São Paulo School of Advanced Science on Nanophotonics (July 18, 2016)

Class 1: introduction to nonlinear optics

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Instituto de Física "Gleb Wataghin"

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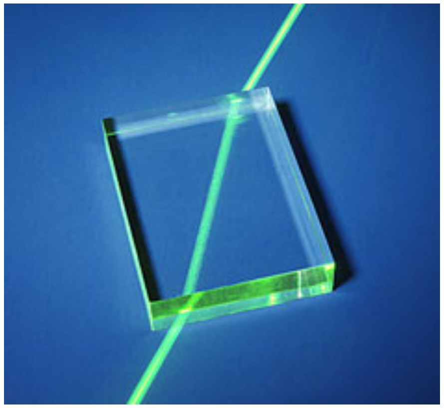
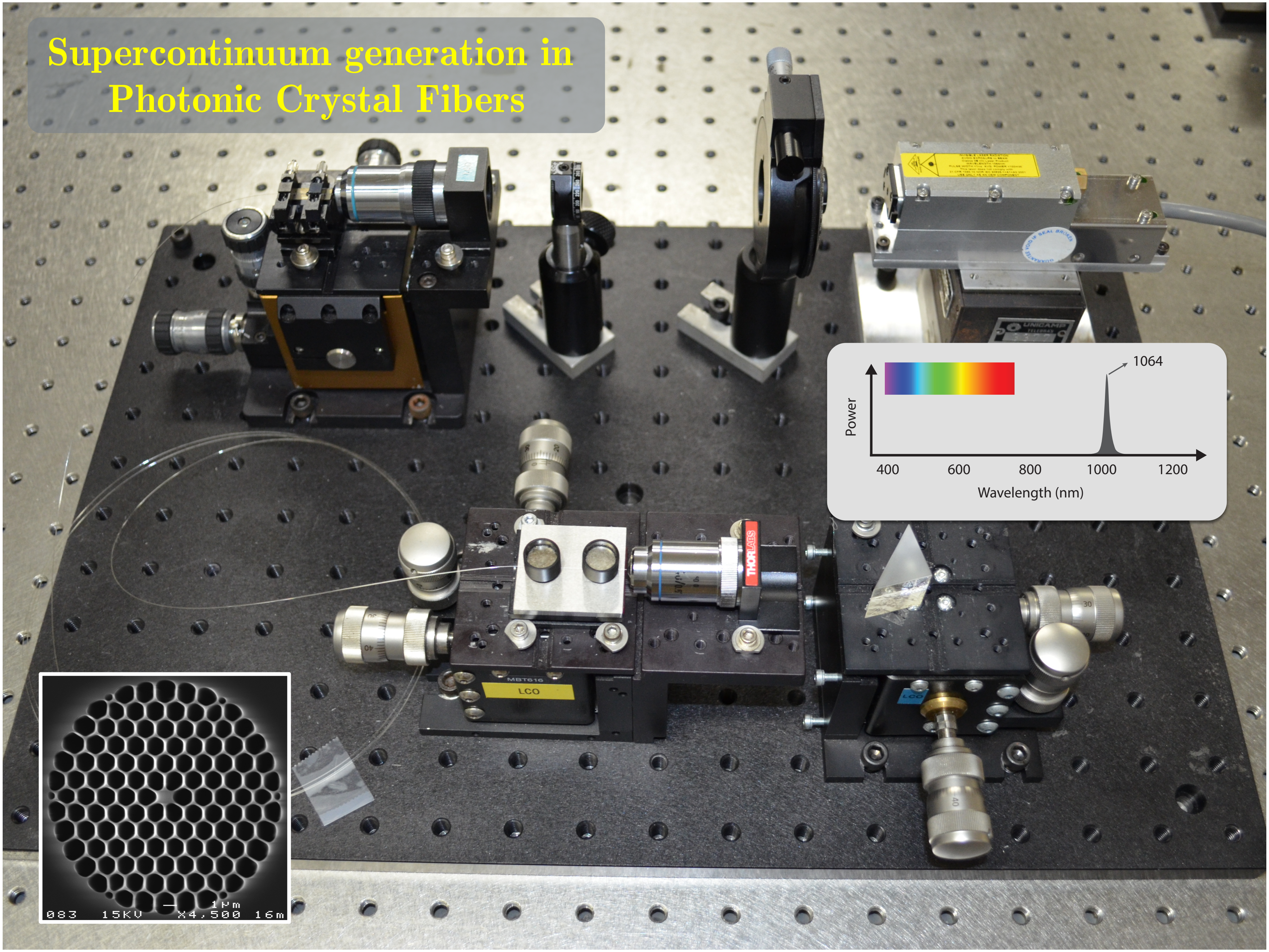
Linear optics

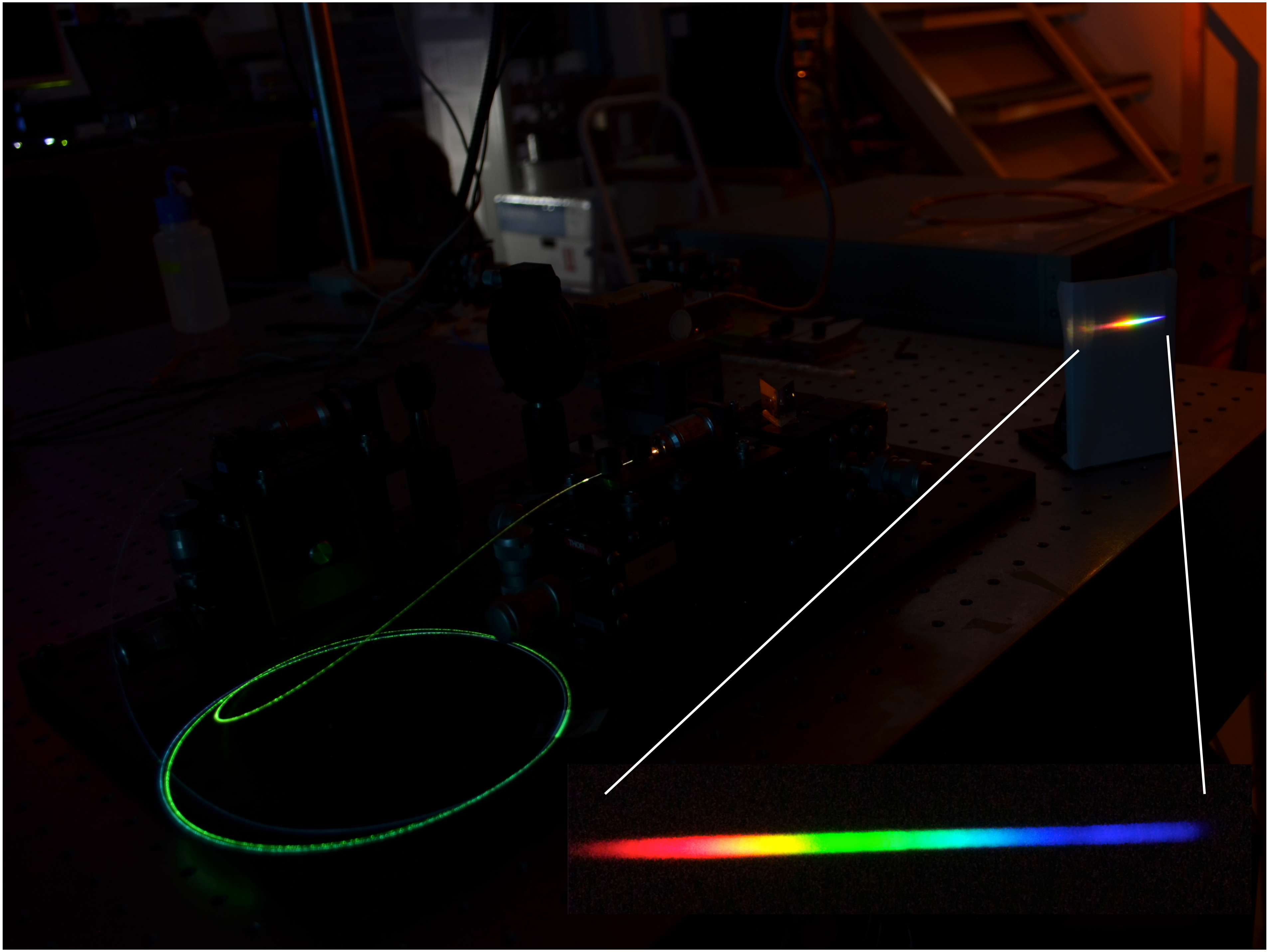


$$\frac{c}{n}$$

the (linear) refractive index

What is "linear" about it?
Where does it come from?


Supercontinuum generation in
Photonic Crystal Fibers
1064
Power
400
600
800
1000
1200
Wavelength (nm)
THORLABS
LCO
083 15KV X4,500 16m
1μm




1064
Power
400
600
800
1000
1200
Wavelength (nm)

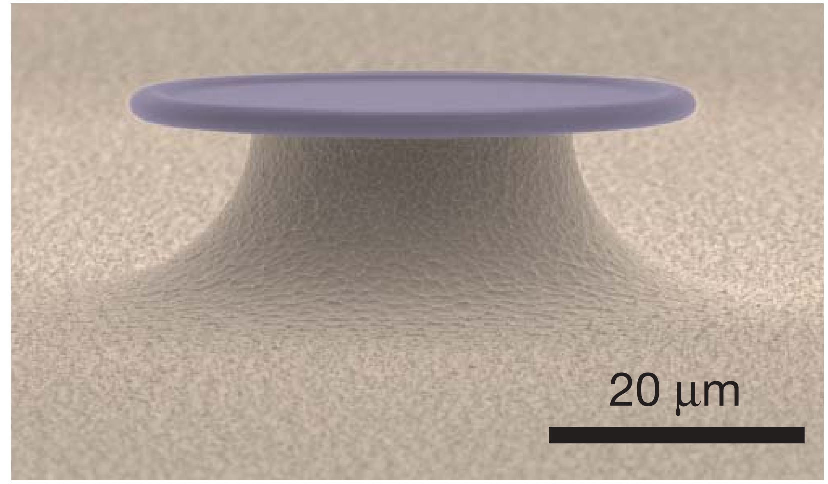
Optical frequency comb generation in a toroid microcavity

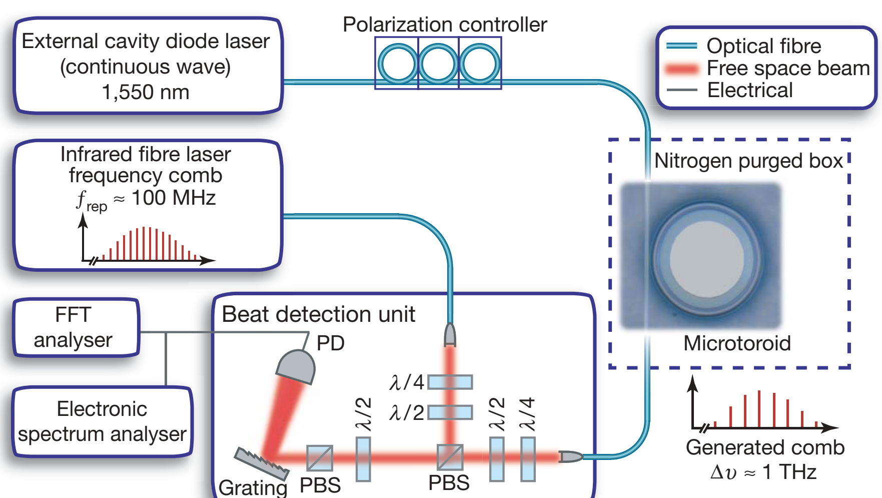
nature Vol 450|20/27 December 2007|doi:10.1038/nature06401

LETTERS

Optical frequency comb generation from a monolithic microresonator

P. Del'Haye[1], A. Schliesser[1], O. Arcizet[1], T. Wilken[1], R. Holzwarth[1] & T. J. Kippenberg[1]





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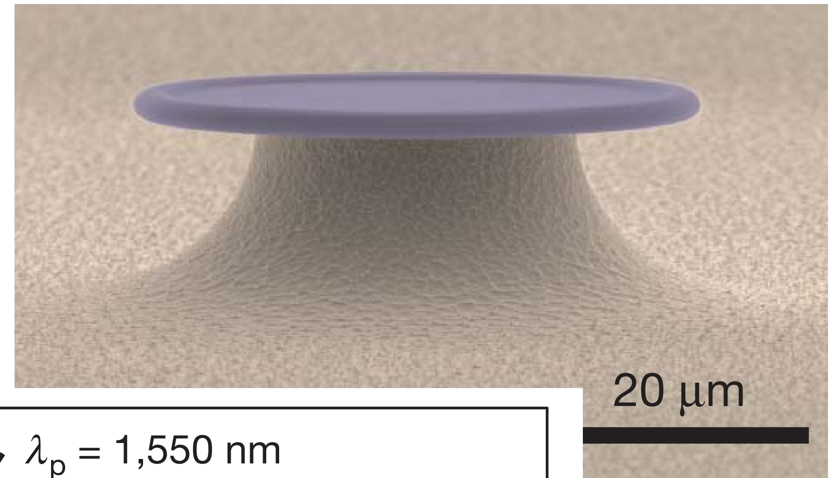
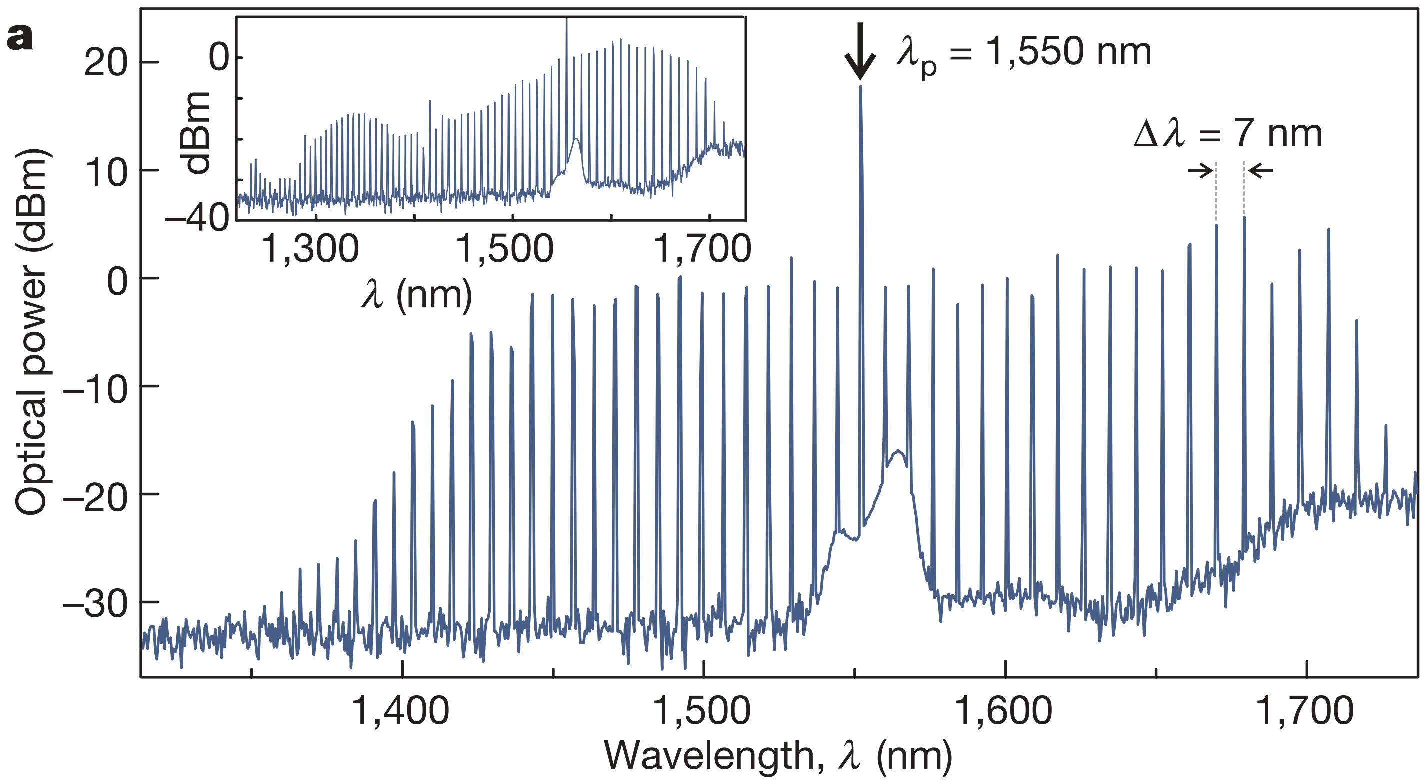
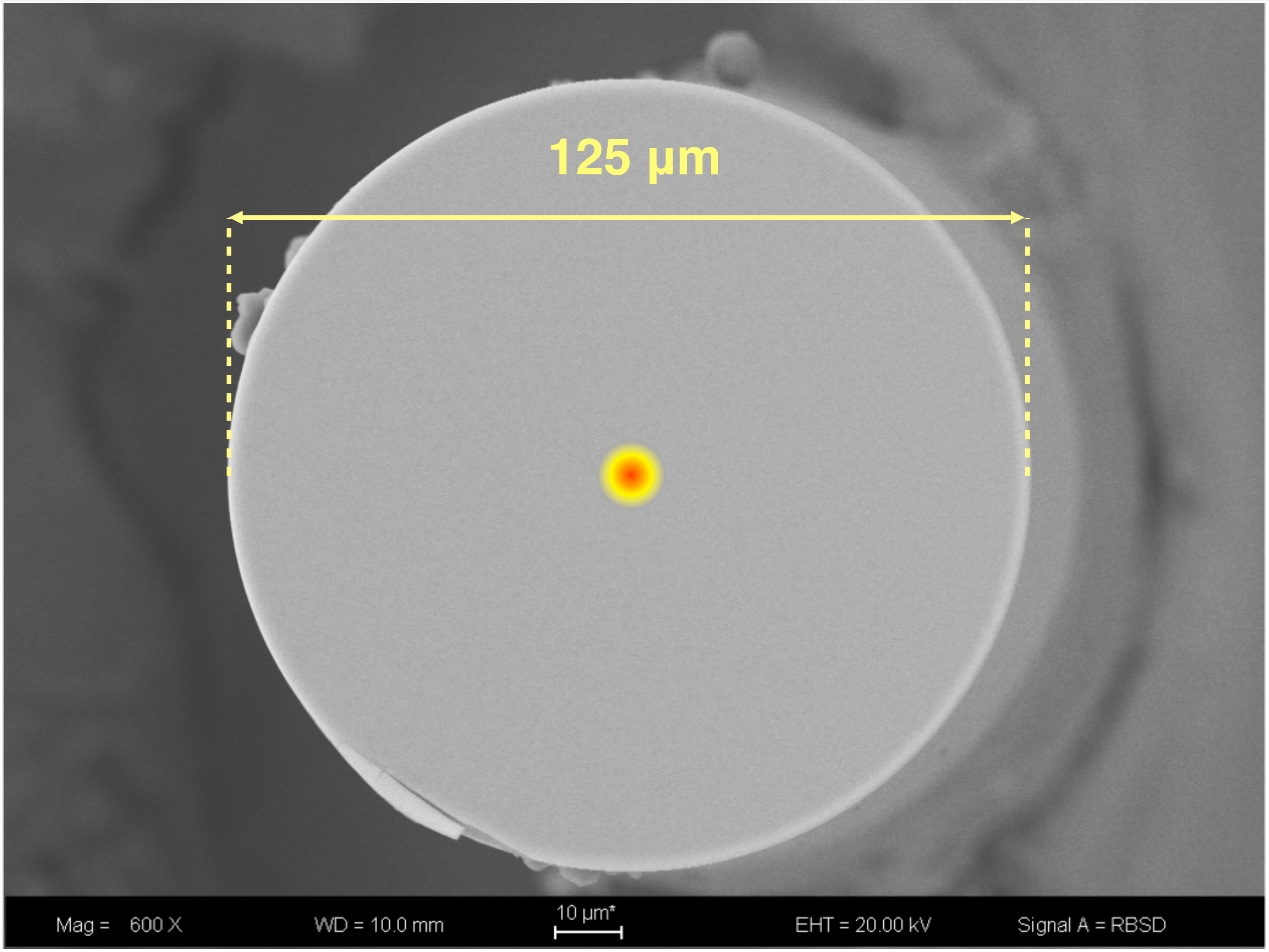
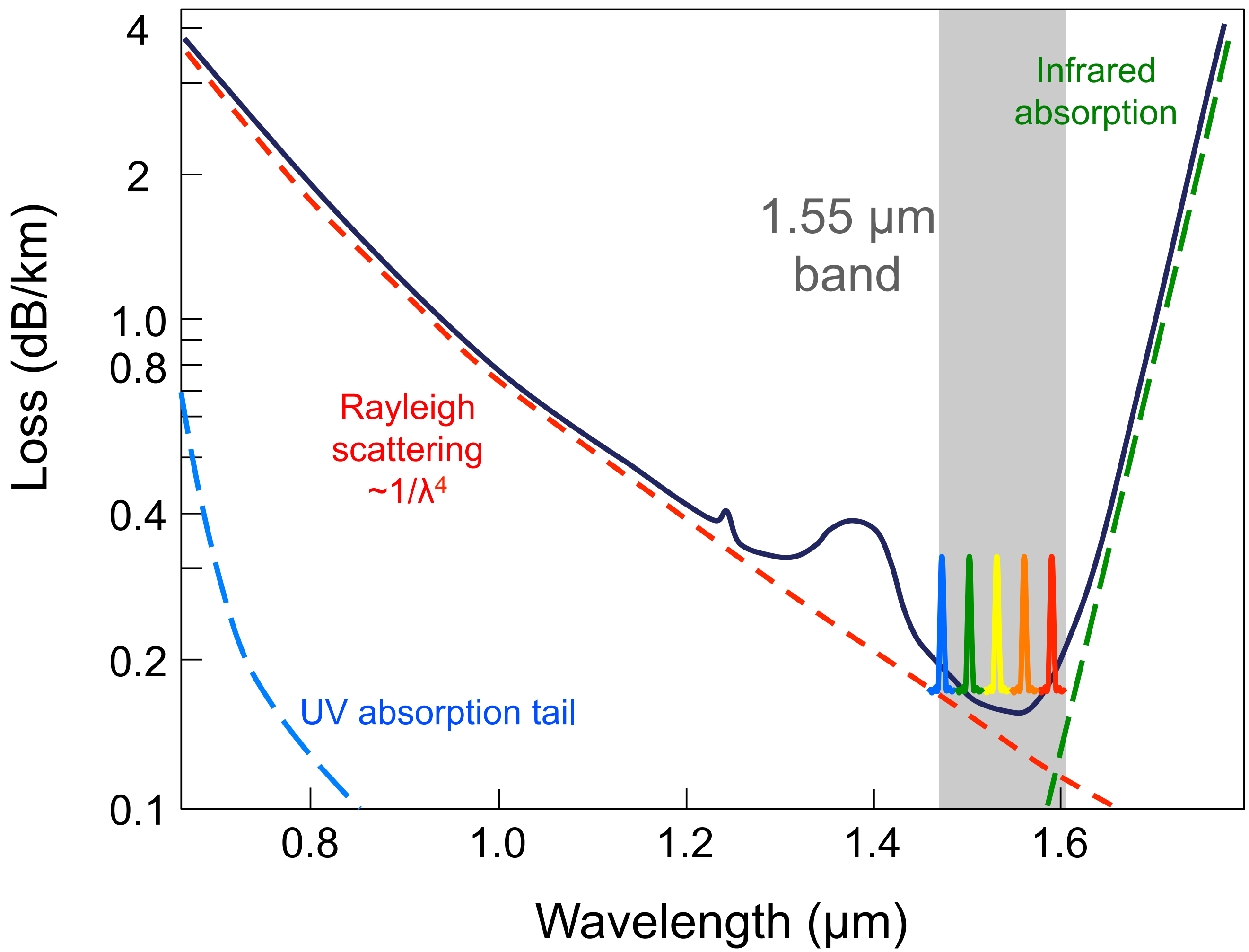




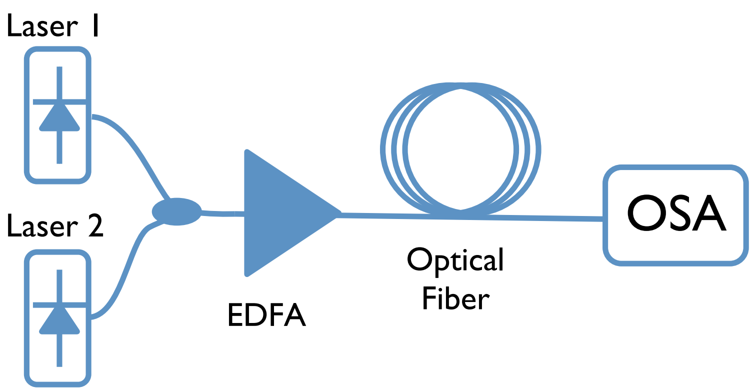
Four-Wave Mixing in Optical Communications Systems

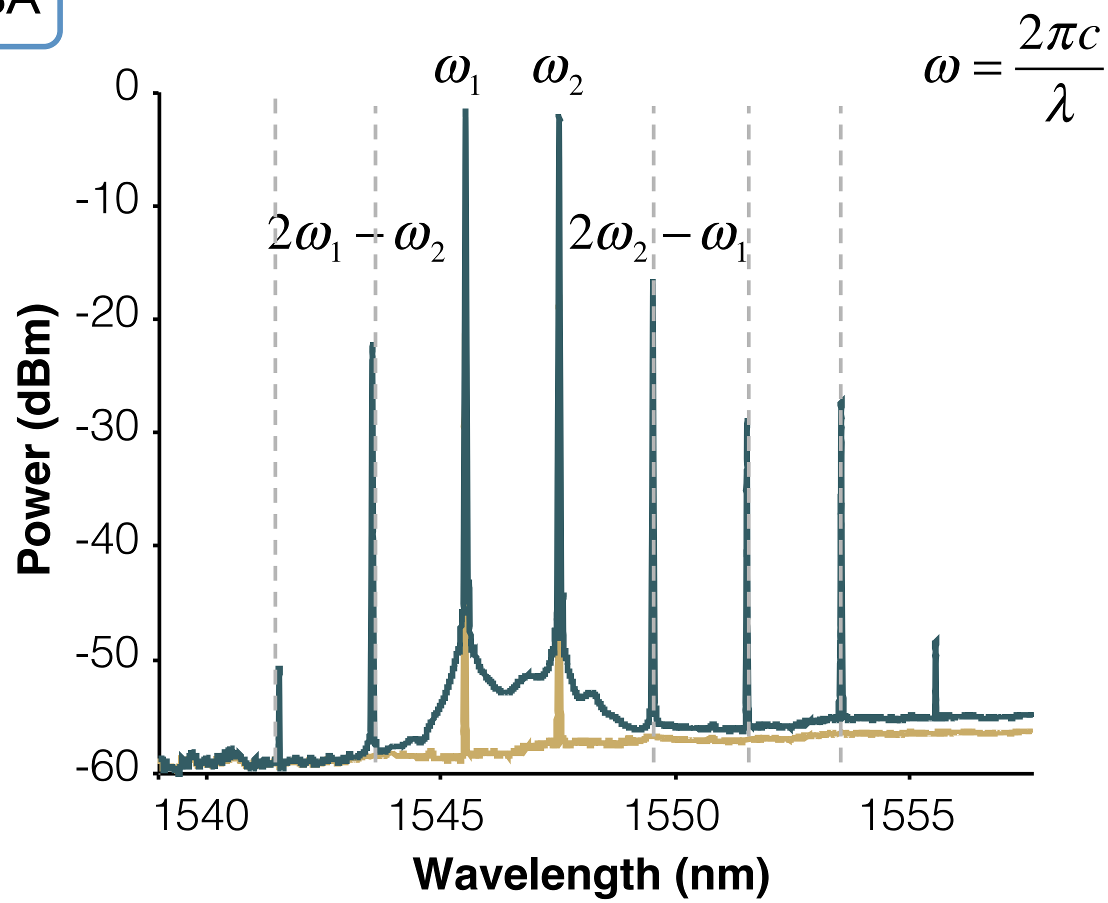


Four-Wave Mixing in Optical Communications Systems



Four-Wave Mixing in Optical Communications Systems



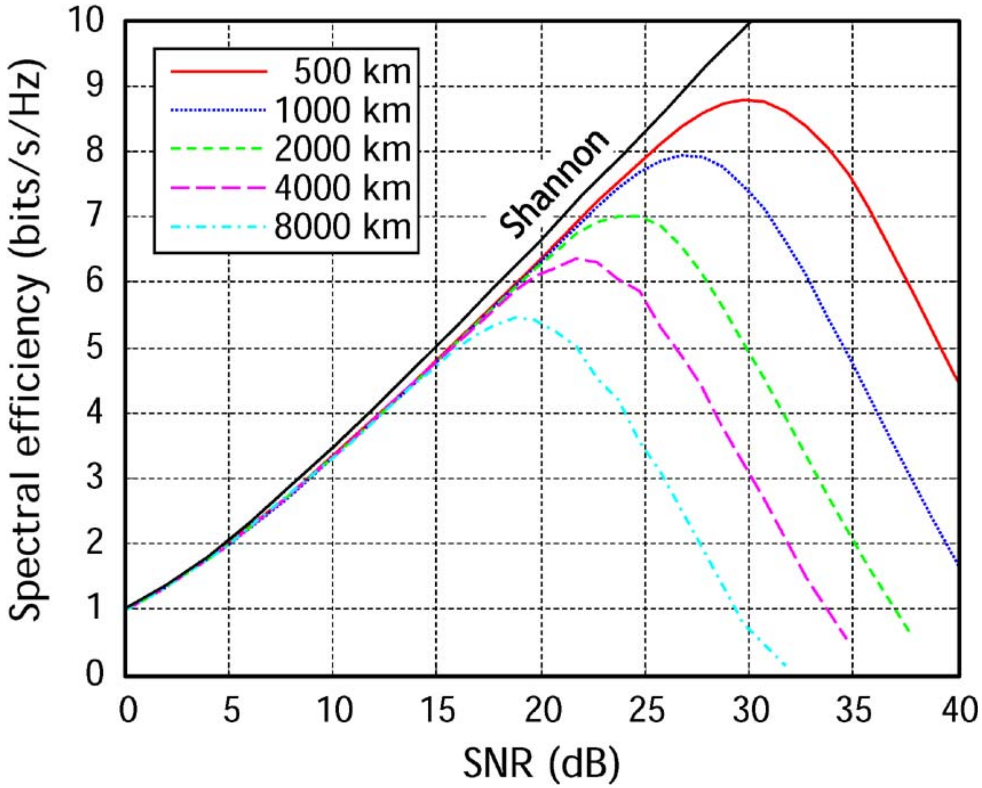


Measurements performed by A. Gil

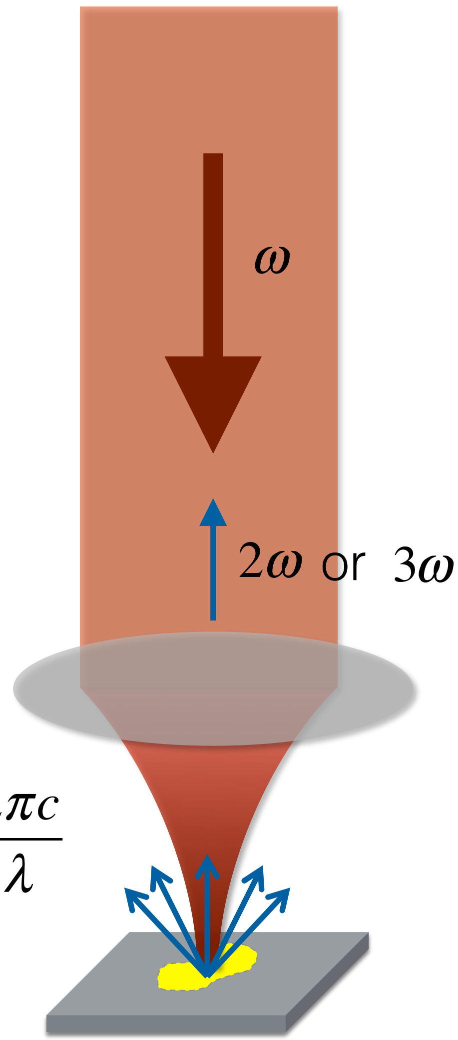
Glass nonlinear response is the fundamental mechanism limiting single-mode fiber capacity

Essiambre, R.-J. and R.W. Tkach, "Capacity Trends and Limits of Optical Communication Networks", Proceedings of the IEEE, 2012

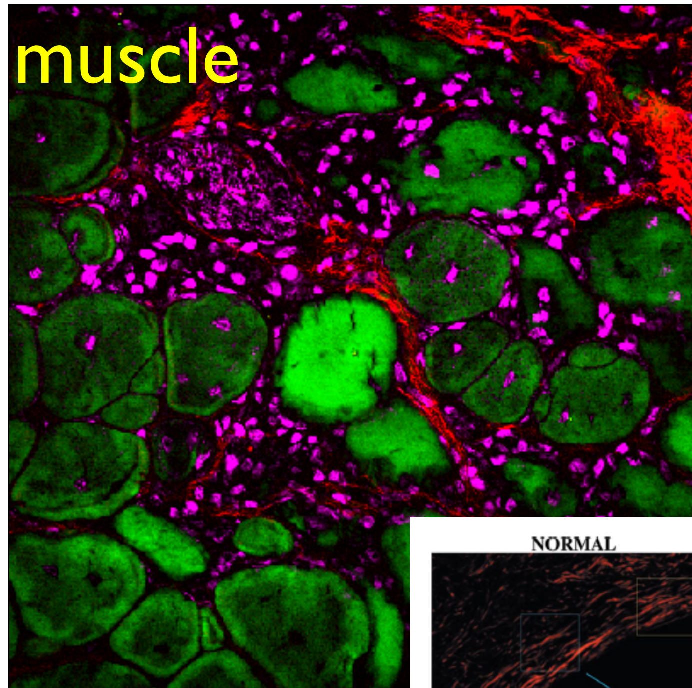
Spectral Efficiency = C/B = $\log_2(1+P/N)$



Second- and Third-harmonic in Biological structures

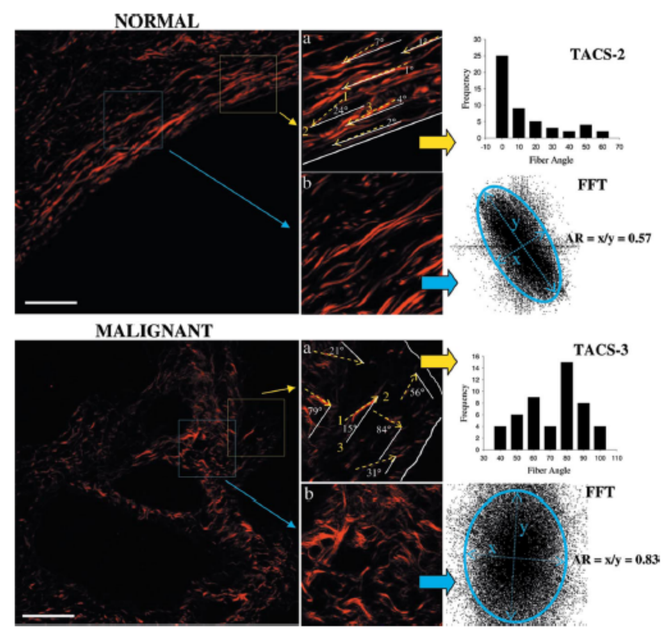


$$\omega = \frac{2\pi c}{\lambda}$$



Green – TPA fluorescence (adipose)
Red – SHG (collagen)
Magenta – THG (nucleus)

SHG



J. Biophotonics 1–13 (2012) / **DOI** 10.1002/jbio.201200108

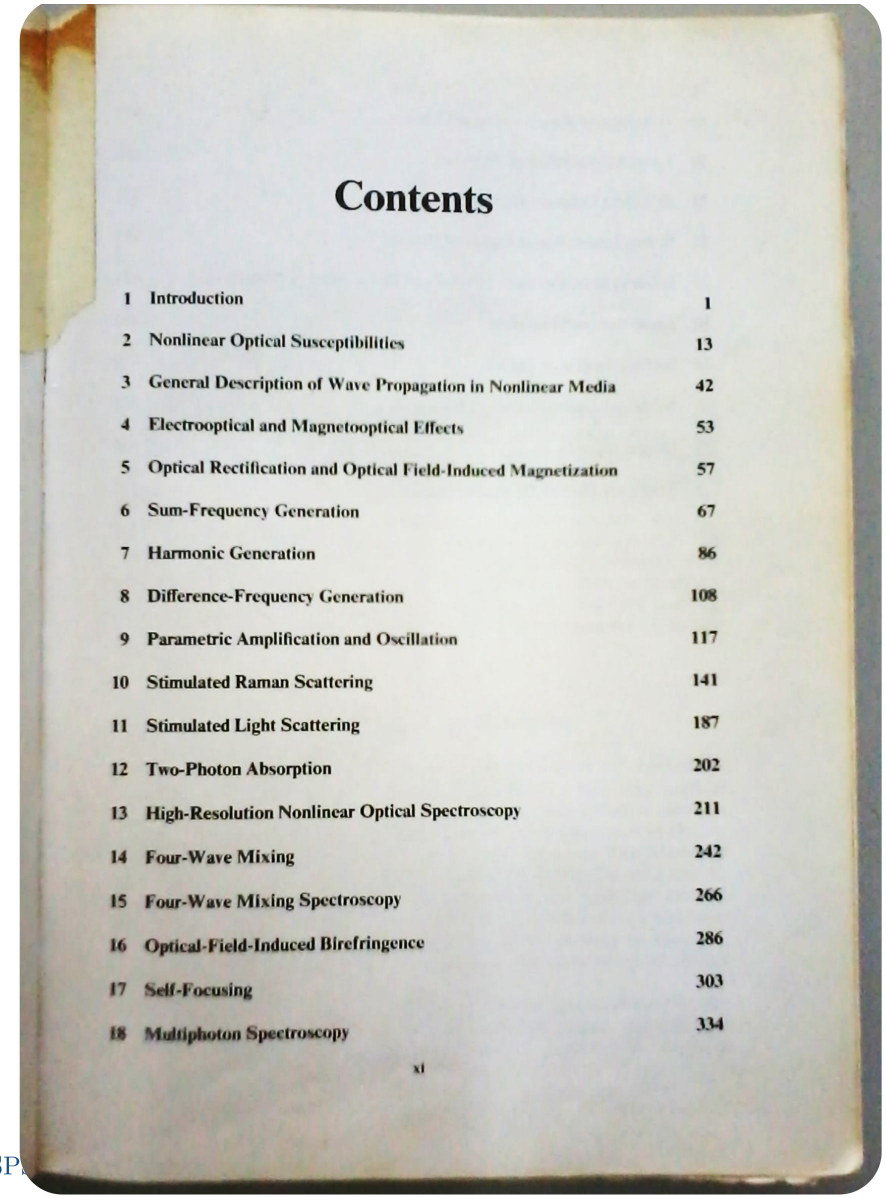
"Physics would be dull and life most unfulfilling if all physical phenomena around us were linear. Fortunately we are living in an nonlinear world…

…the study of nonlinear electromagnetic phenomena in the optical region which normally occur with high-intensity laser beams…

…In the optical region, however, nonlinear optics became a subject of great common interest only after the laser was invented. It has since contributed a great deal to the rejuvenation of the old science of optics".

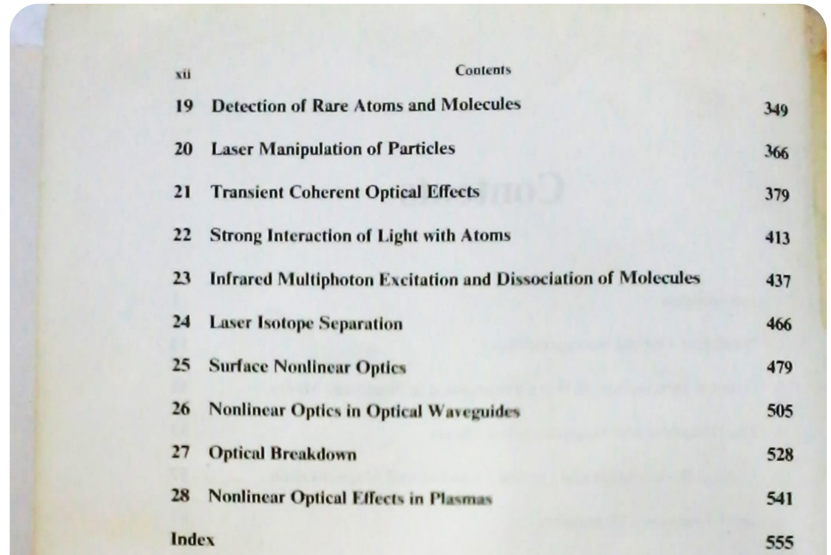
First Paragraph on Page 1
The Principles of Nonlinear Optics, Y. R. Shen

Nonlinear Optics is a broad area

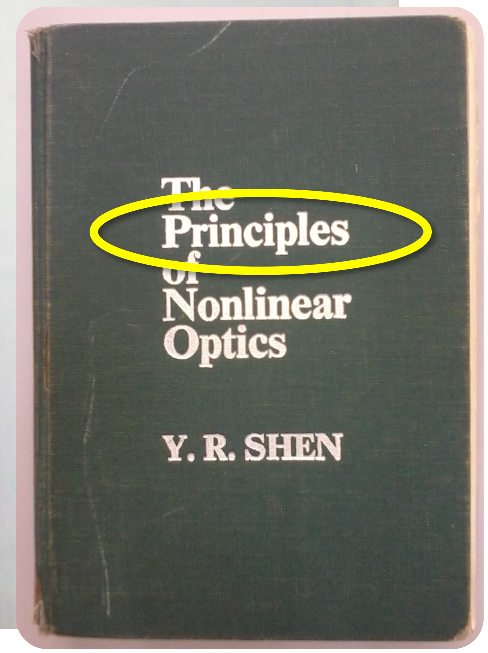


Contents

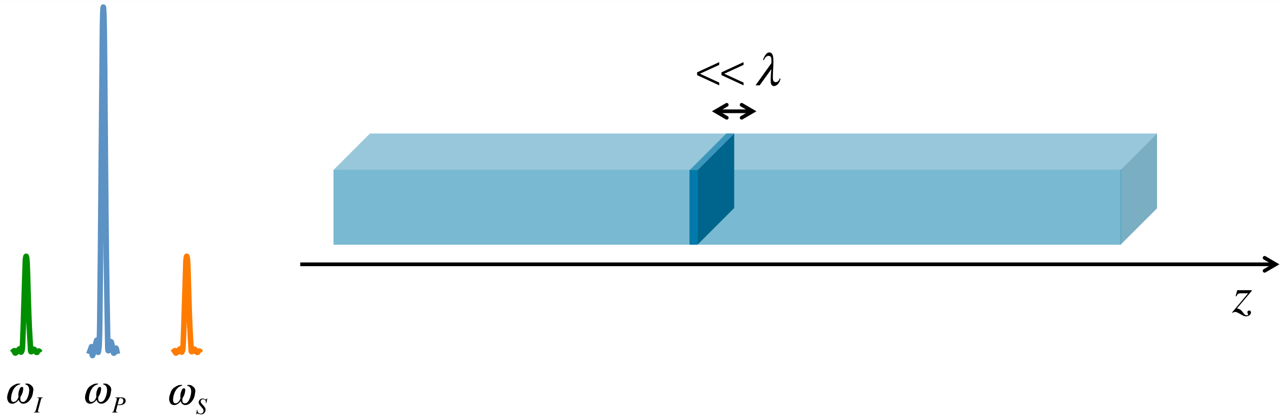
xi



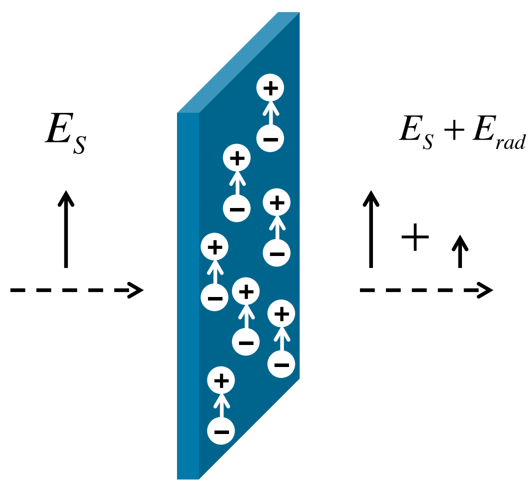
xii Contents



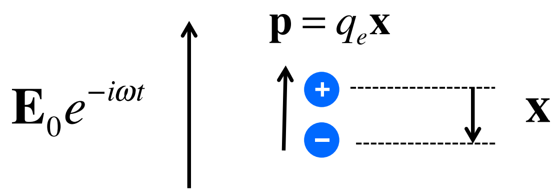
This class



FWM in a waveguide



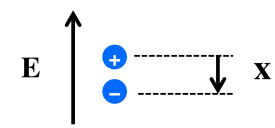
Macroscopic in 2D but "one atom"-thick

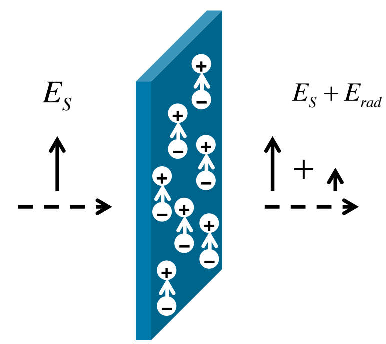


Microscopic (an atom)

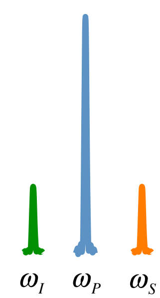
Outline

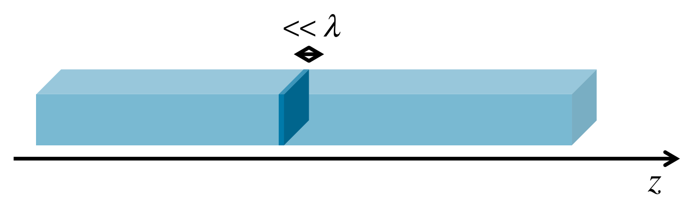
- Origin of (electronic) nonlinearity
  - Lorentz model
  - Anharmonic oscillations
  - Nonlinear polarization





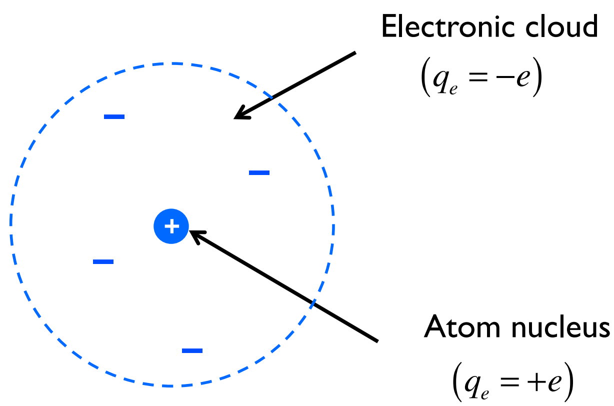
- Maxwell equations in the presence of nonlinearity
  - Wave equation: perturbative solution
  - Self- and Cross-phase modulation
  - Parametric Frequency Mixing

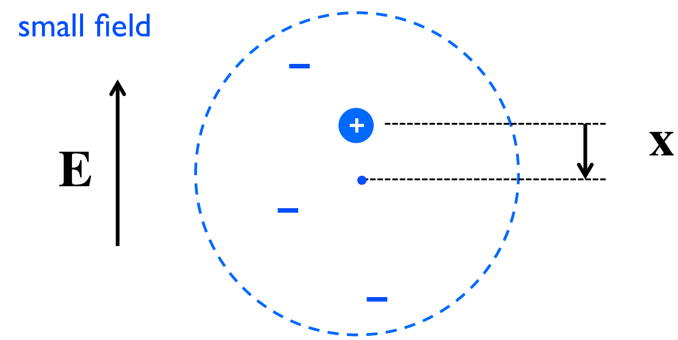




- Examples:
  - Parametrig Gain and Wavelength Conversion
  - Phase-sensitive Amplification
  - Modulation Instability
  - Hands-on: nonlinear coefficient (γ) and second order dispersion (β_2) characterization in optical fibers using Modulation Instability

Lorentz classical oscillator model





Damping term

$$m\left(\partial_t^2 + \gamma\,\partial_t + \omega_0^2\right)\mathbf{x} = q_e\mathbf{E}$$

Applied force

Linear restoring force

$$\mathbf{f} = -k\mathbf{x} = -m\omega_0^2\mathbf{x}$$

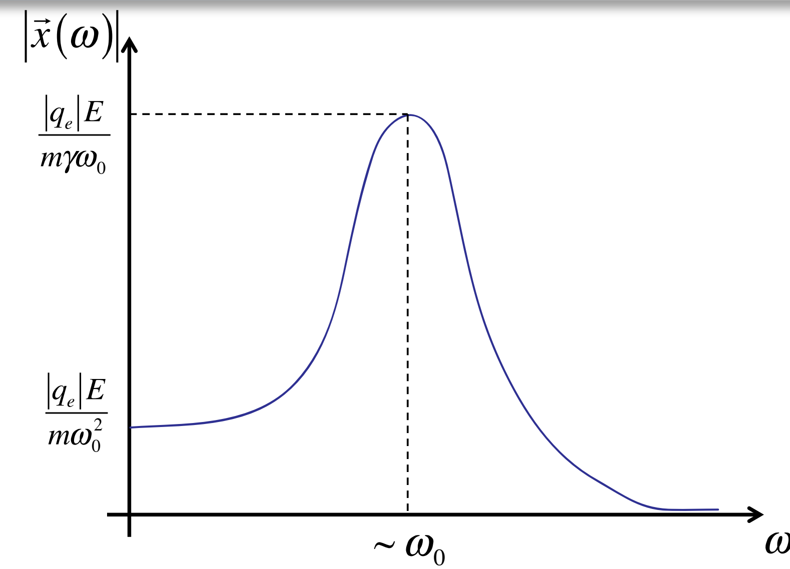
Lorentz classical oscillator model

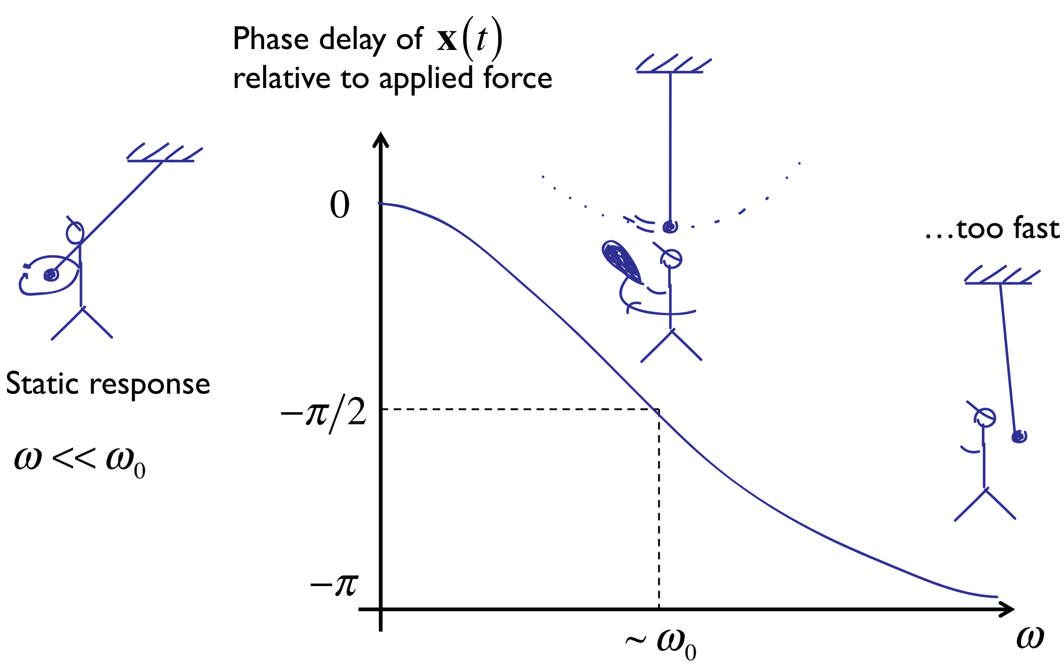
Easily solved in the frequency domain

$$\mathbf{E}(t) = \mathbf{E}_0 e^{-i\omega t}$$

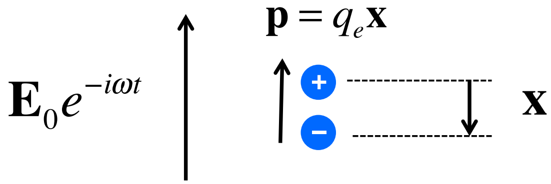
$$\mathbf{x}(t) = \mathbf{x}_0 e^{-i\omega t}$$

$$\mathbf{x}_0 = \frac{q_e \mathbf{E}_0}{m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

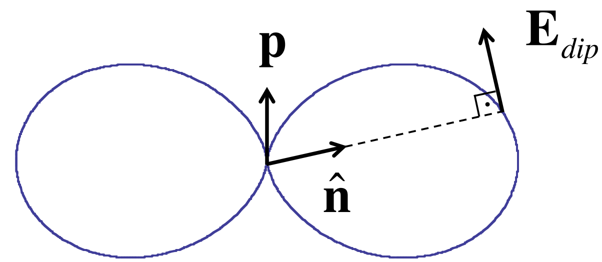




Induced molecular electric dipole



Dipole radiation field (implicit $e^{-i\omega t}$)



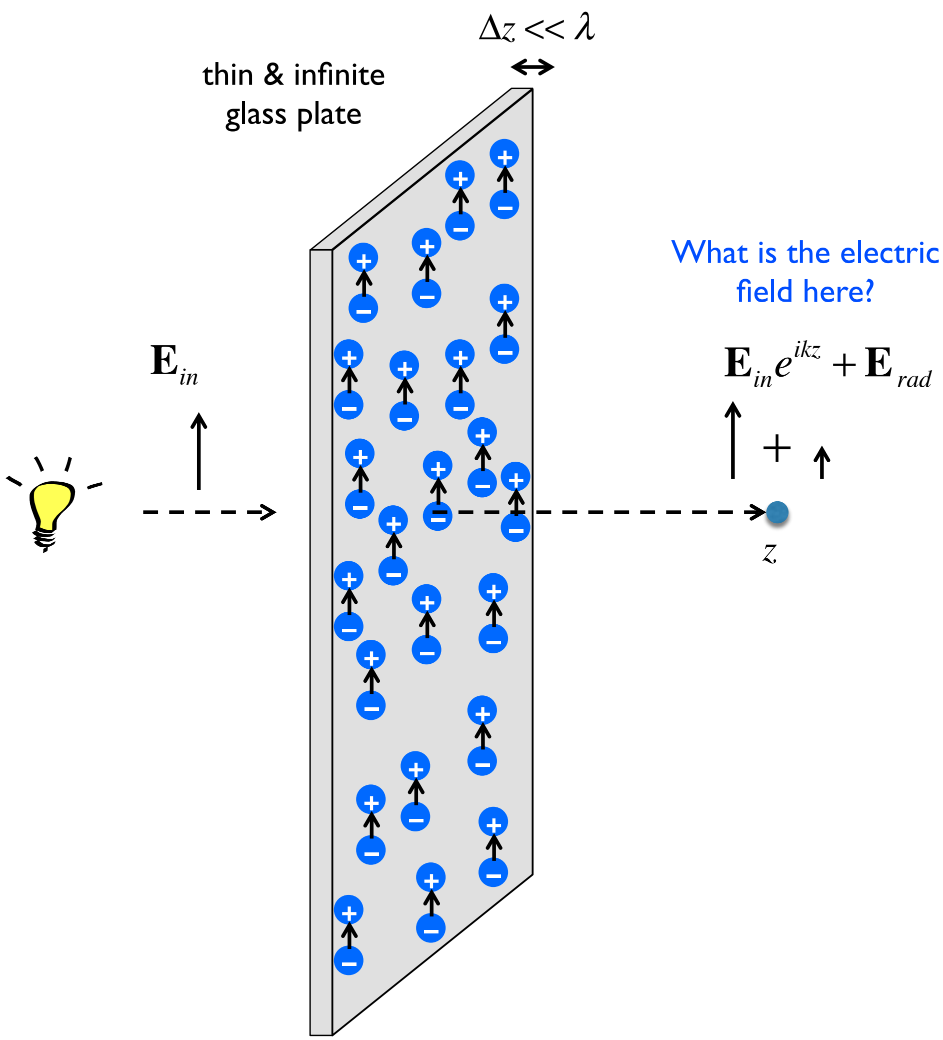
$$\mathbf{p} = \mathbf{p}_0 e^{-i\omega t}$$

$$\mathbf{p}_0 = q_e \mathbf{x}_0 = \frac{q_e^2 \mathbf{E}_0}{m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$\mathbf{E}_{dip} = \frac{k^2}{4\pi\varepsilon_0} [\hat{\mathbf{n}} \times \mathbf{p}] \times \hat{\mathbf{n}} \frac{e^{ikr}}{r}$$

- induced molecular dipole is linear on E
  - oscillates at the same frequency as incident field
  - but with a phase delay

Origin of refractive index



$$\mathbf{E}_{rad} = \sum \mathbf{E}_{dip} = \frac{iN\Delta z k}{2\varepsilon_0}\mathbf{p}e^{ikz}$$

$$= \frac{iN\Delta z k}{2\varepsilon_0}\frac{e^2\,\mathbf{E}_{in}e^{ikz}/m}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$\mathbf{E}_{out} = \mathbf{E}_{in}e^{ikz}\left(1 + \frac{iN\Delta z k}{2\varepsilon_0}\frac{e^2/m}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)$$

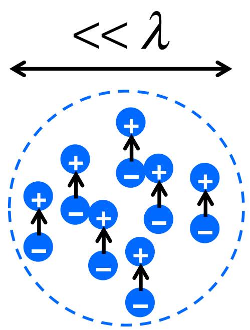
Now, we define the refractive index in the usual manner

$$\mathbf{E}_{out} = \mathbf{E}_{in}e^{ikz}e^{i(n-1)k\Delta z} \approx \mathbf{E}_{in}e^{ikz}\left[1 + i(n-1)k\Delta z\right]$$

$$n = 1 + \frac{1}{2}\frac{N q_e^2/\varepsilon_0 m}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Feynman Lectures (Vol I, chapter 31)

Macroscopic polarization



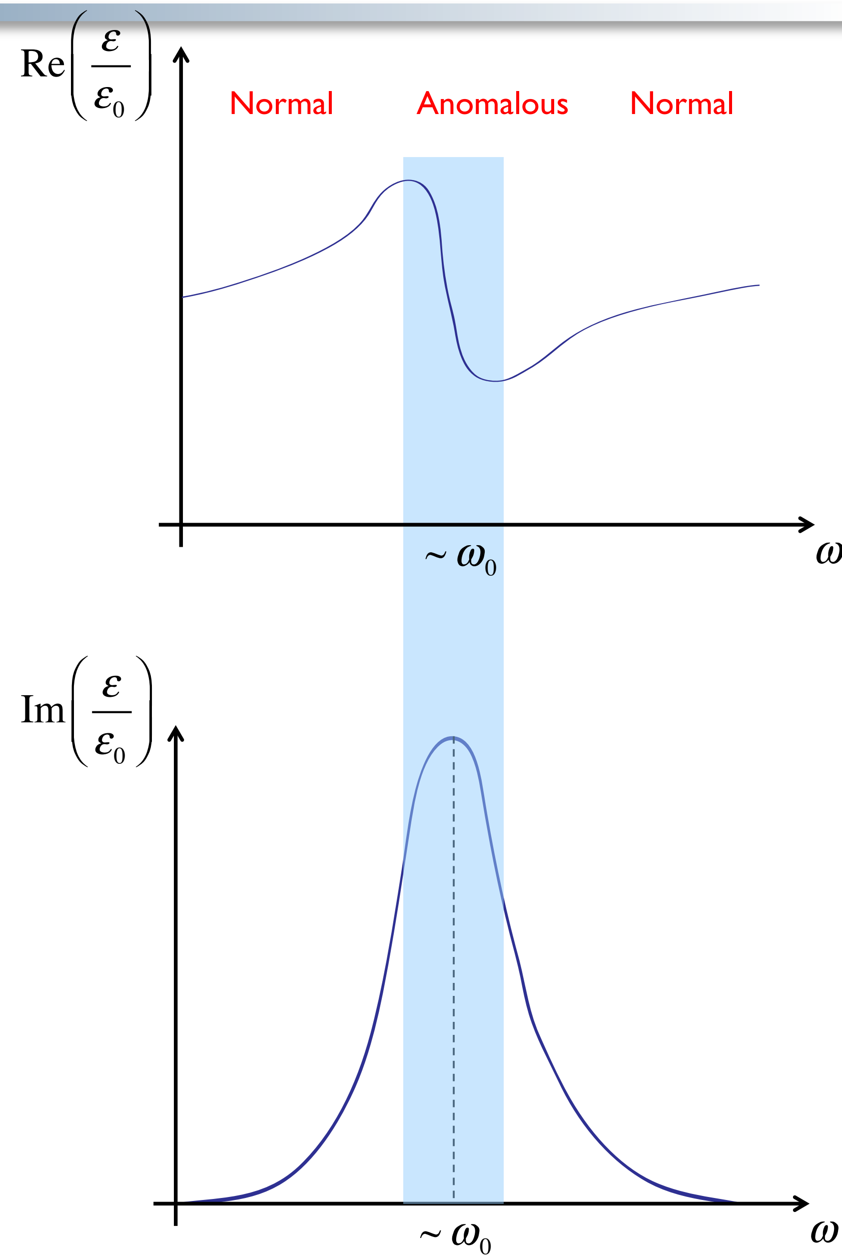
Atoms per unit volume

$$\mathbf{P} = N\,\mathbf{p} = \varepsilon_0 \chi^{(1)} \mathbf{E}$$

First order susceptibility: $$\chi^{(1)} = \frac{N q_e^2 / \varepsilon_0 m}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Electric displacement: $$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

Refractive index: $$n^2 = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi^{(1)} = 1 + \frac{N q_e^2 / \varepsilon_0 m}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

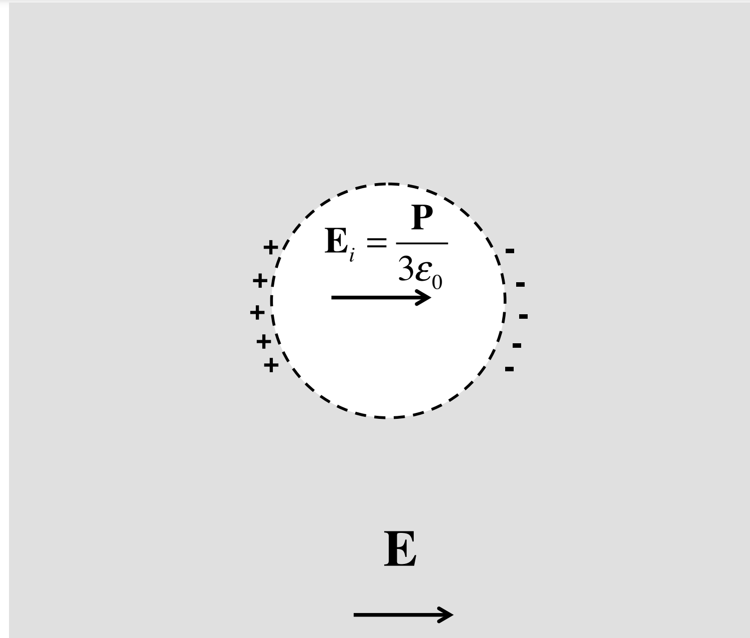


Local field correction

$$\mathbf{P} = N\alpha\mathbf{E}_{local} = N\alpha\left(\mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0}\right)$$

Solving for $\mathbf{P}$ and using that $\mathbf{P} = \varepsilon_0\chi^{(1)}\mathbf{E}$

$$\chi^{(1)} = \frac{N\alpha}{1 - \frac{N\alpha}{3}}$$



Lorentz oscillator model: key learnings

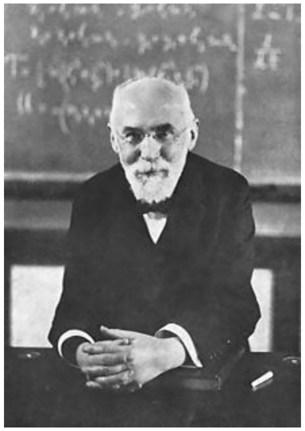
- Electric field induces an electric dipole (linear on E)
- The dipole oscillates at the same frequency as incident field
- Oscillating dipole radiates an electromagnetic wave
- Part of the energy is absorbed by the dipole (mechanical energy)
- Dipole's radiation **interferes** with the original field (refractive index)
- Medium's response:

$$\mathbf{P} = \varepsilon_0 \chi^{(1)} \mathbf{E}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\partial_t \mathbf{B} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \partial_t \mathbf{D} \end{aligned}$$

- …all linear optics is here



Hendrik Antoon Lorentz (18 July 1853 – 4 February 1928) was a Dutch physicist who shared the 1902 Nobel Prize in Physics with Pieter Zeeman for the discovery and theoretical explanation of the Zeeman effect. He also derived the transformation equations subsequently used by Albert Einstein to describe space and time.

What if the restoring force is not linear?

$$P = \varepsilon_0 \chi^{(1)} E$$

$$P = \varepsilon_0 \left(\chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \right) = P^{(1)} + P^{(2)} + P^{(3)} + \ldots$$

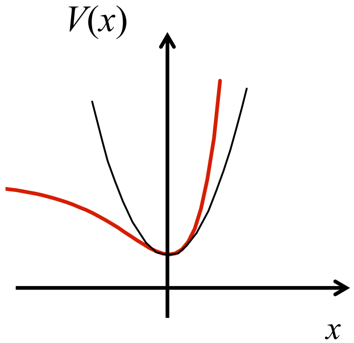
Third order susceptibility

Second order susceptibility

Anharmonic classical oscillator (real potentials)

Non Centro-symmetric

$$V(x) = \frac{m\omega_0^2}{2}x^2 + \frac{ma}{3}x^3$$

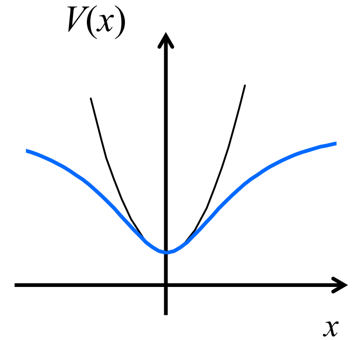


$$f = -m\left(\omega_0^2 x - ax^2\right)$$

$$m\left(\ddot{x} + \gamma\dot{x} + \omega_0^2 x + ax^2\right) = q_e E$$

Centro-symmetric

$$V(x) = \frac{m\omega_0^2}{2}x^2 - \frac{mb}{4}x^4$$



$$f = -m\left(\omega_0^2 x - bx^3\right)$$

$$m\left(\ddot{x} + \gamma\dot{x} + \omega_0^2 x - bx^3\right) = q_e E$$

Rayleigh-Schrödinger perturbation method

Iterative method $x = \lambda x^{(1)} + \lambda^2 x^{(2)} + \ldots$ and $E \to \lambda E$

Non Centro-symmetric

$$\ddot{x}^{(1)} + \gamma \dot{x}^{(1)} + \omega_0^2 x^{(1)} = q_e E/m$$

$$\ddot{x}^{(2)} + \gamma \dot{x}^{(2)} + \omega_0^2 x^{(2)} = -a\left[x^{(1)}\right]^2$$

Centro-symmetric

$$\ddot{x}^{(1)} + \gamma \dot{x}^{(1)} + \omega_0^2 x^{(1)} = q_e E/m$$

$$\ddot{x}^{(3)} + \gamma \dot{x}^{(3)} + \omega_0^2 x^{(3)} = b\left[x^{(1)}\right]^3$$

Example: single-frequency incident field $E(t) = E_\omega e^{-i\omega t} + c.c.$ ➡ $x^{(1)}(t) = x_\omega e^{-i\omega t} + c.c.$

with $x_\omega = \frac{q_e E_\omega}{m} \frac{1}{D_\omega}$ and $D_\omega = \omega_0^2 - \omega^2 - i\gamma\omega$

$$\left[x^{(1)}(t)\right]^2 = |x_\omega|^2 + x_\omega^2 e^{-2i\omega t} + c.c.$$

$$\left[x^{(1)}(t)\right]^3 = 3|x_\omega|^2 x_\omega e^{-i\omega t} + x_\omega^3 e^{-3i\omega t} + c.c.$$

Rayleigh-Schrödinger perturbation method

Non Centro-symmetric $\ddot{x}^{(2)} + \gamma \dot{x}^{(2)} + \omega_0^2 x^{(2)} = -a\left(\left|x_\omega\right|^2 + x_\omega^2 e^{-2i\omega t} + c.c.\right)$

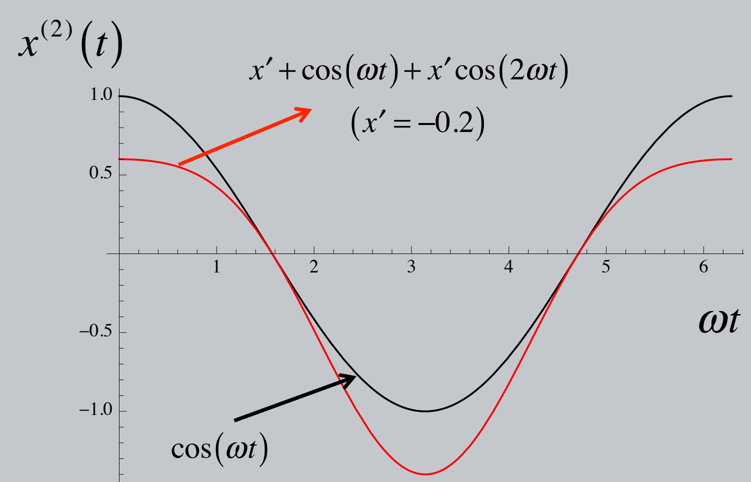
$$x^{(2)}(t) = x_{DC}^{(2)} + x_{2\omega}^{(2)} e^{-i2\omega t} + c.c.$$

$$x_{DC}^{(2)} = \frac{-a\left|x_\omega\right|^2}{D_0}$$

$$x_{2\omega}^{(2)} = \frac{-a x_\omega^2}{D_{2\omega}}$$

Example: single frequency incident field $(\omega << \omega_0)$: $x^{(2)}(t) = \frac{2q_e E}{m\omega_0^2}\left[x' + \cos(\omega t) + x' \cos(2\omega t)\right]$

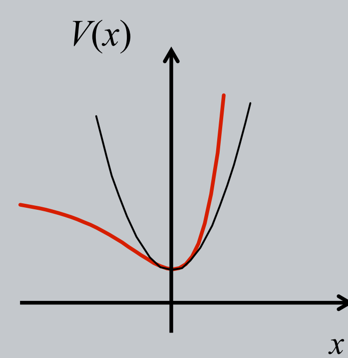
$$x' = -\frac{2a}{\omega_0^2}\left(\frac{q_e E}{m\omega_0^2}\right)$$



Penetrate less into positive x

Center shifted towards negative x

Penetrates deeper into negative x



Rayleigh-Schrödinger perturbation method

Centro-symmetric $\quad \ddot{x}^{(3)} + \gamma \dot{x}^{(3)} + \omega_0^2 x^{(3)} = b\left(3|x_\omega|^2 x_\omega e^{-i\omega t} + x_\omega^3 e^{-3i\omega t} + c.c.\right)$

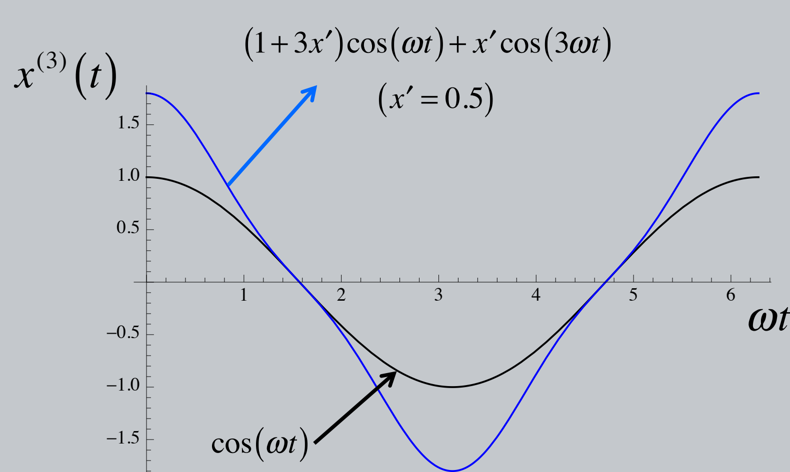
$$x^{(3)}(t) = x_\omega^{(3)} e^{-i\omega t} + x_{3\omega}^{(3)} e^{-i3\omega t} + c.c.$$

$$x_\omega^{(3)} = \frac{3b|x_\omega|^2 x_\omega}{D_\omega}$$

$$x_{3\omega}^{(3)} = \frac{b x_\omega^3}{D_{3\omega}}$$

Example: single frequency incident field $(\omega << \omega_0)$: $\quad x^{(3)}(t) = \frac{2q_e E}{m\omega_0^2}\left[(1+3x')\cos(\omega t) + x'\cos(3\omega t)\right]$

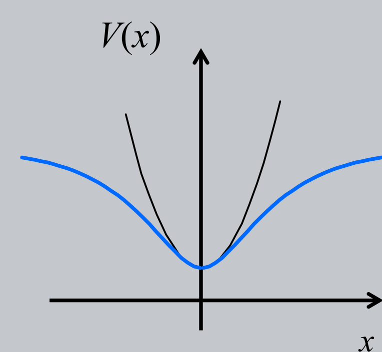
$$x' = \frac{3b}{\omega_0^2}\left(\frac{q_e E}{m\omega_0^2}\right)^2$$



Penetrates deeper into positive x

Centered at x=0!

Penetrates deeper into negative x



Generalization to multiple input frequency

Centro-symmetric $E(t)=E_{\omega_1}e^{-i\omega_1 t}+E_{\omega_2}e^{-i\omega_2 t}+E_{\omega_3}e^{-i\omega_3 t}+c.c.$

$$\ddot{x}^{(1)}+\gamma\dot{x}^{(1)}+\omega_0^2x^{(1)}=q_eE/m$$

$$\ddot{x}^{(3)}+\gamma\dot{x}^{(3)}+\omega_0^2x^{(3)}=b\left[x^{(1)}\right]^3$$

$\Rightarrow$

$$x^{(1)}(t)=\sum_j x_{\omega_j}e^{-i\omega_j t}+c.c. \qquad x_{\omega_j}=\frac{q_eE_{\omega_j}}{m}\frac{1}{D_{\omega_j}}$$

$$x^{(3)}(t)=\sum_j x^{(3)}_{\omega_j}e^{-i\omega_j t}+c.c.$$

$$\begin{aligned}\left[x^{(1)}(t)\right]^3 &= \underline{3}\sum_j\left|x_{\omega_j}\right|^2x_{\omega_j}e^{-i\omega_j t}\\ &\underline{+6}\sum_{i\neq j}\left|x_{\omega_i}\right|^2x_{\omega_j}e^{-i\omega_j t}\\ &+\sum_j x^3_{\omega_j}e^{-3i\omega_j t}\\ &\underline{+3}\sum_{i\neq j}x^2_{\omega_i}x^*_{\omega_j}e^{-i(2\omega_i-\omega_j)t}\\ &\underline{+3}\sum_{i\neq j}x^2_{\omega_i}x_{\omega_j}e^{-i(2\omega_i+\omega_j)t}\\ &\underline{+6}\sum_{i\neq j\neq k}x_{\omega_i}x_{\omega_j}x^*_{\omega_k}e^{-i(\omega_i+\omega_j-\omega_k)t}\\ &\underline{6}\sum_{i\neq j\neq k}x_{\omega_i}x_{\omega_j}x_{\omega_k}e^{-i(\omega_i+\omega_j+\omega_k)t}\\ &+c.c.\end{aligned}$$

degeneracy factor

$$x^{(3)}_{\omega_j}=b\frac{q_e^3}{m^3}\left(\frac{3\left|E_{\omega_j}\right|^2E_{\omega_j}}{D_{\omega_j}D_{\omega_j}D_{\omega_{-j}}D_{\omega_j}}+\sum_{i\neq j}\frac{6\left|E_{\omega_i}\right|^2E_{\omega_j}}{D_{\omega_j}D_{\omega_i}D_{\omega_{-i}}D_{\omega_j}}\right)$$

$$x^{(3)}_{3\omega_j}=b\frac{q_e^3}{m^3}\frac{E^3_{\omega_j}}{D_{3\omega_j}D_{\omega_j}D_{\omega_j}D_{\omega_j}}$$

$$x^{(3)}_{2\omega_i-\omega_j}=b\frac{q_e^3}{m^3}\frac{3E^2_{\omega_i}E^*_{\omega_j}}{D_{2\omega_i-\omega_j}D_{\omega_i}D_{\omega_i}D_{-\omega_j}}$$

$$x^{(3)}_{2\omega_i+\omega_j}=b\frac{q_e^3}{m^3}\frac{3E^2_{\omega_i}E_{\omega_j}}{D_{2\omega_i+\omega_j}D_{\omega_i}D_{\omega_i}D_{\omega_j}}$$

$$x^{(3)}_{\omega_i+\omega_j-\omega_k}=b\frac{q_e^3}{m^3}\frac{6E_{\omega_i}E_{\omega_j}E^*_{\omega_k}}{D_{\omega_i+\omega_j-\omega_k}D_{\omega_i}D_{\omega_j}D_{-\omega_k}}$$

$$x^{(3)}_{\omega_i+\omega_j+\omega_k}=b\frac{q_e^3}{m^3}\frac{6E_{\omega_i}E_{\omega_j}E_{\omega_k}}{D_{\omega_i+\omega_j+\omega_k}D_{\omega_i}D_{\omega_j}D_{\omega_k}}$$

Third order polarization and susceptibility

$$P = \varepsilon_0\left(\chi^{(1)}E + \chi^{(3)}E^3 + \ldots\right) = P^{(1)} + P^{(3)}$$

$$P^{(i)}(t) = Nq_e\sum_j x_{\omega_j}^{(i)}e^{-i\omega_j t} + c.c. = \sum_j P_{\omega_j}^{(i)}e^{-i\omega_j t} + c.c$$

Linear polarization

$$P_{\omega_j}^{(1)} = \varepsilon_0\chi_{\omega_j}^{(1)}E_{\omega_1}$$

$$\chi_{\omega_j}^{(1)} = \frac{Nq_e^2}{m\varepsilon_0}\frac{1}{D_{\omega_j}}$$

Self-phase Modulation (SPM)

Cross-Phase Modulation (XPM)

$$P_{\omega_j}^{(3)} = 3\varepsilon_0\left(\chi_{\omega_j,\omega_j,-\omega_j,\omega_j}^{(3)}\left|E_{\omega_j}\right|^2 + 2\sum_{i\neq j}\chi_{\omega_j,\omega_i,-\omega_i,\omega_j}^{(3)}\left|E_{\omega_i}\right|^2\right)E_{\omega_j}$$

Third Harmonic Generation (THG)

$$P_{3\omega_j}^{(3)} = \varepsilon_0\chi_{3\omega_j,\omega_j,\omega_j,\omega_j}^{(3)}E_{\omega_j}^3$$

Four-Wave Mixing Processes

$$P_{2\omega_i-\omega_j}^{(3)} = 3\varepsilon_0\chi_{2\omega_i-\omega_j,\omega_i,\omega_i,-\omega_j}^{(3)}E_{\omega_i}^2E_{\omega_j}^*$$

$$P_{2\omega_i+\omega_j}^{(3)} = 3\varepsilon_0\chi_{2\omega_i+\omega_j,\omega_i,\omega_i,\omega_j}^{(3)}E_{\omega_i}^2E_{\omega_j}$$

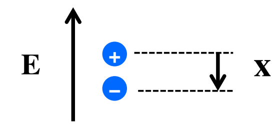
$$P_{\omega_i+\omega_j-\omega_k}^{(3)} = 6\varepsilon_0\chi_{\omega_i+\omega_j-\omega_k,\omega_i,\omega_j,-\omega_k}^{(3)}E_{\omega_i}E_{\omega_j}E_{\omega_k}^*$$

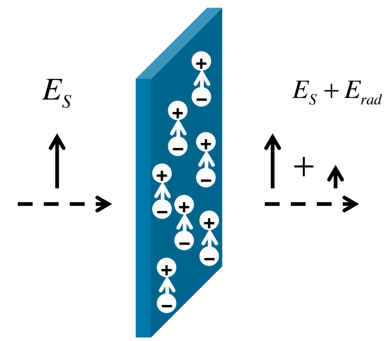
$$P_{\omega_i+\omega_j+\omega_k}^{(3)} = 6\varepsilon_0\chi_{\omega_i+\omega_j+\omega_k,\omega_i,\omega_j,\omega_k}^{(3)}E_{\omega_i}E_{\omega_j}E_{\omega_k}$$

$$\chi_{\omega_i+\omega_j+\omega_k,\omega_i,\omega_j,\omega_k}^{(3)} = \frac{bm\varepsilon_0^3}{N^3q_e^5}\chi_{\omega_i+\omega_j+\omega_k}^{(1)}\chi_{\omega_i}^{(1)}\chi_{\omega_j}^{(1)}\chi_{\omega_k}^{(1)}$$

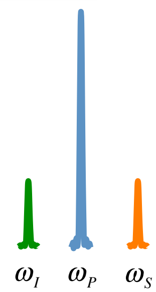
Outline

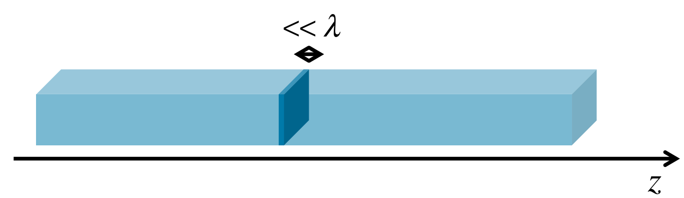
- Origin of (electronic) nonlinearity
  - Lorentz model
  - Anharmonic oscillations
  - Nonlinear polarization





- Maxwell equations in the presence of nonlinearity
  - Wave equation: perturbative solution
  - Self- and Cross-phase modulation
  - Parametric Frequency Mixing





- Examples:
  - Parametrig Gain and Wavelength Conversion
  - Phase-sensitive Amplification
  - Modulation Instability
  - Hands-on: nonlinear coefficient (γ) and second order dispersion (β_2) characterization in optical fibers using Modulation Instability

Maxwell Equations

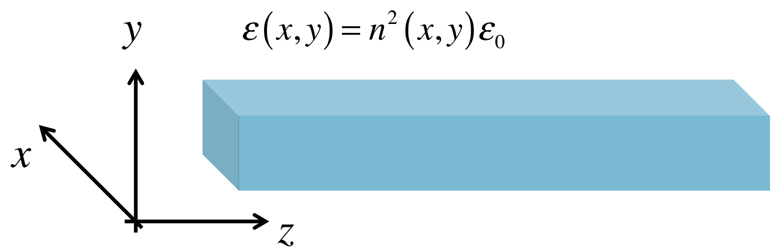
Maxwell Equations in dielectric material

$$\nabla \cdot \mathbf{D} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$
$$\nabla \times \mathbf{H} = \partial_t \mathbf{D}$$

Nonlinear polarization

$$\begin{aligned}\mathbf{D} &= \varepsilon_0 \mathbf{E} + \mathbf{P} \\ &= \varepsilon_0 \mathbf{E} + \mathbf{P}_L + \mathbf{P}_{NL} \\ &= \varepsilon_0 \left(1 + \chi^{(1)}\right) \mathbf{E} + \mathbf{P}_{NL} \\ &= \varepsilon \mathbf{E} + \mathbf{P}_{NL}\end{aligned}$$

$$\varepsilon(x, y) = n^2(x, y)\, \varepsilon_0$$



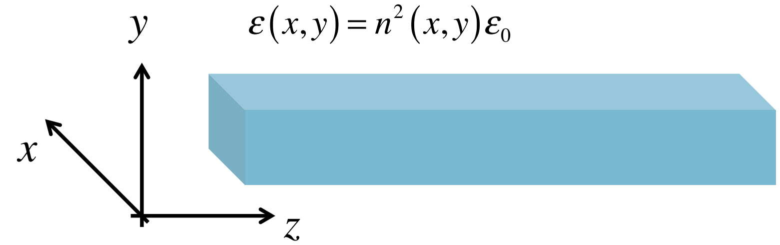
Slowly Varying Envelope Approximation (SVEA)

Wave equation

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \partial_t^2 \mathbf{D}$$

$$\mathbf{D} = \varepsilon \mathbf{E} + \mathbf{P}_{NL}$$

$$\varepsilon(x,y) = n^2(x,y)\varepsilon_0$$



Harmonic expansion

$$\mathbf{P}_{NL} = \sum_j \mathbf{P}_{\omega_j} e^{-i\omega_j t} + c.c$$

$$\mathbf{E}(t) = \sum_j \mathbf{E}_j e^{-i\omega_j t} + c.c. = \sum_j A_j(z)\mathbf{e}_j(x,y) e^{i(\beta_j z - \omega_j t)} + c.c.$$

Eigenmodes ($\mathbf{P}_{NL} = 0$)

$$\mathbf{E}_j = A_j(z)\mathbf{e}_j(x,y)e^{i\beta_j z}$$

Slowly varying envelope

Rapid phase front (wavelength scale)

Slowly Varying Envelope Approximation (SVEA)

Wave equation $$\nabla\times\nabla\times\left[A_j(z)\mathbf{e}_j(x,y)e^{i(\beta_j z-\omega_j t)}\right]=-\mu_0\partial_t^2\left[\varepsilon A_j(z)\mathbf{e}_j(x,y)e^{i(\beta_j z-\omega_j t)}+\mathbf{P}_{\omega_j}e^{-i\omega_j t}\right]$$

$$A_j\nabla\times\nabla\times\left[\mathbf{e}_j e^{i\beta_j z}\right]+2\partial_z A_j\hat{\mathbf{z}}\times\nabla\times\left[\mathbf{e}_j e^{i\beta_j z}\right]+\partial_z^2 A_j\left[\hat{\mathbf{z}}\times\hat{\mathbf{z}}\times\mathbf{e}_j e^{i\beta_j z}\right]=\mu_0\omega_j^2\left[\varepsilon A_j\mathbf{e}_j e^{i\beta_j z}+\mathbf{P}_{\omega_j}\right]$$

SVEA

Eigenmode $\left(\mathbf{P}_{\omega_j}=0\right)$ $$\nabla\times\nabla\times\left[\mathbf{e}_j e^{i\beta_j z}\right]=\varepsilon\mu_0\omega_j^2\mathbf{e}_j e^{i\beta_j z}$$

$$2\partial_z A_j\hat{\mathbf{z}}\times\nabla\times\left[\mathbf{e}_j e^{i\beta_j z}\right]=\mu_0\omega_j^2\mathbf{P}_{\omega_j}$$

Curl equation $$\nabla\times\left[\mathbf{e}_j e^{i\beta_j z}\right]=i\omega_j\mu_0\mathbf{h}_j e^{i\beta_j z}$$

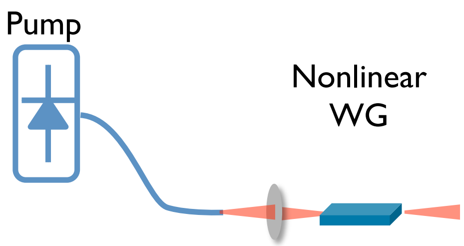
$$2\partial_z A_j\hat{\mathbf{z}}\times\mathbf{h}_j e^{i\beta_j z}=-i\omega_j\mathbf{P}_{\omega_j}$$

Eigenmode normalization $$\mathcal{N}_j=2\int\hat{\mathbf{z}}\cdot\left[\mathbf{e}_j^*\times\mathbf{h}_j\right]da$$

$$\partial_z A_j\left(2\int\mathbf{e}_j^*\cdot\left[\hat{\mathbf{z}}\times\mathbf{h}_j\right]da\right)e^{i\beta_j z}=-i\omega_j\int\mathbf{e}_j^*\cdot\mathbf{P}_{\omega_j}\,da$$

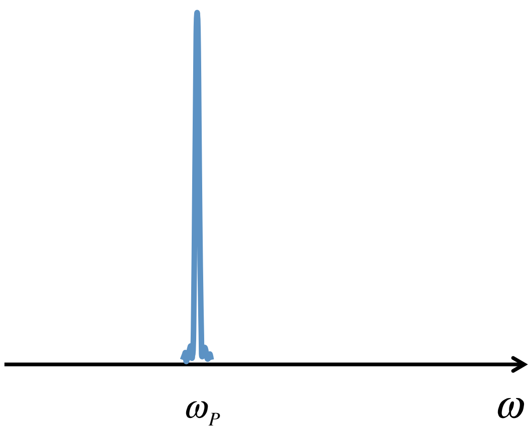
$$\partial_z A_j=\frac{i\omega_j}{\mathcal{N}_j}\left[\int\mathbf{e}_j^*\cdot\mathbf{P}_{\omega_j}\,da\right]e^{-i\beta_j z}$$

Self-Phase Modulation (SPM)



$$\gamma, \beta(\omega)$$

$$E(t) = E_P e^{-i\omega_P t} + c.c.$$



$$\partial_z A_P = \frac{i\omega_P}{\mathcal{N}}\left[\int \mathbf{e}^* \cdot \mathbf{P}_{\omega_P}\, da\right] e^{-i\beta_P z}$$

Nonlinear polarization

$$P_{\omega_P}^{(3)} = 3\varepsilon_0 \chi^{(3)} |E_P|^2 E_P$$

Weakly guidance approximation

$$\mathbf{e} = \hat{\mathbf{x}}\psi(x,y) \quad \mathbf{h} = nc\varepsilon_0 \hat{\mathbf{y}}\psi(x,y)$$

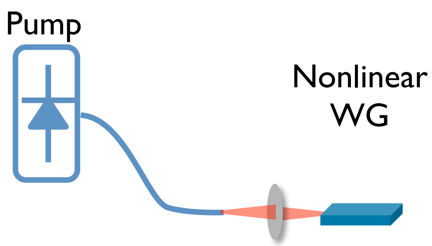
$$\mathcal{N} = 2\varepsilon_0 cn \int \psi^2\, da \qquad \mathcal{P}_P = |A_P|^2 \mathcal{N}$$

$$E_P = A_P(z)\psi(x,y)e^{i\beta_P z}$$

$$\mathbf{e}_P^* \cdot \mathbf{P}_{\omega_P} = 3\varepsilon_0 \chi^{(3)} |A_P|^2 A_P \psi^4 e^{i\beta_P z}$$

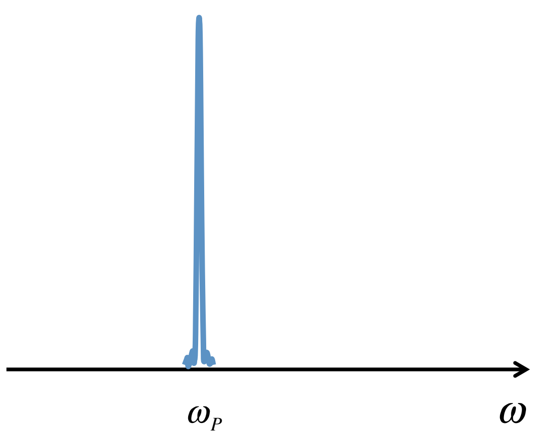
$$\partial_z A_P = i\frac{3\omega_P \varepsilon_0 \chi^{(3)}}{\mathcal{N}}\left[\int \psi^4\, da\right]|A_P|^2 A_P$$

Self-Phase Modulation (SPM)



$$\gamma, \beta(\omega)$$

$$E(t) = E_P e^{-i\omega_P t} + c.c.$$



$$\partial_z A_P = i\frac{3\omega_P \varepsilon_0 \chi^{(3)}}{\mathcal{N}}\left[\int \psi^4 \, da\right]|A_P|^2 A_P$$

$$\partial_z A_P = i\gamma \mathcal{P}_P A_P$$

$$\gamma_P = \frac{3\omega_P \chi^{(3)}}{4\varepsilon_0 c^2 n^2 A_{eff}} \qquad A_{eff} = \frac{\left[\int \psi^2 \, da\right]^2}{\int \psi^4 \, da}$$

$$|A_P|^2 = A_P A_P^*$$

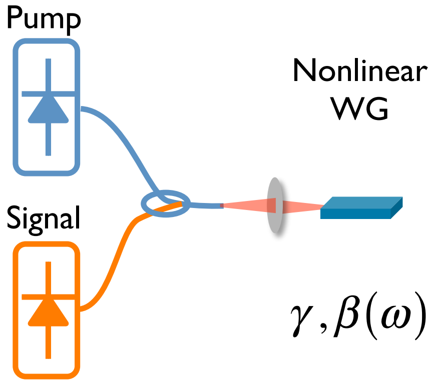
$$\partial_z |A_P|^2 = A_P^* \partial_z A_P + A_P \partial_z A_P^*$$

$$= A_P^*\left(i\gamma \mathcal{P}_P A_P\right) + A_P\left(-i\gamma \mathcal{P}_P A_P^*\right) = 0$$

$$A_P = A_P e^{i\gamma \mathcal{P}_P z}$$

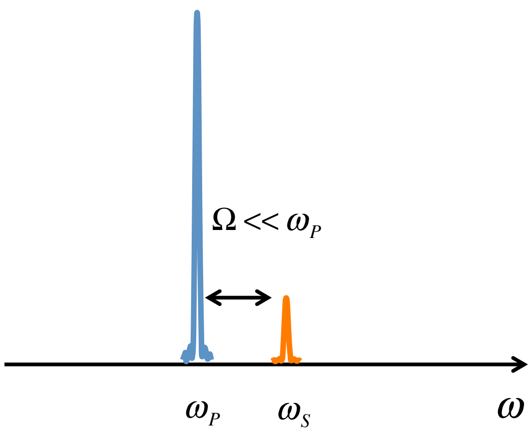
$$E_P = A_P(z)\psi(x,y)e^{i(\beta_P + \gamma \mathcal{P}_P)z}$$

Cross-Phase Modulation (XPM)



$$\gamma, \beta(\omega)$$

$$E(t) = E_P e^{-i\omega_P t} + E_S e^{-i\omega_S t} + c.c.$$



$$\partial_z A_{P,S} = \frac{i\omega_{P,S}}{\mathcal{N}} \left[\int \mathbf{e}^* \cdot \mathbf{P}_{\omega_{P,S}} \, da \right] e^{-i\beta_{P,S} z}$$

$$E_j = A_j(z)\psi(x,y)e^{i\beta_j z}$$

$$\mathcal{P}_j = |A_j|^2 \mathcal{N}$$

Nonlinear polarization

$$P^{(3)}_{\omega_P} = 3\varepsilon_0 \chi^{(3)} \left(|E_P|^2 + 2|E_S|^2 \right) E_P$$

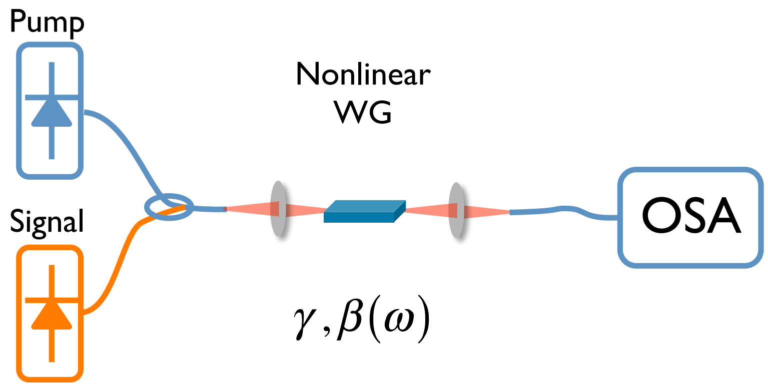
$$P^{(3)}_{\omega_S} = 3\varepsilon_0 \chi^{(3)} \left(|E_S|^2 + 2|E_P|^2 \right) E_S$$

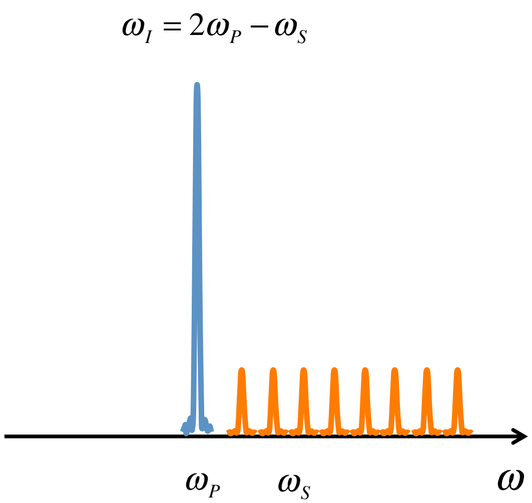
$$\partial_z A_P = i\gamma(\mathcal{P}_P + 2\mathcal{P}_S) A_P$$
$$\partial_z A_S = i\gamma(\mathcal{P}_S + 2\mathcal{P}_P) A_S$$

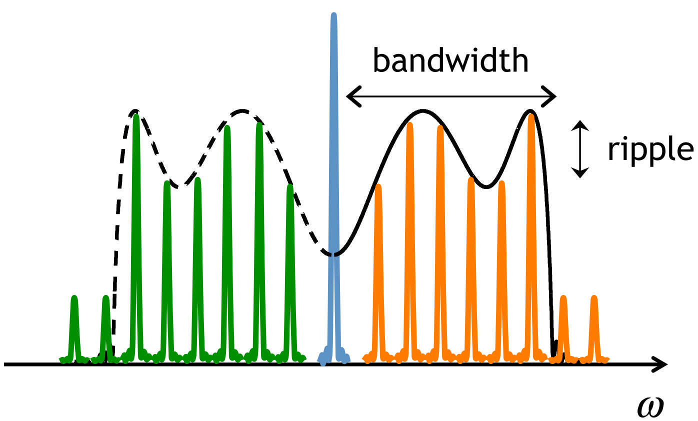
$$\gamma_P \approx \gamma_I$$

$$E_{P,S} = A_{P,S}(0)\psi(x,y)e^{i\left[\beta_{P,S} + \gamma(\mathcal{P}_{P,S} + 2\mathcal{P}_{S,P})\right]z}$$

Four-Wave Mixing (degenerate case)

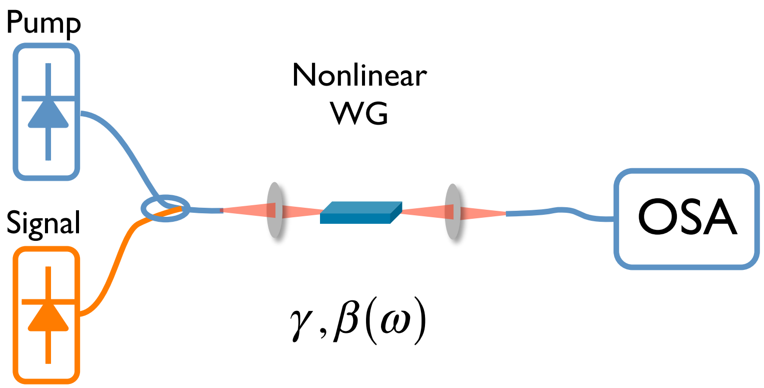






$$P(\omega = 2\omega_1 - \omega_2) = \varepsilon_0 \chi^{(3)} E_1^2 E_2^*$$

Four-Wave Mixing (degenerate case)



$$\partial_z A_j = \frac{i\omega_j}{\mathcal{N}}\left[\int \mathbf{e}^* \cdot \mathbf{P}_{\omega_j}\, da\right] e^{-i\beta_j z}$$

$$E(t) = E_P e^{-i\omega_P t} + E_S e^{-i\omega_S t} + E_I e^{-i\omega_I t} + c.c.$$

$$E_j = A_j(z)\psi(x,y)e^{i\beta_j z} \qquad \mathcal{P}_j = |A_j|^2 \mathcal{N}$$

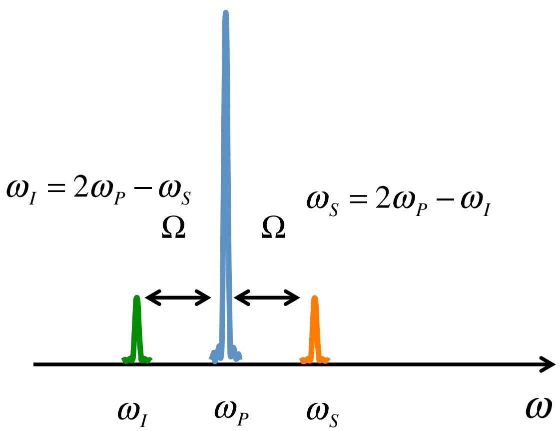
Nonlinear polarization

$$P_{\omega_P}^{(3)} = 3\varepsilon_0 \chi^{(3)}\left[\left(|E_P|^2 + 2|E_S|^2\right)E_P + 2E_I E_S E_P^*\right]$$

$$E_{\omega_S}^{(3)} = 3\varepsilon_0 \chi^{(3)}\left[\left(|E_S|^2 + 2|E_P|^2\right)E_S + E_P^2 E_I^*\right]$$

$$P_{\omega_I}^{(3)} = 3\varepsilon_0 \chi^{(3)}\left[\left(|E_I|^2 + 2|E_P|^2\right)E_I + E_P^2 E_S^*\right]$$

$$\omega_P = \omega_I + \omega_S - \omega_P$$



$$\partial_z A_P = i\left[\left(\gamma_P \mathcal{P}_P + 2\gamma_S \mathcal{P}_S + 2\gamma_I \mathcal{P}_I\right)A_P + 2\gamma_P A_I A_S A_P^* e^{-i\Delta\beta z}\right]$$

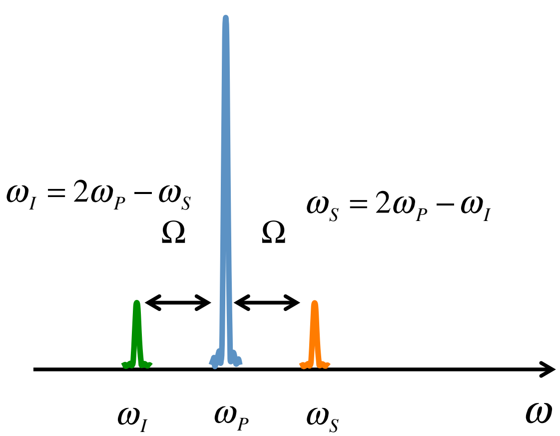
$$\partial_z A_S = i\left[\left(\gamma_S \mathcal{P}_S + 2\gamma_P \mathcal{P}_P + 2\gamma_I \mathcal{P}_I\right)A_S + \gamma_S A_P^2 A_I^* e^{i\Delta\beta z}\right]$$

$$\partial_z A_I = i\left[\left(\gamma_I \mathcal{P}_I + 2\gamma_P \mathcal{P}_P + 2\gamma_S \mathcal{P}_S\right)A_I + \gamma_I A_P^2 A_S^* e^{i\Delta\beta z}\right]$$

$$\Delta\beta = \beta_I + \beta_S - 2\beta_P$$ Linear phase mismatch

Photon flux conservation

$$\omega_P = \omega_I + \omega_S - \omega_P$$



<u>Manley-Rowe Relations</u>

$$F_j = \frac{P_j}{\hbar\omega_j A_{eff}} = \frac{\mathcal{N}\left|A_j\right|^2}{\hbar\omega_j A_{eff}}$$

Photon flux at ω_j

$$\partial_z F_j = \frac{\mathcal{N}}{\hbar A_{eff}} \frac{1}{\omega_j}\left(A_j \partial_z A_j^* + A_j^* \partial_z A_j\right)$$

$$\gamma_j = \frac{n_2 \omega_j}{c A_{eff}}$$

$$\partial_z F_P = -2\partial_z F_S = -2\partial_z F_I$$

$$\partial_z F_S + \partial_z F_I + \partial_z F_P = 0$$

$$\partial_z A_P = i\left[\left(\gamma_P \mathcal{P}_P + 2\gamma_S \mathcal{P}_S + 2\gamma_I \mathcal{P}_I\right) A_P + 2\gamma_P A_I A_S A_P^* e^{-i\Delta\beta z}\right]$$

$$\partial_z A_S = i\left[\left(\gamma_S \mathcal{P}_S + 2\gamma_P \mathcal{P}_P + 2\gamma_I \mathcal{P}_I\right) A_S + \gamma_S A_P^2 A_I^* e^{i\Delta\beta z}\right]$$

$$\partial_z A_I = i\left[\left(\gamma_I \mathcal{P}_I + 2\gamma_P \mathcal{P}_P + 2\gamma_S \mathcal{P}_S\right) A_I + \gamma_I A_P^2 A_S^* e^{i\Delta\beta z}\right]$$

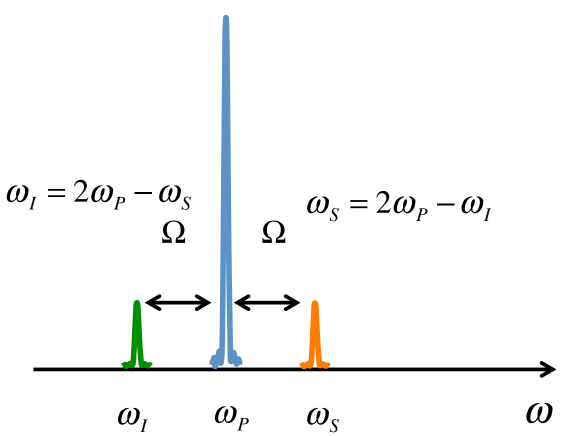
Four-Wave Mixing (degenerate case)

$$\partial_z A_P = i\left[\left(\gamma_P \mathcal{P}_P + 2\gamma_S \mathcal{P}_S + 2\gamma_I \mathcal{P}_I\right) A_P + 2\gamma_P A_I A_S A_P^* e^{-i\Delta\beta z}\right]$$

$$\partial_z A_S = i\left[\left(\gamma_S \mathcal{P}_S + 2\gamma_P \mathcal{P}_P + 2\gamma_I \mathcal{P}_I\right) A_S + \gamma_S A_P^2 A_I^* e^{i\Delta\beta z}\right]$$

$$\partial_z A_I = i\left[\left(\gamma_I \mathcal{P}_I + 2\gamma_P \mathcal{P}_P + 2\gamma_S \mathcal{P}_S\right) A_I + \gamma_I A_P^2 A_S^* e^{i\Delta\beta z}\right]$$

$$\omega_P = \omega_I + \omega_S - \omega_P$$



Strong pump (un-depleted approximation)

$$\mathcal{P}_P >> \mathcal{P}_{S,I} \qquad \partial_z A_P = i\gamma \mathcal{P}_P A_P \qquad A_P = A_P(0) e^{i\gamma \mathcal{P}_P z} = \sqrt{\mathcal{P}_P} e^{i\gamma \mathcal{P}_P z}$$

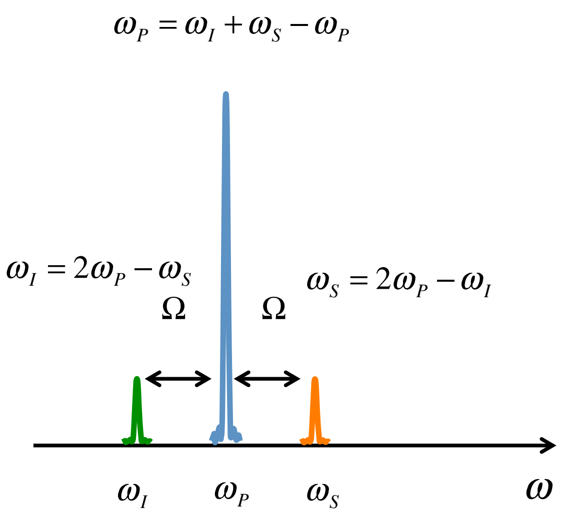
Small frequency shifts

$$\Omega << \omega_P \qquad \gamma_P = \gamma_I = \gamma_S$$

$$\partial_z A_S = i\gamma\left[2\mathcal{P}_P A_S + \mathcal{P}_P A_I^* e^{i(\Delta\beta + 2\gamma \mathcal{P}_P)z}\right]$$

$$\partial_z A_I = i\gamma\left[2\mathcal{P}_P A_I + \mathcal{P}_P A_S^* e^{i(\Delta\beta + 2\gamma \mathcal{P}_P)z}\right]$$

Four-Wave Mixing (degenerate case)



Strong pump (un-depleted approximation)

$\mathcal{P}_P >> \mathcal{P}_{S,I}$ $\quad \partial_z A_P = i\gamma \mathcal{P}_P A_P$ $\quad A_P = A_P(0) e^{i\gamma \mathcal{P}_P z} = \sqrt{\mathcal{P}_P} e^{i\gamma \mathcal{P}_P z}$

Small frequency shifts

$\Omega << \omega_P$ $\quad \gamma_P = \gamma_I = \gamma_S$

Now it is easy…

$$\partial_z A_S = i\gamma\left[2\mathcal{P}_P A_S + \mathcal{P}_P A_I^* e^{i(\Delta\beta + 2\gamma\mathcal{P}_P)z}\right]$$

$$\partial_z A_I = i\gamma\left[2\mathcal{P}_P A_I + \mathcal{P}_P A_S^* e^{i(\Delta\beta + 2\gamma\mathcal{P}_P)z}\right]$$

$$A_{S,I} = a_{S,I} e^{i2\gamma\mathcal{P}_P z}$$

$$\partial_z a_I = i\gamma \mathcal{P}_P a_S^* e^{i(\Delta\beta + 2\gamma\mathcal{P}_P)z}$$

$$\partial_z a_S = i\gamma \mathcal{P}_P a_I^* e^{i(\Delta\beta + 2\gamma\mathcal{P}_P)z}$$

$$\left[\partial_z^2 + i(\Delta\beta + 2\gamma\mathcal{P}_P)\partial_z - (\gamma\mathcal{P}_P)^2\right] a_{S,I} = 0$$

$$a_{S,I}(0) = A_{S,I}(0)$$

$$\partial_z a_{S,I}(0) = i\gamma\mathcal{P}_P A_{I,S}^*(0)$$

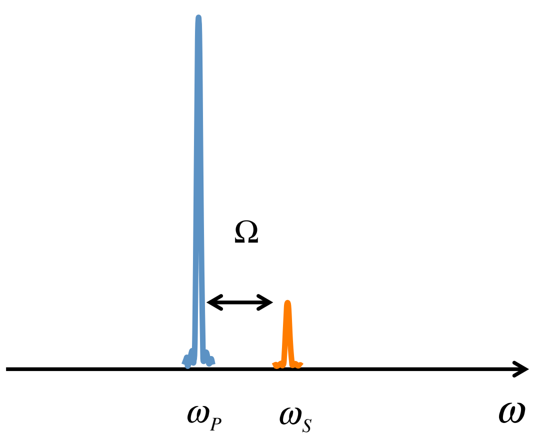
Four-Wave Mixing (degenerate case)

$$A_{S,I}(z)=e^{\frac{i}{2}(\Delta\beta+6\gamma\mathcal{P}_P)z}\left\{A_{S,I}(0)\left[\cosh\left(\frac{gz}{2}\right)-i\left(\frac{\Delta\beta+2\gamma\mathcal{P}_P}{g}\right)\sinh\left(\frac{gz}{2}\right)\right]+i\frac{2\gamma\mathcal{P}_P}{g}A_{I,S}^{*}(0)\sinh\left(\frac{gz}{2}\right)\right\}$$

$$g=\sqrt{(2\gamma P)^2-(\Delta\beta+2\gamma P)^2}=\sqrt{-\Delta\beta(\Delta\beta+4\gamma P)}$$

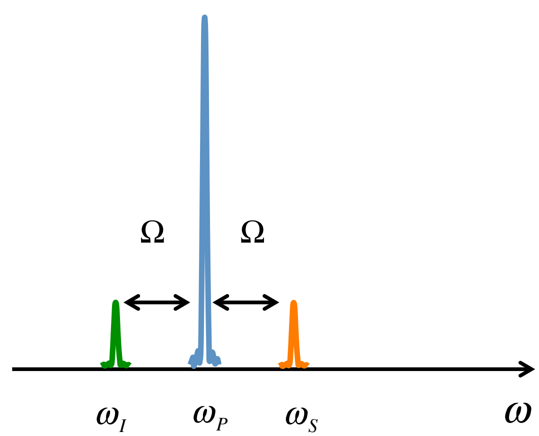
Case I: Parametric Gain and Wavelength Conversion

$$A_S(0)=\sqrt{\mathcal{P}_S(0)}$$
$$A_I(0)=0$$



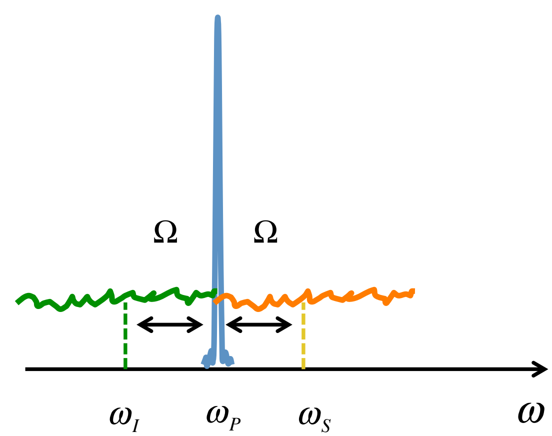
Case II: Phase-Sensitive Amplification

$$A_S(0)=\sqrt{\mathcal{P}}$$
$$A_I(0)=\sqrt{\mathcal{P}}e^{i\phi}$$



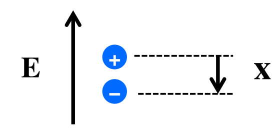
Case III: Modulation Instability

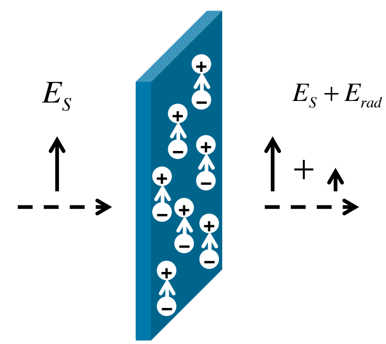
$$A_S(0)=\sqrt{\mathcal{P}}$$
$$A_I(0)=\sqrt{\mathcal{P}}e^{i\phi\_random}$$



Outline

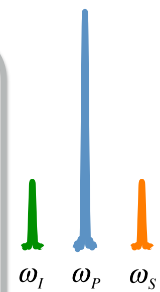
- Origin of (electronic) nonlinearity
  - Lorentz model
  - Anharmonic oscillations
  - Nonlinear polarization

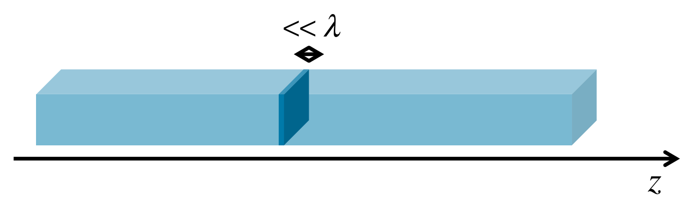




- Maxwell equations in the presence of nonlinearity
  - Wave equation: perturbative solution
  - Self- and Cross-phase modulation
  - Parametric Frequency Mixing

- Examples:
  - Parametrig Gain and Wavelength Conversion
  - Phase-sensitive Amplification
  - Modulation Instability
  - Hands-on: nonlinear coefficient (γ) and second order dispersion (β_2) characterization in optical fibers using Modulation Instability

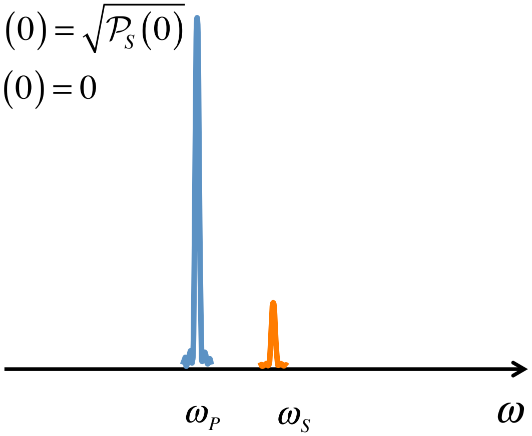




Parametric Gain and Wavelength Conversion

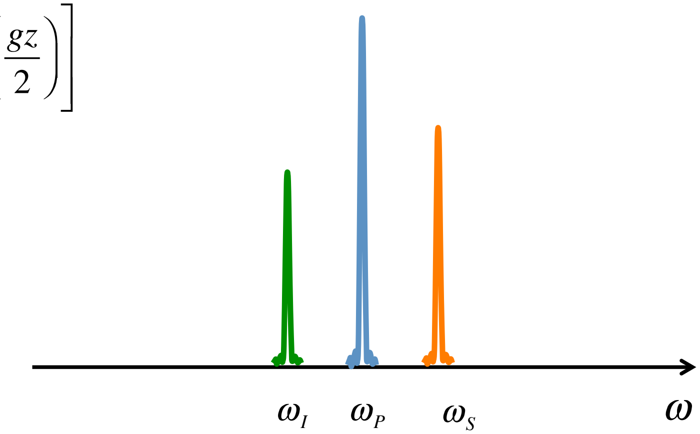
$A_S(0)=\sqrt{\mathcal{P}_S(0)}$

$A_I(0)=0$

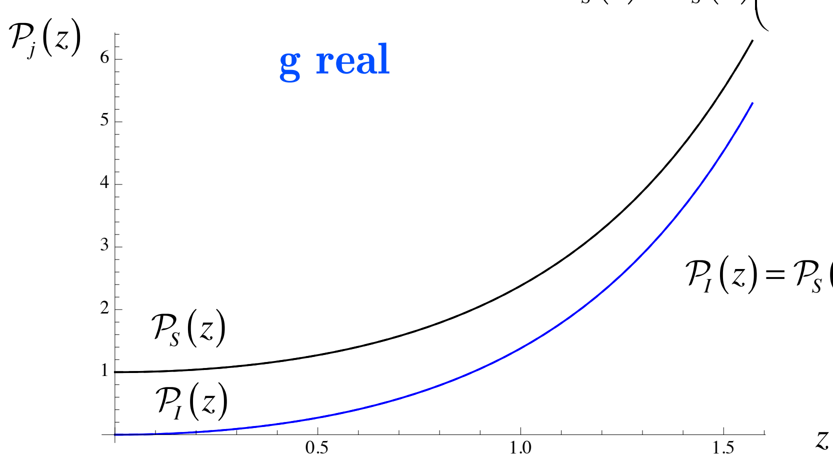


$$\mathcal{P}_S(z)=\mathcal{P}_S(0)\left[1+\left(\frac{2\gamma\mathcal{P}_P}{g}\right)^2\sinh^2\left(\frac{gz}{2}\right)\right]$$

$$\mathcal{P}_I(z)=\mathcal{P}_S(0)\left(\frac{2\gamma\mathcal{P}_P}{g}\right)^2\sinh^2\left(\frac{gz}{2}\right)$$



$$\mathcal{P}_S(z)=\mathcal{P}_S(0)\left(1+\frac{e^{2\gamma\mathcal{P}_P z}}{4}\right)$$



$$\mathcal{P}_I(z)=\mathcal{P}_S(0)\frac{e^{2\gamma\mathcal{P}_P z}}{4}$$

Maximum gain:
Phase-matching is fully satisfied!

$$g=\sqrt{(2\gamma P)^2-(\Delta\beta+2\gamma P)^2}=\sqrt{-\Delta\beta(\Delta\beta+4\gamma P)}$$

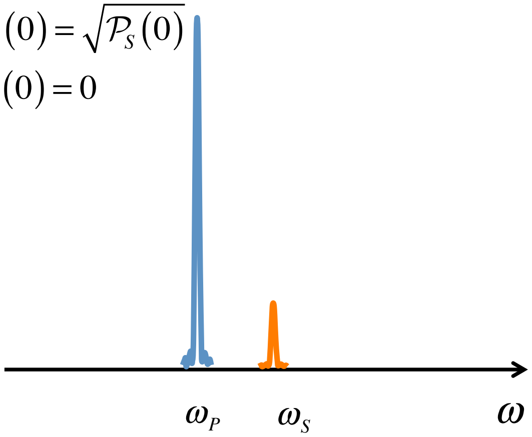
$$\Delta\beta+2\gamma P=0$$

$$g_0=2\gamma P$$

Parametric Gain and Wavelength Conversion

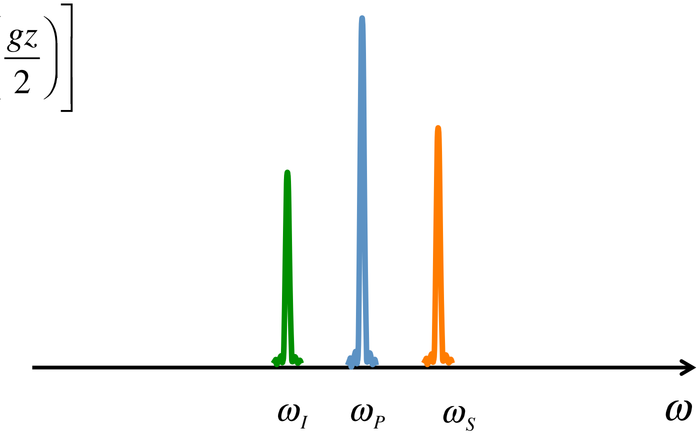
$A_S(0)=\sqrt{\mathcal{P}_S(0)}$

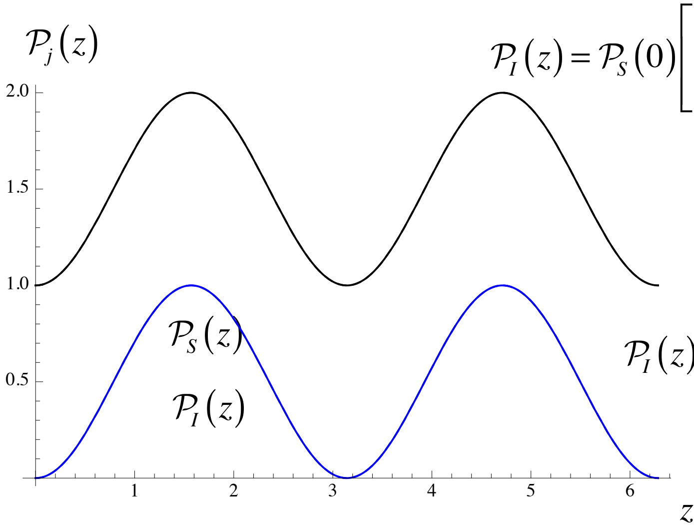
$A_I(0)=0$



$$\mathcal{P}_S(z)=\mathcal{P}_S(0)\left[1+\left(\frac{2\gamma\mathcal{P}_P}{g}\right)^2\sinh^2\left(\frac{gz}{2}\right)\right]$$

$$\mathcal{P}_I(z)=\mathcal{P}_S(0)\left(\frac{2\gamma\mathcal{P}_P}{g}\right)^2\sinh^2\left(\frac{gz}{2}\right)$$





$$\mathcal{P}_I(z)=\mathcal{P}_S(0)\left[1+\left(\frac{2\gamma\mathcal{P}_P}{g'}\right)^2\sin^2\left(\frac{g'z}{2}\right)\right]$$

$$\mathcal{P}_I(z)=\mathcal{P}_S(0)\left(\frac{2\gamma\mathcal{P}_P}{g'}\right)^2\sin^2\left(\frac{g'z}{2}\right)$$

Phase-matching is not satisfied

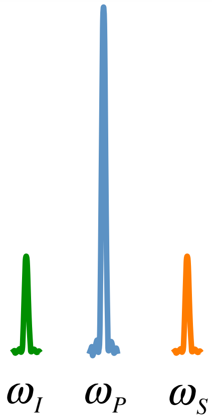
Oscillatory behavior

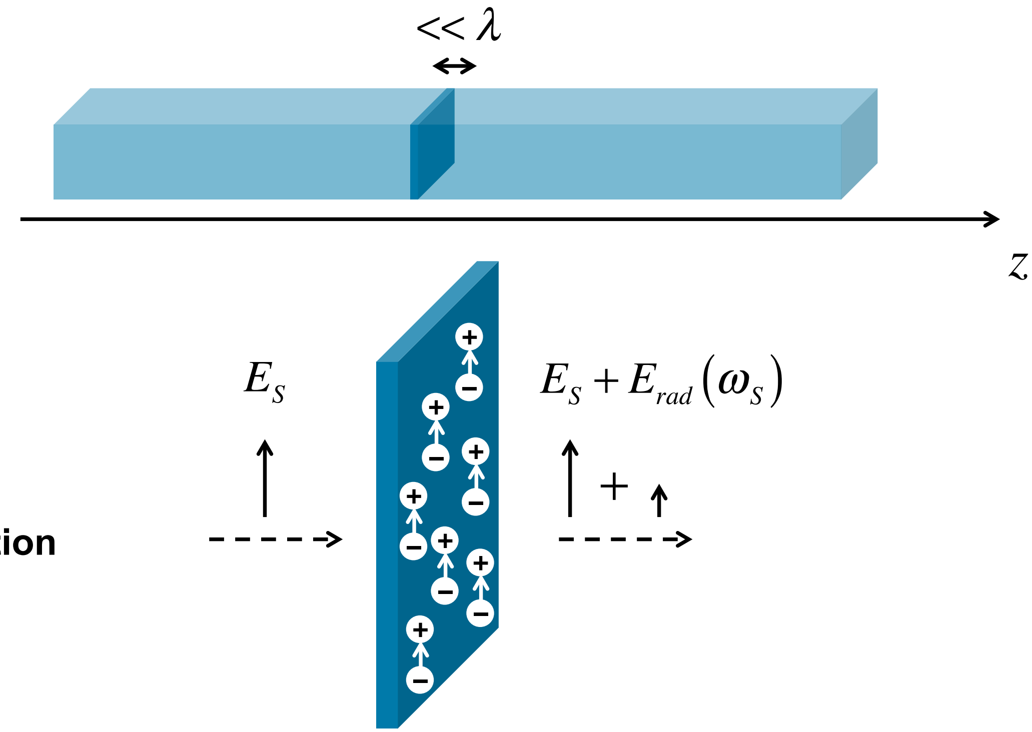
$$g=\sqrt{-\Delta\beta(\Delta\beta+4\gamma P)}$$

g pure imag

$$\boxed{g=ig'}$$

Phase-matching condition





Self- and Cross-phase modulation

$$E_p \propto e^{i[\gamma P+\beta(\omega_P)]z}$$

$$E_{S,I} \propto e^{i[2\gamma P+\beta(\omega_{S,I})]z}$$

Phases of interfering fields

$$E_{rad}(\omega_S) \propto P^{(3)} \propto E_P^2 E_I^* \propto e^{i[2\beta(\omega_P)-\beta(\omega_I)]z}$$

$$E_S \propto e^{i[2\gamma P+\beta(\omega_S)]z}$$

Phase-matching

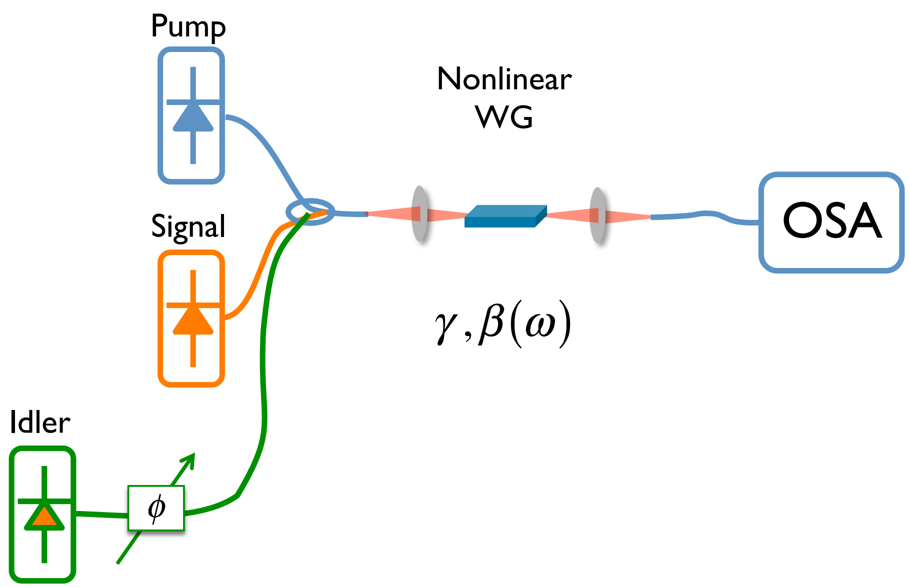
$$2\gamma P + \Delta\beta = 0$$

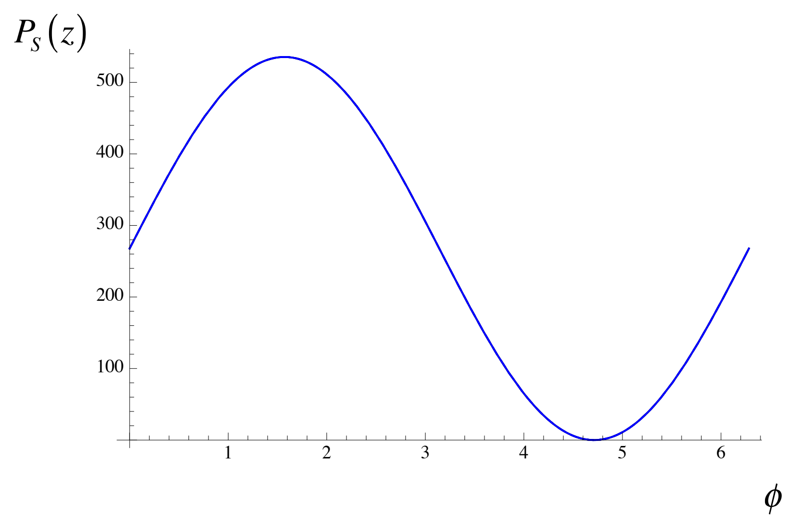
$$\Delta\beta = \beta(\omega_I) + \beta(\omega_S) - 2\beta(\omega_P)$$

Phase-sensitive amplification

$$A_S(0) = \sqrt{\mathcal{P}_S(0)}$$
$$A_I(0) = \sqrt{\mathcal{P}_I(0)}e^{i\phi}$$

$$P_S(z) = \mathcal{P}_S(0)\left[1 + \left(\frac{2\gamma\mathcal{P}_P}{g}\right)^2 \sinh^2\left(\frac{gz}{2}\right)\right] + \left(\frac{2\gamma\mathcal{P}_P}{g}\right)^2 \mathcal{P}_I(0)\sinh^2\left(\frac{gz}{2}\right)$$
$$-2\sqrt{\mathcal{P}_S(0)\mathcal{P}_I(0)}\,\frac{2\gamma\mathcal{P}_P}{g}\sinh\left(\frac{gz}{2}\right)\left[\cosh\left(\frac{gz}{2}\right)\sin(\phi) - \left(\frac{\Delta\beta + 2\gamma\mathcal{P}_P}{g}\right)\sinh\left(\frac{gz}{2}\right)\cos(\phi)\right]$$





Modulation Instability

$$A_S(0) = \sqrt{\mathcal{P}}$$
$$A_I(0) = \sqrt{\mathcal{P}} e^{i\phi\_random}$$

Maximum gain:
Phase-matching is fully satisfied!

$$\langle \cos(\phi) \rangle = \langle \sin(\phi) \rangle = 0$$

$$g = \sqrt{(2\gamma P)^2 - (\Delta\beta + 2\gamma P)^2} = \sqrt{-\Delta\beta(\Delta\beta + 4\gamma P)}$$

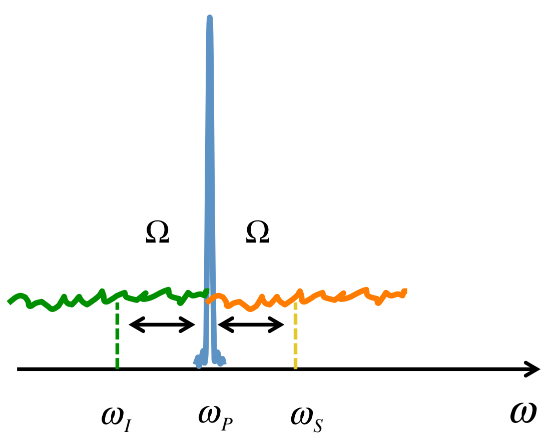
$$\Delta\beta + 2\gamma P = 0$$

$$g_0 = 2\gamma P$$

$$P_{S,I}(z) = \mathcal{P}\left[1 + 2\left(\frac{2\gamma \mathcal{P}_P}{g}\right)^2 \sinh^2\left(\frac{gz}{2}\right)\right]$$

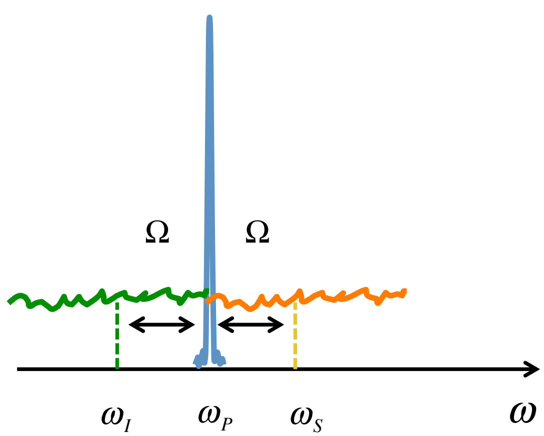
$$\mathcal{P}_{S,I}(z) = \mathcal{P}\left(1 + \frac{e^{2\gamma \mathcal{P}_P z}}{2}\right)$$

$$\mathcal{P}_S(z) = \mathcal{P}_S(0)\frac{e^{2\gamma \mathcal{P}_P z}}{4}$$



Modulation Instability: Gain Spectrum

$$P_{S,I}(z) = \mathcal{P}\left[1 + 2\left(\frac{2\gamma\mathcal{P}_P}{g}\right)^2 \sinh^2\left(\frac{gz}{2}\right)\right]$$



How does G depend on Ω?

$$g = \sqrt{(2\gamma P)^2 - (\Delta\beta + 2\gamma P)^2} = \sqrt{-\Delta\beta(\Delta\beta + 4\gamma P)}$$

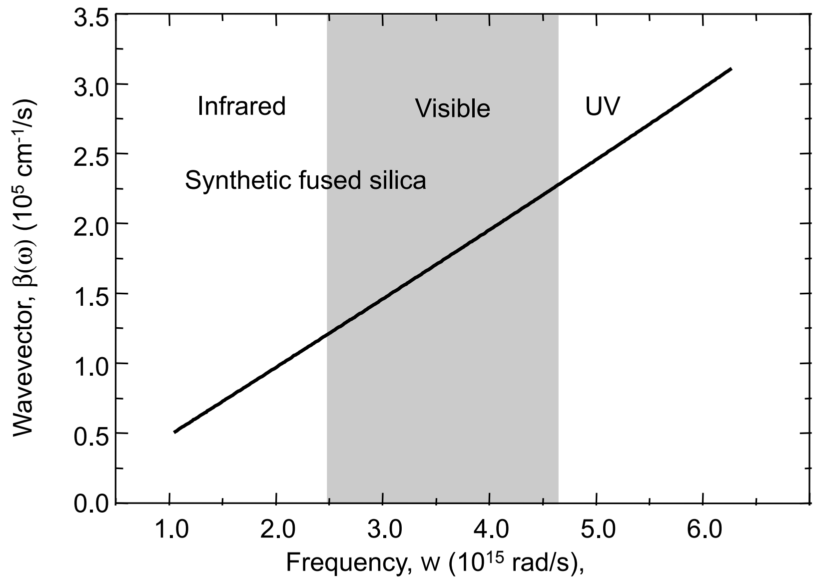
$$\Delta\beta + 2\gamma P = 0$$

$$g_0 = 2\gamma P$$

Frequency dependence is hidden in the **linear phase mismatch**

$$\Delta\beta = \beta(\omega_I) + \beta(\omega) - 2\beta(\omega_P)$$

Linear phase mismatch



$$\beta(\omega) = \beta_P + \beta_1(\omega - \omega_P) + \tfrac{1}{2}\beta_2(\omega - \omega_P)^2 + \tfrac{1}{3!}\beta_3(\omega - \omega_P)^3 + \tfrac{1}{4!}\ \beta_4(\omega - \omega_P)^4 \cdots$$

$$\beta_P = \beta(\omega_P)$$

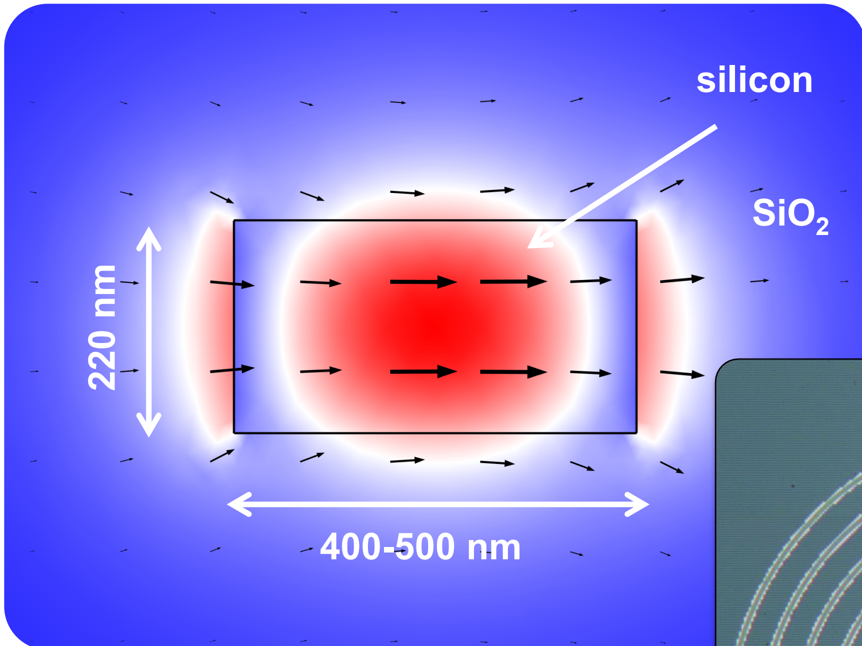
$$\beta_n = \left(\frac{d^n\beta}{d\omega^n}\right)_{\omega_P}$$

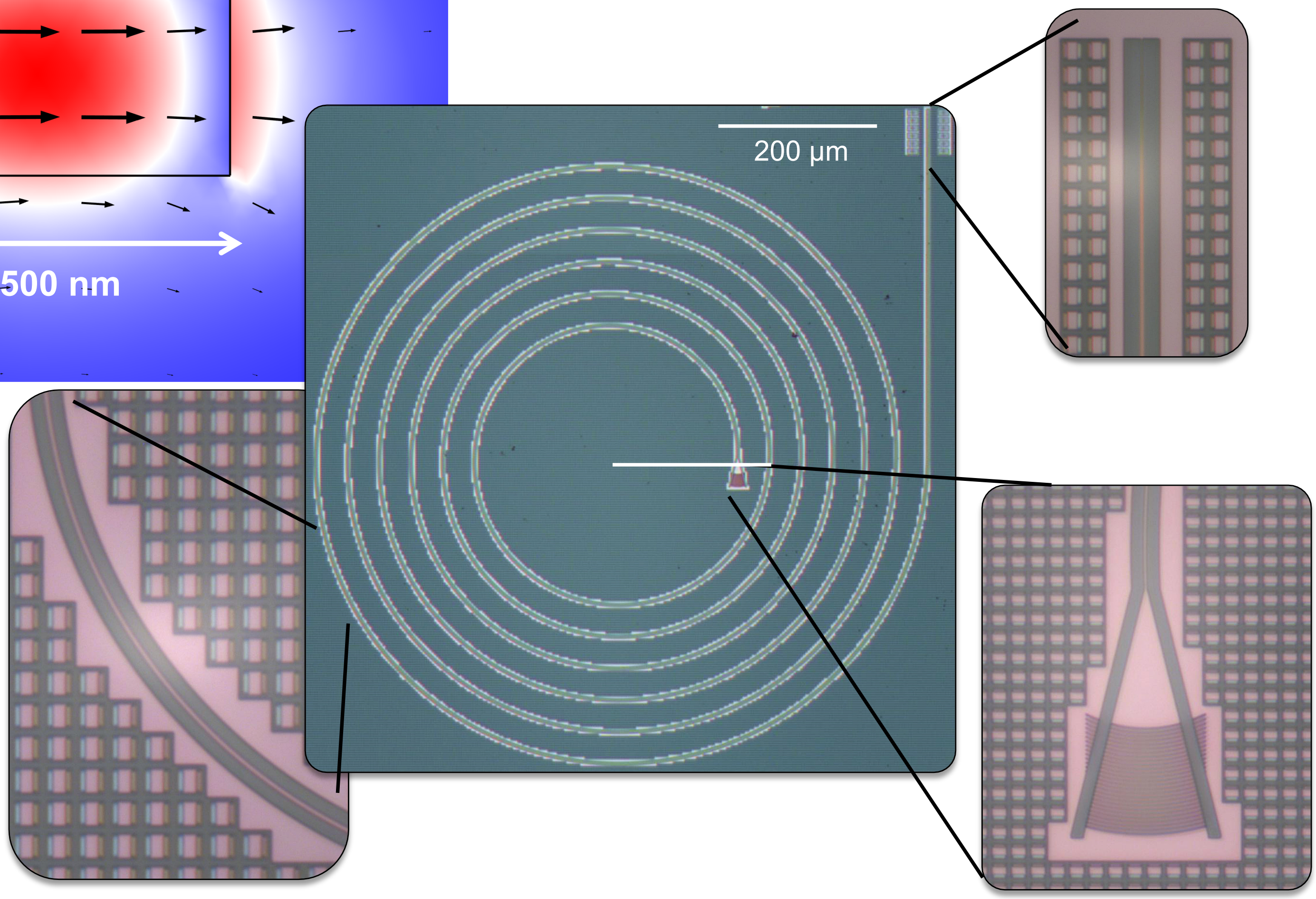
$$\Delta\beta = \beta_I + \beta_S - 2\beta_P = \beta_2\Omega^2 + \frac{\beta_4}{12}\Omega^4 + \ldots$$

Fourth-order dispersion coefficient

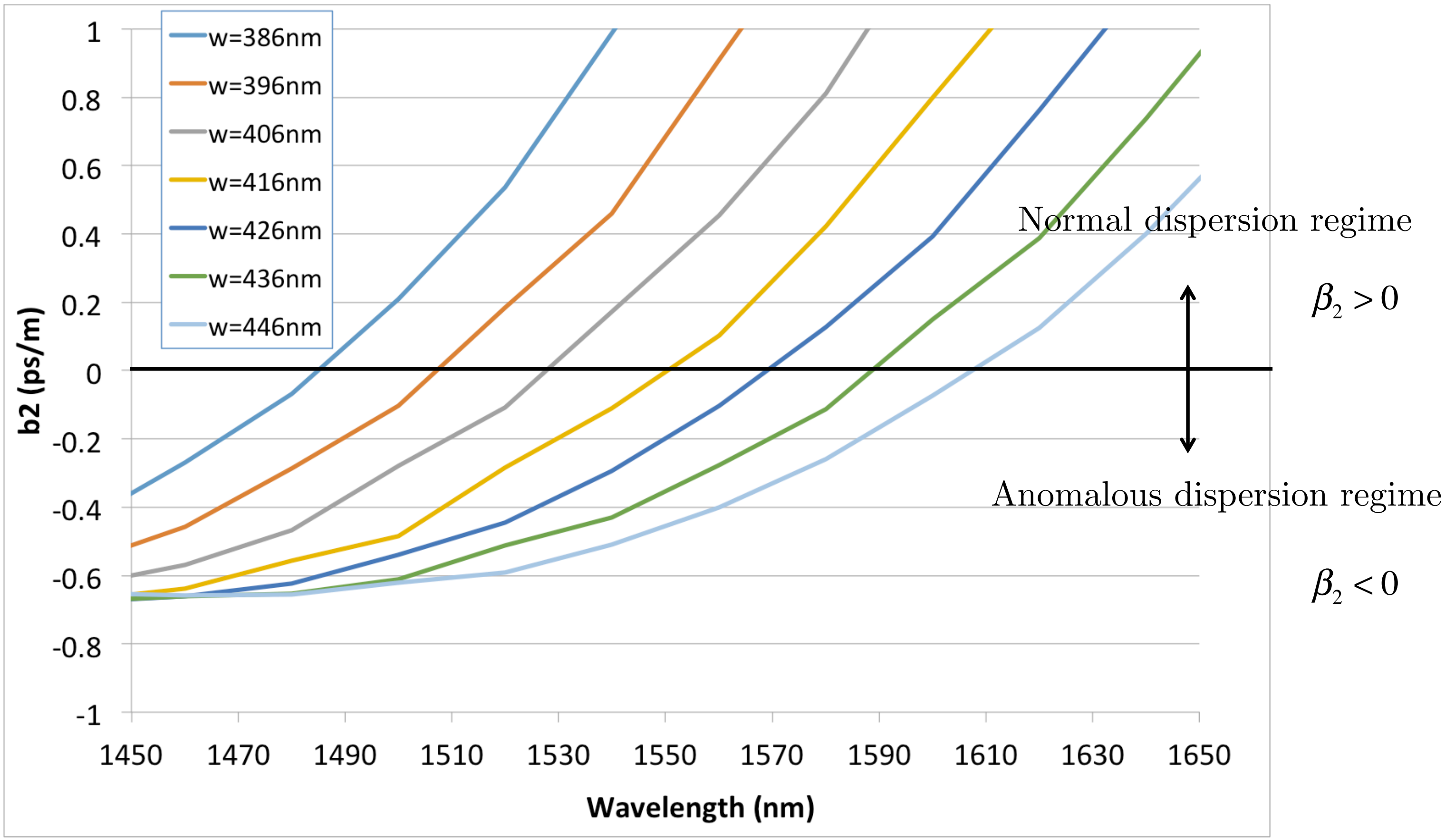
Second-order dispersion coefficient

Dispersion coefficients: an example (Silicon Waveguide)



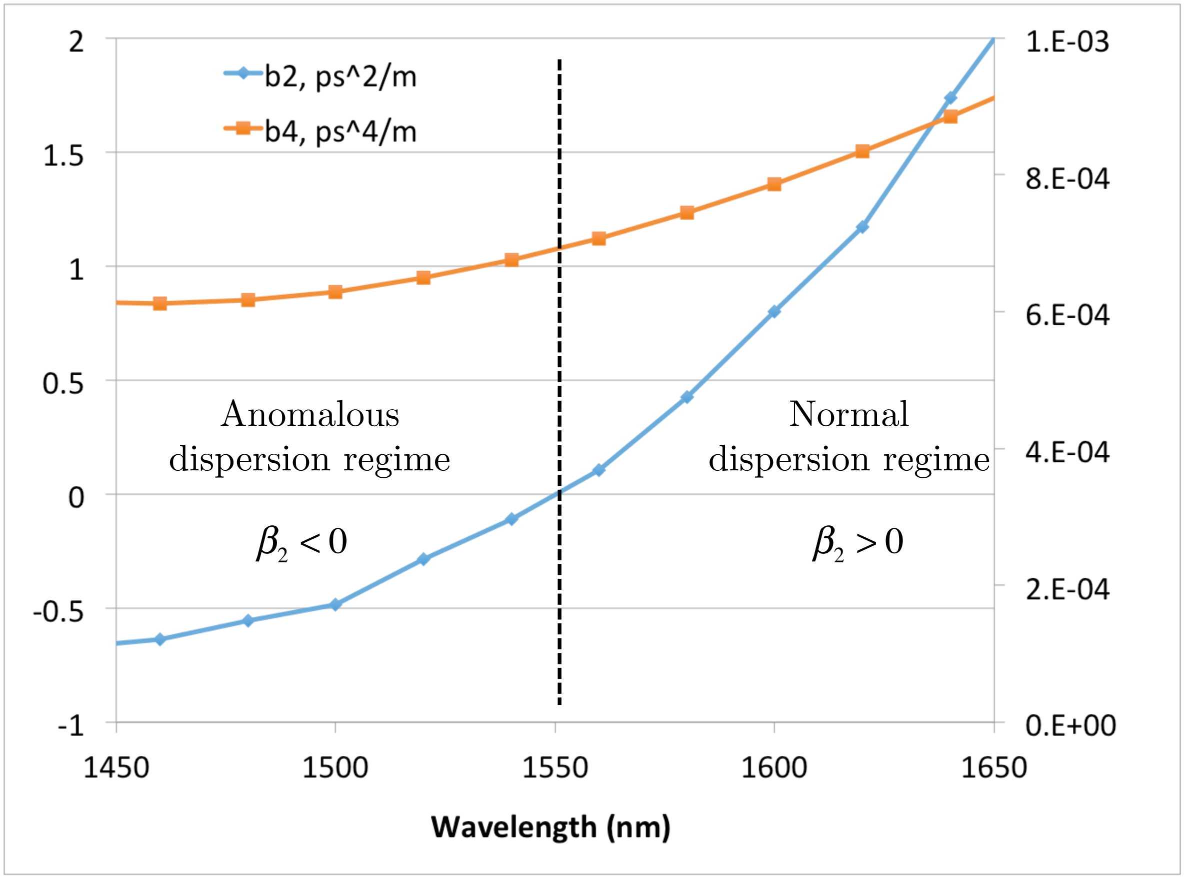


Second-order dispersion coefficients

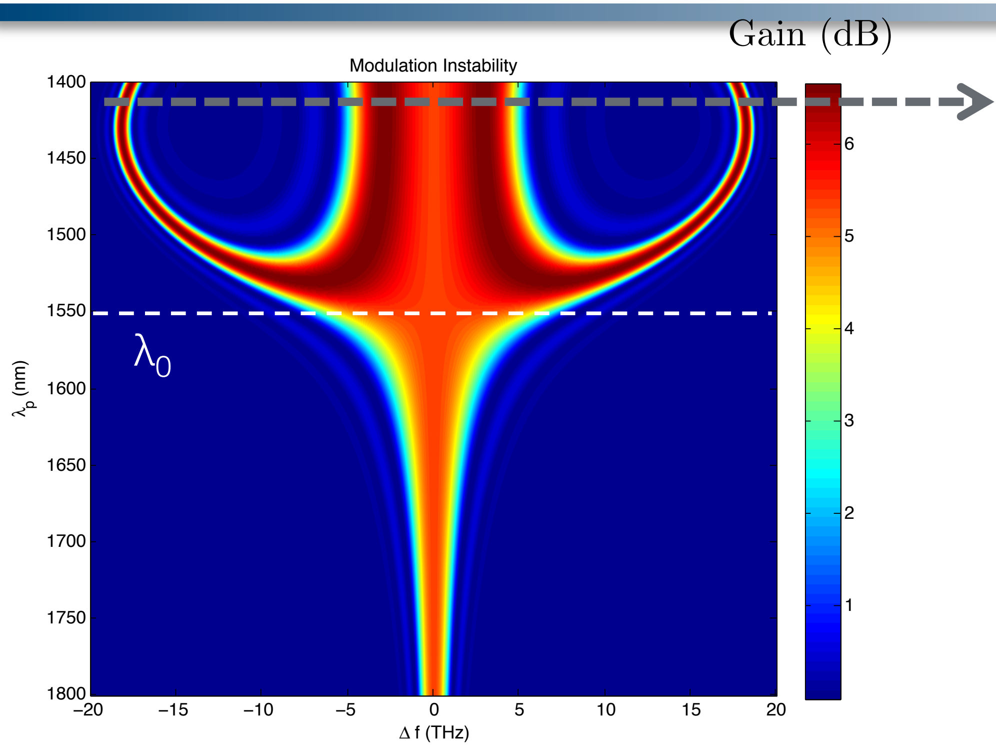


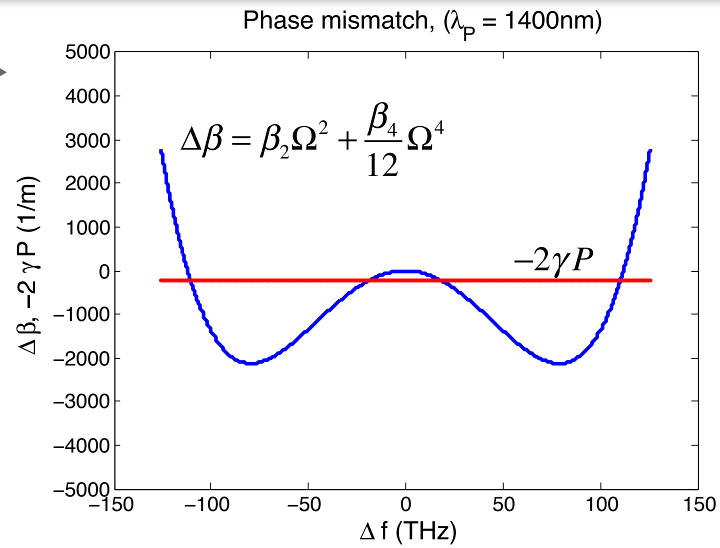
Second- and Fourth-order dispersion coefficients

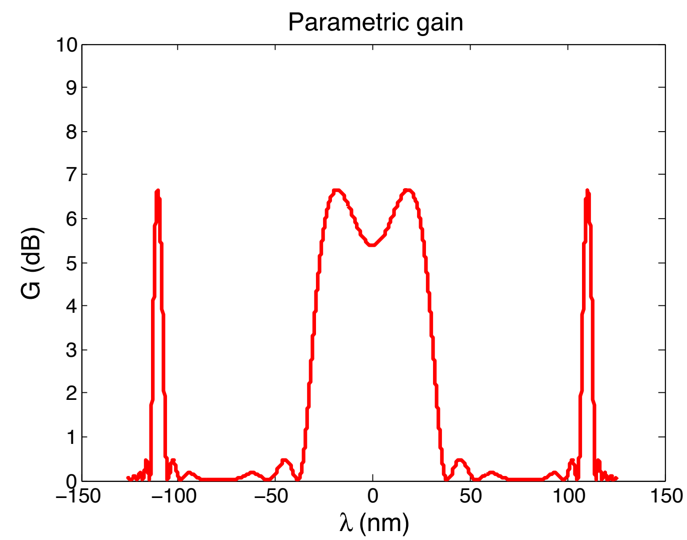
$\beta_4 > 0$



Modulation Instability

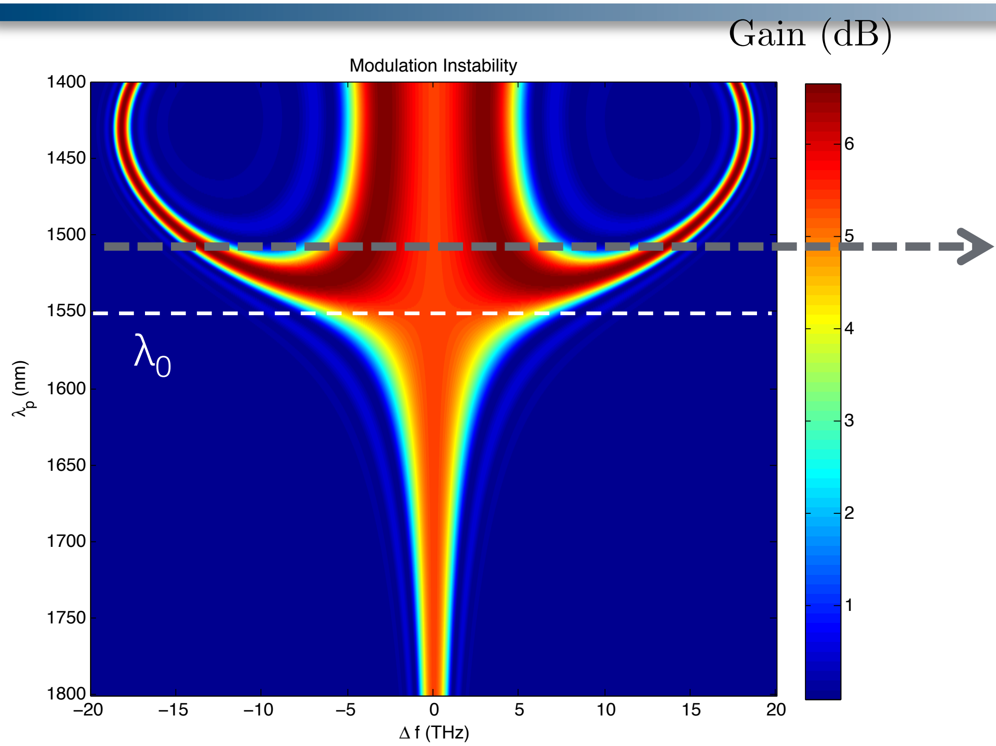


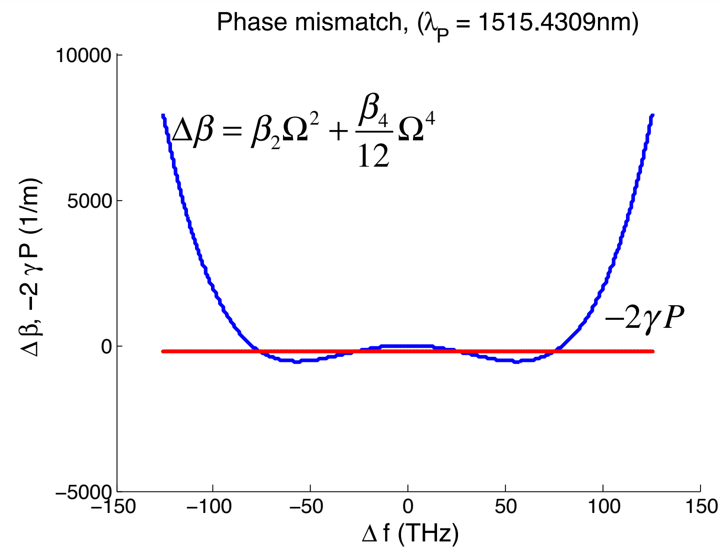


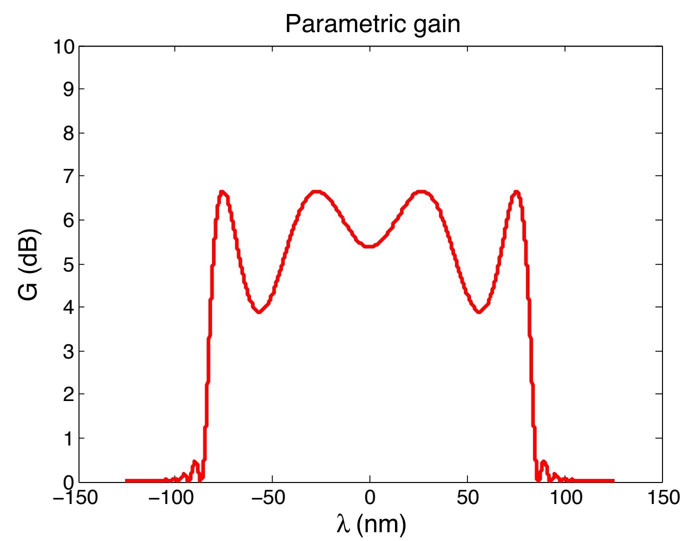


$$\Delta\beta + 2\gamma P = 0$$

Modulation Instability

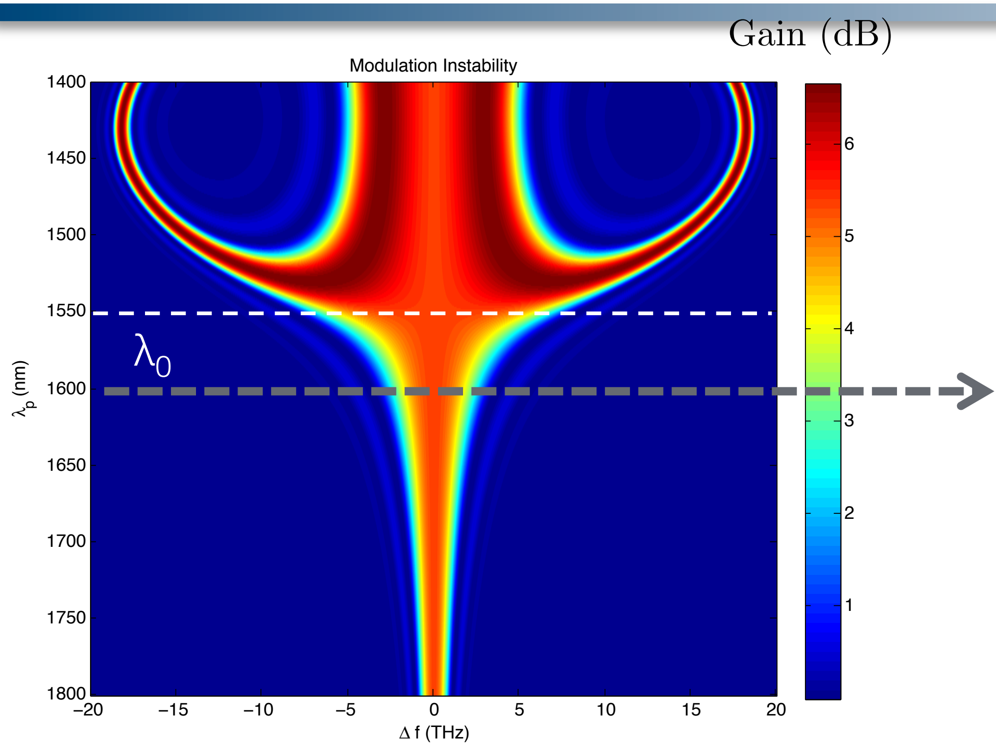


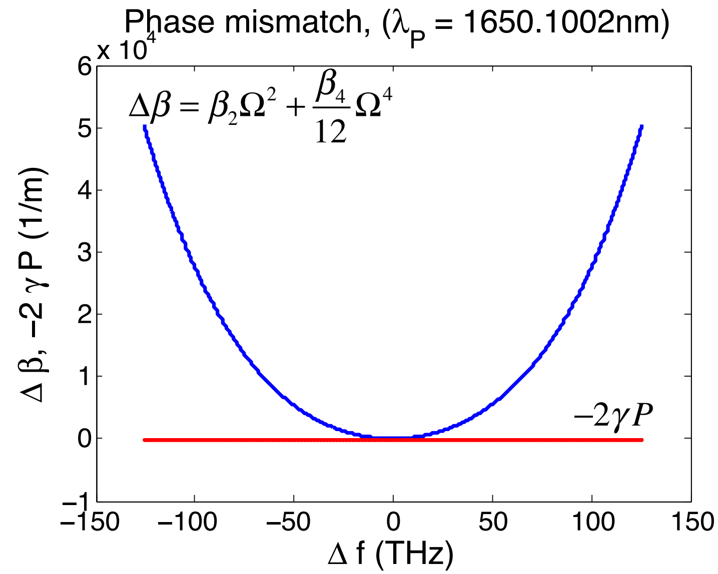


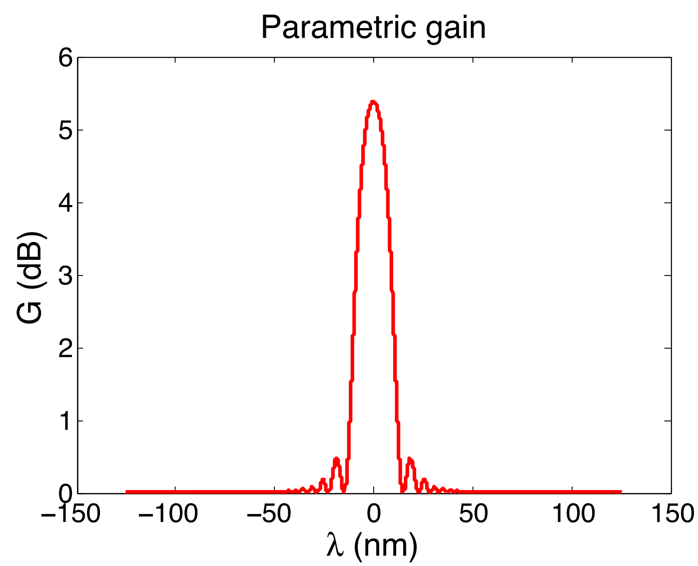


$$\Delta\beta + 2\gamma P = 0$$

Modulation Instability

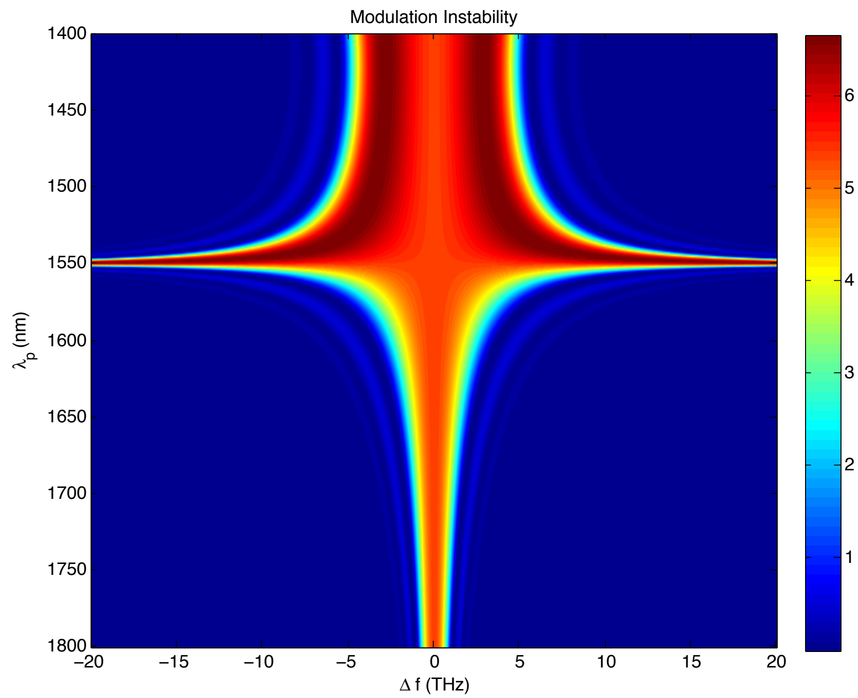




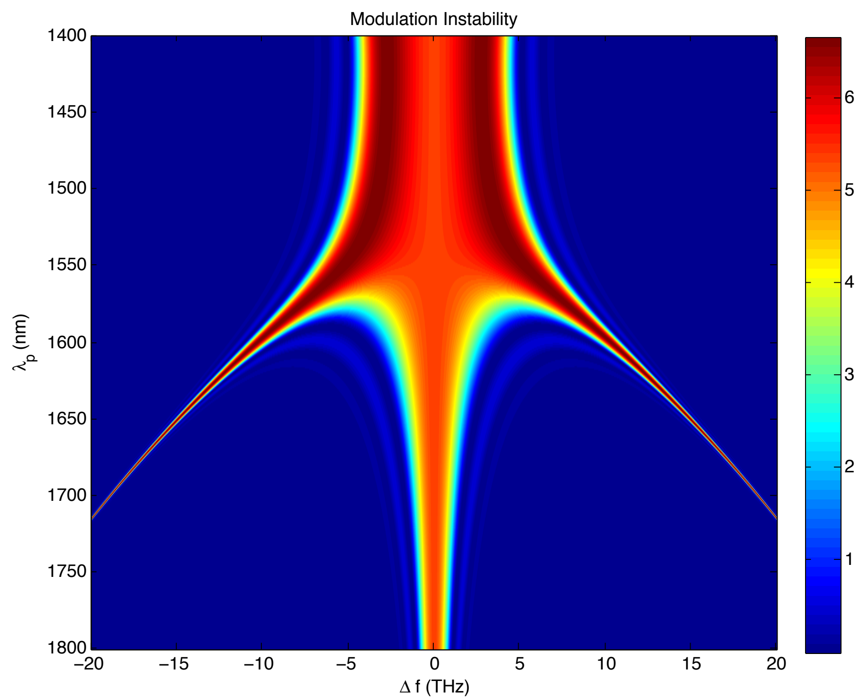


$$\Delta\beta + 2\gamma P = 0$$

Beta 4 zero


Modulation Instability
1400
1450
1500
1550
1600
1650
1700
1750
1800
λp (nm)
−20
−15
−10
−5
0
5
10
15
20
Δ f (THz)
6
5
4
3
2
1

Beta 4 negative



Hands-on 1: experiment on Modulation Instability

- Work out the equations and answer…
- Can we obtain the waveguide parameter β_2 and γ from the Modulation Instability spectrum?

Modulation Instability in Optical Fibers
(measured by A. Gil)