GLOBAL THERMODYNAMIC VARIABLES FOR TRAPPED QUANTUM SYSTEM

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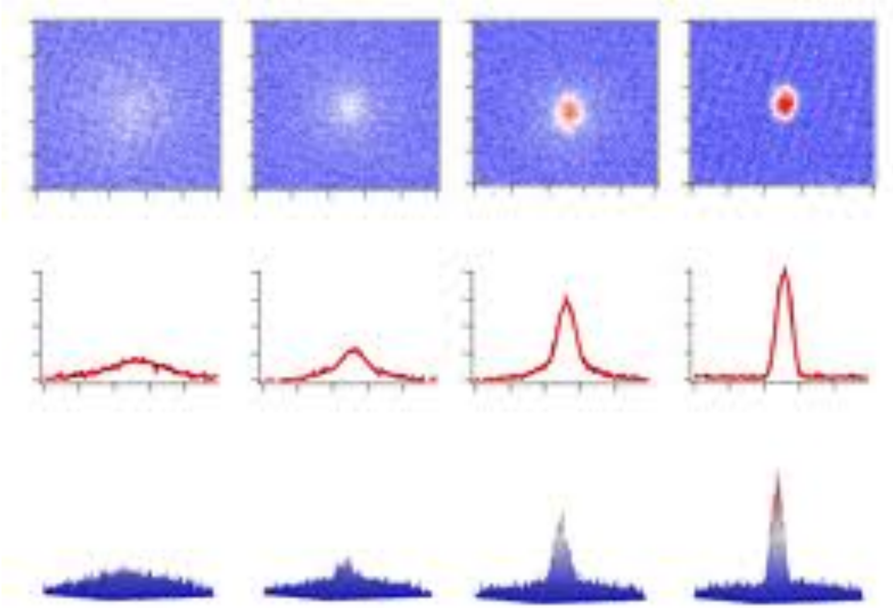
http://cepof.ifsc.usp.br

THERMODYNAMICS FOR COLD TRAPPED ATOMS

LOCAL DENSITY APPROXIMATION - LDA



$$U(\vec{r}) \leftrightarrow n(\vec{r})$$

$$\Downarrow$$

$$n(U)$$

$$P(U) = \int_U^\infty dU' n(U', T)$$

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \Rightarrow K_T = -\frac{1}{n^2(U)}\left(\frac{\partial n}{\partial U}\right)$$

$$\boxed{K_T(P)}$$

$$U(r) = \frac{1}{2} m\omega^2 r^2$$

$$n(r) = n_0 e^{-\frac{1}{2}\frac{m\omega^2 r^2}{KT}}$$

$$n(U) = n_0 e^{-\frac{U}{KT}}$$

$$P = \int_U^{\infty} n_0 e^{-\frac{U}{KT}} dU' = nKT$$

$$\boxed{P = nKT}$$

$$T > T_c$$

$$n(r) = N\left(\frac{1}{2}\frac{m\omega^2}{\pi KT}\right)e^{-\frac{1}{2}\frac{m\omega^2 r^2}{KT}}$$

$$T < T_c$$

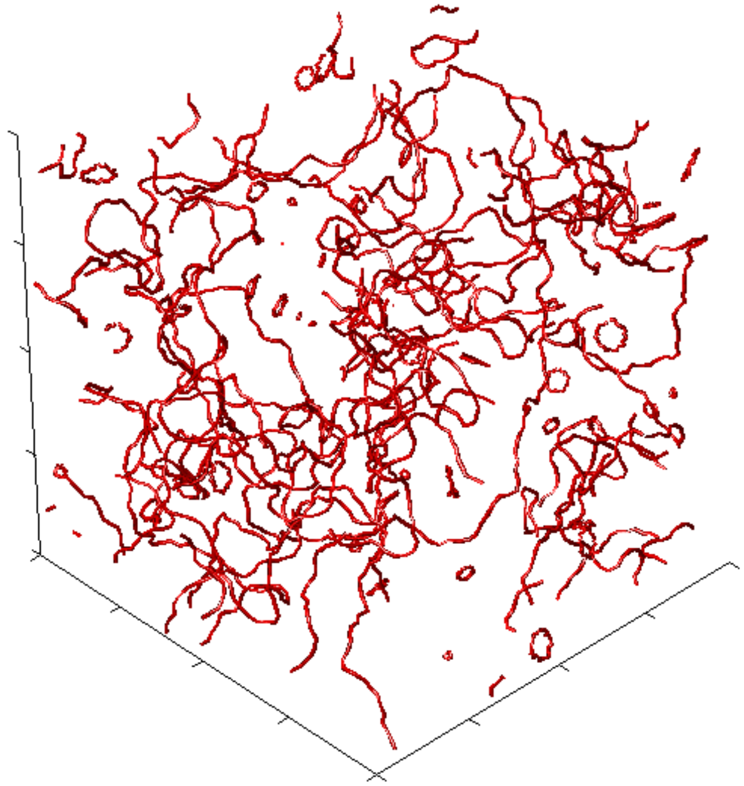
$$n(r) = Gaussian + ThomasFermi$$

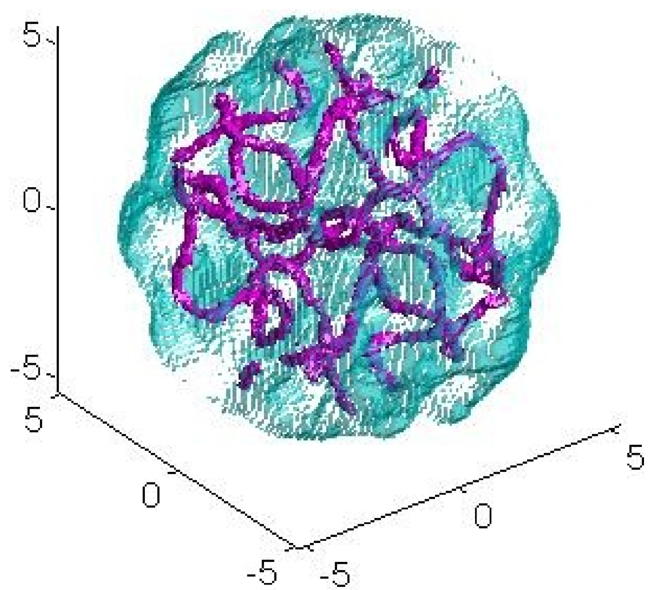
$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$dP = dU n(U) \qquad V = \frac{N}{n} \rightarrow dV = -\frac{N}{n^2}dn$$

$$K_T = -\frac{1}{\cancel{N}/\cancel{n}}\frac{\left(\cancel{-}\cancel{N}/n^2\right)dn}{\cancel{n}dU} = \frac{1}{n^2}\frac{dn}{dU}$$

turbulent regime in superfluids





1955: **Feynman** proposed that "superfluid turbulence" consists of a tangle of quantized vortices.

THERMODYNAMICS WITH GLOBAL VARIABLES FOR COLD TRAPPED ATOMS

Can we describe the trapped system with global variables , in steady of local variables?

Volume – Extensive variable
Pressure - Intensive variable

$$\Omega(\mu, T, \omega) = \textit{Extensive} \text{ x } \textit{Intensive}$$

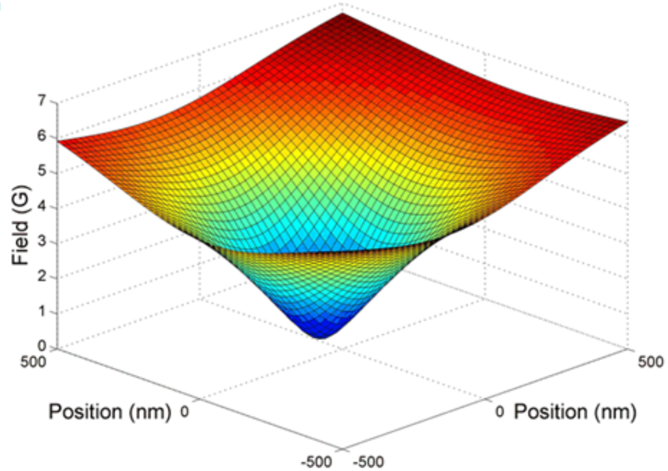
For extensive: $\surd \rightarrow \lambda\surd \quad \rightarrow \rightarrow$

$N \rightarrow \lambda N$

$E \rightarrow \lambda E$

$S \rightarrow \lambda S$

:



Critical temperature:

$$k_B T_C^{ho} = 0.94 \cdot \hbar \overline{\omega} \cdot N^{1/3}$$

Harmonic trap:

thermodynamic limit: $T_c^3 \propto \frac{N}{1/\omega^3}$

$$\mathcal{V} = \frac{1}{\omega^3}$$

Volume parameter: $\mathcal{V} \equiv \frac{1}{(\omega_x \omega_y \omega_z)} = \frac{1}{\bar{\omega}^3}$

Thermodynamic limit: $\begin{cases} N \to \infty \\ \mathcal{V} \to \infty \end{cases}$ $T_c^{ho} = const$

Ideal gas

Box vs. harmonic potential (Romero-Rochín PRL94, 130601 (2005) [?])

Grand-canonical partition function

$$\Xi = \mathrm{Tr}\left[\mathrm{e}^{-\left(\hat{H}_N - \mu\hat{N}\right)/k_B T}\right],$$

The Hamiltonian is

$$\hat{H}_N = \sum_{i=1}^{N}\left(\frac{\hat{p}_i^2}{2m} + \hat{U}\left(\hat{\mathbf{r}}_i\right)\right).$$

$$\Omega\left(\mu, T, V\right) = -PV$$

$$\Omega\left(\mu, T, \mathcal{V}\right) = -\Pi\mathcal{V}$$

Conjugate variables

$$V = L_x L_y L_z, \quad P = \frac{k_B T}{\lambda_{dB}^3} g_{5/2}\left(z\right)$$

Conjugate variables

$$\mathcal{V} = \frac{1}{\omega_x\omega_y\omega_z}, \quad \Pi = \frac{k_B^4 T^4}{\hbar^3} g_4\left(z\right),$$

Grand-canonical partition function

$$\Xi = \mathrm{Tr}\left[\mathrm{e}^{-\hat{K}/k_B T}\right],$$

The Hamiltonian is

$$\hat{H}_N = \sum_{i=1}^{N}\left(\frac{\hat{p}_i^2}{2m} + \hat{U}\left(\hat{\mathbf{r}}_i\right)\right) + \sum_{i<j=1}^{N}\hat{V}\left(\mathbf{r}_{ij}\right).$$

Global parameter volume

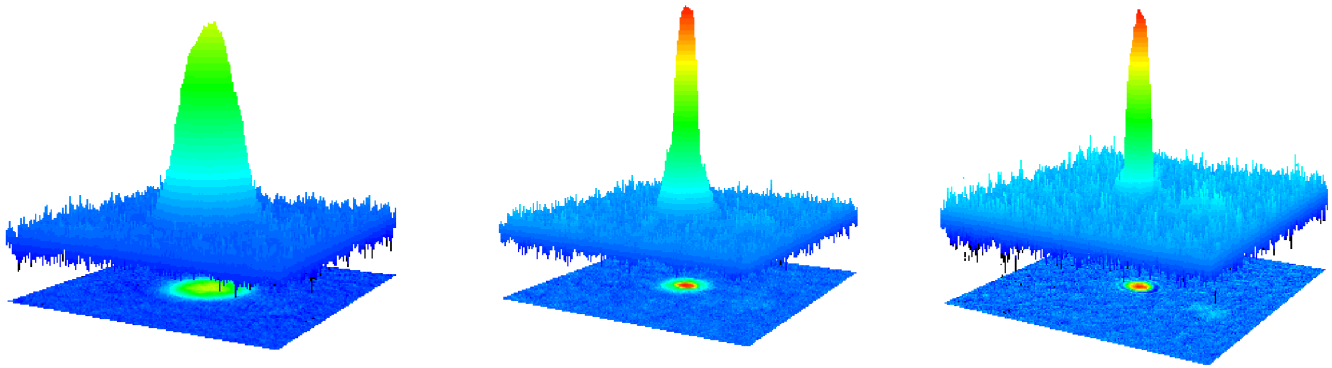
$$\mathcal{V} = \frac{1}{\omega_x \omega_y \omega_z}$$

Global parameter pressure

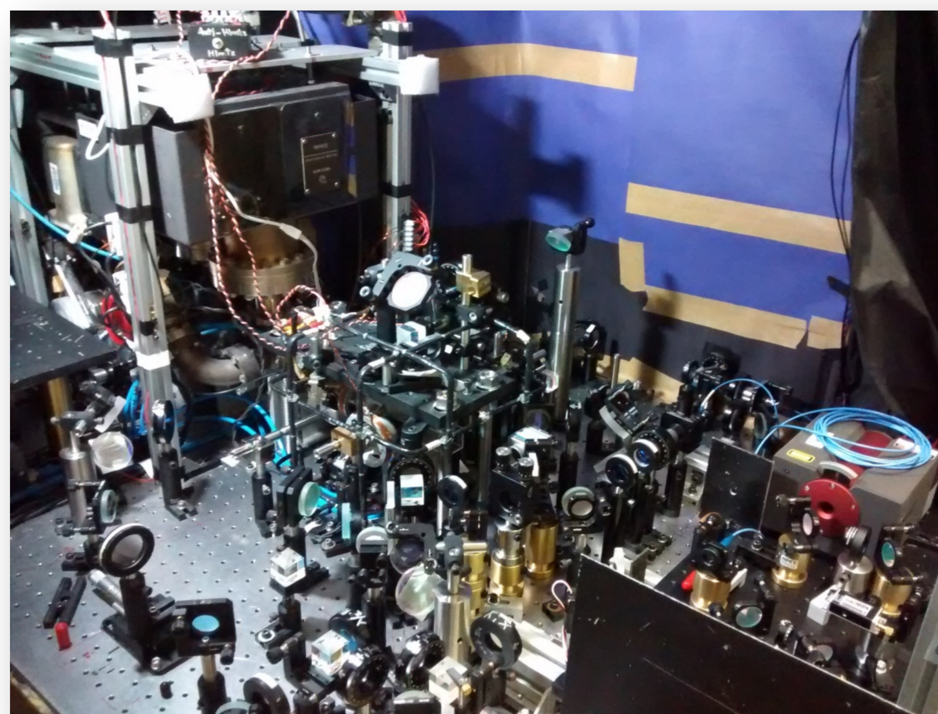
$$\Pi = -\left(\frac{\partial \Omega}{\partial \mathcal{V}}\right)_{\mu,T} = \frac{m}{3\mathcal{V}}\int d^3r\, n\left(\mathbf{r}\right)\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

Pressure conjugated to volume….

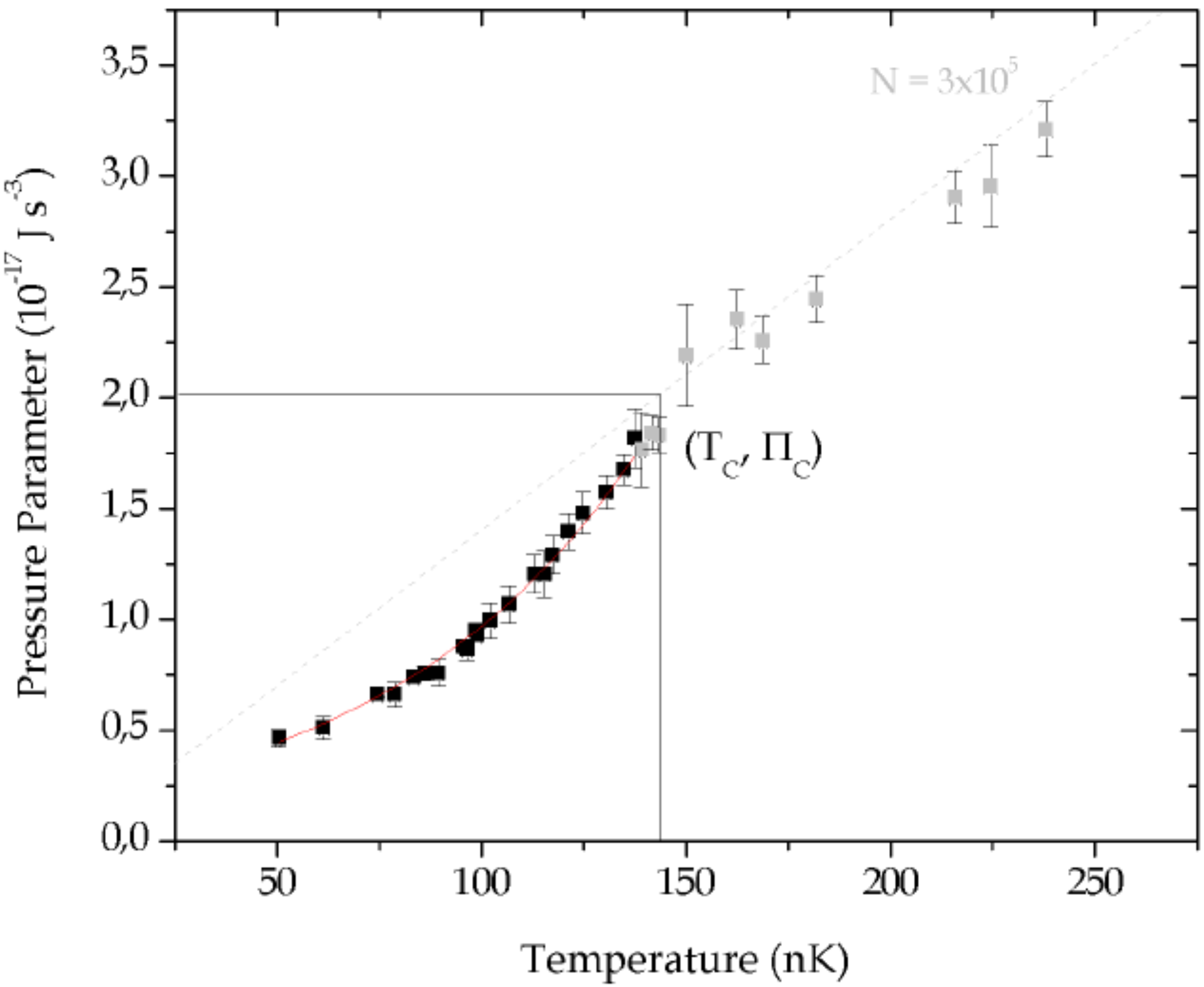
$$\Pi \equiv -\left(\frac{\partial F}{\partial \mathcal{V}}\right)_{N,\beta}$$



$T > T_c$ $T < T_c$ $T \ll T_c$

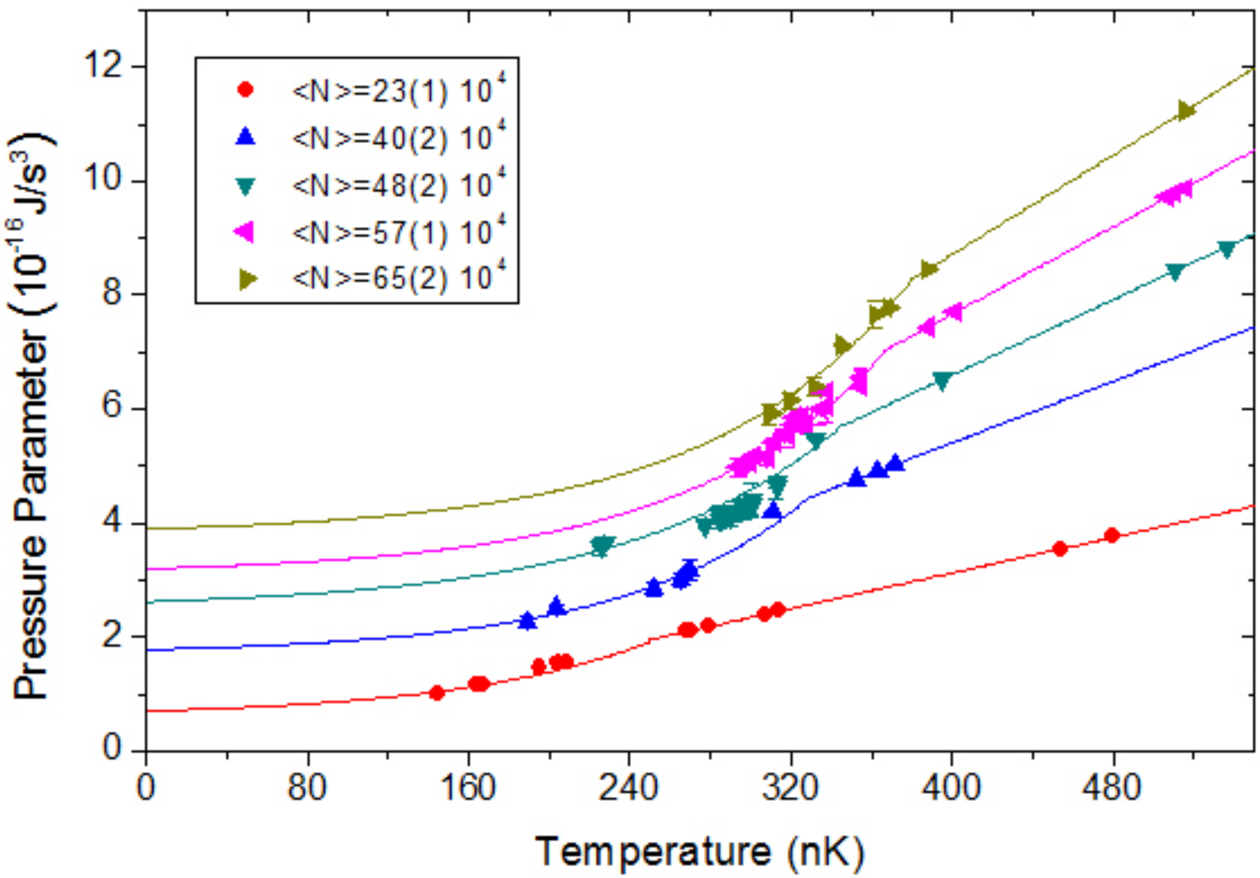


Volume Parameter 4 - Isodensity Curve



Isodensity curves:

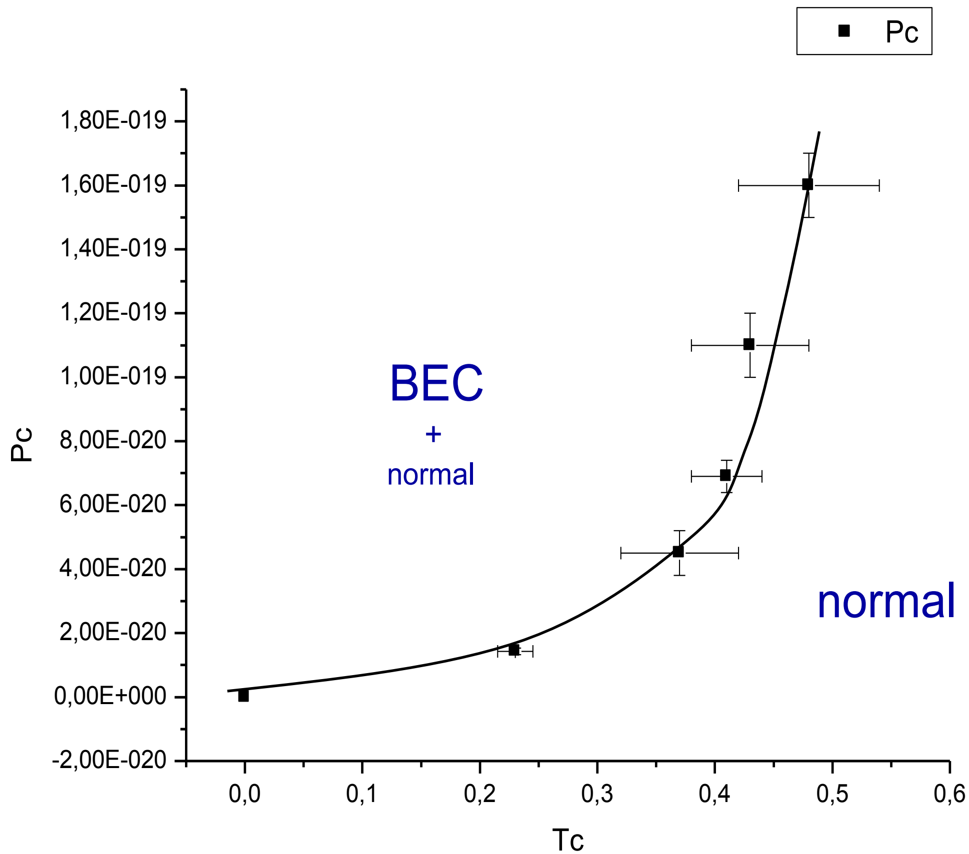
$$\Pi(N/\mathcal{V}, T)$$

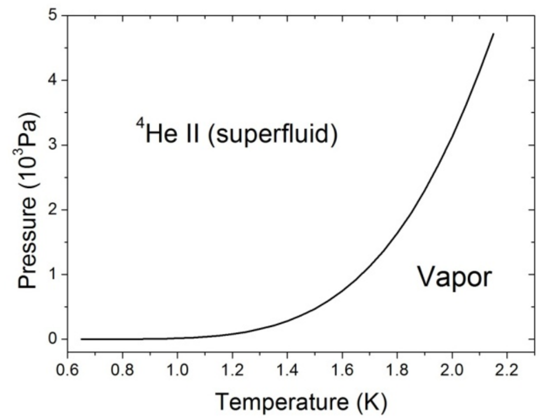


Phys. Rev. A, v. 85, 023632, 2012.

The transition line P vs T – Phase Diagram

It occurs from the discontinuity of the derivative of Pc vs Tc





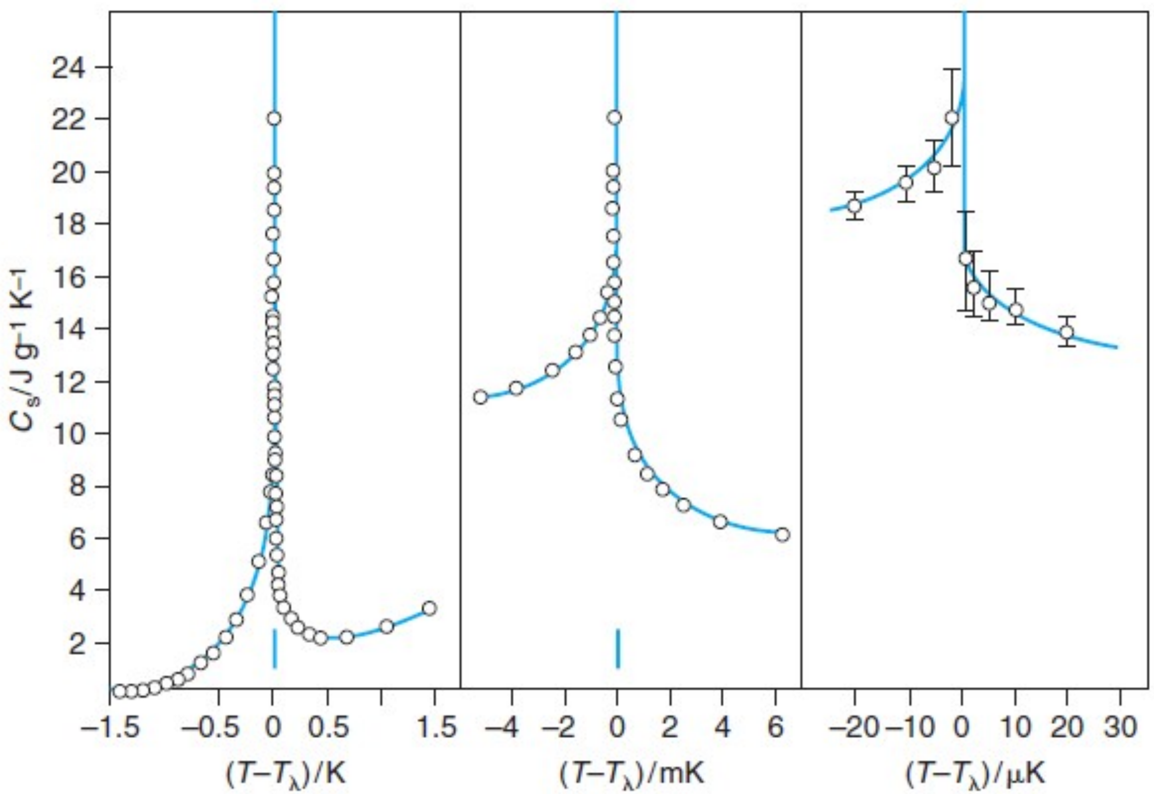
$$P_c \,\alpha\, T_c^{\delta}$$

Phys. Rev. A 85, 023632 (2012)

Heat Capacity

In He:

➢ **λ transition He-I => He-II**



With the macroscopic parameters:

$$C_{\mathcal{V}} \equiv \left(\frac{\partial U}{\partial T}\right)_{\mathcal{V},N}$$

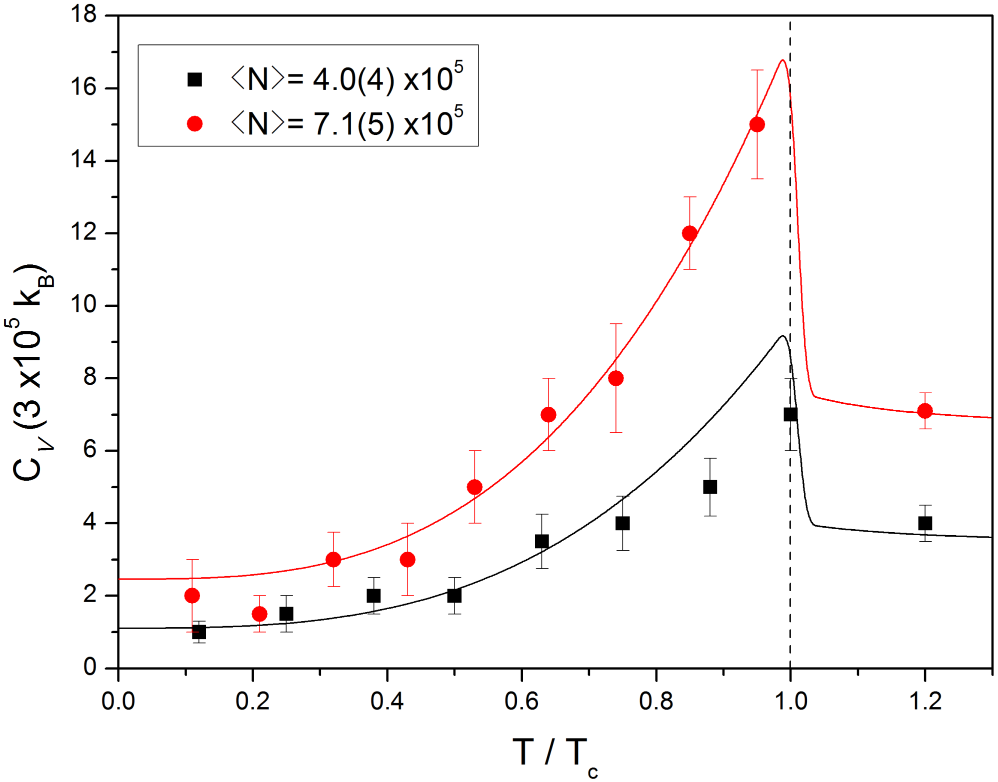
$$U_{th} = 3\Pi\mathcal{V} \quad \text{(pure thermal)}$$

$$U_0 = \tfrac{5}{2}\Pi\mathcal{V} \quad \text{(pure BEC)}$$

$$U = 3\Pi\mathcal{V} - \cancel{\frac{1}{2}\Pi_0\mathcal{V}}$$

$$\frac{\partial \Pi_0}{\partial T} \ll \frac{\partial \Pi}{\partial T} \Longrightarrow C_{\mathcal{V}} \simeq 3\mathcal{V}\underbrace{\left(\frac{\partial \Pi}{\partial T}\right)_{\mathcal{V},N}}_{\Pi(N/\mathcal{V},\,T)}$$

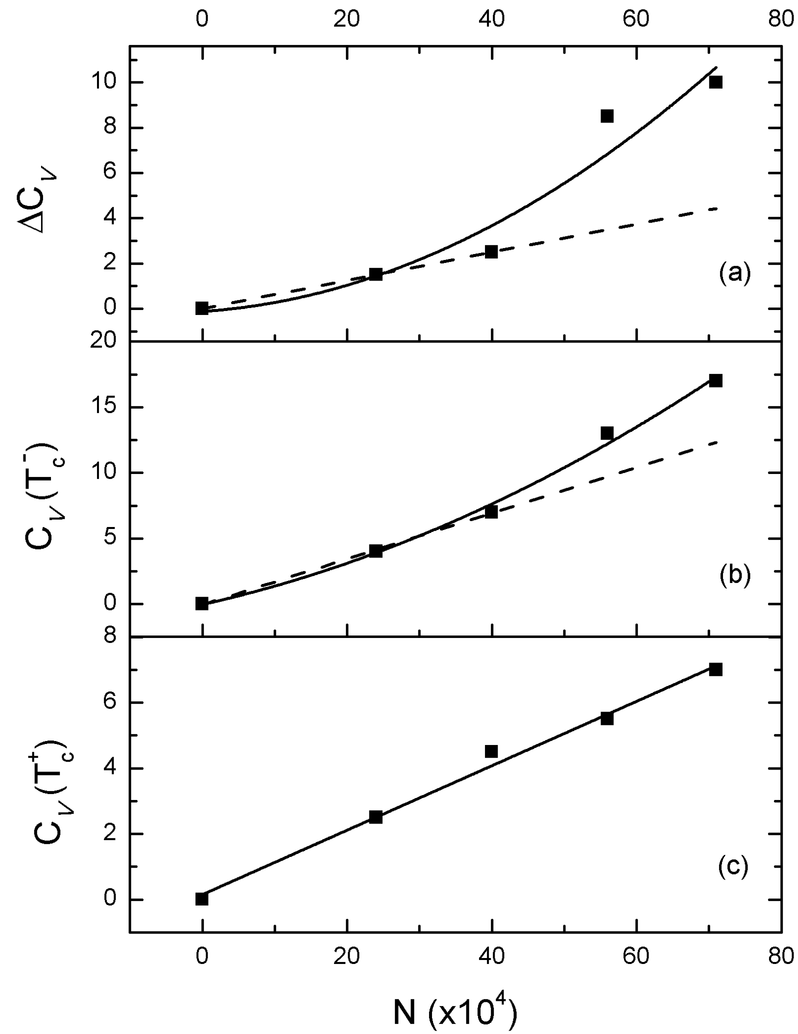
R.F. Shiozaki *et al.*, Phys. Rev. A. (2014)



S. Grossmann and M. Holthaus, Phys. Lett. A, v. 208, p. 188, 1995

- Effect of interactions.

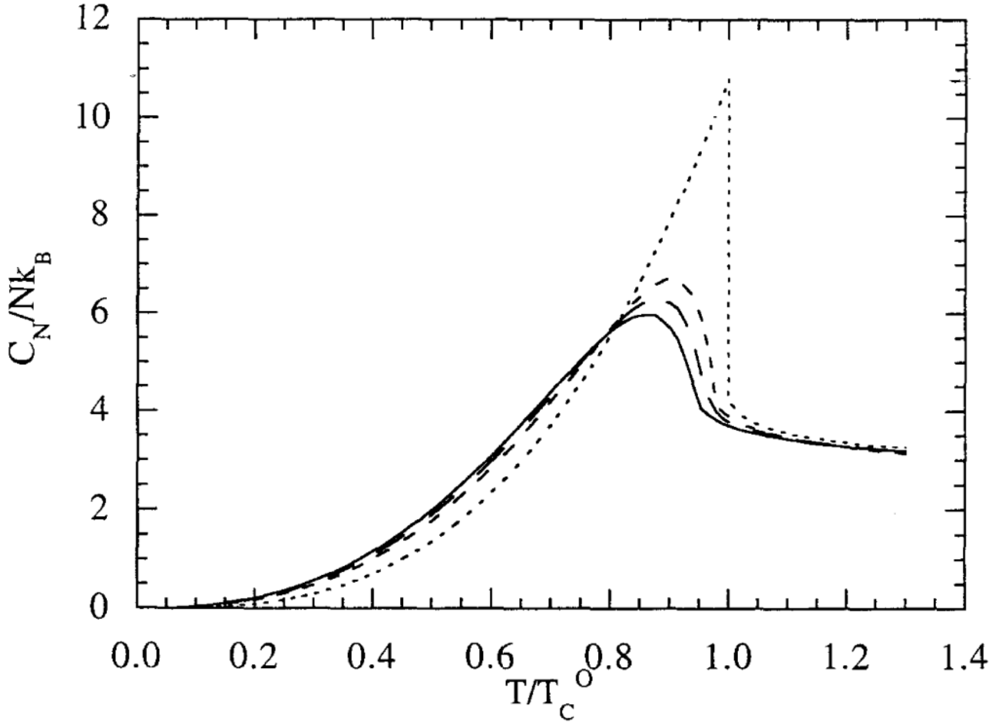
$$\frac{C_<}{Nk} = 12\frac{\zeta(4)}{\zeta(3)}\left(\frac{T}{T_0}\right)^3 + 6\gamma\zeta(3)^{1/3}\frac{1}{N^{1/3}}\left(\frac{T}{T_0}\right)^2$$

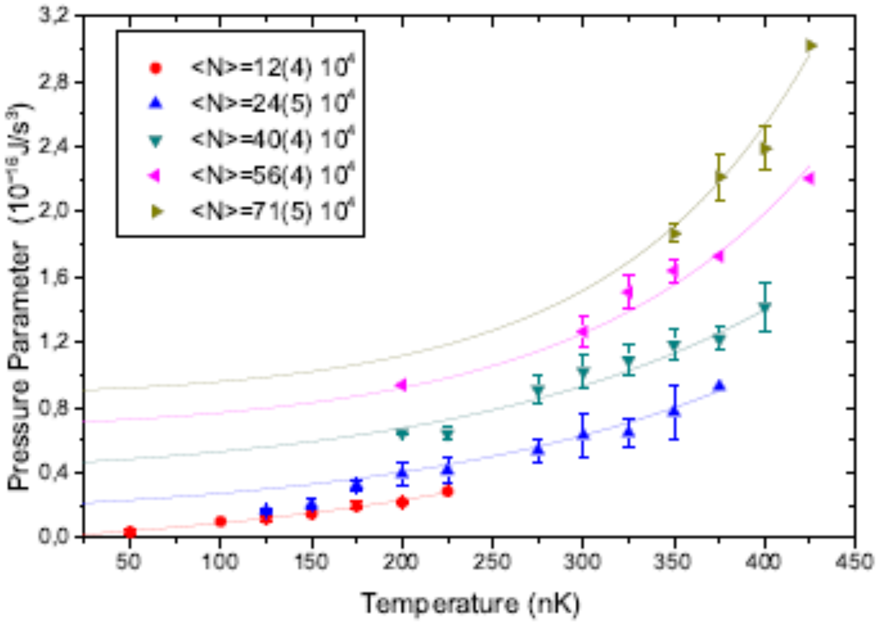


Phys. Rev. A 90, 043640 (2014)

Normalized C_V :

S. Giorgini *et al.*, J. Low Temp. Phys., v. 109, 309, 1997.



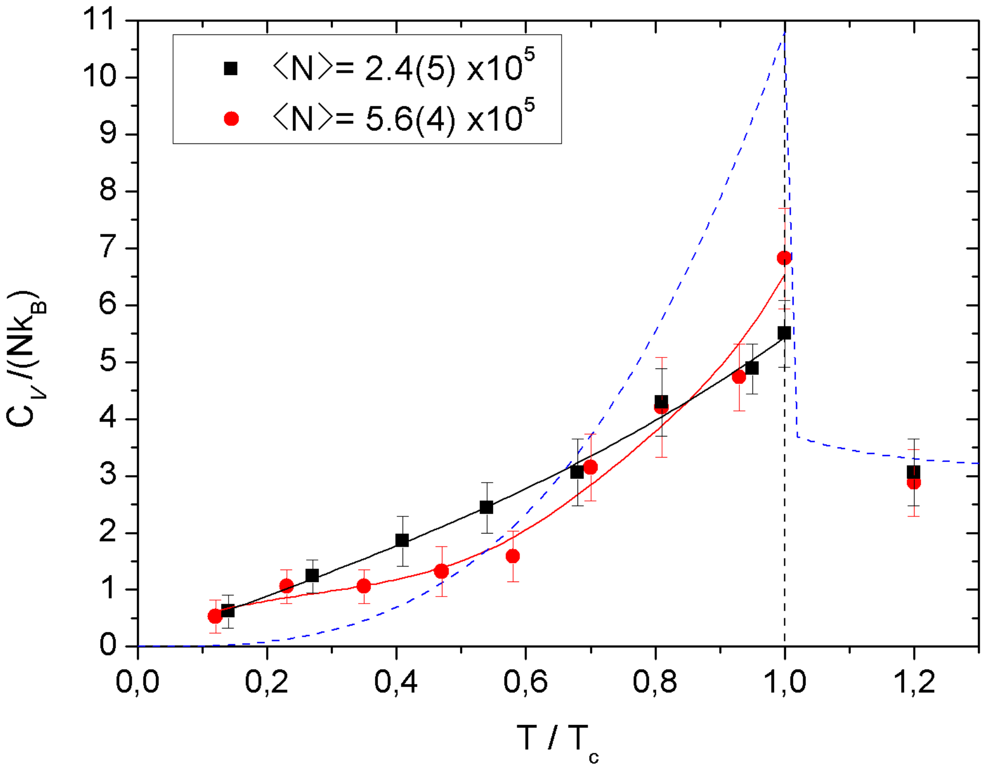


$$\Pi_{bec} = \frac{1}{7}\left[15 a_s \hbar^2 \sqrt{m}\left[\frac{N}{\mathcal{V}} - \left(\frac{k_B T}{0.94\hbar}\right)^3\right]^{7/2}\right]^{2/5}$$

Limit $T \to 0$

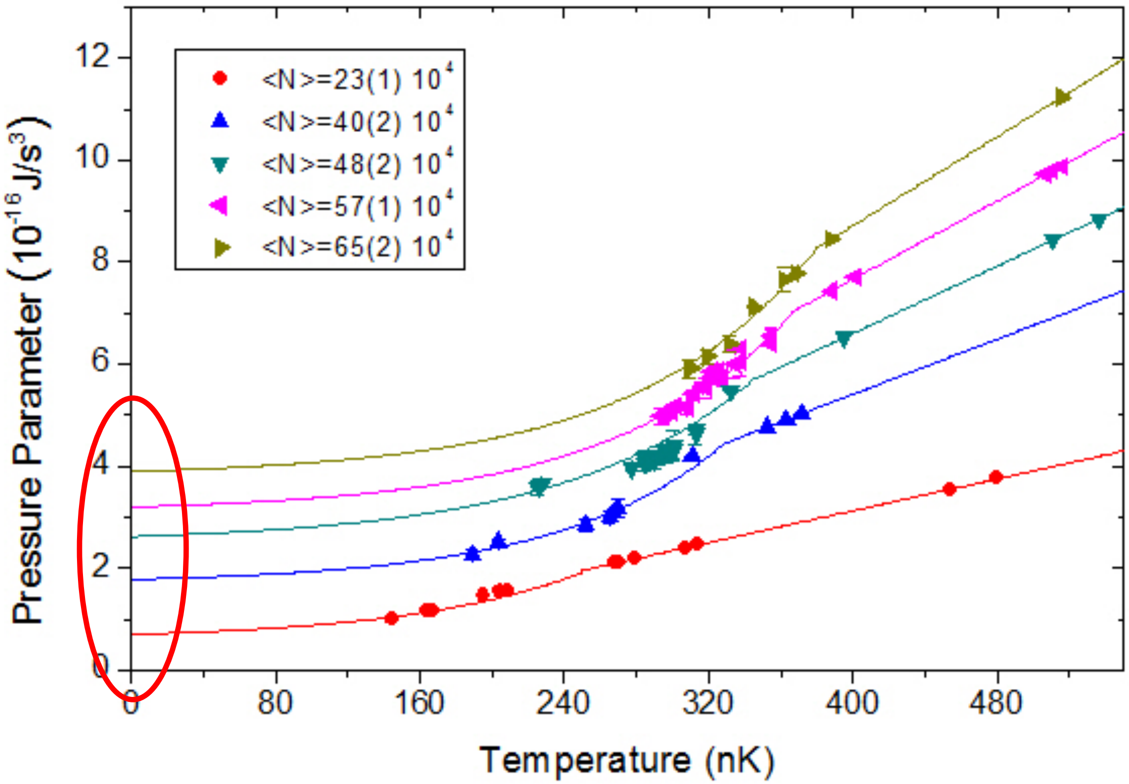
$$\Pi_0\,(T \to 0) = \frac{1}{7}\left(15 a_s \hbar^2 \sqrt{m}\right)^{2/5}\left(\frac{N}{\mathcal{V}}\right)^{7/5}$$

Results



$$\frac{C_V}{Nk_B} = \frac{4a}{\lambda}\frac{\Delta}{k_BT}e^{-2\Delta/k_BT},$$

$$\Delta\,(\propto N)$$



$$\Pi_{T=0} \propto (N/\mathcal{V})^{\delta}$$

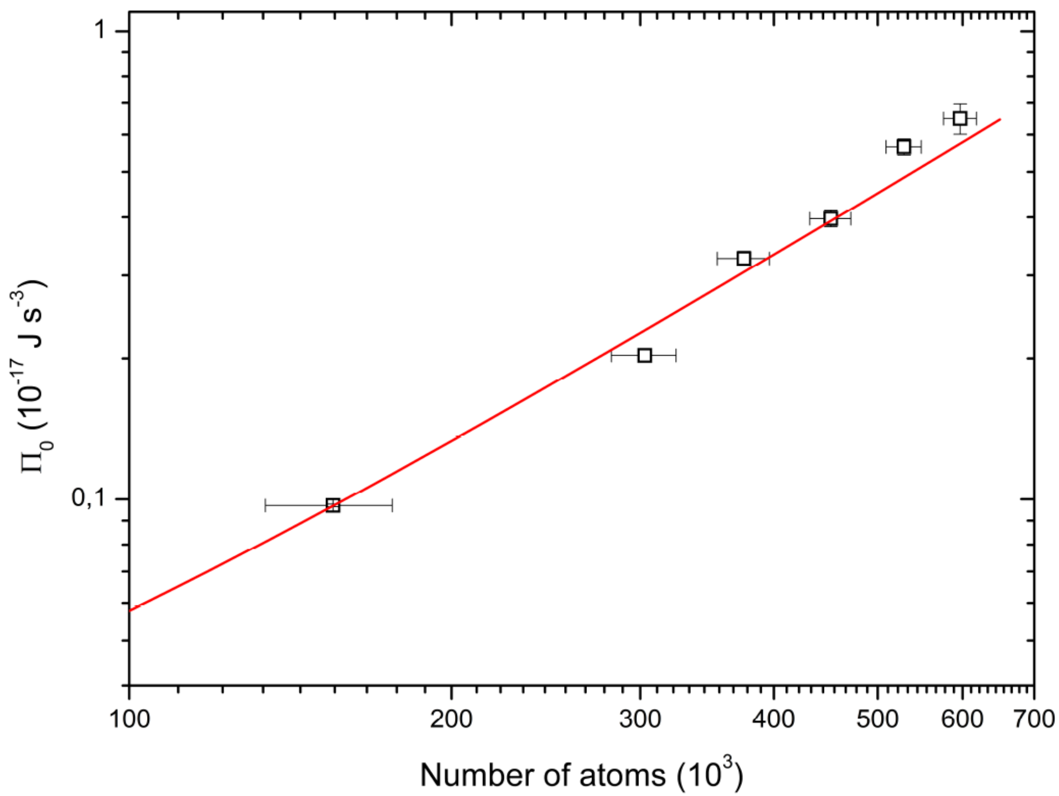
$$\delta = 1.5^{+0.3}_{-0.2}$$

$$a_s = 94 \pm 32 \text{Å}$$

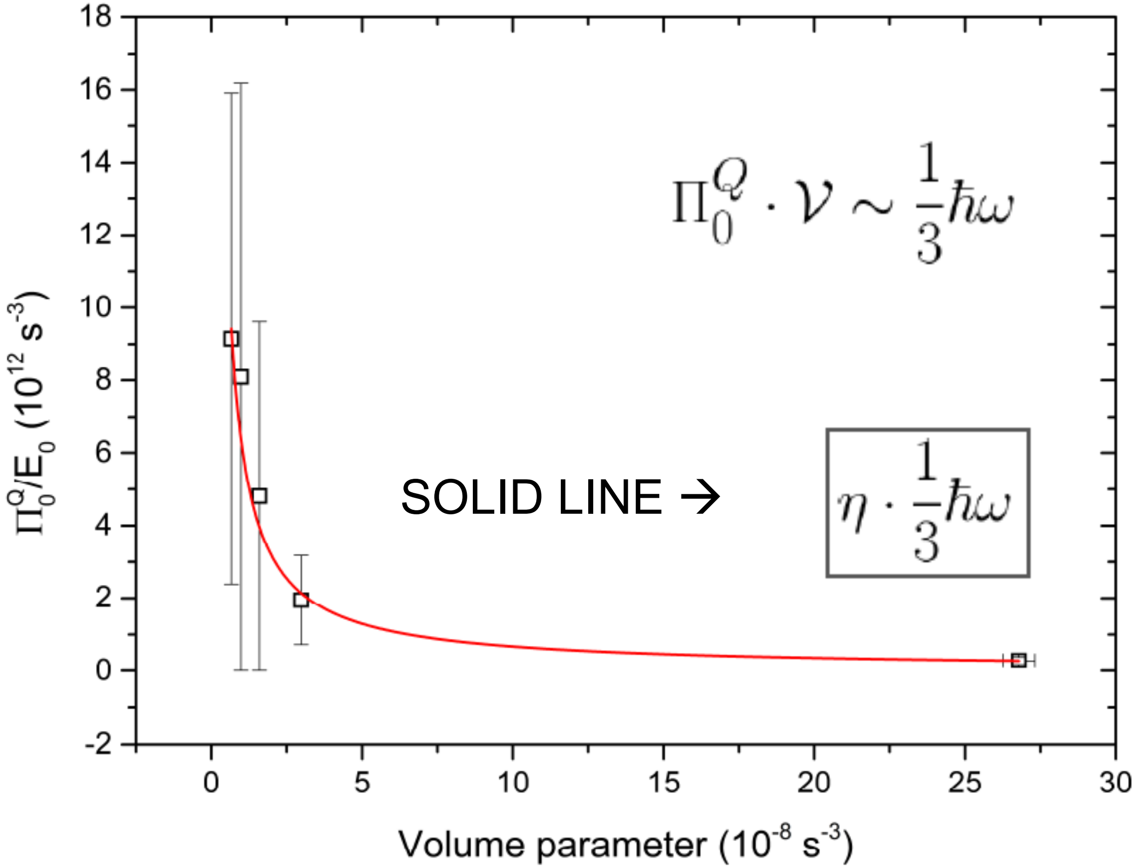
$$\Pi_{T=0} = \frac{1}{7}\left(15\hbar^2 a_s \sqrt{m}\right)^{2/5}\left(\frac{N}{\mathcal{V}}\right)^{7/5}$$

., Phys. Rev. A, v. 85, 023632, 2012.

Limit of N→ 0 and T→ 0



HEISEMBERG LIMITTED STATE EQUATION



Bohr´s remarkers in his 1930 Faraday Lecture concerning complementarity between pairs of thermodynamics variables: P and V. One infers a kind of thermodynamics uncertainty principle.

C. Moller – Matematisk –Fysiske Meddelelser 87, 4 (1969)

“ ….pressure is complementary to volume, in much the same way that momentum and position…. It is true that for systems of ponderable size where N is very large, the complemmentary character of the mentioned quantities is usually not apparent, but in principle……..”

Compressibility (isothermal)

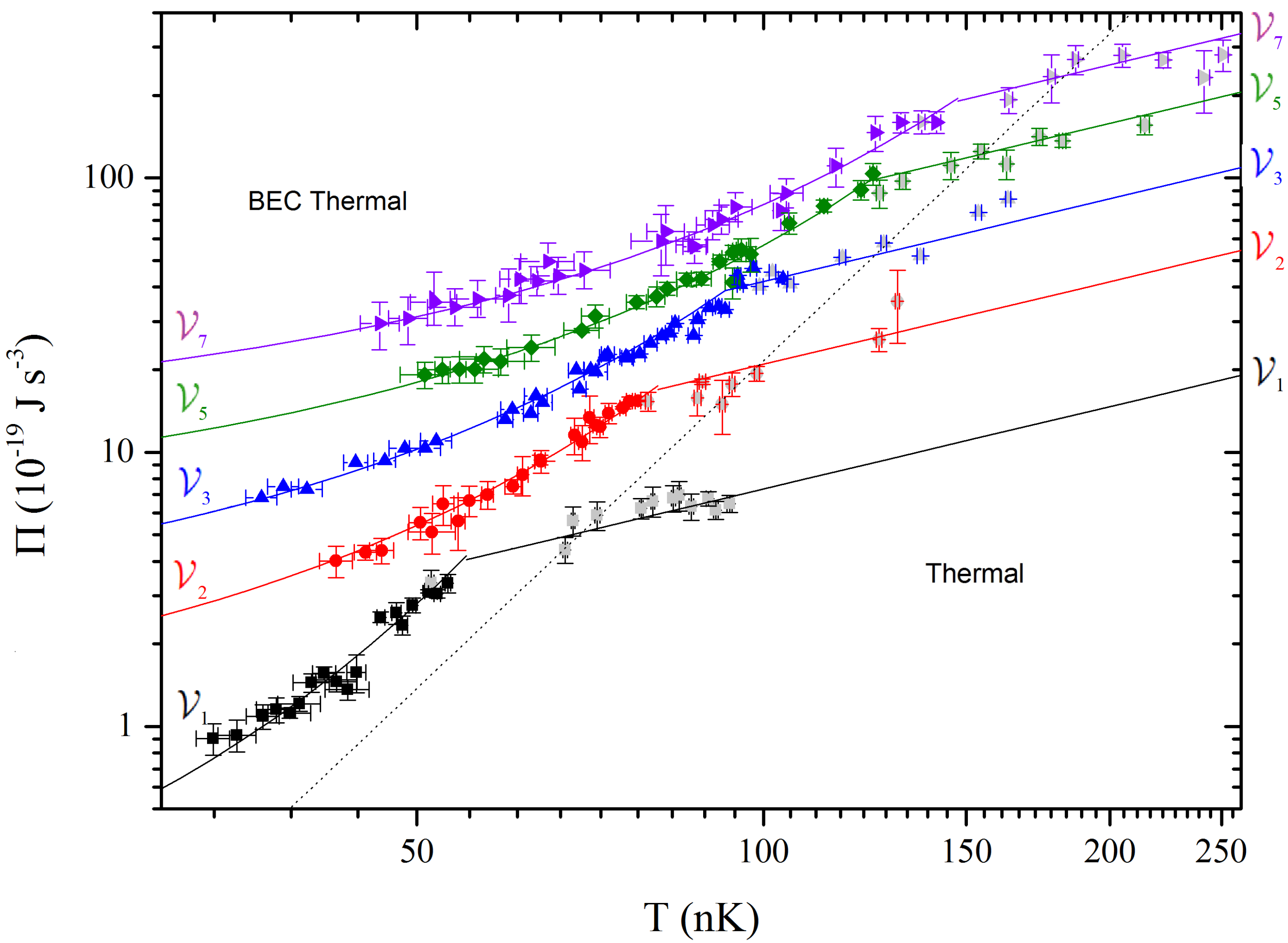
$$\kappa_T = -\frac{1}{\mathcal{V}}\left(\frac{\partial \mathcal{V}}{\partial \Pi}\right)_T$$

Isothermal compressibility determination across Bose-Einstein condensation

F. J. Poveda-Cuevas, Patricia C. M. Castilho,
E. D. Mercado-Gutierrez, A. R. Fritsch, Eleonora Lucioni, and V. S. Bagnato
Instituto de Fisica de São Carlos, C.P. 369, 13560-970 São Carlos, SP, Brazil

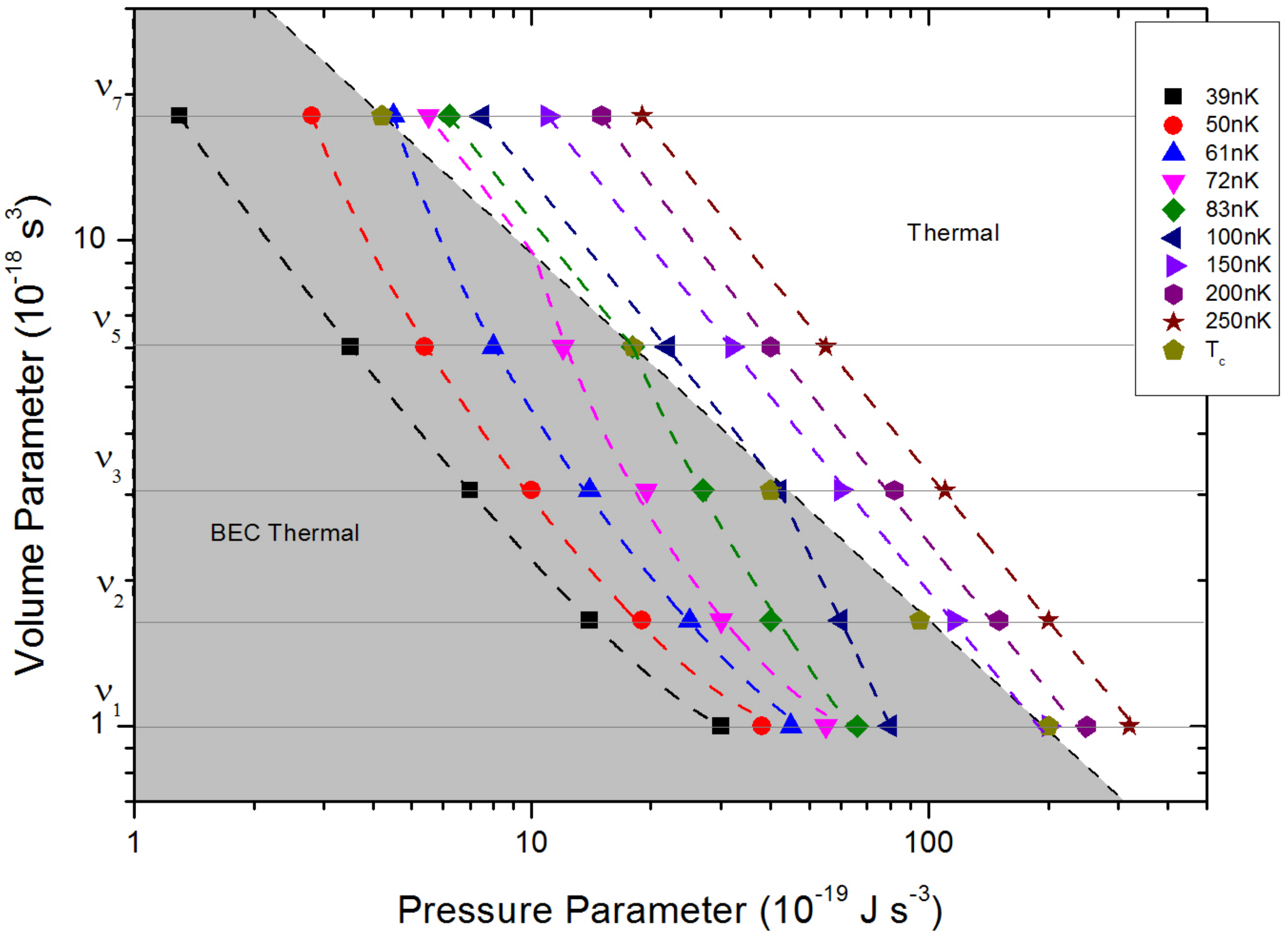
Date: November 25, 2014

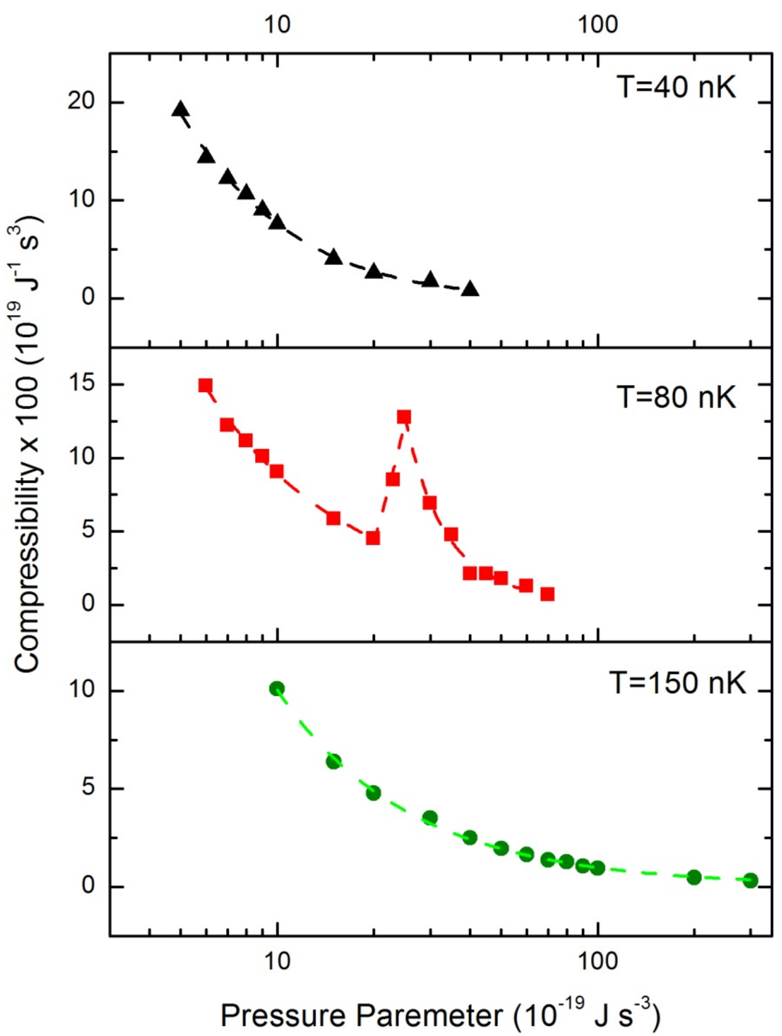
We investigate the thermodynamics of a bosonic gas confine in a harmonic potential across the Bose Einstein condensation. We apply the global thermodynamical variables approach to experimentally determine the isothermal compressibility k_T of the trapped Bose gas crossing the phase transition. We demonstrate the universal behavior of k_T and its behavior around the critical pressure, revealing the second order nature of the uncondensed-condensed phase transition. The use of global variables shows advantages with respect to local density approximation method in many situations discussed.



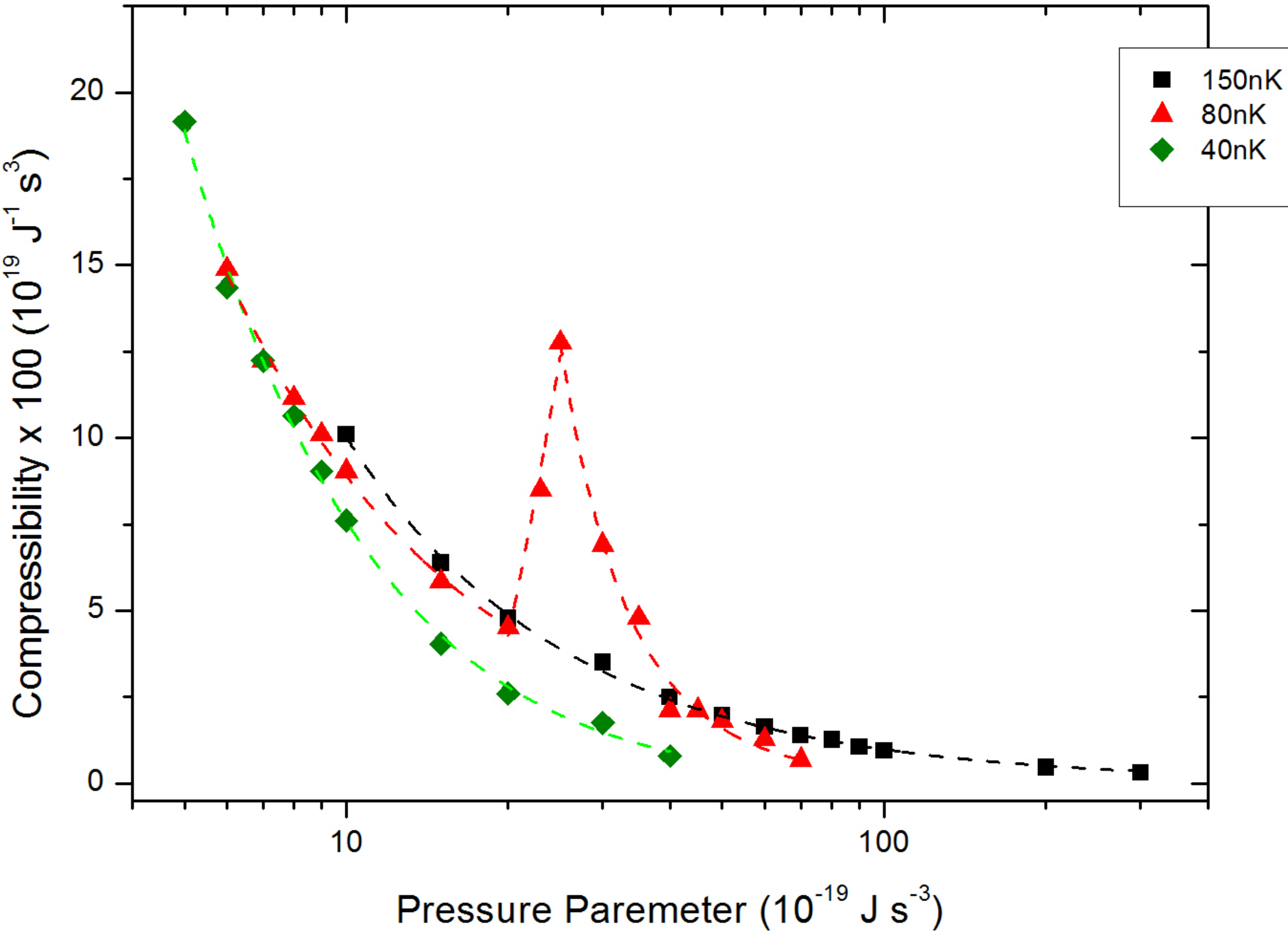
$\Pi\ (10^{-19}\ \mathrm{J\ s^{-3}})$
T (nK)
BEC Thermal
Thermal
ν_1
ν_2
ν_3
ν_5
ν_7
1
10
100
50
100
150
200
250

NEAR CRITICAL TEMPERATURE



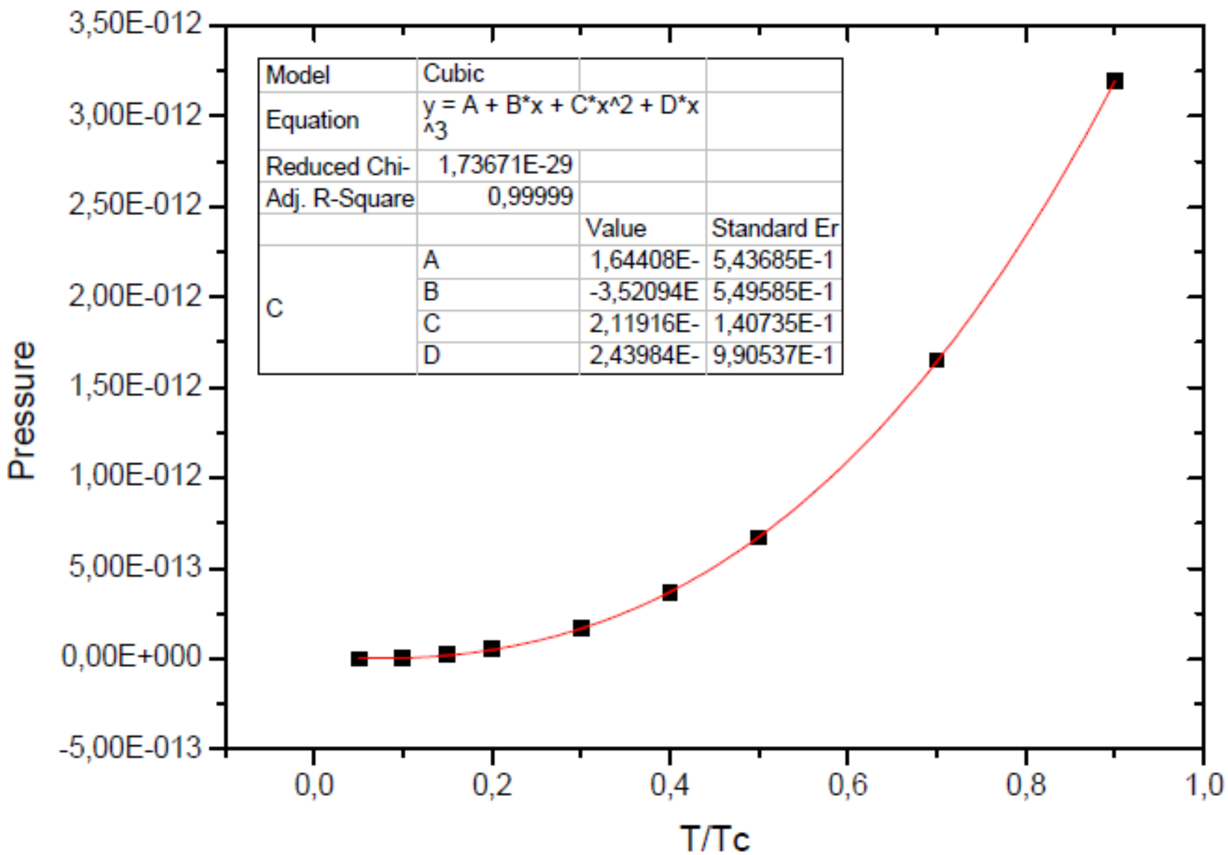


T=40 nK
T=80 nK
T=150 nK
Compressibility x 100 (10^{19} J^{-1} s^{3})
Pressure Paremeter (10^{-19} J s^{-3})
10
100
20
10
0
15
10
5
0
10
5
0
10
100

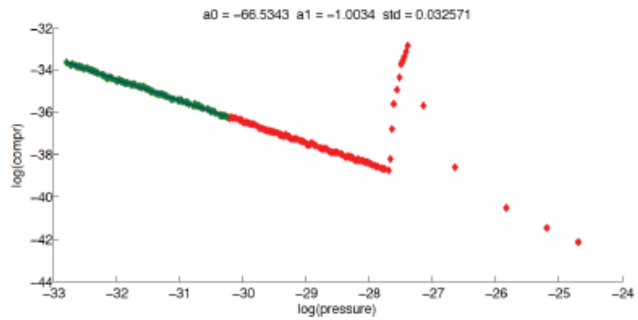


150nK
80nK
40nK
Compressibility x 100 (10^{19} J^{-1} s^3)
Pressure Paremeter (10^{-19} J s^{-3})
0
5
10
15
20
10
100

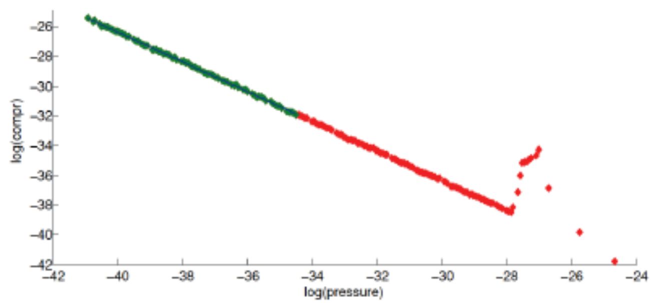
Pressure to have the peak in the compressibility



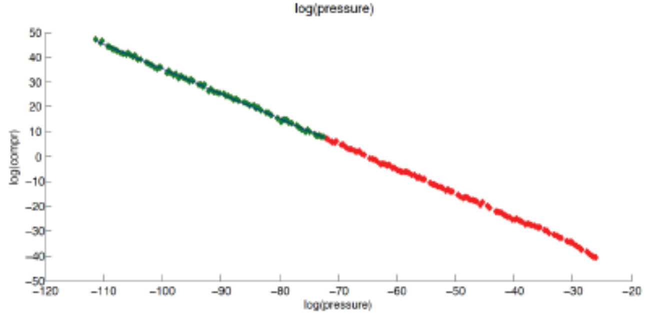
PROBLEM WITH LDA - TECHNICAL RESOLUTION



Mag=8

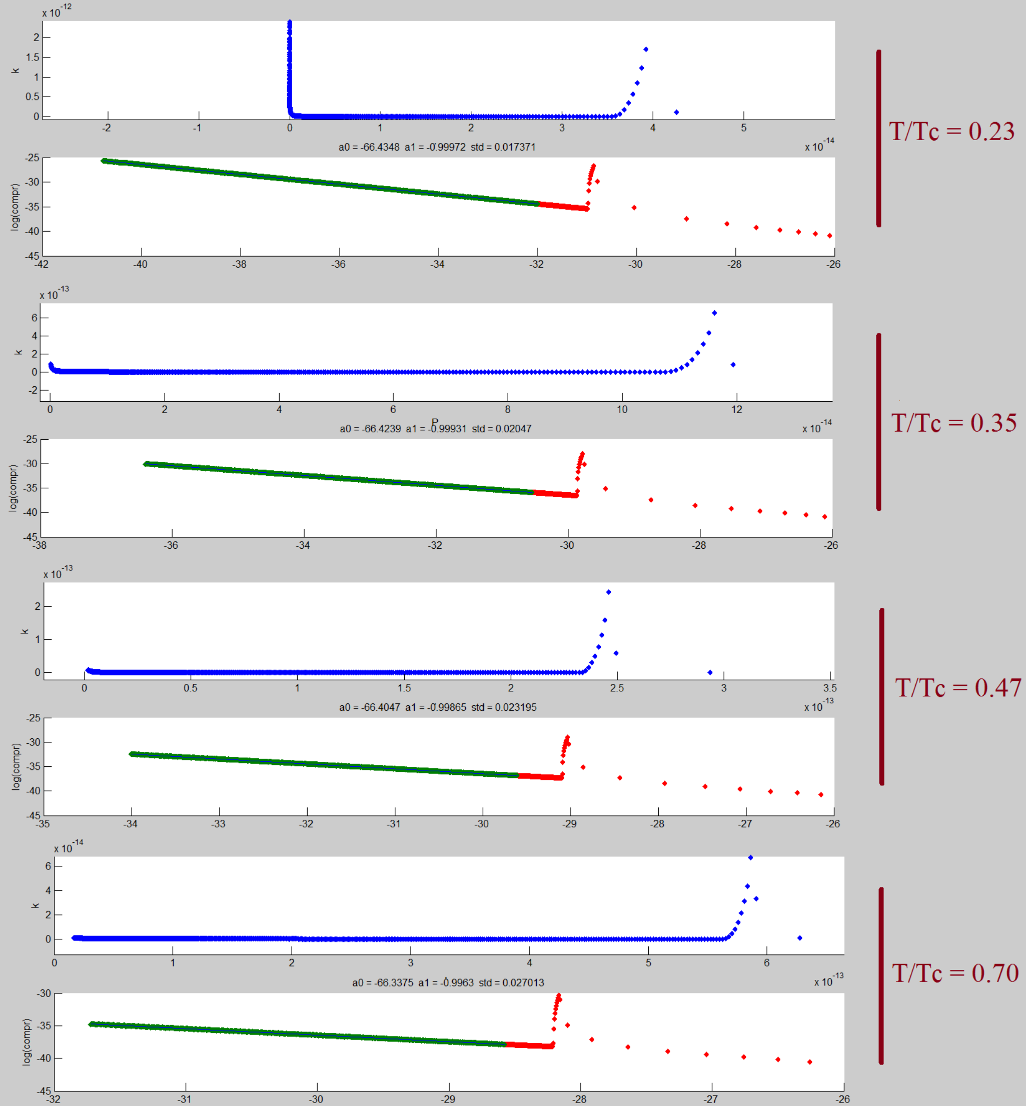


Mag=5



Mag=2

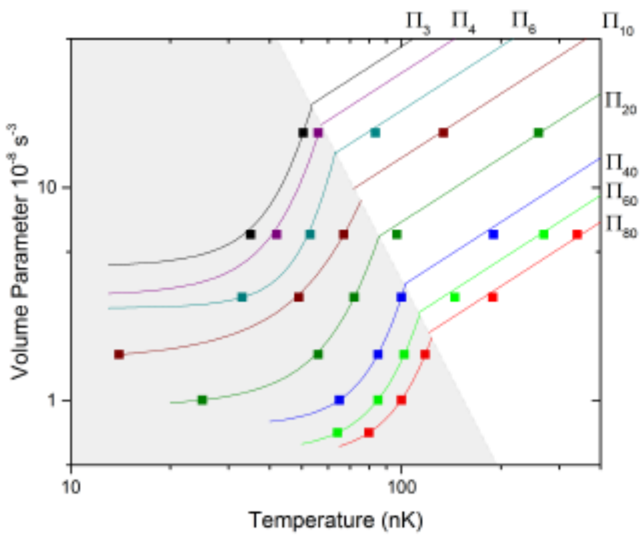
Number of atoms = 3x10^5
Trap freq = 100Hz
CCD pixel = 6.45x10^(-6)m
Mag = 14;

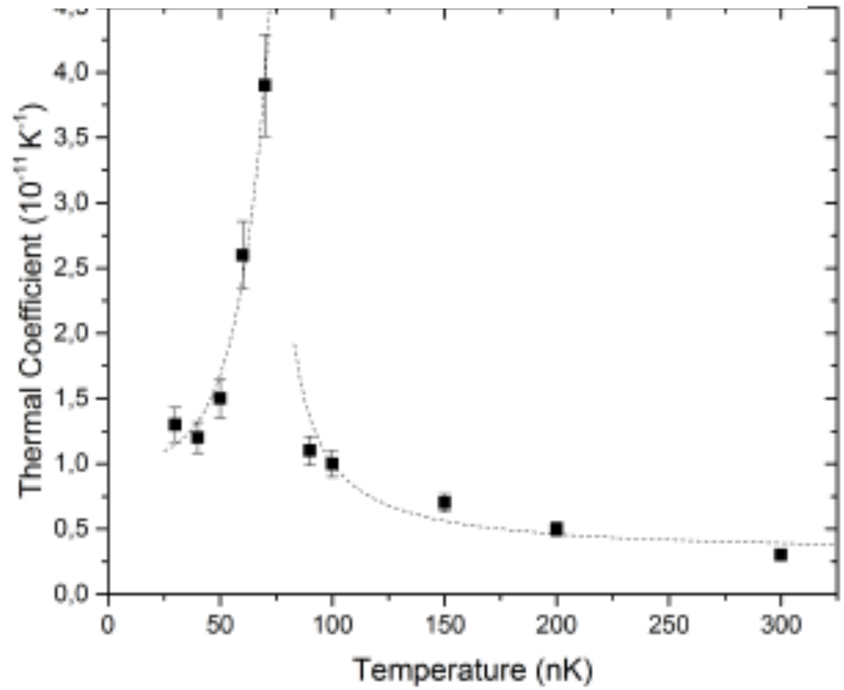


Compressibility on the turbulent regime

$$\mathcal{K}_T = -\frac{1}{\mathcal{V}}\left(\frac{\partial \mathcal{V}}{\partial \Pi}\right)_T$$

Measurement of the global thermal expansion coefficient near Bose-Einstein condensation and scaling properties for a harmonic trapped gas





$$\beta = \frac{1}{V}\left(\frac{dV}{dT}\right)_{p,N}$$

$$\alpha = 0.15 \pm 0.09$$

$$d = 2.76 \pm 0.16$$

$$d\nu = 2 - \alpha$$

$$\beta \sim t_r^{-\alpha}$$

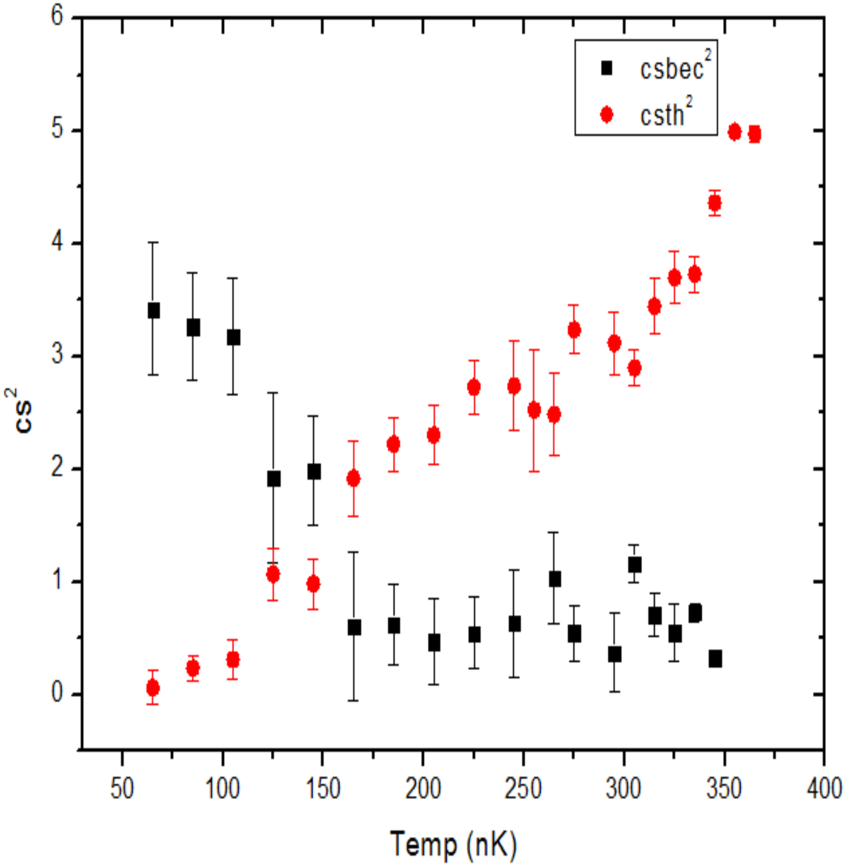
Sound Velocity using global variables.

Using *total* number of atoms

Reduced scale

$$cs_{bec}^2 = \frac{1}{m\omega^3}\left(\frac{\partial \Pi_{bec}}{\partial N_{Tot}}\right)$$

$$cs_{th}^2 = \frac{1}{m\omega^3}\left(\frac{\partial \Pi_{th}}{\partial N_{Tot}}\right)$$

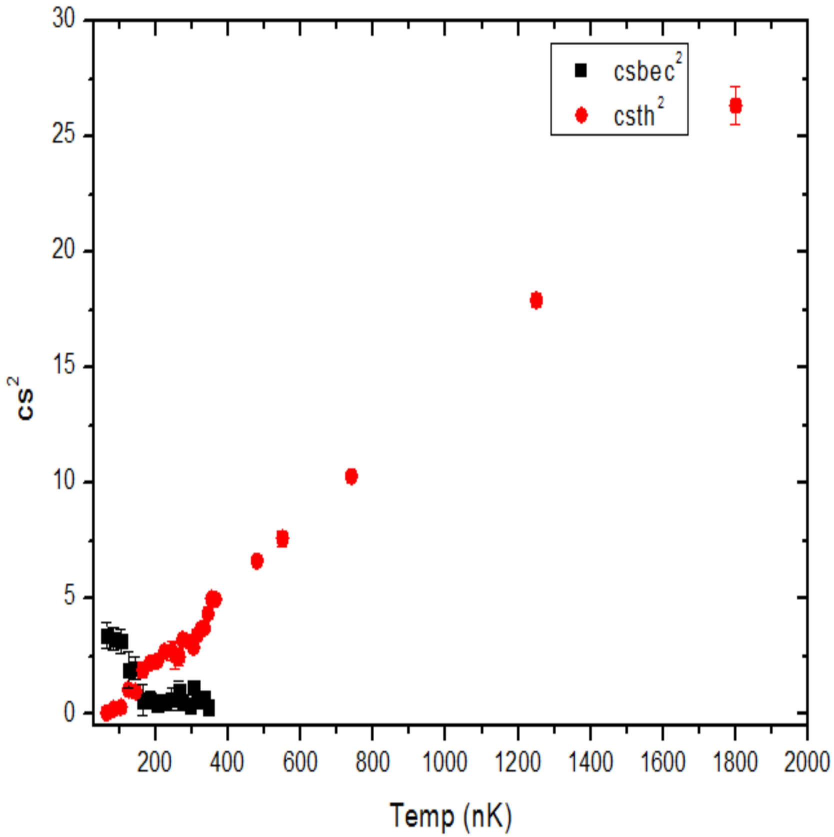


Using *total* number of atoms

Full scale

$$cs_{bec}^2 = \frac{1}{m\omega^3}\left(\frac{\partial \Pi_{bec}}{\partial N_{Tot}}\right)$$

$$cs_{th}^2 = \frac{1}{m\omega^3}\left(\frac{\partial \Pi_{th}}{\partial N_{Tot}}\right)$$

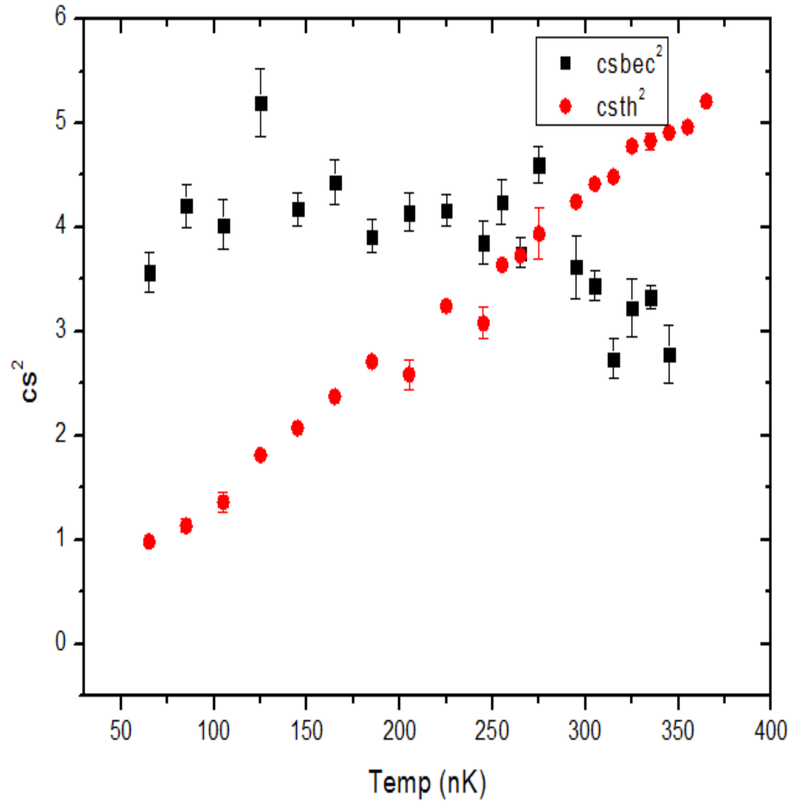


Using *relative* number of atoms

Reduced scale

$$cs_{bec}^2 = \frac{1}{m\omega^3}\left(\frac{\partial\Pi_{bec}}{\partial N_{bec}}\right)$$

$$cs_{th}^2 = \frac{1}{m\omega^3}\left(\frac{\partial\Pi_{th}}{\partial N_{th}}\right)$$

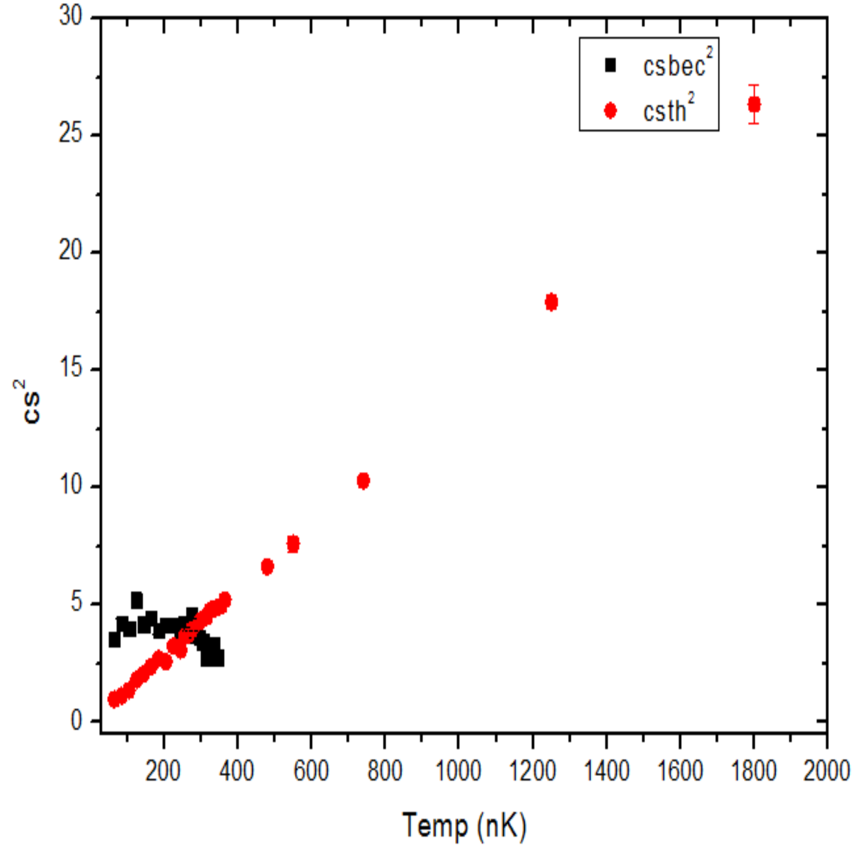


Using *relative* number of atoms

Full scale

$$cs_{bec}^2 = \frac{1}{m\omega^3}\left(\frac{\partial \Pi_{bec}}{\partial N_{Tot}}\right)$$

$$cs_{th}^2 = \frac{1}{m\omega^3}\left(\frac{\partial \Pi_{th}}{\partial N_{Tot}}\right)$$



- *Universalidade:* O fenômeno de diferentes sistemas exibindo o mesmo comportamento critico.

- Exponentes críticos $\alpha, \beta, \gamma, \delta$ são universais.

$C_p = C_{\pm}\|t_r\|^{-\alpha}$	Capacidade térmica (pressão constante)
$k_T = k_{\pm}\|t_r\|^{-\gamma}$	Compressibilidade isotérmica
$\rho_L - \rho_G = \rho_c(-t_r)^{\beta}$	Densidade (Liquido - Gás)
$\beta = \beta_{\pm}\|t_r\|^{-\alpha}$	Coeficiente isobárico de expansão térmica

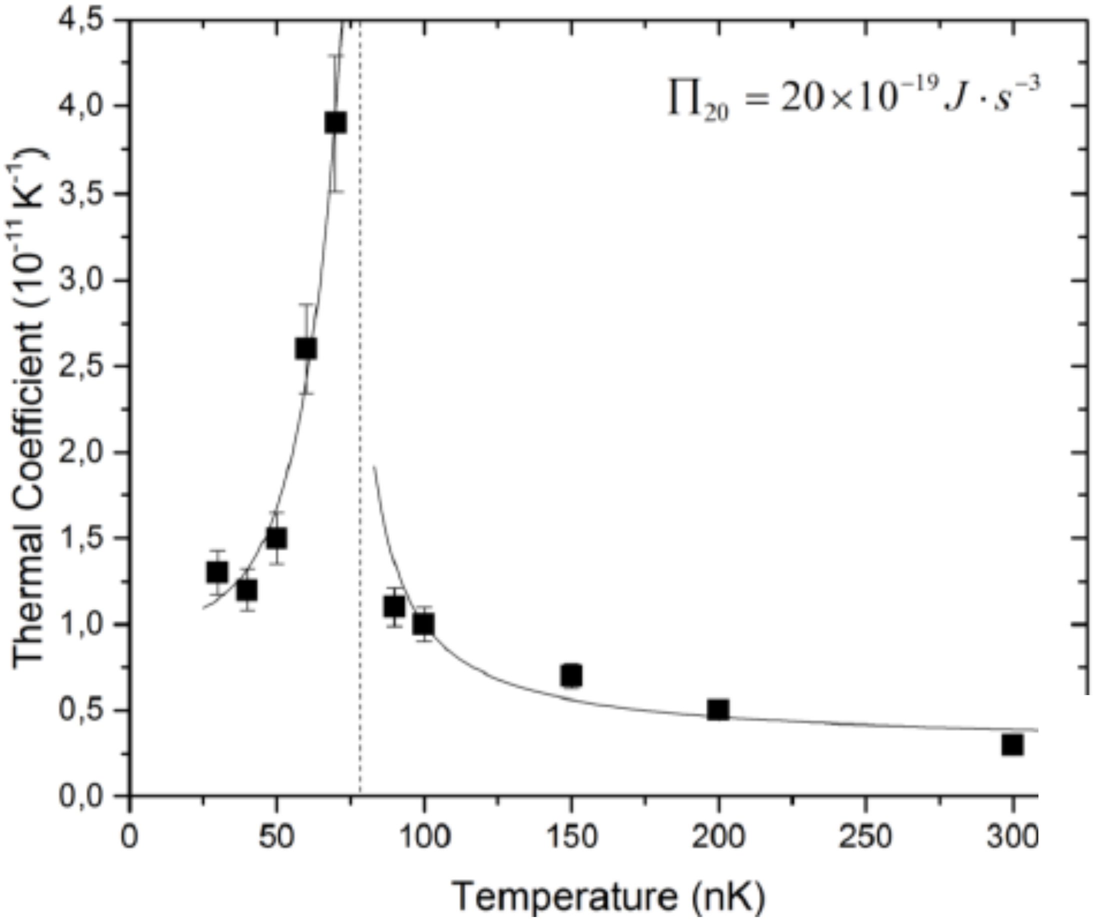
Não todos os exponentes críticos são independentes

$$\alpha + 2\beta + \gamma = 2$$

$$\alpha + \beta(\delta + 1) = 2$$

$$2 - \alpha = d\nu$$

$$2\beta + \gamma = d\nu$$



$$d\nu = 2 - \alpha$$

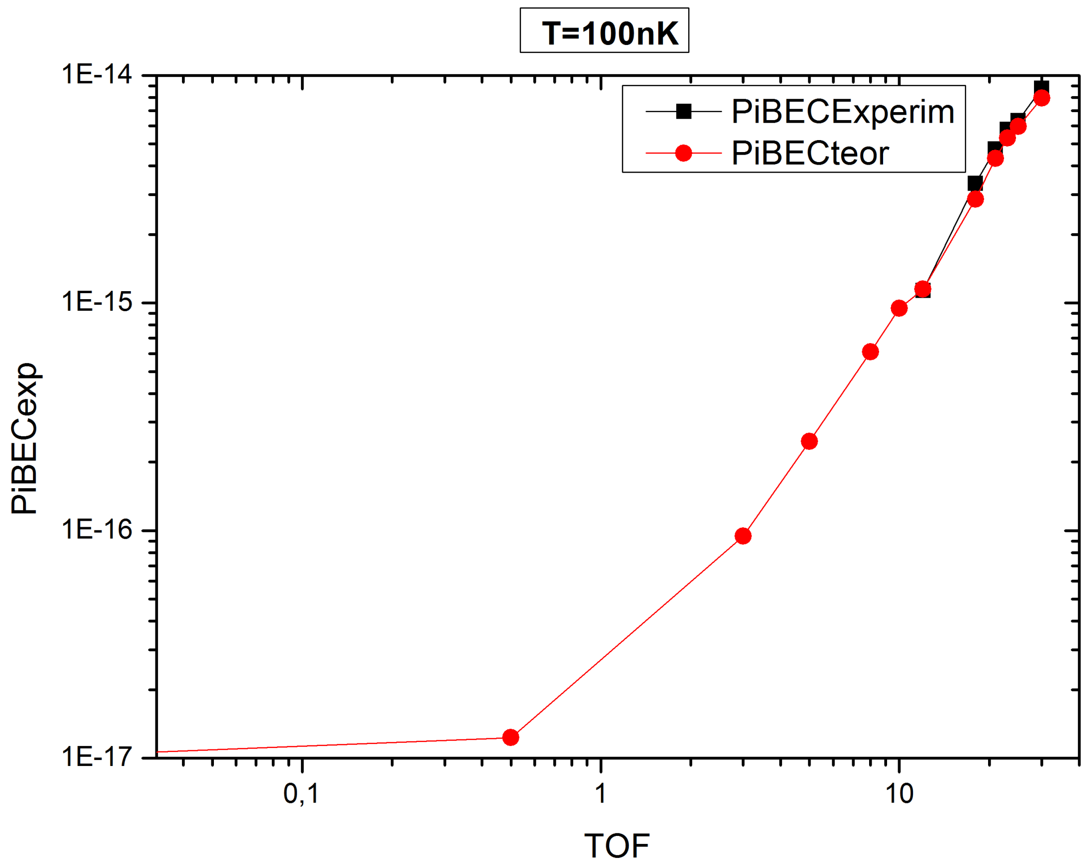
SCIENCE VOL 315 16 MARCH 2007

$$\nu = 0.67 \pm 0.13$$

$$\boxed{d = 2.76 \pm 0.16}$$

$$d \sim 3?$$

Thermodynamics for an expanding cloud



THANKS