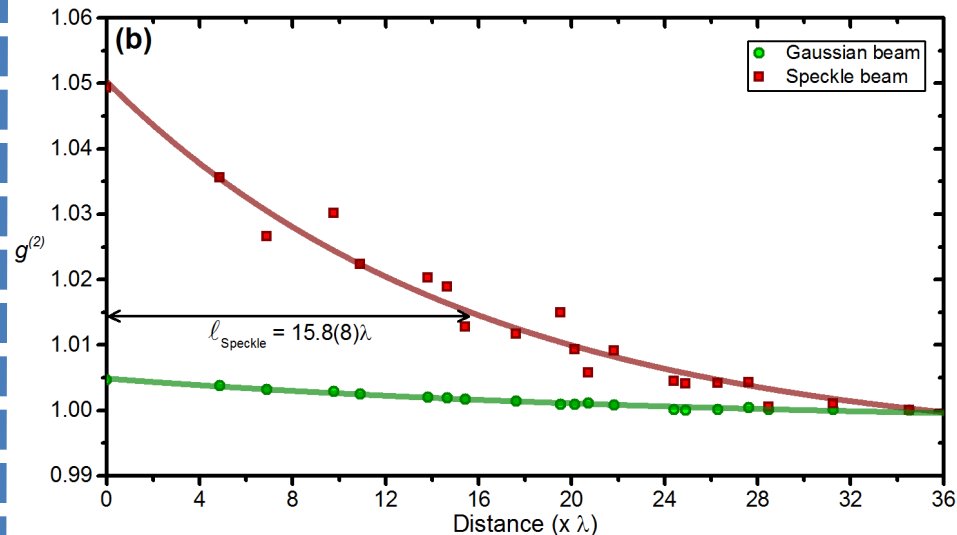
$$g^{(2)}(\Delta ij) = \frac{\langle \text{OD}(i, j) \text{OD}(i + \Delta i, j + \Delta j) \rangle_{\Delta ij}}{\langle \text{OD}(i, j) \rangle_{\Delta ij} \langle \text{OD}(i + \Delta i, j + \Delta j) \rangle_{\Delta ij}}$$

# $g^{(2)}$ for Atomic Cloud and Optical Beam



$$R_x = 63(3) \mu\text{m} \quad R_y = 198(6) \mu\text{m}$$

$$\ell_{\text{Turb}} = 8.5(2) \mu\text{m}$$

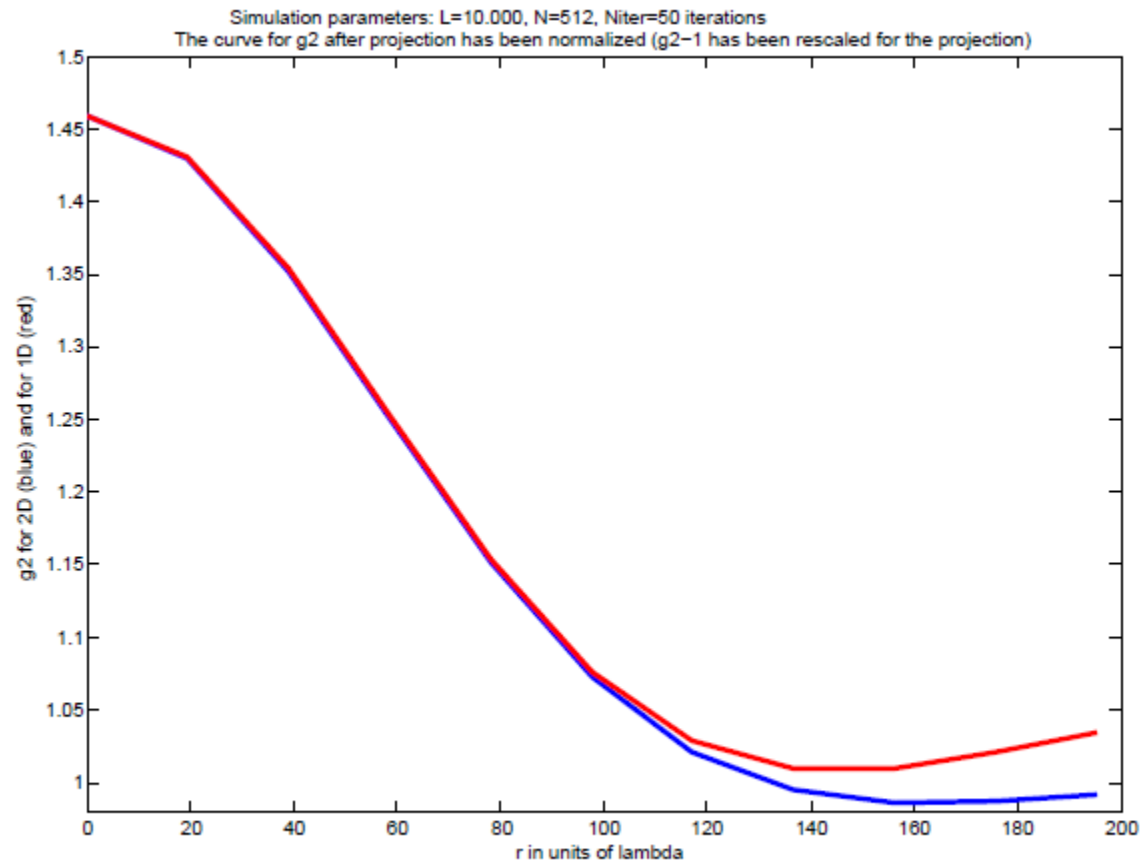


$$w_x = 1876 \lambda \quad w_y = 2528 \lambda$$

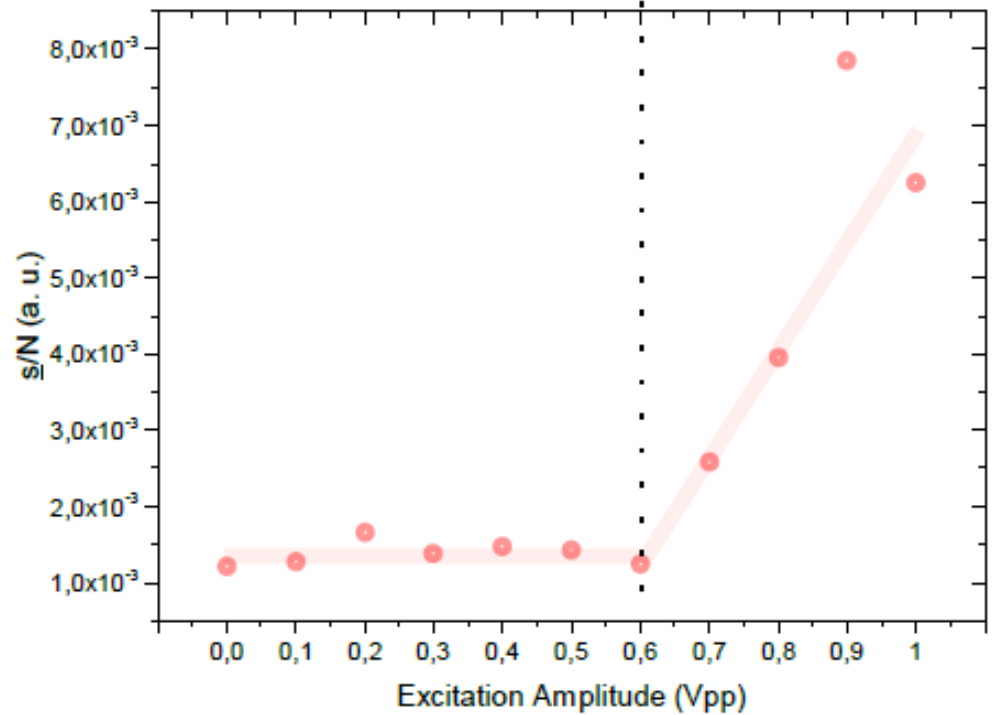
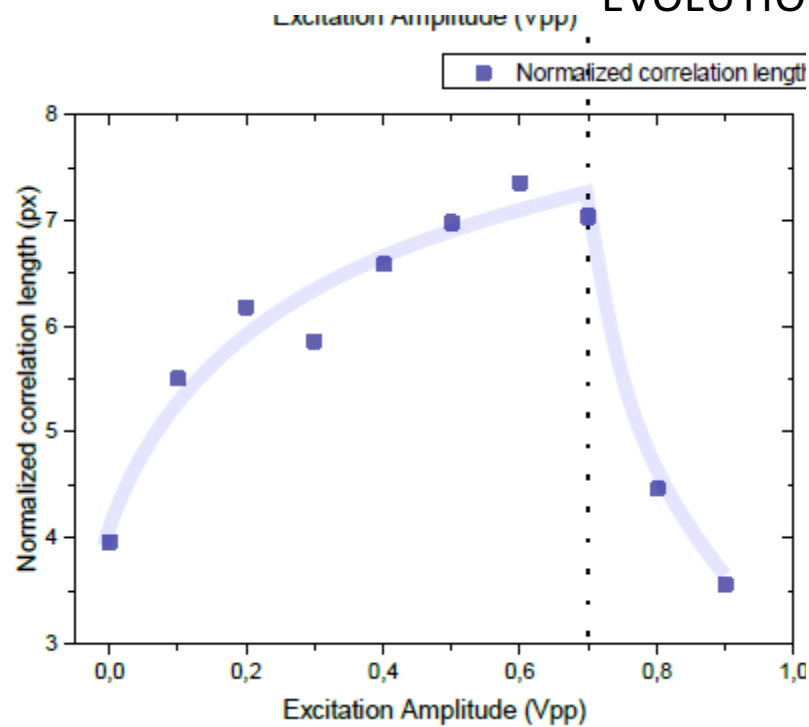
$$\ell_{\text{Speckle}} = 15.8(8) \lambda$$

$$\ell_i < w_i$$

## SPECKLE FIELD – PROJECTED CORRELATION



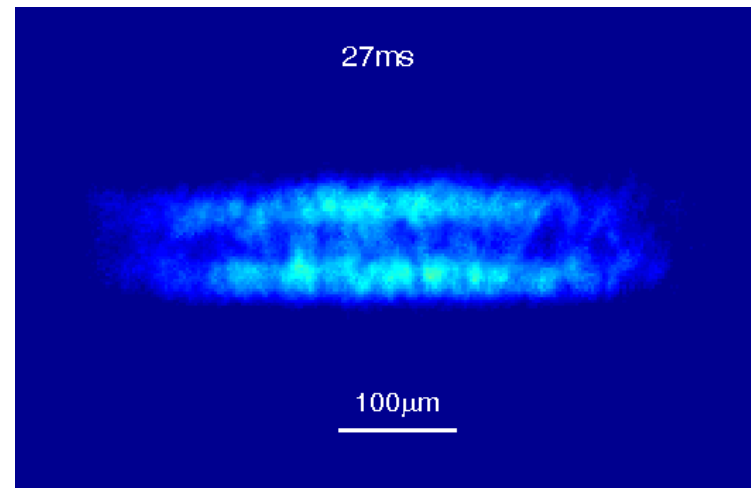
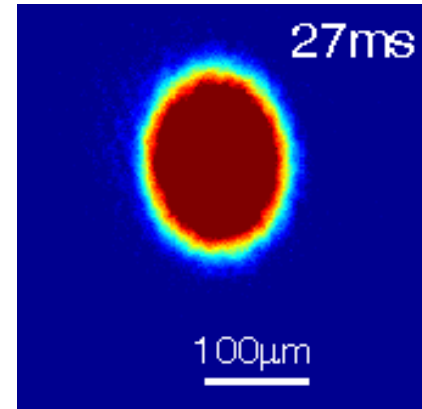
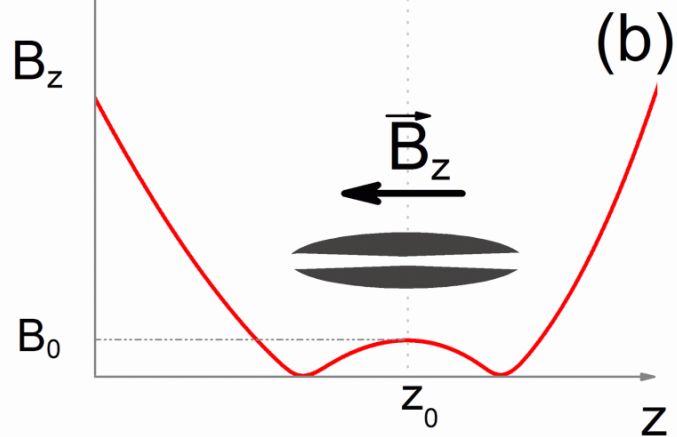
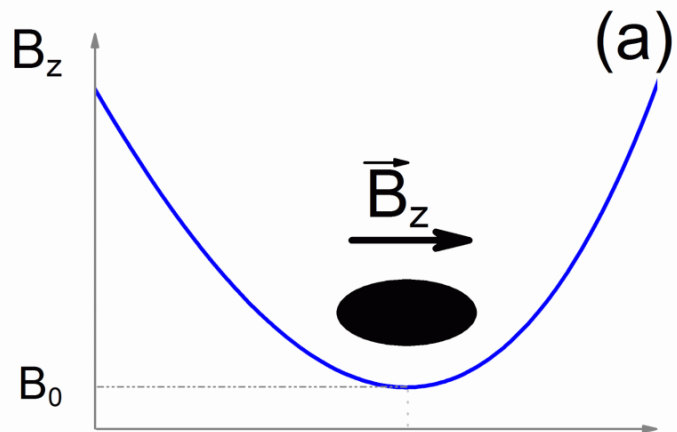
## EVOLUTION OF THE DISORDER

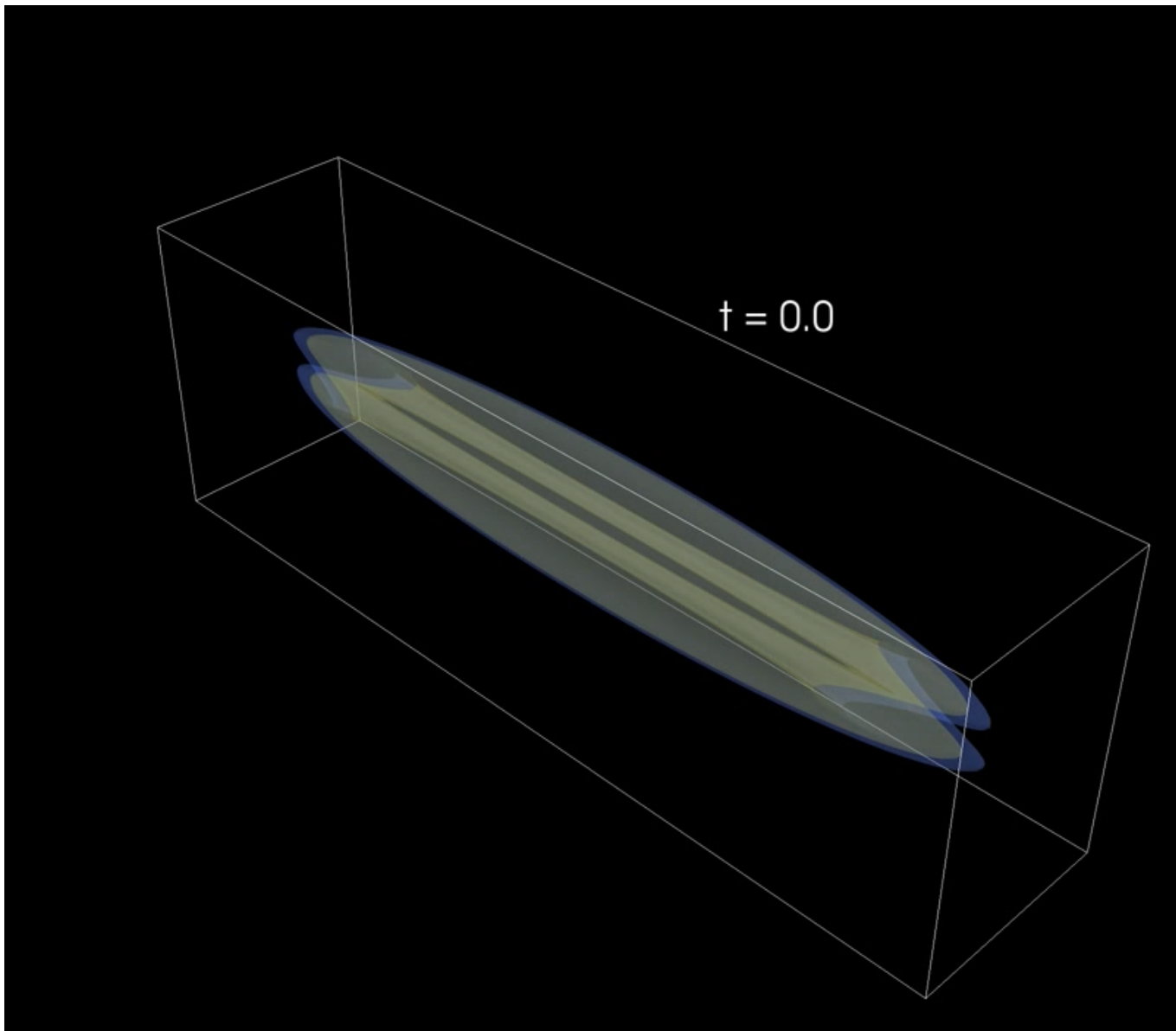


$$S = -k \int n(\vec{r}) \ln n(\vec{r}) d^3 r$$

$$\int n(\vec{r}) d^3 r = 1$$

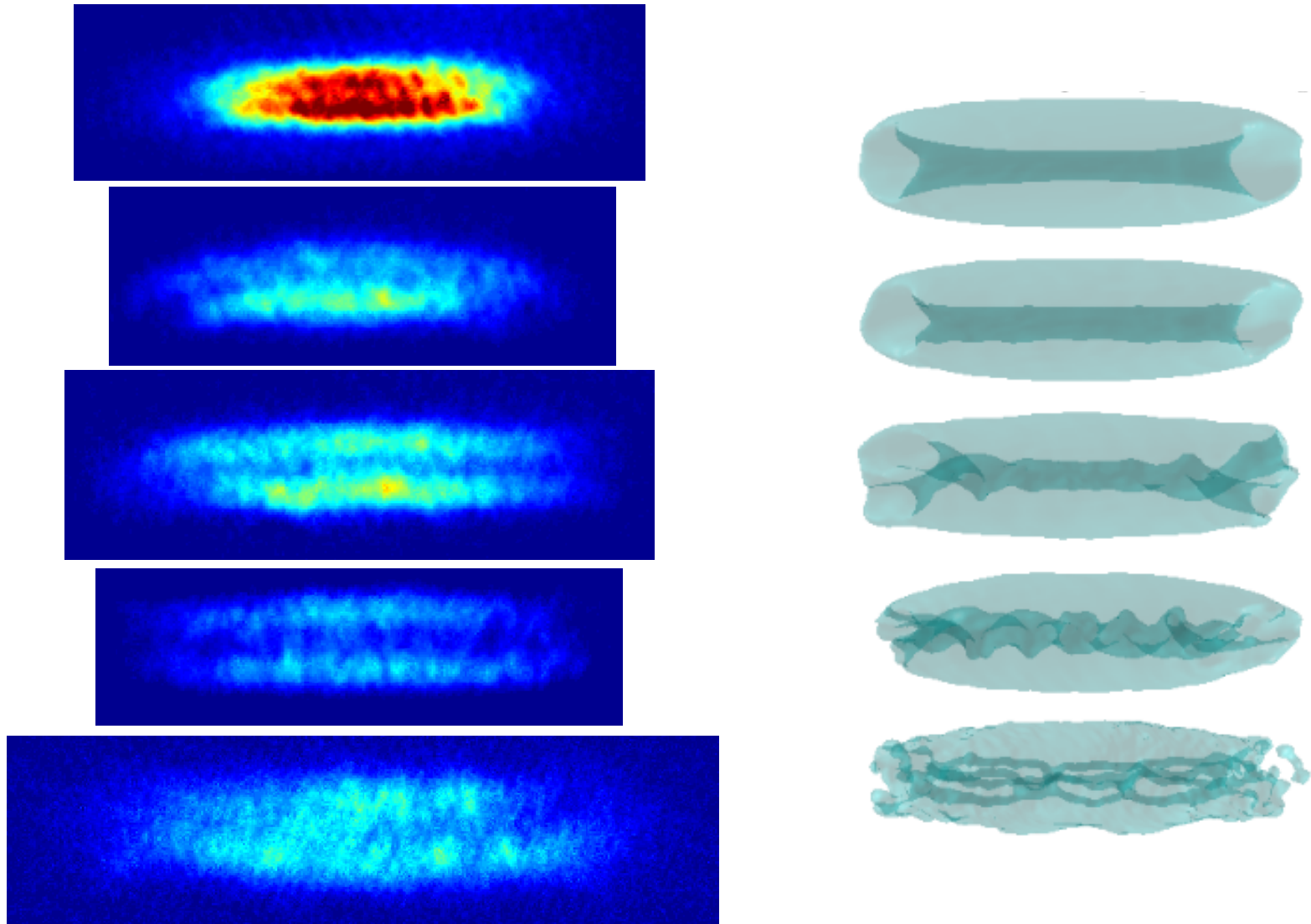
# PRODUCTION OF MULTIPLE CHARGED VORTEX



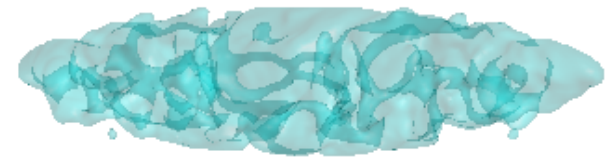
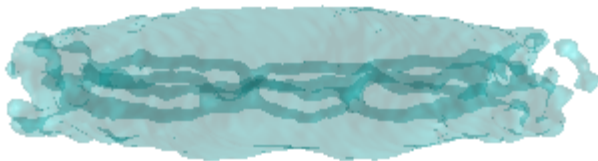


Collaboration: C. Barenghi  
PhD of Andre Cidrim

# Multicharged vortex: initial results

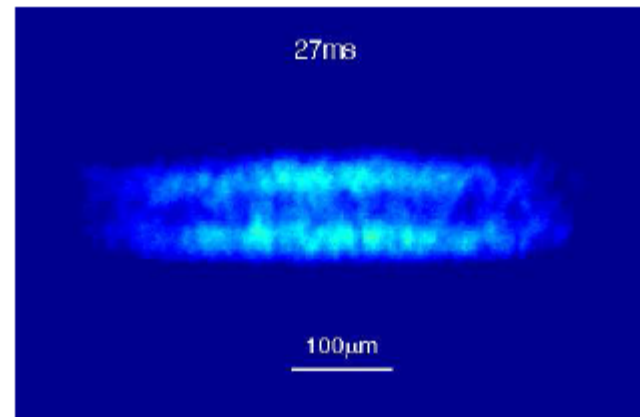


# Collaboration: Newcastle & São Carlos



**Twisted Unwinding...:**

[arXiv:1505.00616](https://arxiv.org/abs/1505.00616)



# MODULATION OF SCATT. LENGTH

$$a(B) = a_{BC} \left( 1 - \frac{\Delta}{B - B_{\infty}} \right),$$



$$B(t) = B_{av} + \delta B \cos(\Omega t),$$

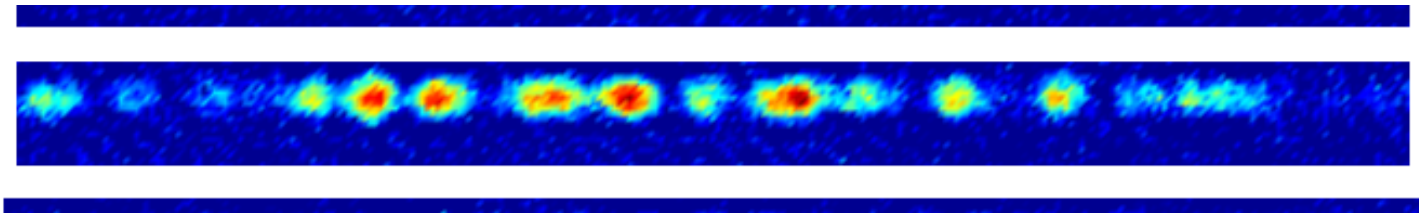
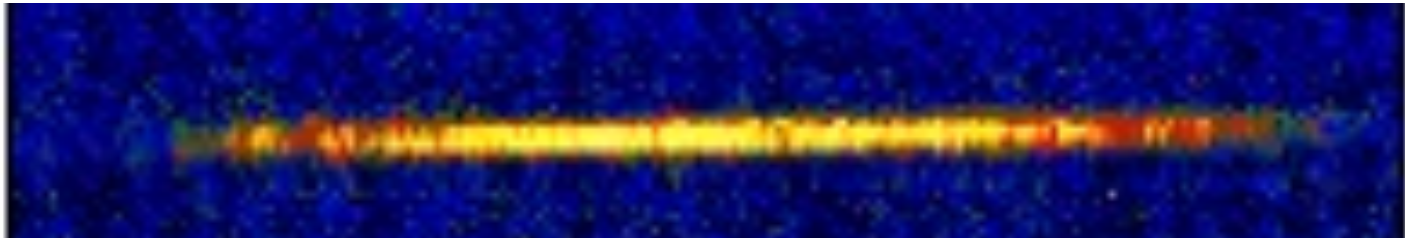


$$a(t) \simeq a_{av} + \delta a \cos(\Omega t),$$

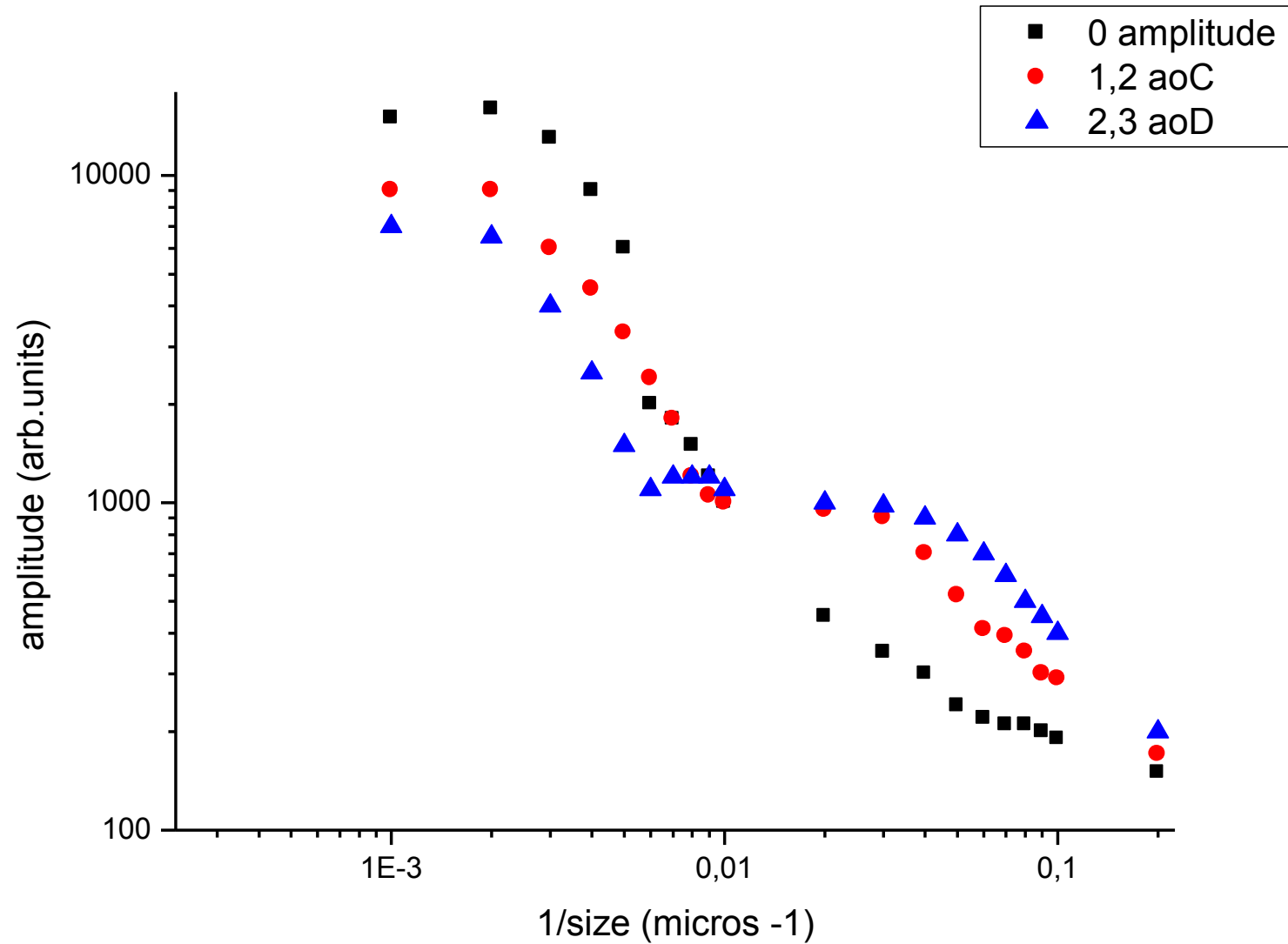
Collaboration with R. Hulet - Texas

- PhysRevA.85.033608(2012)

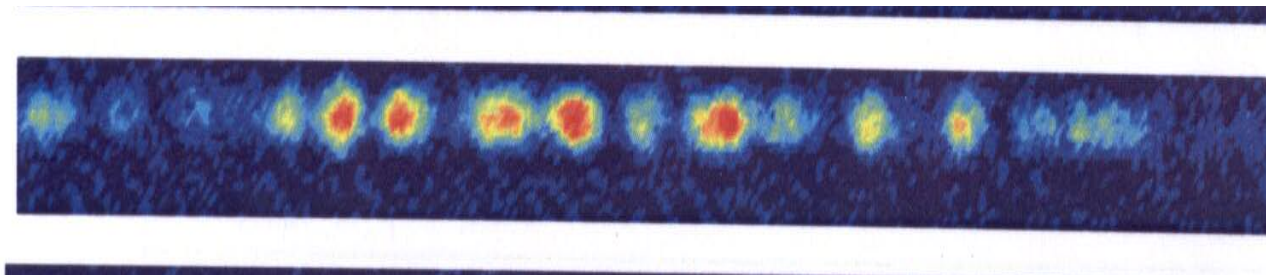
- Higher amplitudes take to “granulation”



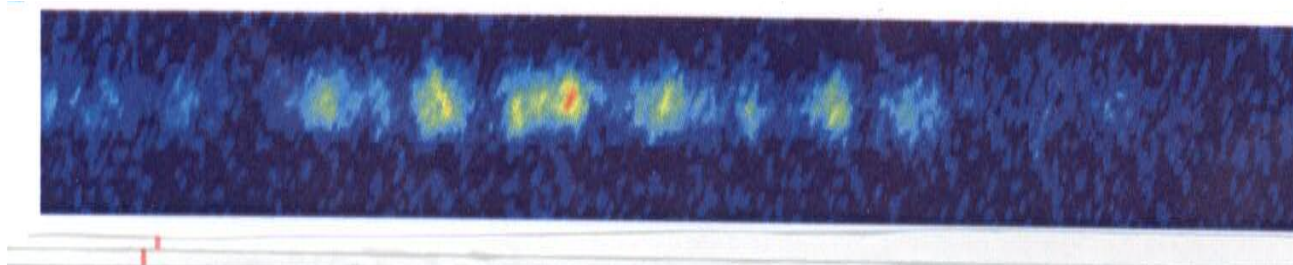
Higher amplitudes → fined size grains



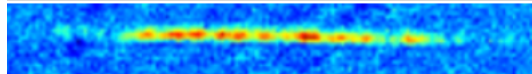
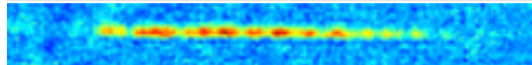
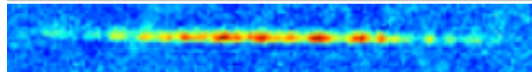
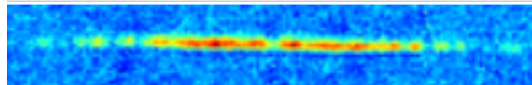
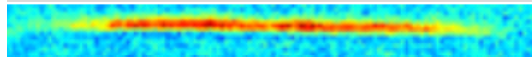
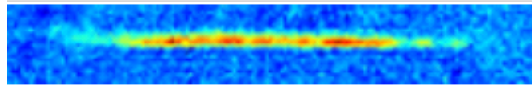
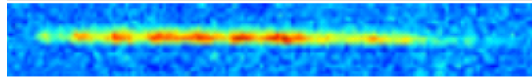
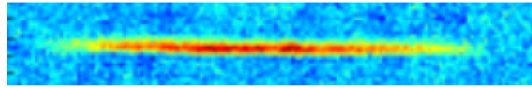
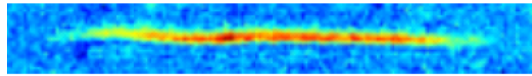
**Long living after excitation!!!!**

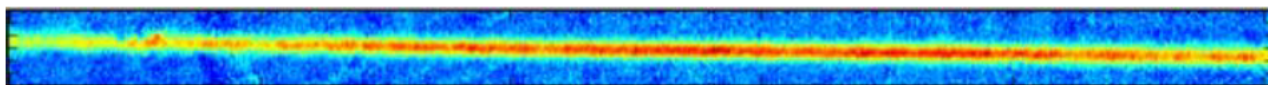


**> 5sec**

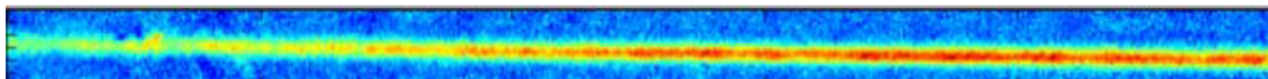


Time of  
excitation

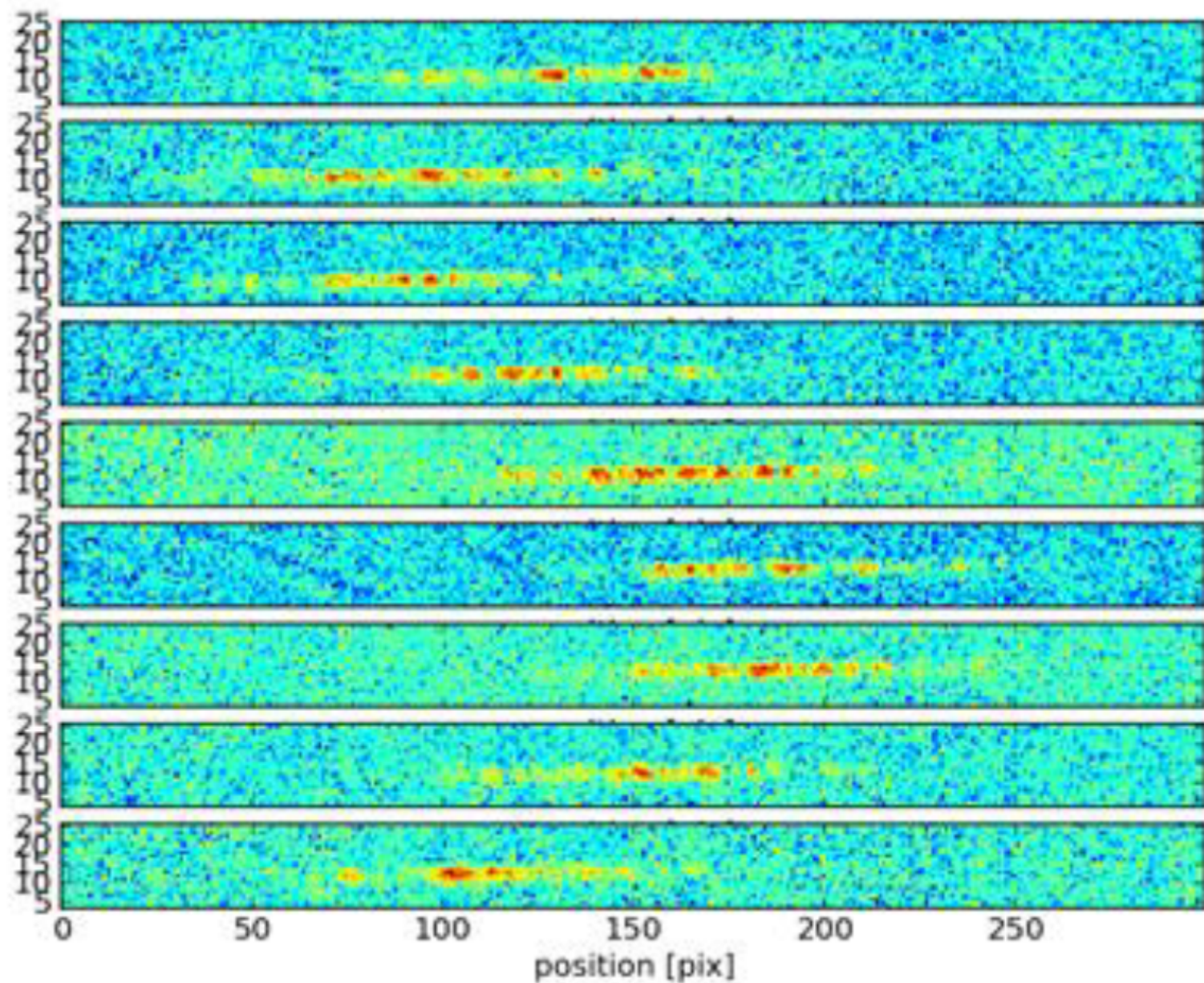




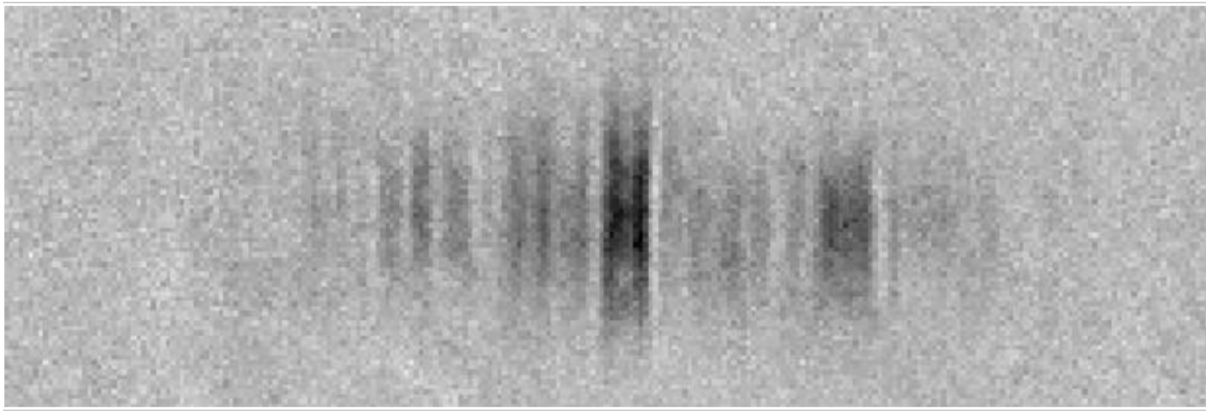
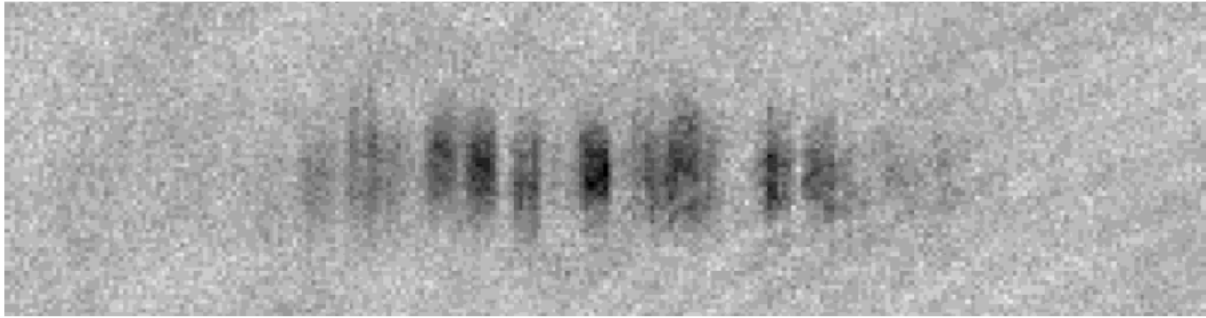
Now what happens when we modulate? Shot number 57788:



10/17/2014

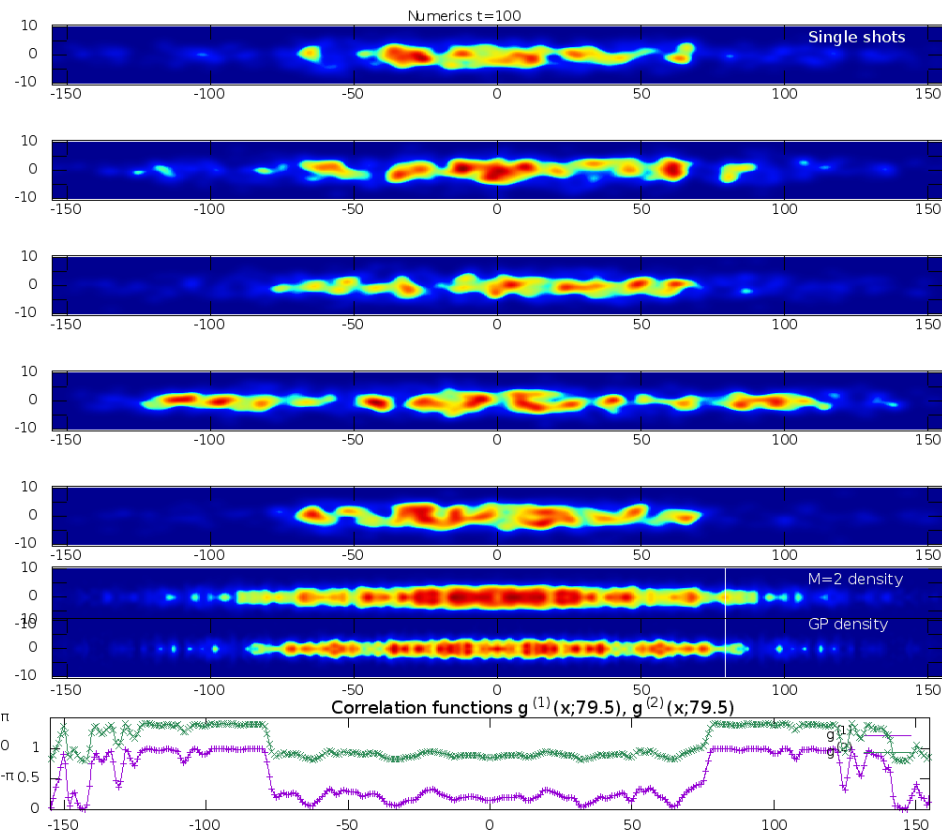
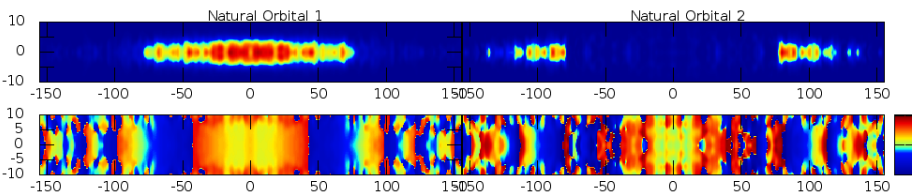
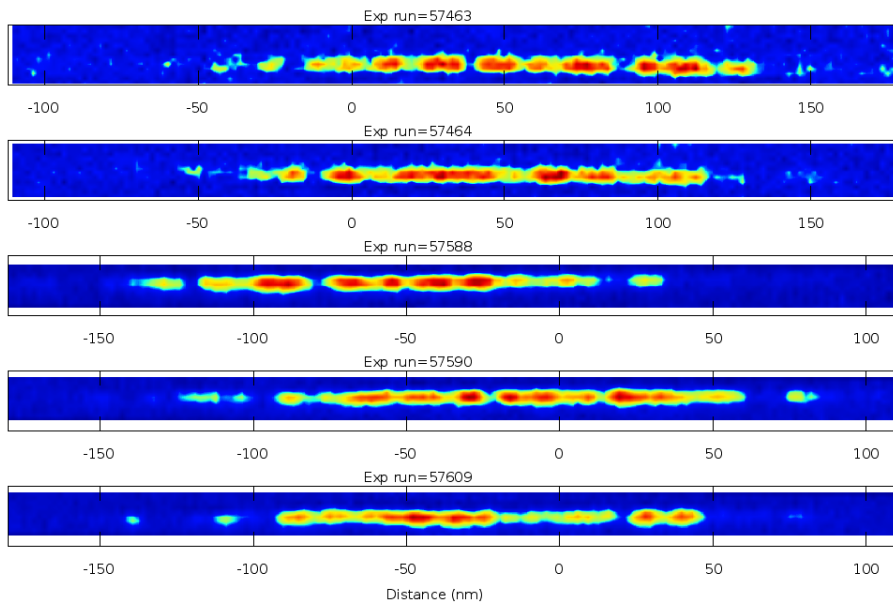


image



# Beyond mean field .....

$t=250+250$  ms     $\Omega=10\omega_x = 80\text{Hz}$





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# Quantum turbulence in trapped atomic Bose–Einstein condensates

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