Quantum Information with Continuous Variables of Light

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FAPESP

CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

- Representations of the quantized field state.
- Displacing, squeezing and splitting: unitary operations of the field.
- Measuring the field: interferometric techniques.
- Entanglement with continuous variables.
- Sources of non-classical states: squeezers and entanglers.
- Teleporters and other quantum machines.

Short introduction to Quantum Optics

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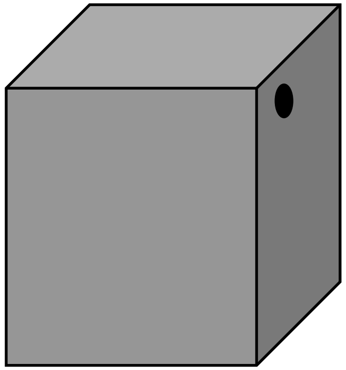
CAPES

Quantum Mechanics

Birth of a revolution at the dawn of the 20th Century



Introduction of the concept of “quanta”

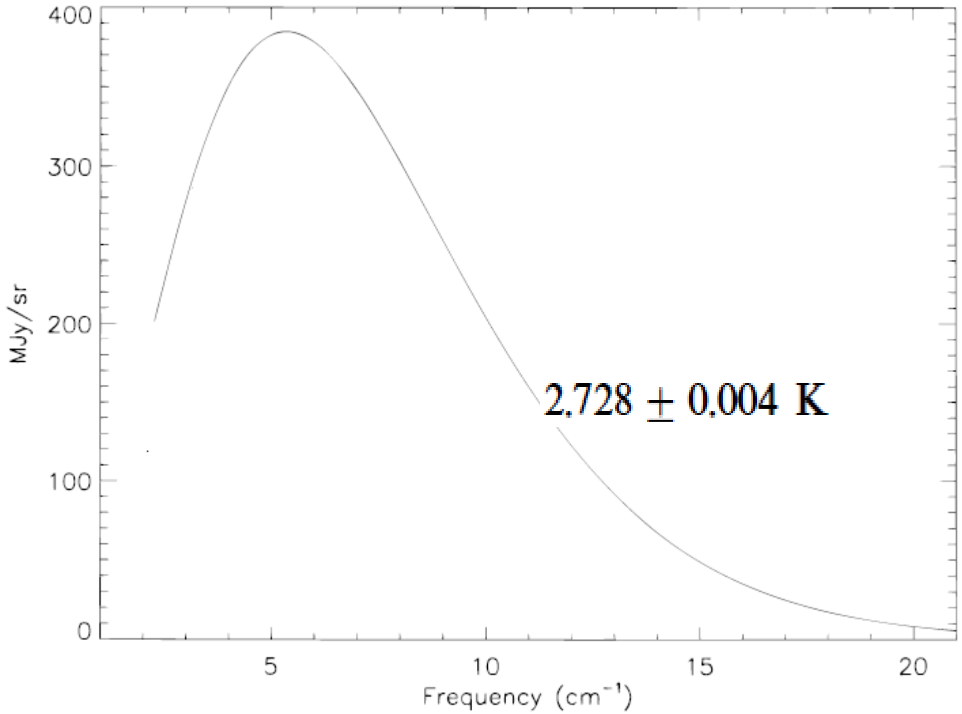


FIG. 4.—Uniform spectrum and fit to Planck blackbody (T). Uncertainties are a small fraction of the line thickness.

Energy per unit volume per unit wavelength $$S_\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

Energy per unit volume per unit frequency $$S_\upsilon = \frac{8\pi h}{c^3}\frac{\upsilon^3}{e^{h\upsilon/kT}-1}$$

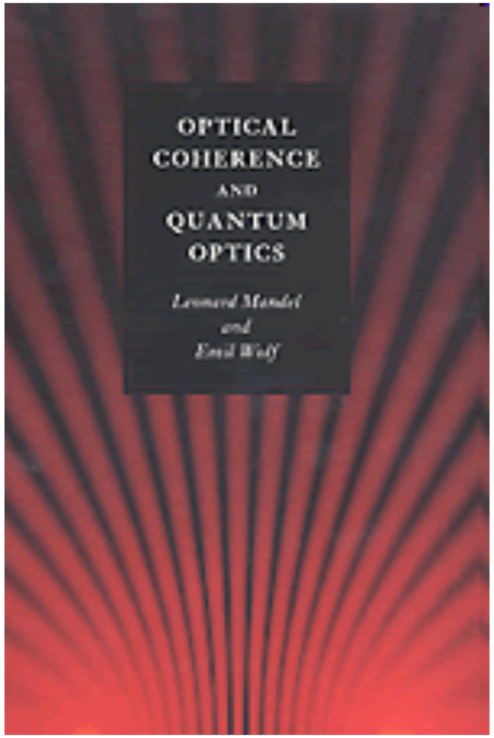
THE COSMIC MICROWAVE BACKGROUND SPECTRUM FROM THE FULL *COBE*[1] FIRAS DATA SET

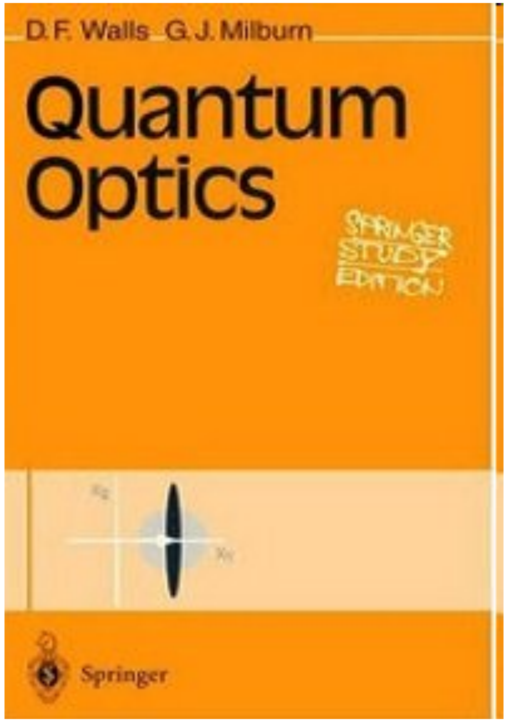
D. J. FIXSEN,[2] E. S. CHENG,[3] J. M. GALES,[2] J. C. MATHER,[3] R. A. SHAFER,[3] AND E. L. WRIGHT[4]

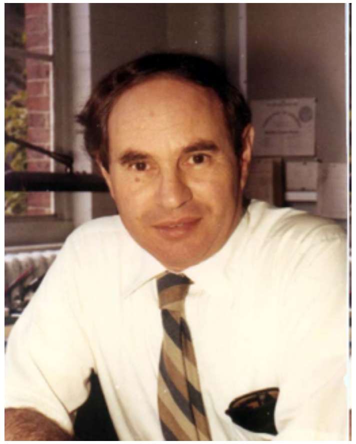
THE ASTROPHYSICAL JOURNAL, 473:576–587, 1996 December 20

Quantum Optics

Quantization of the Electromagnetic Field (on the shoulders...)







Optics

Maxwell Equations

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}.$$

Solution in a Box

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{\sqrt{\epsilon_0 L^3}} \sum_{\mathbf{k}} \sum_{s} i\omega \left[u_{\mathbf{k}s}(t) \boldsymbol{\epsilon}_{\mathbf{k}s} e^{i\mathbf{k}\cdot\mathbf{r}} - u^*_{\mathbf{k}s}(t) \boldsymbol{\epsilon}^*_{\mathbf{k}s} e^{-i\mathbf{k}\cdot\mathbf{r}} \right],$$

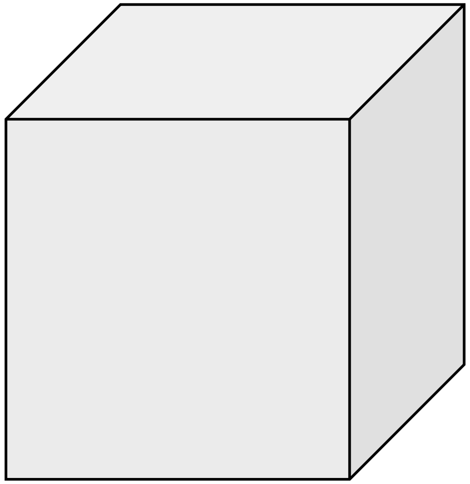
$$\mathbf{B}(\mathbf{r},t) = \frac{i}{\sqrt{\epsilon_0 L^3}} \sum_{\mathbf{k}} \sum_{s} \left[u_{\mathbf{k}s}(t)(\mathbf{k} \times \boldsymbol{\epsilon}_{\mathbf{k}s}) e^{i\mathbf{k}\cdot\mathbf{r}} - u^*_{\mathbf{k}s}(t)(\mathbf{k} \times \boldsymbol{\epsilon}^*_{\mathbf{k}s}) e^{-i\mathbf{k}\cdot\mathbf{r}} \right]$$

Wavevector

$$k_j = 2\pi n_j / L$$

Angular Frequency

$$\omega = c|\mathbf{k}|$$



Amplitude

$$u_{\mathbf{k}s}(t) = c_{\mathbf{k}s} e^{-i\omega t}$$

Polarization

$$\boldsymbol{\epsilon}^*_{\mathbf{k}s} \cdot \boldsymbol{\epsilon}_{\mathbf{k}s'} = \delta_{ss'}$$

$$\boldsymbol{\epsilon}^*_{\mathbf{k}1} \times \boldsymbol{\epsilon}_{\mathbf{k}2} = \mathbf{k}/k$$

Optics

Energy of the EM Field

$$\mathcal{H} = \frac{1}{2}\int_V \left[\epsilon_0 \mathbf{E}^2(\mathbf{r},t) + \frac{\mathbf{B}^2(\mathbf{r},t)}{\mu_0}\right] dv = 2\sum_{\mathbf{k}}\sum_s \omega^2 |u_{\mathbf{k}s}(t)|^2$$

Canonical Variables: going into Hamiltonian formalism

$$q_{\mathbf{k}s}(t) = u_{\mathbf{k}s}(t) + u^*_{\mathbf{k}s}(t)$$
$$p_{\mathbf{k}s}(t) = -i\omega\left[u_{\mathbf{k}s}(t) - u^*_{\mathbf{k}s}(t)\right]$$

Energy of the EM Field

$$\mathcal{H} = \frac{1}{2}\sum_{\mathbf{k}}\sum_s \left[p^2_{\mathbf{k}s}(t) + \omega^2 q^2_{\mathbf{k}s}(t)\right]$$

A very familiar Hamiltonian!

Sum over independent harmonic oscillators

Energy of the EM Field

$$\hat{\mathcal{H}} = \frac{1}{2}\sum_{\mathbf{k}}\sum_{s}\left[\hat{p}^2_{\mathbf{k}s}(t) + \omega^2\hat{q}^2_{\mathbf{k}s}(t)\right]$$

Using creation and annihilation operators, associated with amplitudes u_{ks}

$$\hat{q}_{\mathbf{k}s}(t) = \sqrt{\frac{\hbar}{2\omega}}\left[\hat{a}_{\mathbf{k}s}(t) + \hat{a}^{\dagger}_{\mathbf{k}s}(t)\right]$$

$$\hat{p}_{\mathbf{k}s}(t) = i\sqrt{\frac{\hbar\omega}{2}}\left[\hat{a}_{\mathbf{k}s}(t) - \hat{a}^{\dagger}_{\mathbf{k}s}(t)\right]$$

$$\left[\hat{a}_{\mathbf{k}s}(t), \hat{a}^{\dagger}_{\mathbf{k}'s'(t)}\right] = \delta^3_{\mathbf{k}\mathbf{k}'}\delta_{ss'}$$

$$\left[\hat{a}_{\mathbf{k}s}(t), \hat{a}_{\mathbf{k}'s'(t)}\right] = 0$$

$$\left[\hat{a}^{\dagger}_{\mathbf{k}s}(t), \hat{a}^{\dagger}_{\mathbf{k}'s'(t)}\right] = 0.$$

$$\hat{a}_{\mathbf{k}s}(t) = \hat{a}_{\mathbf{k}s}e^{-i\omega t} \qquad \hat{a}^{\dagger}_{\mathbf{k}s}(t) = \hat{a}^{\dagger}_{\mathbf{k}s}e^{i\omega t}$$

Energy of the EM Field

$$\hat{\mathcal{H}} = \sum_{\mathbf{k}} \sum_{s} \hbar\omega_{\mathbf{k}} \left(\hat{a}^{\dagger}_{\mathbf{k}s} \hat{a}_{\mathbf{k}s} + \frac{1}{2} \right)$$

Amplitudes of Electric and Magnetic Fields

$$\hat{\mathbf{E}}(\mathbf{r},t) = \frac{1}{L^{3/2}} \sum_{\mathbf{k}} \sum_{s} \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \left[i\hat{a}_{\mathbf{k}s} \boldsymbol{\epsilon}_{\mathbf{k}s} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} - i\hat{a}^{\dagger}_{\mathbf{k}s} \boldsymbol{\epsilon}^{*}_{\mathbf{k}s} e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \right]$$

$$\hat{\mathbf{B}}(\mathbf{r},t) = \frac{1}{L^{3/2}} \sum_{\mathbf{k}} \sum_{s} \sqrt{\frac{\hbar}{2\omega\epsilon_0}} \left[i\hat{a}_{\mathbf{k}s} (\mathbf{k}\times\boldsymbol{\epsilon}_{\mathbf{k}s}) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} - i\hat{a}^{\dagger}_{\mathbf{k}s} (\mathbf{k}\times\boldsymbol{\epsilon}^{*}_{\mathbf{k}s}) e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \right].$$

Field Quadratures – Classical Description

- Classical Description of the Electromagnectic Field:

Fresnel Representation of a single mode

$$E(t)=\mathrm{Re}\{\alpha\ \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]\}$$

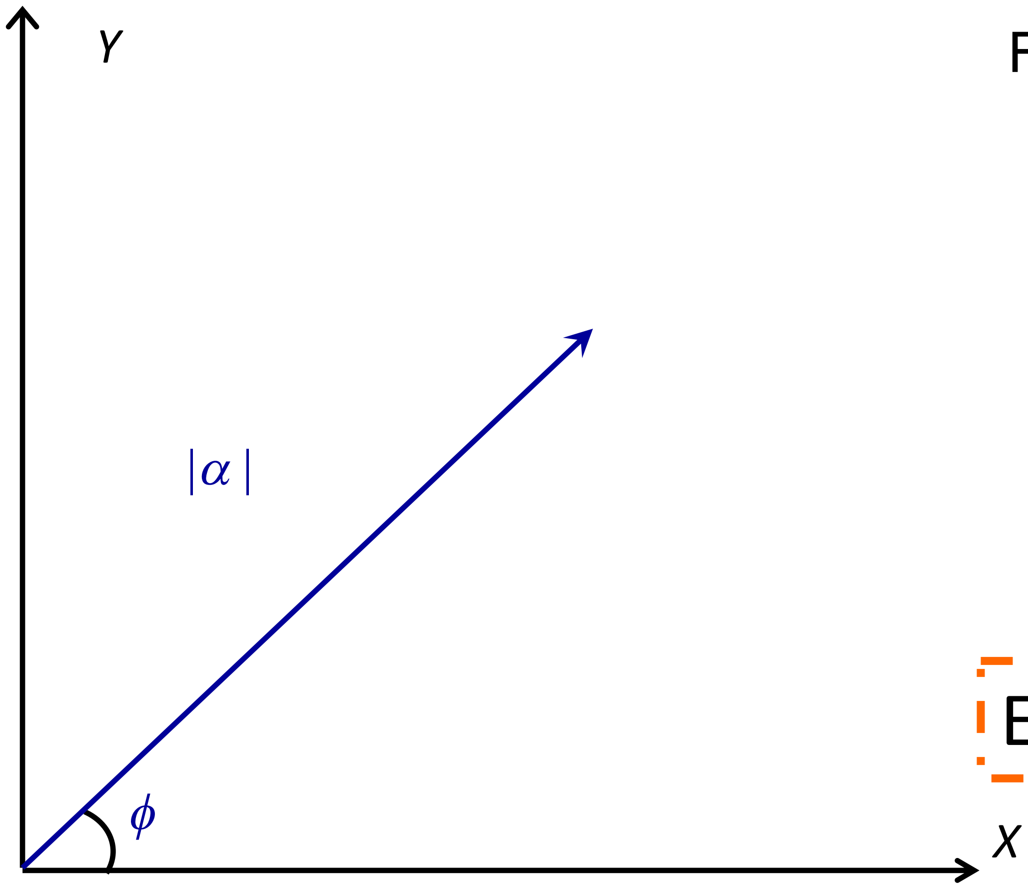
$$\mathbf{E}(\mathbf{r},t) = \frac{1}{\sqrt{\epsilon_0 L^3}} \sum_{\mathbf{k}} \sum_{s} i\omega \left[u_{\mathbf{k}s}(t) \boldsymbol{\epsilon}_{\mathbf{k}s} e^{i\mathbf{k}\cdot\mathbf{r}} - u^*_{\mathbf{k}s}(t) \boldsymbol{\epsilon}^*_{\mathbf{k}s} e^{-i\mathbf{k}\cdot\mathbf{r}} \right],$$

$$\mathbf{B}(\mathbf{r},t) = \frac{i}{\sqrt{\epsilon_0 L^3}} \sum_{\mathbf{k}} \sum_{s} \left[u_{\mathbf{k}s}(t) (\mathbf{k} \times \boldsymbol{\epsilon}_{\mathbf{k}s}) e^{i\mathbf{k}\cdot\mathbf{r}} - u^*_{\mathbf{k}s}(t) (\mathbf{k} \times \boldsymbol{\epsilon}^*_{\mathbf{k}s}) e^{-i\mathbf{k}\cdot\mathbf{r}} \right]$$

Field Quadratures – Classical Description

- Classical Description of the Electromagnectic Field:

Fresnel Representation of a single mode



For a fixed position

$$E(t)=\mathrm{Re}[\alpha\ \exp(i\omega t)]$$

$$\alpha = X + i\,Y$$

$$E(t)=X\cos(\omega t)+ Y\,\mathrm{sen}(\omega t)$$

Field Quadratures – Quantum Optics

The electric field can be decomposed as

$$\hat{\mathbf{E}}^{(+)} = \frac{i}{L^{3/2}} \sum_{\mathbf{k}} \sum_{s} \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \left[\hat{a}_{\mathbf{k}s} \mathbf{u}_{\mathbf{k}s}(\mathbf{r}) e^{-i\omega t}\right] \quad ; \quad \hat{\mathbf{E}}^{(-)} = \left[\hat{\mathbf{E}}^{(+)}\right]^{\dagger}$$

And also as

$$\hat{\mathbf{E}} = \frac{2i}{L^{3/2}} \sum_{\mathbf{k}} \sum_{s} \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \epsilon \left[\hat{X} \cos(\mathbf{k}\cdot\mathbf{r} - \omega t) + \hat{Y} \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)\right]$$

X and Y are the field quadrature operators, satisfying

$$\hat{X}_\theta(t) = \mathrm{e}^{-i\theta}\, \hat{a}(t) + \mathrm{e}^{i\theta}\, \hat{a}^\dagger(t)\ , \qquad \hat{Y}_\theta(t) = -i\left[\mathrm{e}^{-i\theta}\, \hat{a}(t) - \mathrm{e}^{i\theta}\, \hat{a}^\dagger(t)\right]$$

$$\left[\hat{X}(\theta), \hat{X}\left(\theta + \frac{\pi}{2}\right)\right] = 2i$$

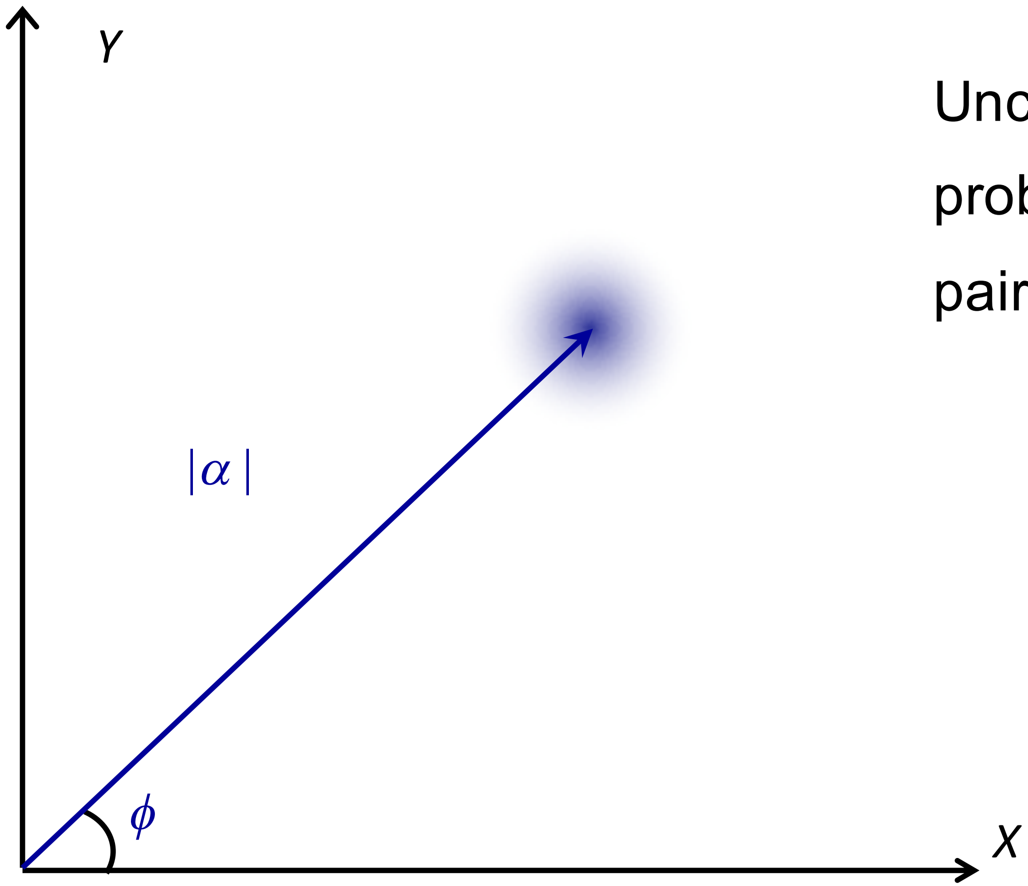
Thus,

$$\Delta X \Delta Y \geq 1$$

Field Quadratures – Quantum Optics

$$\left[\hat{X}(\theta), \hat{X}\left(\theta + \frac{\pi}{2}\right)\right] = 2i$$

Thus, $\Delta X \Delta Y \geq 1$



Uncertainty relation implies in a probability distribution for a given pair of quadrature measurements

Field quadratures behave just as position and momentum operators!

Now we know that:

- the description of the EM field follows that of a set of harmonic oscillators,
- the quadratures of the electric field are observables, and
- they must satisfy an uncertainty relation.

But how to describe different states of the EM field?

Can we find appropriate basis for the description of the field?

Or alternatively, can we describe it using density operators?

And how to characterize these states?

Quantum Optics – Number States

Eigenstates of the number operator

$$\hat{n}_{\mathbf{k}s} = \hat{a}^{\dagger}_{\mathbf{k}s}\hat{a}_{\mathbf{k}s} \qquad \hat{n}_{\mathbf{k}s}|n_{\mathbf{k}s}\rangle = n_{\mathbf{k}s}|n_{\mathbf{k}s}\rangle$$

Number of excitations in a given harmonic oscillator →

number of excitations in a given mode of the field →

number of photons in a given mode!

Annihilation and creation operators:

$$\hat{a}_{\mathbf{k}s}|n_{\mathbf{k}s}\rangle = \sqrt{n_{\mathbf{k}s}}|n_{\mathbf{k}s}-1\rangle,$$
$$\hat{a}^{\dagger}_{\mathbf{k}s}|n_{\mathbf{k}s}\rangle = \sqrt{n_{\mathbf{k}s}+1}|n_{\mathbf{k}s}+1\rangle,$$
$$\hat{a}_{\mathbf{k}s}|0\rangle = 0.$$

Fock States:
Eigenvectors of the Hamiltonian

$$|\{n\}\rangle = \prod_{\mathbf{k}s} |n_{\mathbf{k}s}\rangle$$

$$\hat{\mathcal{H}}|\{n\}\rangle = \left[\sum_{\mathbf{k}s}(n_{\mathbf{k}s}+1/2)\hbar\omega\right]|\{n\}\rangle$$

$$\mathcal{E} = \sum_{\mathbf{k}s}\left[\hbar\omega_{\mathbf{k}}\left(\hat{n}_{\mathbf{k}}+\frac{1}{2}\right)\right]$$

Quantum Optics – Number States

Complete, orthonormal, discrete basis

$$\langle n_{\mathbf{k}s}|m_{\mathbf{k}s}\rangle = \delta_{n_{\mathbf{k}s}m_{\mathbf{k}s}} \Rightarrow \langle\{n\}|\{m\}\rangle = \prod_{\mathbf{k}s} \delta_{n_{\mathbf{k}s}m_{\mathbf{k}s}},$$

$$\sum_{n_{\mathbf{k}s}=0}^{\infty} |n_{\mathbf{k}s}\rangle\langle n_{\mathbf{k}s}| = 1 \Rightarrow \sum_{\{n\}} |\{n\}\rangle\langle\{n\}| = 1.$$

Disadvantage: except for the vacuum mode it is quite an unusual state of the field.

Can we find something better?

Quantum Optics – Coherent States

Eigenvalues of the annihilation operator: $a_{\mathbf{k}s}|\alpha_{\mathbf{k}s}\rangle = \alpha_{\mathbf{k}s}|\alpha_{\mathbf{k}s}\rangle$

In the Fock State Basis:

$$|\alpha_{\mathbf{k}s}\rangle = e^{-|\alpha_{\mathbf{k}s}|^2/2} \sum_{n_{\mathbf{k}s}=0}^{\infty} \frac{\alpha_{\mathbf{k}s}^{n_{\mathbf{k}s}}}{\sqrt{n_{\mathbf{k}s}!}} |n_{\mathbf{k}s}\rangle$$

Completeness:

$$\frac{1}{\pi}\int |\alpha\rangle\langle\alpha| d^2\alpha = 1$$

but is not orthonormal

$$\langle\alpha|\alpha'\rangle = exp\left(-\frac{1}{2}|\alpha|^2 + \alpha'\alpha^* - \frac{1}{2}|\alpha'|^2\right)$$

Over-complete!

Moreover:

- corresponds to the state generated by a classical current,
- reasonably describes a monomode laser well above threshold,
- it is the closest description of a “classical” state.

Quantum Optics – Number States

Precise number of photons

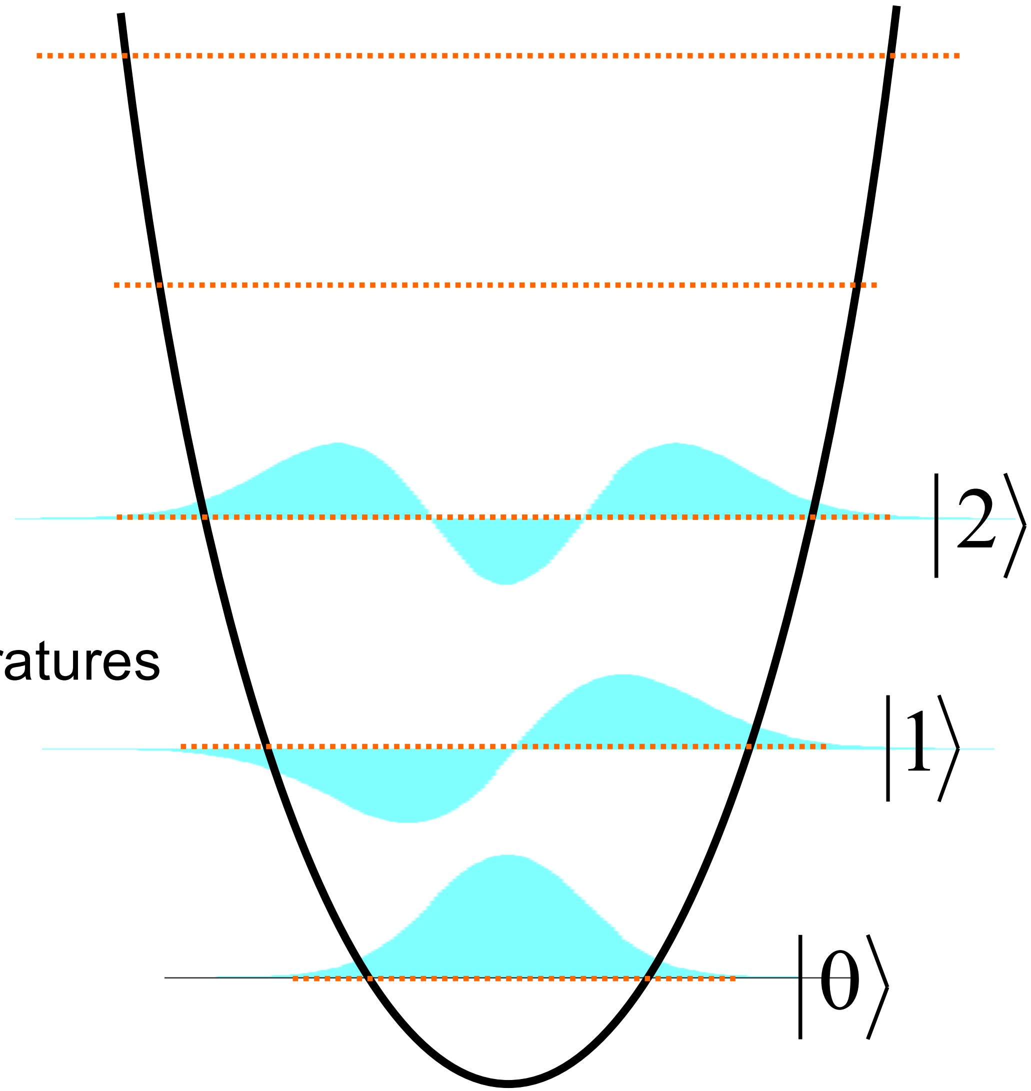
$$\langle \hat{n} \rangle = n$$

$$\Delta \hat{n}^2 = 0$$

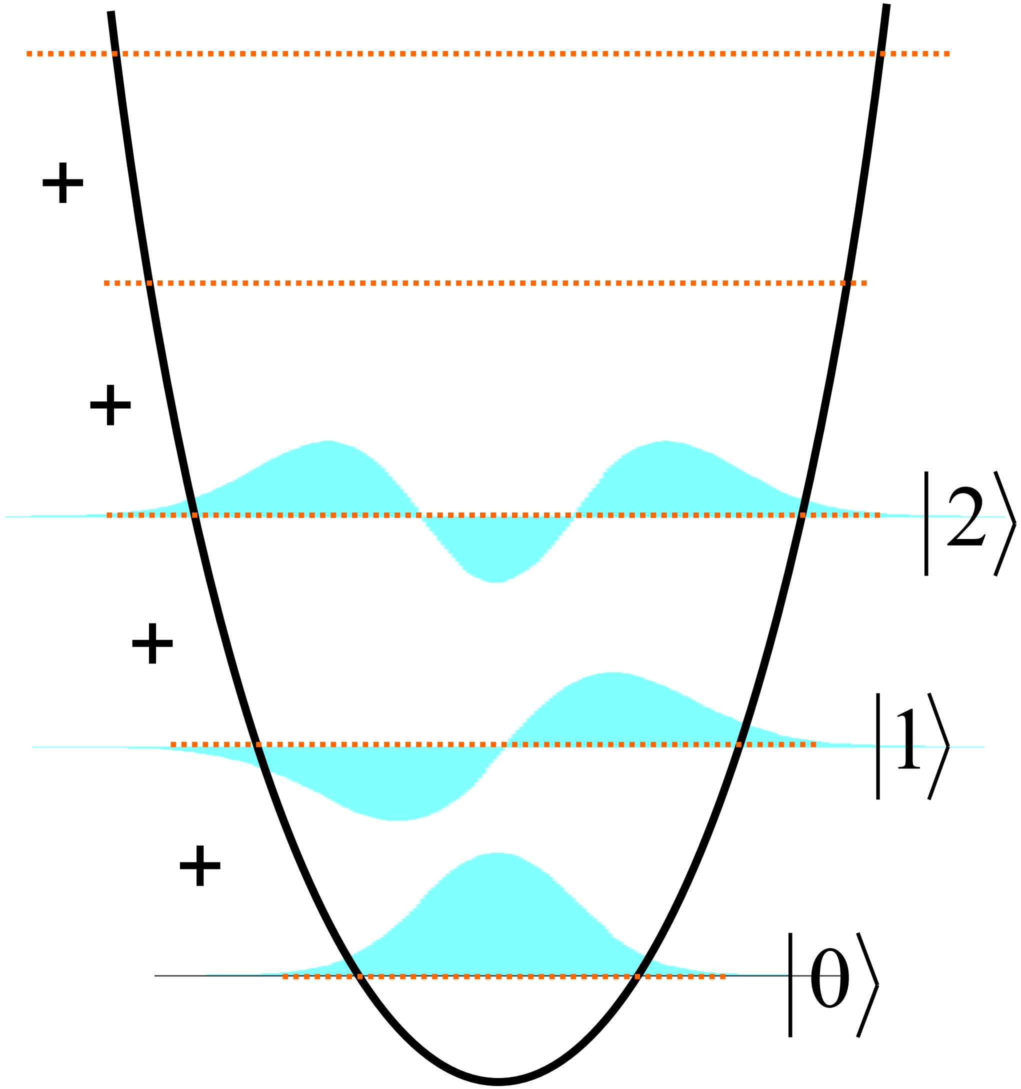
Growing dispersion of the quadratures

$$\langle \hat{X} \rangle = \langle \hat{Y} \rangle = 0$$

$$\langle \hat{X}^2 \rangle = \langle \hat{Y}^2 \rangle = 2n + 1$$

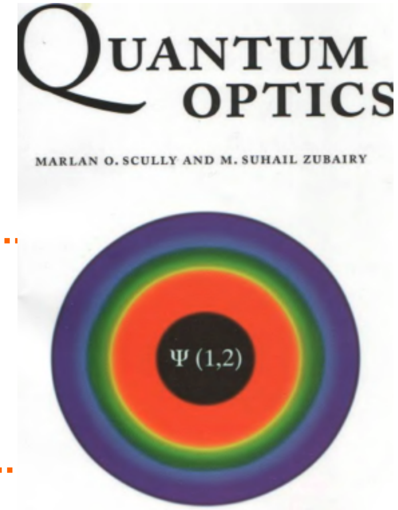


Quantum Optics – Coherent State



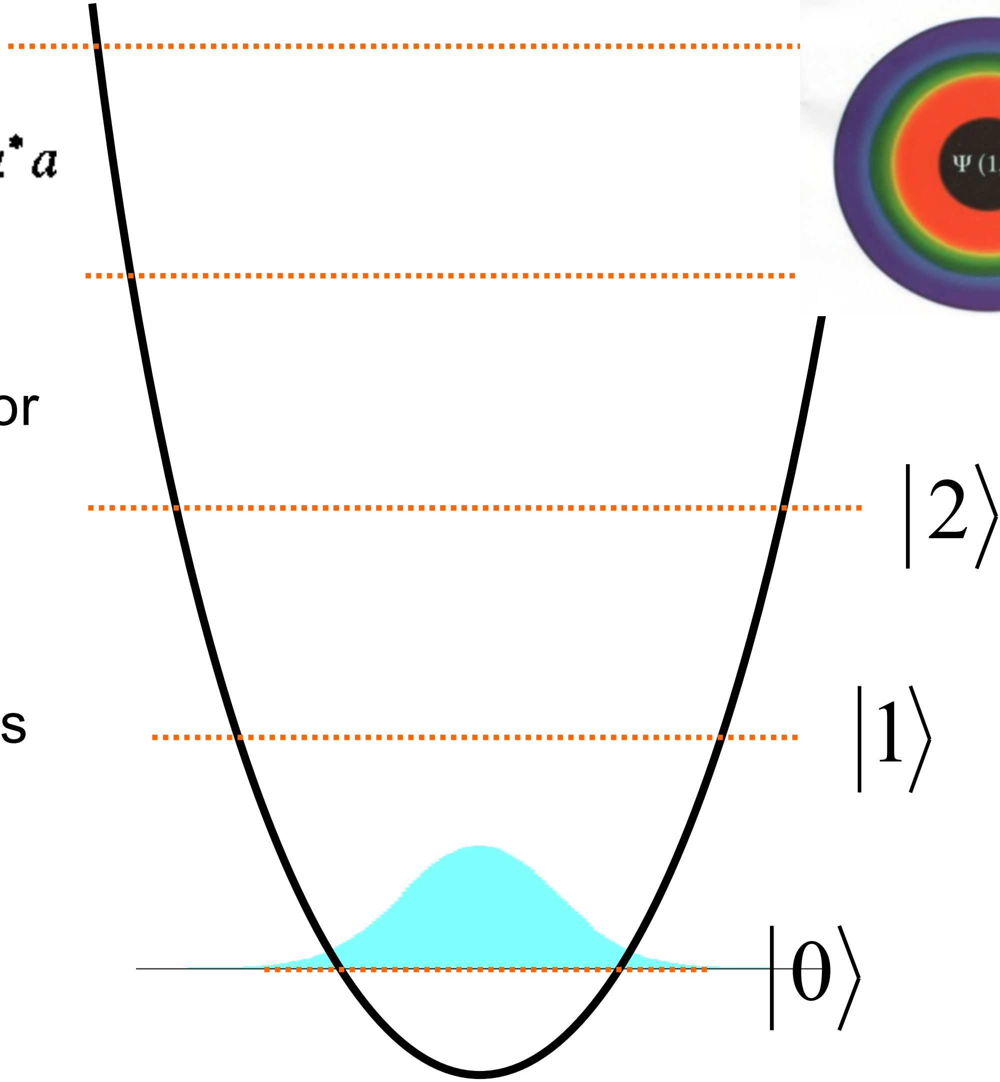
$$|\alpha_{\mathbf{k}s}\rangle = e^{-|\alpha_{\mathbf{k}s}|^2/2} \sum_{n_{\mathbf{k}s}=0}^{\infty} \frac{\alpha_{\mathbf{k}s}^{n_{\mathbf{k}s}}}{\sqrt{n_{\mathbf{k}s}!}} |n_{\mathbf{k}s}\rangle$$

Quantum Optics – Coherent State



$$|\alpha\rangle = D(\alpha)|0\rangle$$

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$$



Mean value of number operator

$$\langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$$

Poissonian distribution of photons

$$p(n) = \frac{\langle n\rangle^n e^{-\langle n\rangle}}{n!}$$

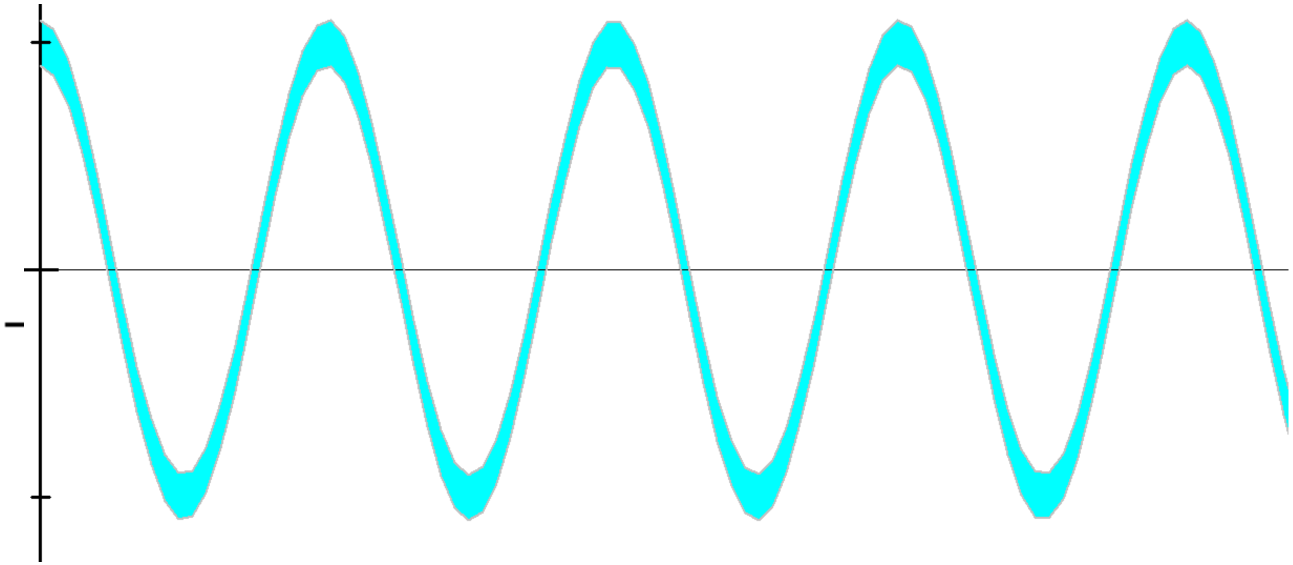
Therefore, variance of photon number is equal to the mean number!

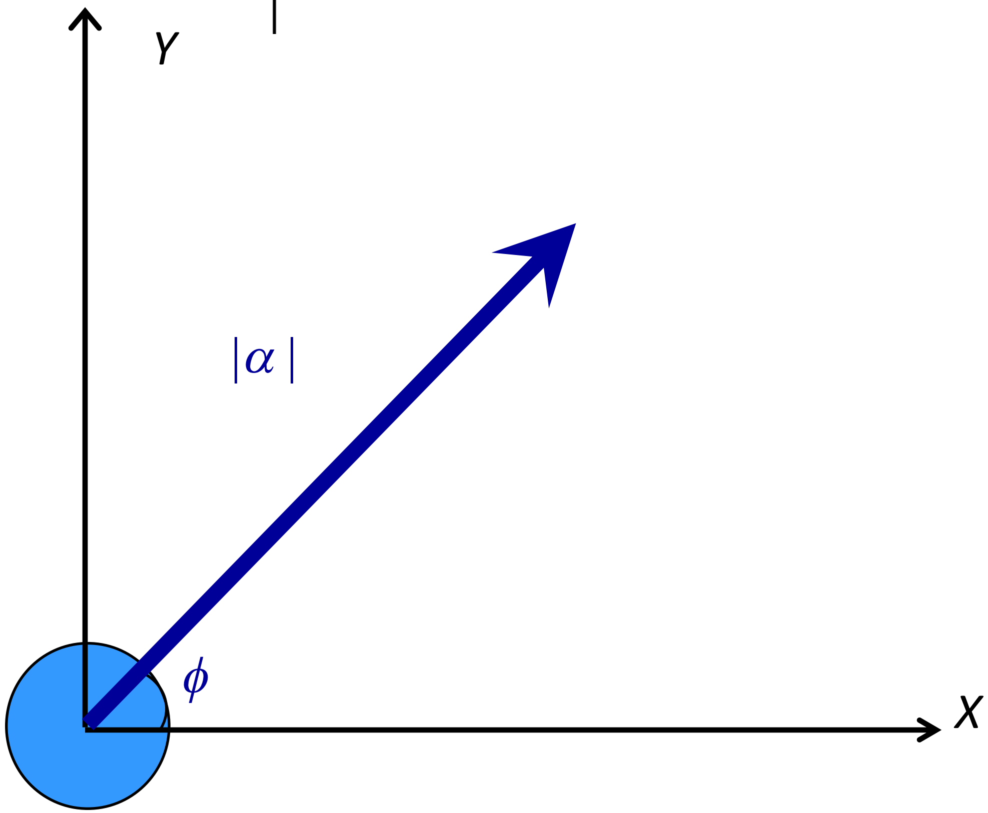
$$\Delta^2\hat{n} = \langle\hat{n}^2\rangle - \langle\hat{n}\rangle^2 = |\alpha|^2$$

Quantum Optics – Coherent State

$$|\alpha\rangle = D(\alpha)|0\rangle$$

$$D(\alpha) = \exp(\alpha a^{\dagger} -$$





Quantum Optics – Coherent Squeezed States

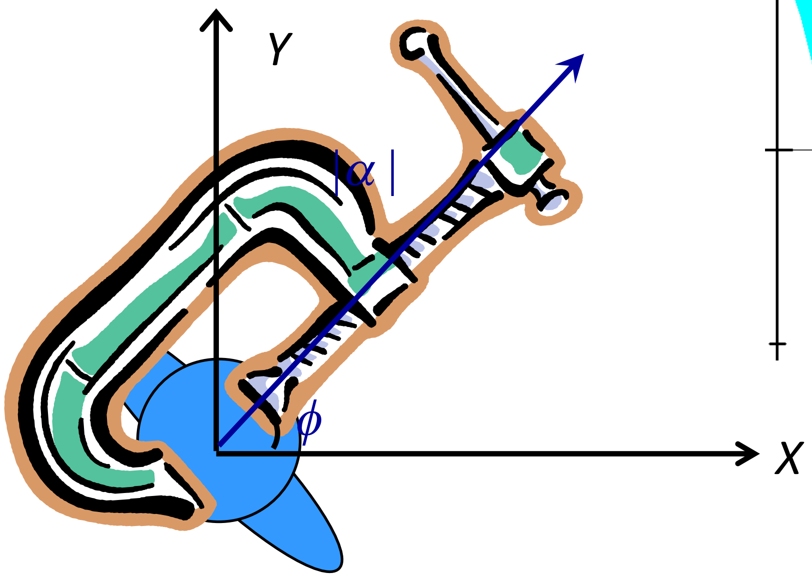
$$|\alpha\rangle = D(\alpha)|0\rangle$$

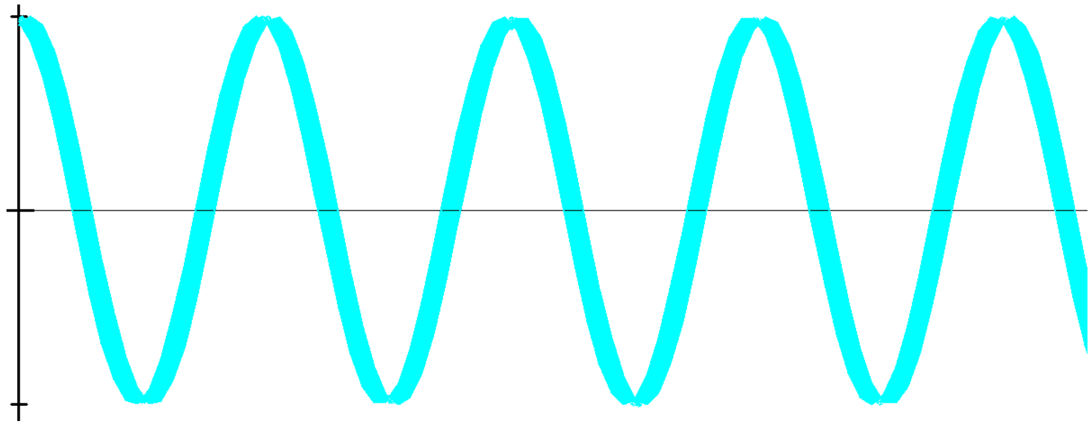
$$D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^* a)$$

$$S(\varepsilon) = \exp(1/2\varepsilon^* a^2 - 1/2\varepsilon a^{\dagger 2})$$

$$\varepsilon = re^{2i\phi}$$

$$|\alpha, \varepsilon\rangle = D(\alpha)S(\varepsilon)|0\rangle$$





Squeezed States: quadratic creation and annihilation operators → paired photons in the mode

Quantum Optics – Density Operators

Pure X Mixed States

$$|\psi\rangle = \sum c_n |a_n\rangle$$

$$\sum |a_m\rangle\langle a_m| = 1$$

$$c_n = \langle a_n|\psi\rangle$$

$$\langle a_m|a_n\rangle = \delta_{mn}$$

$$\langle A\rangle = \langle\psi|A|\psi\rangle$$

$$\langle a_m|A|a_n\rangle = A_{mn}$$

Introducing the density operator (von Neumann – 1927)

$$c_n c_m^* = \rho_{nm}$$

$$\rho = |\psi\rangle\langle\psi|$$

Quantum Optics – Density Operators

$$\langle A \rangle = \sum \langle a_n | \rho | a_m \rangle \langle a_m | A | a_n \rangle$$

$$= \sum \langle a_n | \rho A | a_n \rangle = Tr\{\rho A\}$$

Now we can represent a statistical mixture of pure states!

$$\rho = \sum p_k \rho_k \qquad \sum p_k = 1$$

$$\langle A \rangle = Tr\{\rho A\} \qquad Tr\rho = 1$$

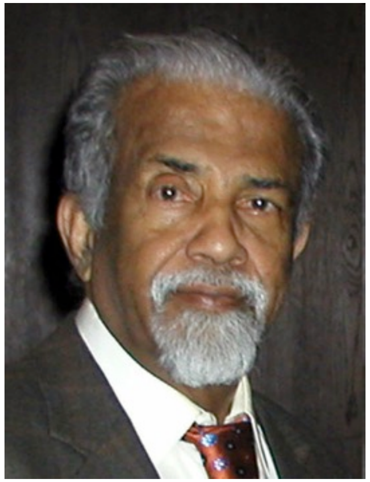
$$Tr\rho \geq Tr\rho^2$$

Quantum Optics – Density Operators

Coherent States $|\alpha\rangle$ $$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|\,\mathrm{d}^2\alpha$$

P(α) : representation of the density operator:

Glauber and Sudarshan



Quantum Optics – Density Operators

Coherent States $|\alpha\rangle$ $$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|\,\mathrm{d}^2\alpha$$

P(α) : representation of the density operator:

Glauber and Sudarshan

Representations of the density operators provide a simple way to describe the state of the field as a function of dimension 2N, where N is the number of modes involved.

P representation is a good way to present “classical” states, like thermal light or coherent states.

Good for calculating normally ordered operators.

$$\langle a^{\dagger n} a^m \rangle = \int P(\alpha)\alpha^{*n}\alpha^m \mathrm{d}^2\alpha$$

But it is singular for “non classical states” (e.g. Fock and squeezed states).

Quasi-Probability Representations

P- Glauber - Sudarshan $\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|\,\mathrm{d}^2\alpha$

Wigner

$$W(\alpha) = \frac{2}{\pi}\int P(\beta)\exp\left(-2|\beta-\alpha|^2\right)\mathrm{d}^2\beta$$

$$\bar{W}(\bar{x},\bar{p}) = \frac{1}{\pi\hbar}\int \mathrm{d}y\,\langle \bar{x}-y|\rho|\bar{x}+y\rangle \exp(2\mathrm{i}y\bar{p}/\hbar)$$

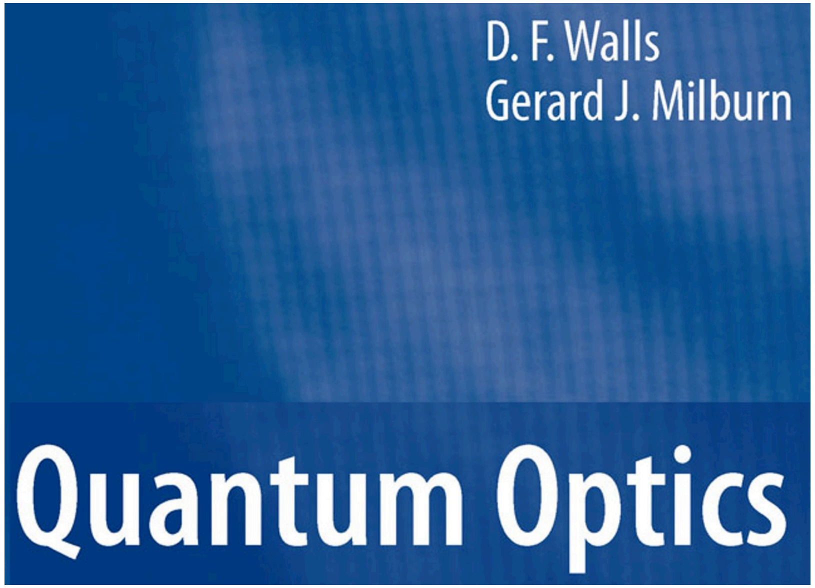
- It is non-singular.
- It is bounded.
- *It may be negative!*
- It is good for calculating statistics of measurements (marginal distribution).

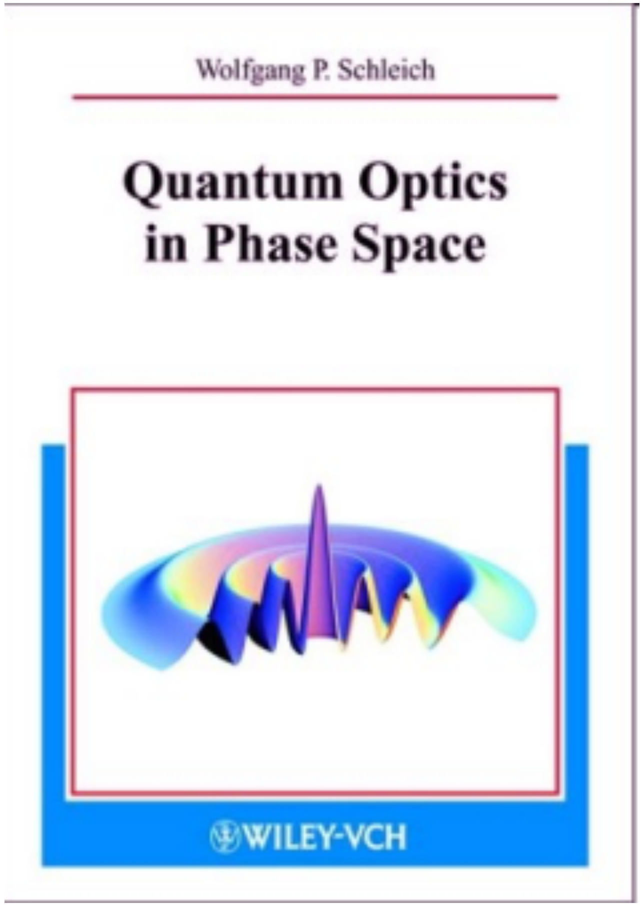
$$P(x_k) = \int_{-\infty}^{\infty} \mathrm{d}x_{\bar{k}} W(x_1, x_2)$$

- Is is good for calculating variances (simple operator ordering)

$$\langle\{a^r(a^\dagger)^s\}_{\mathrm{sym}}\rangle = \int d^2\alpha\, \alpha^r(\alpha^*)^s W(\alpha,\alpha^*).$$

Quasi-Probability Representations





C.W. Gardiner P. Zoller

Quantum Noise

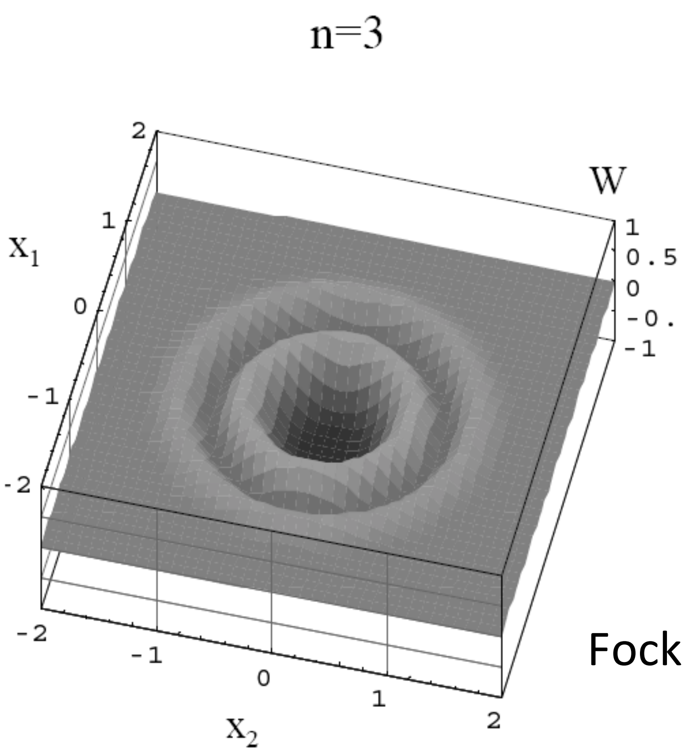
A Handbook
of Markovian and Non-Markovian
Quantum Stochastic Methods
with Applications to Quantum Optics

Wigner Representation

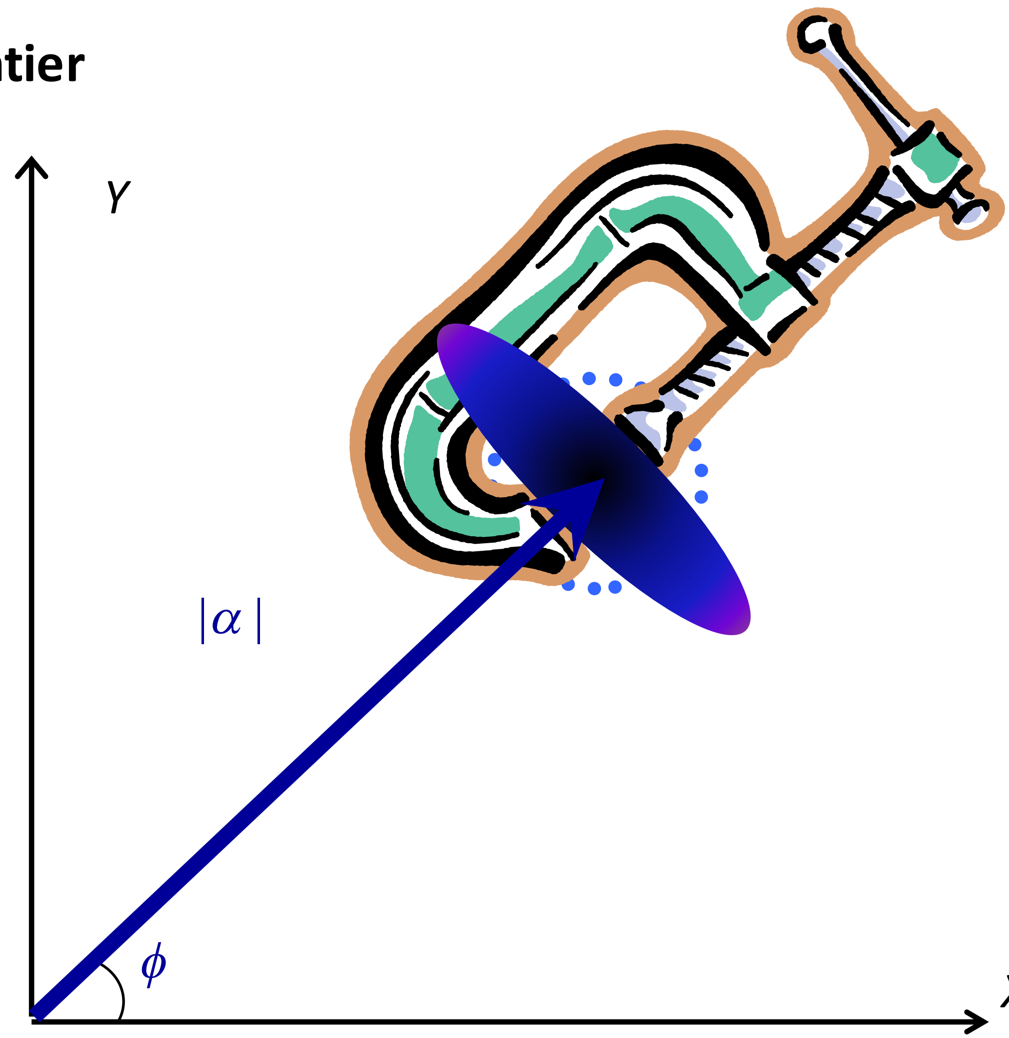
Evident quantum/ classical frontier

Squeezed states

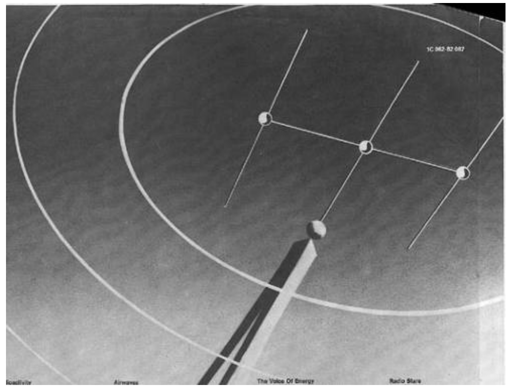
States with W<0



Fock states $|n\rangle$



Quantum Optics – Measurement of the Field



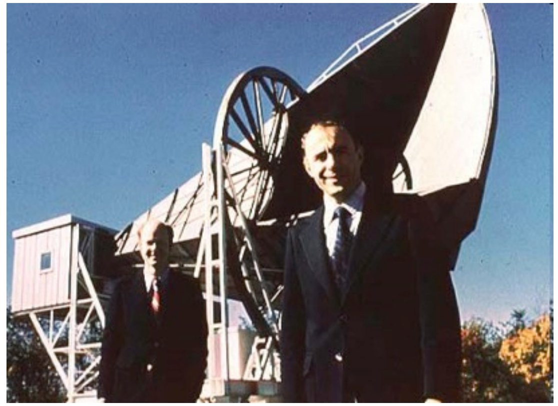
Slow varying EM Field can be detected by an antenna:

→ conversion of electric field in electronic displacement.

→amplification, recording, analysis of the signal.

→electronic readly available.

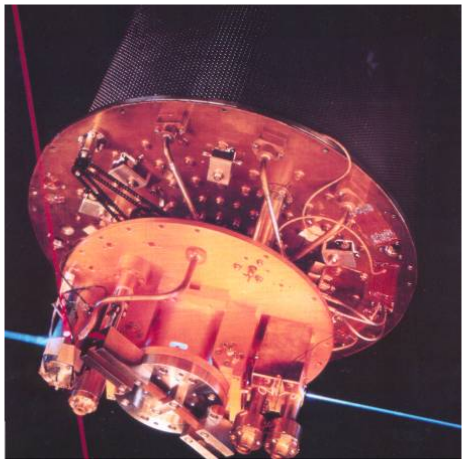
Example: 3 K cosmic background (Penzias & Wilson).

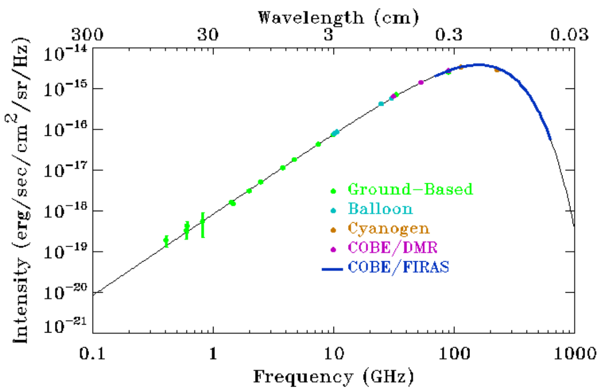


Problems:

→Even this tiny field accounts for a strong photon density.

→Every measurement needs to account for thermal background (e.g. Haroche *et al.*).





Quantum Optics – Measurement of the Field

Fast varying EM Field cannot be measured directly.

We often detect the mean value of the Poynting vector: $\mathbf{S} = \varepsilon_0 \mathbf{E} \times \mathbf{B}$

Photoelectric effect converts photons into ejected electrons

We measure photo-electrons

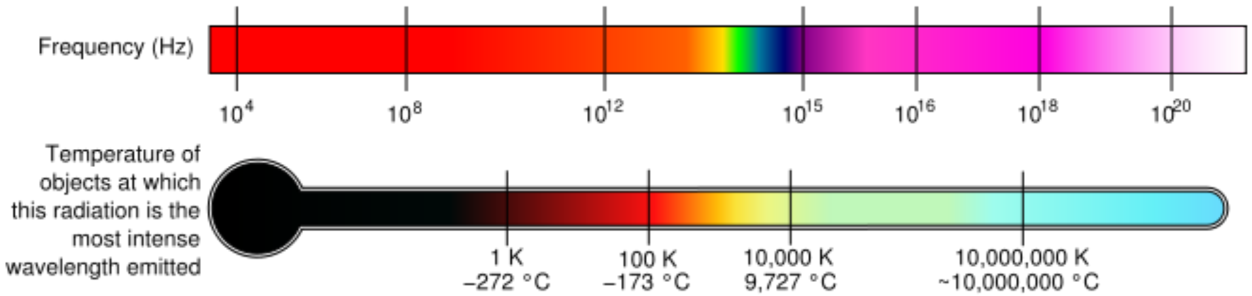
→individually with APDs or photomultipliers – a single electron is converted in a strong pulse – discrete variable domain,

→in a strong flux with photodiodes, where the photocurrent is converted into a voltage – continuous variable domain.

Advantages: in this domain, photons are energetic enough:

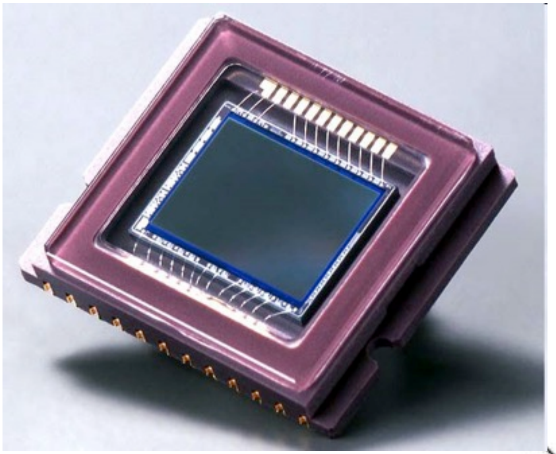
→in a small flux, every photon counts.

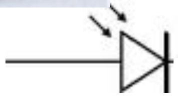
→for the eV region (visible and NIR), presence of background photons is negligible: measurements are nearly the same in L-He or at room temperature.

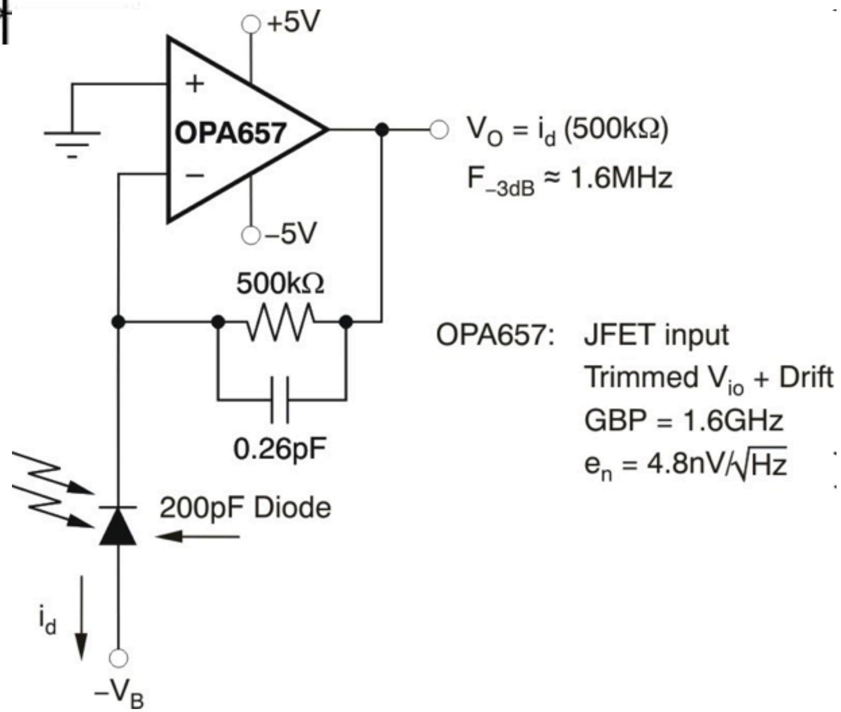


Quantum Optics – Measurement of the Field

And detectors are cheap!



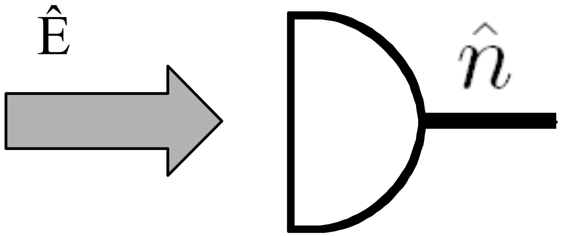




Quantum Optics – Measurement of the Intense Field

We can easily measure photon flux: field intensity

(or more appropriate, optical power)



$$I = \langle E^* \, E \rangle = \alpha^* \, \alpha$$

$$\hat{n} = \hat{a}^\dagger \hat{a}$$

$$\hat{a}^\dagger = \alpha + \delta \hat{a}^\dagger$$

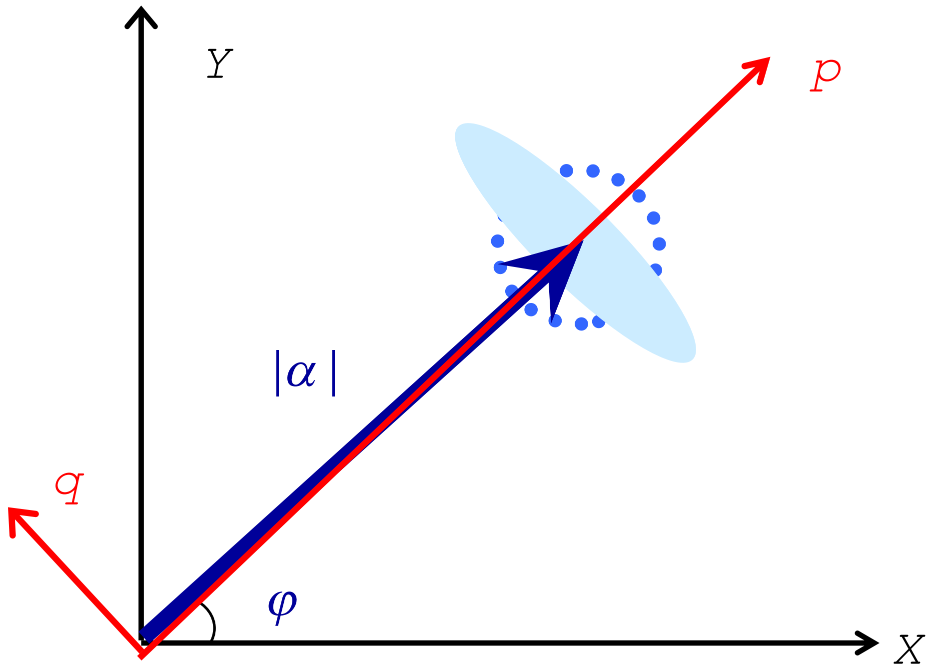
$$\alpha = |\alpha| exp(i\varphi)$$

$$\hat{n} = |\alpha|^2 + |\alpha| e^{i\varphi} \delta \hat{a}^\dagger + |\alpha| e^{-i\varphi} \delta \hat{a} + \delta \hat{a}^\dagger \delta \hat{a}$$

$$\hat{n} = |\alpha|^2 + |\alpha| \delta \hat{p} + O(2)$$

Quantum Optics – Measurement of the Intense Field

$$\hat{n} = |\alpha|^2 + |\alpha|\delta\hat{p}$$



$$
\begin{aligned}
\Delta^2 \hat{n} &= \langle (\hat{n} - \langle \hat{n} \rangle)^2 \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \\
&= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 + \langle \hat{a}^\dagger \hat{a} \rangle \\
&= \langle : \hat{n}^2 : \rangle - \langle \hat{n} \rangle^2 + \langle \hat{n} \rangle
\end{aligned}
$$

"Classical" Variance

Shot noise !

- For a Poissonian photon distribution, the "classical variance" is zero.
- A coherent state is an example of light beam with Poissonian distribution.
- But a Poissonian distribution is not sufficient for coherence!

$$
|\alpha\rangle = exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle
$$

$$
p_n = |\langle \hat{n} | \alpha \rangle|^2 = exp(-|\alpha|^2) \frac{|\alpha|^{2n}}{n!}
$$

Quantum Optics – Measurement of the Intense Field

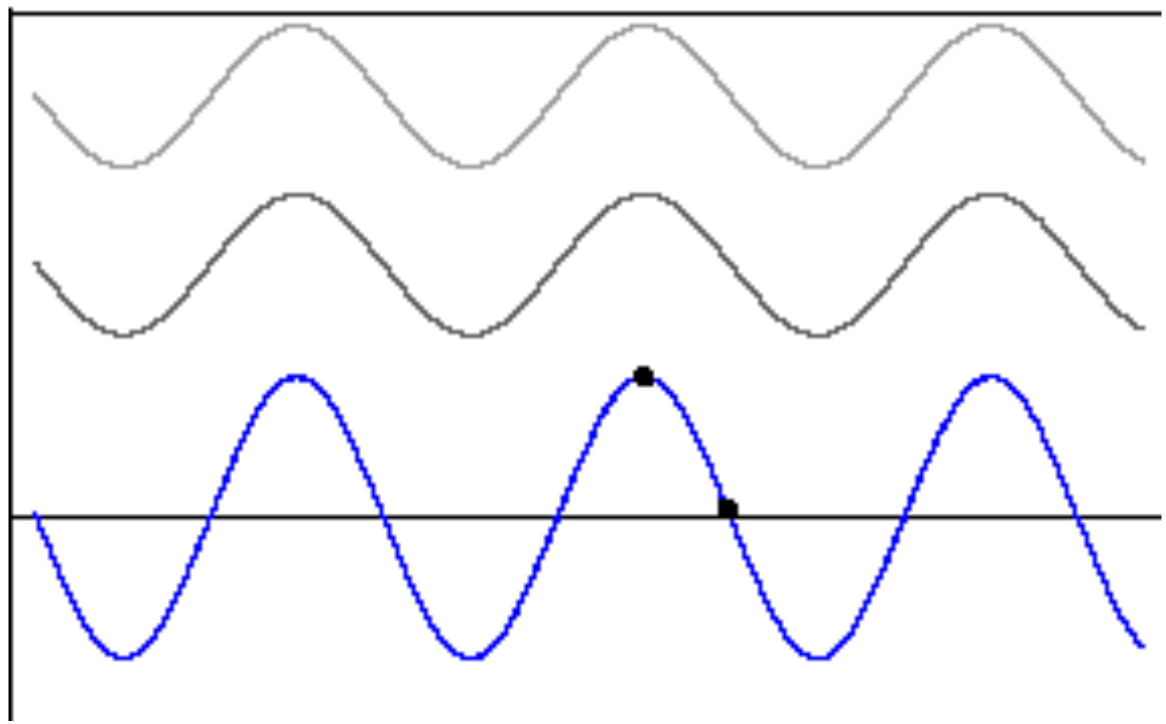
OK, we got the amplitude measurement, but that is only part of the history!

Amplitude is directly related to the measurement of the number of photon, (or the photon counting rate, if you wish).

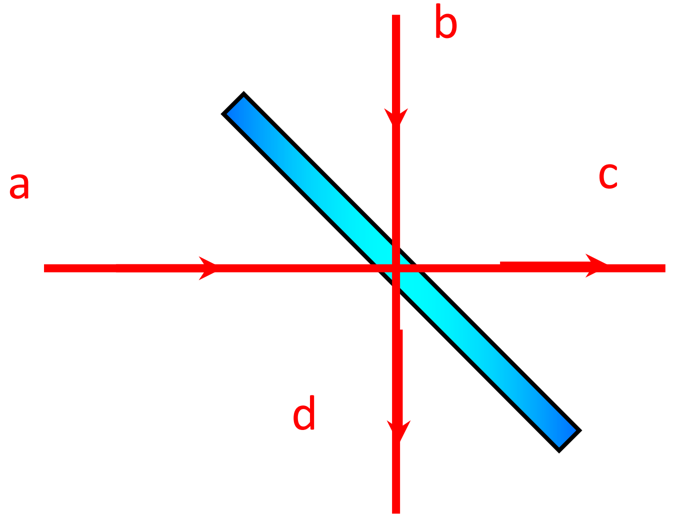
This leaves an unmeasured quadrature, that can be related to the phase of the field.

But there is not such an evident “phase operator”!

Still, there is a way to convert phase into amplitude: interference and interferometers.



Building an Interferometer – The Beam Splitter



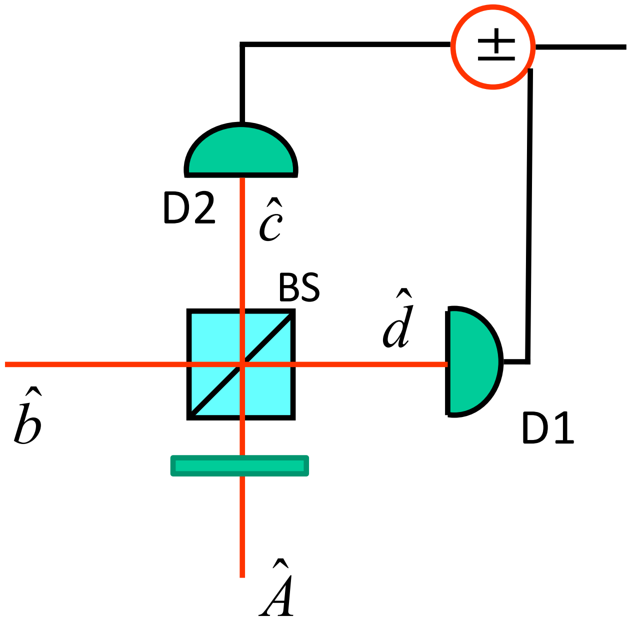
$$\hat{c} = \frac{1}{\sqrt{2}}\left(\hat{a} + \hat{b}\right)$$

$$\hat{d} = \frac{1}{\sqrt{2}}\left(\hat{b} - \hat{a}\right)$$

$$\hat{n}_c = \frac{\hat{n}_a + \hat{n}_b + \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}}{2}$$

$$\hat{n}_d = \frac{\hat{n}_a + \hat{n}_b - \hat{a}^\dagger \hat{b} - \hat{b}^\dagger \hat{a}}{2}$$

Building an Interferometer – The Beam Splitter



$$\hat{n}_c = \frac{\hat{n}_a + \hat{n}_b + \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}}{2}$$

$$\hat{n}_d = \frac{\hat{n}_a + \hat{n}_b - \hat{a}^\dagger \hat{b} - \hat{b}^\dagger \hat{a}}{2}$$

$$\hat{n}_+ = \hat{n}_a + \hat{n}_b$$

$$\hat{n}_- = \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}$$

<u>**Homodyning**</u>

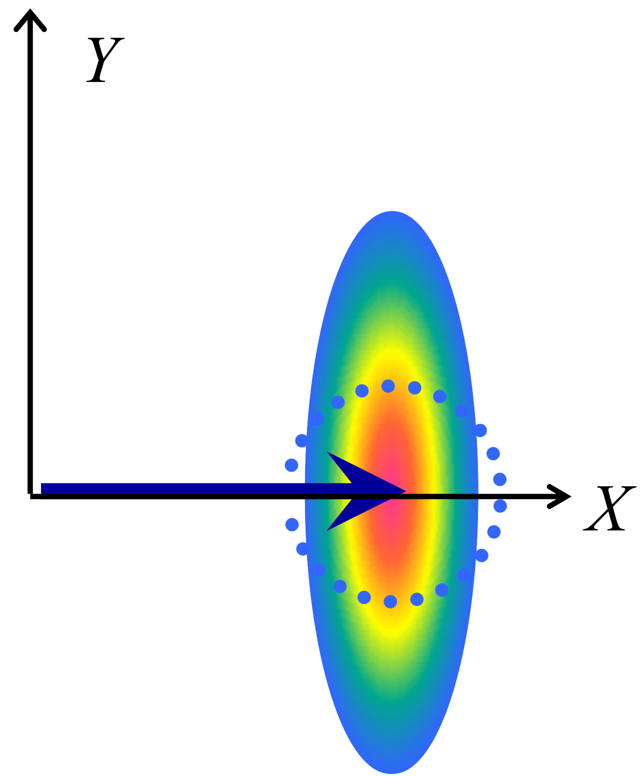
if $<|\hat{a}|> \;<<\; <|\hat{b}|>$

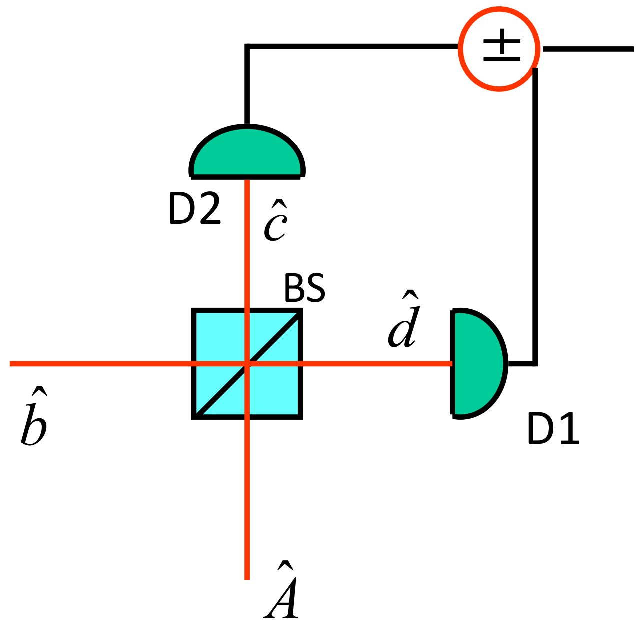
$$\hat{n}_-(t) = |\beta|\left(\hat{A}(t)e^{-i\theta} + \hat{A}^\dagger(t)e^{i\theta}\right)$$

Quadrature Operator !

Homodyning

$$\hat{n}_-(t) = |\beta|\left(\hat{A}(t)e^{-i\theta} + \hat{A}^\dagger(t)e^{i\theta}\right)$$





Vacuum Homodyning

$$\hat{n}_+ = \hat{n}_b$$

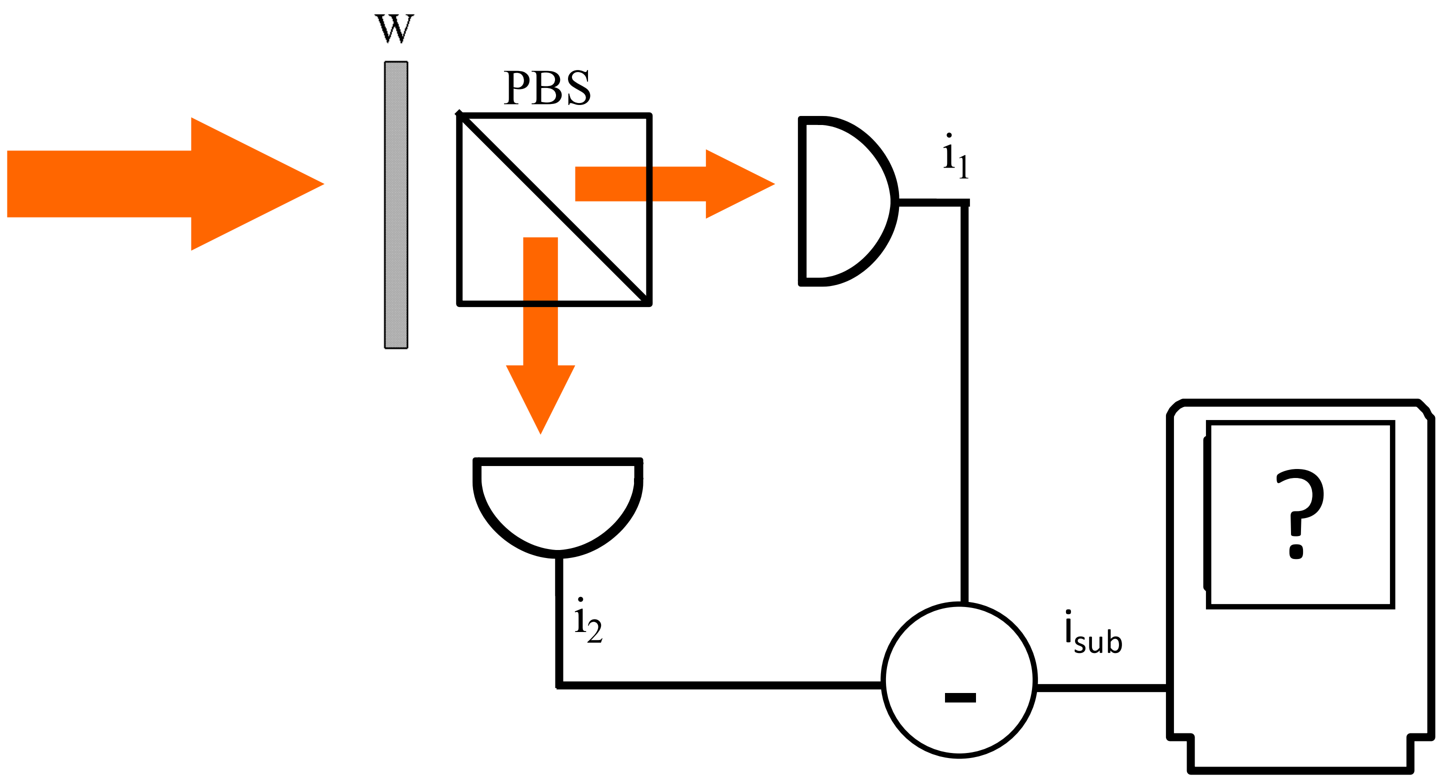
$$\langle \hat{n}_- \rangle = 0$$

$$\Delta^2 \hat{n}_- = \langle \hat{n}_b \rangle$$

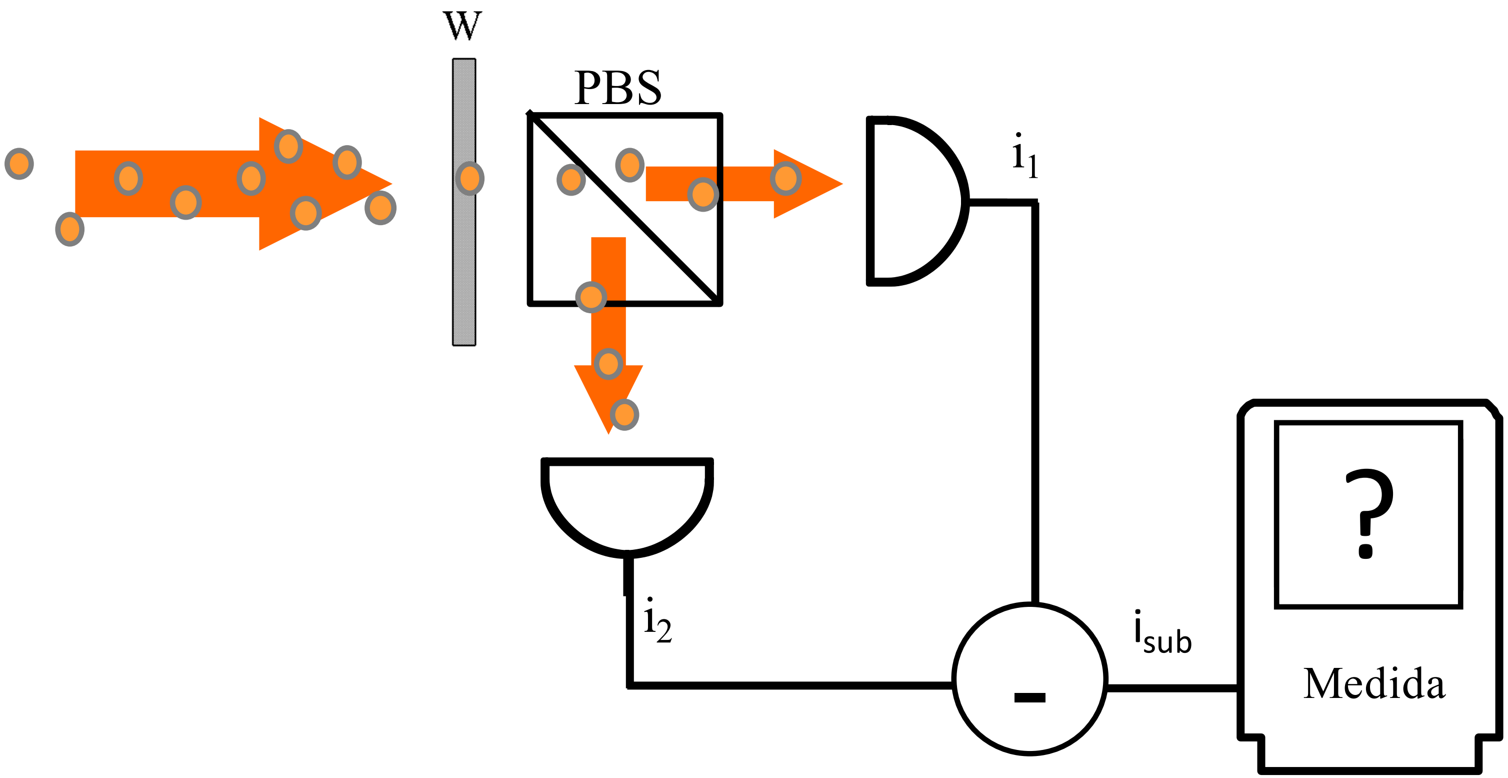
Calibration of the Standard Quantum Level

Question:

Dividing the incident beam in two "equal" parts, what will be the result?



Answer:



Classically: $i_{sub} = 0$

Quantically: “photons are clicks on photodetectors” (A. Zeilinger)

$$<i_{sub}> = 0, \qquad \Delta^2 i_{sub} > 0 \; !$$

Entanglement with continuous variables of the field

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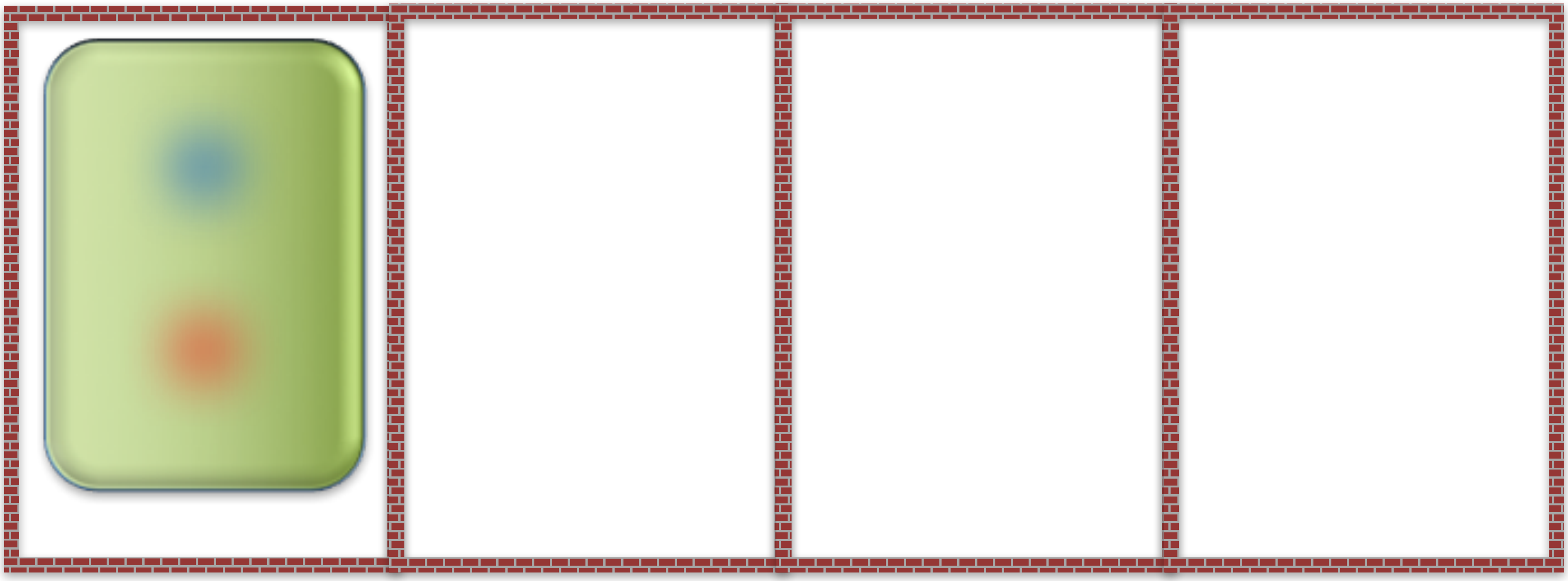
Games we may play within the rules...

A two balls quantum billiard game

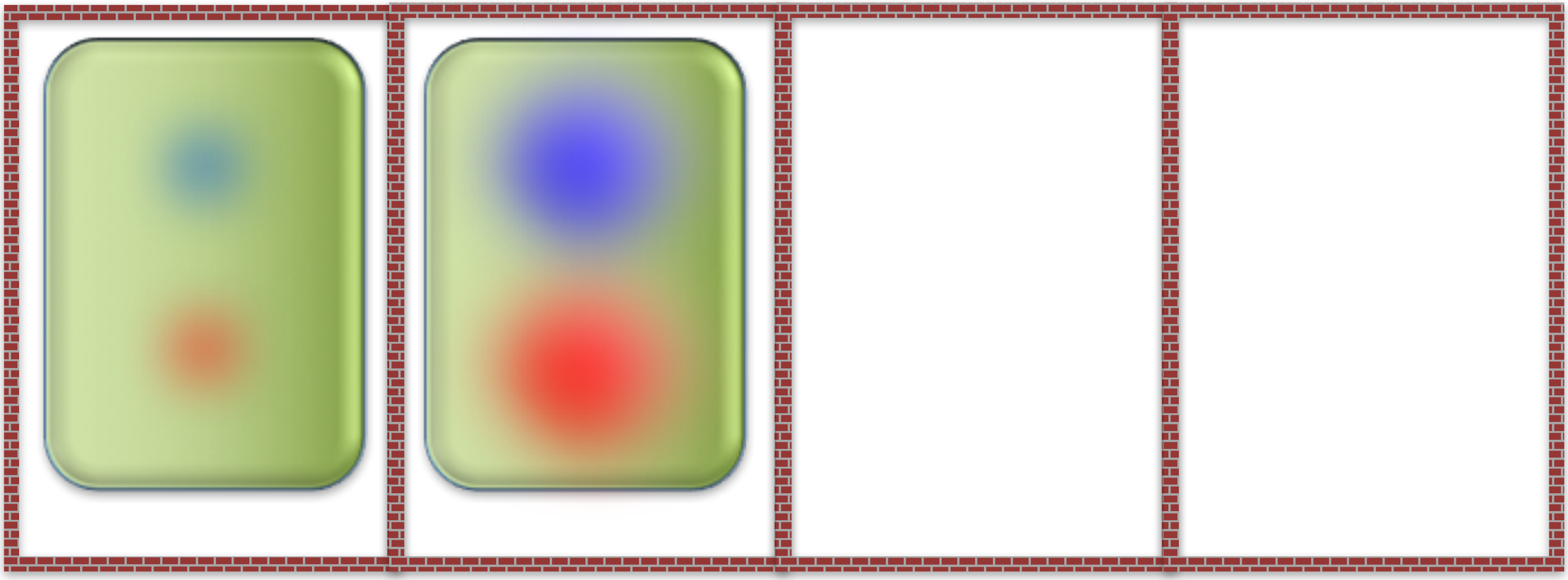


Beginning from the minimum uncertainty condition,

or maximal information for each ball.

Games we may play within the rules...

A two balls quantum billiard game

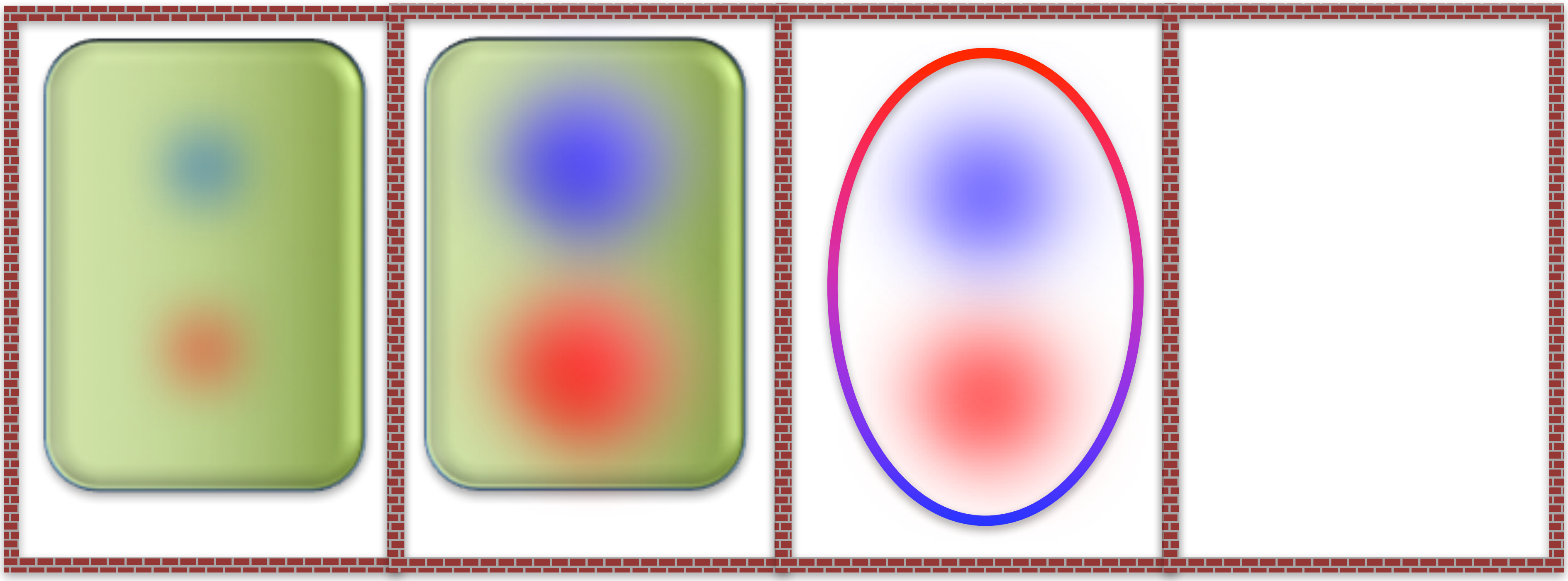


Under time evolution, their interaction will

degrade their individual information.

Games we may play within the rules...

A two balls quantum billiard game



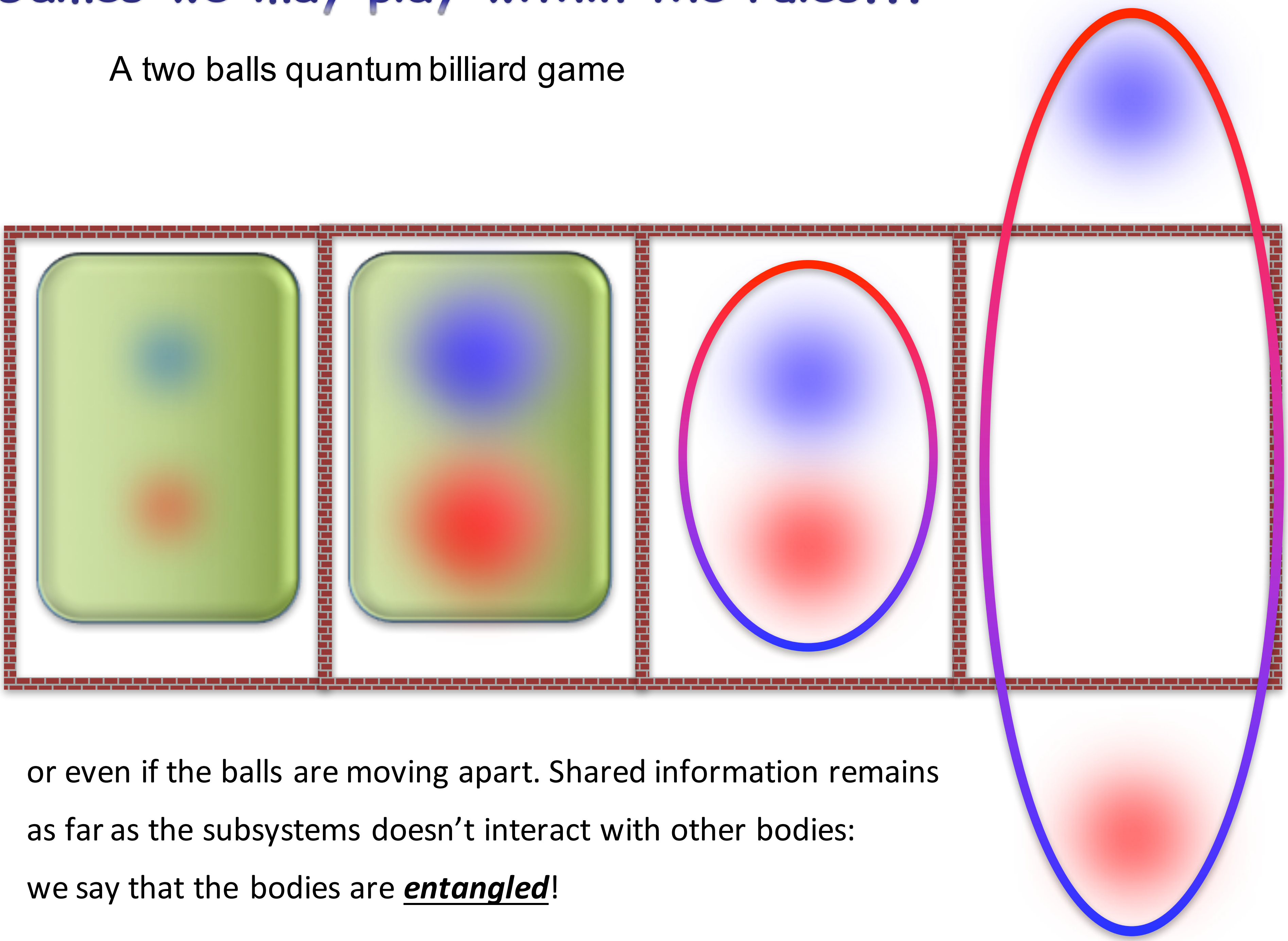
Without external interference, globalinformation is kept: the ball

share information at *quantum level*!

Even if we remove the billiard table...

Games we may play within the rules...

A two balls quantum billiard game



or even if the balls are moving apart. Shared information remains as far as the subsystems doesn’t interact with other bodies: we say that the bodies are ***entangled***!

The New York Times

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

PRINCETON, N. J., May 3.—Professor Albert Einstein will attack science's important theory of quantum mechanics, a theory of which he was a sort of grandfather. He concludes that while it is "correct" it is not "complete."

With two colleagues at the Institute for Advanced Study here, the noted scientist is about to report to the American Physical Society what is wrong with the theory of quantum mechanics, it has been learned exclusively by Science Service.

The quantum theory, with which science predicts with some success inter-atomic happenings, does not meet the requirements for a satisfactory physical theory, Professor Einstein will report in a joint paper with Dr. Boris Podolsky and Dr. N. Rosen.

EPR and Entanglement

Anybody who is not shocked by quantum theory has not understood it.

Niels Bohr

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality.

EPR's example

$$W(x_1, p_1, x_2, p_2) \cong \delta(x_1 - x_2 - L)\delta(p_1 + p_2)$$

→localized in $x_1 - x_2$ and $p_1 + p_2$

We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

A measurement of x_1 yields x_2, as well as a measurement of p_1 gives p_2. But x_2 and p_2 *don't commute*! $\leftrightarrow$ $[x, p] = i\hbar$

A tale of two systems

For strong entanglement, local information should vanish.

Meanwhile. global information is maximally kept

(bounded by the Uncerntainty Principle)!

Although there is a limitation for information in the quantum world, we are allowed to have extreme nonlocal correlations.

Testing the entanglement...

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Bohr's reply

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

$$[q_1 p_1] = [q_2 p_2] = ih/2\pi,$$
$$[q_1 q_2] = [p_1 p_2] = [q_1 p_2] = [q_2 p_1] = 0,$$

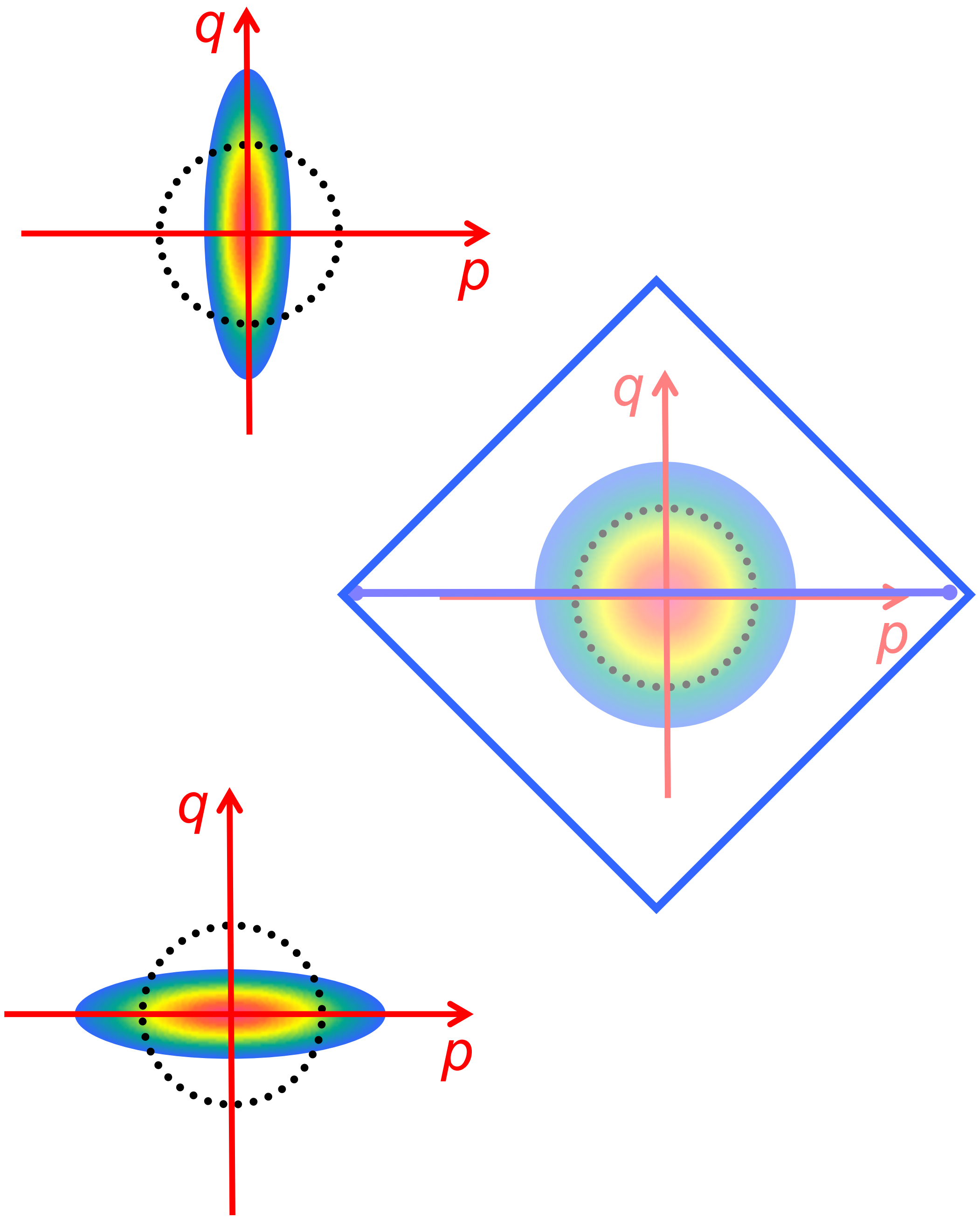
$$q_1 = Q_1 \cos\theta - Q_2 \sin\theta \qquad p_1 = P_1 \cos\theta - P_2 \sin\theta$$
$$q_2 = Q_1 \sin\theta + Q_2 \cos\theta \qquad p_2 = P_1 \sin\theta + P_2 \cos\theta.$$

$$[Q_1 P_1] = ih/2\pi, \qquad [Q_1 P_2] = 0,$$

$$Q_1 = q_1 \cos\theta + q_2 \sin\theta,$$

$$P_2 = -p_1 \sin\theta + p_2 \cos\theta,$$

Entanglement Generation

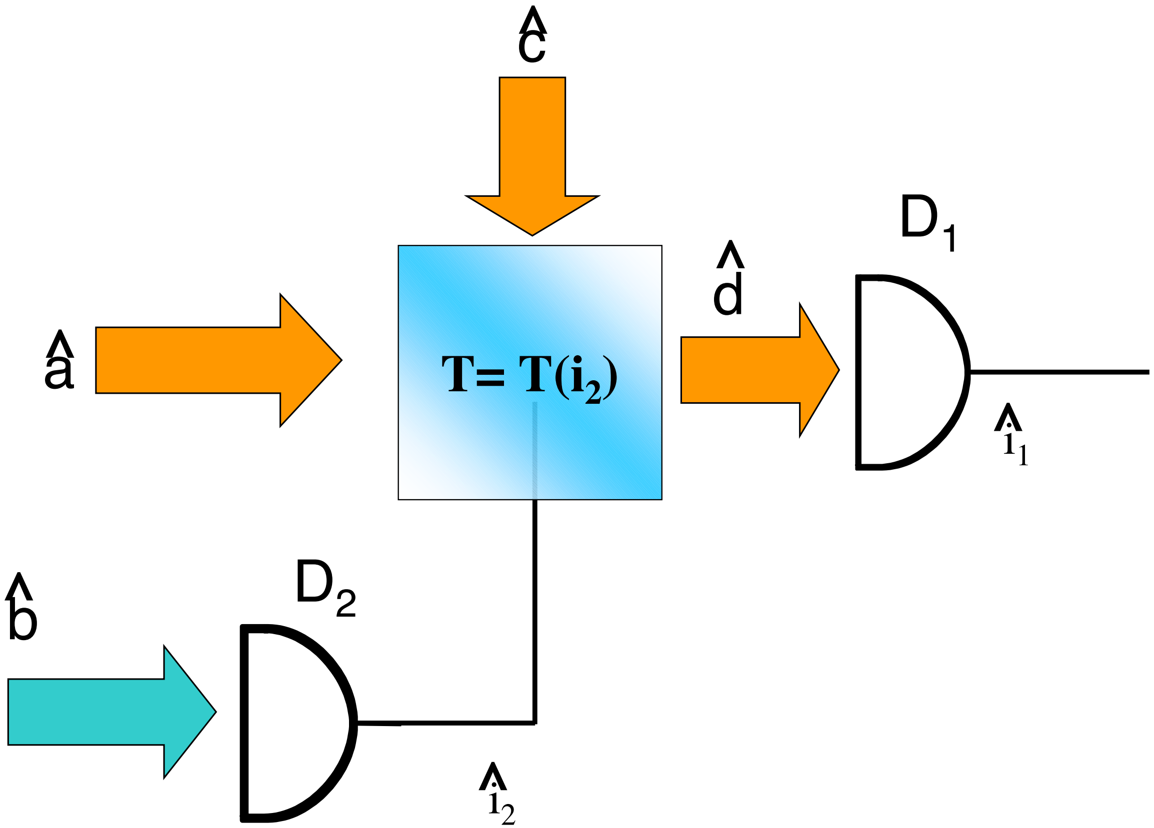


p_1+p_2 ,
q_1+q_2

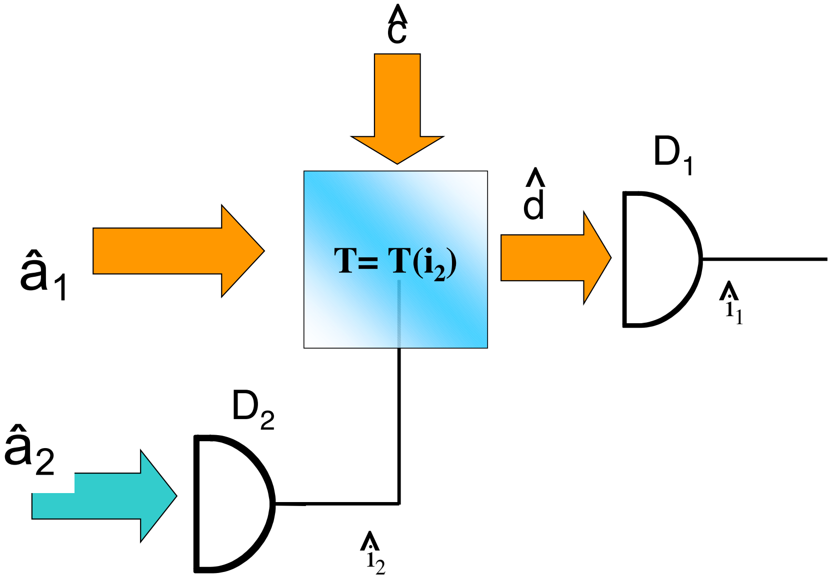
p_1-p_2 ,
q_1-q_2

Few words about entanglement characterization

• "EPR" criterion [M. D. Reid, PRA **40**, 913 (1989), M. D. Reid and P. D. Drummond, PRL **60**, 2731 (1988) & PRA **40**, 4493 (1989)]



$$\delta\hat{p}_i \;=\; \hat{p}_i - \langle\hat{p}_i\rangle$$



$$\Delta^2\hat{p}_{\rm inf} = \Delta^2\hat{p}_1\left(1 - \frac{\langle\delta\hat{p}_1\delta\hat{p}_2\rangle^2}{\Delta^2\hat{p}_1\Delta^2\hat{p}_2}\right)$$

$$\Delta^2\hat{p}_{\rm inf}\,\Delta^2\hat{q}_{\rm inf} \geq 1$$

Entanglement Test - DGCZ

•DGCZ separability criterion:

$$\rho = \sum_i p_i \, \rho_i = \sum_i p_i \, \rho_i^1 \otimes \rho_i^2 \qquad [\hat{q}_i, \hat{p}_j] = 2i\delta_{ij}$$

$$\hat{u} = a\hat{q}_1 + \frac{1}{a}\hat{q}_2,$$
$$\hat{v} = a\hat{p}_1 - \frac{1}{a}\hat{p}_2,$$

Separability $\Rightarrow \langle(\Delta\hat{u})^2\rangle_\rho + \langle(\Delta\hat{v})^2\rangle_\rho \geq 2\,(a^2 + \frac{1}{a^2})$

Lu-Ming Duan, G. Giedke, J.I. Cirac, P. Zoller,
Inseparability criterion for continuous variable systems, Phys. Rev. Lett. **84**, 2722 (2000).

•After some (simple) algebra:

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \geq 0;$$

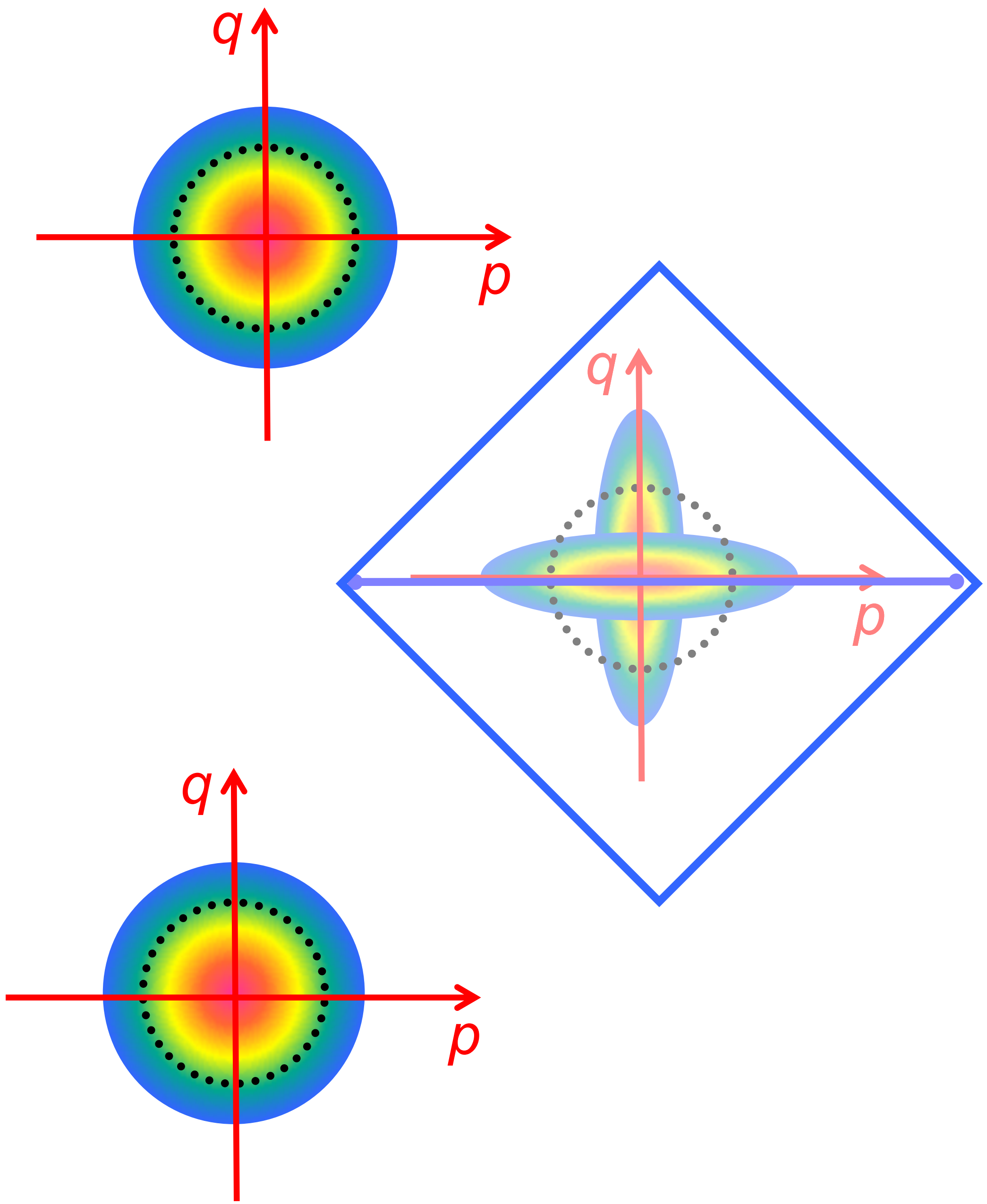
Entanglement Test - DGCZ

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \geq 0;$$

Entanglement Test - DGCZ



p_1+p_2 ,
q_1+q_2

p_1-p_2 ,
q_1-q_2

Entanglement Test – Peres & Horodecki

- Positivity under Partial Transposition (discrete variables)

Separability Criterion for Density Matrices

Asher Peres*

PRL **77**, 1413 (1996)

$$\rho = \sum_A w_A \rho'_A \otimes \rho''_A \quad \Rightarrow \quad \sigma = \sum_A w_A (\rho'_A)^T \otimes \rho''_A$$

non-negative eigenvalues -> Separability

Entanglement Test - Simon

- Continuous variables:

Peres-Horodecki Separability Criterion for Continuous Variable Systems

R. Simon

PRL **84**, 2726 (2000)

$$PT: \quad W(q_1, p_1, q_2, p_2) \to W(q_1, p_1, q_2, -p_2)$$

$$V + \frac{i}{2}\Omega \geq 0 \quad \Rightarrow \quad \tilde{V} + \frac{i}{2}\Omega \geq 0$$

$$\tilde{V} = \Lambda V \Lambda$$

$$\Omega = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix} \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Lambda = \mathrm{diag}(1, 1, 1, -1)$$

Simplectic Eigenvalues >1

Diagonalize: $-(\Omega\tilde{V})^2$

Entanglement Test - Simon

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle$$

$$S_{xj} = C_{xjxj}$$

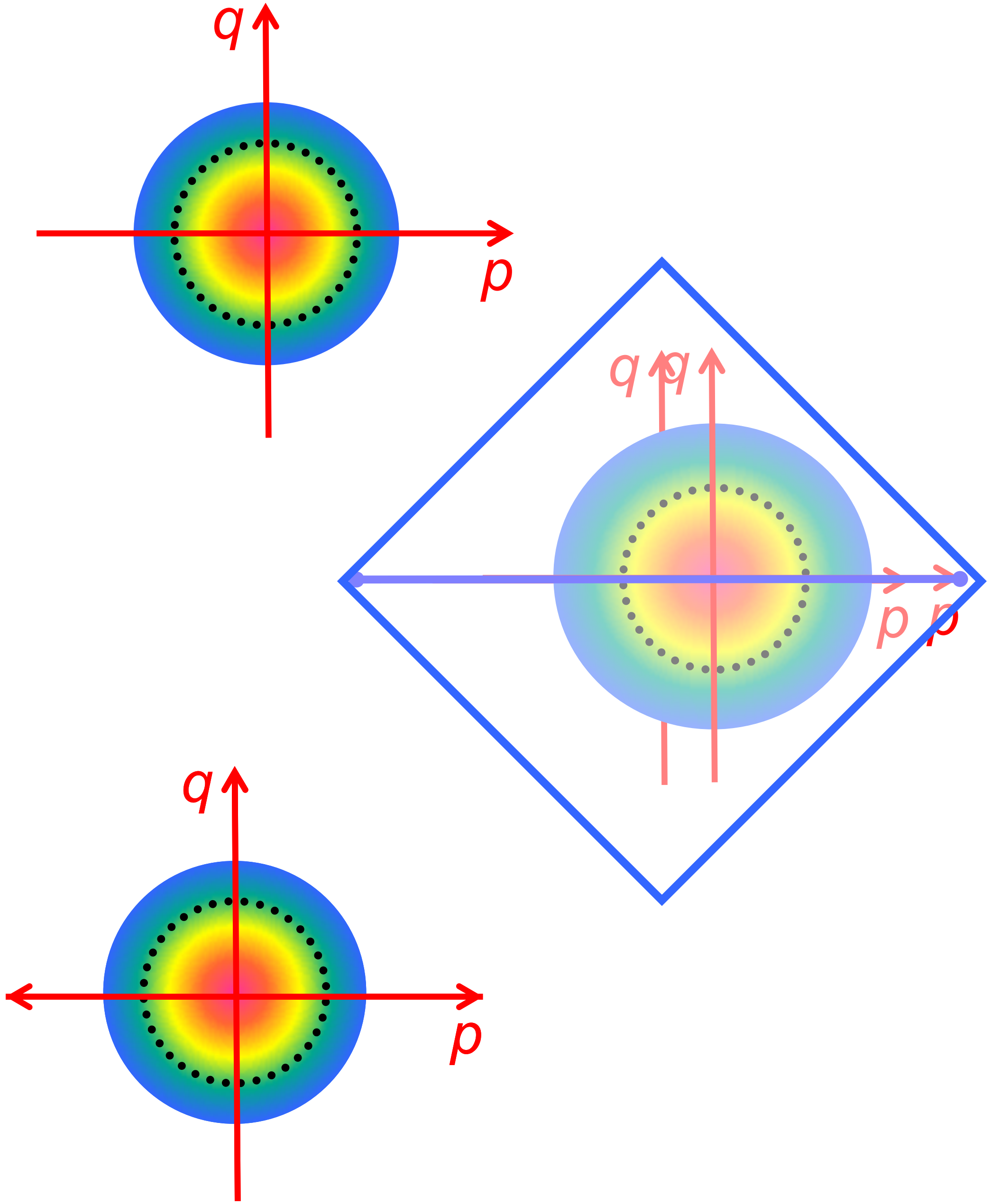
Entanglement Test - Simon

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & -C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & -C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & -C_{p2q2} \\ -C_{p1q2} & -C_{q1q2} & -C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i \rangle \langle x_j \rangle$$

$$S_{xj} = C_{xjxj}$$

Entanglement Test - Simon



$p_1 - p_2$,
$q_1 + q_2$

$p_1 + p_2$,
$q_1 - q_2$

Tripartite Entanglement

- Extend DGCZ criterion to three variables

Detecting genuine multipartite continuous-variable entanglement

PHYSICAL REVIEW A **67**, 052315 (2003)
Peter van Loock[1] and Akira Furusawa[2]

$$\hat{u} \equiv h_1 \hat{x}_1 + h_2 \hat{x}_2 + h_3 \hat{x}_3, \quad \hat{v} \equiv g_1 \hat{p}_1 + g_2 \hat{p}_2 + g_3 \hat{p}_3,$$

$$\langle (\Delta \hat{u})^2 \rangle_\rho + \langle (\Delta \hat{v})^2 \rangle_\rho \geq f(h_1, h_2, h_3, g_1, g_2, g_3),$$

- Apply PPT to multiple partitions

Bound Entangled Gaussian States

R. F. Werner* and M. M. Wolf†

PHYSICAL REVIEW LETTERS
VOLUME 86, NUMBER 16
DOI: 10.1103/PhysRevLett.86.3658

Gaussian states of $1 \times N$ systems

ppt implies separability.

Generating non-classical states of light

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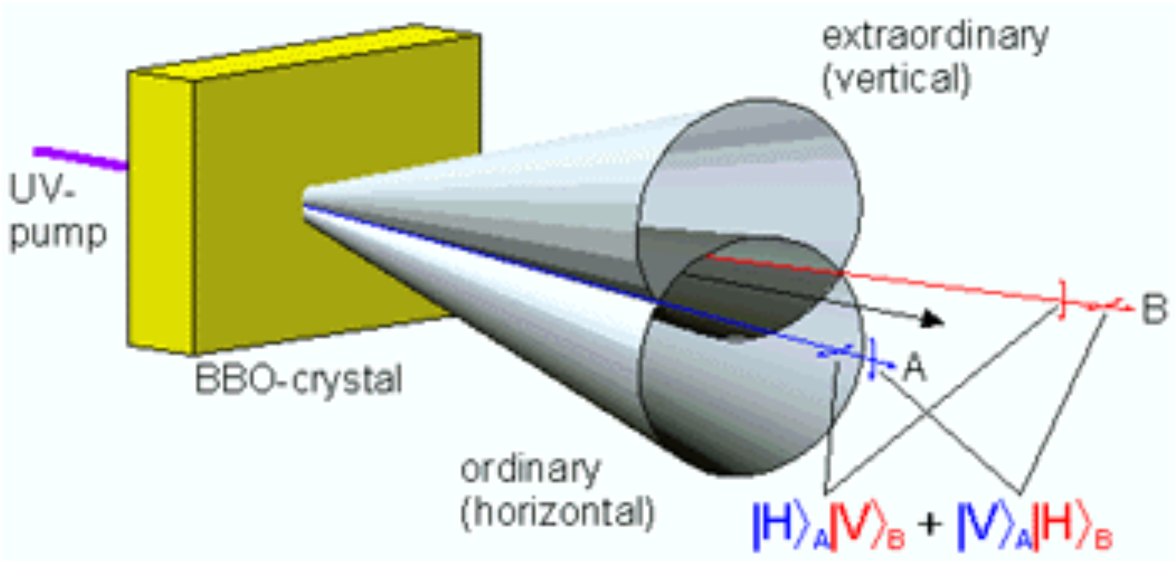
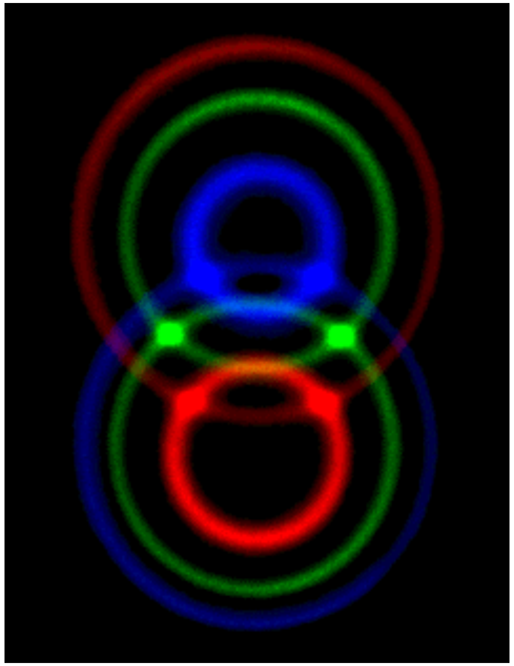
Parametric Down Conversion





$$P = \chi^{(1)} E + \chi^{(2)} E^2$$

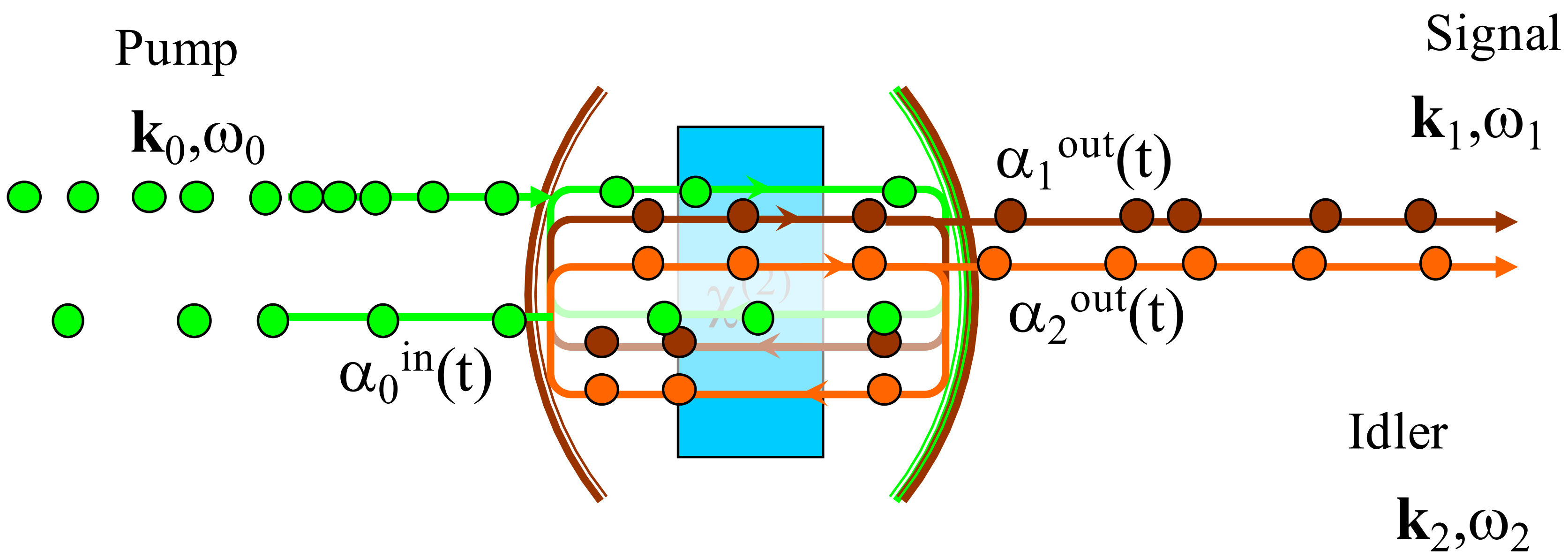
Energy and momentum conservation

$\omega_0 = \omega_1 + \omega_2$ $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$

Polarization and transverse momentum correlations

Optical Parametric Oscillator

PDC + Cavity

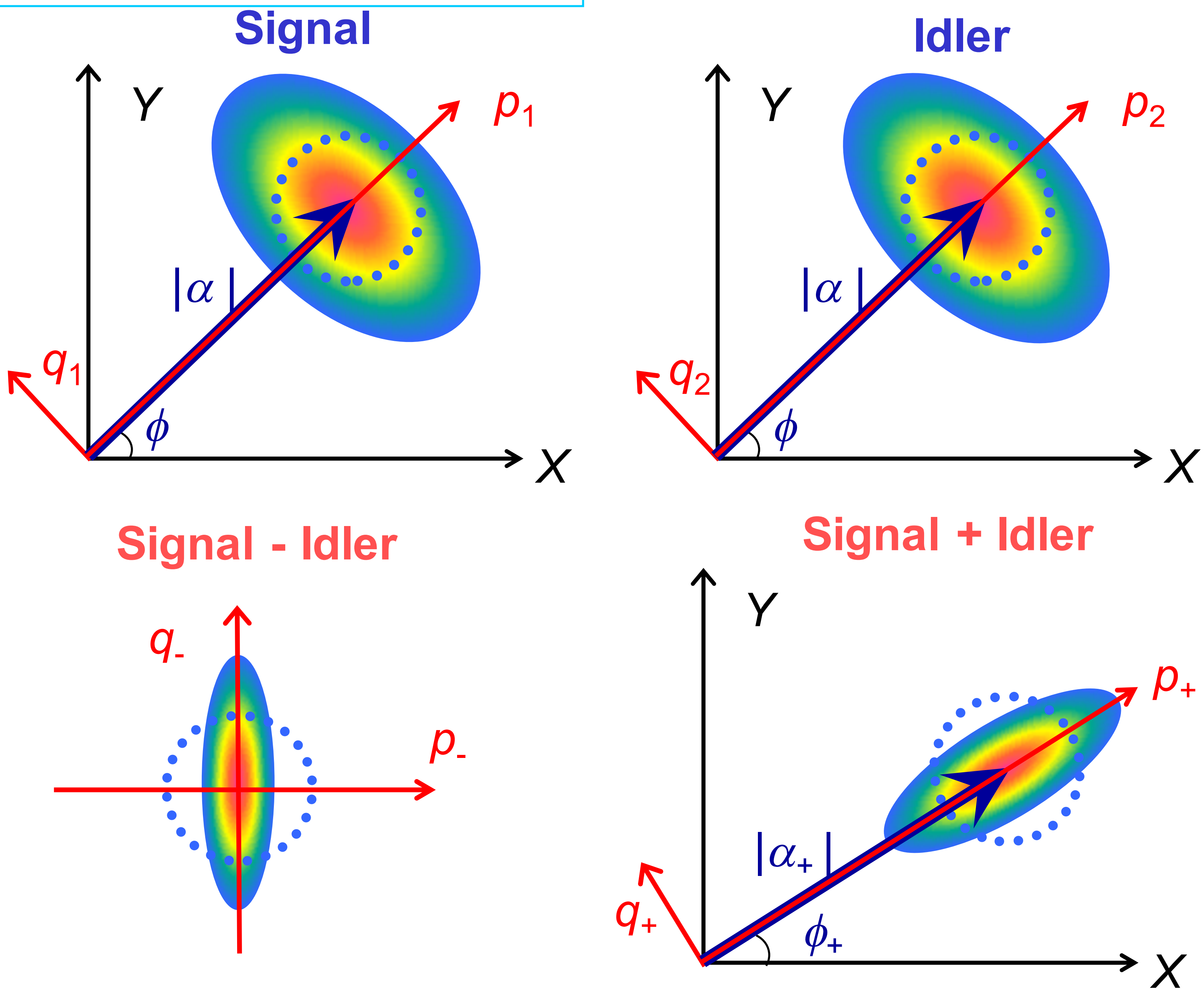


Twin photons + phase correlation

- Sub-threshold
  - squeezed vacuum (degenerate case) - OPA
  - entangled fields (non-degenerate case)

- Above threshold: intense entangled fields

Noise correlations



Energy Conservation

$$\omega_1 + \omega_2 = \omega_0$$

$$\delta I_1 - \delta I_2 = 0$$

Intensity Correlation

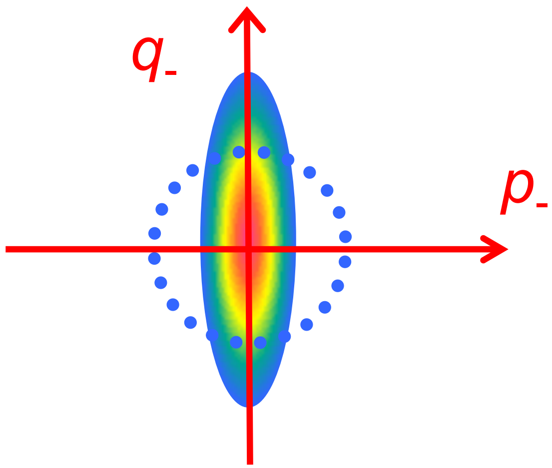
A. Heidmann *et al.*, PRL. **59**, 2555 (1987)

$$\delta\varphi_1 + \delta\varphi_2 = \delta\varphi_0$$

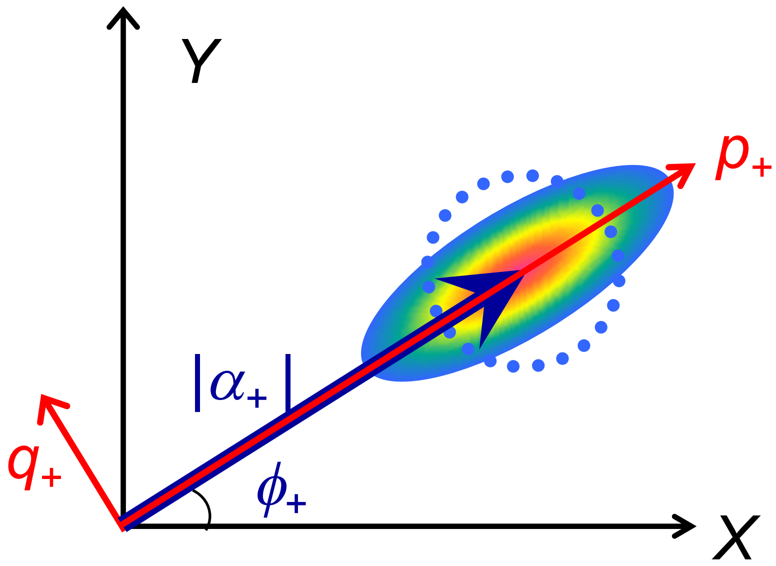
Phase Anti-correlation

A. S. Villar *et al.*, PRL **95**, 243603 (2005)

Signal - Idler



Signal + Idler



Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} & C_{p1p0} & C_{p1q0} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} & C_{q1p0} & C_{q1q0} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} & C_{p2p0} & C_{p2q0} \\ C_{p1q2} & C_{q1q0} & C_{p2q2} & S_{q2} & C_{q2p0} & C_{q2q0} \\ C_{p1p0} & C_{q1p0} & C_{p2p0} & C_{q2p0} & S_{p0} & C_{p0q0} \\ C_{p1q0} & C_{q1q0} & C_{p2q0} & C_{q2q0} & C_{p0q0} & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

36 independent terms !

Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q0} & 0 & S_{q2} & 0 & C_{q2q0} \\ C_{p1p0} & 0 & C_{p2p0} & 0 & S_{p0} & 0 \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

18 independent terms !

Twin beams ($P_0 > P_{th}$)

Squeezed vacuum ($P_0 < P_{th}$, degenerate)

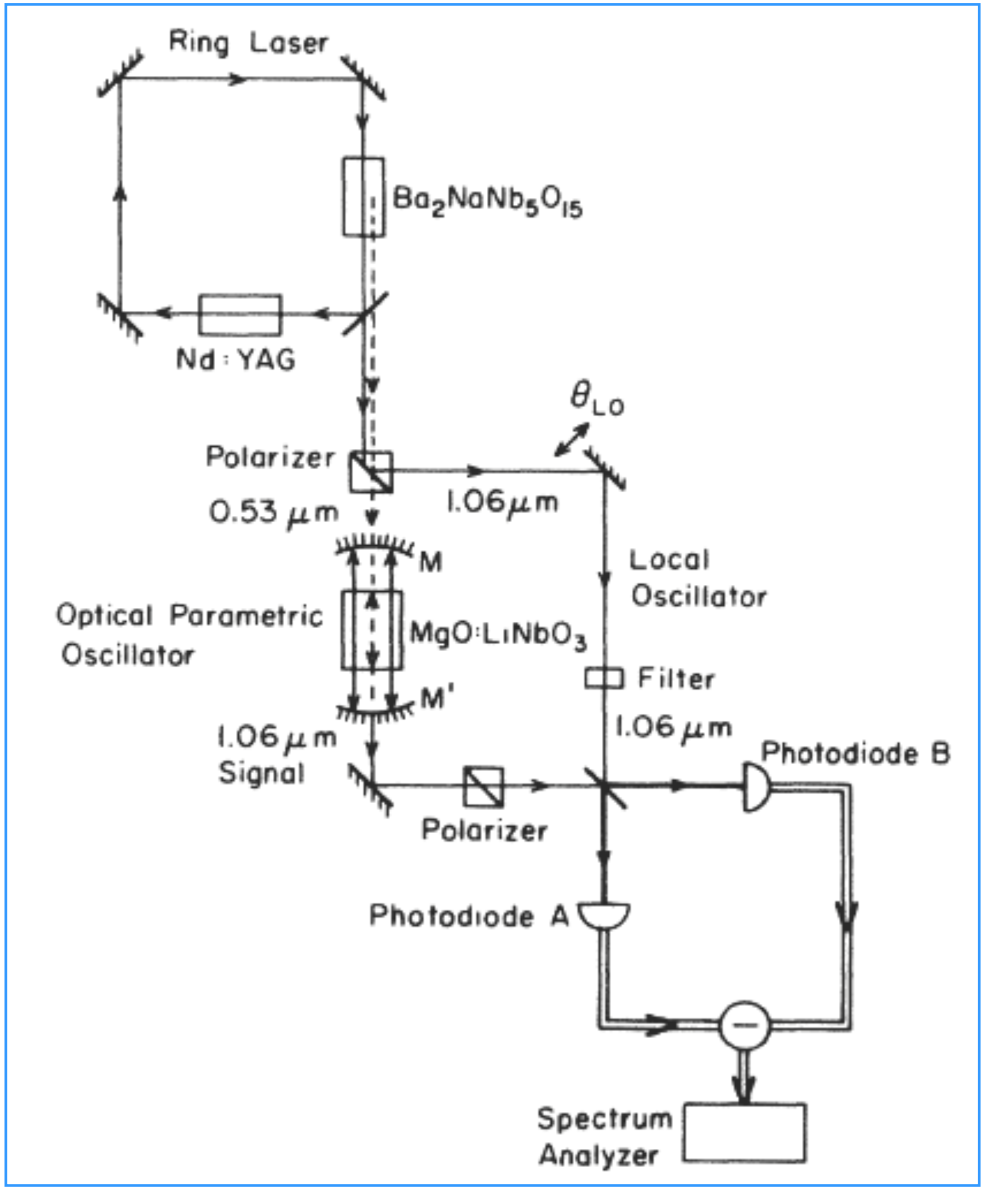
$$V = \begin{bmatrix} S_{p-} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{q-} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{p+} & 0 & C_{p+p0} & 0 \\ 0 & 0 & 0 & S_{q+} & 0 & C_{q+q0} \\ 0 & 0 & C_{p+p0} & 0 & S_{p0} & 0 \\ 0 & 0 & 0 & C_{q+q0} & 0 & S_{q0} \end{bmatrix}$$

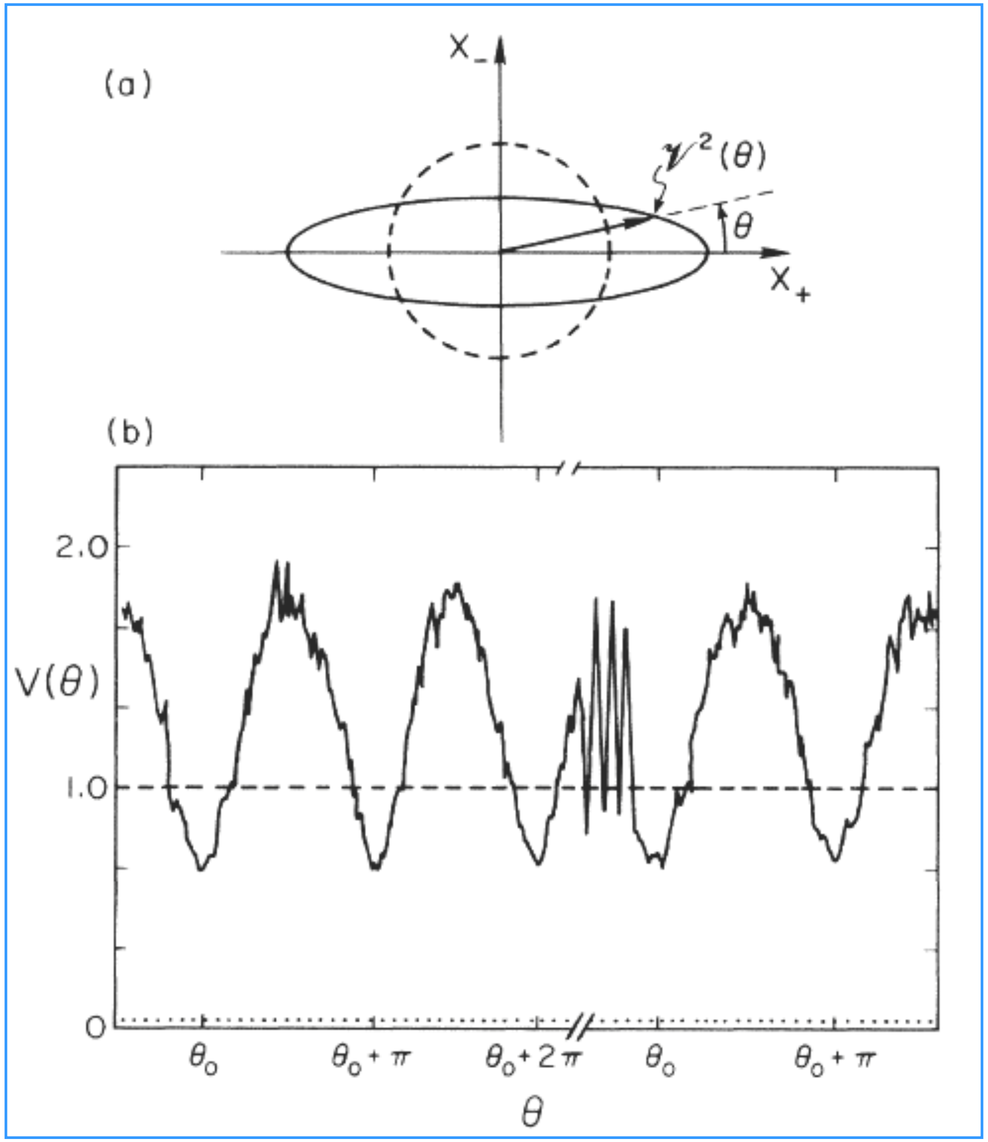
Entangled fields
- Vacuum ($P_0 < P_{th}$, maximum entanglement)
- Intense beams ($P_0 > P_{th}$)

Pump Squeezing ($P_0 > P_{th}$)

Generation of Squeezed States by Parametric Down Conversion

Ling-An Wu, H. J. Kimble, J. L. Hall,[a] and Huifa Wu
Department of Physics, University of Texas at Austin, Austin, Texas 78712
(Received 11 September 1986)





VOLUME 57, NUMBER 20 PHYSICAL REVIEW LETTERS 17 NOVEMBER 1986

Generation of Squeezed States by Parametric Down Conversion

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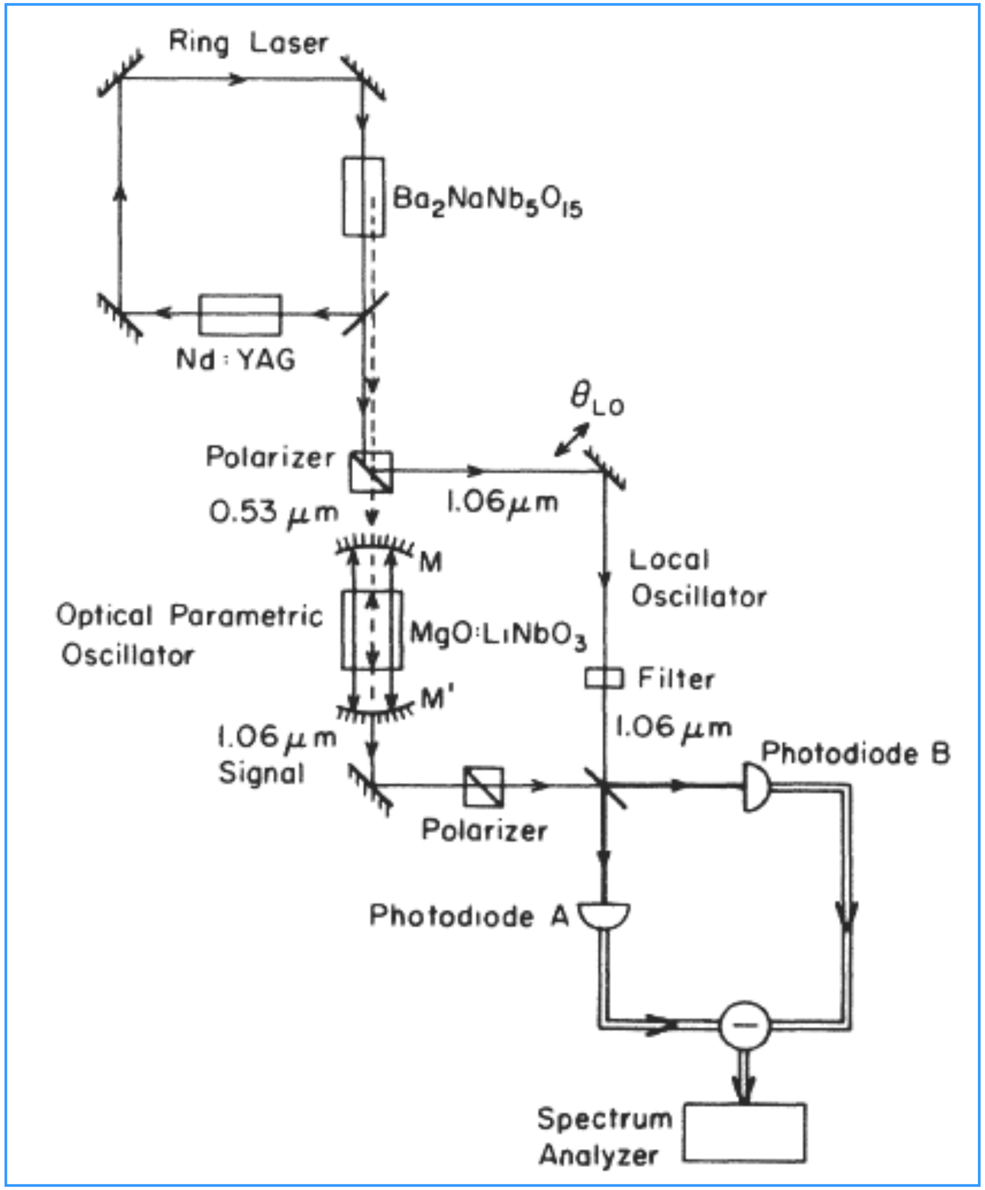
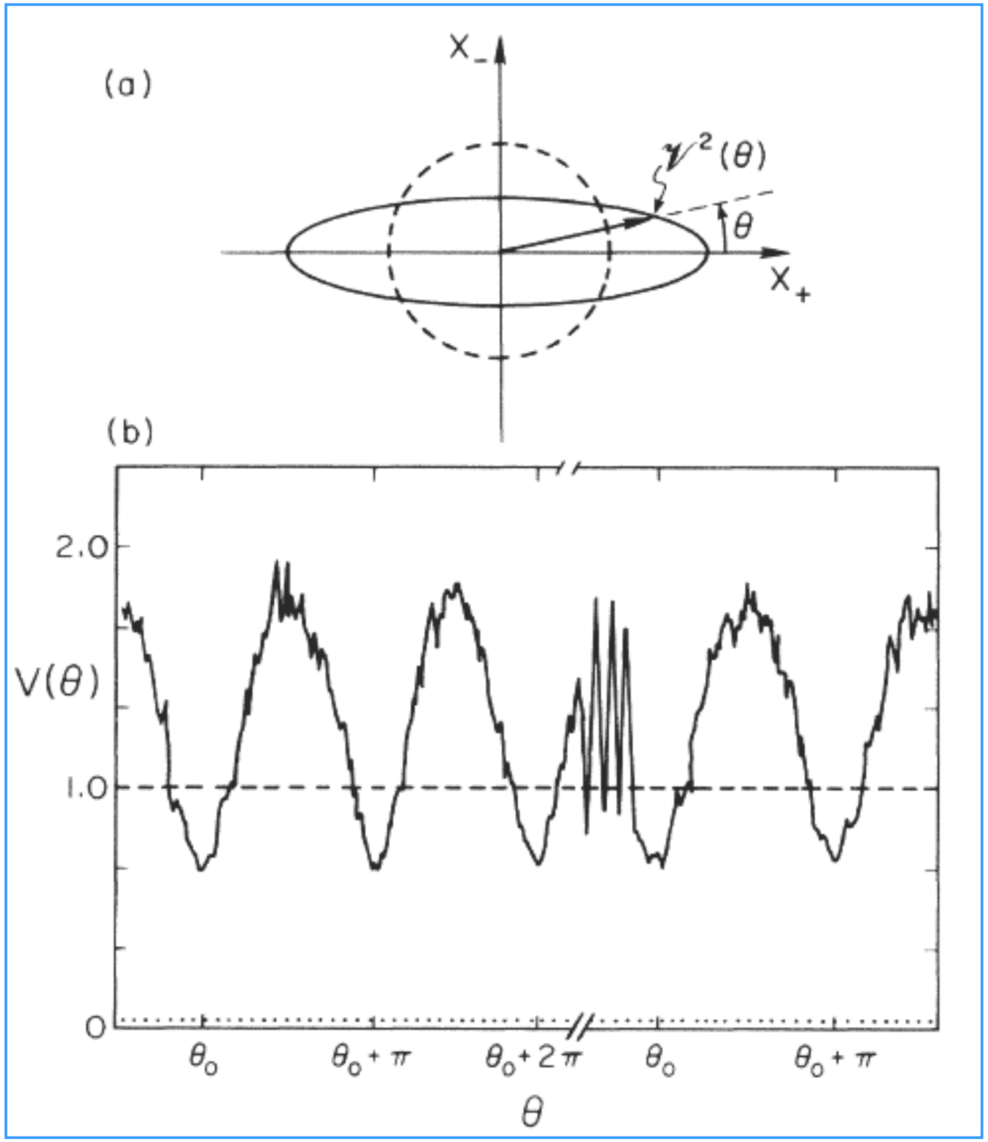


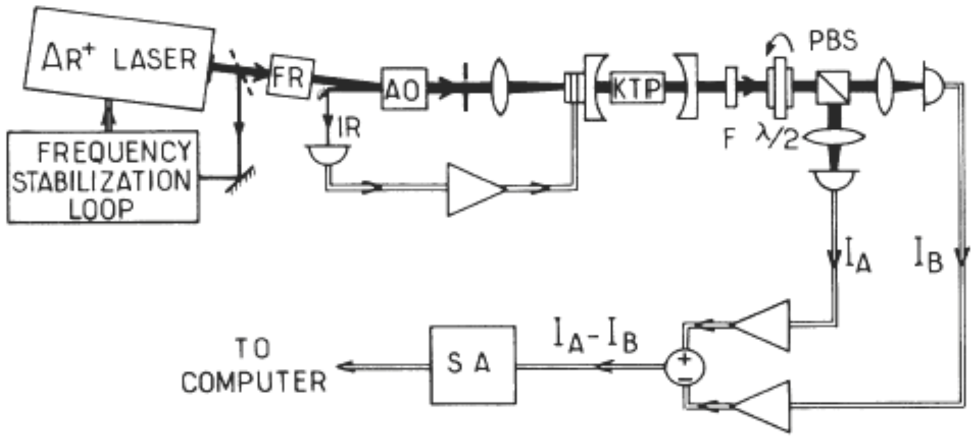


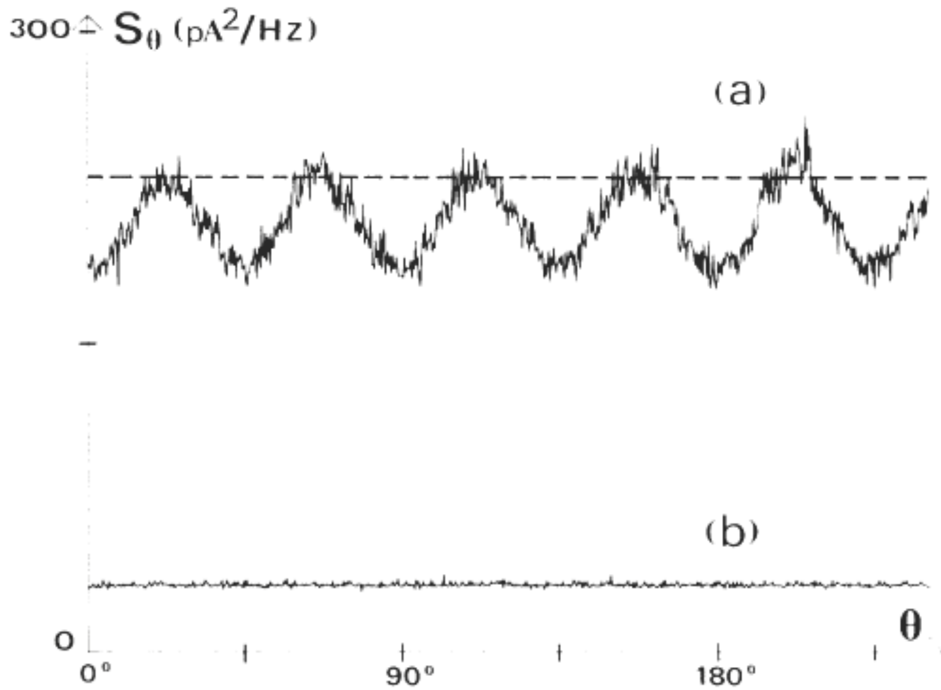
Observation of Quantum Noise Reduction on Twin Laser Beams

A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, and C. Fabre
Laboratoire de Spectroscopie Hertzienne de l'Ecole Normale Supérieure, Université Pierre et Marie Curie, 75252 Paris Cedex 05, France

and

G. Camy
Laboratoire de Physique des Lasers, Université de Paris Nord, 93430 Villetaneuse, France
(Received 3 August 1987)





Europhys. Lett., **40** (1), pp. 25-30 (1997)

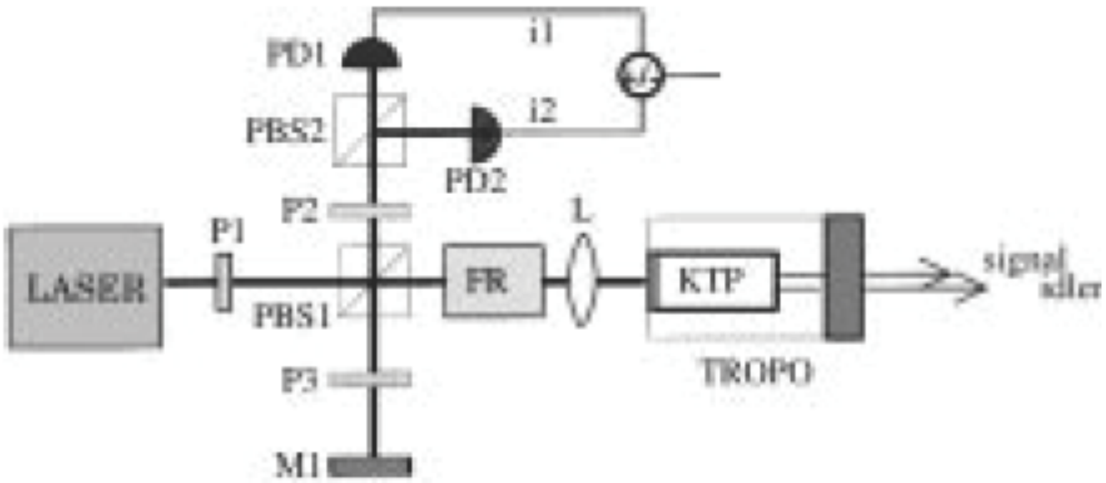
Observation of squeezing using cascaded nonlinearity

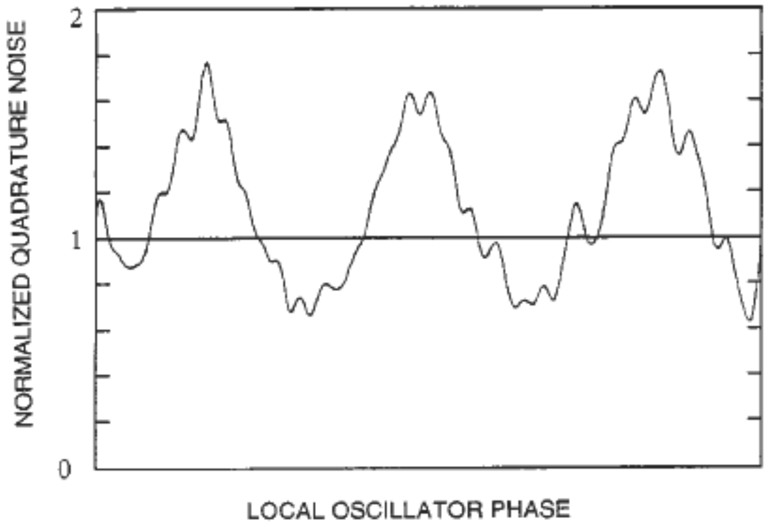
K. Kasai(*), Gao Jiangrui(**) and C. Fabre

Laboratoire Kastler Brossel (***) UPMC - Case 74 75252 Paris Cedex 05, France

(received 20 January 1997; accepted in final form 2 September 1997)

Abstract. – We have observed that the pump beam reflected by a triply resonant optical parametric oscillator, after a cascaded second-order nonlinear interaction in the crystal, is significantly squeezed. The maximum measured squeezing in our device is 30% (output beam squeezing inferred: 48%). The direction of the noise ellipse depends on the cavity detuning and can be adjusted from intensity squeezing to phase squeezing.





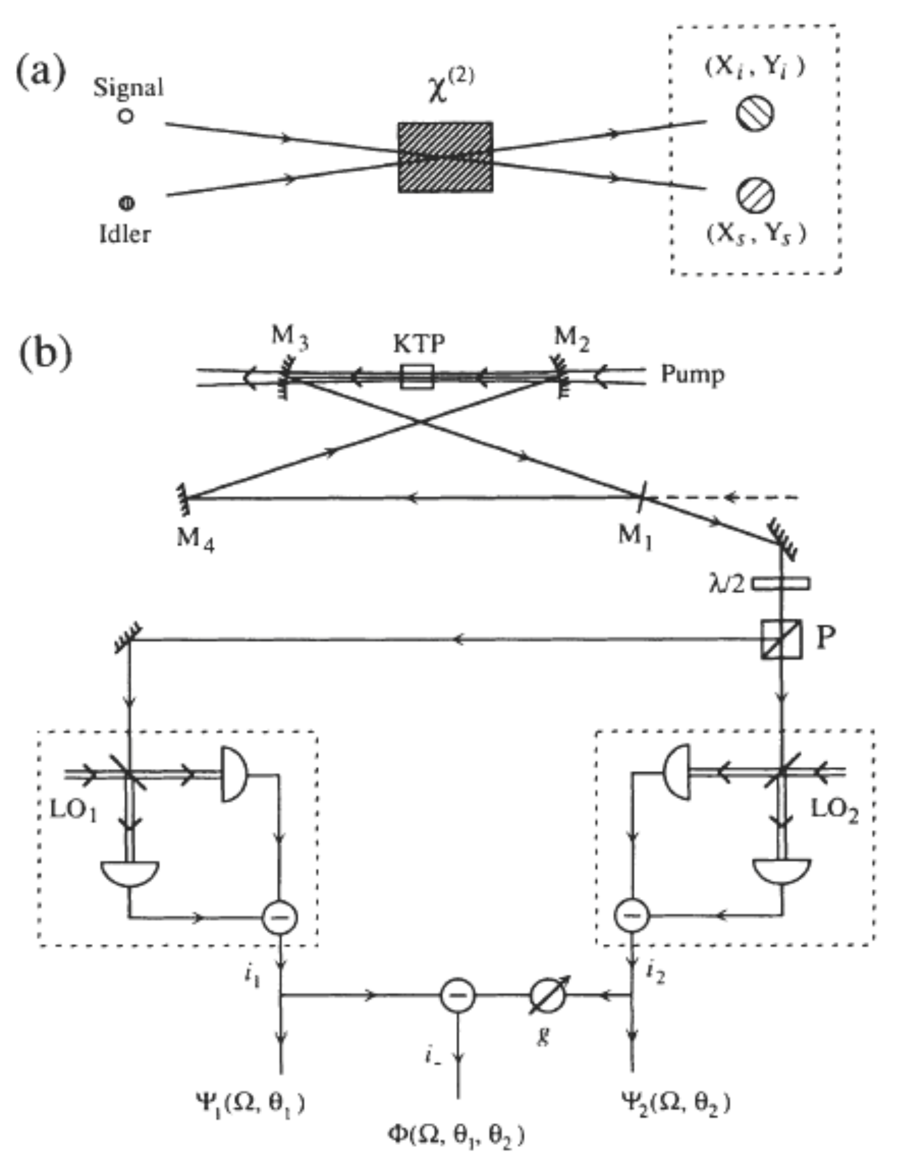
Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables

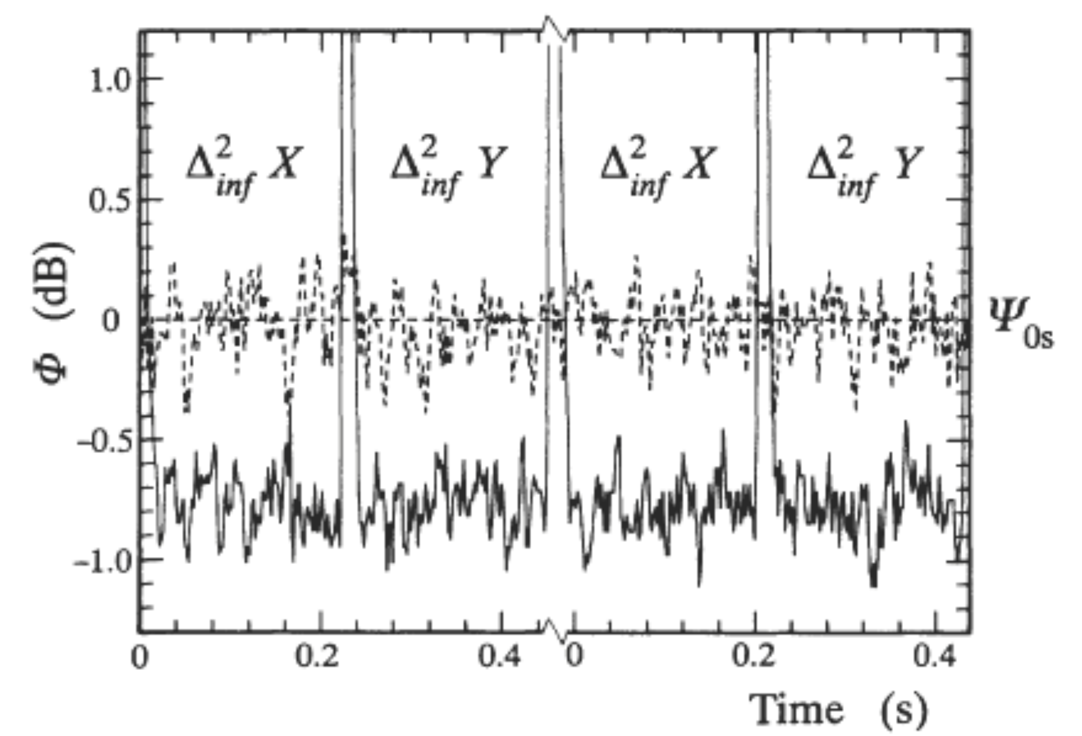
Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng[(a)]
Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125
(Received 20 February 1992)

The Einstein-Podolsky-Rosen paradox is demonstrated experimentally for dynamical variables having a continuous spectrum. As opposed to previous work with discrete spin or polarization variables, the continuous optical amplitudes of a signal beam are inferred in turn from those of a spatially separated but strongly correlated idler beam generated by nondegenerate parametric amplification. The uncertainty product for the variances of these inferences is observed to be 0.70 ± 0.01, which is below the limit of unity required for the demonstration of the paradox.

Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables

Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng[a]





$$\Delta^2_{inf}X\,\Delta^2_{inf}Y = 0.70 \pm 0.01$$

But if we look for a complete characterization of the OPO, we have to measure three fields of diferent colors!

Is it possible to perform a homodyne measurement without a local oscillator?

Multipartite entanglement in the OPO

Marcelo Martinelli

LMCAL - IFUSP

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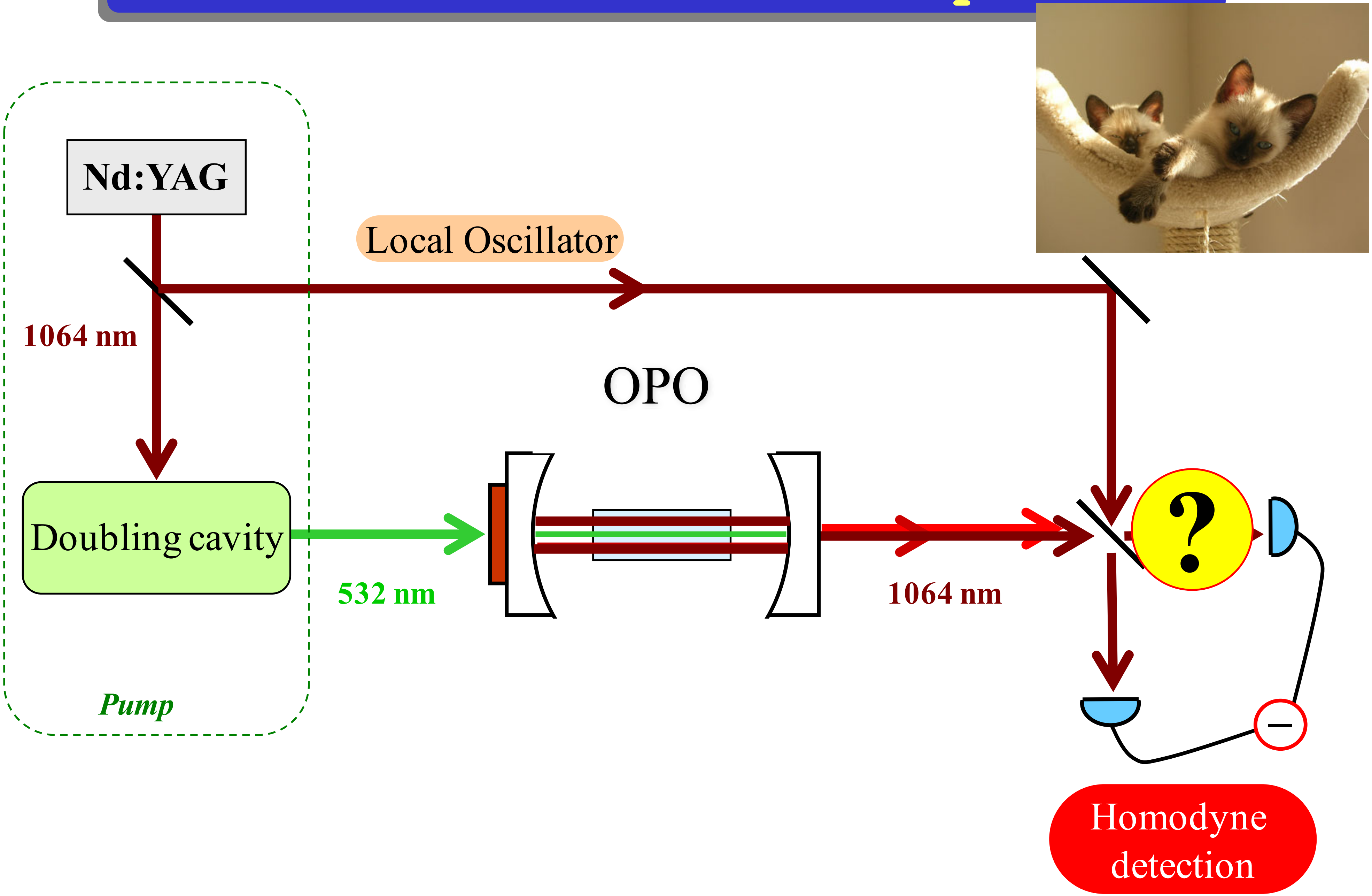
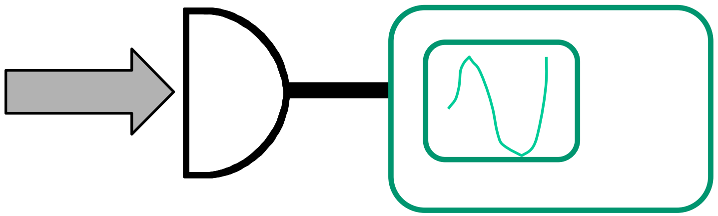
How can we measure the phase?

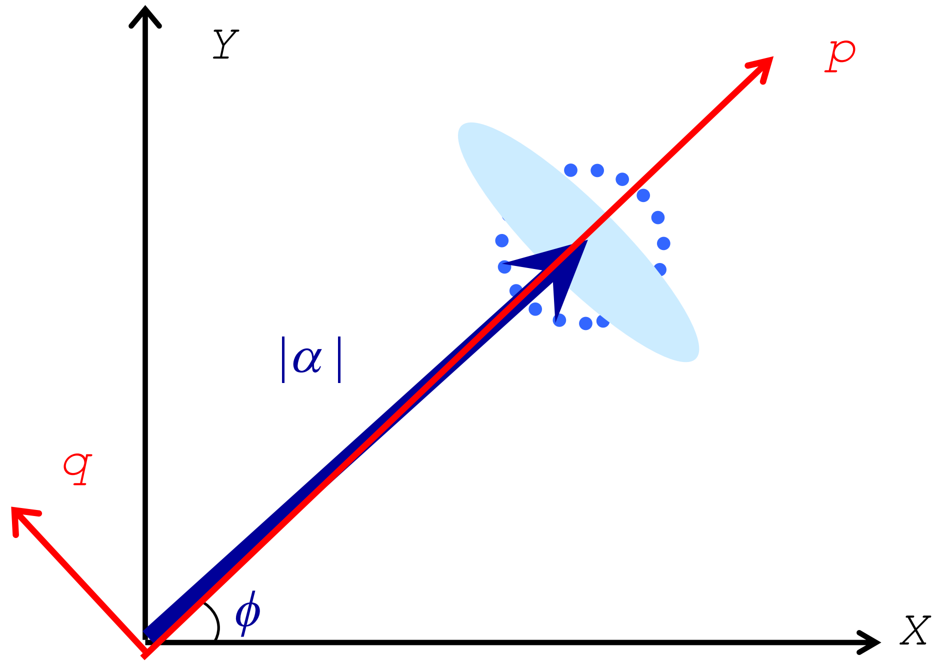


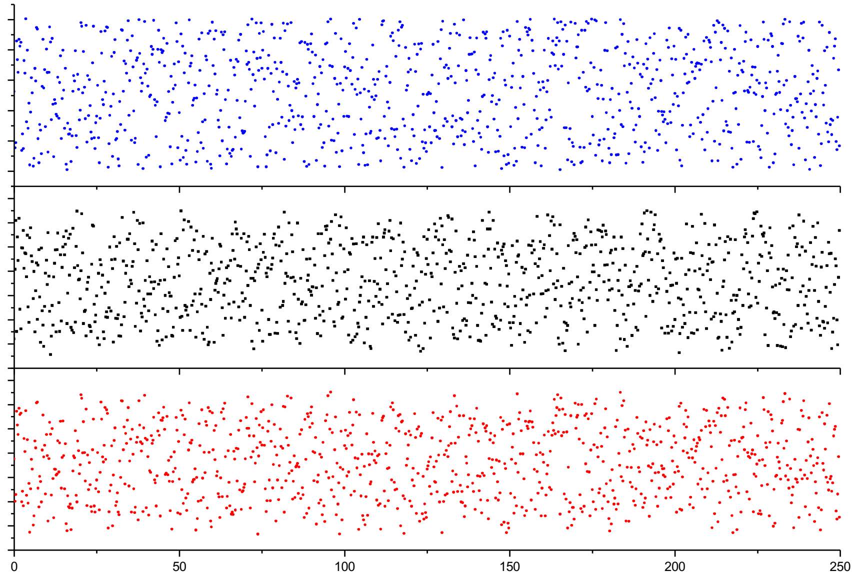

ONLY THEIR MOTHER CAN TELL THEM APART.
TWINS
AN IVAN REITMAN FILM
ARNOLD SCHWARZENEGGER DANNY DEVITO "TWINS" KELLY PRESTON CHLOE WEBB BONNIE BARTLETT WRITTEN BY WILLIAM DAVIES &

Measurement of the Field in the time domain

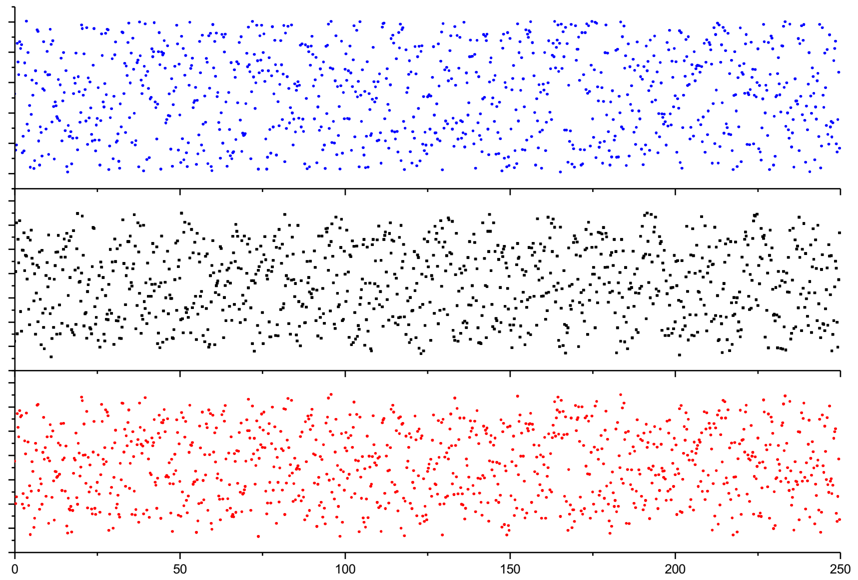


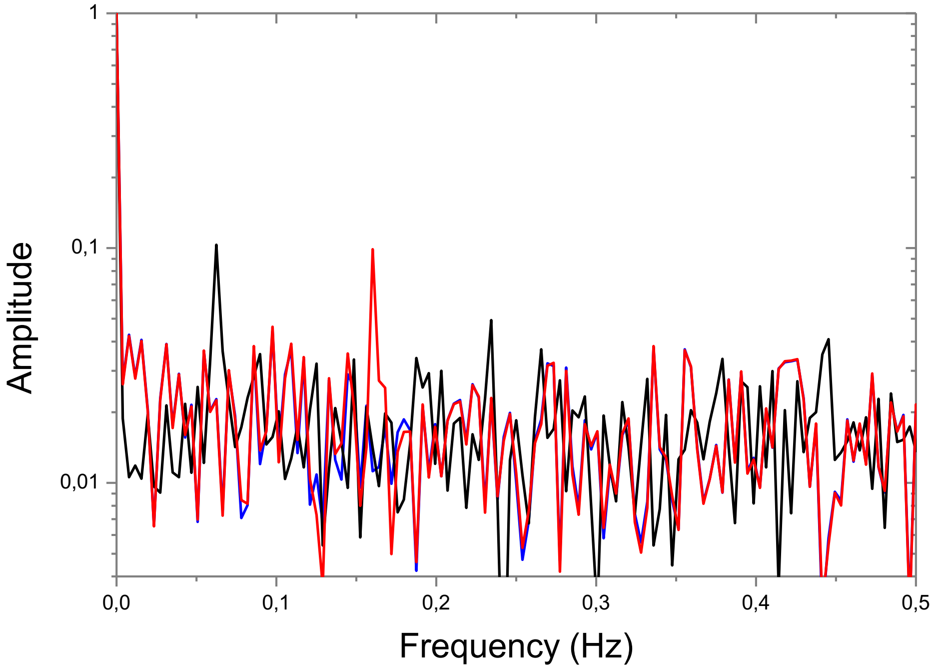
$$\hat{n} = |\alpha|^2 + |\alpha|\delta\hat{p}$$





Measurement of the Field in the frequency domain





Measurement of the Field in the frequency domain

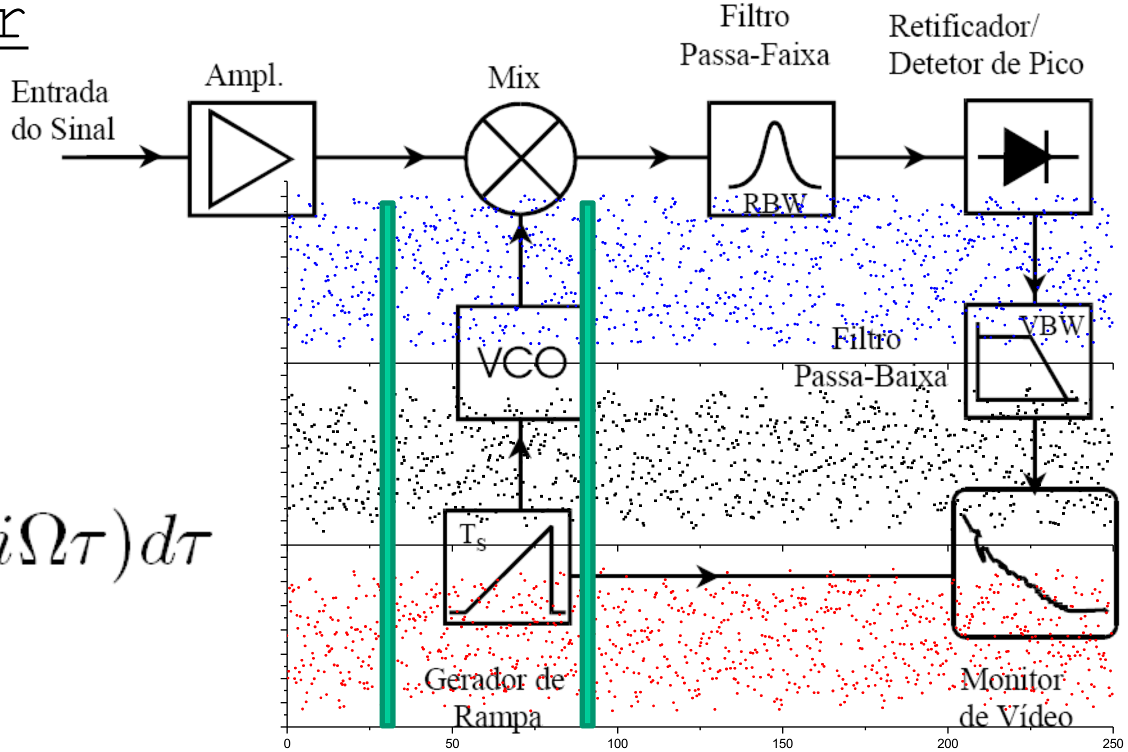
$$\hat{a}(t) = \int_{-\infty}^{\infty} \hat{a}(\Omega) \exp(-i\Omega t) \, d\Omega.$$

$$\hat{a}(\Omega) = \hat{x}(\Omega) + i\hat{y}(\Omega)$$

Spectrum Analyser

$$\langle \hat{X}(t)\hat{X}(t+\tau)\rangle =$$

$$\int \hat{X}(\Omega)\hat{X}(-\Omega)exp(i\Omega\tau)d\tau$$



Measurement of the Field in the frequency domain

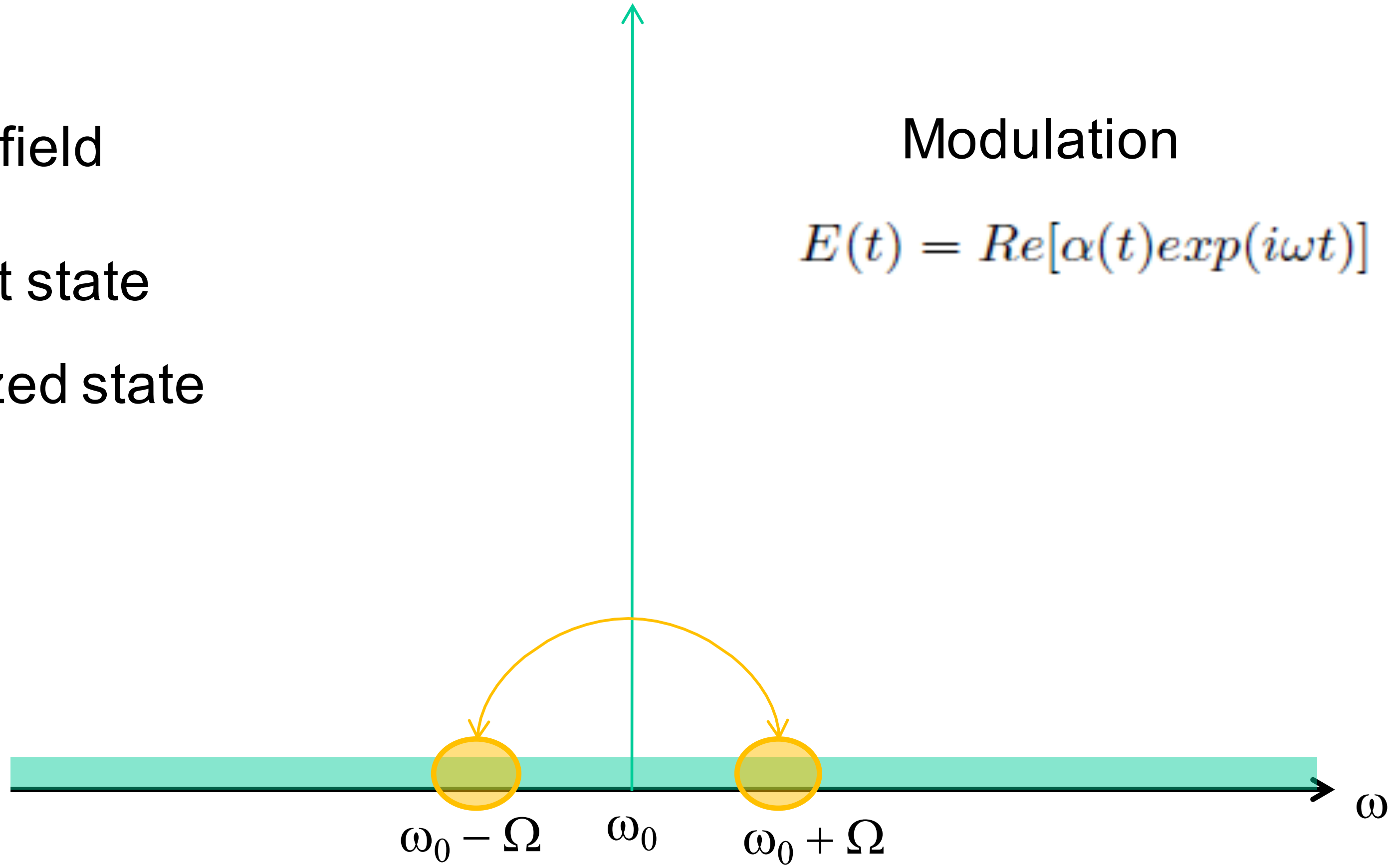
A classic field

Coherent state

Squeezed state

Modulation

$$E(t) = Re[\alpha(t) exp(i\omega t)]$$



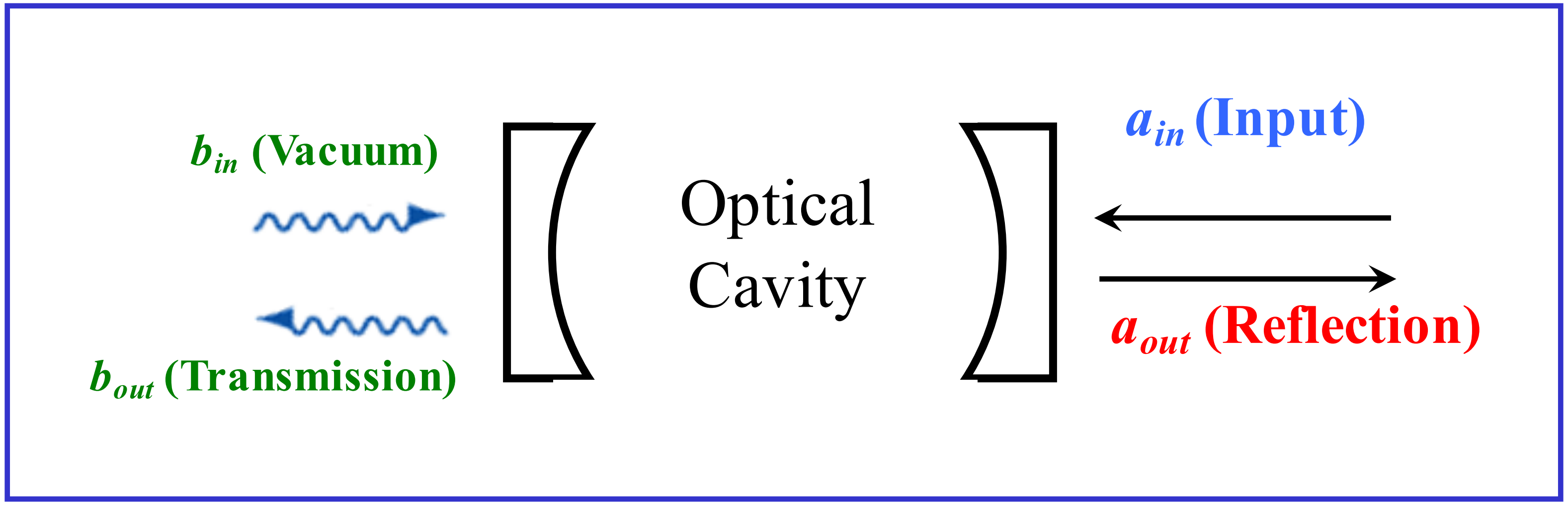
Amplitude $\alpha(t) = A[1 + 2\kappa cos(\Omega t)]$

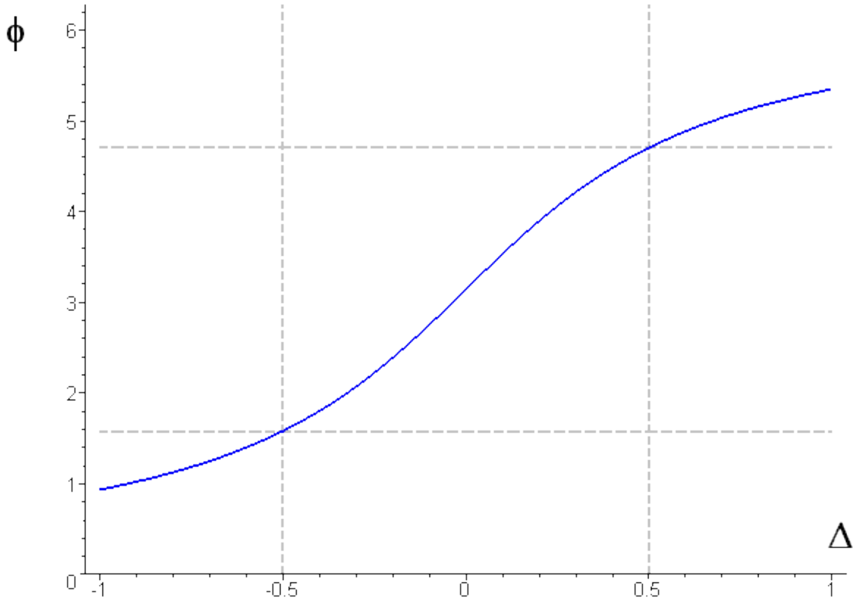
$$E(t) = A\, Re\{\kappa exp[i(\omega - \Omega)t] + exp(i\omega t) + \kappa exp[i(\omega + \Omega)t]\}$$

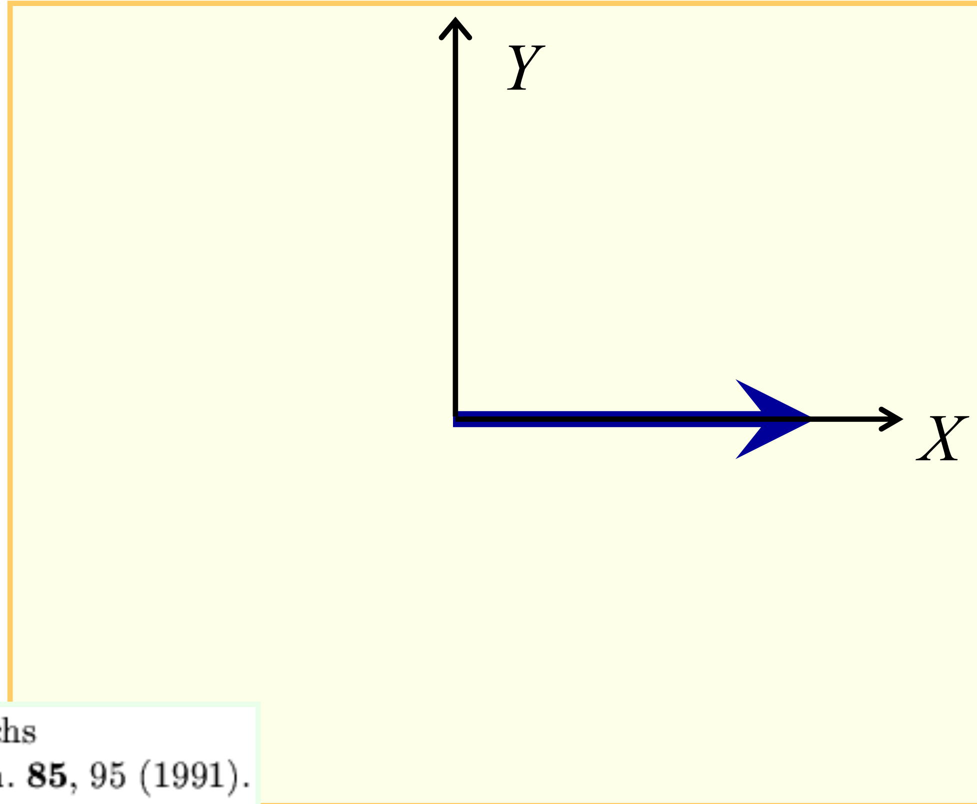
Phase $\alpha(t) = A exp[2i\kappa cos(\Omega t)] \simeq A[1 + 2i\kappa cos(\Omega t)]$

$$E(t) = A\, Re\{i\kappa exp[i(\omega - \Omega)t] + exp(i\omega t) + i\kappa exp[i(\omega + \Omega)t]\}$$

Phase Rotation of Noise Ellipse





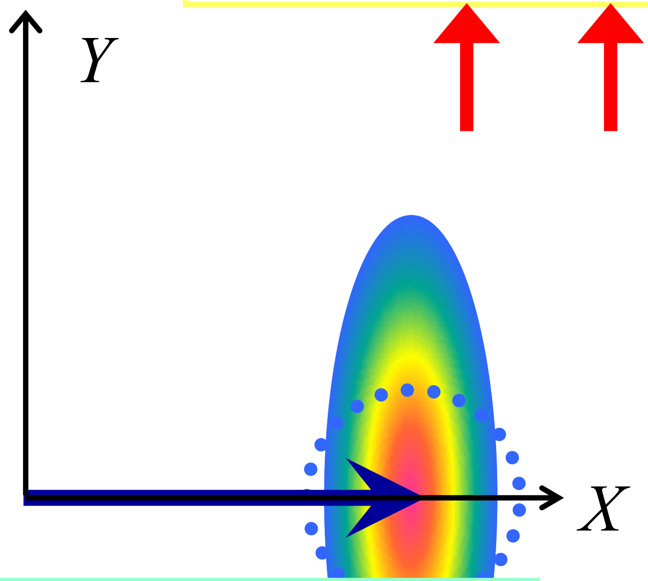


P. Galatola, L.A. Lugiato, M.G. Porreca, P. Tombesi e G. Leuchs
System control by variation of the squeezing phase, Opt. Comm. **85**, 95 (1991).

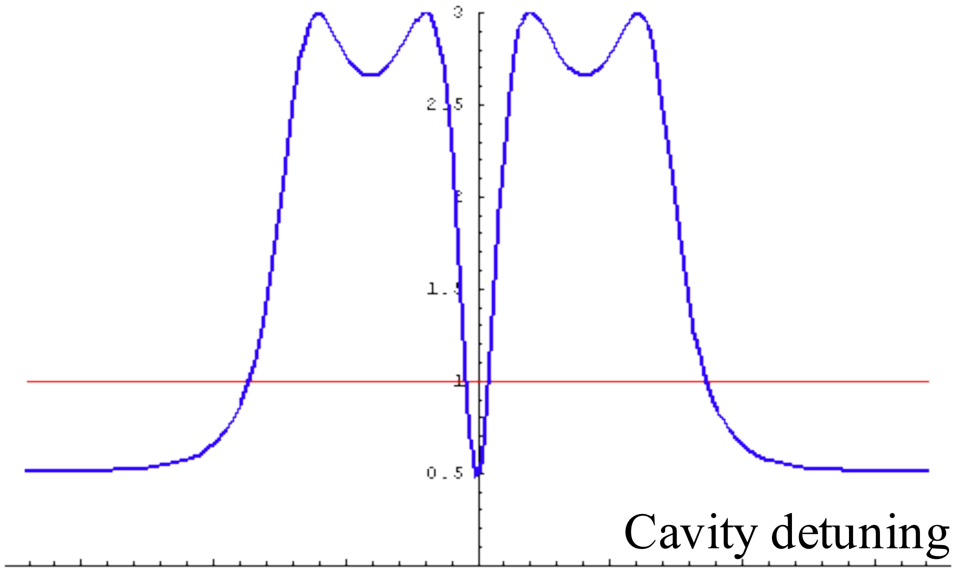
Alessandro S. Villar, *The conversion of phase to amplitude fluctuations of a light beam by an optical cavity*
American Journal of Physics **76**, pp. 922-929 (2008).

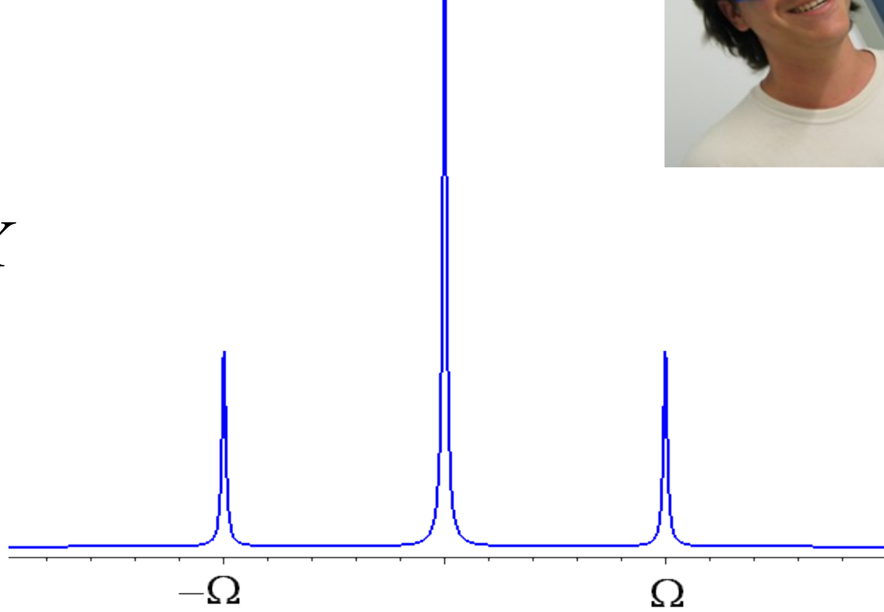
$$\delta p_{in} = \delta a_{in}(\Omega) + \delta a^{\dagger}_{in}(-\Omega)$$

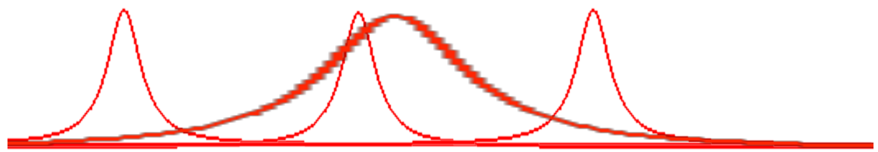
$$\delta p_{out} = \dots\ \dots\dots\ \delta a_{in}(\Omega) + \dots\dots\ \delta a^{\dagger}_{in}(-\Omega)$$



Reflected Beam
Amplitude Noise







Covariance Matrix

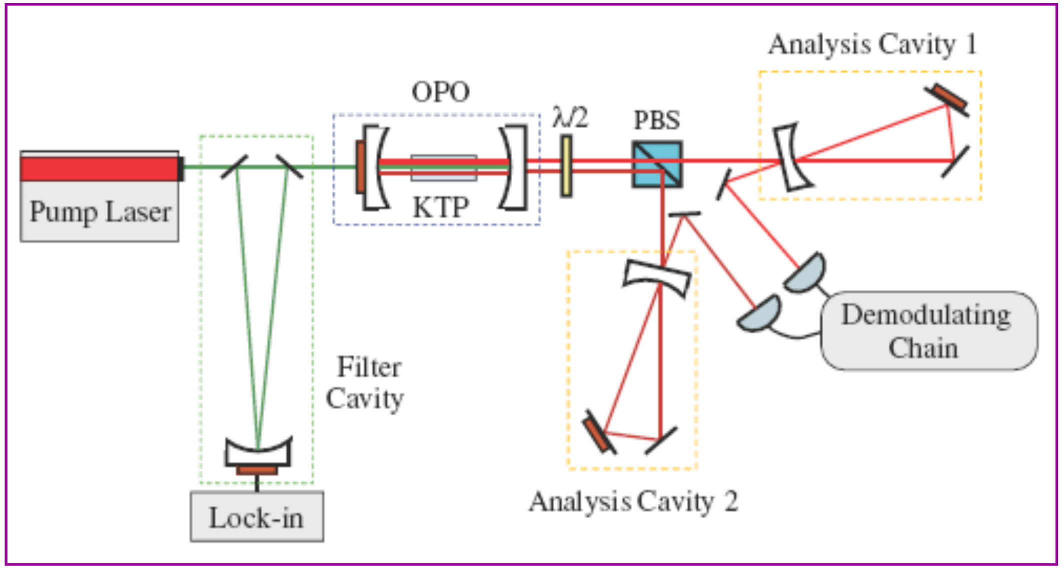
$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q0} & 0 & S_{q2} & 0 & C_{q2q0} \\ C_{p1p0} & 0 & C_{p2p0} & 0 & S_{p0} & 0 \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

Generation of Bright Two-Color Continuous Variable Entanglement

A. S. Villar, L. S. Cruz, K. N. Cassemiro, M. Martinelli, and P. Nussenzveig*
Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, São Paulo, Brazil

We present the first measurement of squeezed-state entanglement between the twin beams produced in an optical parametric oscillator operating above threshold. In addition to the usual squeezing in the intensity difference between the twin beams, we have measured squeezing in the sum of phase quadratures. Our scheme enables us to measure such phase anticorrelations between fields of different frequencies. In the present measurements, wavelengths differ by ≈ 1 nm. Entanglement is demonstrated according to the Duan *et al.* criterion [Phys. Rev. Lett. **84**, 2722 (2000)] $\Delta^2\hat{p}_- + \Delta^2\hat{q}_+ = 1.41(2) < 2$. This experiment opens the way for new potential applications such as the transfer of quantum information between different parts of the electromagnetic spectrum.



Generation of Bright Two-Color Continuous Variable Entanglement

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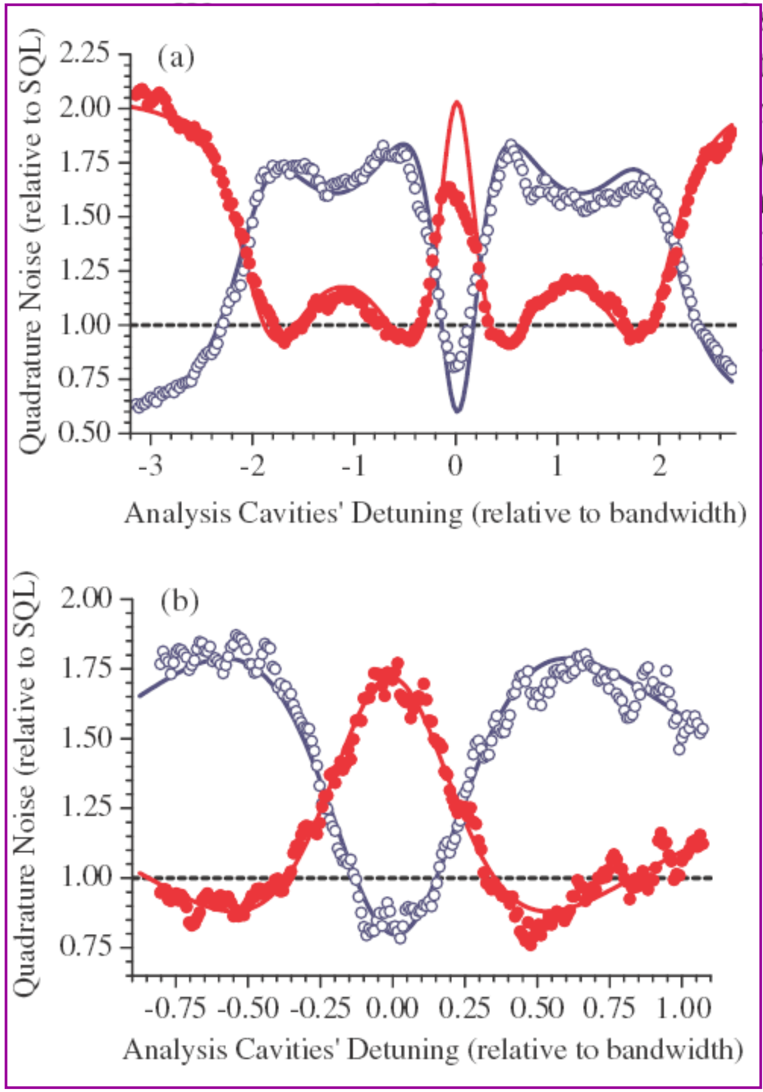

squeezed-state entanglement between the twin beams produced in
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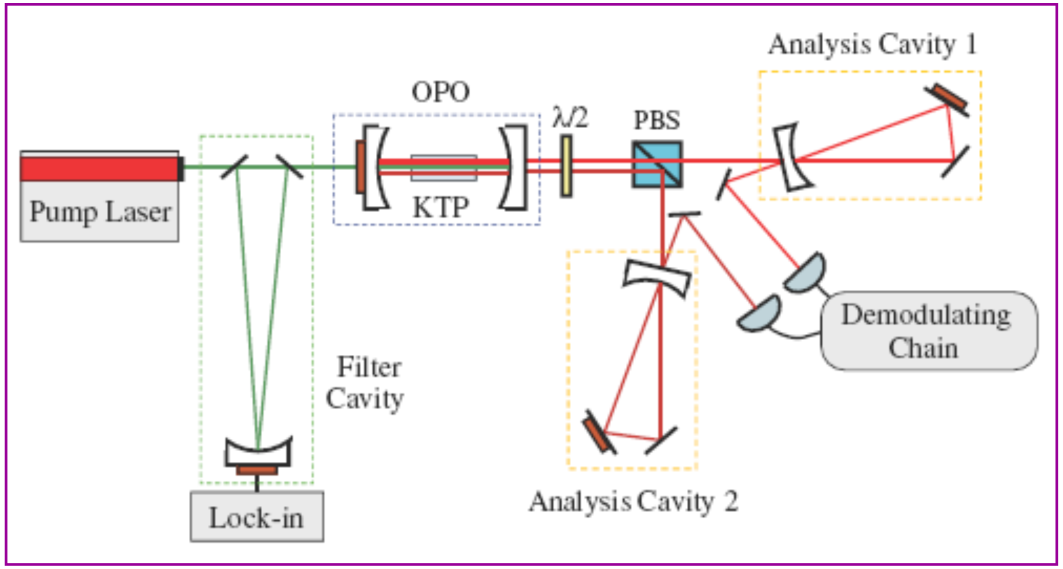
agnetic spectrum.



Two, three, many entangled modes

Marcelo Martinelli

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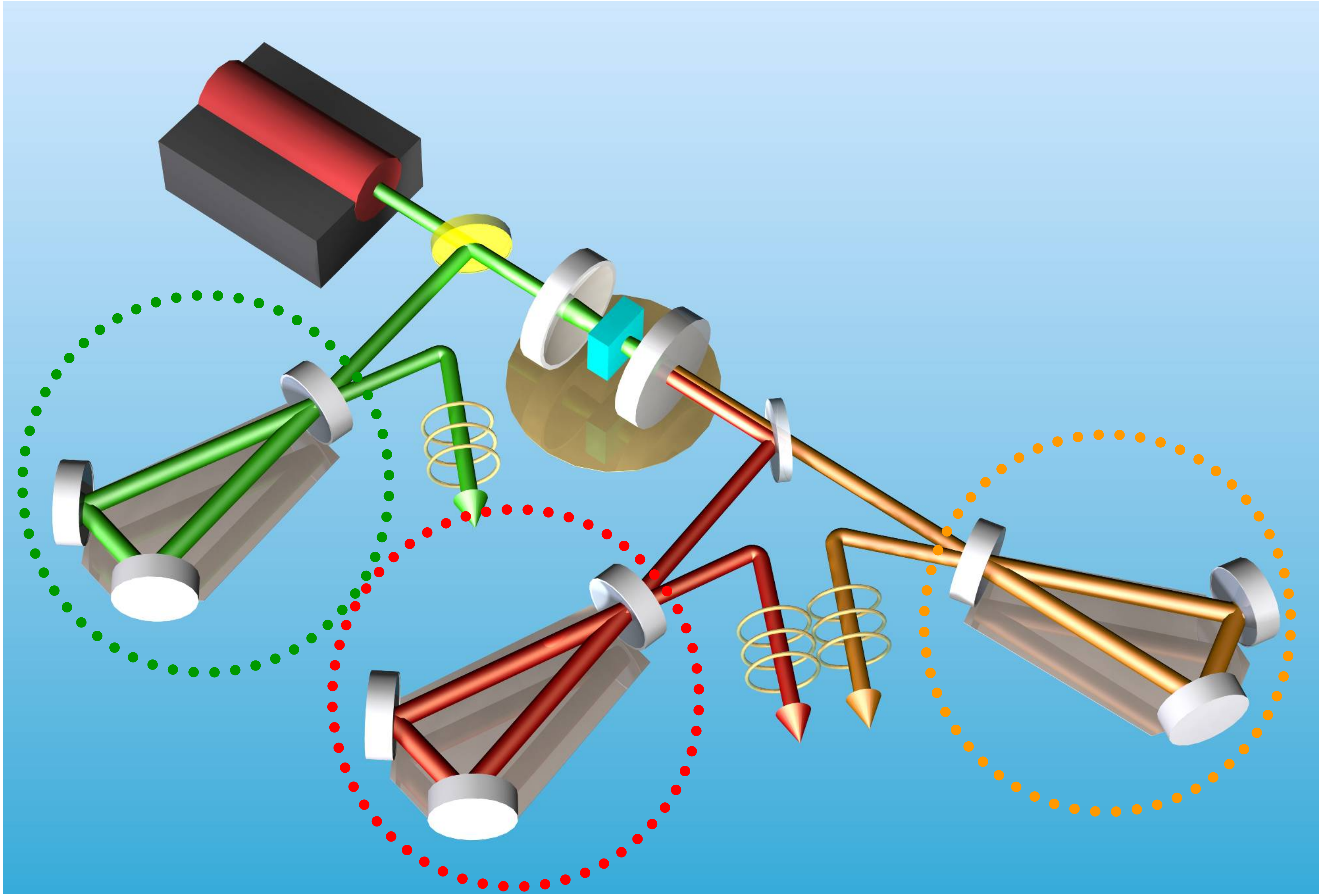
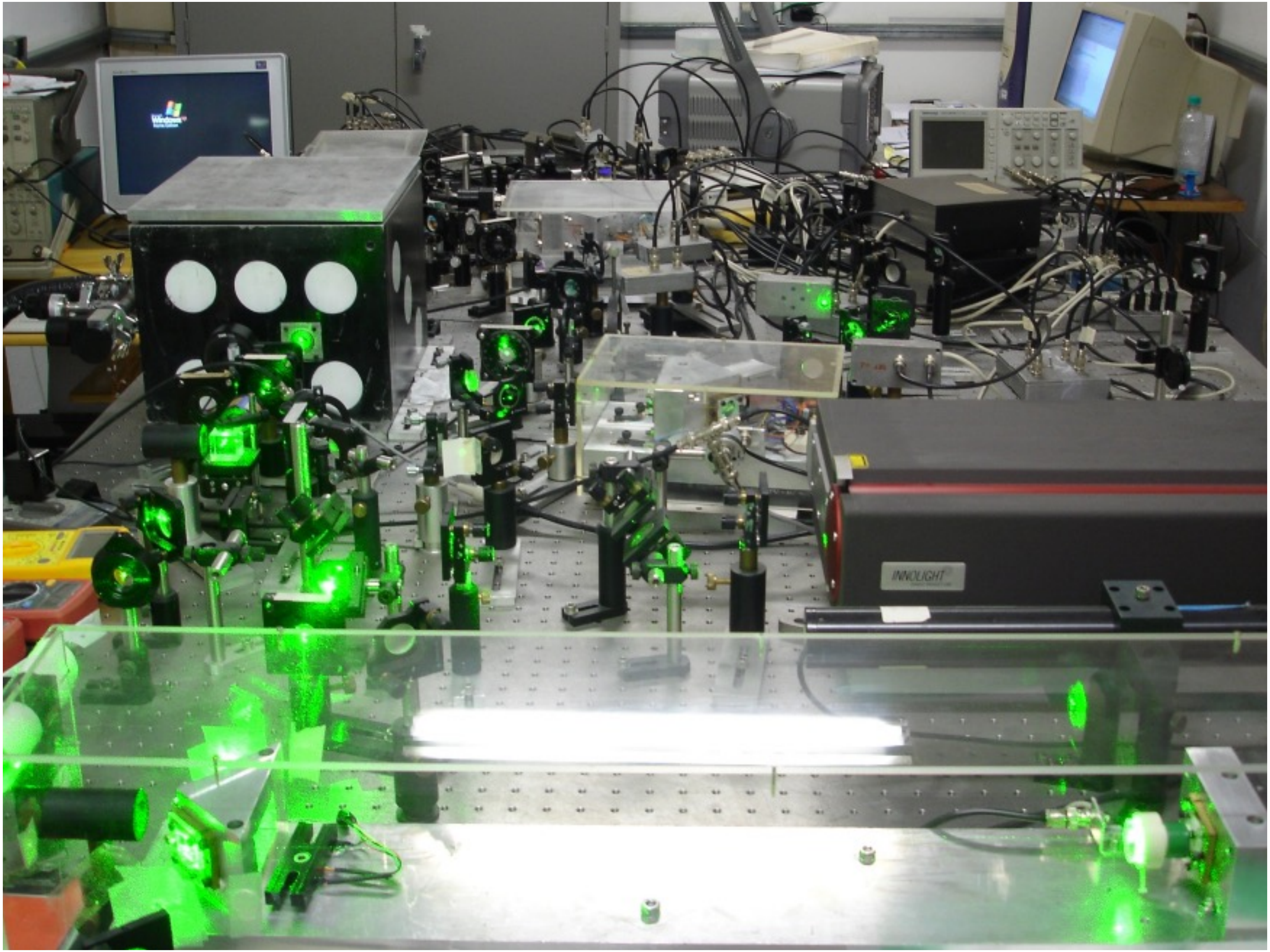
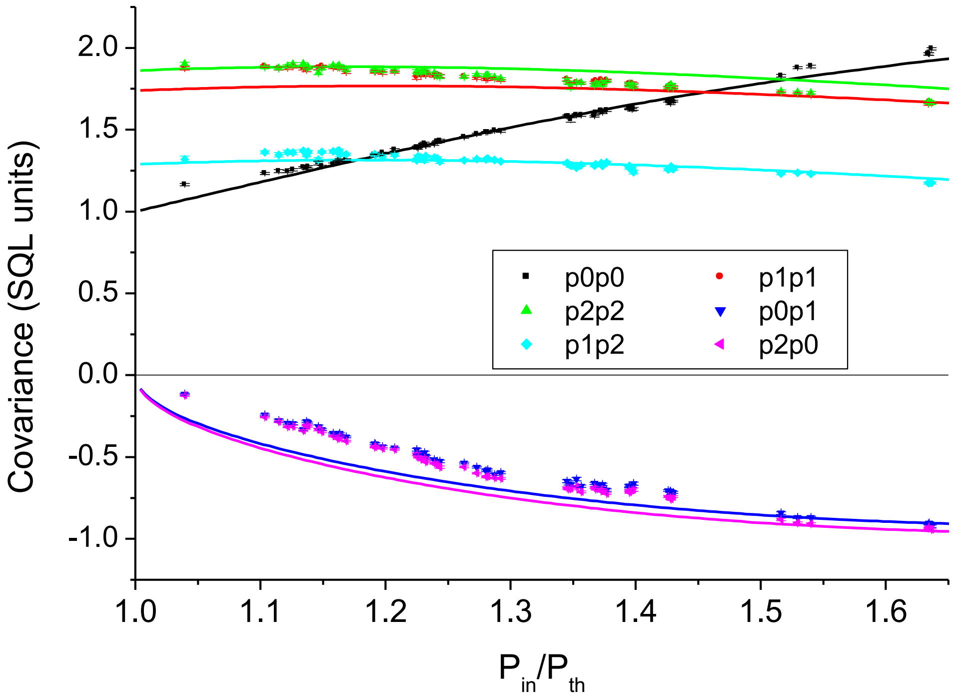
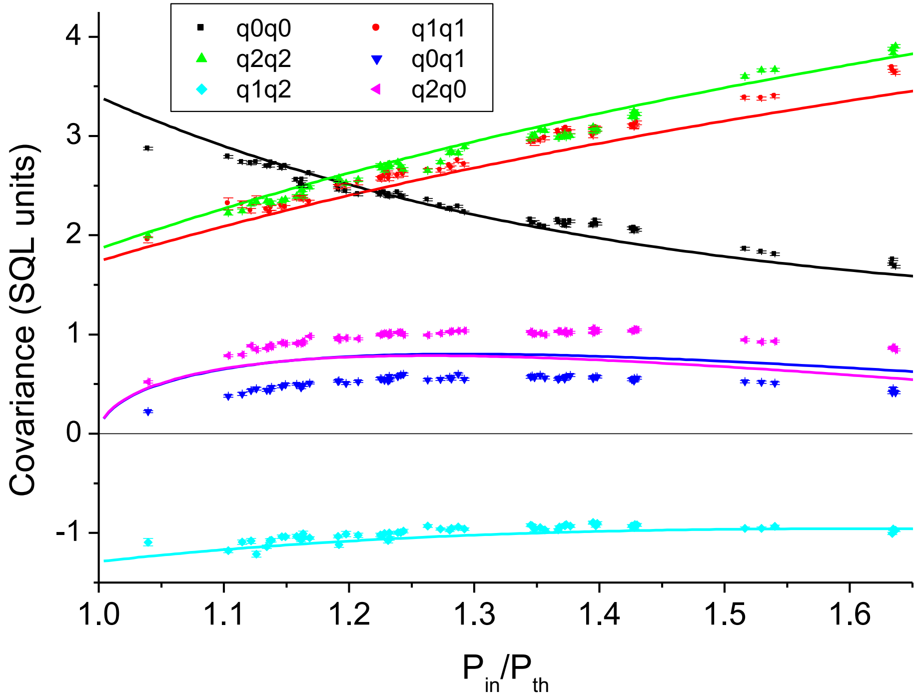
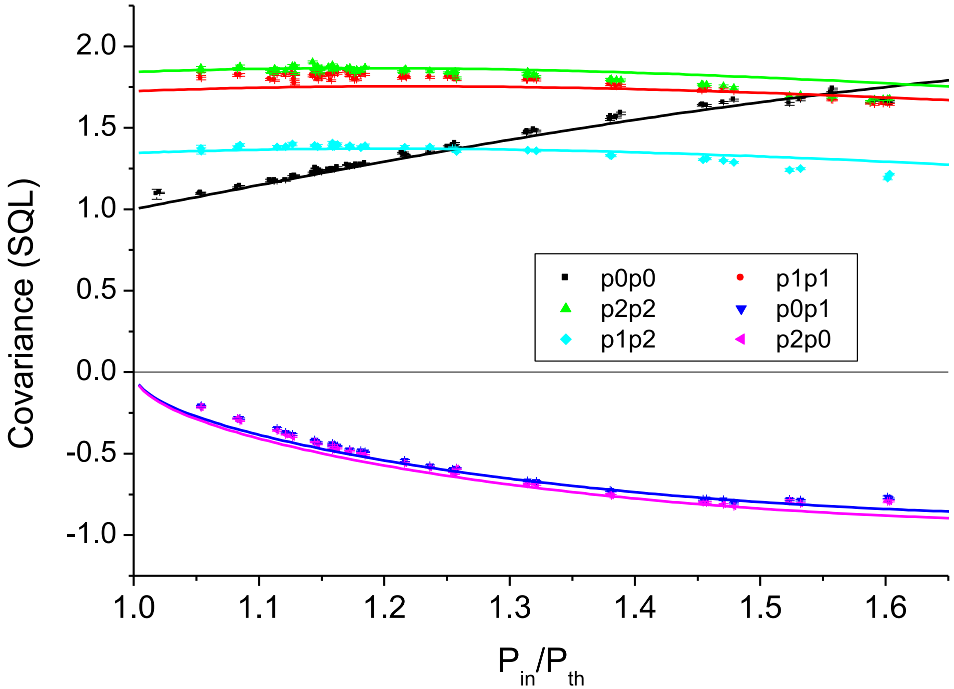
Full characterization of the OPO

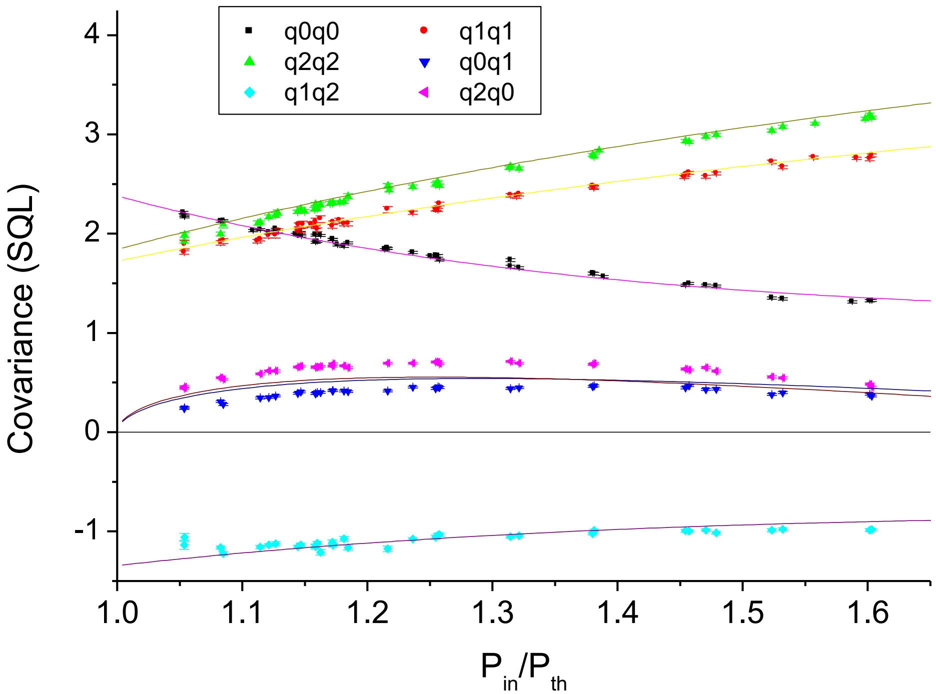



INNOLIGHT


Covariance (SQL units)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
1.0 1.1 1.2 1.3 1.4 1.5 1.6
P_in/P_th
p0p0
p1p1
p2p2
p0p1
p1p2
p2p0


Covariance (SQL units)
4
3
2
1
0
-1
1.0 1.1 1.2 1.3 1.4 1.5 1.6
P_in/P_th
q0q0
q1q1
q2q2
q0q1
q1q2
q2q0


Covariance (SQL)
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
1.0 1.1 1.2 1.3 1.4 1.5 1.6
P_in/P_th
p0p0
p1p1
p2p2
p0p1
p1p2
p2p0


Covariance (SQL)
4
3
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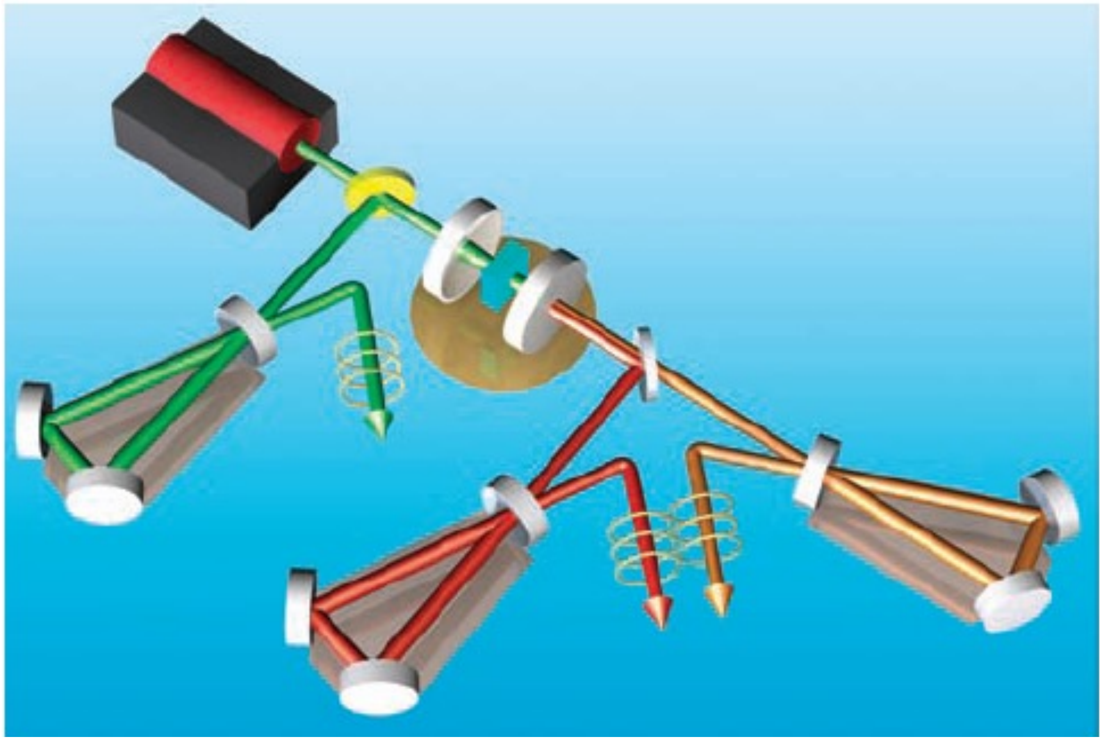
1.0 1.1 1.2 1.3 1.4 1.5 1.6
P_in/P_th
q0q0
q1q1
q2q2
q0q1
q1q2
q2q0

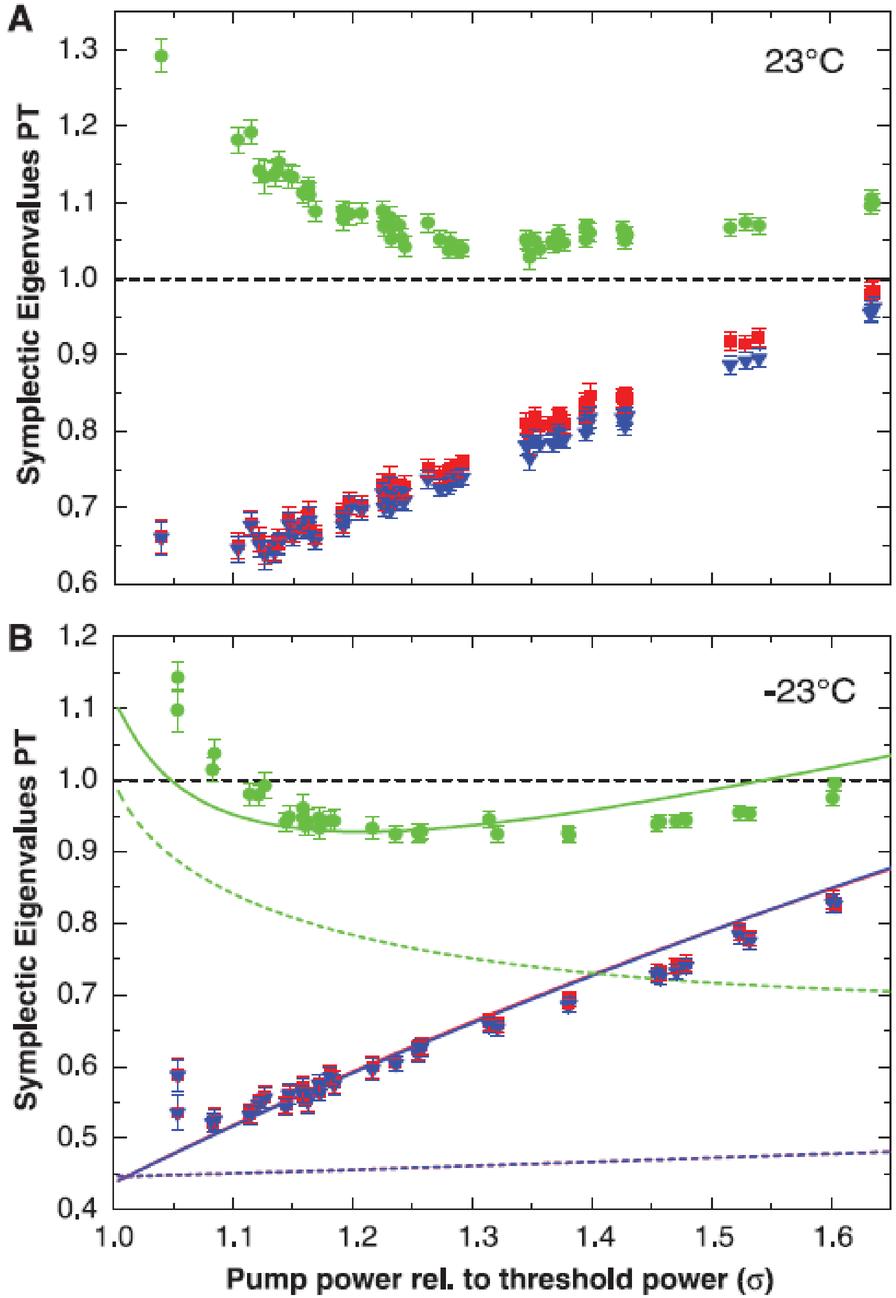
Three-Color Entanglement
A. S. Coelho, *et al.*
Science **326**, 823 (2009);

Three-Color Entanglement

A. S. Coelho,[1] F. A. S. Barbosa,[1] K. N. Cassemiro,[2] A. S. Villar,[2,3] M. Martinel

Entanglement is an essential quantum resource for the acceleration of inforn
well as for sophisticated quantum communication protocols. Quantum inform
expected to convey information from one place to another by using entangle
demonstrated the generation of entanglement among three bright beams of
wavelengths (532.251, 1062.102, and 1066.915 nanometers). We also obser
for finite channel losses, the continuous variable counterpart to entanglemer





Learning from entanglement

Marcelo Martinelli

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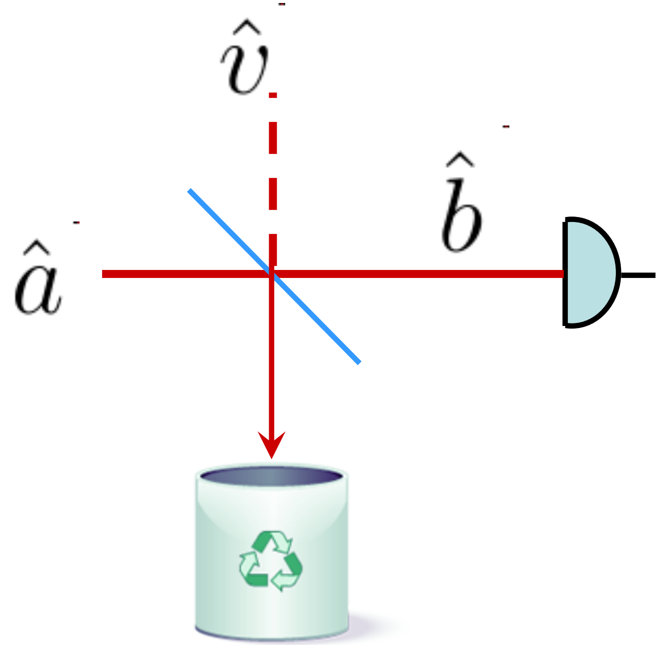
The effect of losses



$$\hat{b} \;=\; \sqrt{\eta}\,\hat{a} + \sqrt{1-\eta}\,\hat{v}$$

$$\hat{X}_{b,\varphi} \;=\; \sqrt{\eta}\,\hat{X}_{a,\varphi} + \sqrt{1-\eta}\,\hat{X}_{b,\varphi}$$

$$\Delta\hat{X}^2_{b,\varphi} \;=\; \eta\Delta\hat{X}^2_{a,\varphi} + (1-\eta)\Delta\hat{X}^2_{v,\varphi}$$

$$\Delta\hat{X}^2_{b,\varphi} - 1 = \eta\left(\Delta\hat{X}^2_{a,\varphi} - 1\right)$$

The problem of decoherence

Is the main problem for an eventual quantum computer, operating over many entangled qubits.

What is the limit for this entanglement?

Interaction with the environment!

Why producing and keeping them is a hard task?

Decoherence: as if the environment where continuously measuring the system!

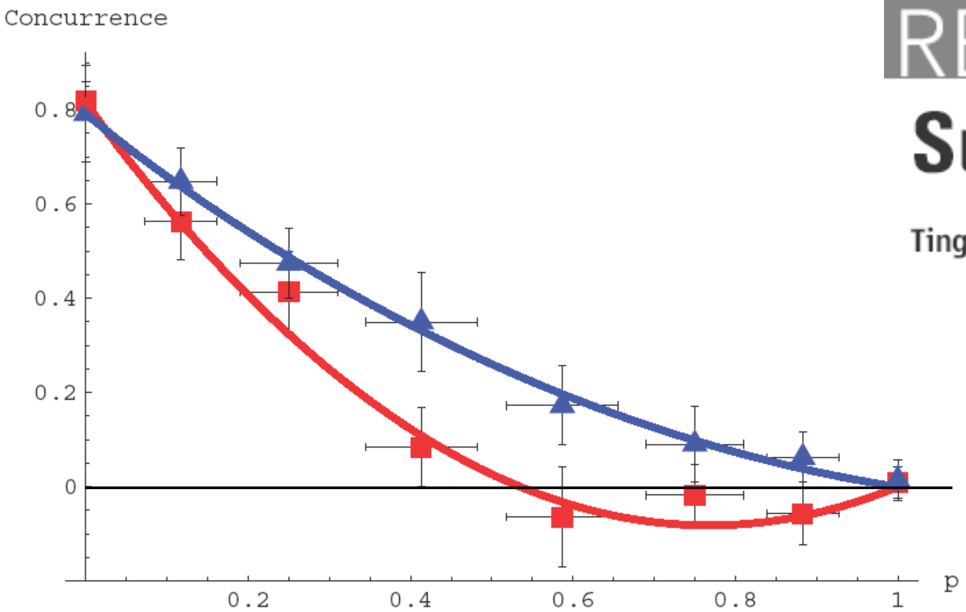
Famous example:
Schrödinger Cat Paradox (1935).

Also an entangled state

REPORTS SCIENCE VOL 316 27 APRIL 2007

Environment-Induced Sudden Death of Entanglement

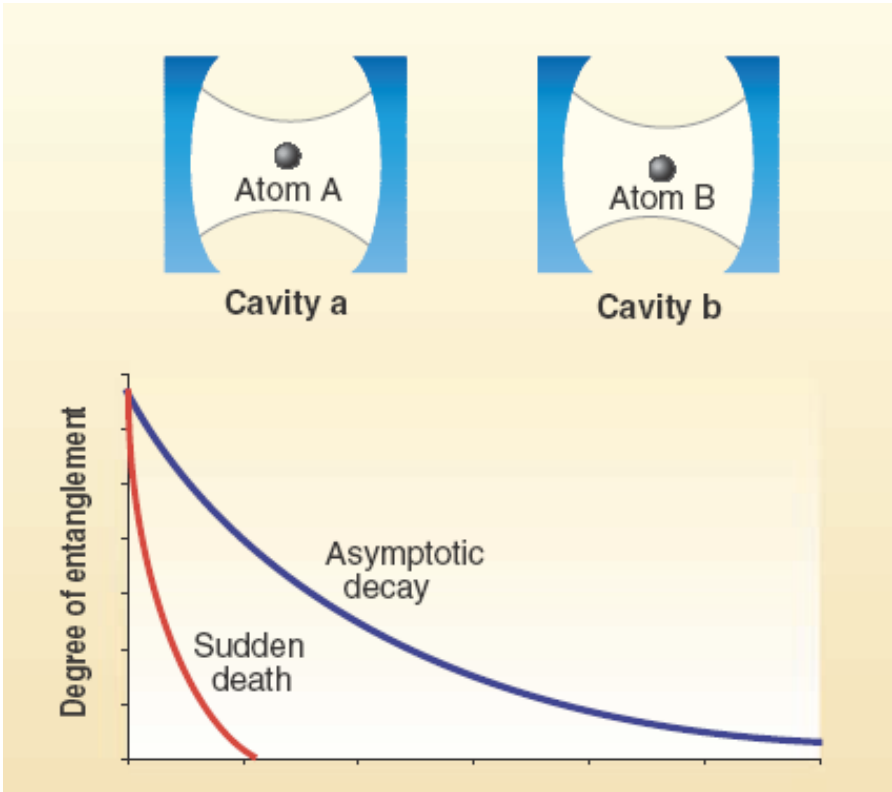
M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, L. Davidovich*



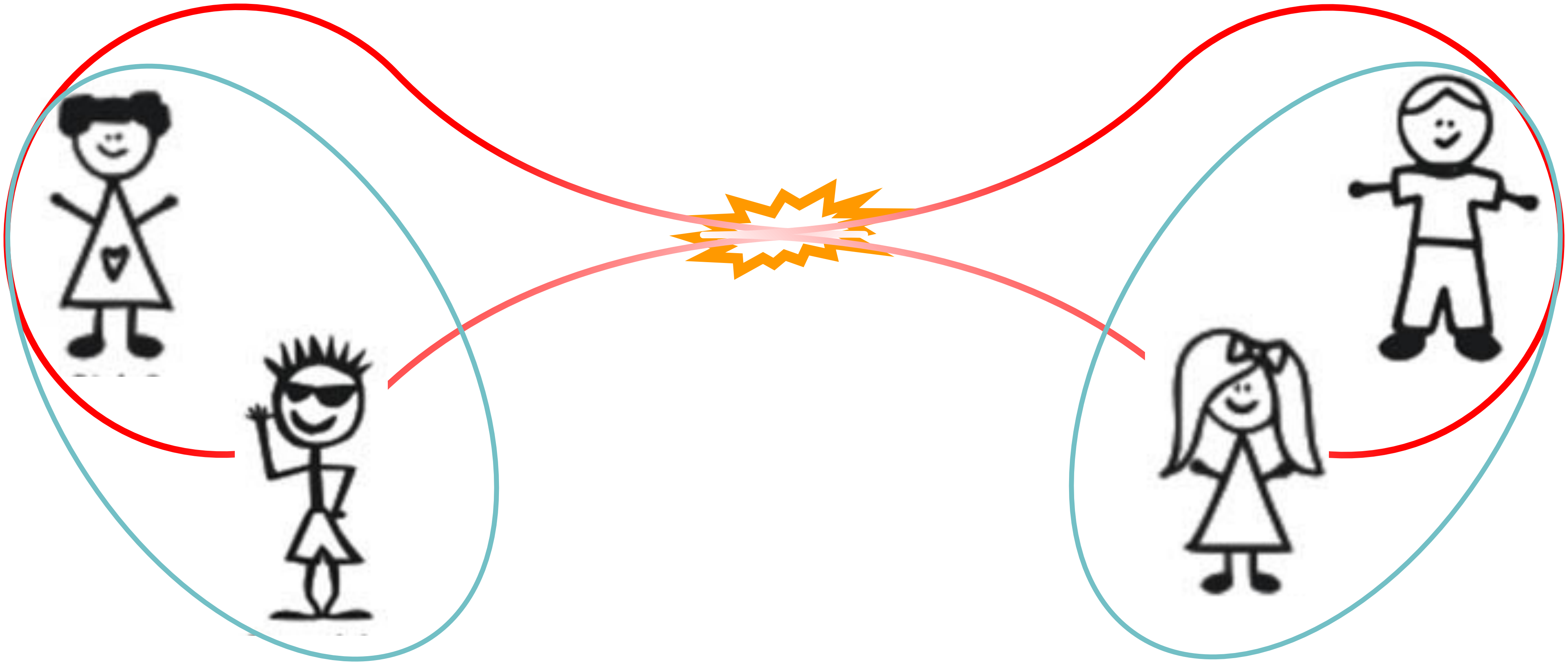
REVIEW 30 JANUARY 2009 VOL 323 SCIENCE

Sudden Death of Entanglement

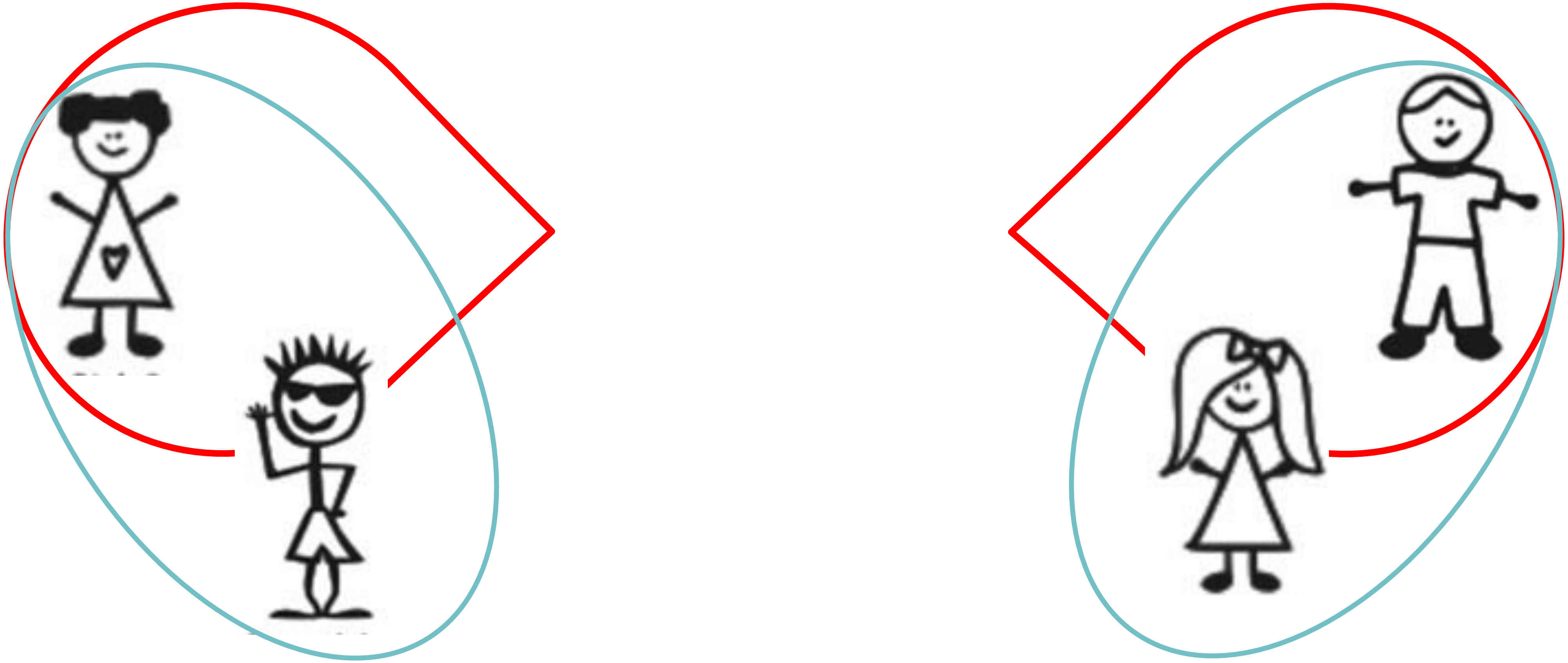
Ting Yu[1*] and J. H. Eberly[2*]

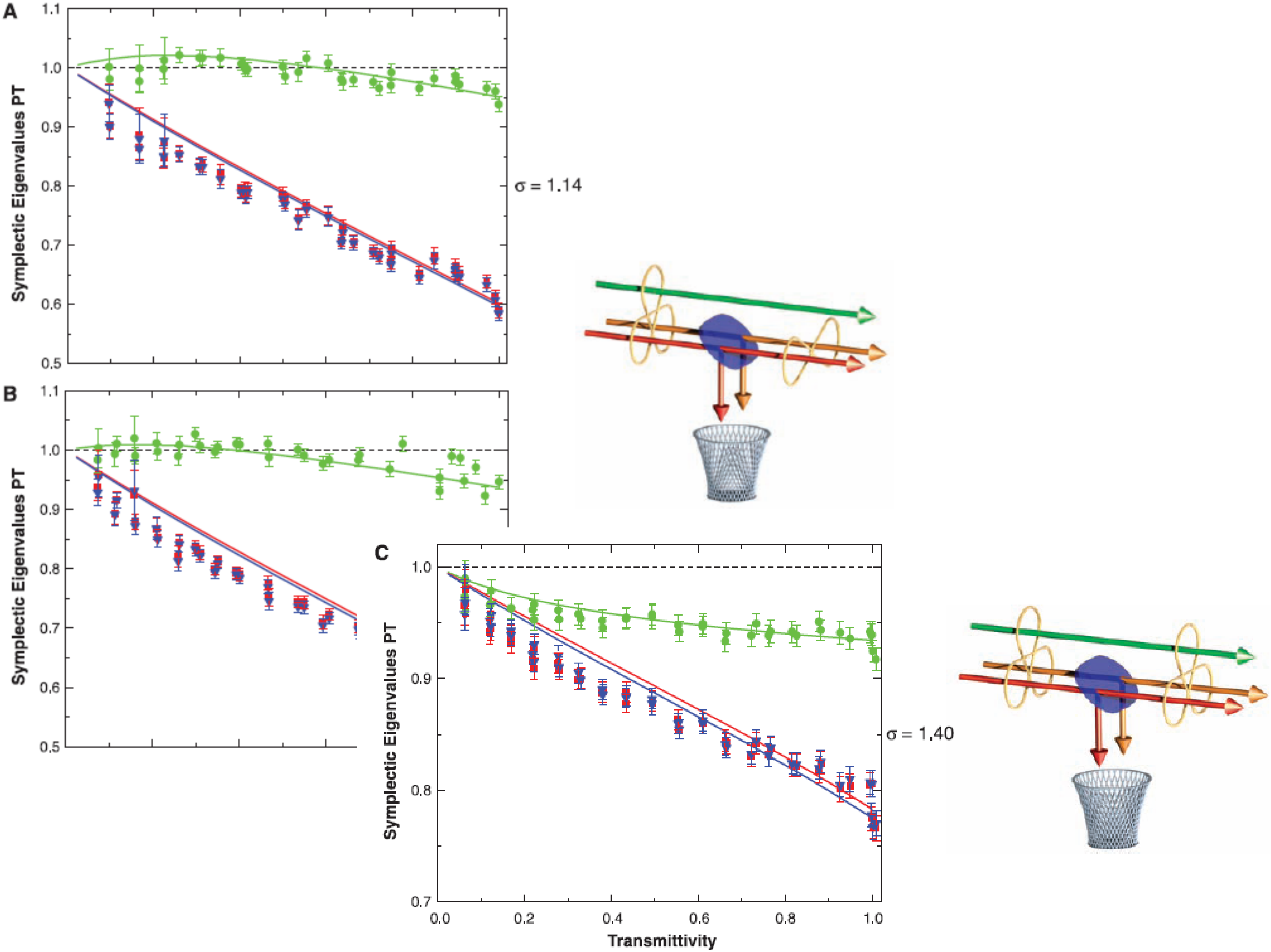


EPR's example

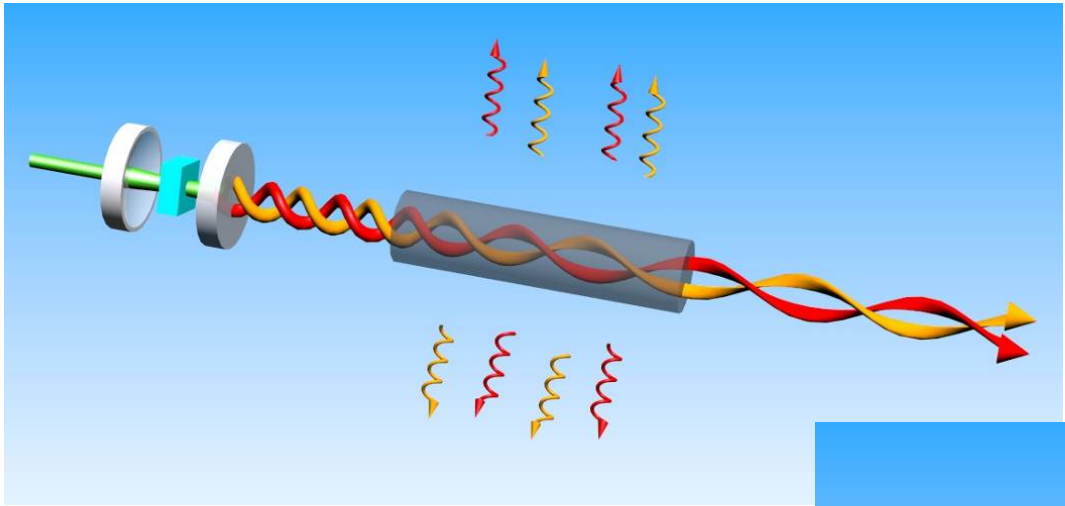


EPR's example



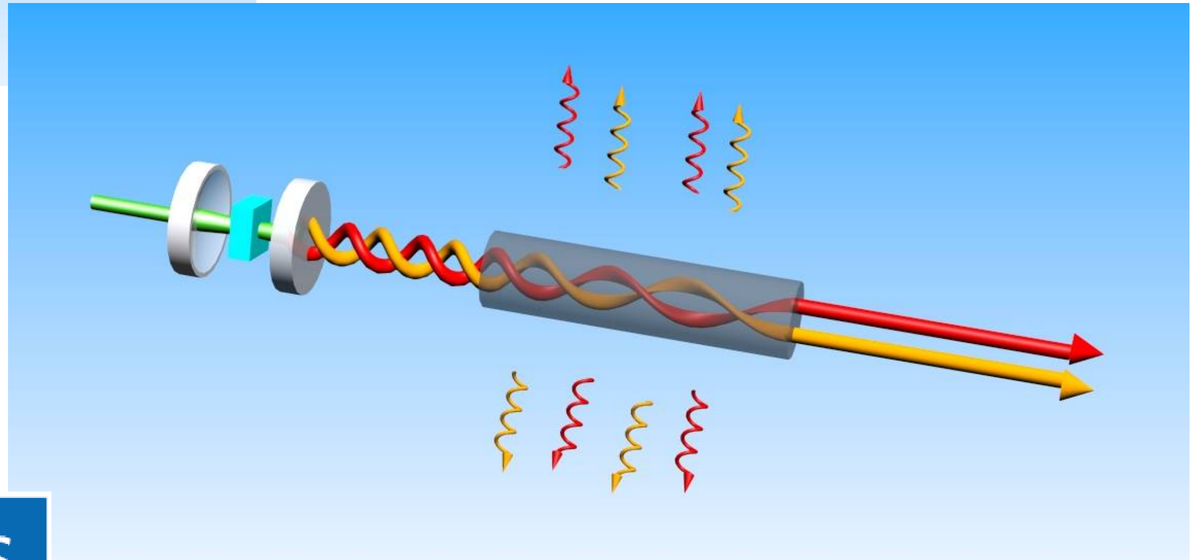

A
B
C
Symplectic Eigenvalues PT
Transmittivity
σ = 1.14
σ = 1.40
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.0
0.2
0.4
0.6
0.8
1.0

Disentanglement for a Bipartite & Gaussian state



Scenario (1):
robust entanglement

Scenario (2):
disentanglement



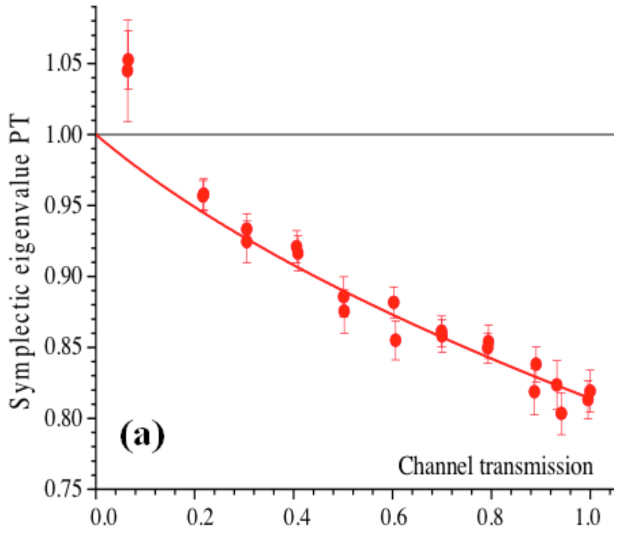
nature photonics

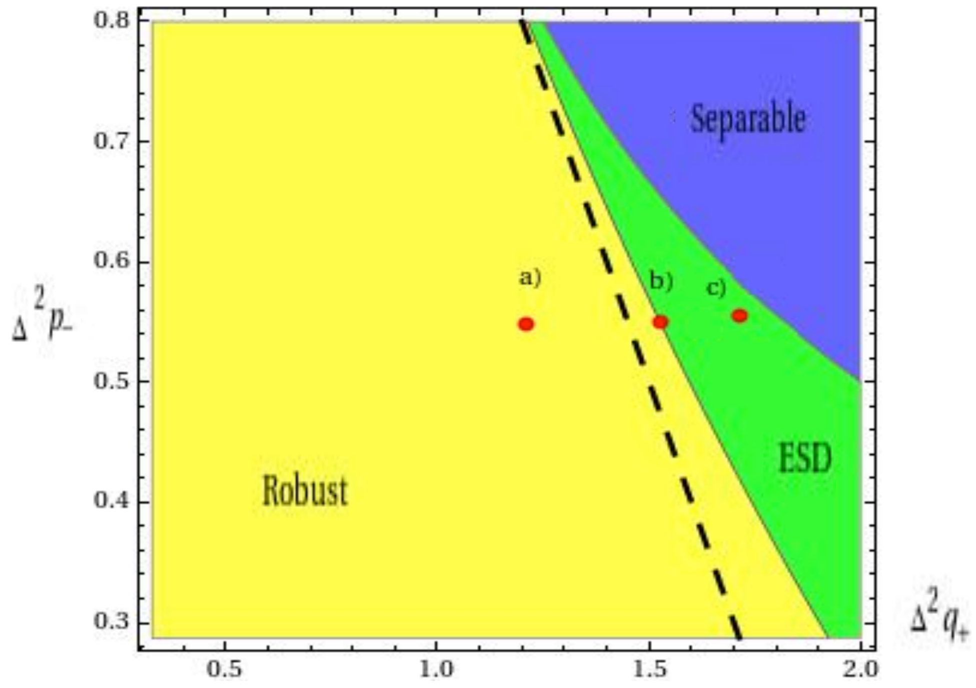
LETTERS | DOI: 10.1038/NPHOTON.2010.222

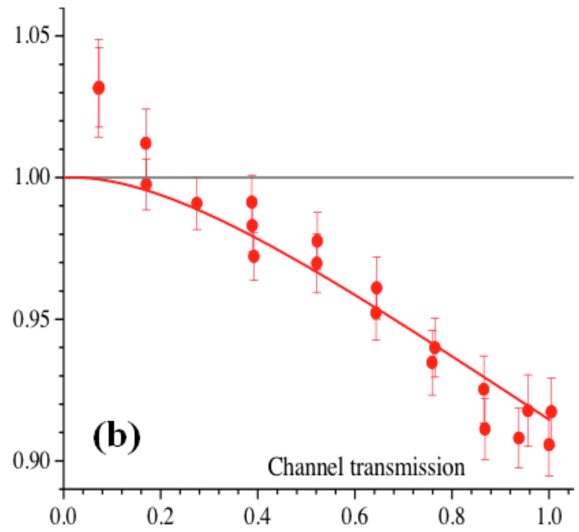
Robustness of bipartite Gaussian entangled beams propagating in lossy channels

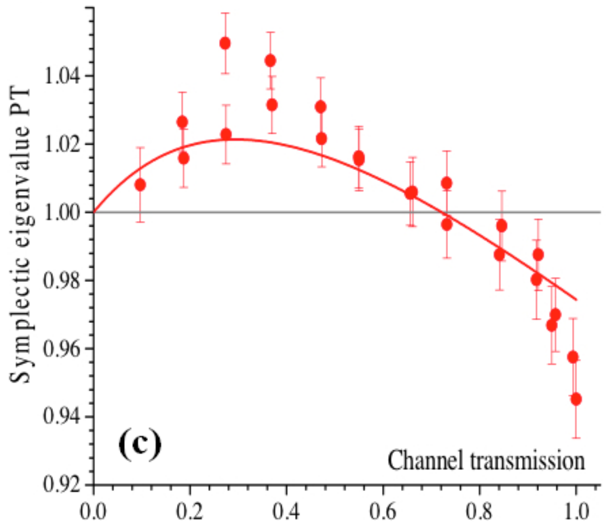
F. A. S. Barbosa[1], A. S. Coelho[1], A. J. de Faria[1], K. N. Cassemiro[2], A. S. Villar[2,3], P. Nussenzveig[1] and M. Martinelli[1]*

Disentanglement for a simpler model: Attenuation on a single beam









Tighter conditions for transmission of quantum entanglement!

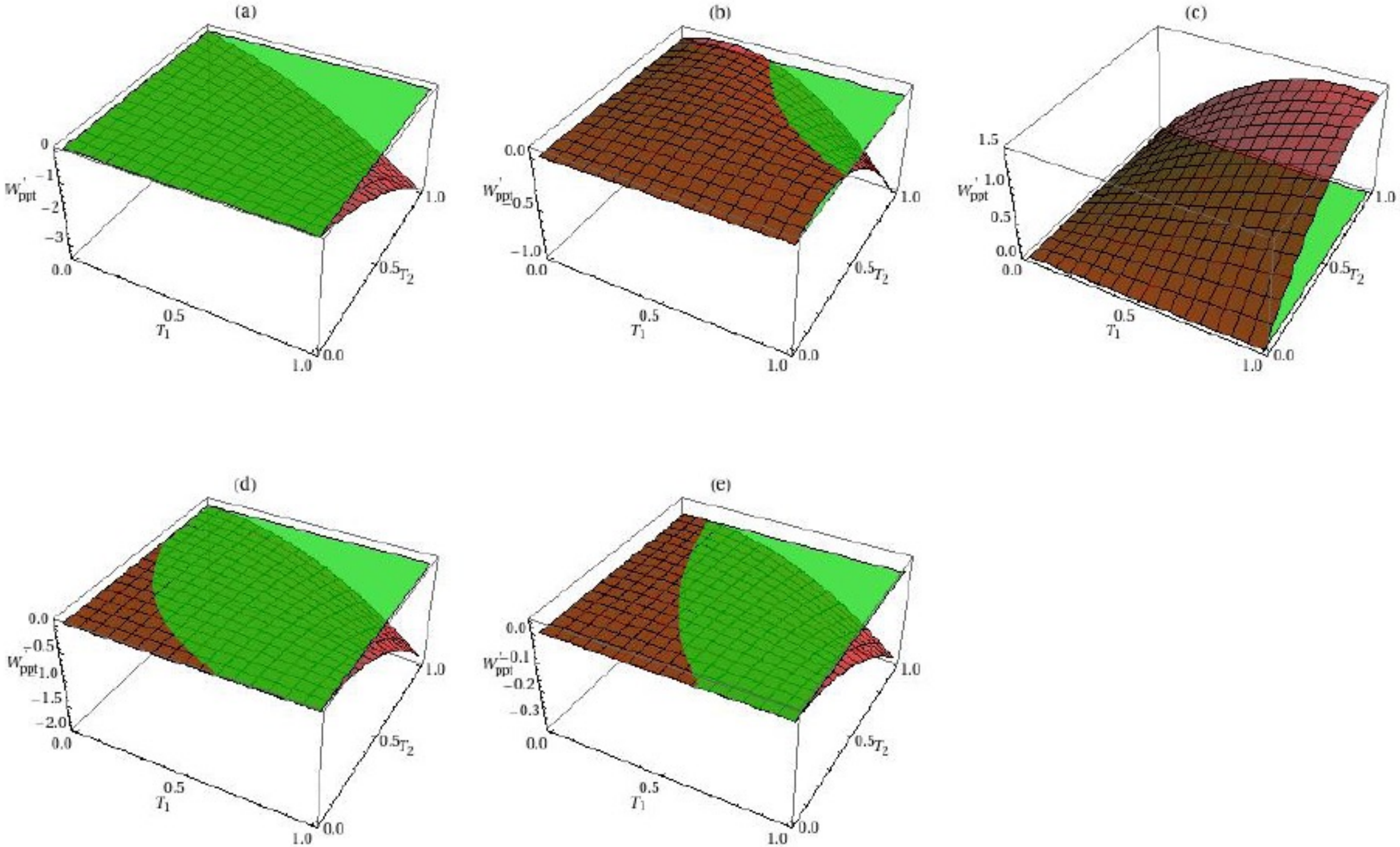
PHYSICAL REVIEW A **84**, 052330 (2011)

Disentanglement in bipartite continuous-variable systems

F. A. S. Barbosa,[1] A. J. de Faria,[2] A. S. Coelho,[1] K. N. Cassemiro,[1,3] A. S. Villar,[1,3,4] P. Nussenzveig,[1] and M. Martinelli[1,*]

Duan (optimized)

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \geq 0;$$



Using entanglement for teleportation

Marcelo Martinelli

LMCAL - IFUSP

USP

INCT-IQ

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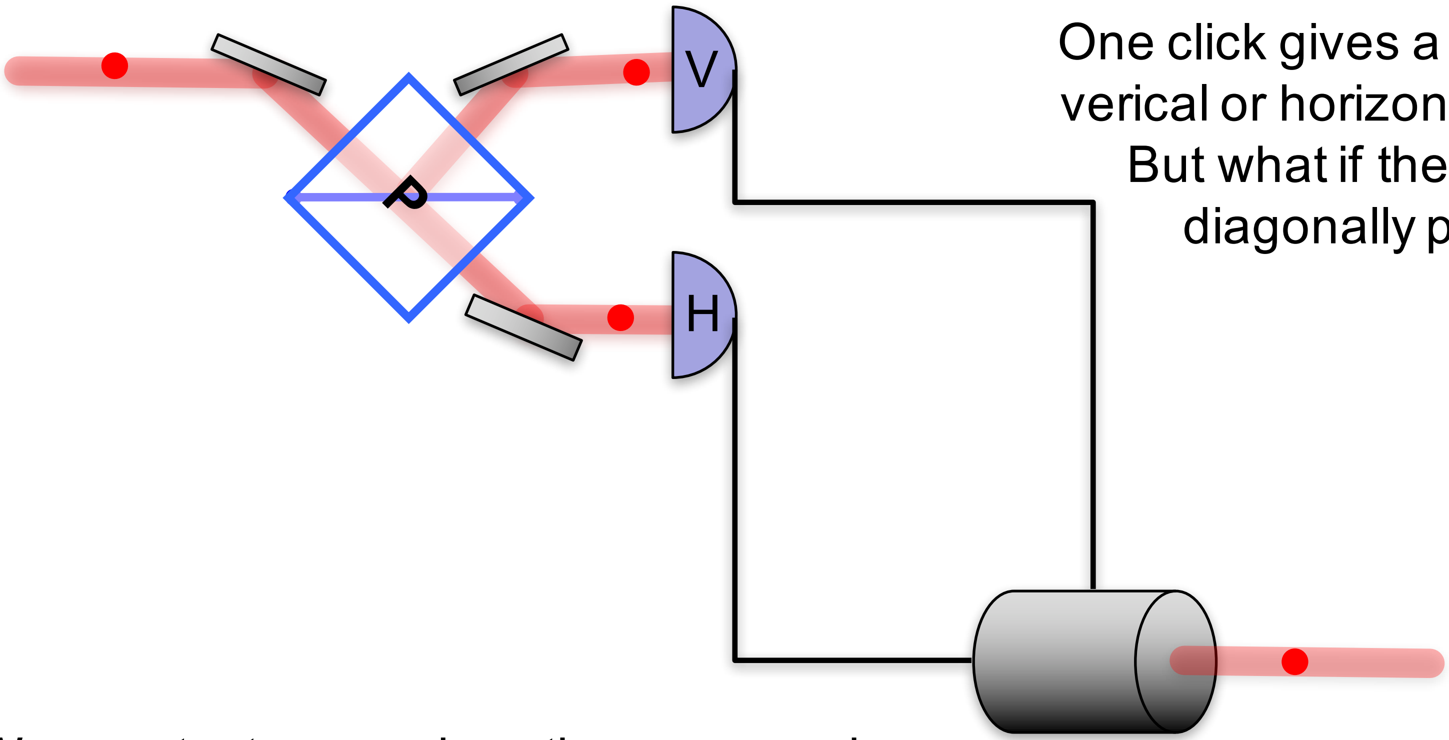
Teleportation:

Entanglement can build a channel for quantum information.

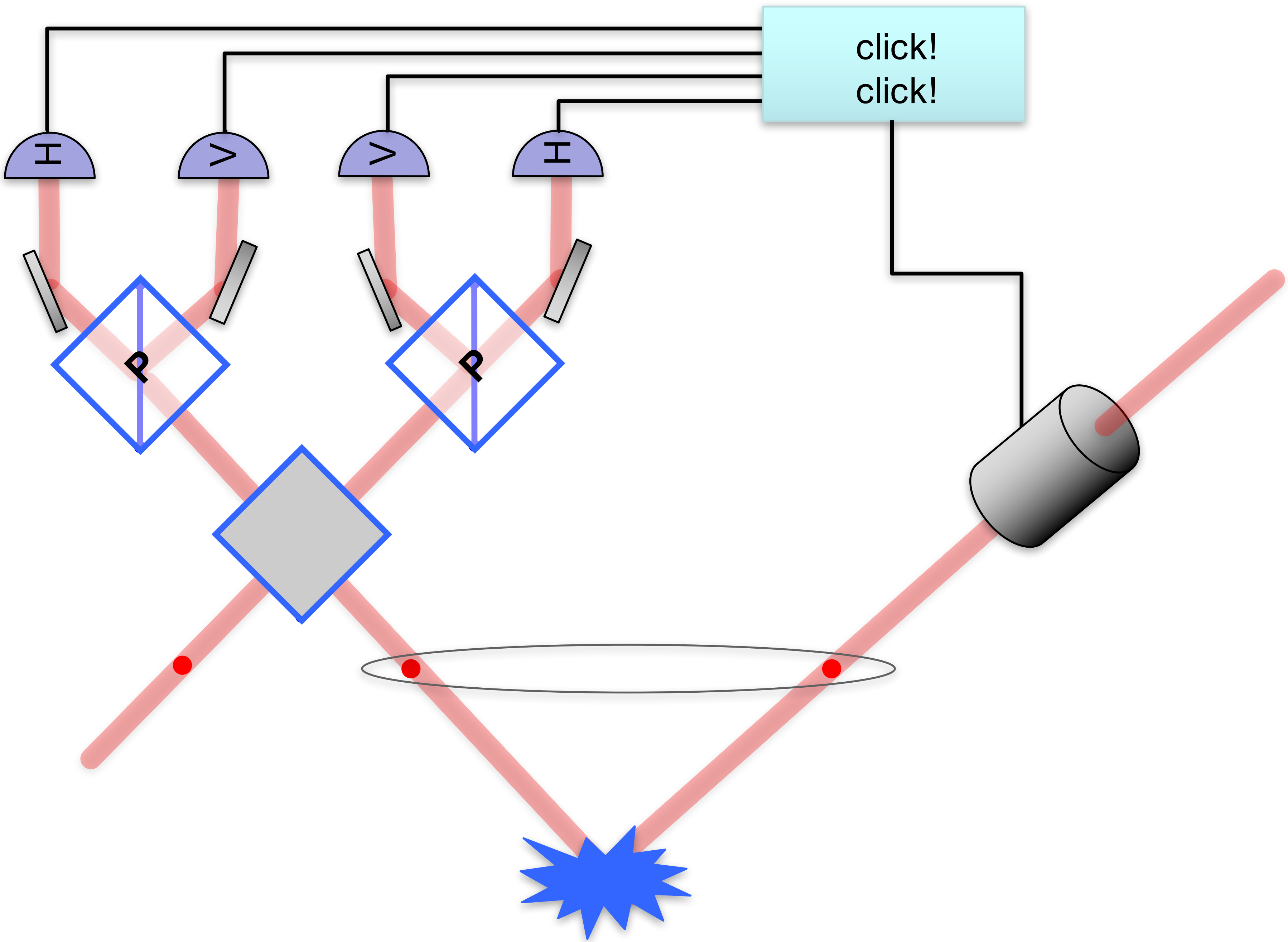
Consider that we want to tranfer the state of a photon between two stations, without tranfering the photon.



One click gives a measuremtn at verical or horizontal polarization. But what if the photon was diagonally polarized?

We may try to reproduce the measured photon from the gained information, but odds are high that we are mistaken (we cannot fully recover a state from a single measurement).

Teleportation: suscess by ignorance!



Teleportation

VOLUME 70, NUMBER 13 PHYSICAL REVIEW LETTERS 29 MARCH 1993

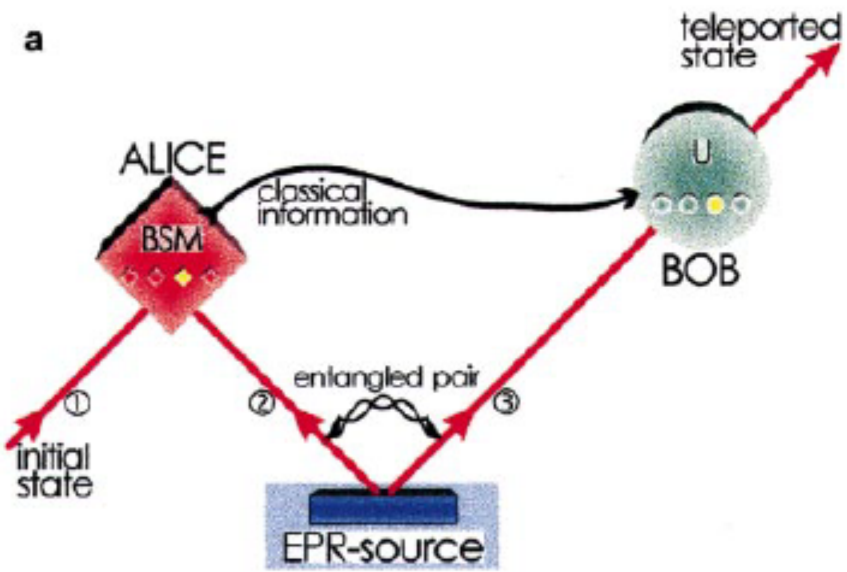
Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

Charles H. Bennett,[(1)] Gilles Brassard,[(2)] Claude Crépeau,[(2),(3)] Richard Jozsa,[(2)] Asher Peres,[(4)] and William K. Wootters[(5)]

NATURE | VOL 390 | 11 DECEMBER 1997 575

Experimental quantum teleportation

Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger



- ✓ Doesn't transfer the original photon.
- ✓ It isn't a copying machine: photon information is lost.
- ✓ Is limited by light speed: knowledge about the action upon the entangled pair is available after the first measurement.

Teleportation

VOLUME 70, NUMBER 13 PHYSICAL REVIEW LETTERS 29 MARCH 1993

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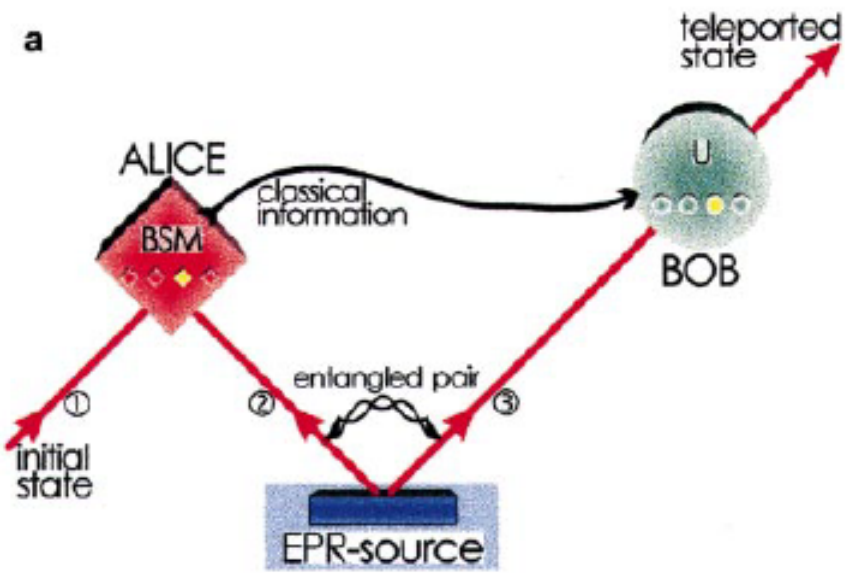
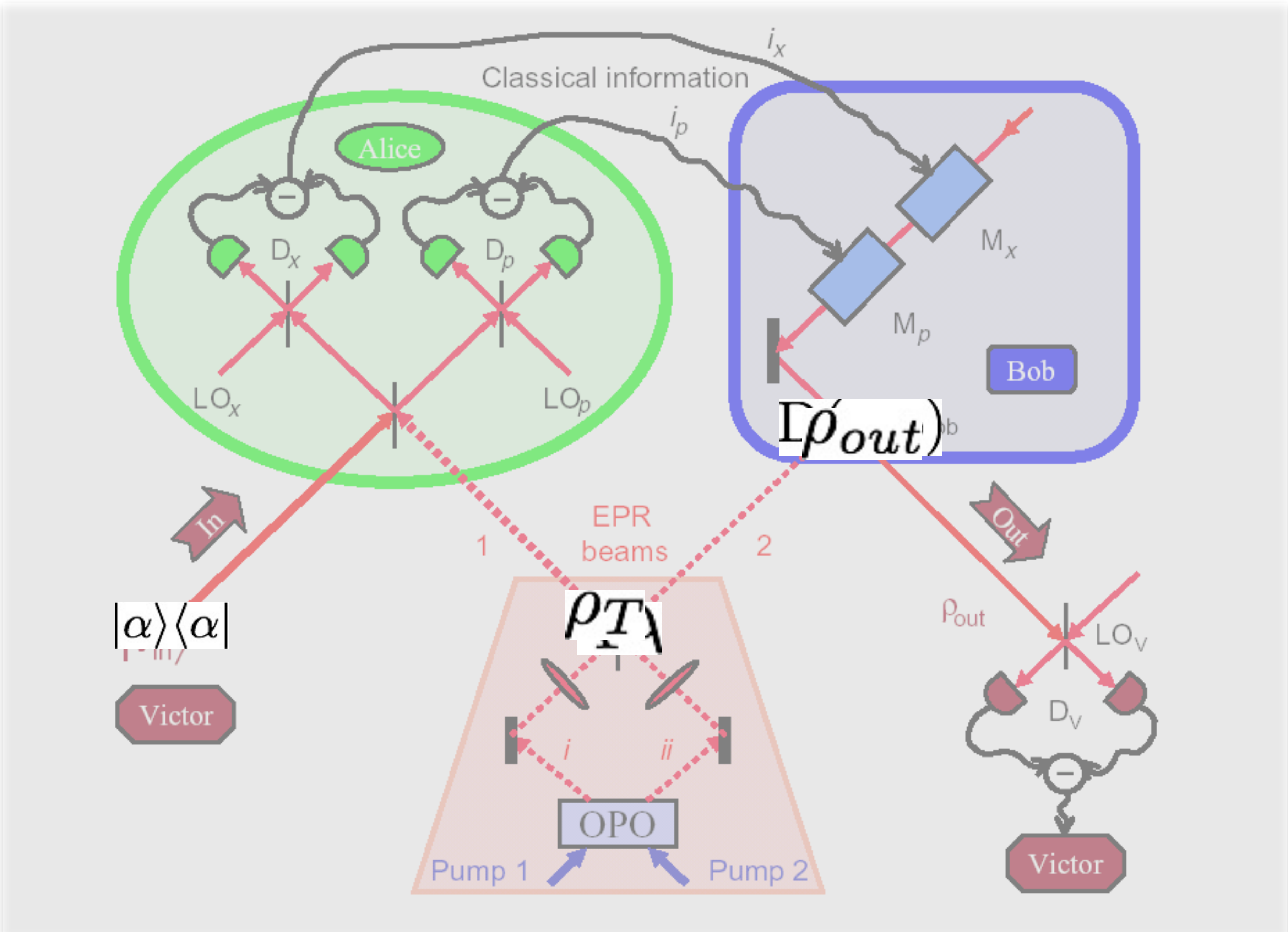


It is a useful tool for transferring the state,

- ✓ for quantum information processing,
- ✓ for cryptography.
- ✓ for QM testing!

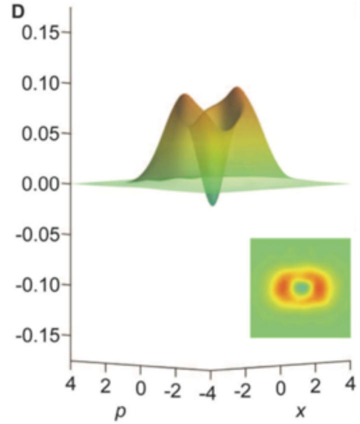
Teleportation with CV

Unconditional Quantum Teleportation
A. Furusawa, *et al.*
Science **282**, 706 (1998);



Teleportation with CV meets DV!

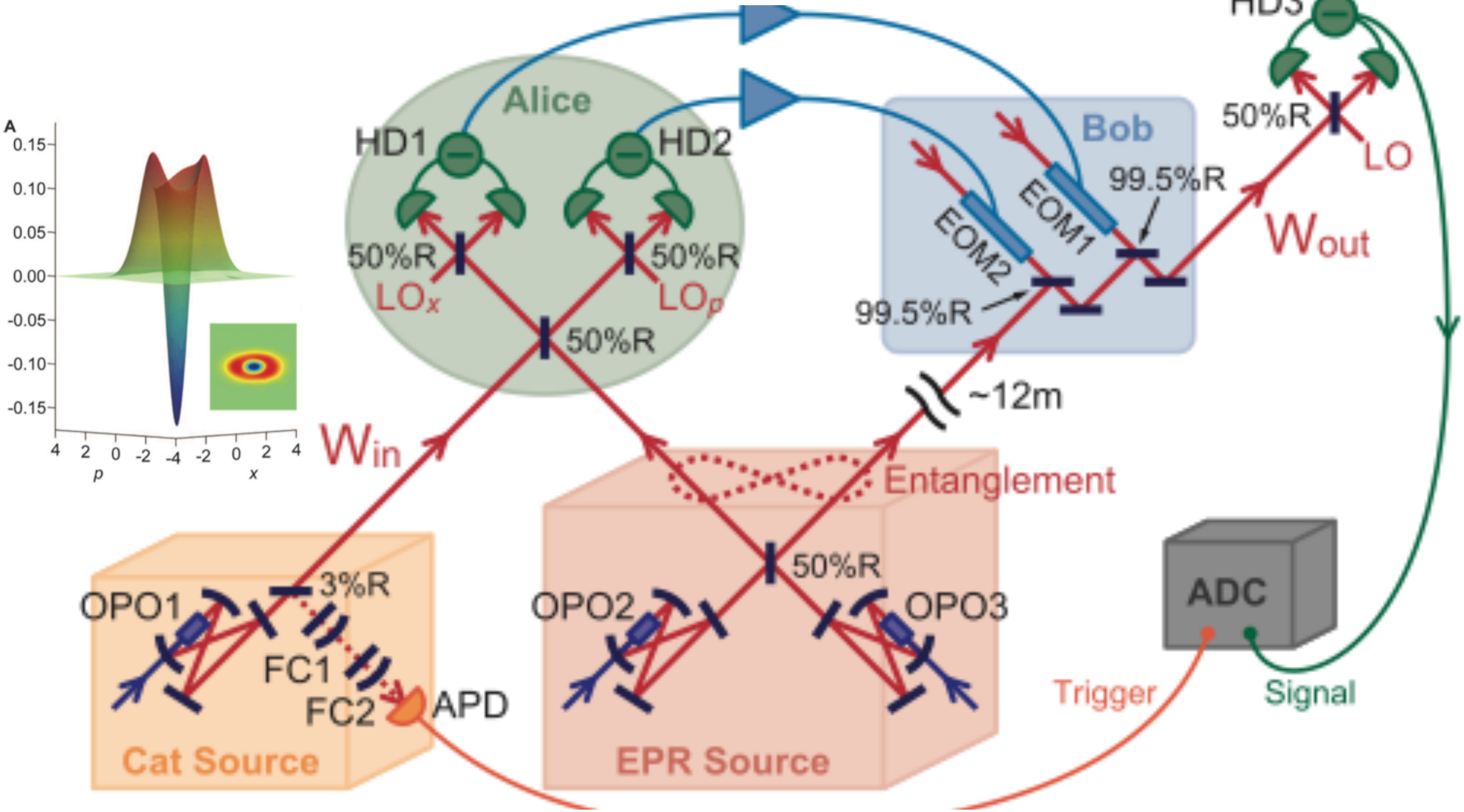
Teleportation of Nonclassical Wave Packets of Light
Noriyuki Lee, *et al.*
Science **332**, 330 (2011);
DOI: 10.1126/science.1201034



15 AUGUST 2013 | VOL 500 | NATURE | 315

Deterministic quantum teleportation of photonic quantum bits by a hybrid technique

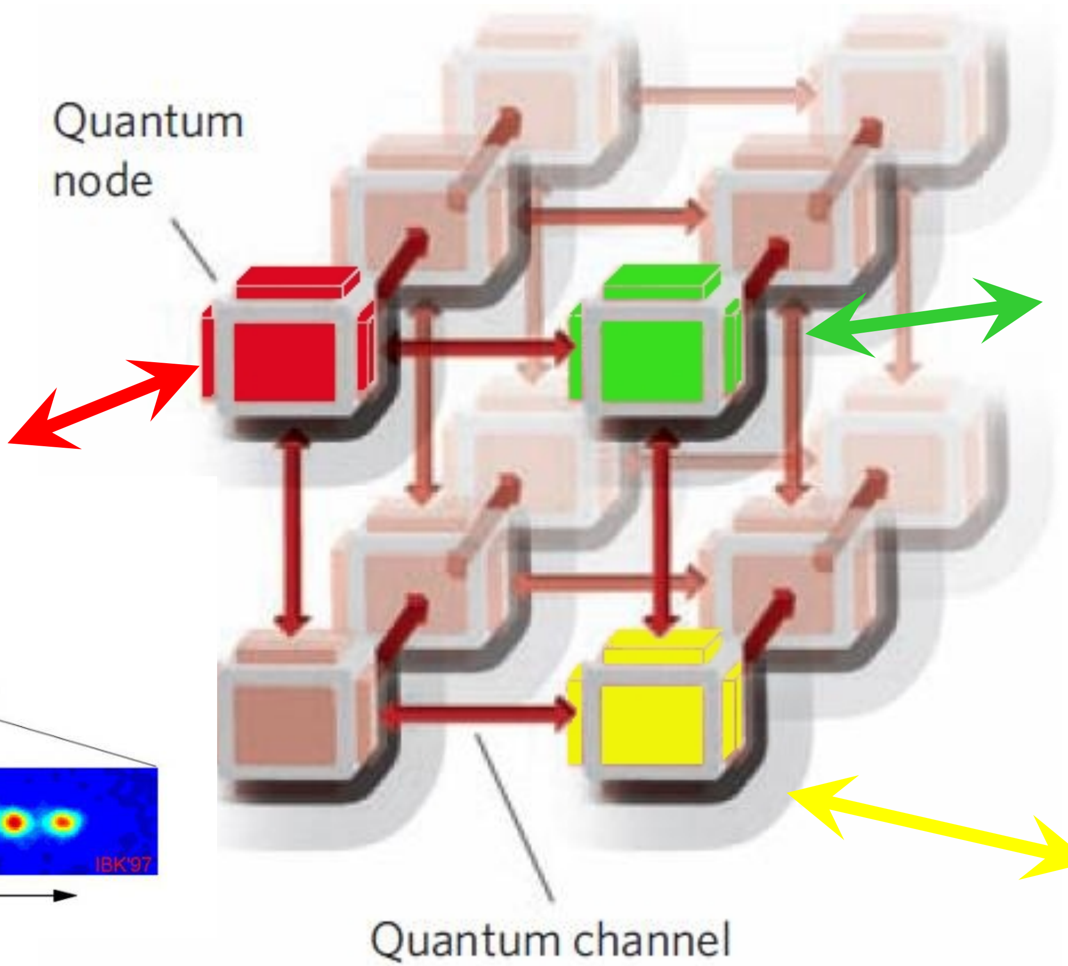
Shuntaro Takeda[1], Takahiro Mizuta[1], Maria Fuwa[1], Peter van Loock[2] & Akira Furusawa[1]

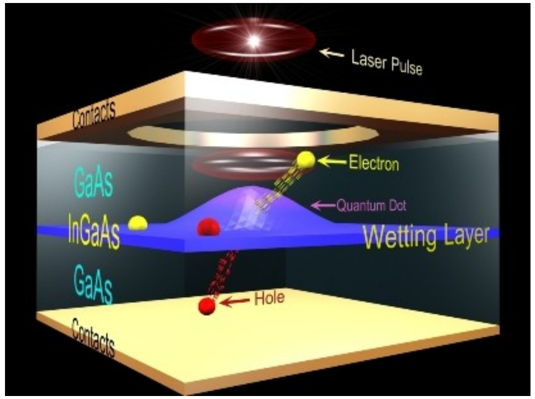


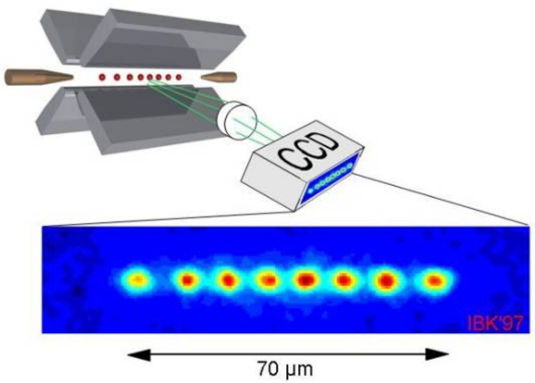
NATURE|Vol 453|19 June 2008|doi:10.1038/nature07127

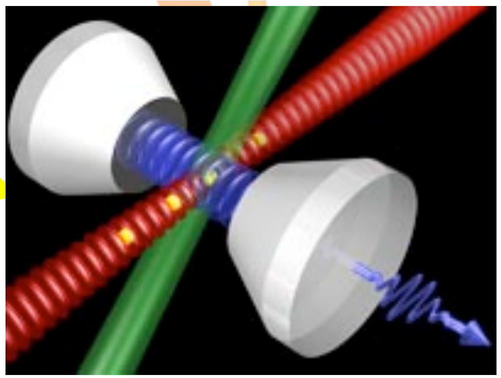
The quantum internet

H. J. Kimble[1]









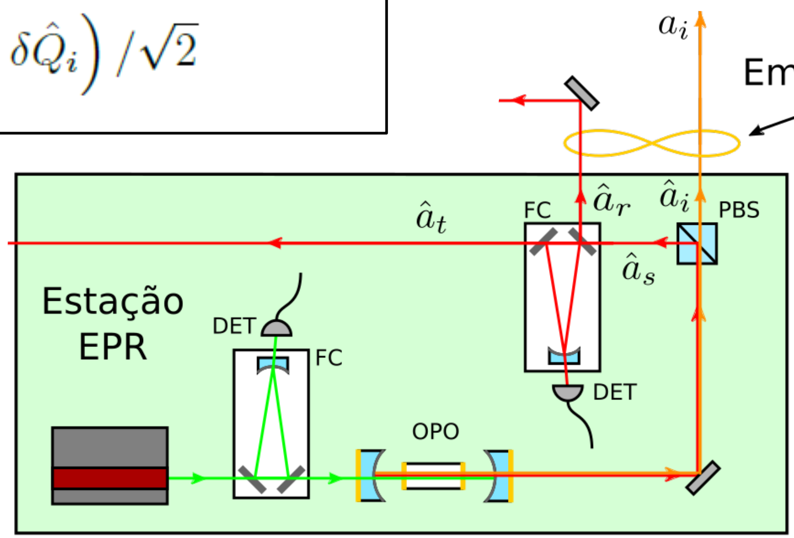
Multicolor teleportation project at LMCAL

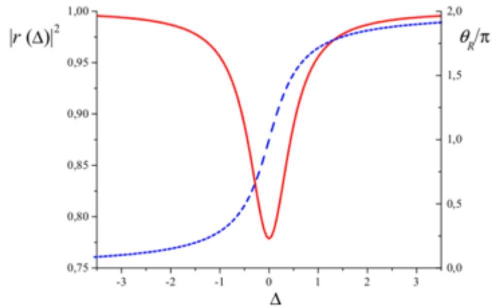
- EPR Station

$$
\begin{aligned}
S_{\hat{p}-} &= \left\langle \delta \hat{P}_{-}(\Omega)\, \delta \hat{P}_{-}(-\Omega) \right\rangle \rightarrow 0 \\
S_{\hat{q}+} &= \left\langle \delta \hat{Q}_{+}(\Omega)\, \delta \hat{Q}_{+}(-\Omega) \right\rangle \rightarrow 0
\end{aligned}
$$

$$
\begin{aligned}
\delta \hat{P}_{-} &= \left(\delta \hat{P}_{s} - \delta \hat{P}_{i} \right) / \sqrt{2} \\
\delta \hat{Q}_{+} &= \left(\delta \hat{Q}_{s} + \delta \hat{Q}_{i} \right) / \sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
\hat{a}_r &= \hat{a}_{s-l} + \hat{a}_{s-u} + \sqrt{p}\alpha_s = \delta \hat{P}_s + i \delta \hat{Q}_s + \sqrt{p}\alpha_s \\
\hat{a}_t &= \sqrt{f}\alpha_s + \hat{a}_{vac}
\end{aligned}
$$



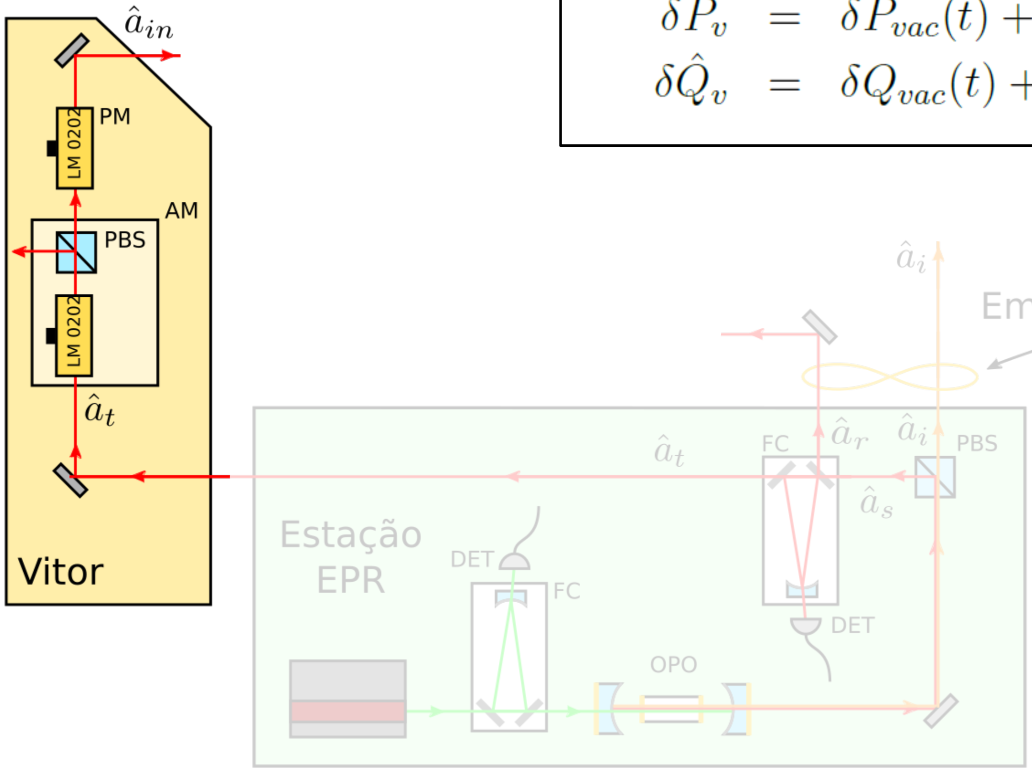


Multicolor teleportation project at LMCAL

- Vitor

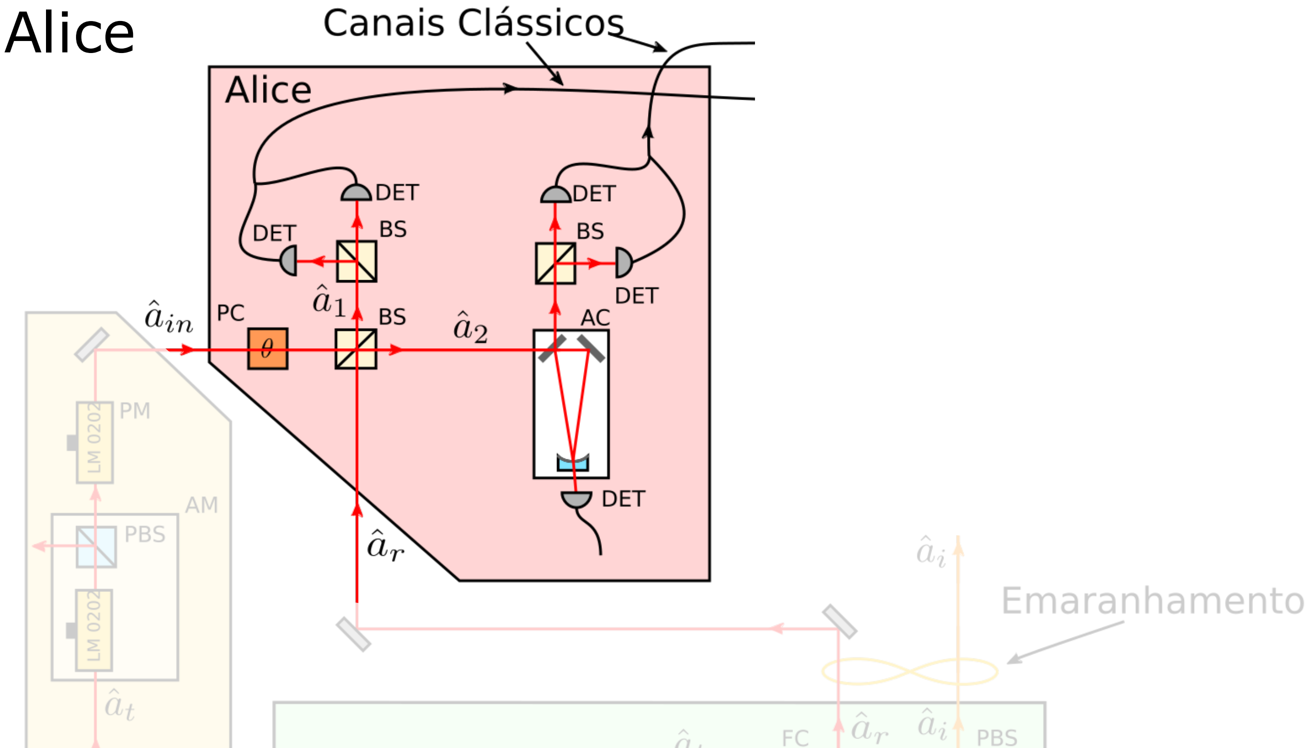
$$\hat{a}_{in} = \delta\hat{a}_v + \alpha_v = \delta\hat{P}_v + i\delta\hat{Q}_v + \alpha_v$$

$$\delta\hat{P}_v = \delta P_{vac}(t) + 2\gamma_{am}(t)\alpha_0$$
$$\delta\hat{Q}_v = \delta Q_{vac}(t) + 2\phi_{pm}(t)\alpha_0$$



Multicolor teleportation project at LMCAL

- Alice



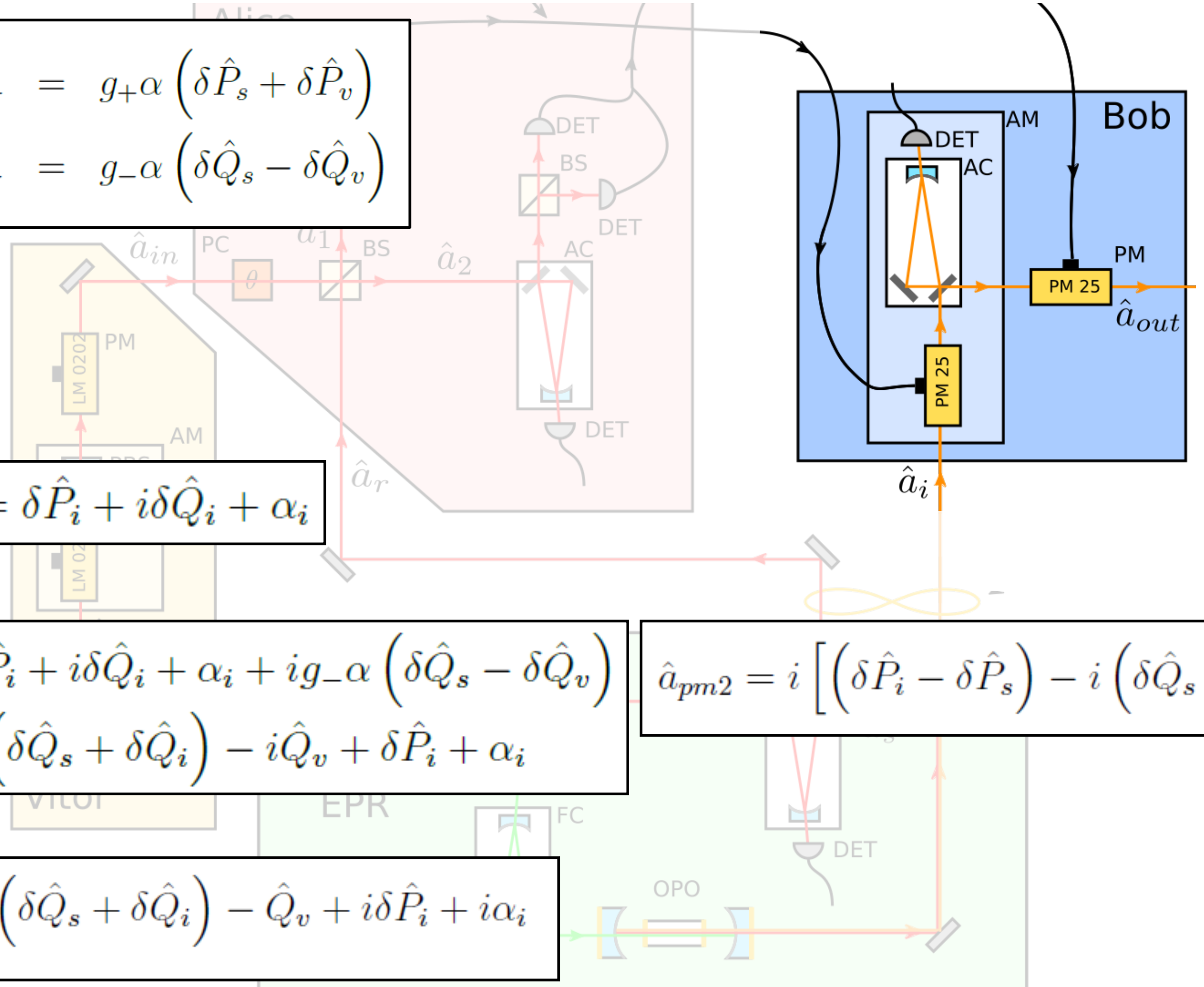
$$\sqrt{2}\hat{a}_1 = \delta\hat{P}_s + \delta\hat{P}_v + i\underbrace{\left(\delta\hat{Q}_s + \delta\hat{Q}_v\right)}_{\text{medida de } Q} + \underbrace{(\sqrt{p}\alpha_s + \alpha_v)}_{\text{máxima intensidade}}$$

$$\sqrt{2}\hat{a}_2 = \underbrace{\left(\delta\hat{P}_s - \delta\hat{P}_v\right)}_{\text{medida de } P} + i\left(\delta\hat{Q}_s - \delta\hat{Q}_v\right) + \underbrace{(\sqrt{p}\alpha_s - \alpha_v)}_{\text{mínima intensidade}}$$

Multicolor teleportation project at LMCAL

- Bob

$$\begin{aligned}\delta I_+ &= g_+\alpha\left(\delta\hat{P}_s + \delta\hat{P}_v\right)\\ \delta I_- &= g_-\alpha\left(\delta\hat{Q}_s - \delta\hat{Q}_v\right)\end{aligned}$$



$$\hat{a}_{out} = i\left[-\left(\delta\hat{P}_v - i\delta\hat{Q}_v\right) + \alpha_i\right]$$

$$\hat{a}_i = \delta\hat{P}_i + i\delta\hat{Q}_i + \alpha_i$$

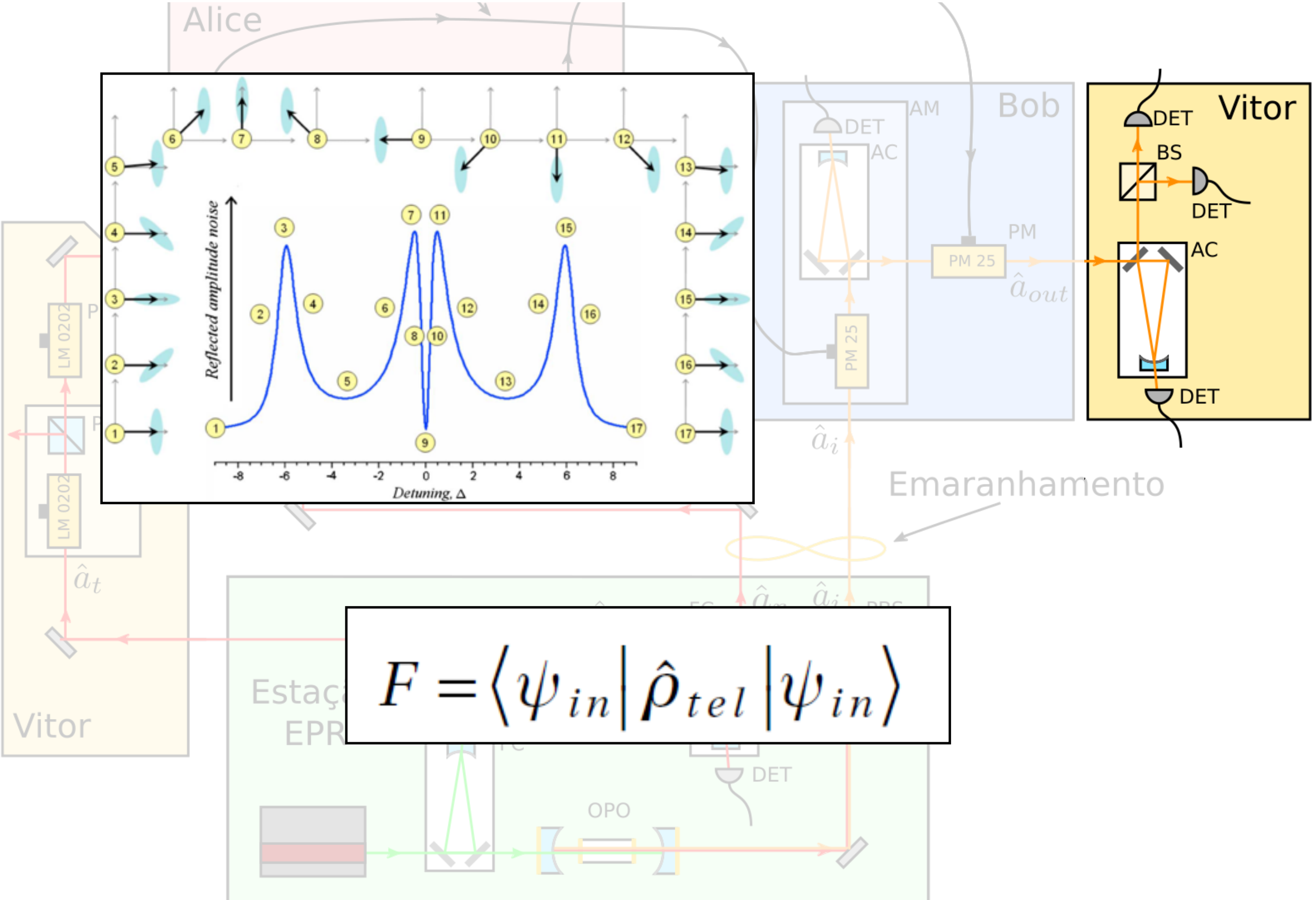
$$\begin{aligned}\hat{a}_{pm1} &= \delta\hat{P}_i + i\delta\hat{Q}_i + \alpha_i + ig_-\alpha\left(\delta\hat{Q}_s - \delta\hat{Q}_v\right)\\ &= i\left(\delta\hat{Q}_s + \delta\hat{Q}_i\right) - i\hat{Q}_v + \delta\hat{P}_i + \alpha_i\end{aligned}$$

$$\hat{a}_{pm2} = i\left[\left(\delta\hat{P}_i - \delta\hat{P}_s\right) - i\left(\delta\hat{Q}_s + \delta\hat{Q}_i\right) - \left(\delta\hat{P}_v - i\hat{Q}_v\right) + \alpha_i\right]$$

$$\hat{a}_{re} = \left(\delta\hat{Q}_s + \delta\hat{Q}_i\right) - \hat{Q}_v + i\delta\hat{P}_i + i\alpha_i$$

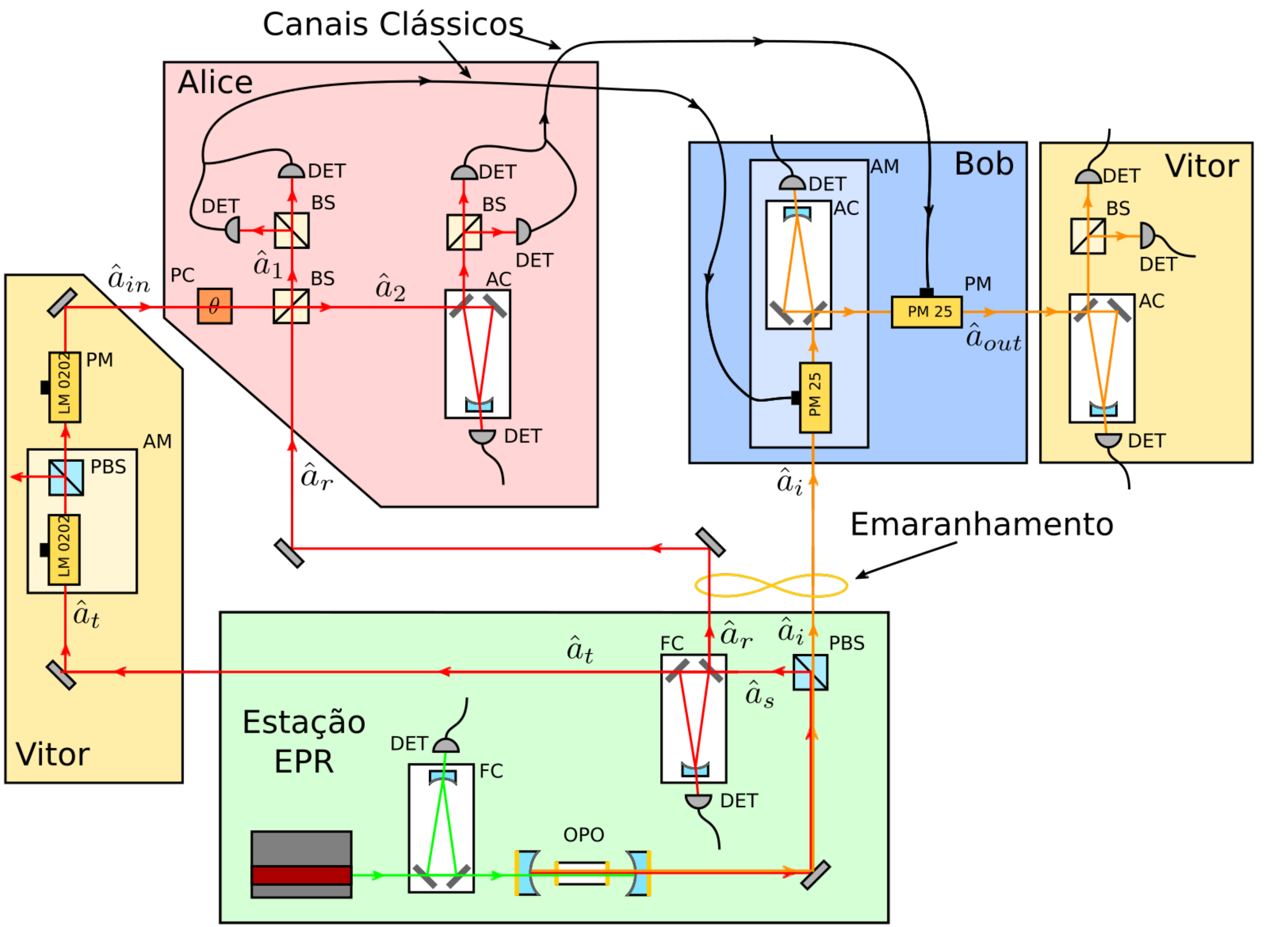
Multicolor teleportation project at LMCAL

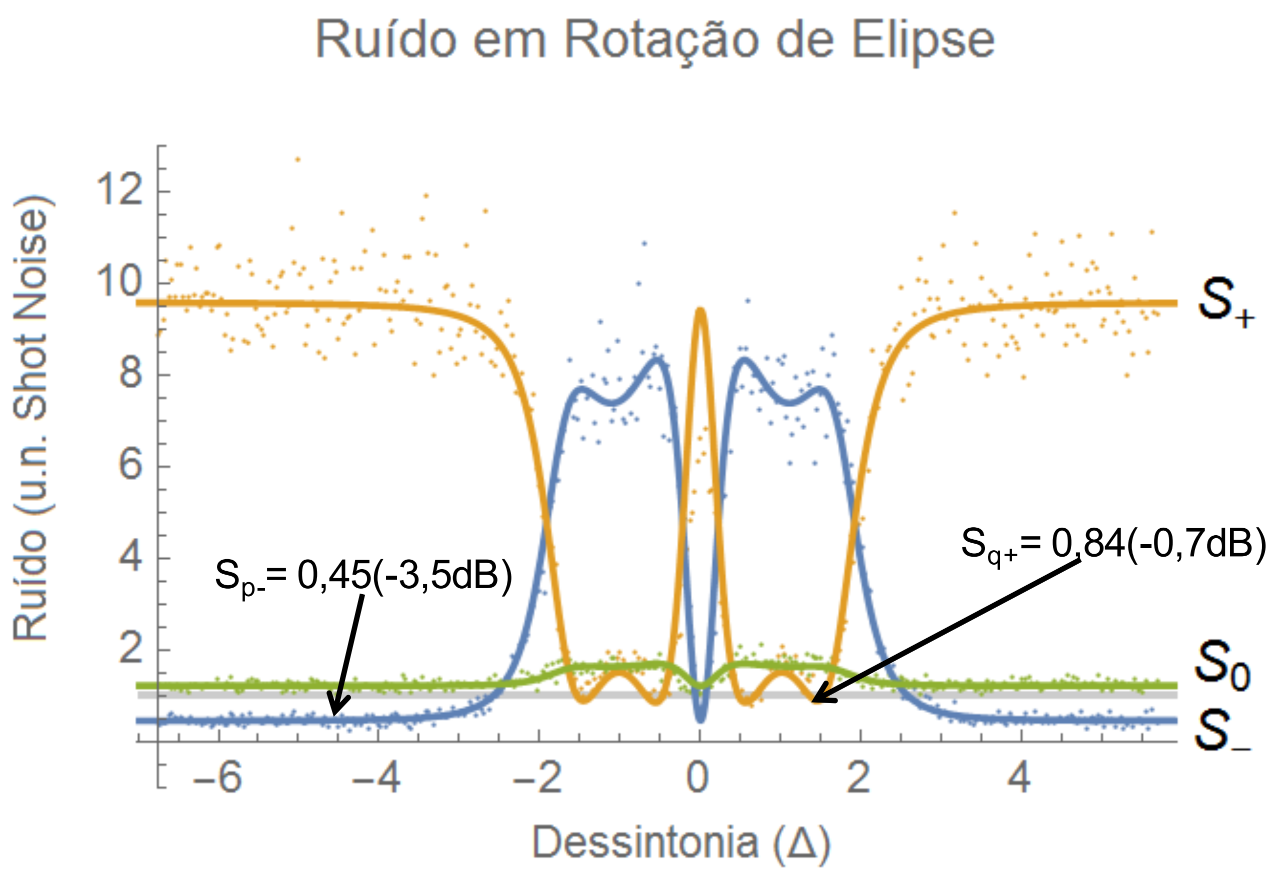
- Vitor evaluates the fidelity



$$F = \langle \psi_{in} | \hat{\rho}_{tel} | \psi_{in} \rangle$$

Multicolor teleportation project at LMCAL



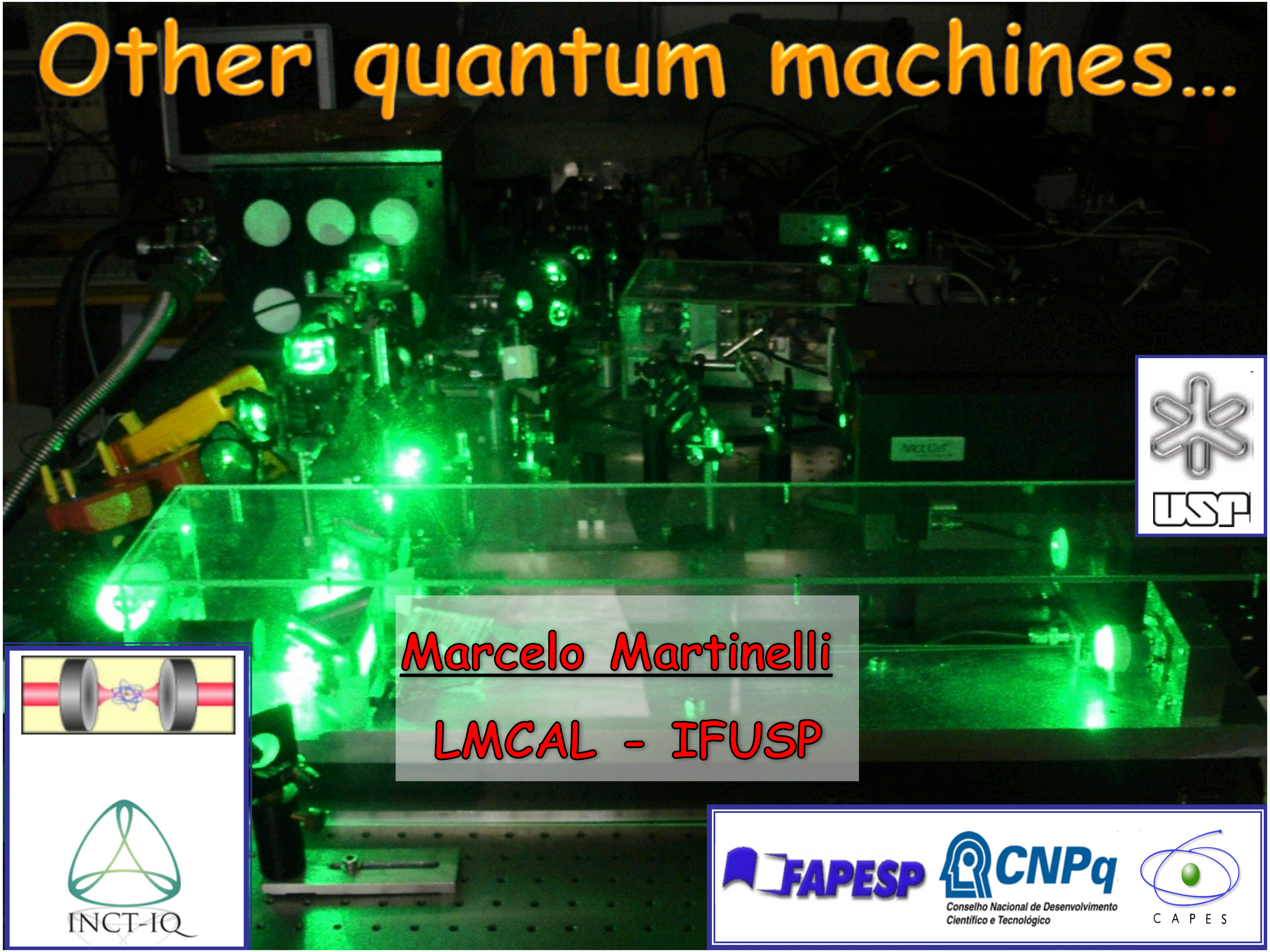


- Pth=80 mW
- P=84 mW
- σ=1.05
- $T_{cristal}$=2ºC
- P=1 atm
- Duan: $p^2 + \quad q_+^2 = 1{,}29 \pm 0{,}10(\quad 1{,}9dB)$
- VLF: $p^2 + \quad (q_+ \qquad q_0)^2 = 1{,}27 \pm 0{,}10(\quad 2{,}0dB)$

1. Noise compression → Necessary **Entanglement**

	Ideal	Single "copy"	Classic	Best possible resutl	From data
S_{p-}	0	½	1	0,2	0,45
S_{q+}	0	½	1	0,45	0,84
F	1	2/3	½	0,75	0,6

Other quantum machines…



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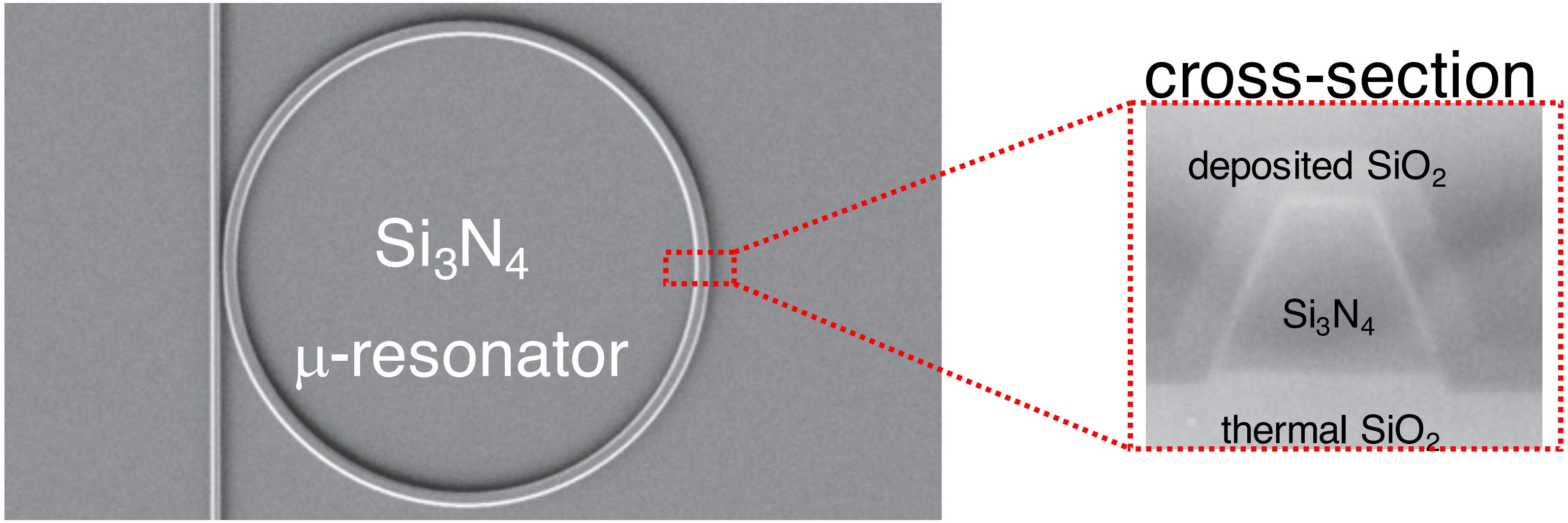
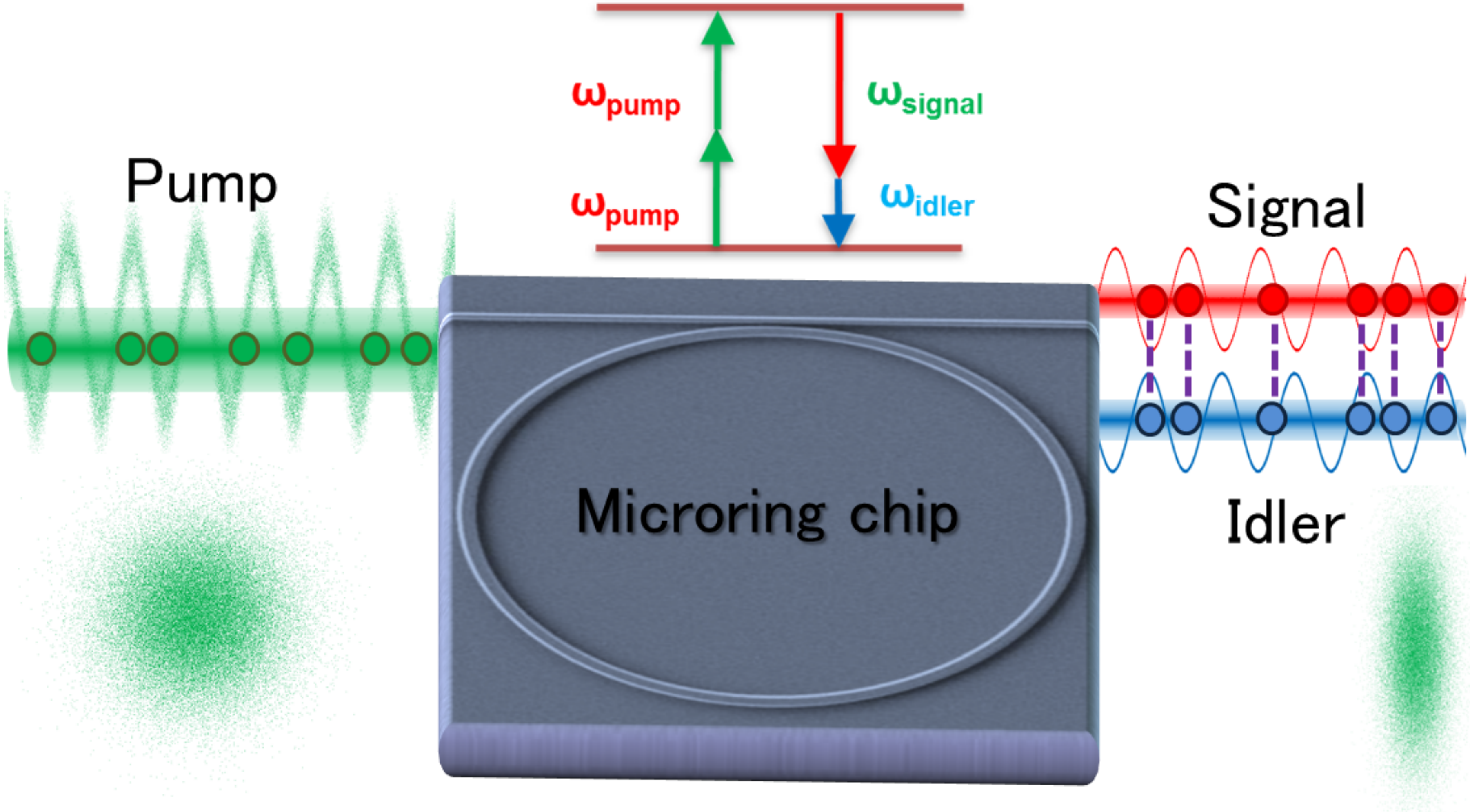
Chip-Based Silicon Nitride Microrings for Parametric Oscillators

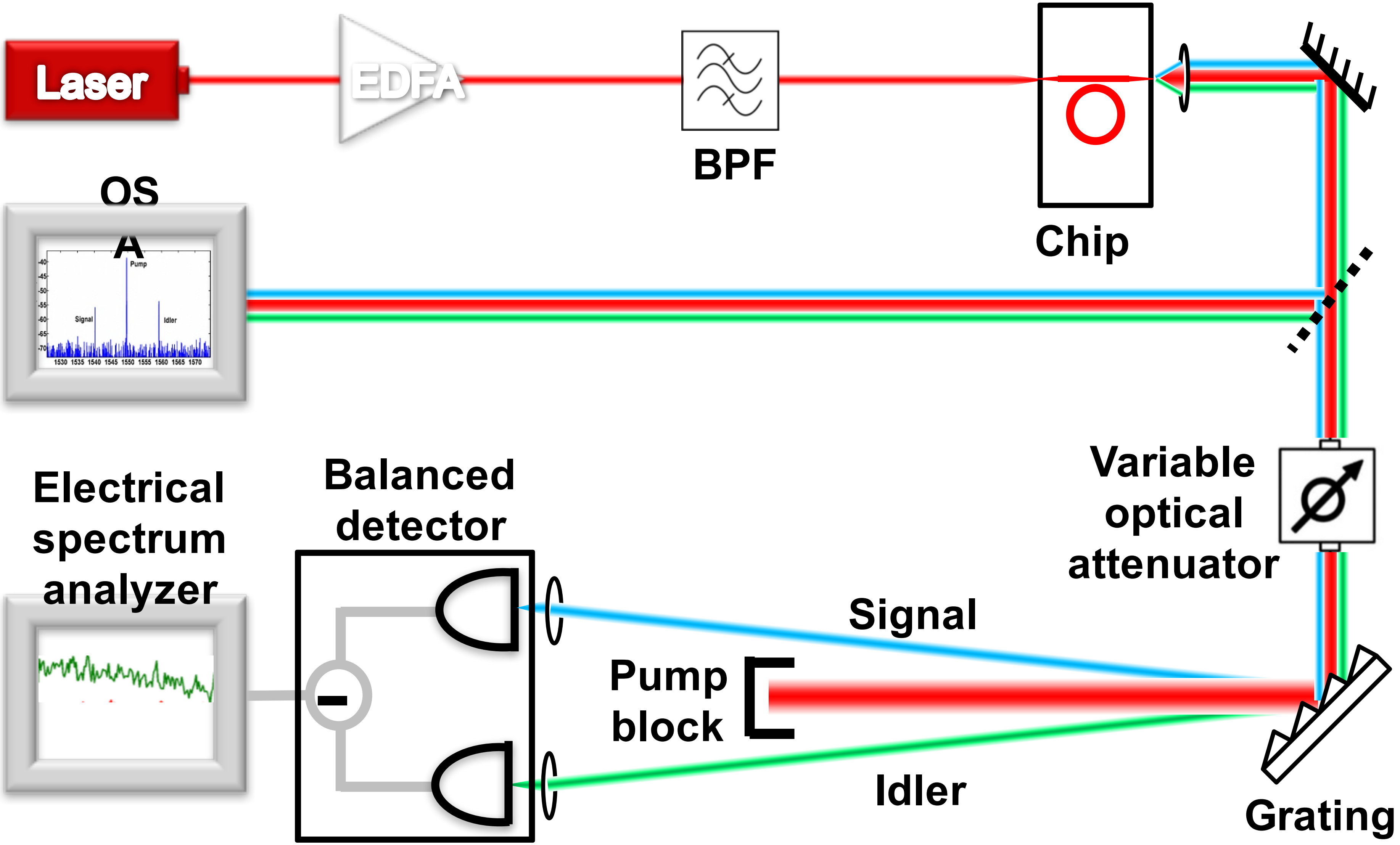


- CMOS-compatible material
- Fully monolithic and sealed structures and couplers
- High-*Q* resonators → $Q = \sim 1 \times 10^6$ [Gondarenko, et al., *Opt. Express* (2009).]
- High nonlinearity → $n_2 \sim 10 \times$ silica [Ikeda, et al., Opt. Express (2008).]
- Waveguide dispersion can be engineered [Turner-Foster, et al., *Opt. Express* (2006); Tan, Ikeda, Sun, and Fainman, Appl. Phys. Lett. (2010).]

Signal – idler correlations



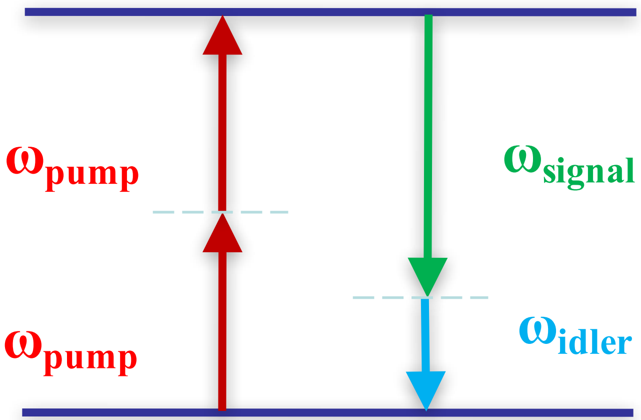
Experimental setup

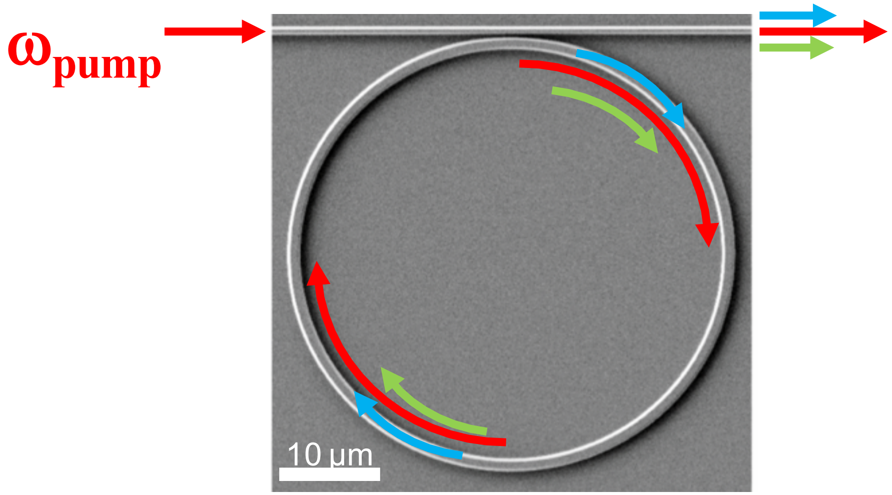


80 GHz FSR rings

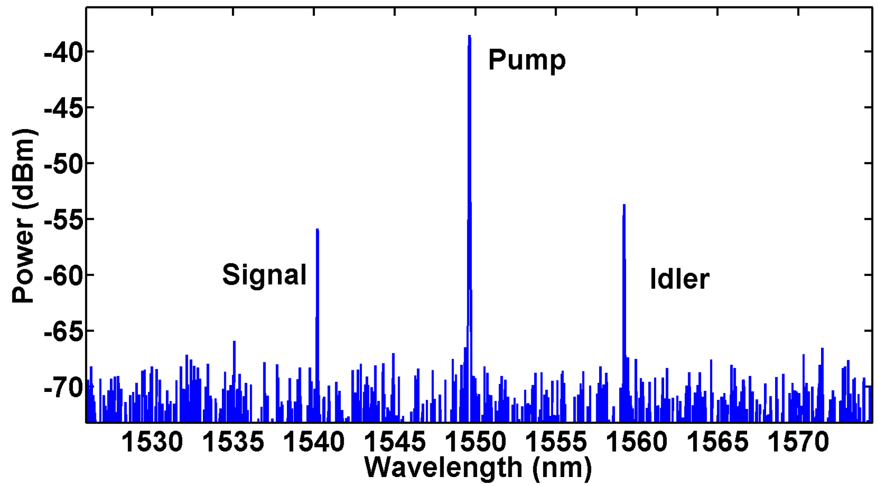
- FSR = 80 GHz
- Cavity length: 1800 microns
- Dimensions of waveguide: 1700 nm x 800 nm
- Intrinsic Q : 2 million
- Loaded Q: 200,000

SiN OPO



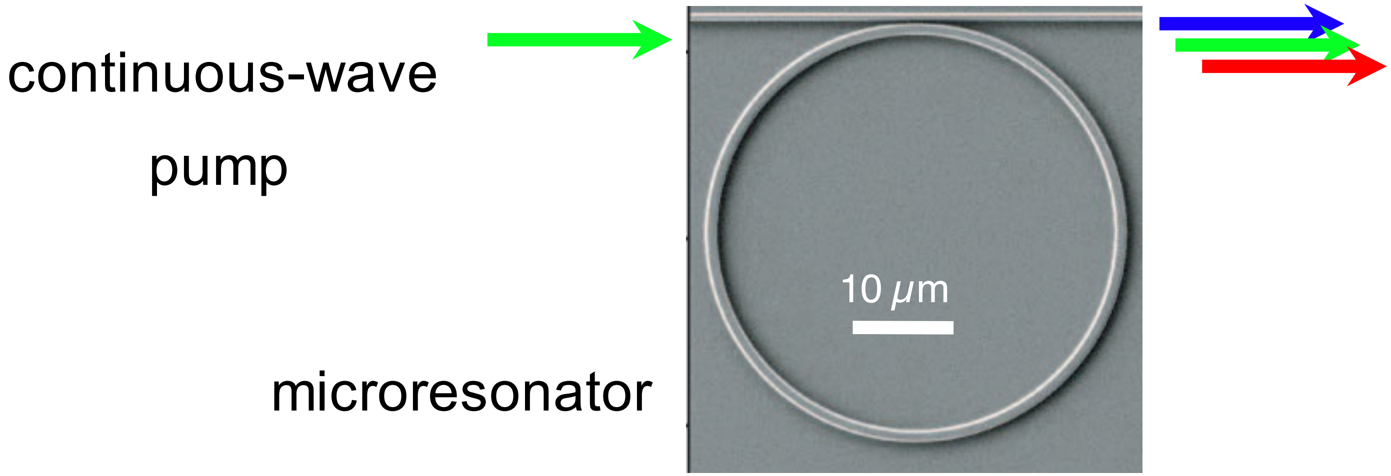


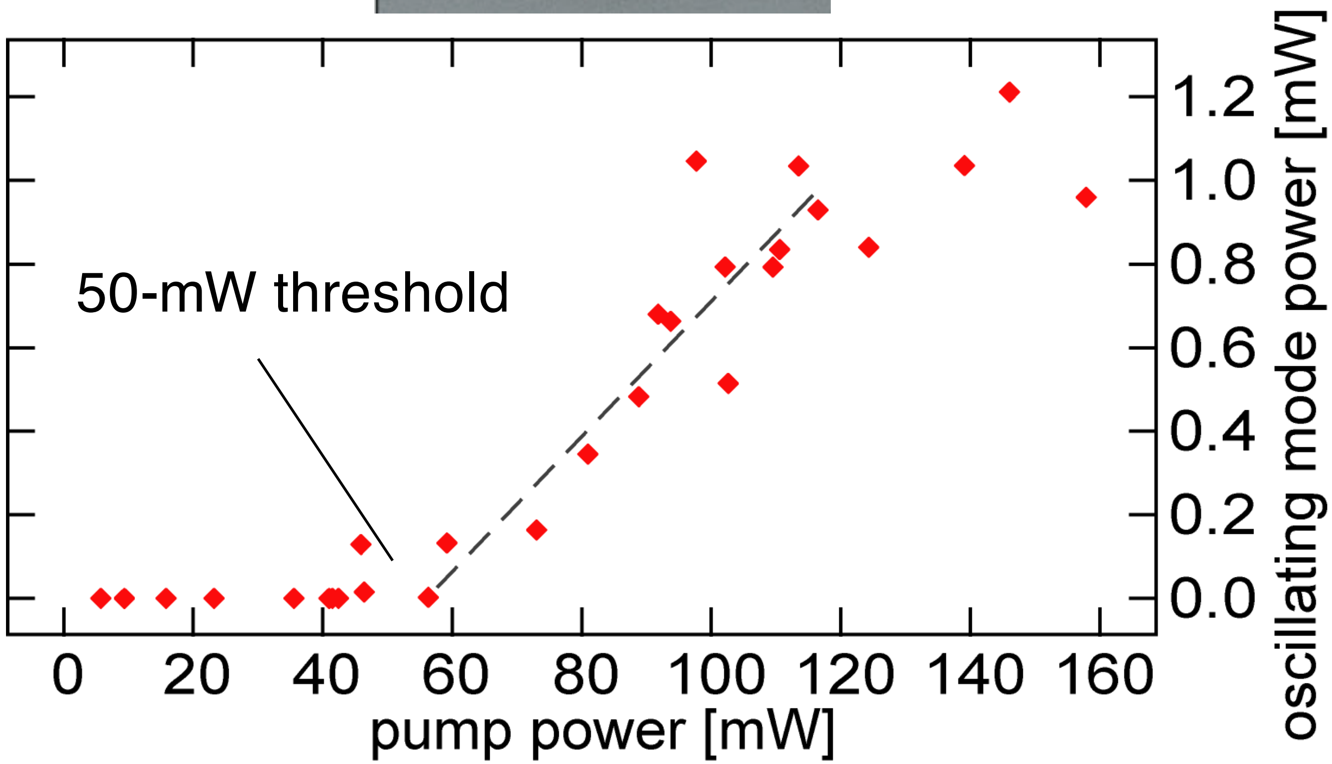
$$\omega_{signal} + \omega_{idler} = 2\omega_{pump}$$



Levy, Gaeta, Lipson *et al.* “CMOS-compatible multiple-wavelength oscillator for on-chip optical interconnects,” Nature Photonics 4, 37 (2010).

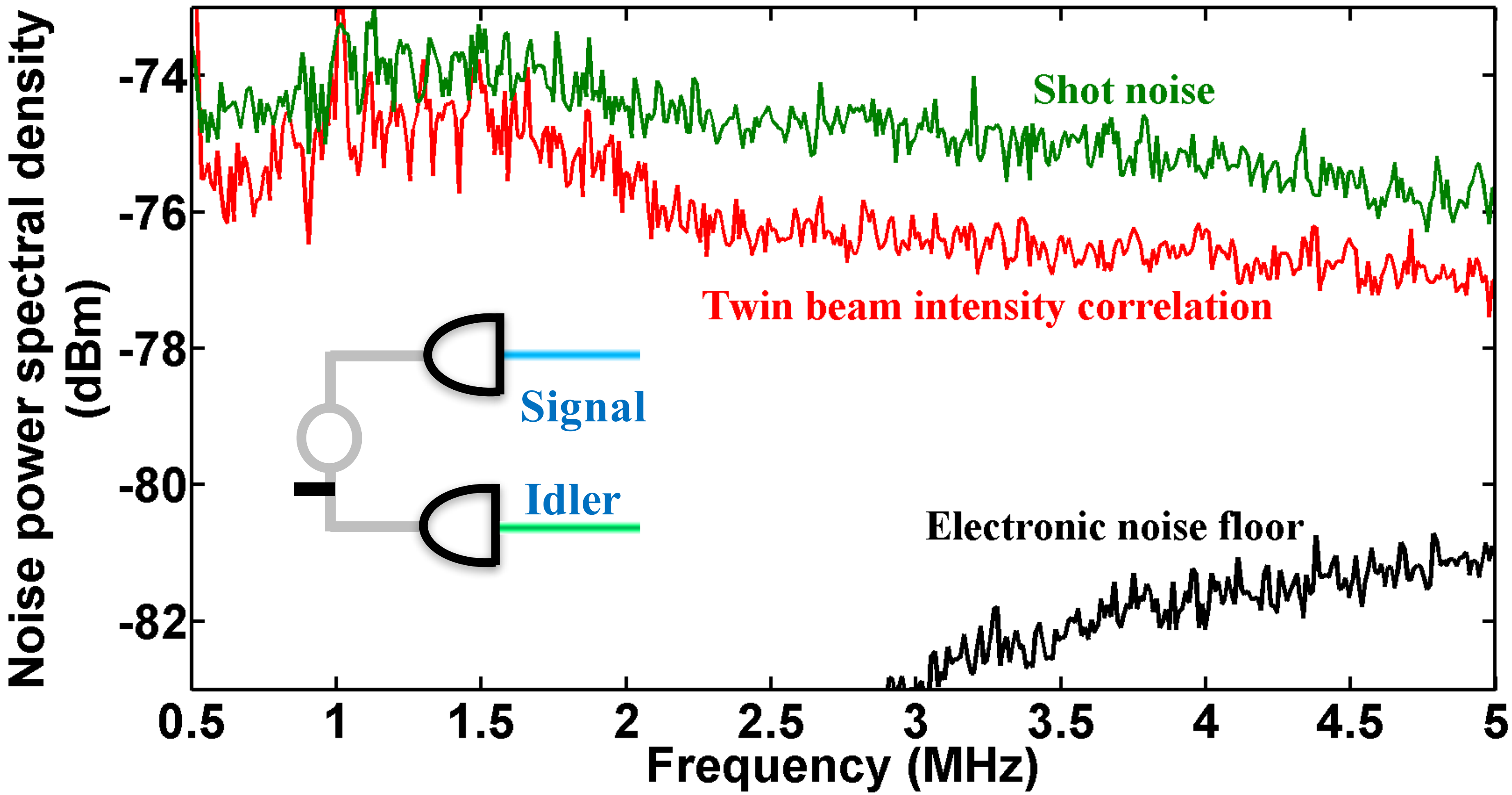
Threshold for Oscillation in SiN Microring





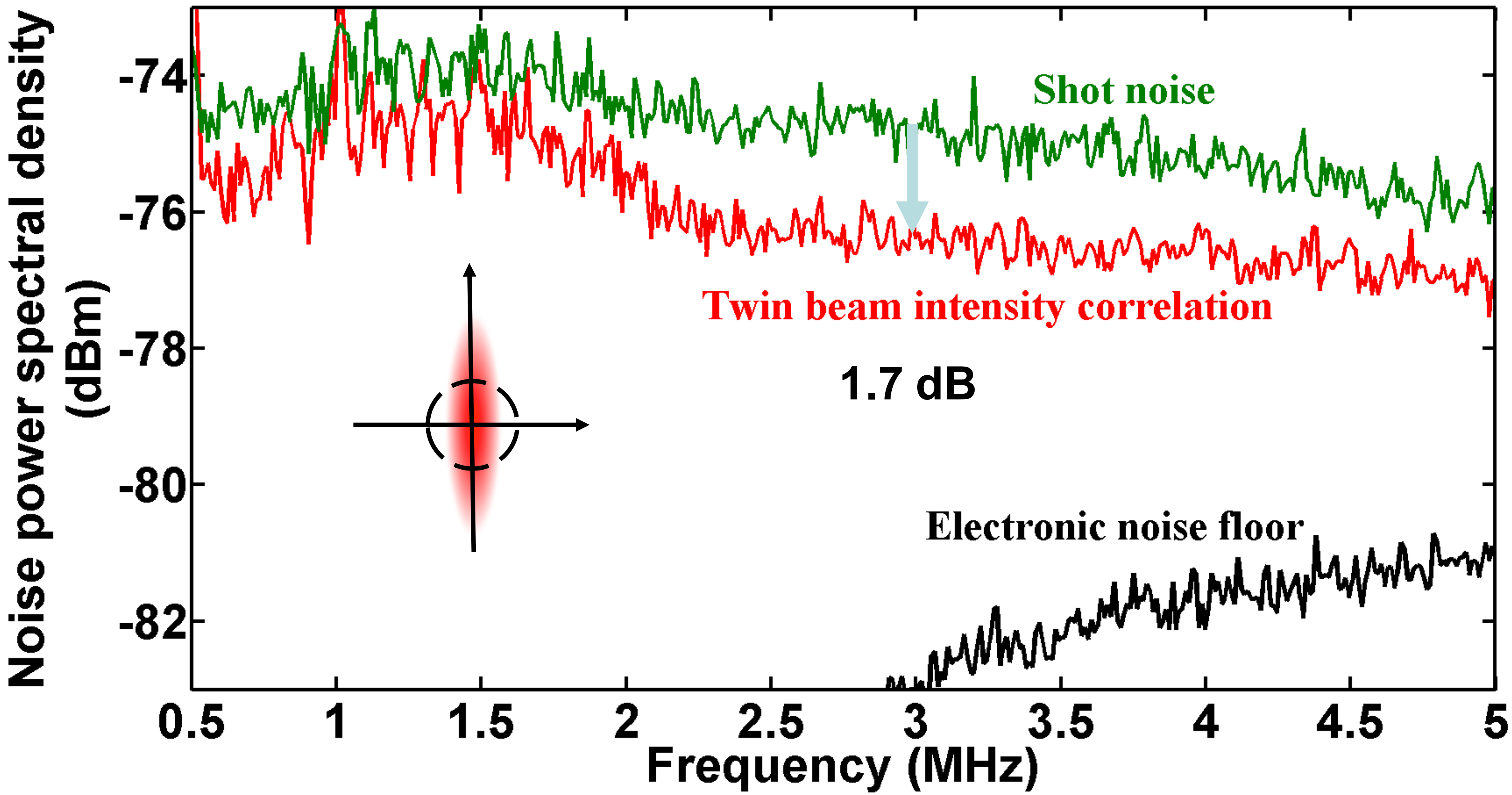
Review article: Kippenberg, Holzwarth, & Diddams, *Science* (2011)

Results



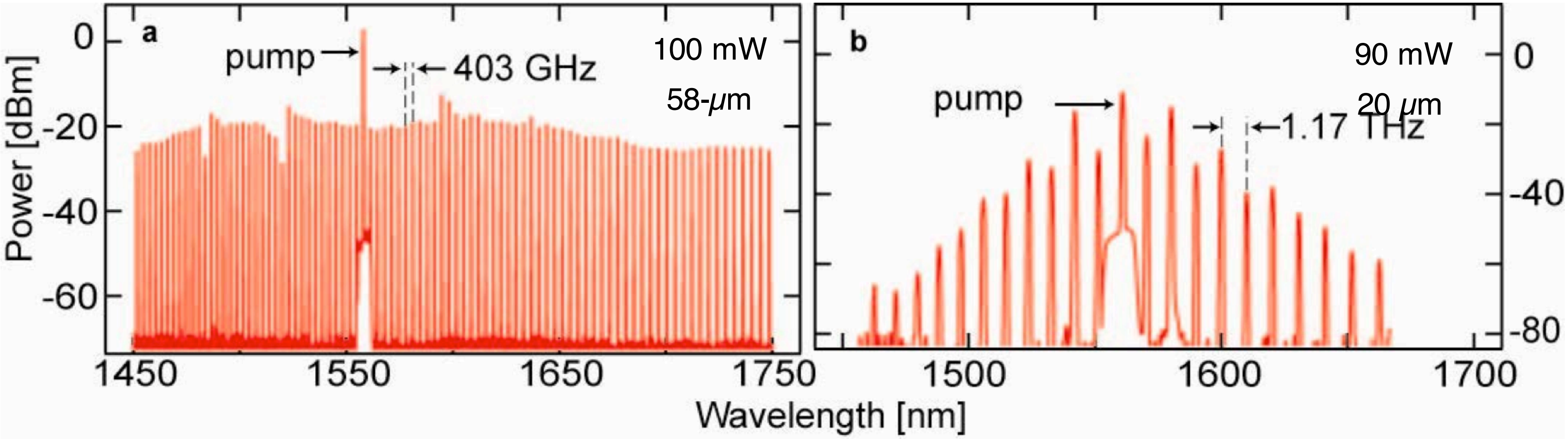
A.Dutt, K. Luke, S. Manipatruni, A. L. Gaeta, 5, P. A. Nussenzveig, Michal Lipson, Demonstration of Squeezing on chip, CLEO postdeadline 2013

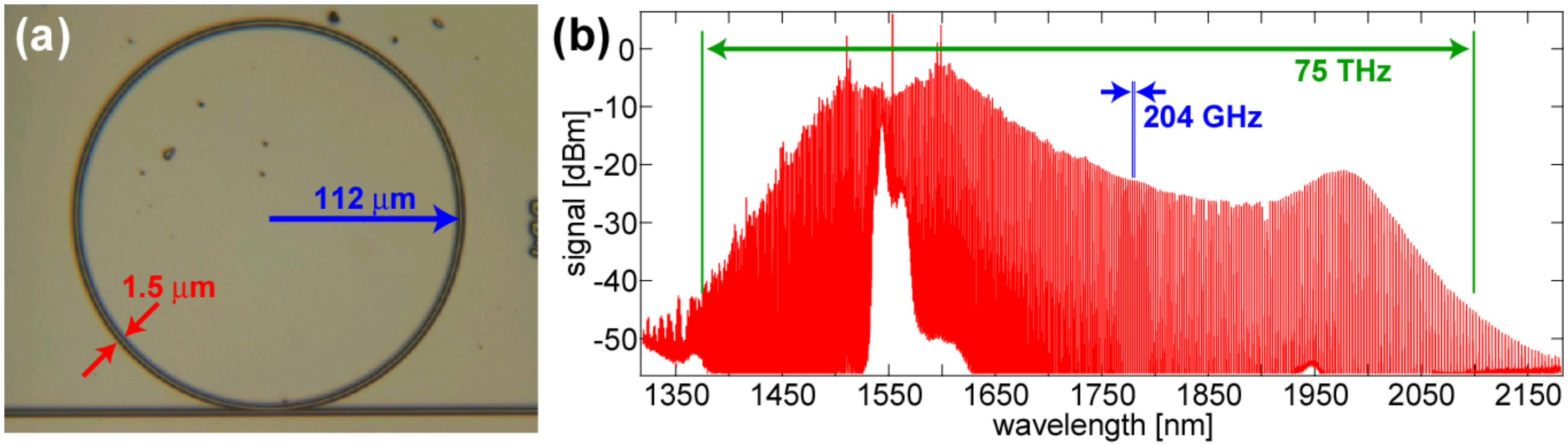
Results



A.Dutt, K. Luke, S. Manipatruni, A. L. Gaeta,, P. A. Nussenzveig, Michal Lipson, Demonstration of Squeezing on chip, CLEO postdeadline 2013

Chip-Based FWM Frequency Comb



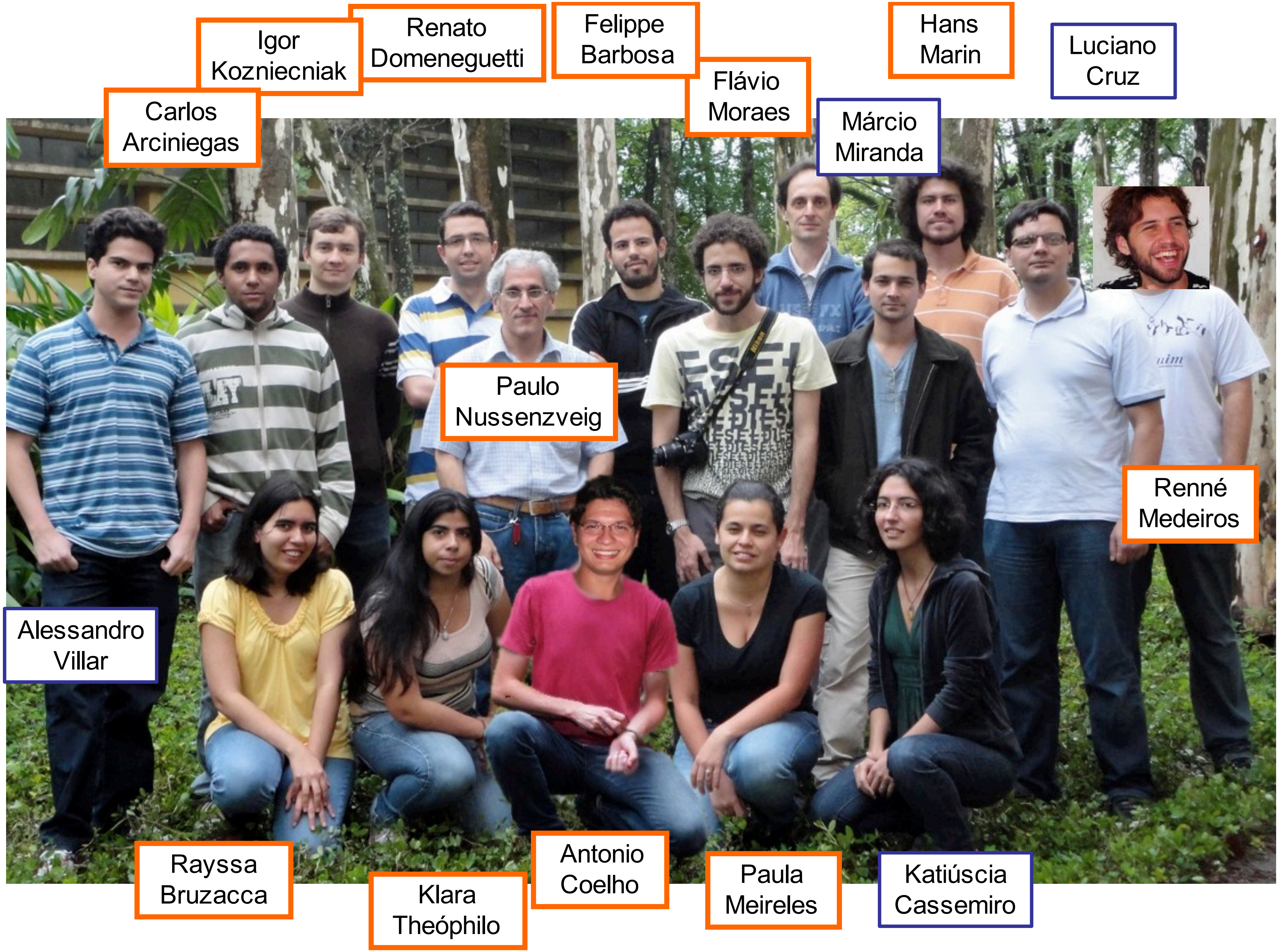


- Octave-spanning comb possible with suitable waveguide design and sufficiently high powers (~ 500 mW).

Levy, Gondarenko, Foster, Turner-Foster, Gaeta, and Lipson, *Nature Photon.* **4**, 37 (2010).

- While looking for the purity of the OPO, we found relevant information about our systems and our detection methods.
- We can still treat the tripartite entanglement involving linear combinations of sidebands: a valid mode of the system.
- But there is more space behind: we have hexapartite entanglement in the OPO.
- Gaussianity in photocurrent is a strong evidence of the generation of Gaussian states of the field.
- This ensures the validity of the linearized approach of strong carriers and weak sidebands.
- We have a tunable source of entanglement available for quantum communication, conveying information over the electromagnetic spectra.

- OPO above threshold can be used either as a bipartite or a tripartite source of tunable entangled states, spanning more than one octave in the frequency domain.
- These Gaussian states provide a tool for quantum communication among different quantum "pieces of hardware".
- Entanglement of continuous variables can suffer of "sudden death", just like qubits.
- It can lead to particular states, where there is tripartite entanglement without bipartite entanglement.
- But that's not the complete story…
- **How can we use this?**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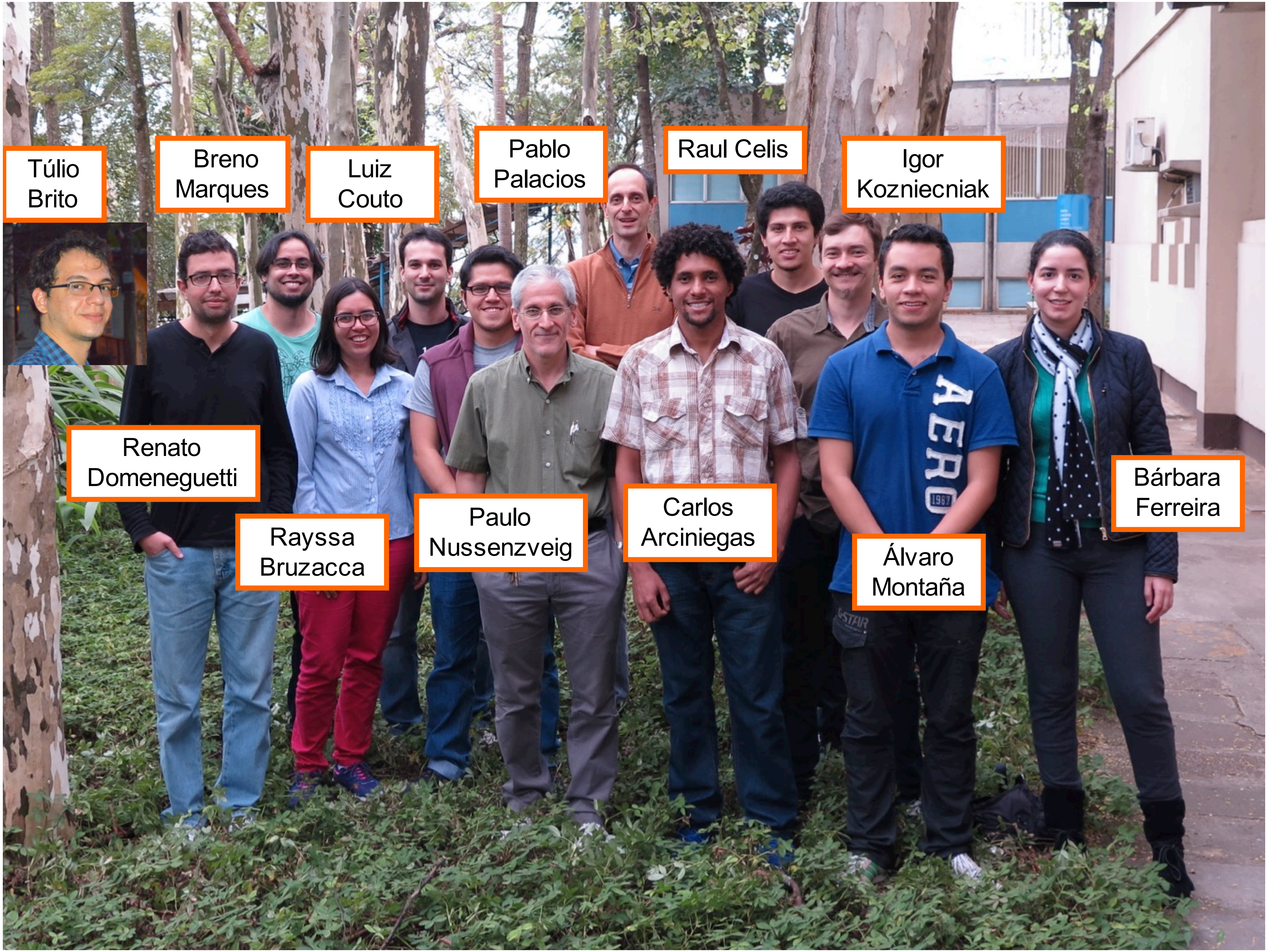

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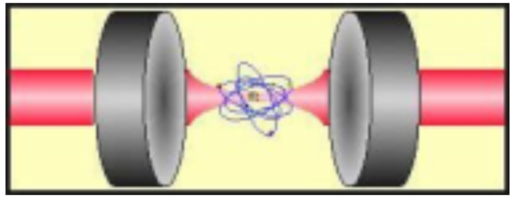

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Scholarships/
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Funding (equipment)
R$ 250 k/ year

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