From quantum to classical: Photons, entanglement, and decoherence

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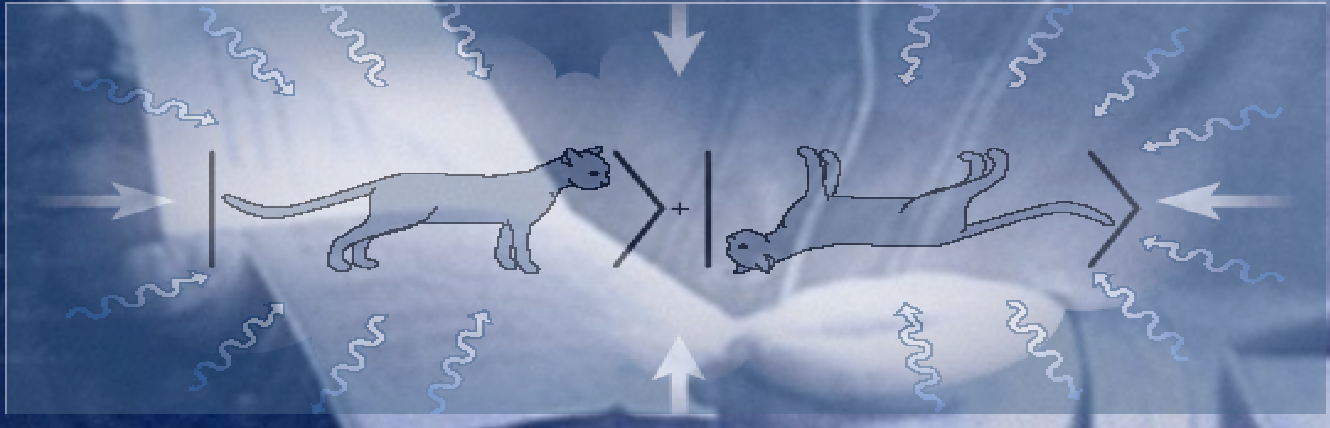


Outline of the lectures

LECTURE 1

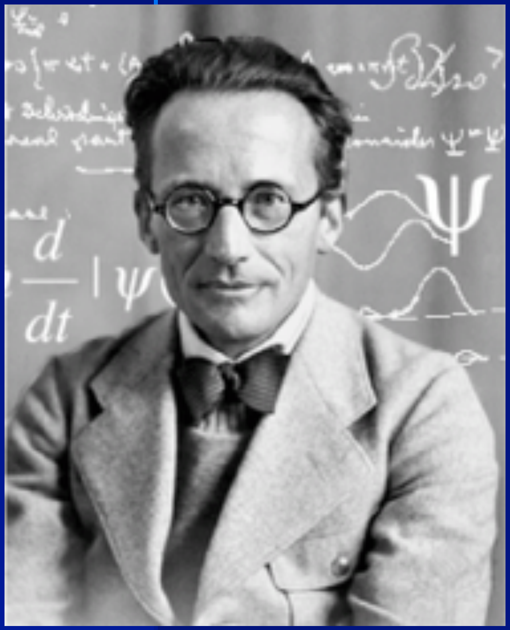
- Decoherence and the classical limit of the quantum world
- Cavities, fields, and Schrödinger cats

LECTURE 2

- Introduction to entanglement
- Entanglement and decoherence: new experimental results
- Multiparticle systems and decoherence

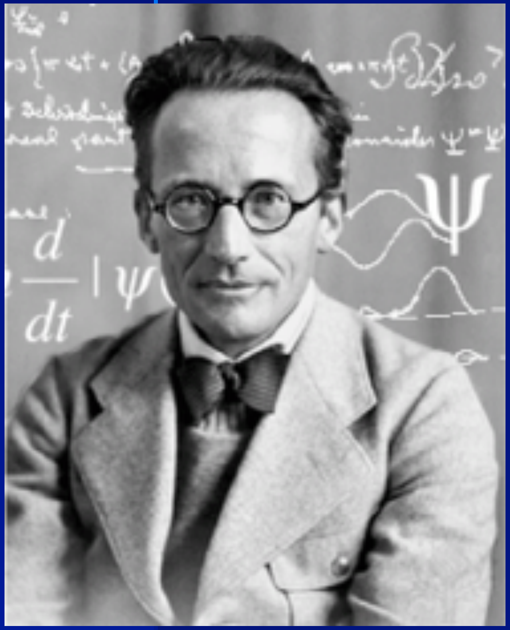
FIRST LECTURE

Schrödinger on the classical limit



- 1926: "At first sight it appears very strange to try to describe a process, which we previously regarded as belonging to particle mechanics, by a system of such proper vibrations.'' Demonstrates that "a group of proper vibrations" of high quantum number n and of relatively small quantum number differences may represent a particle executing the motion expected from usual mechanics, i. e. oscillating with a constant frequency.

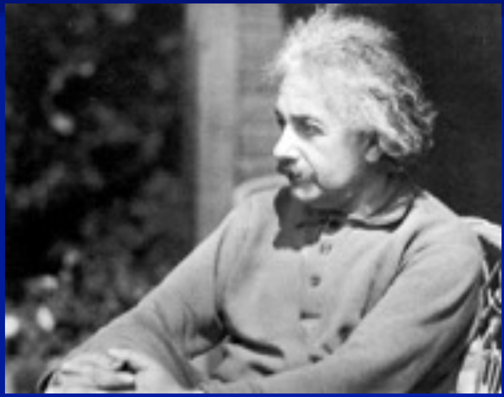
Schrödinger on the classical limit



- 1935: “An uncertainty originally restricted to the atomic domain has become transformed into a macroscopic uncertainty, which can be resolved through direct observation... This inhibits us from accepting in a naive way a `blurred model' as an image of reality...There is a difference between a shaky or not sharply focused photograph and a photograph of clouds and fogbanks.”

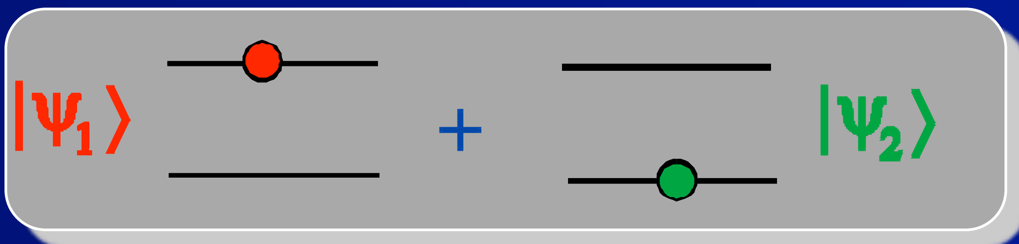
Quantum physics and localization

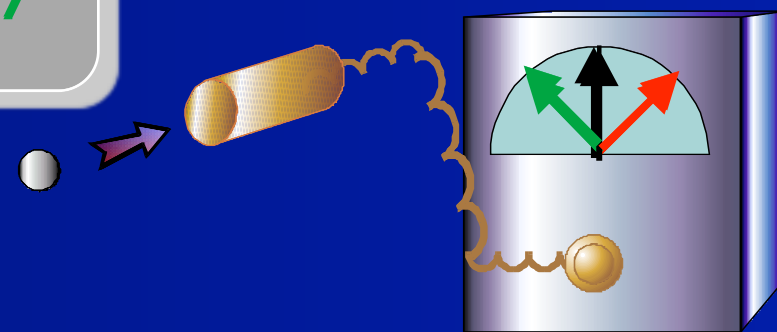
- "Let Ψ_1 and Ψ_2 be two solutions of the same Schrödinger equation. Then $\Psi = \Psi_1+\Psi_2$ also represents a solution of the Schrödinger equation, with equal claim to describe a possible real state. **When the system is a macrosystem, and when Ψ_1 and Ψ_2 are \`narrow' with respect to the macro-coordinates, then in by far the greater number of cases, this is no longer true for Ψ. Narrowness in regard to macro-coordinates is a requirement which is not only independent of the principles of quantum mechanics, but, moreover, incompatible with them."**



Letter from Einstein to Born, January 1, 1954

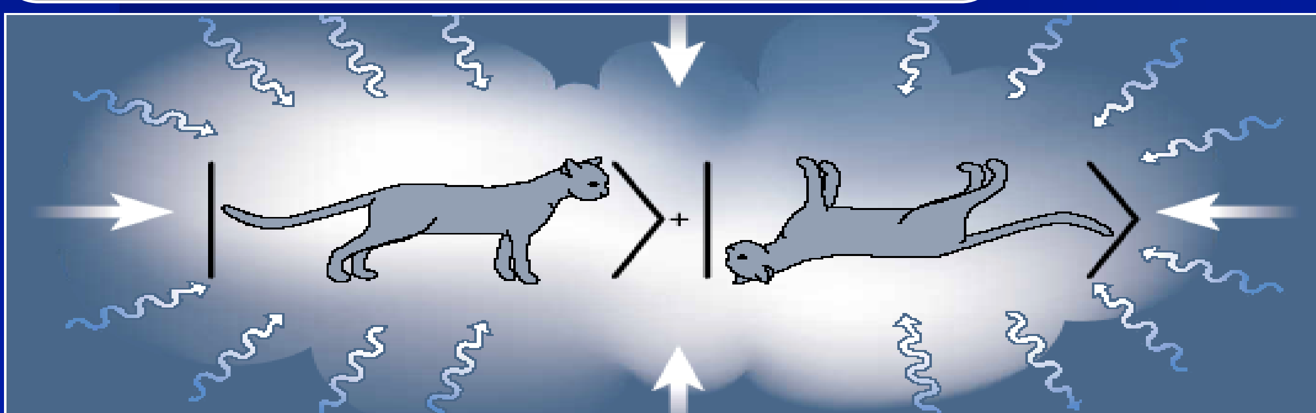
Quantum measurement





Linear evolution:

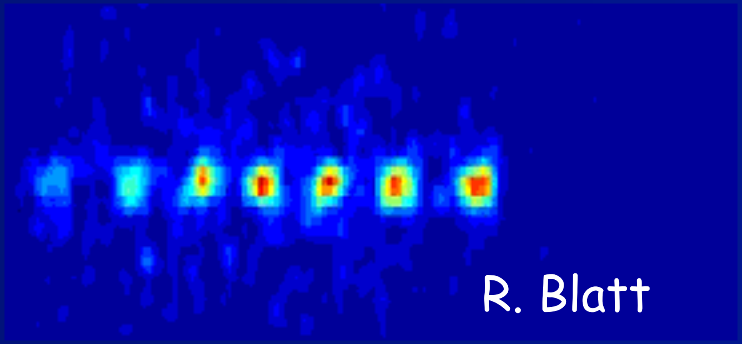
$$|\text{BEFORE}\rangle = \left(|\psi_1\rangle + |\psi_2\rangle\right)|\uparrow\rangle/\sqrt{2}$$



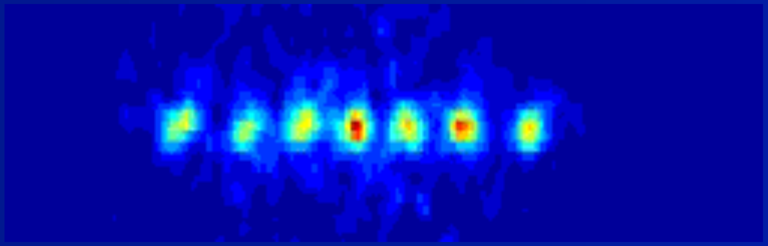
Why interference cannot be seen?

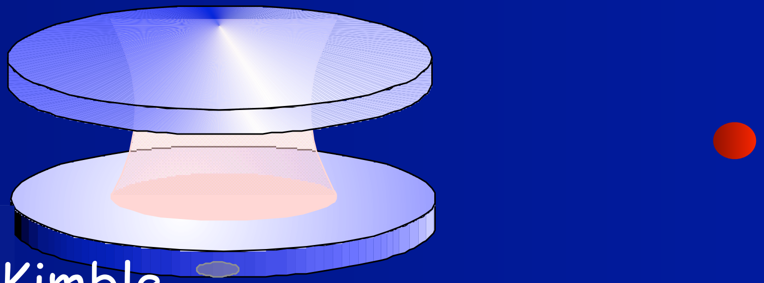
- Decoherence: entanglement with the environment - same process by which quantum computers become classical computers!
- Dynamics of decoherence: related to elusive boundary between quantum and classical world
- Important reference: work by Caldeira and Leggett

End of XX century: Advent of Quantum Technology



Ion traps (Blatt, Wineland)

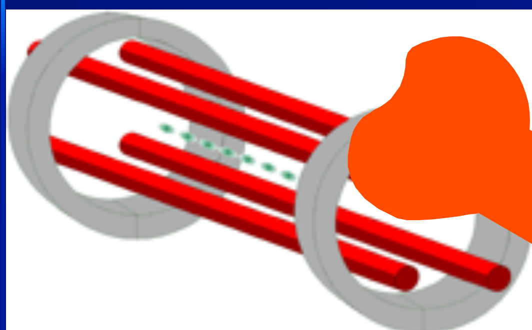




Chapman, Haroche, Kimble, Rempe, Walther

Decoherence and the quest for quantum computers

Ion traps

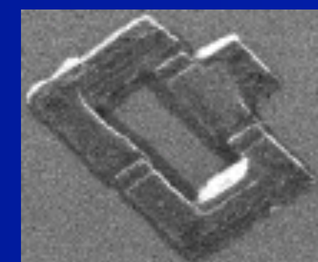


Wineland, Blatt

Quantum dots

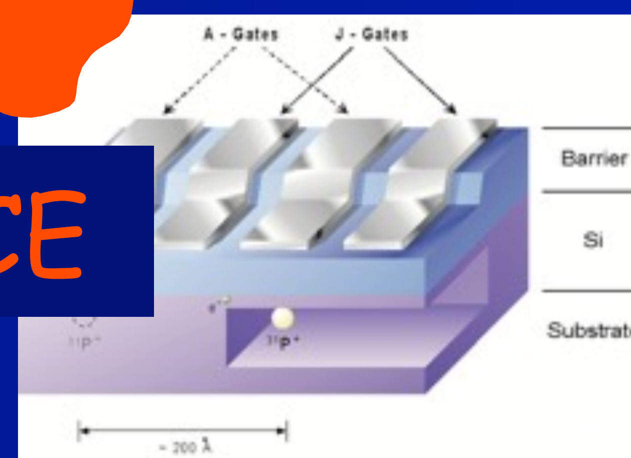
halom

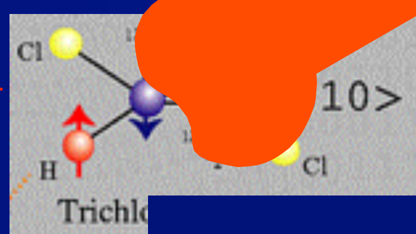
Josephson junctions



Van der Wal, Nakamura

Silicon





Nuclear M

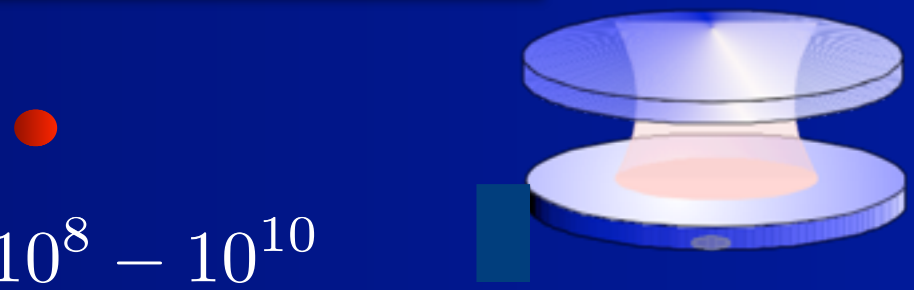
Resor

DECOHERENCE

I. Chuang, D. Cory,
R. Laflamme, E. Knill

Cavity QED

Microwave domain: Haroche, Walther
Optical domain: Kimble, Rempe



$$Q = \omega\tau \approx 10^8 - 10^{10}$$

τ up to a fraction of a second

Field manipulation and measurement: Rydberg atoms (n=50 - 60), $\ell = n - 1$, lifetime 30 ms. Transition frequencies: 20 - 100 GHz. Transit time: 20 - 100 μs
Transition dipoles: 1000 atomic units

Quantized electromagnetic field

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$\hat{H}_F = \hbar\omega\left(\hat{N} + \frac{1}{2}\right),\ \hat{N} = \hat{a}^\dagger\hat{a} \rightarrow \textit{number operator}$$

$$\left[\hat{a}, \hat{a}^\dagger\right] = 1,\ \hat{N}|n\rangle = n|n\rangle,\ n = 0, 1, 2, \ldots$$

Spatial dependence of the field mode

Electric field: $\vec{\hat{E}} = E_\omega\left[\hat{a}u(\vec{r})\vec{\varepsilon} + \hat{a}^\dagger u^*(\vec{r})\vec{\varepsilon}^{\,*}\right]$

$$u(\vec{r}) = |u(\vec{r})|e^{i\phi(\vec{r})}$$

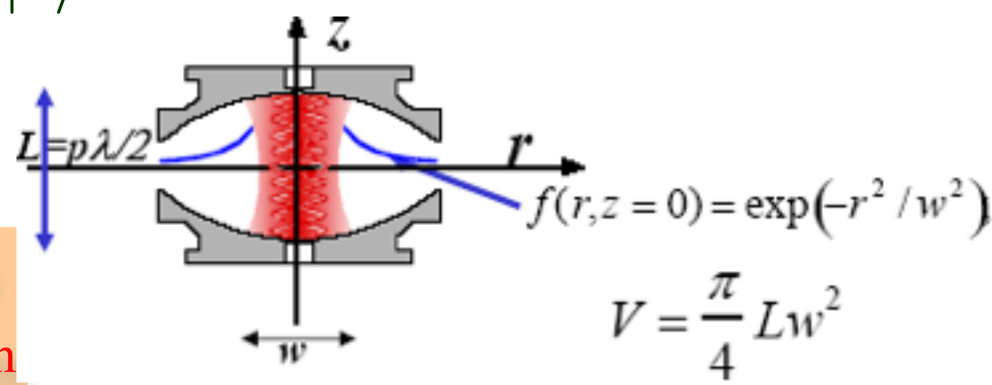
Polarization vector

$$E_\omega = \sqrt{h\omega / V} \rightarrow \textit{field per photon}$$

$V = \int |u(\vec{r})|^2 d^3r \rightarrow$ effective volume of mode,

defined so that $(1/4\pi)\int \langle 0|\vec{\hat{E}}(\vec{r})^2|0\rangle d^3r = \hbar\omega/2$

Note that $\langle n|\vec{\hat{E}}|n\rangle = 0$



Microwaves: $V \approx 0.7\ \text{cm}^3,\ E_\omega = 1.5\ \text{mV/m}$

Optical fields: $V \approx 10^3\ \mu\text{m}^3,\ E_\omega = 150\ \text{V/cm}$

Coherent states

"Quasi-classical states." Require that

$$\langle \alpha | \hat{\vec{E}}(\vec{r}) | \alpha \rangle = \sqrt{h\omega / V}\, u(\vec{r})\,\vec{\varepsilon}\,\alpha + c.c$$

$\Rightarrow$ Classical energy $E_{cl} = \hbar\omega |\alpha|^2$

$$H_F = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

Require also that $\langle \hat{n} \rangle = \langle \alpha | \hat{a}^\dagger \hat{a} | \alpha \rangle = |\alpha|^2$, so that $\langle \alpha | \hat{H}_F | \alpha \rangle$ coincides with the classical energy when $|\alpha| \gg 1$

Then $\langle \alpha | (\hat{a}^\dagger - \alpha^*)(\hat{a} - \alpha) | \alpha \rangle$

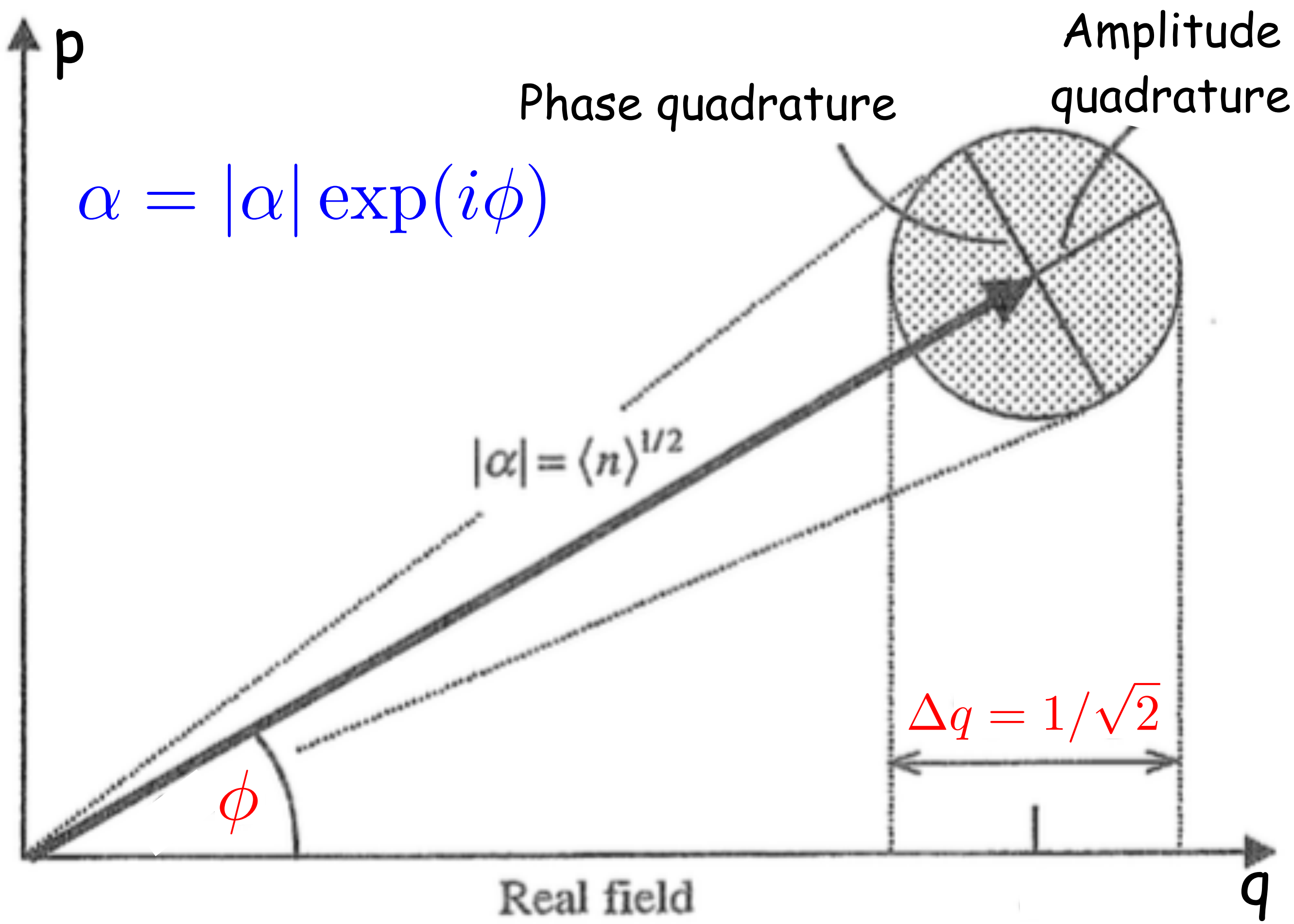
$$= \langle \alpha | \hat{a}^\dagger \hat{a} | \alpha \rangle - \alpha^* \langle \alpha | \hat{a} | \alpha \rangle - \alpha \langle \alpha | \hat{a}^\dagger | \alpha \rangle + |\alpha|^2 = 0$$

$\Rightarrow \hat{a}|\alpha\rangle = \alpha|\alpha\rangle \rightarrow$ Eigenstates of $\hat{a}$

Expansion in terms of eigenstates of $\hat{N} = \hat{a}^{\dagger}\hat{a}$:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \Rightarrow \langle n-1|\hat{a}|\alpha\rangle = \sqrt{n}\langle n|\alpha\rangle = \alpha\langle n-1|\alpha\rangle$$

$$\Rightarrow \langle n|\alpha\rangle = \frac{\alpha^n}{\sqrt{n!}}\langle 0|\alpha\rangle \Rightarrow \boxed{|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle}$$



Two-level atom interacting with one mode of the electromagnetic field

$$\hat{H} = \hat{H}_F + \hat{H}_A + \hat{H}_I, \quad |e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boxed{\delta = \omega_0 - \omega}$$

$$\hat{H}_F = \hbar\omega\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right), \quad \hat{H}_A = \frac{\hbar\omega_0}{2}\hat{\sigma}_3 = \frac{\hbar\omega_0}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\boxed{\text{Two-levels: } |\delta| \ll \omega, \omega_0}$$

Rabi frequency

$\hat{H}_I = \dfrac{\hbar\Omega_0}{2}\left(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger\right)$ (Ω_0 real, rotating-wave and dipole approximation - Jaynes-Cummings Hamiltonian), $\boxed{\Omega_0 \ll \omega, \omega_0}$

$$\hat{\sigma}_+ = \left(\hat{\sigma}_-\right)^\dagger = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}\left(\sigma_1 + i\sigma_2\right), \quad \hat{\sigma}_+|g\rangle = |e\rangle, \quad \hat{\sigma}_-|e\rangle = |g\rangle$$

$\hbar\Omega_0 / 2 = -\vec{d}_{eg} \cdot \vec{\varepsilon}\sqrt{\hbar\omega / V}\, u\left(\vec{R}\right)$, $\vec{R} \rightarrow$ atomic center-of-mass

Semiclassical approximation - Bloch equations

Go to interaction picture corresponding to

$$\hat{H}_{0R} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2} + \frac{1}{2}\hat{\sigma}_3\right)$$ (rotating frame), and set $\hat{a}_I \to \alpha$

(slowly-varying envelope of classical field):

$$\hat{H}_R = \frac{\hbar\delta}{2}\hat{\sigma}_3 + \frac{\hbar\Omega_0}{2}\left(\hat{\sigma}_{+,I}\alpha + \hat{\sigma}_{-,I}\alpha^*\right) = \boxed{\frac{\hbar}{2}\vec{\hat{\sigma}}_I \cdot \vec{\Omega},}$$

Spin-precession equation

where $\vec{\Omega} = (V_1, V_2, \delta)$, with $\Omega_0\alpha \equiv V \equiv V_1 - iV_2$.

Atomic dynamics may be described in terms of the precession of a pseudo-spin around a pseudo-magnetic field $\vec{\Omega}$.

Set $r_1 \equiv \langle\hat{\sigma}_{1,I}\rangle, r_2 \equiv \langle\hat{\sigma}_{2,I}\rangle, r_3 \equiv \langle\hat{\sigma}_{3,I}\rangle$, then $\boxed{\frac{d\vec{r}}{dt} = \vec{\Omega}\times\vec{r}}$

Bloch vector

SHOW THAT!

Bloch vector and density matrix

Atomic density matrix:

$$\hat{\rho}_A = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}$$

Pure state: $|\psi\rangle = c_e|e\rangle + c_g|g\rangle$

$$\hat{\rho}_A = \begin{pmatrix} |c_e|^2 & c_e c_g^* \\ c_e^* c_g & |c_g|^2 \end{pmatrix}$$

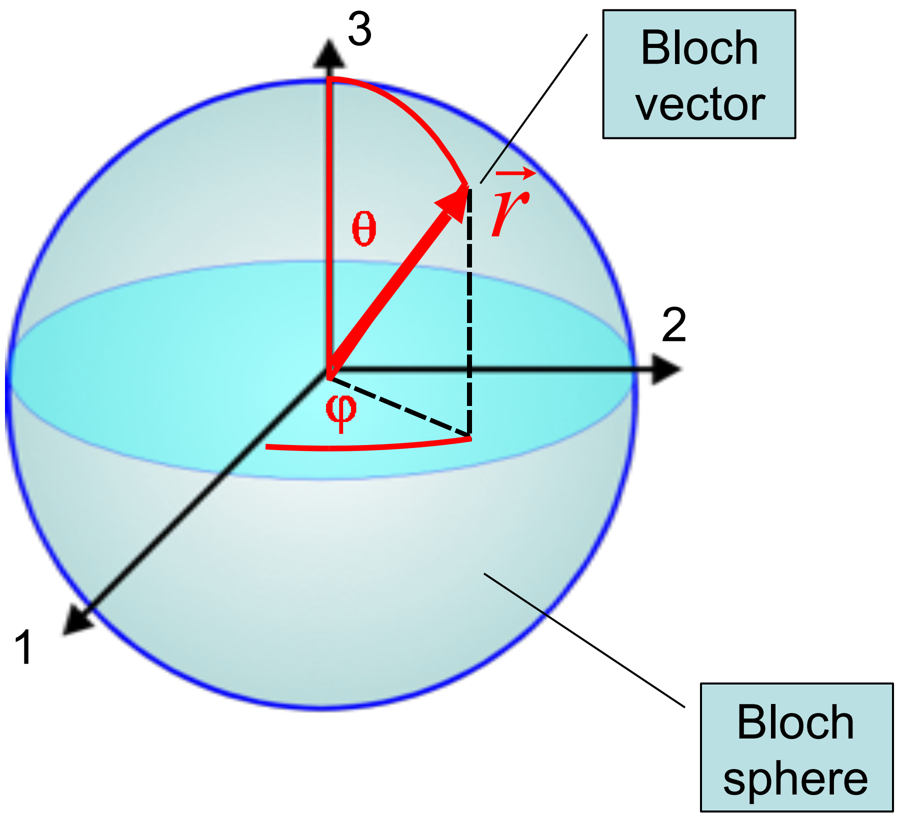
$$\left.\begin{aligned} r_1 &= \mathrm{Tr}\left[\hat{\sigma}_1(0)\hat{\rho}_A^I(t)\right] = \rho_{eg} + \rho_{ge} = 2\,\mathrm{Re}\left(\rho_{ge}\right) \\ r_2 &= \mathrm{Tr}\left[\hat{\sigma}_2(0)\hat{\rho}_A^I(t)\right] = i\left(\rho_{eg} - \rho_{ge}\right) = 2\,\mathrm{Im}\left(\rho_{ge}\right) \end{aligned}\right\} \text{coherences}$$

$$r_3 = \mathrm{Tr}\left[\hat{\sigma}_3(0)\hat{\rho}_A^I(t)\right] = \rho_{ee} - \rho_{gg} \rightarrow \text{population}$$

Exercise: Show that $r_1^2 + r_2^2 + r_3^2 \leq 1$, the equality holding if and only if the state is pure

SHOW THAT!

Bloch sphere



$$|\psi\rangle = c_e|e\rangle + c_g|g\rangle$$

or

$$|\psi\rangle = \cos(\theta/2)e^{-i\varphi/2}|e\rangle + \sin(\theta/2)e^{i\varphi/2}|g\rangle$$

$$r_1 = 2\,\mathrm{Re}\left(c_e^* c_g\right) = \sin\theta\cos\varphi$$

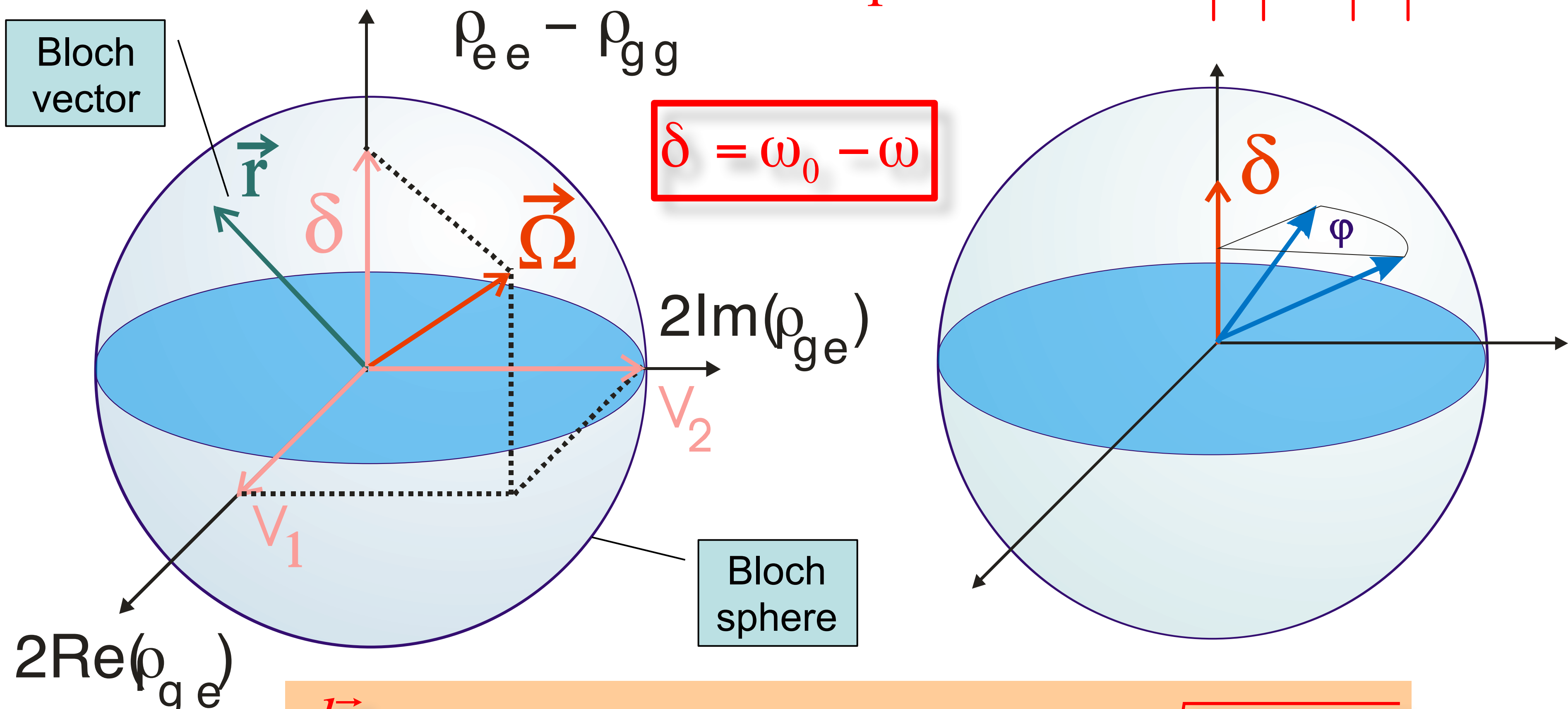
$$r_2 = 2\,\mathrm{Im}\left(c_e^* c_g\right) = \sin\theta\sin\varphi$$

$$r_3 = |c_e|^2 - |c_g|^2 = \cos\theta$$

SHOW THAT!

Geometrical interpretation of atom dynamics

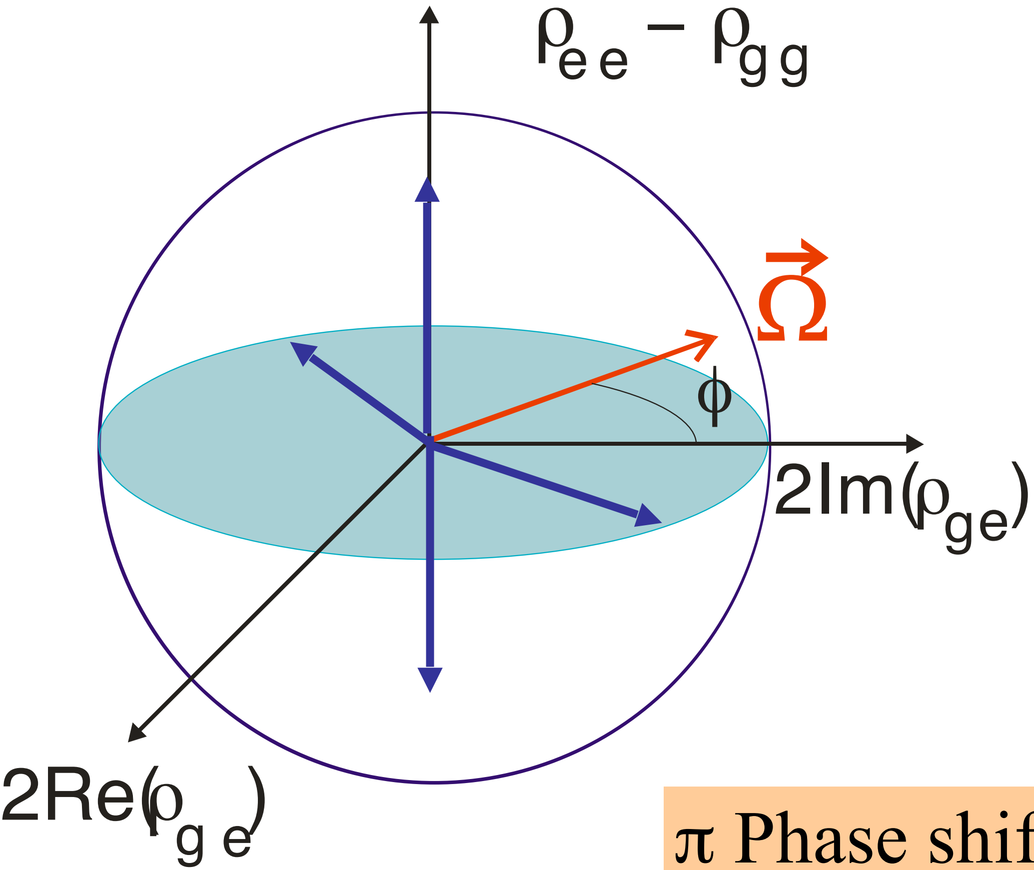
Dispersive case: $|\delta| \gg |V|$



$$\frac{d\vec{r}}{dt} = \vec{\Omega} \times \vec{r}, \ \vec{\Omega} = (V_1, V_2, \delta), \ |\vec{\Omega}| = \sqrt{|V|^2 + \delta^2}$$

Resonant dynamics

$\pi / 2$ rotation



$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|e\rangle + e^{i\phi}|g\rangle\right)$$

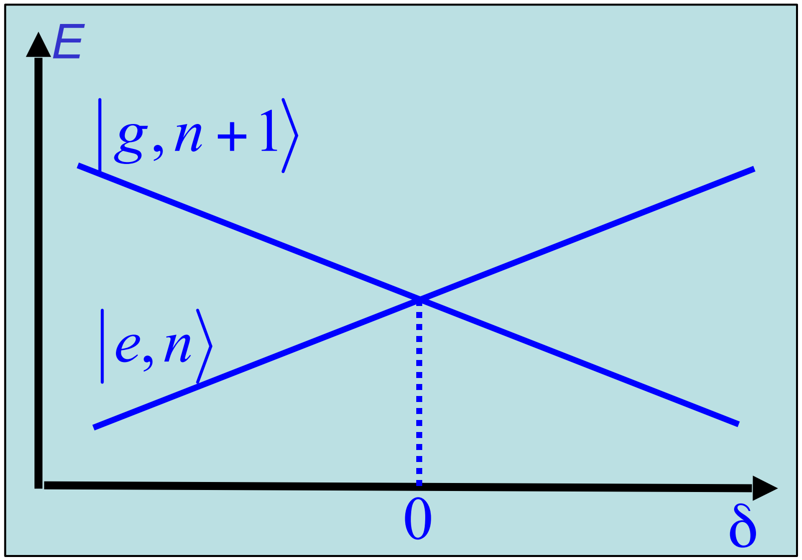
$$\frac{1}{\sqrt{2}}\left(|e\rangle + e^{i\phi}|g\rangle\right) \rightarrow e^{i\phi}|g\rangle$$

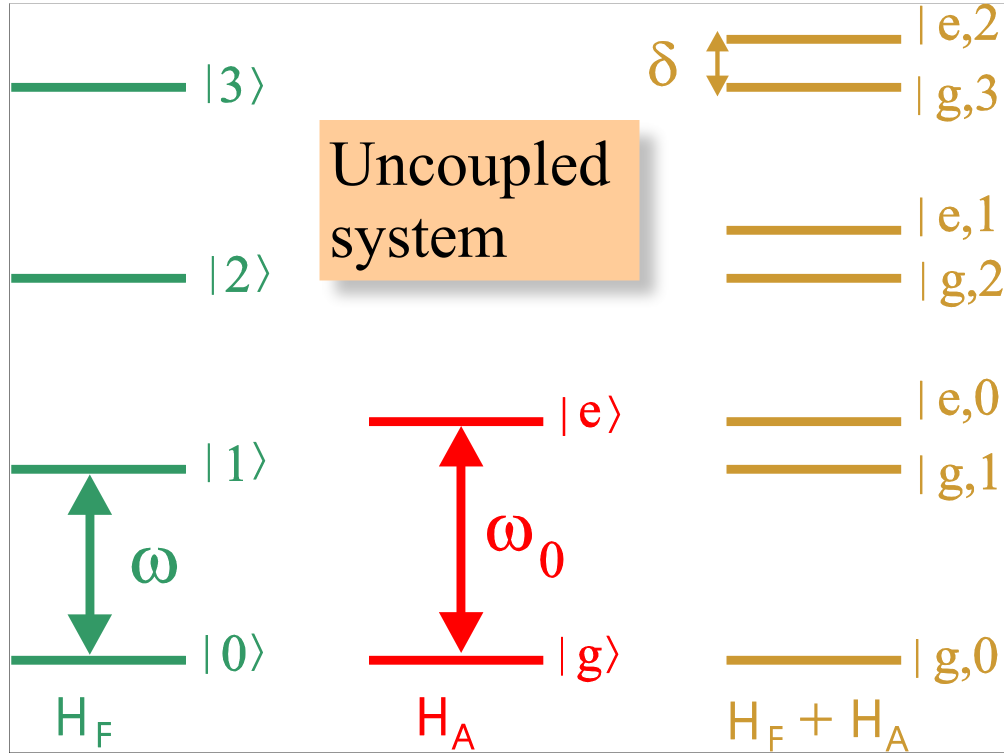
$$|g\rangle \rightarrow \frac{1}{\sqrt{2}}\left(-e^{-i\phi}|e\rangle + |g\rangle\right)$$

$$\frac{e^{i\phi}}{\sqrt{2}}\left(-e^{-i\phi}|e\rangle + |g\rangle\right) \rightarrow -|e\rangle$$

π Phase shift: 2π rotation of a spin 1/2

Atoms and photons: The dressed atom





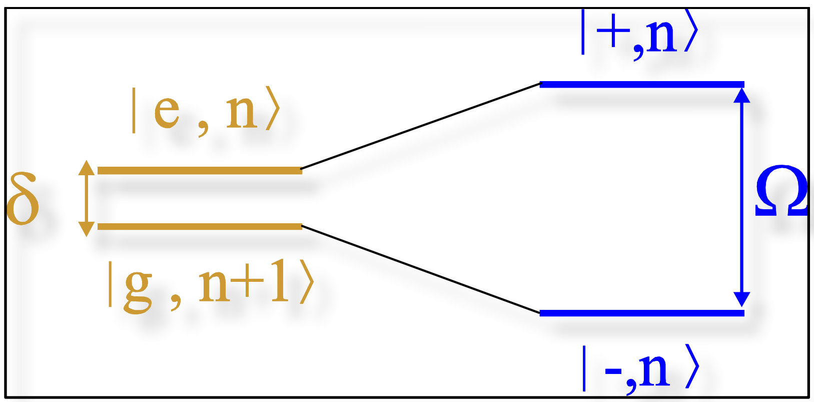
$$\boxed{\delta = \omega_0 - \omega}$$

$$\boxed{\begin{aligned} E_{e,n} &= \hbar\omega(n+1) + \frac{\hbar\delta}{2} \\ E_{g,n+1} &= \hbar\omega(n+1) - \frac{\hbar\delta}{2} \end{aligned}}$$

$$\boxed{H_I = \frac{\hbar\Omega_0}{2}\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger\right)}$$

Couples only states within each doublet!

Effect of interaction

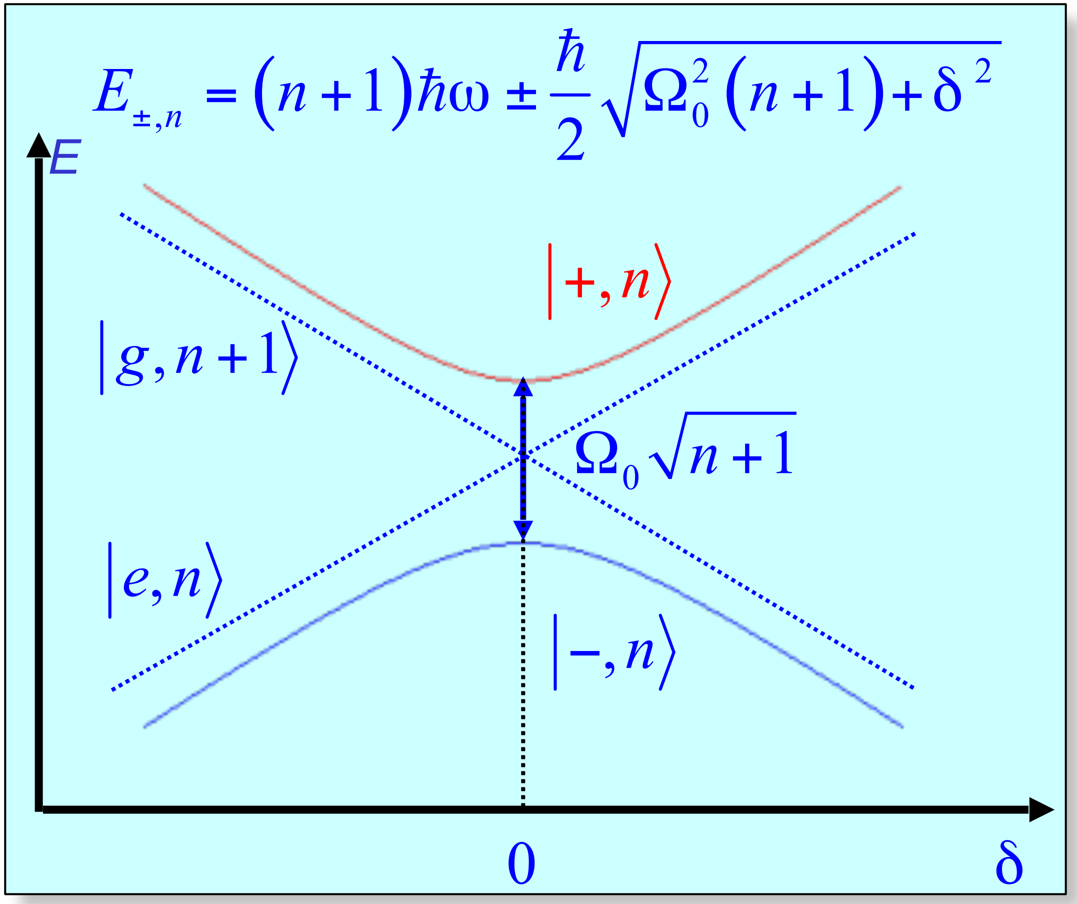


$$[H]_n = \hbar \begin{bmatrix} \omega(n+1) - \frac{\delta}{2} & \Omega_0\sqrt{n+1}/2 \\ \Omega_0\sqrt{n+1}/2 & \omega(n+1) + \frac{\delta}{2} \end{bmatrix}$$

$$\begin{aligned} E_{\pm,n} &= (n+1)\hbar\omega \pm (\hbar\Omega/2)\,, \\ E_{g,0} &= -\hbar\delta/2\,, \end{aligned}$$

$$\Omega = \left[\Omega_0^2(n+1) + \delta^2\right]^{1/2}$$

The dressed atom and the dispersive limit

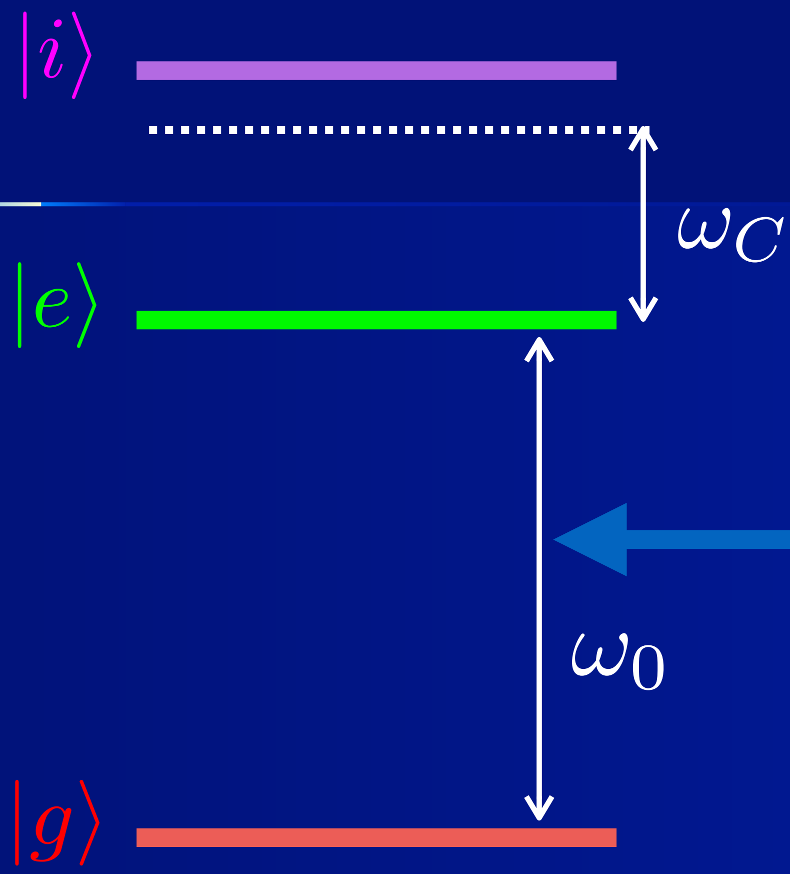


Exercise: Show that, for $|\delta| \gg \Omega_0\sqrt{n+1}$,

AC Stark shift $\Rightarrow$

$$\Delta E_{e,n} \approx \hbar(\Omega_0^2/4\delta)(n+1)\,,$$
$$\Delta E_{g,n} \approx -\hbar(\Omega_0^2/4\delta)n\,.$$

Possible level scheme



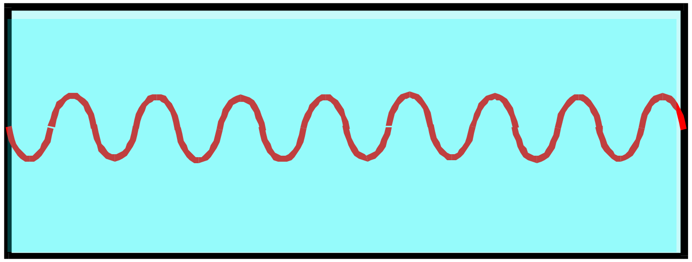
Dispersive interaction between atom and cavity mode

Manipulate through resonant classical fields

$$|\alpha e^{i\phi}\rangle$$

$$|e\rangle|\alpha\rangle = |e\rangle \otimes \exp\left(-|\alpha|^2/2\right)\sum_{n=0}^{\infty} \frac{e^{in\phi}\alpha^n}{\sqrt{n!}}|n\rangle$$

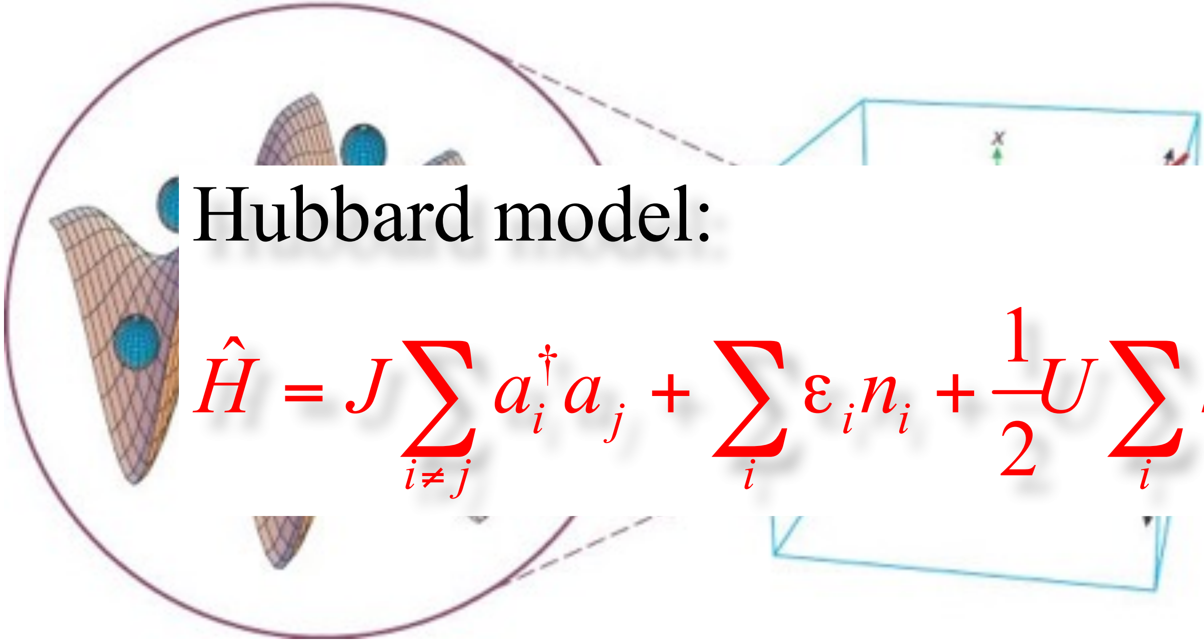
Dispersive interaction in classical physics



$$\boxed{\lambda = \frac{c'}{f'}}$$

Transparent material (dispersive interaction): frequency change $\Rightarrow$ phase change

Application: Optical lattices



Hubbard model:

$$\hat{H} = J\sum_{i\neq j} a_i^{\dagger} a_j + \sum_i \varepsilon_i n_i + \frac{1}{2}U\sum_i \hat{n}_i\left(\hat{n}_i - 1\right)$$

Periodic potential ⇔ Stationary light waves
Electrons ⇔ Atoms

Insulator-superfluid Mott transition (quantum phase transitions: Greiner et al, Nature **415**, 39 (2002)

Decoherence dynamics

VOLUME 71, NUMBER 15 PHYSICAL REVIEW LETTERS 11 OCTOBER 1993

Quantum Switches and Nonlocal Microwave Fields

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(Received 2 July 1993)

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Mesoscopic quantum coherences in cavity QED: Preparation and decoherence monitoring schemes

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(Received 18 April 1995)

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Observing the Progressive Decoherence of the "Meter" in a Quantum Measurement

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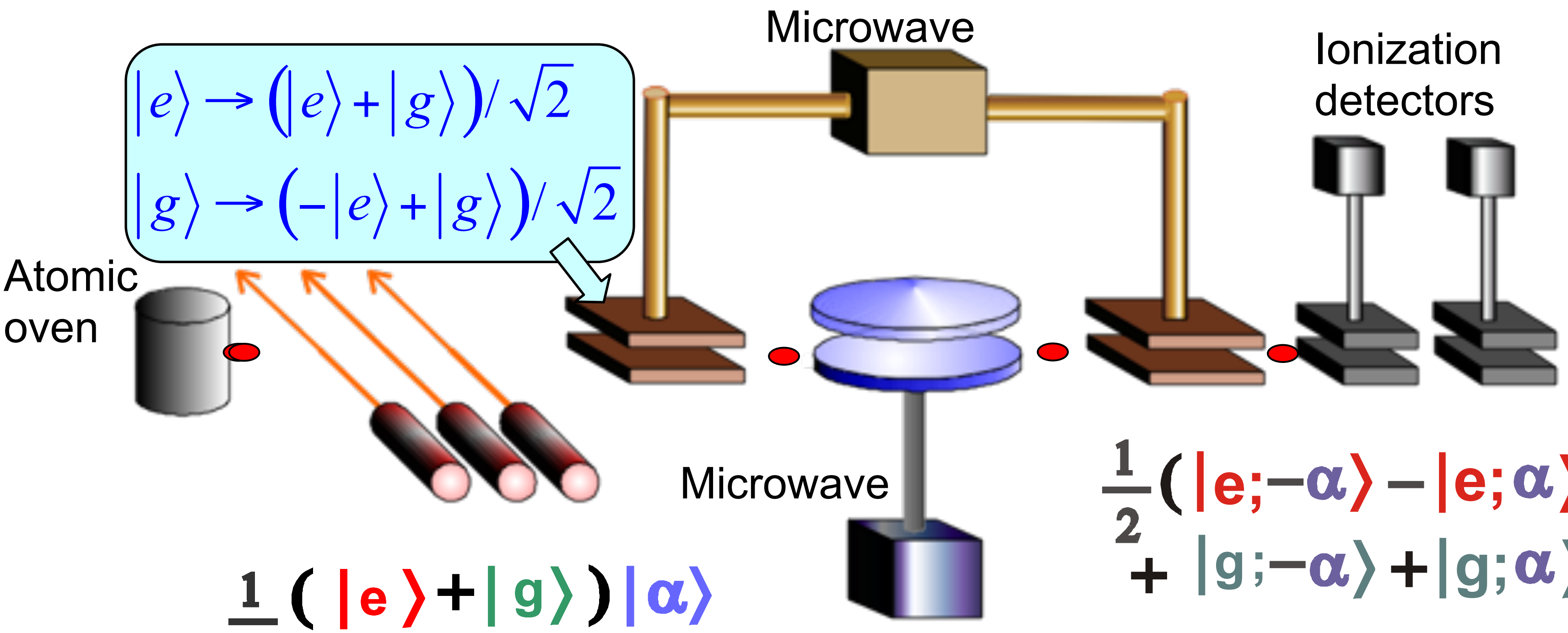
(Received 10 September 1996)

MEASURING DECOHERENCE IN CAVITY QED

$$|e\rangle \rightarrow (|e\rangle + |g\rangle)/\sqrt{2}$$
$$|g\rangle \rightarrow (-|e\rangle + |g\rangle)/\sqrt{2}$$



$$\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)|\alpha\rangle$$

$$\frac{1}{\sqrt{2}}(|e;-\alpha\rangle + |g;\alpha\rangle)$$

$$\frac{1}{2}(|e;-\alpha\rangle - |e;\alpha\rangle + |g;-\alpha\rangle + |g;\alpha\rangle)$$

M. Brune, J.M. Raimond, S. Haroche, L.D. et N. Zagury, PRA **45**, 5193 (1992)

$$\boxed{|\Psi\rangle_F \propto |\alpha\rangle + e^{i\psi}|-\alpha\rangle}$$

$$e \Rightarrow \psi = \pi$$
$$g \Rightarrow \psi = 0$$

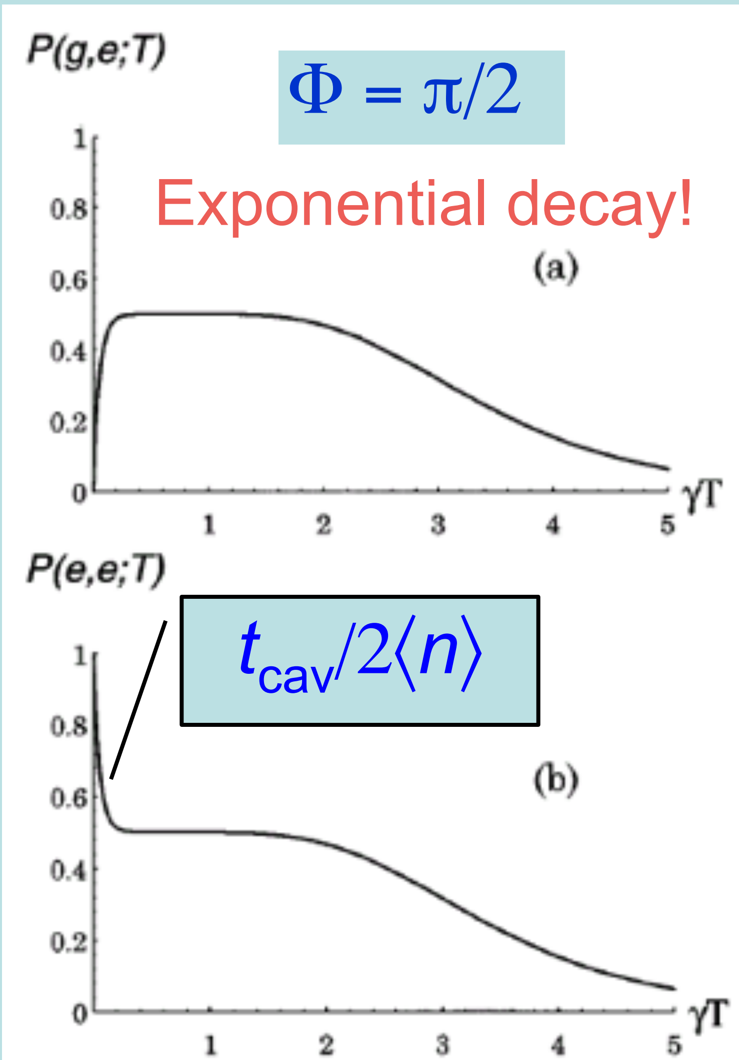
HOW TO DETECT THE COHERENCE?

Send a second atom! [L.D., A. Maali, M. Brune, J.M. Raimond, and S. Haroche, PRL **71**, 2360 (1993); L.D., M. Brune, J.M. Raimond, and S. Haroche, PRA **53**, 1295 (1996)].

Results for phase difference equal to π:

- Coherent superposition: preparation and probing atoms detected in the same state $\rightarrow P_{ee}=1$
- Statistical mixture: second atom detected in $|e\rangle$ or $|g\rangle$ with 50 % chance $\rightarrow P_{ee}=1/2$

EFFECT OF DISSIPATION

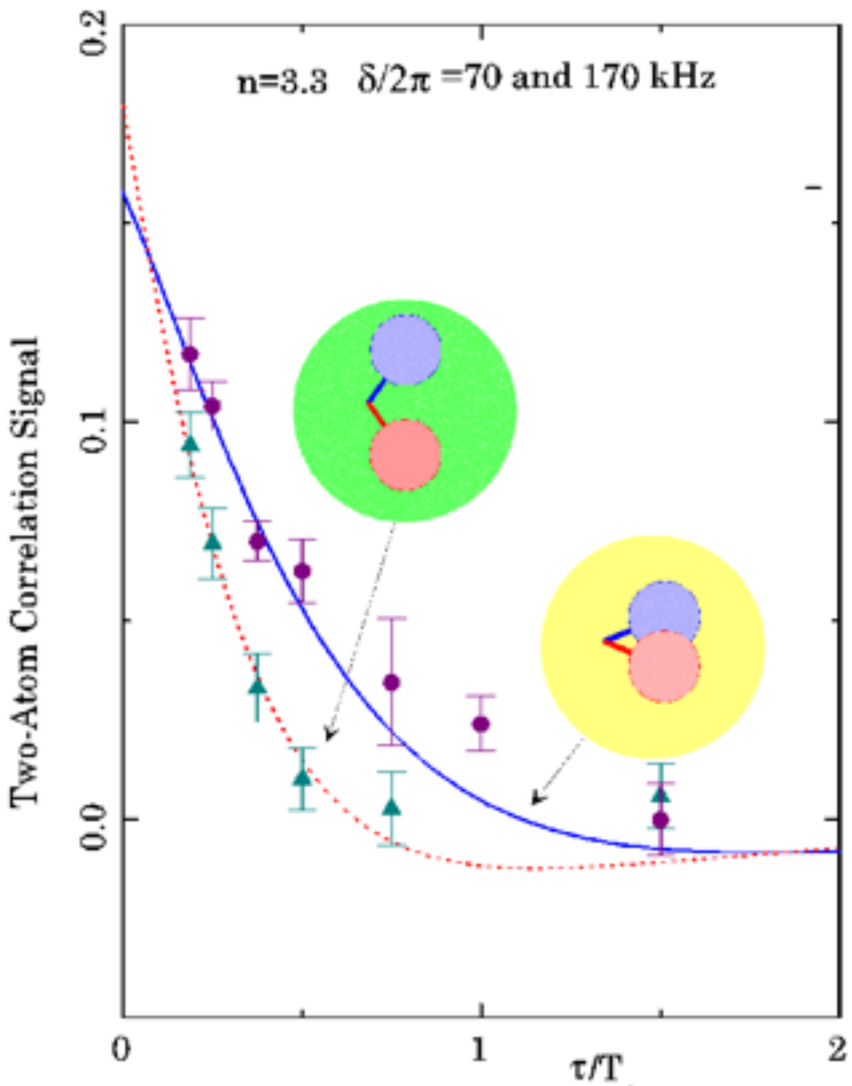


L. D., M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev. A **53**, 1295 (1996).

Decoherence time: t_{cav}/D

$\langle n \rangle$ = average number of photons in cavity

EXPERIMENTAL RESULTS



[Brune et al., PRL **77**, 4887 (1996)]

Plot of $P_{ee} - P_{eg}$

QUIZZ

Where has the coherence gone? Can you describe what happens with the environment?

Why are coherent states more stable than superpositions?

Does this answer Einstein's question?

1935

The New York Times

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

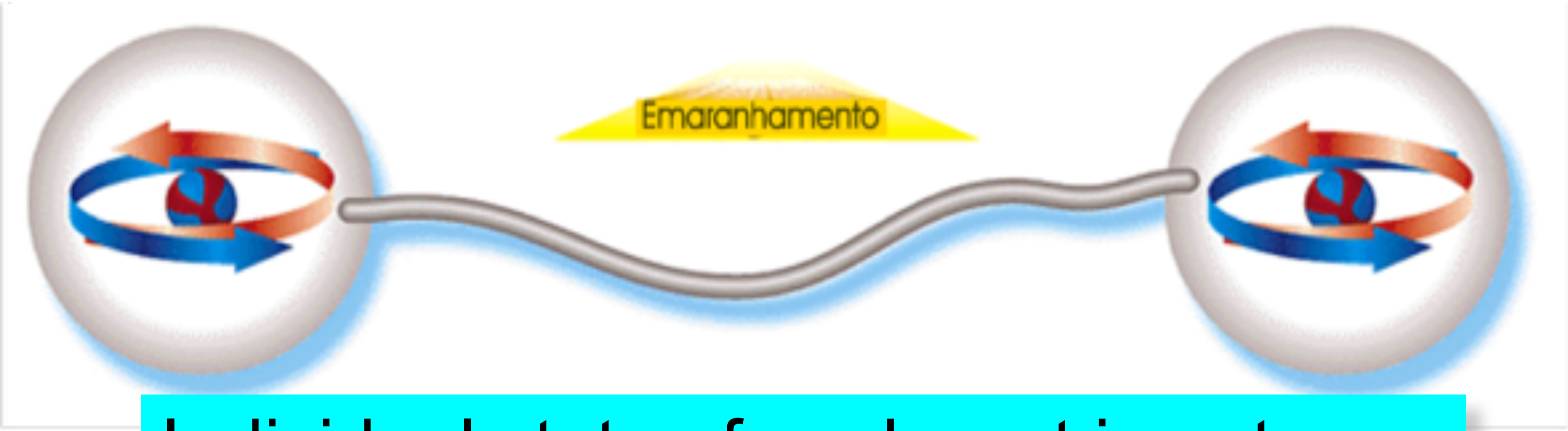
Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

PRINCETON, N. J., May 3.—Professor Albert Einstein will attack science's important theory of quantum mechanics, a theory of which he was a sort of grandfather. He concludes that while it is "correct" it is not "complete."

With two colleagues at the Institute for Advanced Study here, the noted scientist is about to report to the American Physical Society what is wrong with the theory of quantum mechanics, it has been learned exclusively by Science Service.

The quantum theory, with which science predicts with some success inter-atomic happenings, does not meet the requirements for a satisfactory physical theory, Professor Einstein will report in a joint paper with Dr. Boris Podolsky and Dr. N. Rosen.

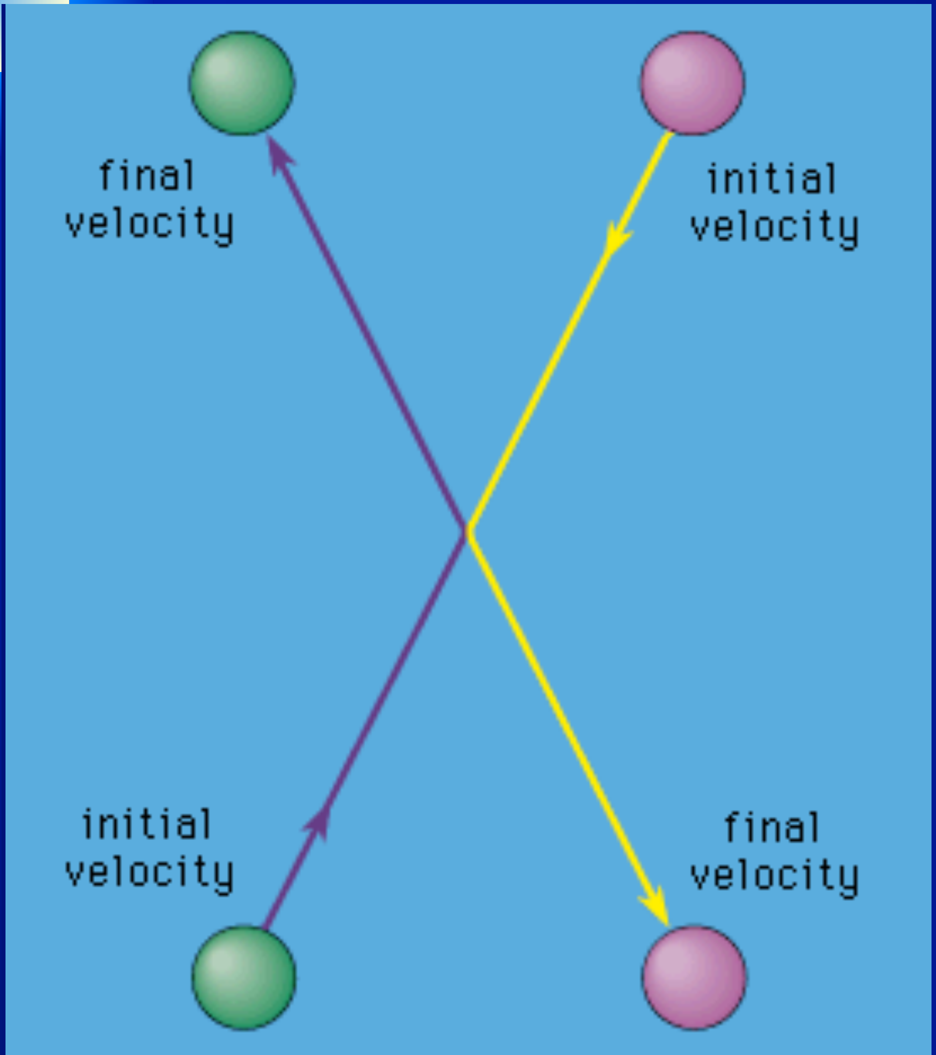
ENTANGLED STATES



Individual state of each part is not known: only global state is known!

State of the system cannot be written as a product of the states of the two systems

Classical X quantum situation



Global properties:
- Center-of-mass momentum = total momentum
- Relative position

Individual properties:
- Momentum of each particle
- Position of each particle

Schrödinger on Entanglement

Naturwissenschaften **23**, 807 (1935)

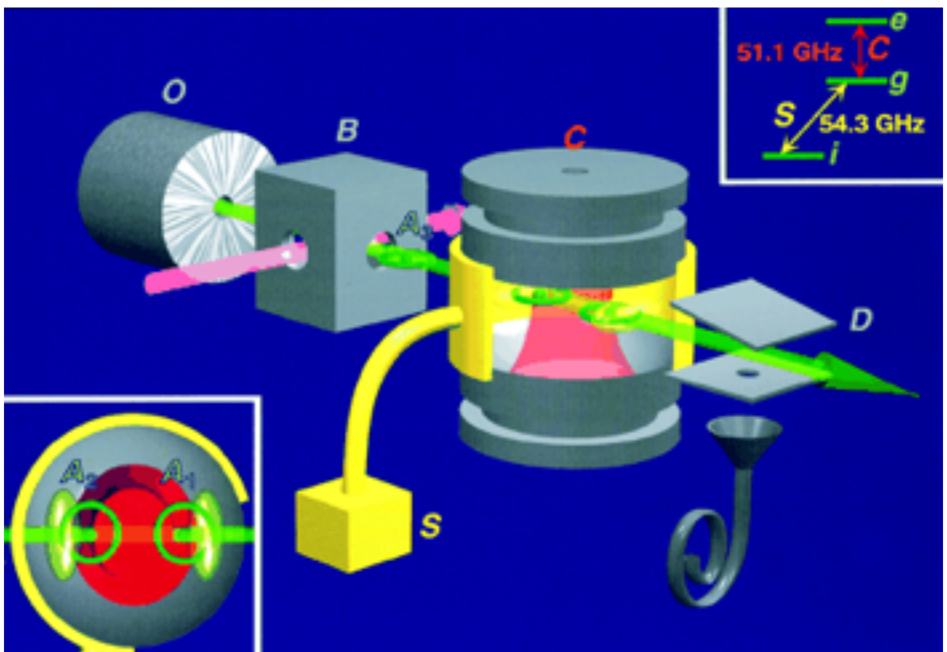
"This is the reason that knowledge of the individual systems can decline to the scantiest, even zero, while that of the combined system remains continually maximal. Best possible knowledge of a whole does not include best possible knowledge of its parts – and that is what keeps coming back to haunt us."

ATOM-PHOTON ENTANGLEMENT

16 JUNE 2000 VOL 288 SCIENCE

Step-by-Step Engineered Multiparticle Entanglement

Arno Rauschenbeutel, Gilles Nogues, Stefano Osnaghi, Patrice Bertet, Michel Brune, Jean-Michel Raimond,* Serge Haroche



Blinov et al, C. *Nature* **428**, 153 (2004)

MULTI-PARTITE ENTANGLEMENT

Vol 438|1 December 2005|doi:10.1038/nature04251 nature

LETTERS

Creation of a six-atom 'Schrödinger cat' state

D. Leibfried[1], E. Knill[1], S. Seidelin[1], J. Britton[1], R. B. Blakestad[1], J. Chiaverini[1]†, D. B. Hume[1], W. M. Itano[1], J. D. Jost[1], C. Langer[1], R. Ozeri[1], R. Reichle[1] & D. J. Wineland[1]

Vol 438|1 December 2005|doi:10.1038/nature04279 nature

LETTERS

Scalable multiparticle entanglement of trapped ions

H. Häffner[1,3], W. Hänsel[1], C. F. Roos[1,3], J. Benhelm[1,3], D. Chek-al-kar[1], M. Chwalla[1], T. Körber[1,3], U. D. Rapol[1,3], M. Riebe[1], P. O. Schmidt[1], C. Becher[1]†, O. Gühne[3], W. Dür[2,3] & R. Blatt[1,3]

Multiparticle entanglement

PRL 106, 130506 (2011) PHYSICAL REVIEW LETTERS week ending 1 APRIL 2011

14-Qubit Entanglement: Creation and Coherence

Thomas Monz,[1] Philipp Schindler,[1] Julio T. Barreiro,[1] Michael Chwalla,[1] Daniel Nigg,[1] William A. Coish,[2,3] Maximilian Harlander,[1] Wolfgang Hänsel,[4] Markus Hennrich,[1,*] and Rainer Blatt[1,4]

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[3]*Department of Physics, McGill University, Montreal, Quebec, Canada H3A 2T8*
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(Received 30 September 2010; published 31 March 2011)

Multiphoton entanglement

letters to nature

NATURE | VOL 430 | 1 JULY 2004 | www.nature.com/nature

Experimental demonstration of five-photon entanglement and open-destination teleportation

Zhi Zhao[1], Yu-Ao Chen[1], An-Ning Zhang[1], Tao Yang[1], Hans J. Briegel[2] & Jian-Wei Pan[1,3]

[1] *Department of Modern Physics and Hefei National Laboratory for Physical Sciences at Microscale, University of Science and Technology of China, Hefei,*

nature physics | VOL 3 | FEBRUARY 2007 | www.nature.com/naturephysics **LETTERS**

Experimental entanglement of six photons in graph states

CHAO-YANG LU[1]*, XIAO-QI ZHOU[1], OTFRIED GÜHNE[2], WEI-BO GAO[1], JIN ZHANG[1], ZHEN-SHENG YUAN[1], ALEXANDER GOEBEL[3], TAO YANG[1] AND JIAN-WEI PAN[1,3]*

MACROSCOPIC SIGNATURE OF ENTANGLEMENT

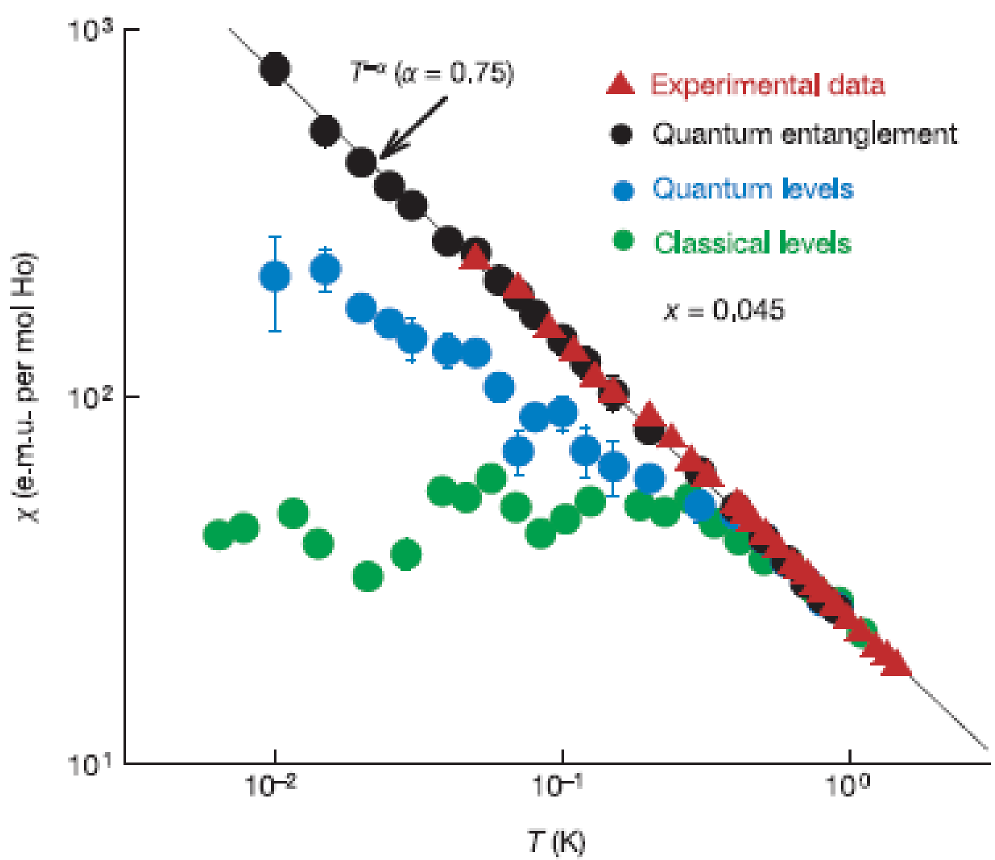
NATURE | VOL 425 | 4 SEPTEMBER 2003 | www.nature.com/nature

Entangled quantum state of magnetic dipoles

S. Ghosh[1], T. F. Rosenbaum[1], G. Aeppli[2] & S. N. Coppersmith[3]

Magnetic salt

$LiHo_xY_{1-x}F_4$



ENTANGLEMENT AS A RESOURCE

- Entanglement is useful for communications and quantum computation

Dynamics of entanglement

- Multiparticle system, initially entangled, with individual couplings of particles to independent environments: each particle undergoes decay, dephasing, diffusion.
- How is local dynamics related to nonlocal loss of entanglement?
- How does loss of entanglement scale with number of particles?
- Need measure of entanglement!

Reminder of density operator

If one measures a complete set of commuting operators, one determines the state of a system

Suppose however one does not measure a complete set, or that the measurements are not precise.

Then one can only say that there is a probability p_i that the state of the system is $|\psi_i\rangle$

Then, the average of any operator A is

$$\langle \hat{A} \rangle = \sum_i p_i \langle \psi_i | \hat{A} | \psi_i \rangle = \mathrm{Tr}\left(\hat{\rho}\hat{A}\right)$$

where $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ → Density operator

Entangled and separable states

- Separable states:

– Pure states:

$$|\Psi_{12\ldots n}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots |\psi_n\rangle$$

– Mixed states (R. F. Werner, PRA, 1989):

$$\rho_{12\ldots n} = \sum_{\mu} p_\mu \rho_1^\mu \otimes \rho_2^\mu \otimes \ldots \rho_n^\mu$$

$$0 \le p_\mu \le 1$$

- Entangled state: non-separable

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$$

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$$

Bell states - Maximally entangled states:

$$\rho_{A,B} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Reminder of partial trace

If one has an entangled state of two parties, and one is interested in calculating the averages of operators acting only on one of the parties, it is useful to consider the density operator corresponding to it, which is obtained by calculating the partial trace of the total density operator with respect to the other party.

Example: $|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$ $\qquad \hat{\rho} = |\psi\rangle_{AB\,AB}\langle\psi|$

$$\hat{\rho} = \frac{1}{2}\left(\begin{array}{cc|cc} 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow \rho_A = \mathrm{Tr}_B\left(|\psi\rangle_{AB\,AB}\langle\psi|\right) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

SHOW THAT!

Measures of entanglement for pure states

Von Neumann entropy

$$S_N(\rho_r) = -\mathrm{Tr}[\rho_r \log_2 \rho_r]$$

$\rho_r \longrightarrow$ reduced density matrix of A or B

Linear entropy

$$S_L(\rho_r) = 2\left(1 - \mathrm{Tr}\rho_r^2\right)$$

Separable state (two qubits):

$$S(\rho_r) = 0$$

Maximally entangled state:

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow S(\rho_A) = 1$$

What about mixed states?

- Let $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|,\ p_i \geq 0$
- Could try to define $M(\rho) = \sum_i p_i M(\Psi_i)$
- But this decomposition is not unique!

$$\rho = \frac{1}{2}|\psi^+\rangle\langle\psi^+| + \frac{1}{2}|\psi^-\rangle\langle\psi^-| = \frac{1}{2}|01\rangle\langle 01| + \frac{1}{2}|10\rangle\langle 10|$$

$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Entanglement of mixed states

- Define

$$M(\rho) = \inf_{(p_i,\Psi_i)} \sum_i p_i M(\Psi_i), \text{ with } p_i \geq 0,$$

$$\text{and } \rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$$

- Difficult to calculate!

A mathematical interlude: partial transposition of a matrix

Transposition: a positive map $T : \rho \to \rho^T$

$$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \begin{pmatrix} \rho_{00} & \rho_{10} \\ \rho_{01} & \rho_{11} \end{pmatrix}$$

Partial transposition: $1_A \otimes T_B : \rho \to \rho^{T_B}$

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$$

Negative eigenvalue!

$$\frac{1}{2}\left(\begin{array}{cc|cc} 1 & 0 & 0 & \pm 1 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \pm 1 & 0 & 0 & 1 \end{array}\right) \to \frac{1}{2}\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 0 & \pm 1 & 0 \\ \hline 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \quad \Lambda = -1/2$$

Mixed states: Separability criterium

- If ρ is separable, then the partially transposed matrix is positive (Asher Peres, PRL, 1996):

$$\rho = \sum_i p_i \rho_i^A \otimes \rho_i^B \Rightarrow \rho^{T_B} = \sum_i p_i \rho_i^A \otimes (\rho_i^B)^T$$

- For 2X2 and 2X3 systems, ρ is separable iff it remains a density operator under the operation of partial transposition (Horodecki family 1996) - that is, it has a partial positive transpose (PPT)

Negativity as a measure of entanglement

$$\boxed{\mathcal{N}(\rho_{AB}) \equiv 2\sum_i |\lambda_{i-}|}$$

$\lambda_{i\_} \rightarrow$ Negative eigenvalues of partially transposed matrix

$\mathcal{N}$=1 for a Bell state

Dimensions higher than 6: $\mathcal{N}$=0 does not imply separability!

Environment-Induced Sudden Death of Entanglement

M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, L. Davidovich*

We demonstrate the difference between local, single-particle dynamics and global dynamics of entangled quantum systems coupled to independent environments. Using an all-optical experimental setup, we showed that, even when the environment-induced decay of each system is asymptotic, quantum entanglement may suddenly disappear. This "sudden death" constitutes yet another distinct and counterintuitive trait of entanglement.

The real-world success of quantum computation (*1*, *2*) and communication (*3*–*9*) relies on the longevity of entanglement in multiparticle quantum states. The presence of decoherence (*10*) in communication channels and computing devices, which stems from the unavoidable interaction between these systems and the environment, degrades the entanglement when the particles propagate or the computation evolves. Decoherence leads to local dynamics, associated with single-particle dissipation, diffusion, and decay, as well as to global dynamics, which may provoke the disappearance of entanglement at a finite time (*11*–*15*). This phenomenon, known as "entanglement sudden death" (*15*), is strikingly different from single-particle dynamics, which occurs asymptotically, and has thus stimulated much recent theoretical work (*11*–*15*). Here we demonstrate the sudden death of entanglement of a two-qubit system under the influence of independent environ-

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PHYSICAL REVIEW A **78**, 022322 (2008)

Experimental investigation of the dynamics of entanglement: Sudden death, complementarity, and continuous monitoring of the environment

A. Salles,[1,*] F. de Melo,[1,2] M. P. Almeida,[1,3] M. Hor-Meyll,[1] S. P. Walborn,[1] P. H. Souto Ribeiro,[1] and L. Davidovich[1]

[1]*Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ 21941-972, Brazil*

[2]*Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, Hermann-Herder-Strasse 3, D-79104 Freiburg, Germany*

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A paradigmatic example: Atomic decay

Our strategy: follow evolution as a function of p, not t

- Qubit states: $|0\rangle \leftrightarrow |g\rangle, |1\rangle \leftrightarrow |e\rangle$
- "Amplitude channel":

$$|g\rangle_S \otimes |0\rangle_E \to |g\rangle_S \otimes |0\rangle_E$$
$$|e\rangle_S \otimes |0\rangle_E \to \sqrt{1-p}|e\rangle_S \otimes |0\rangle_E + \sqrt{p}|g\rangle_S \otimes |1\rangle_E$$

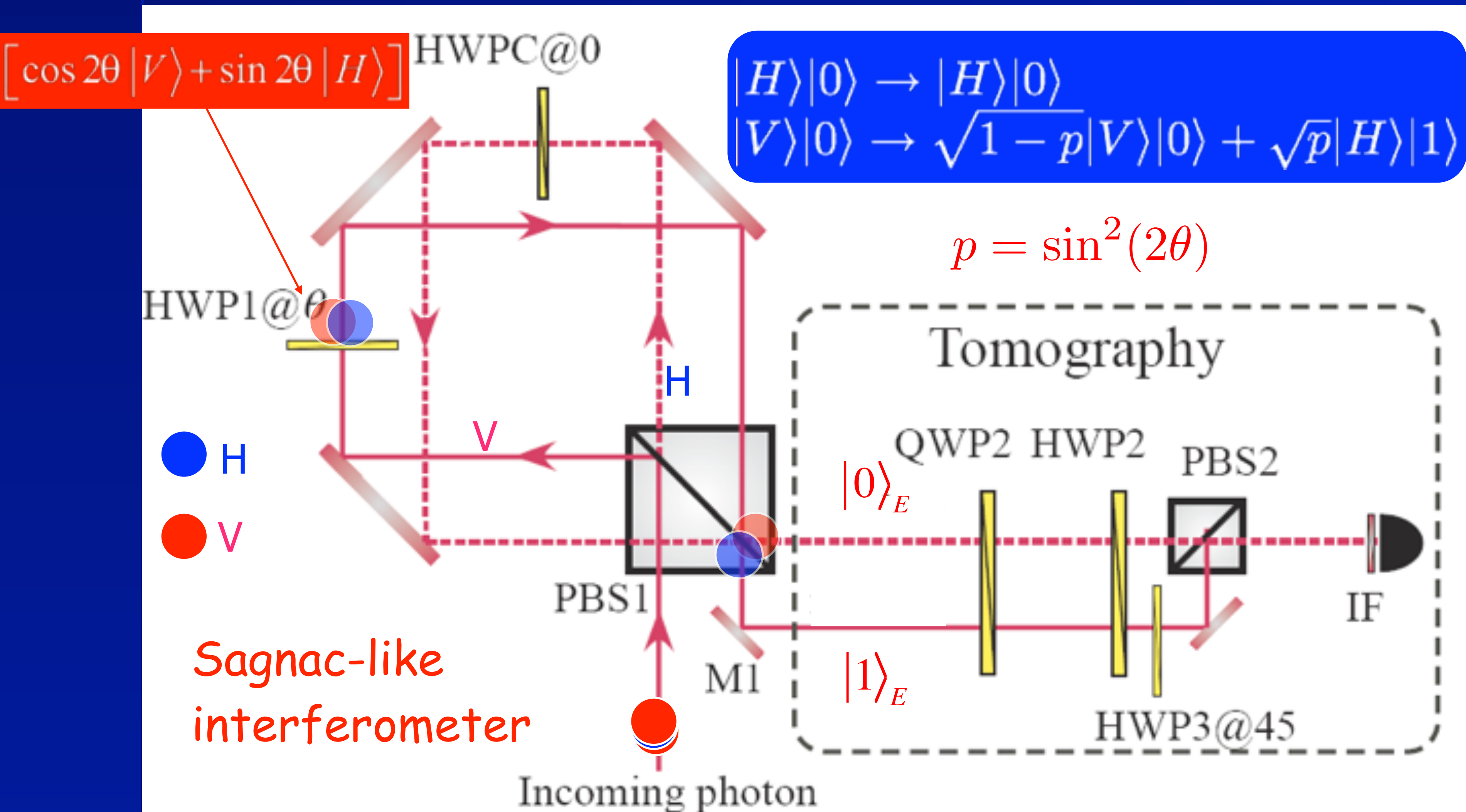
$$p = 1 - \exp(-\Gamma t) \Longrightarrow$$

Usual master equation for decay of two-level atom, upon tracing on environment (Markovian approximation)

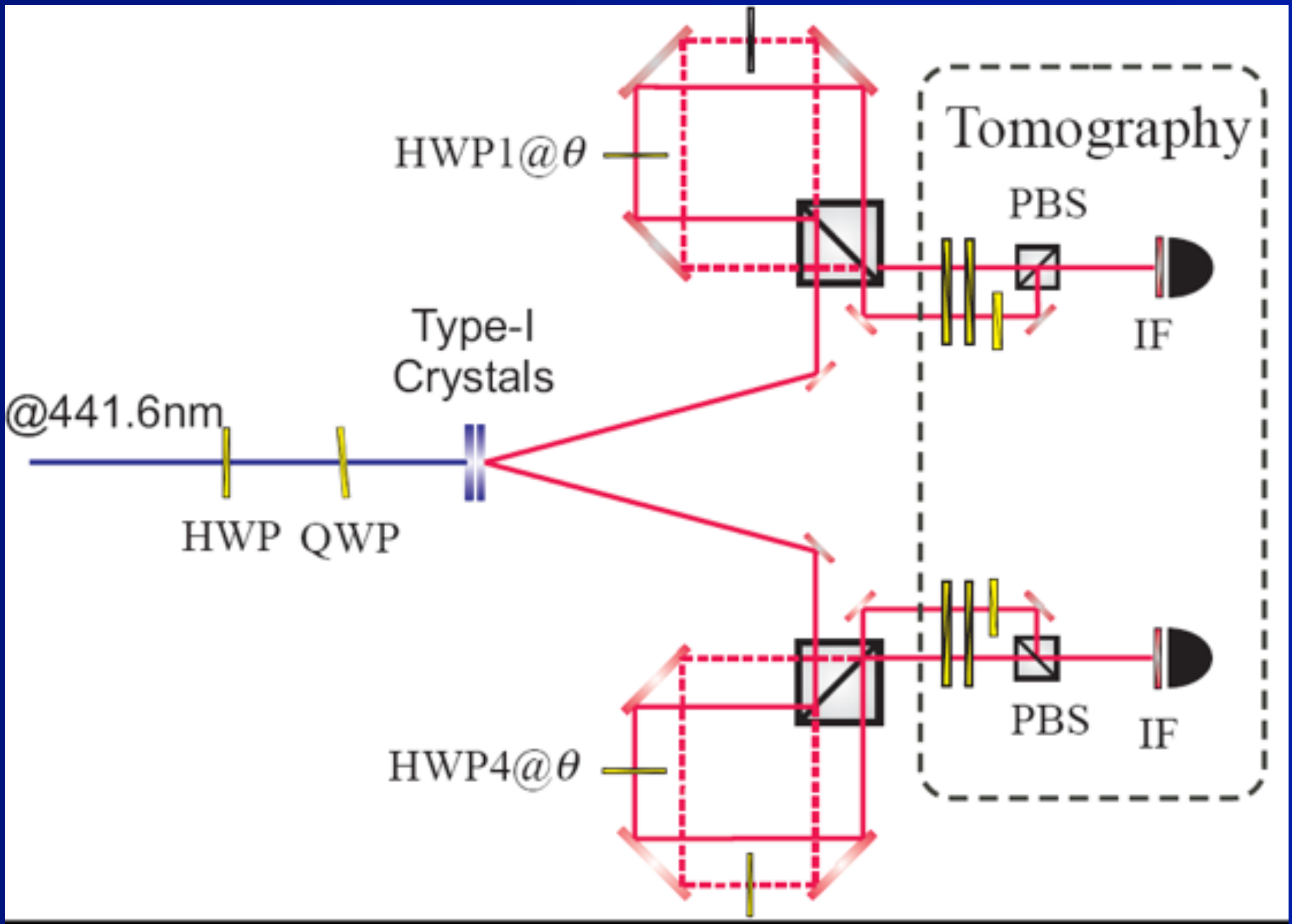
Weisskopf and Wigner (1930)!

Apply evolution to two qubits, take trace with respect to environment degrees of freedom, find evolution of two-qubit reduced density matrix, calculate entanglement

Realization of amplitude map with photons



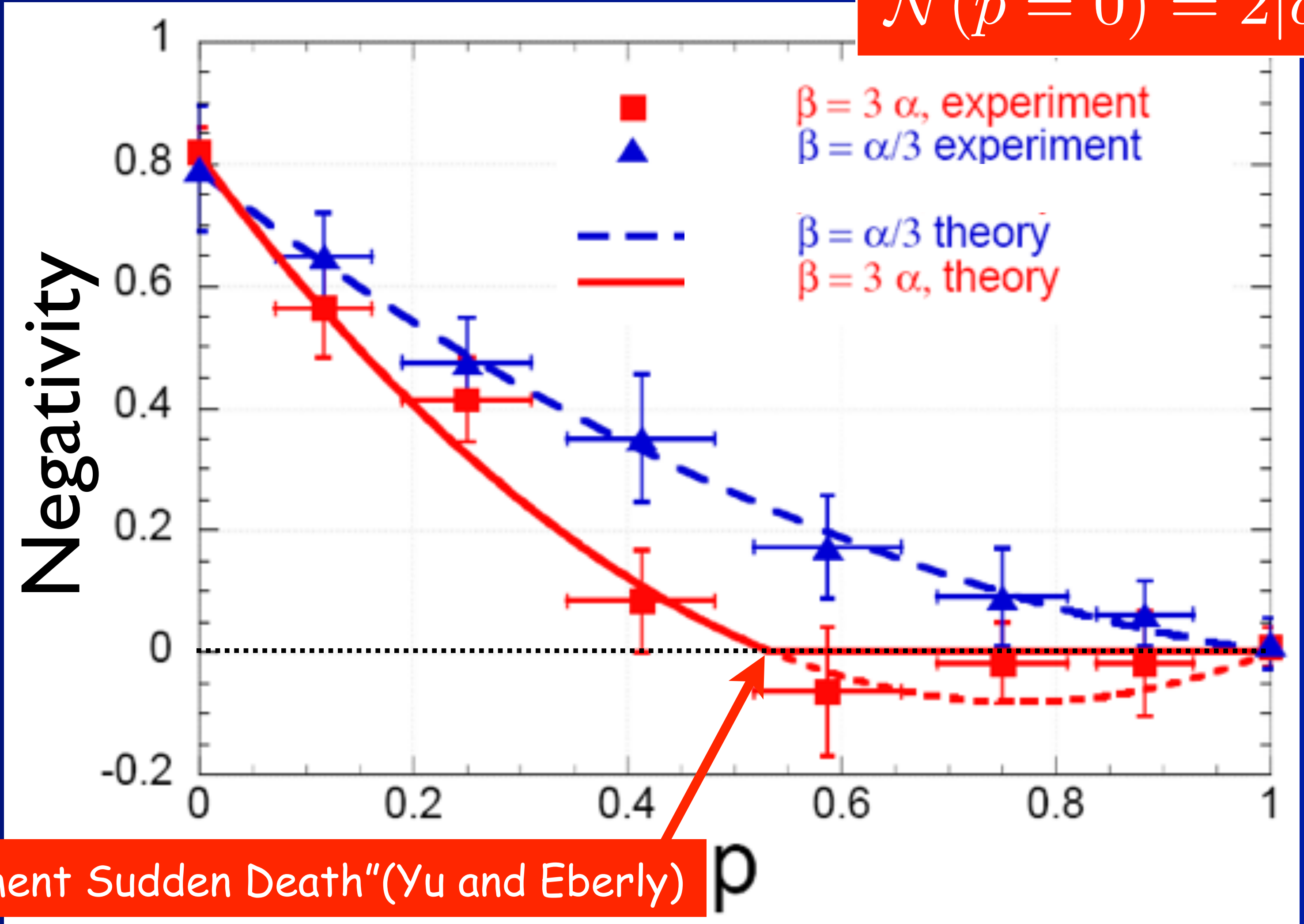
Investigating the dynamics of entanglement



"Sudden death" of entanglement

$$|\Psi(0)\rangle = \alpha|gg\rangle + \beta|ee\rangle$$

$$\mathcal{N}(p=0) = 2|\alpha\beta|$$



"Entanglement Sudden Death"(Yu and Eberly)

Decay of entanglement for N qubits, other environments?

PRL **100,** 080501 (2008) PHYSICAL REVIEW LETTERS week ending 29 FEBRUARY 2008

Scaling Laws for the Decay of Multiqubit Entanglement

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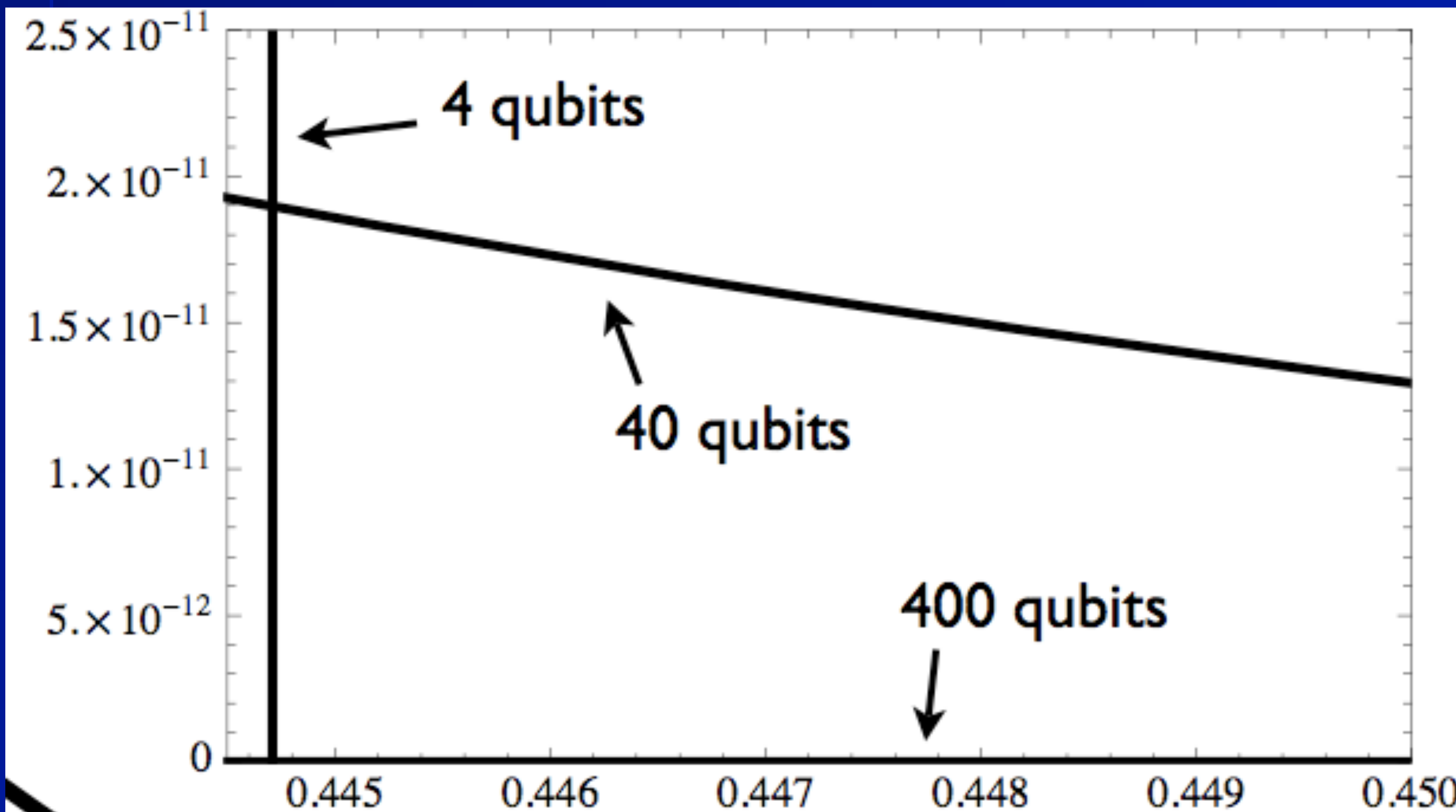
(Received 23 October 2007; published 27 February 2008)

$$|\Psi_0\rangle = \alpha|0\rangle^{\otimes N} + \beta|1\rangle^{\otimes N}$$

- Independent individual environments

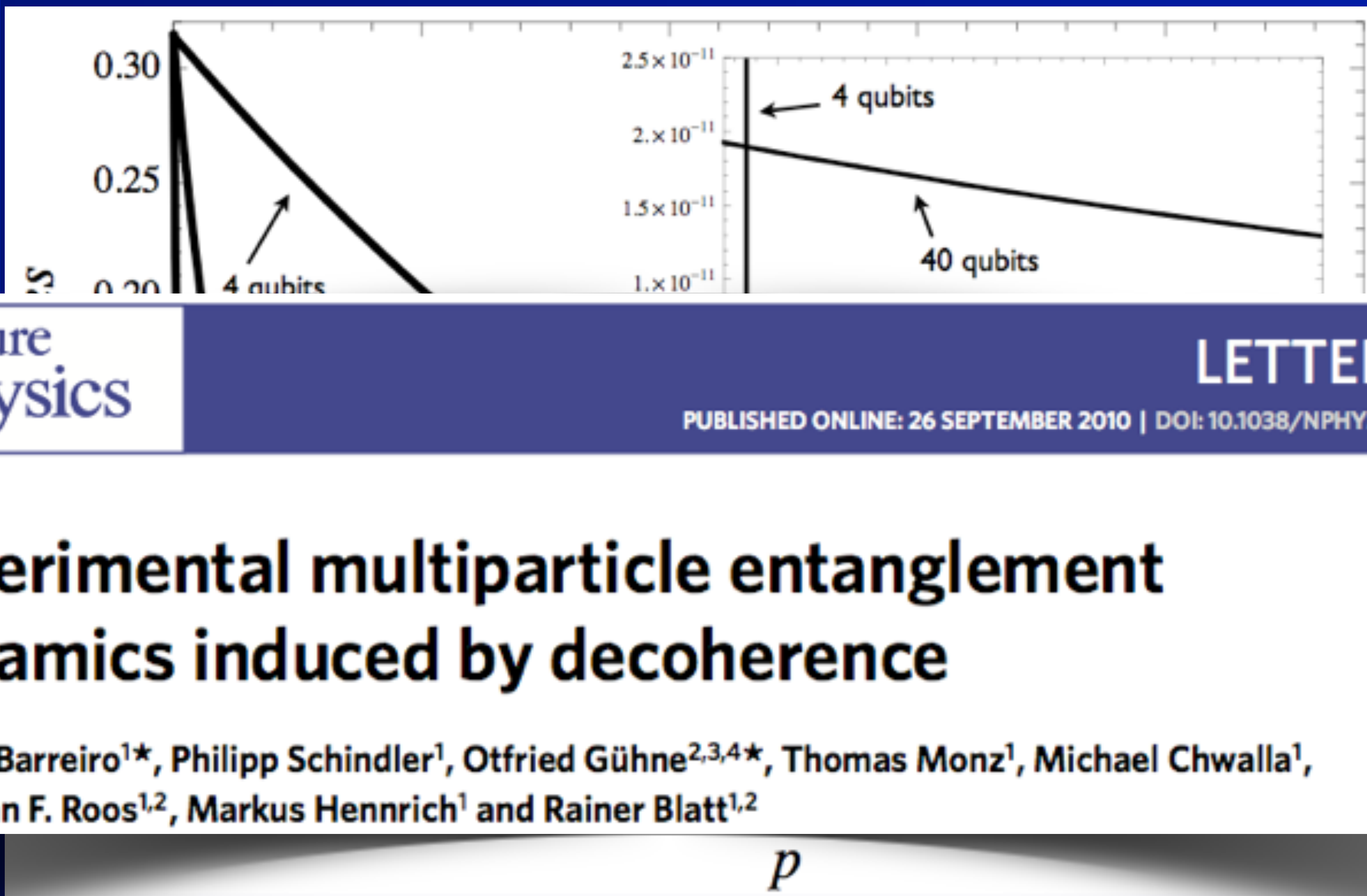
Does entanglement become more robust with increasing N?

$|\Psi_0\rangle = \alpha|0\rangle^{\otimes N} + \beta|1\rangle^{\otimes N}$ $\quad \mathcal{E}_i^D \rho_i = (1-p)\rho_i + (p)1/2$



Is ESD relevant for many particles?

$$|\Psi_0\rangle = \alpha|0\rangle^{\otimes N} + \beta|1\rangle^{\otimes N} \quad \mathcal{E}_i^D \rho_i = (1-p)\rho_i + (p)1/2$$



nature physics

LETTERS

PUBLISHED ONLINE: 26 SEPTEMBER 2010 | DOI: 10.1038/NPHYS1781

Experimental multiparticle entanglement dynamics induced by decoherence

Julio T. Barreiro[1]★, Philipp Schindler[1], Otfried Gühne[2,3,4]★, Thomas Monz[1], Michael Chwalla[1], Christian F. Roos[1,2], Markus Hennrich[1] and Rainer Blatt[1,2]

Role of environment

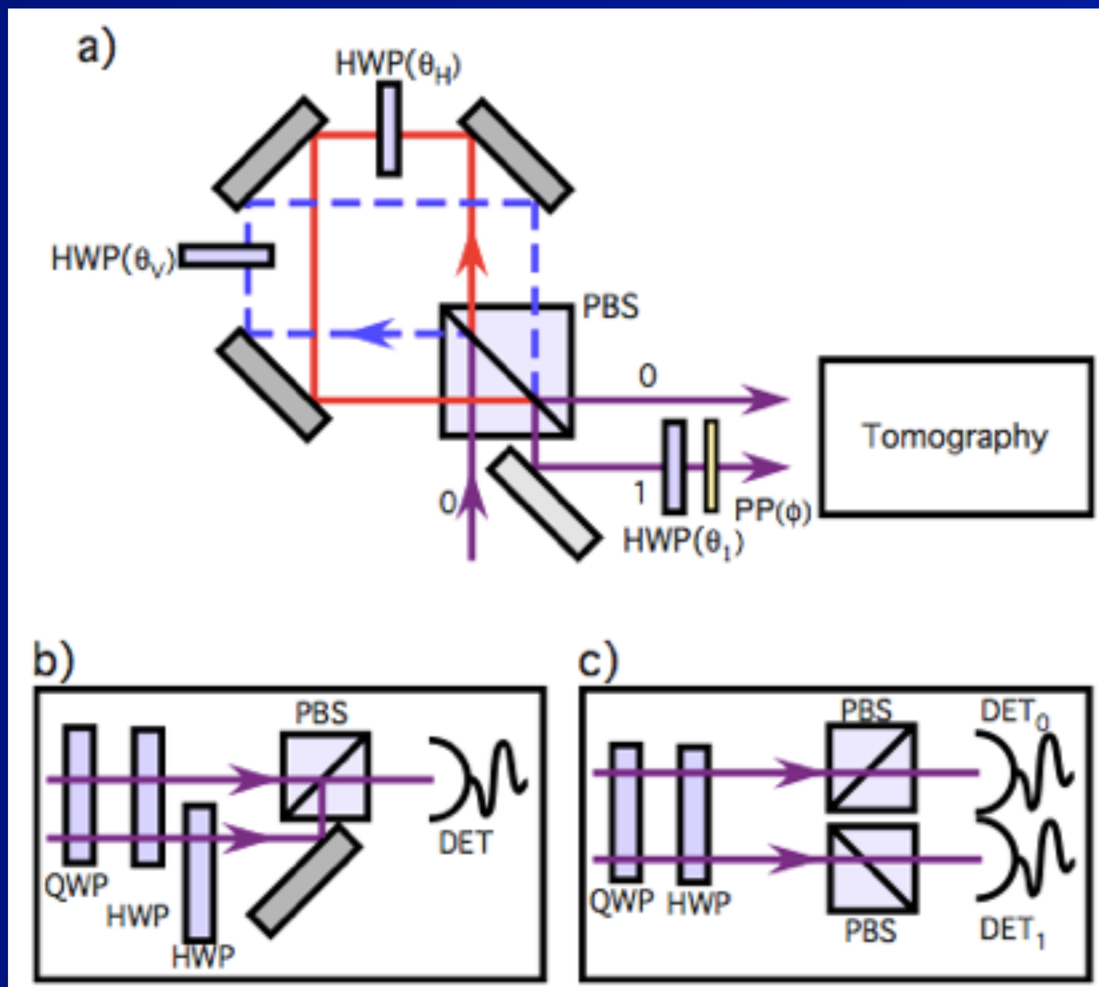
- Usually one traces out environment, and one looks at irreversible evolution of system
- As entanglement decays and eventually disappears, what is its imprint onto the environment?

Measuring the environment?

PHYSICAL REVIEW A 78, 022322 (2008)

Experimental investigation of the dynamics of entanglement: Sudden death, complementarity, and continuous monitoring of the environment

A. Salles,[1,*] F. de Melo,[1,2] M. P. Almeida,[1,3] M. Hor-Meyll,[1] S. P. Walborn,[1] P. H. Souto Ribeiro,[1] and L. Davidovich[1]



PHYSICAL REVIEW A **85**, 012314 (2012)

Experimental investigation of dynamical invariants in bipartite entanglement

O. Jiménez Farías, A. Valdés-Hernández, G. H. Aguilar, P. H. Souto Ribeiro, S. P. Walborn, and L. Davidovich
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Xiao-Feng Qian and J. H. Eberly
Rochester Theory Center and the Department of Physics & Astronomy, University of Rochester, Rochester, New York 14627, USA

PRL **109,** 150403 (2012) PHYSICAL REVIEW LETTERS week ending 12 OCTOBER 2012

Observation of the Emergence of Multipartite Entanglement Between a Bipartite System and its Environment

O. Jiménez Farías, G. H. Aguilar, A. Valdés-Hernández, P. H. Souto Ribeiro, L. Davidovich, and S. P. Walborn
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PHYSICAL REVIEW A **89**, 022339 (2014)

Flow of quantum correlations from a two-qubit system to its environment

G. H. Aguilar,[1] O. Jiménez Farías,[1] A. Valdés-Hernández,[1,2] P. H. Souto Ribeiro,[1] L. Davidovich,[1] and S. P. Walborn[1]
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PRL **113,** 240501 (2014) PHYSICAL REVIEW LETTERS week ending 12 DECEMBER 2014

Experimental Entanglement Redistribution under Decoherence Channels

G. H. Aguilar,[*] A. Valdés-Hernández, L. Davidovich, S. P. Walborn, and P. H. Souto Ribeiro
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Key Issues Review

Open-system dynamics of entanglement: a key issues review

Leandro Aolita[1], Fernando de Melo[2] and Luiz Davidovich[3]

Collaborators: entanglement dynamics


Leandro Aolita


Fernando de Melo


Rafel Chaves

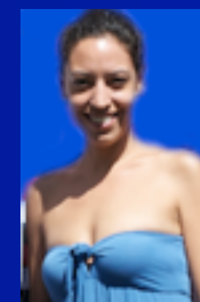

Malena Hor-Meyll


Alejo Salles


Osvaldo Jiménez-Farías

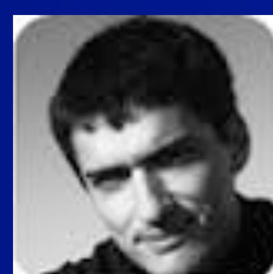

Gabriel Aguillar


Marcelo P. de Almeida


Andrea Valdés-Hernandéz


Paulo Souto Ribeiro


Stephen Walborn


Daniel Cavalcanti


Antonio Acín


Joe Eberly


Xiao-Feng Qian

THANK YOU!