Experimental determination of the Schmidt number of two-photon states entangled in spatial degrees of freedom

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XV Jorge André Swieca School on Nonlinear and Quantum Optics
Campinas - 2016

Entanglement in bipartite systems of photons

Experimental quantification

Tomographic state reconstruction

Low dimensional systems

Direct measurement

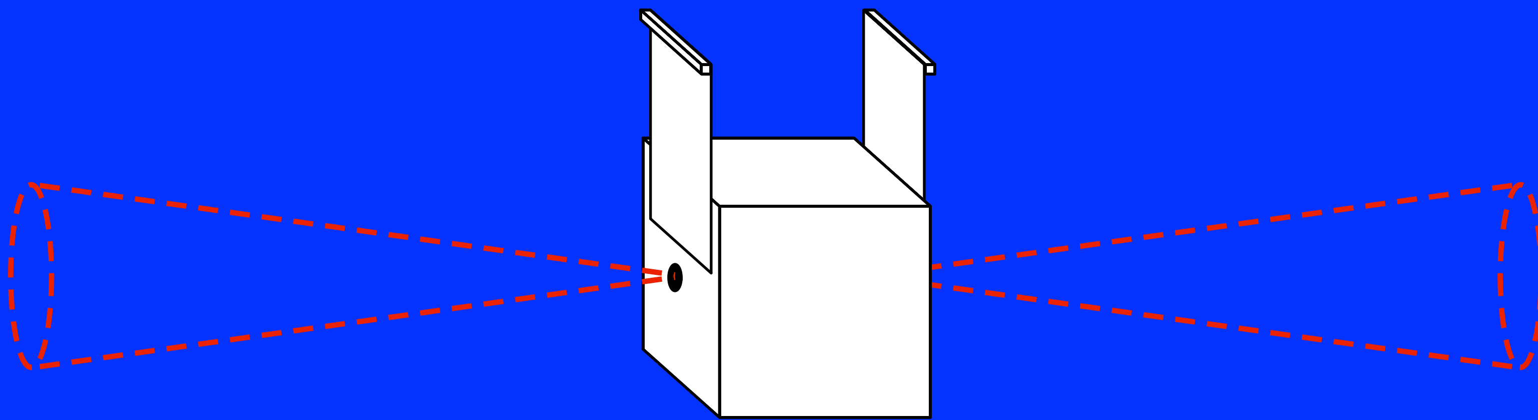
Low dimensional systems

S. P. Walborn *et al.*, Nature 440, 1022 (2006) - pure state
L. Neves *et al.*, Phys. Rev. A 76, 032314 (2007) - pure state
C. Schmid *et al.*, Phys. Rev. Lett. 101, 260505 (2008) - mixed state

High dimensional systems - transverse modes - pure states

M. V. Fedorov *et al.*, Phys. Rev. A 69, 052117 (2004)
M. V. Fedorov *et al.*, Phys. Rev. A 72, 032110 (2005)
M. V. Fedorov *et al.*, Phys. Rev. Lett. 99, 063901 (2007)
M. V. Fedorov *et al.*, Phys. Rev. A 77, 032336 (2008)
J. Janousek *et al.*, Nature Photon. 3, 399 (2009)

Two-photon Sources



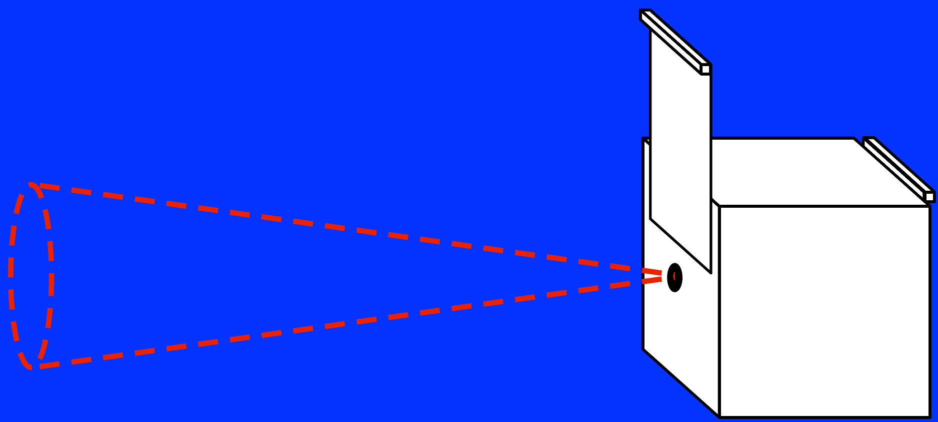
Pure states $\rho_{12} = |\Psi\rangle\langle\Psi|$

Schmidt decomposition

$$|\Psi\rangle = \sum_n \sqrt{\lambda_n}\ \underbrace{|\phi_n\rangle \otimes |\psi_n\rangle}$$

One-photon states
in the Schmidt modes

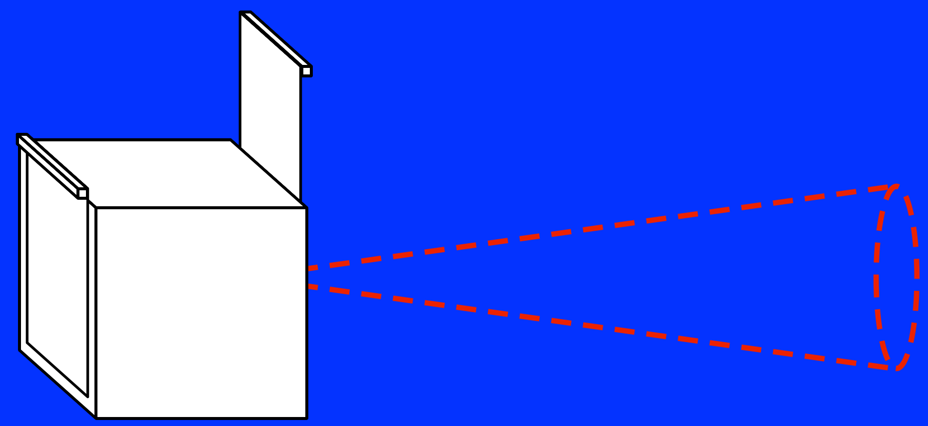
Two-photon Sources



$$\rho_1 = \mathrm{Tr}_2[\rho_{12}] = \sum_n \lambda_n |\phi_n\rangle\langle\phi_n|$$

ρ_{12} entangled $\longrightarrow$ ρ_1 Mixed

Two-photon Sources



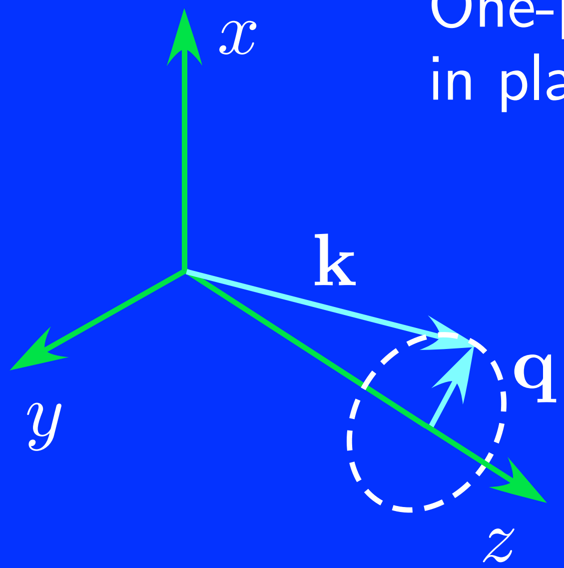
$$\rho_2 = \mathrm{Tr}_1[\rho_{12}] = \sum_n \lambda_n |\psi_n\rangle\langle\psi_n|$$

ρ_{12} entangled $\longrightarrow$ ρ_2 Mixed

Representation of the reduced state operator in terms of plane-wave modes

$$\varrho_1(\mathbf{q}_1, \mathbf{q}_1') = \langle \mathbf{q}_1 | \rho_1 | \mathbf{q}_1' \rangle = \sum_n \lambda_n \langle \mathbf{q}_1 | \phi_n \rangle \langle \phi_n | \mathbf{q}_1' \rangle$$

One-photon states
in plane-wave modes



Schmidt modes for photon 1 in q space

$$\varrho_1(\mathbf{q}_1, \mathbf{q}_1') = \sum_n \lambda_n \; \tilde{\phi}_n(\mathbf{q}_1) \, \tilde{\phi}_n^*(\mathbf{q}_1')$$

$$\phi(\mathbf{x}_1) = \mathscr{F}[\tilde{\phi}(\mathbf{q}_1)]$$ Fourier transform

$$\varrho(\mathbf{q}_1, \mathbf{q}_1') \longrightarrow w(\mathbf{x}_1, \mathbf{x}_1')$$

Schmidt modes for photon 1

$$w(\mathbf{x}_1, \mathbf{x}_1') = \sum_n \lambda_n \, \phi_n(\mathbf{x}_1) \, \phi_n^*(\mathbf{x}_1')$$

$$\phi(\mathbf{x}_1) = \mathscr{F}[\tilde{\phi}(\mathbf{q}_1)]$$ Fourier transform

$$\varrho(\mathbf{q}_1, \mathbf{q}_1') \longrightarrow w(\mathbf{x}_1, \mathbf{x}_1')$$

Schmidt modes for photon 1

$$w(\mathbf{x}_1, \mathbf{x}_1') = \sum_n \lambda_n \, \phi_n(\mathbf{x}_1) \, \phi_n^*(\mathbf{x}_1')$$

Coherent modes for field 1

$$w(\mathbf{x}_1, \mathbf{x}_1') \propto \langle \mathscr{E}_\omega^*(\mathbf{x}_1) \, \mathscr{E}_\omega(\mathbf{x}_1') \rangle$$

Cross-spectral density

L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge)

Quantum mechanics of pure monochromatic two-photon states

$$\varrho_1(\mathbf{q}_1, \mathbf{q}_1') = \sum_n \lambda_n \, \tilde{\phi}_n(\mathbf{q}_1) \, \tilde{\phi}_n^*(\mathbf{q}_1')$$

Reduced (one-photon) density matrix

Schmidt modes

Classical coherence theory

$$w(\mathbf{x}_1, \mathbf{x}_1') = \sum_n \lambda_n \, \phi_n(\mathbf{x}_1) \, \phi_n^*(\mathbf{x}_1')$$

Cross-spectral density function

Coherent modes

The Schmidt number

$$K = \frac{1}{\mathrm{Tr}[\rho_1^2]} = \frac{1}{\mathrm{Tr}[\rho_2^2]} = \frac{1}{\sum_n \lambda_n^2}$$

The overall degree of coherence

$$\bar{\mu}^2 = \iint d\mathbf{x}\, d\mathbf{x}' \, |w(\mathbf{x}, \mathbf{x}')|^2 = \sum_n \lambda_n^2$$

$$K = \frac{1}{\bar{\mu}^2}$$

The Schmidt number

$$K = \frac{1}{\mathrm{Tr}[\rho_1^2]} = \frac{1}{\mathrm{Tr}[\rho_2^2]} = \frac{1}{\sum_n \lambda_n^2}$$

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$$K = \frac{1}{\bar{\mu}^2}$$

May be difficult to measure

Quasi-homogeneous light sources

$$w(\mathbf{x}_1, \mathbf{x}_1') \rightarrow \mu(|\mathbf{x}_1 - \mathbf{x}_1'|)\, P(\tfrac{1}{2}|\mathbf{x}_1 + \mathbf{x}_1'|)$$

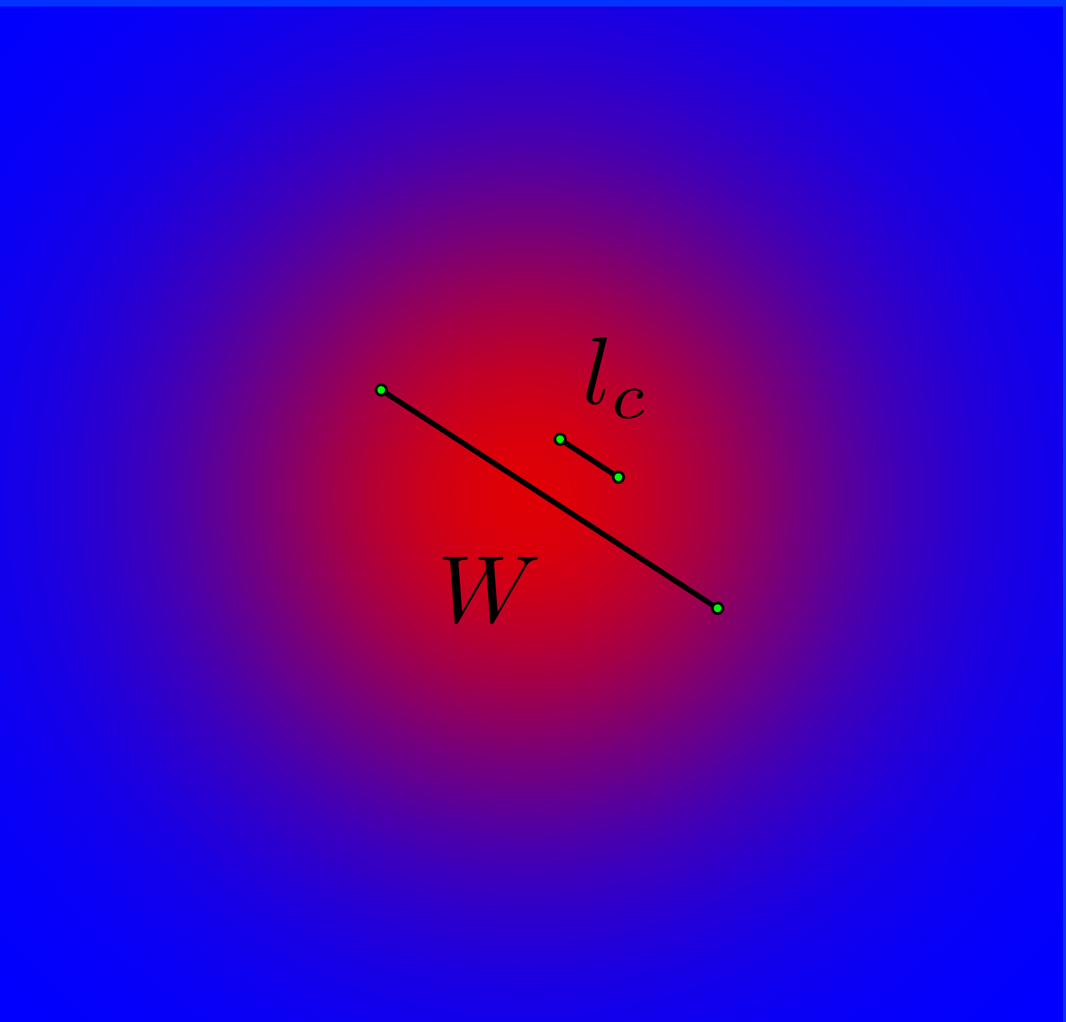
Transverse coherence function

Normalized intensity $$P(\mathbf{x}) = \frac{\mathscr{I}(\mathbf{x})}{\int_{\text{source}} d\mathbf{x}\, \mathscr{I}(\mathbf{x})}$$

L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge)

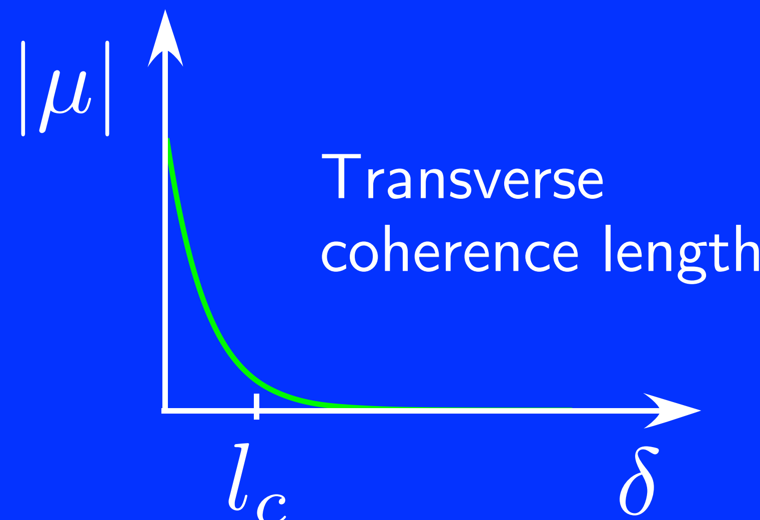
Quasi-homogeneous light sources

$$w(\mathbf{x}_1, \mathbf{x}_1') \to \mu(|\mathbf{x}_1 - \mathbf{x}_1'|)\, P(\tfrac{1}{2}|\mathbf{x}_1 + \mathbf{x}_1'|)$$



Smooth intensity profile

$$l_c \ll W$$

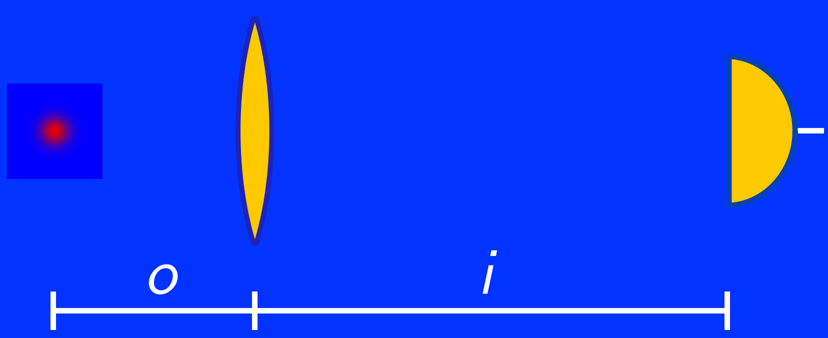


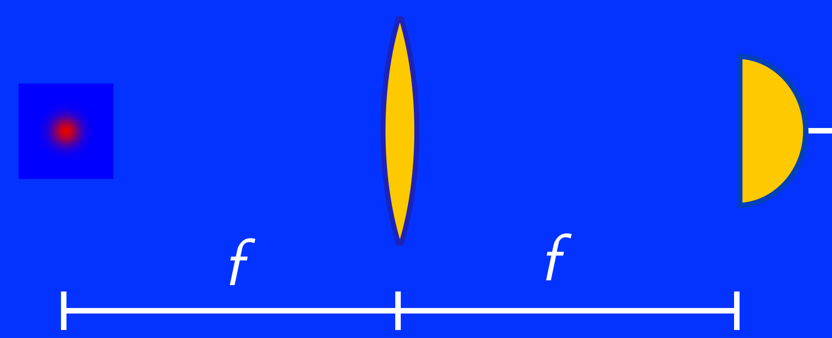
Quasi-homogeneous light sources

$$\bar{\mu}^2 \approx \underbrace{\frac{\int \mathscr{I}^2(\mathbf{x})d\mathbf{x}}{[\int \mathscr{I}(\mathbf{x})d\mathbf{x}]^2}}_{\text{Source}} \times \underbrace{\frac{\int \mathscr{I}^2(\mathbf{q})d\mathbf{q}}{[\int \mathscr{I}(\mathbf{q})d\mathbf{q}]^2}}_{\text{Far field}}$$

$$\bar{\mu}^2 \approx \int P^2(\mathbf{x})d\mathbf{x} \times \int P^2(\mathbf{q})d\mathbf{q}$$

Easy to measure

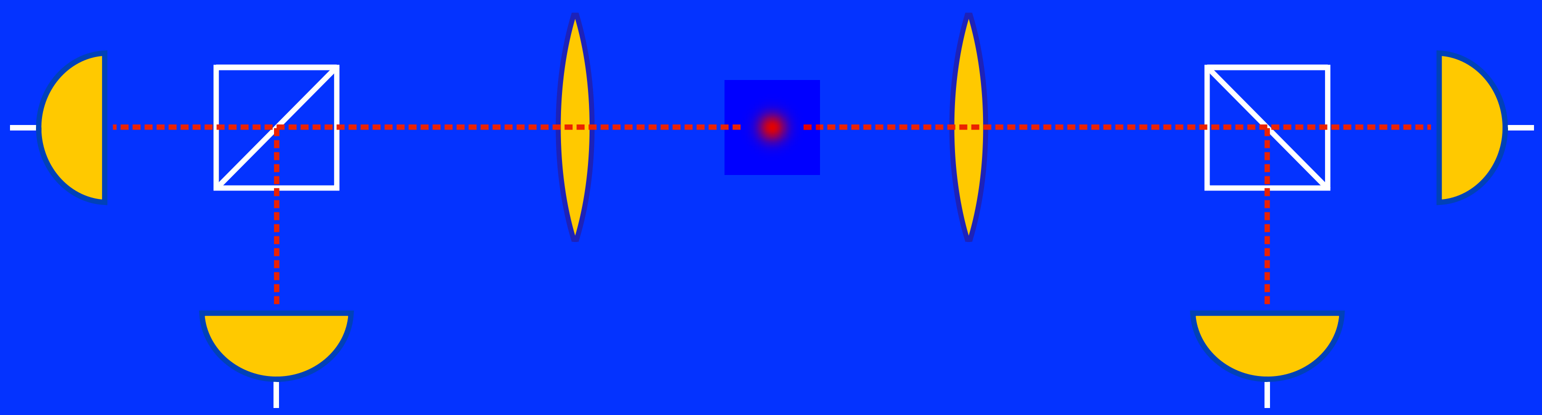




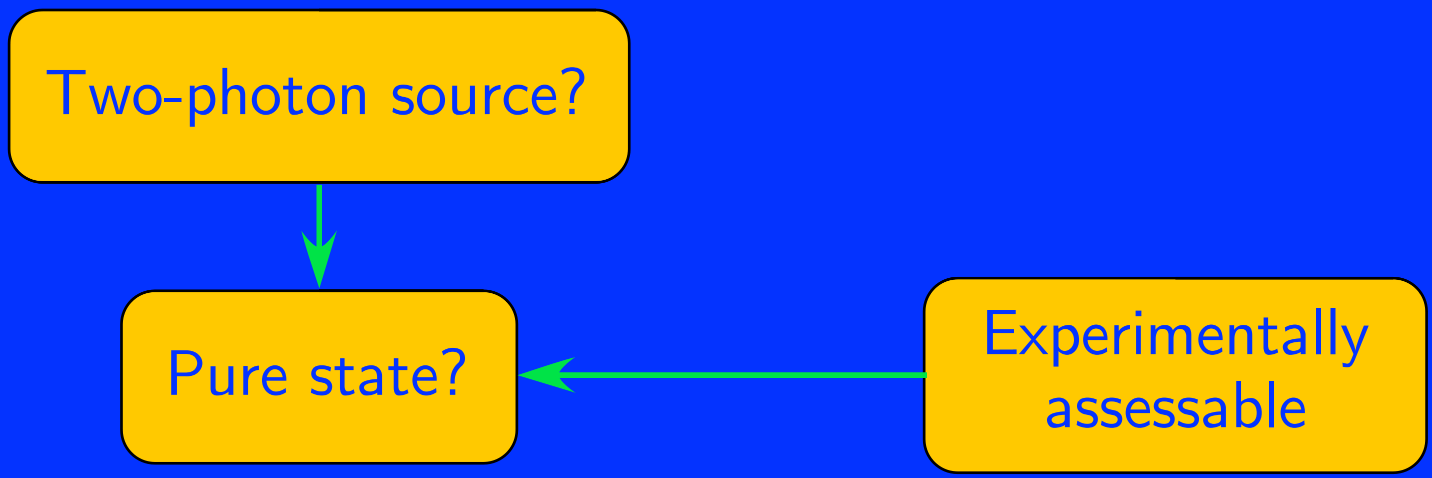
Procedure

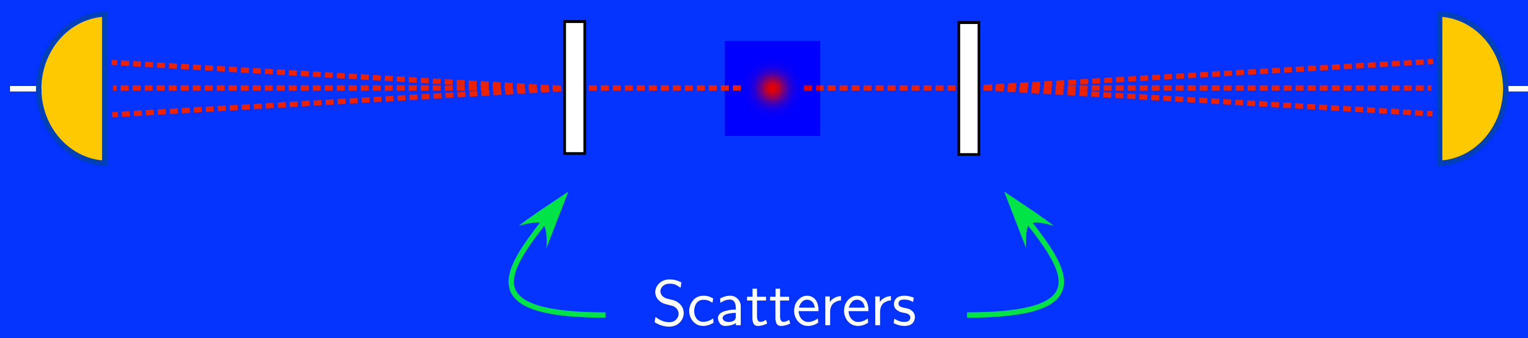
Two-photon source?

Experimentally assessable



Procedure





L. Neves *et al.*, Phys. Rev. A 76, 032314 (2007)
C. W. J. Beenakker *et. al.*, Phys. Rev. Lett. 102, 193601 (2009)

Procedure

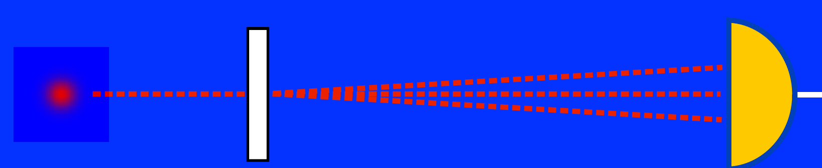
Two-photon source?

↓

Pure state?

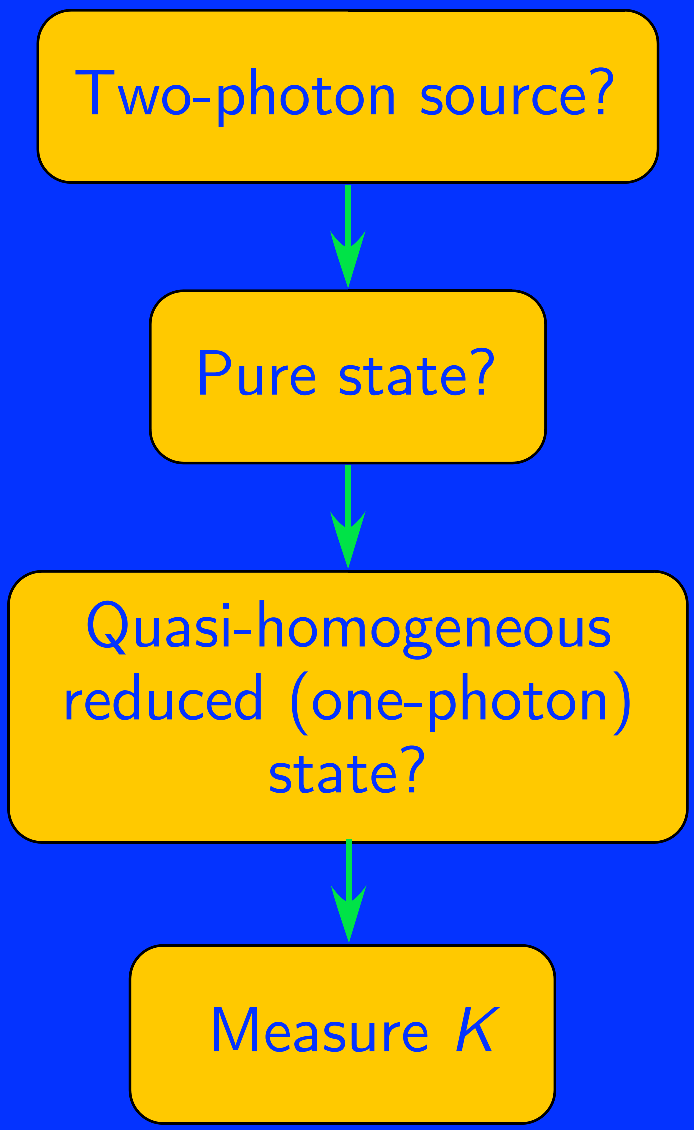
↓

Quasi-homogeneous reduced (one-photon) state? ← Experimentally assessable



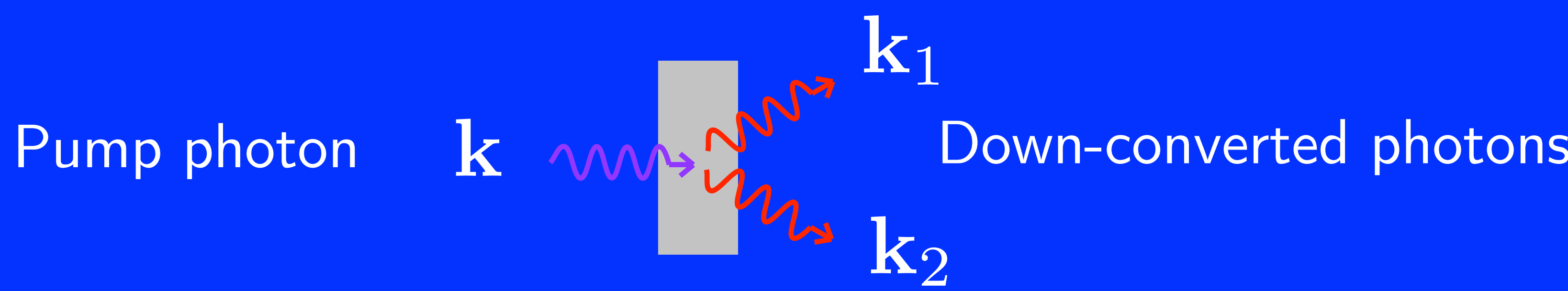
Jannson *et al.*, Opt Lett. 13, 1060 (1988)

Procedure


Two-photon source?
Pure state?
Quasi-homogeneous reduced (one-photon) state?
Measure K

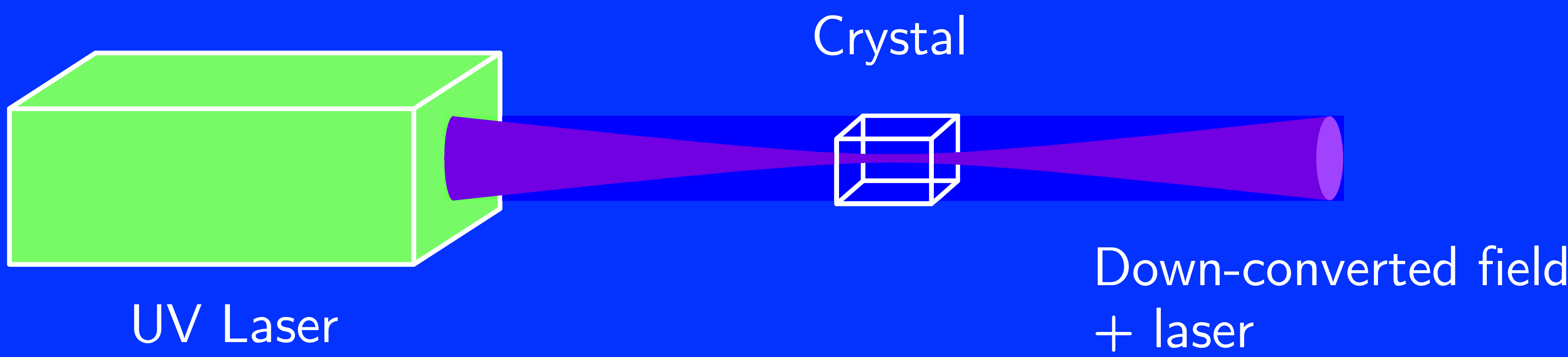
Application

Spontaneous parametric down-conversion

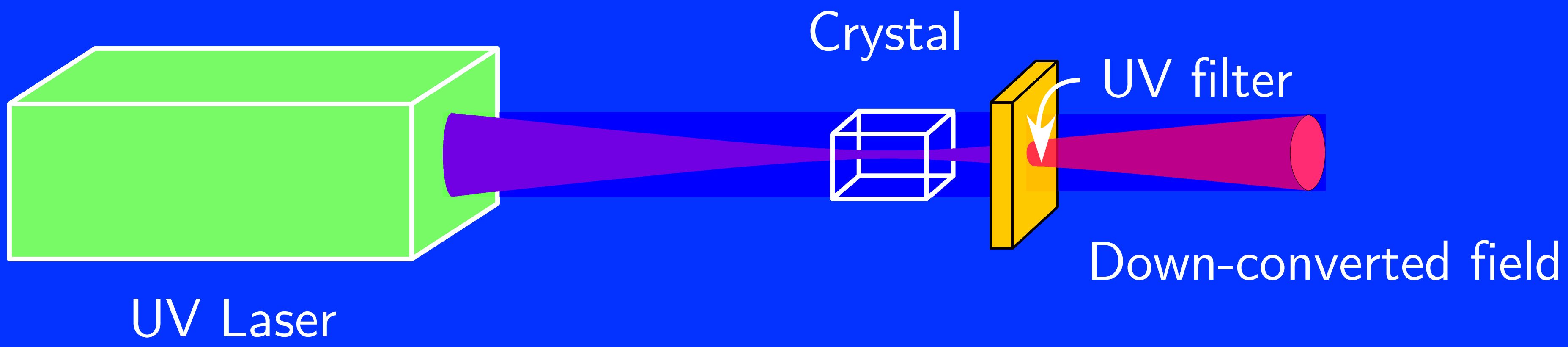


$$a(\mathbf{k})a^\dagger(\mathbf{k}_1)a^\dagger(\mathbf{k}_2)$$

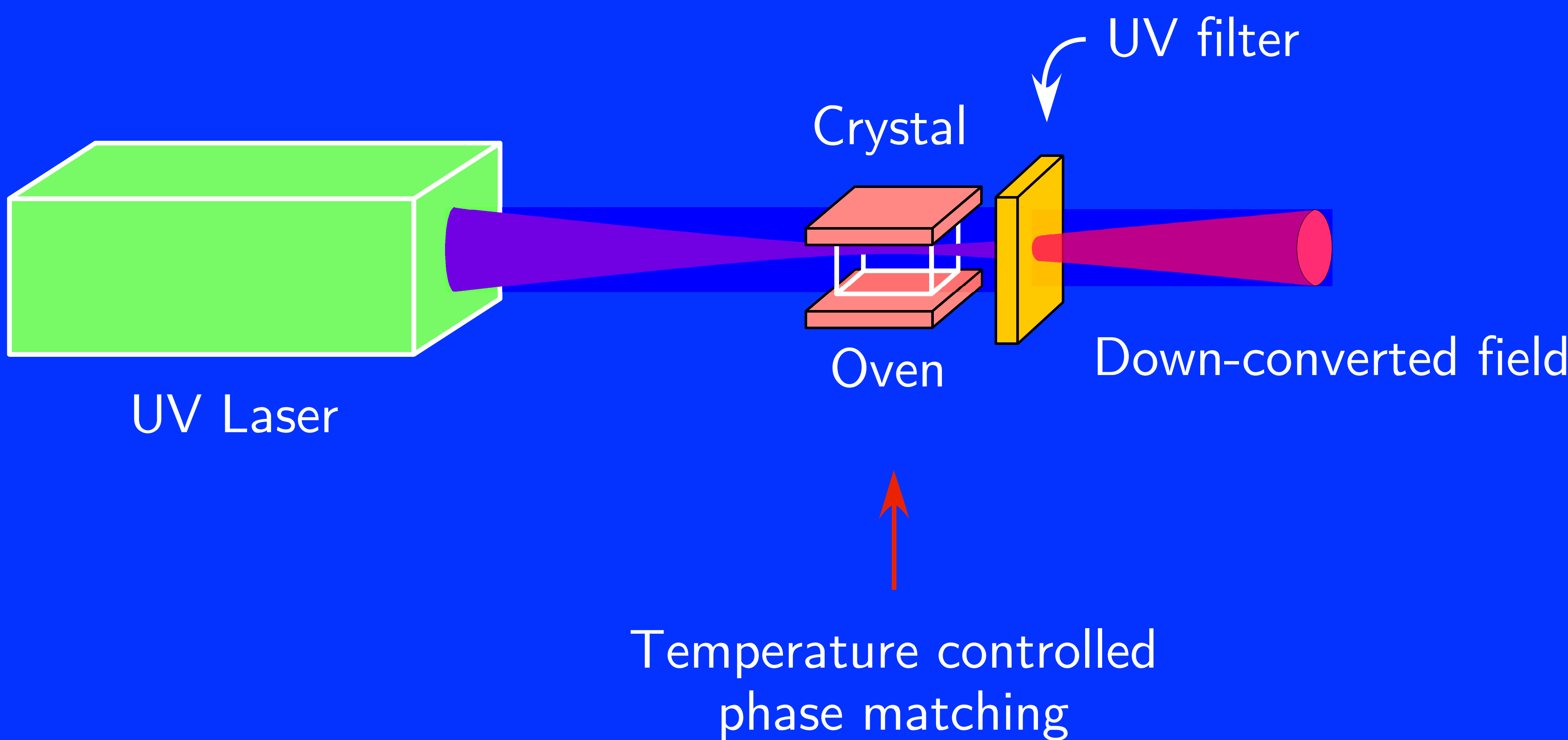
Spontaneous parametric down-conversion



Spontaneous parametric down-conversion

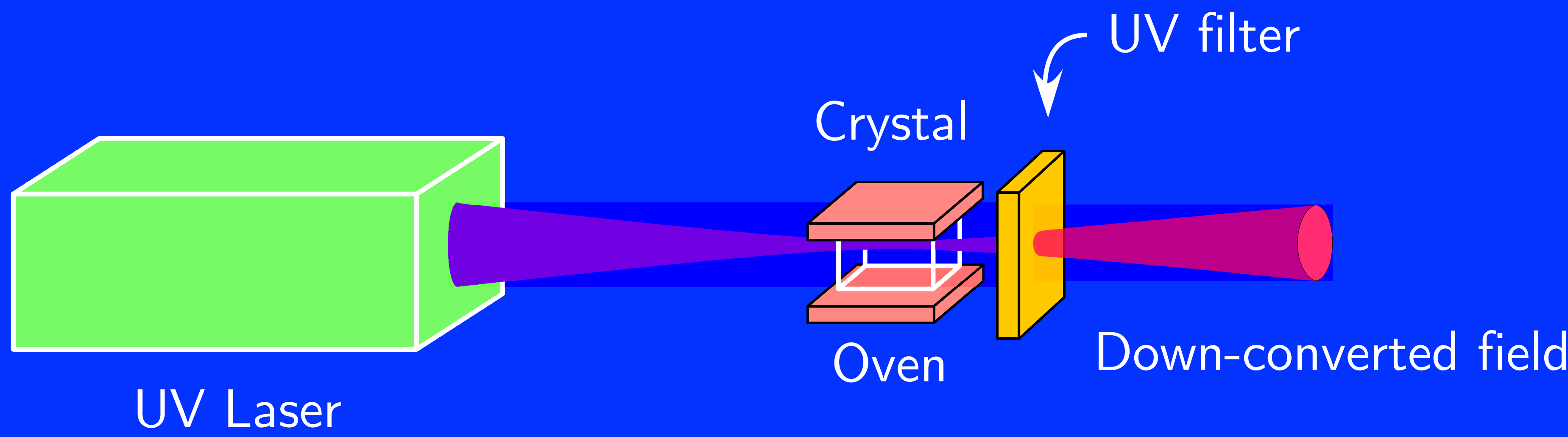


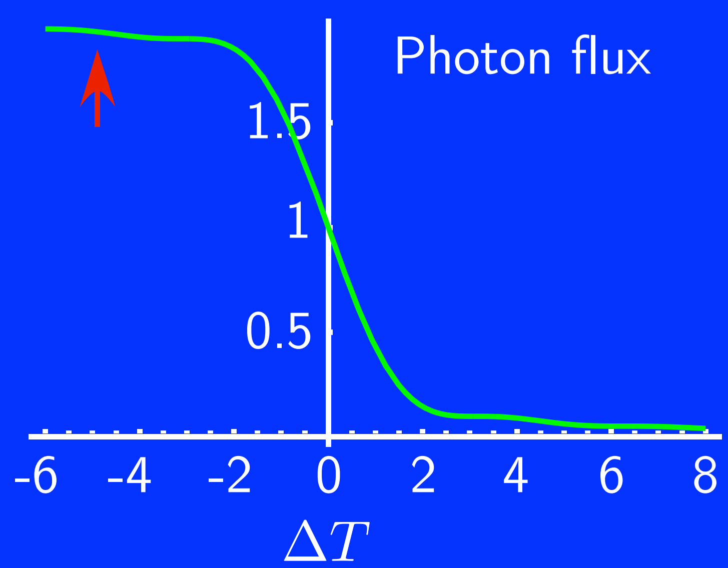
Spontaneous parametric down-conversion

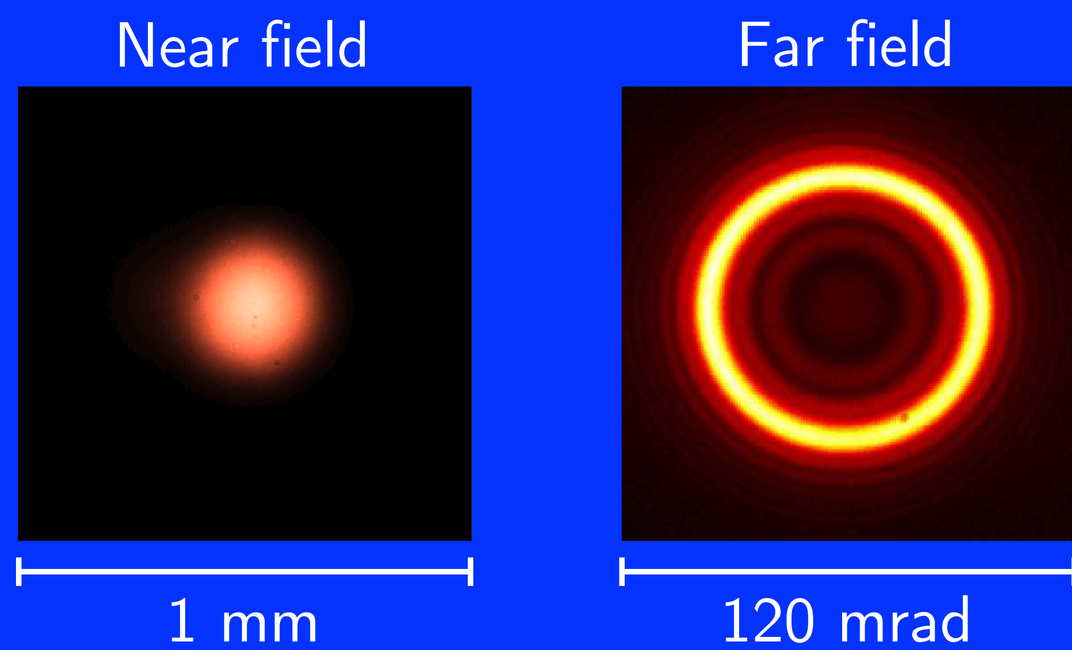


$T_0 = 60\,^{\circ}C$ (in our setup)

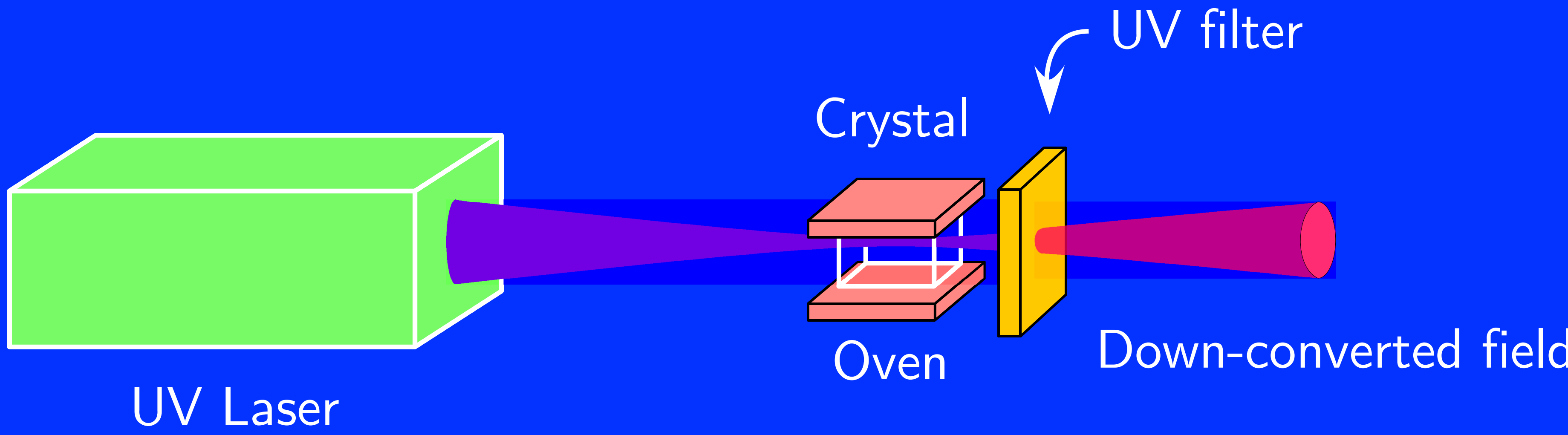
Spontaneous parametric down-conversion

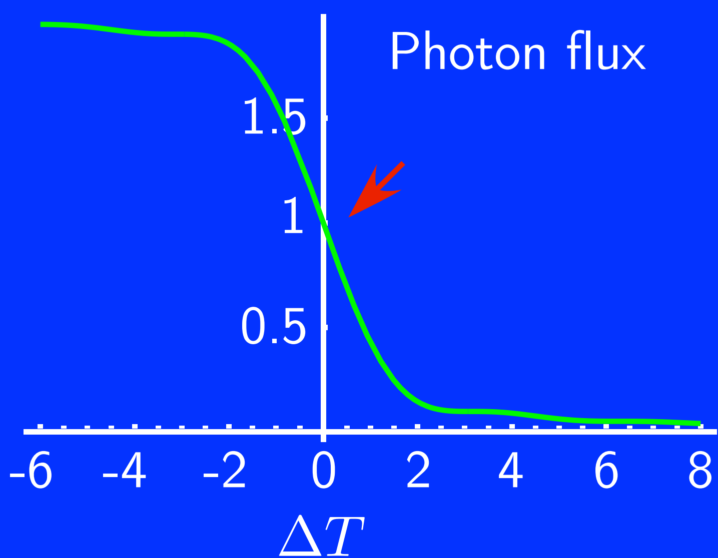




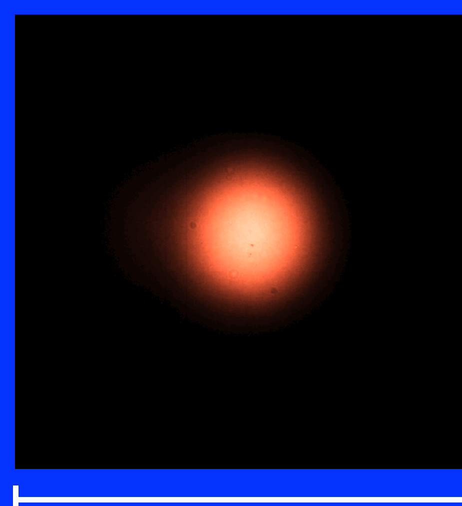


Spontaneous parametric down-conversion

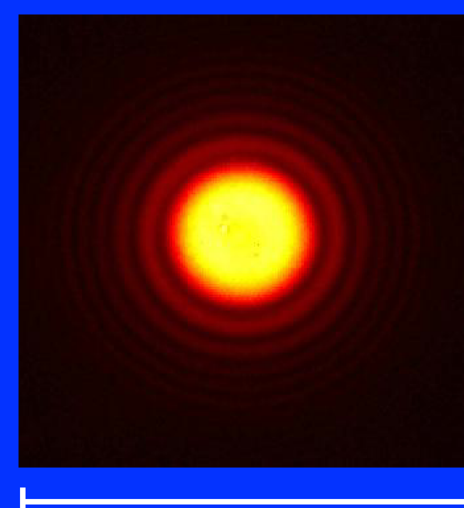




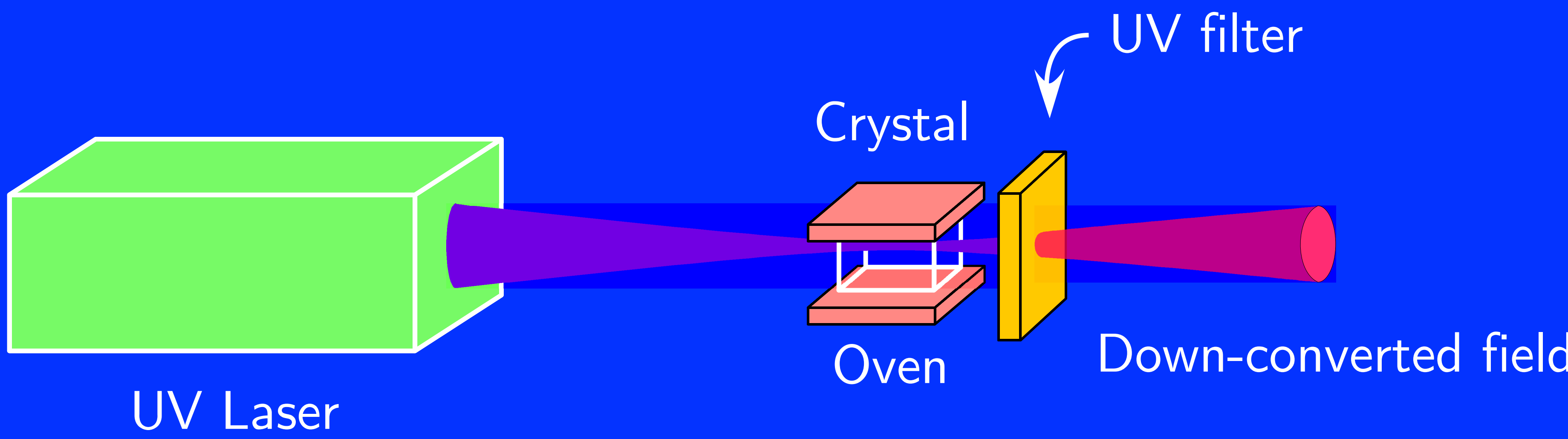
Near field

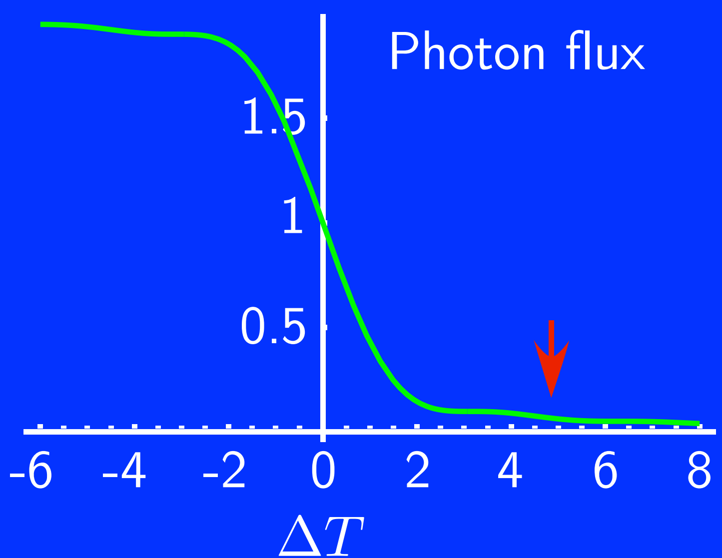


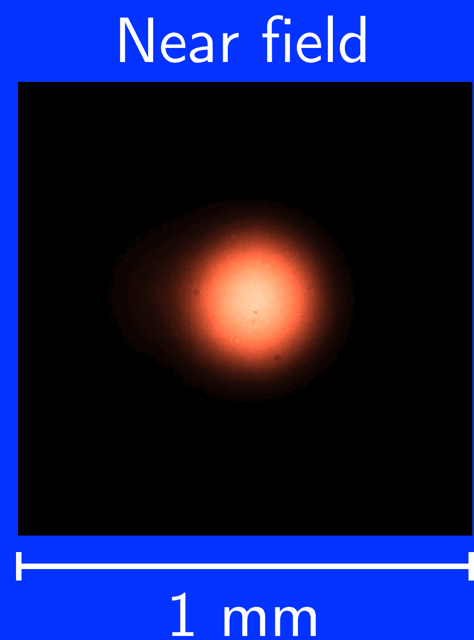
Far field

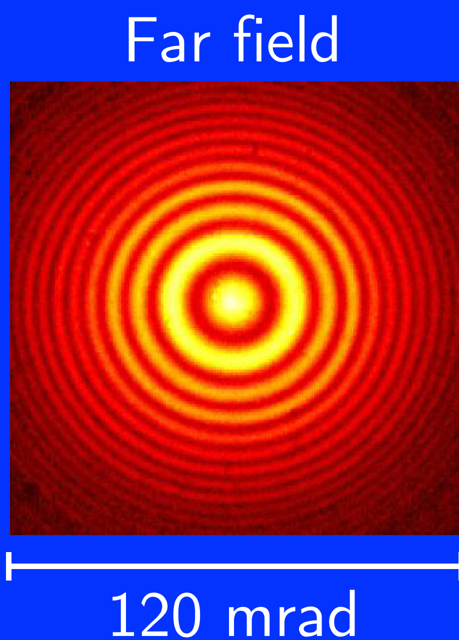


Spontaneous parametric down-conversion

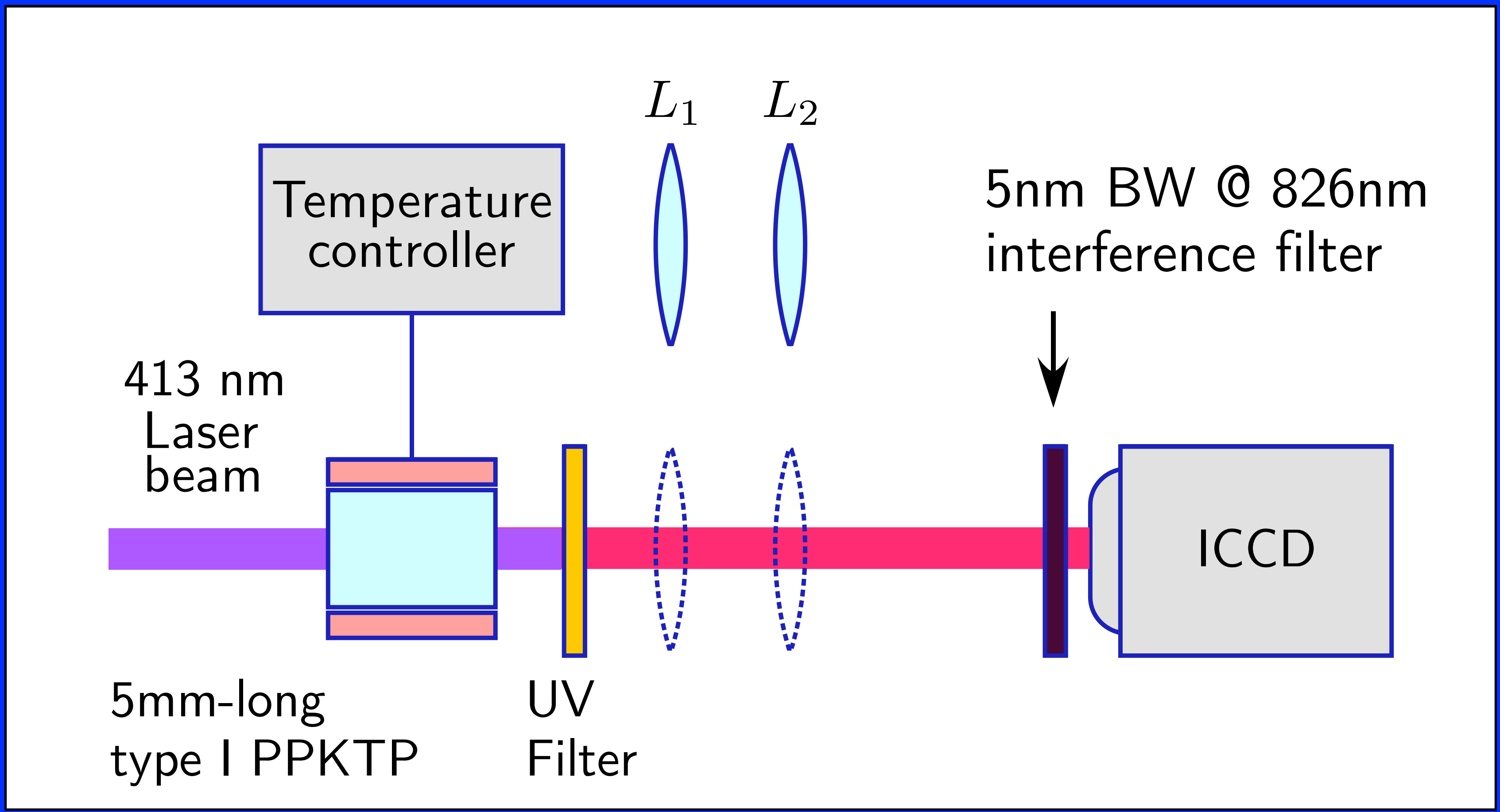








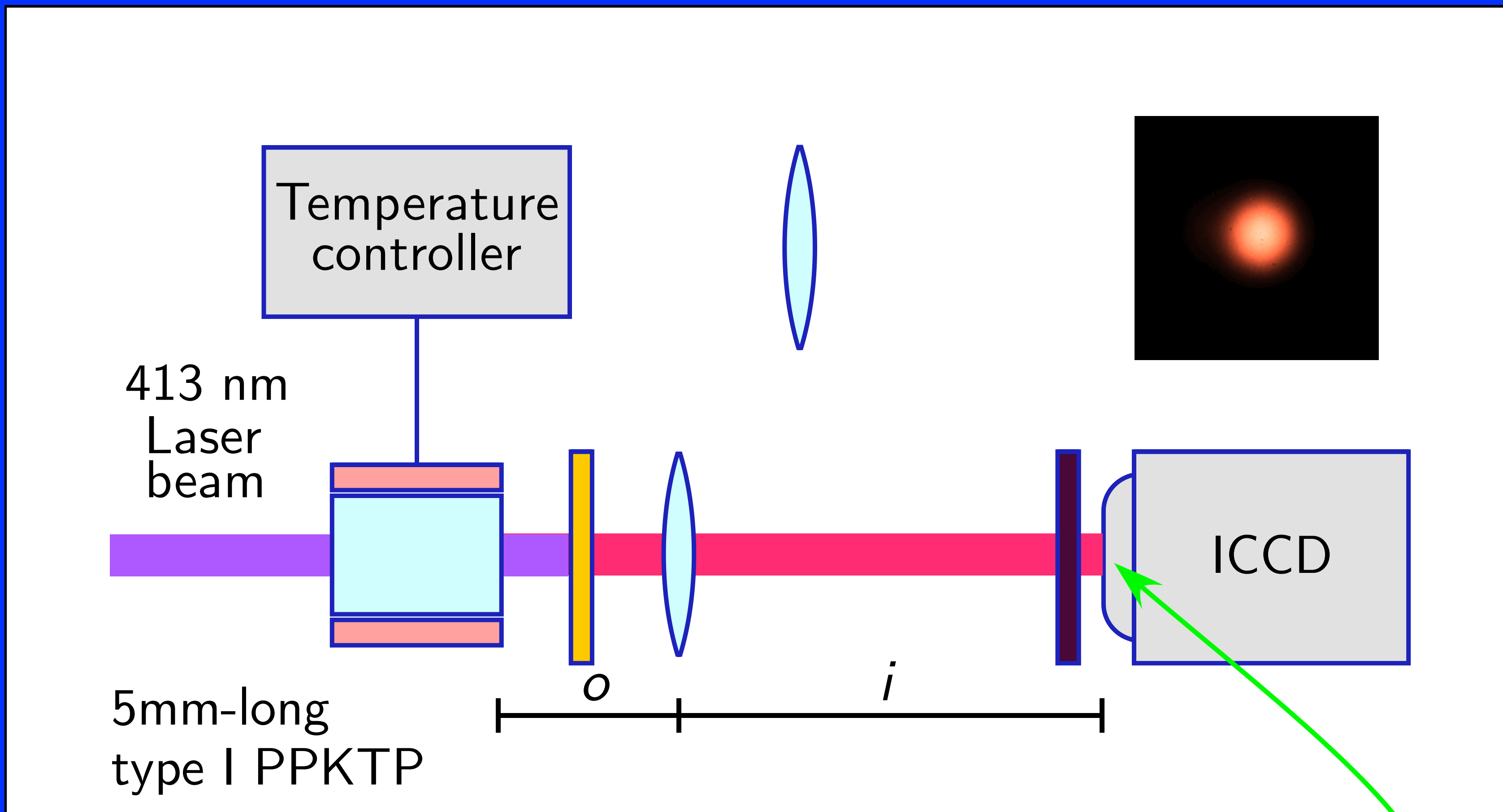
Experimental setup



L_1: f=49mm

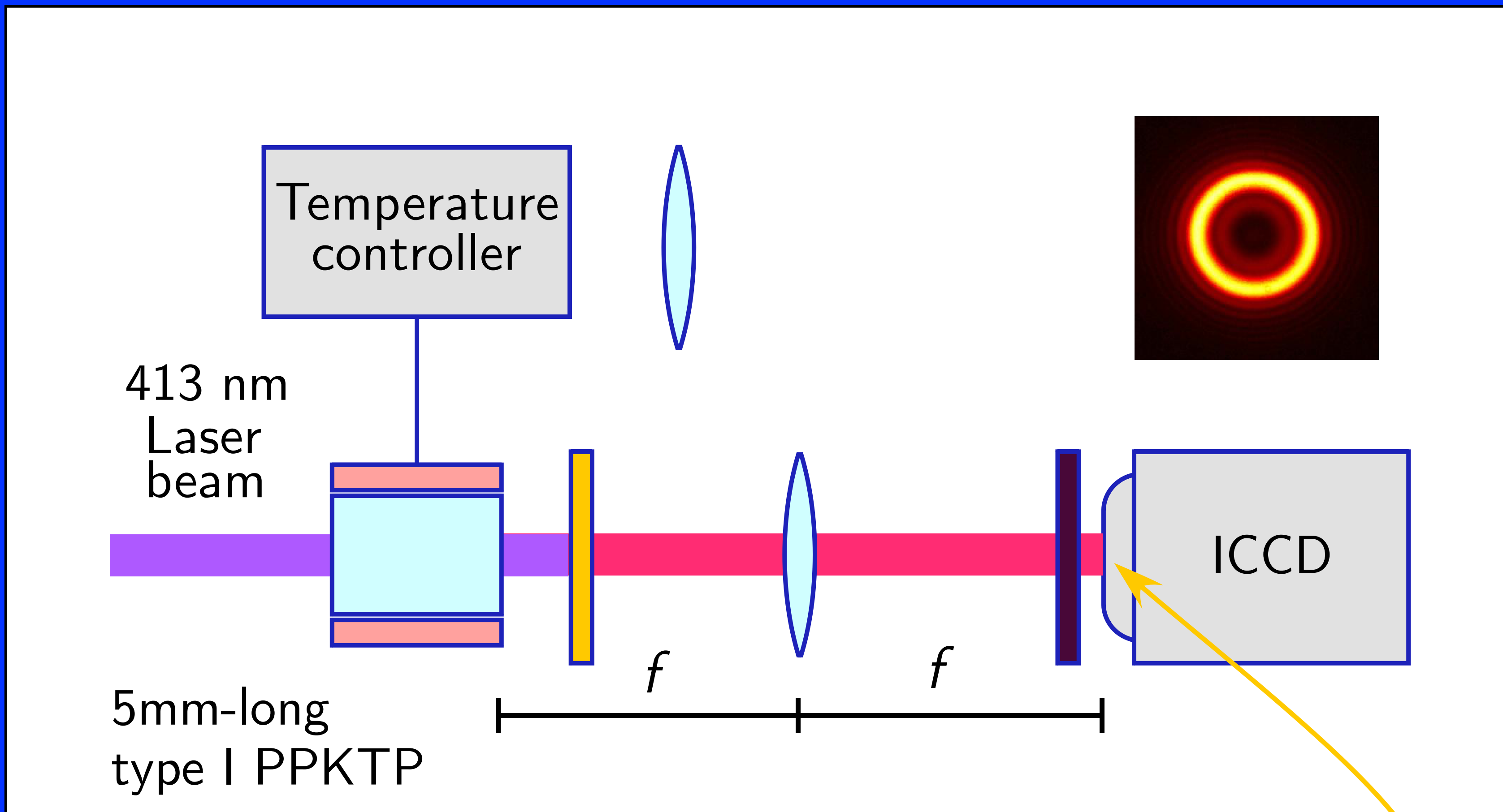
L_2: f=100mm

Experimental setup



12 x image of the source (near field)

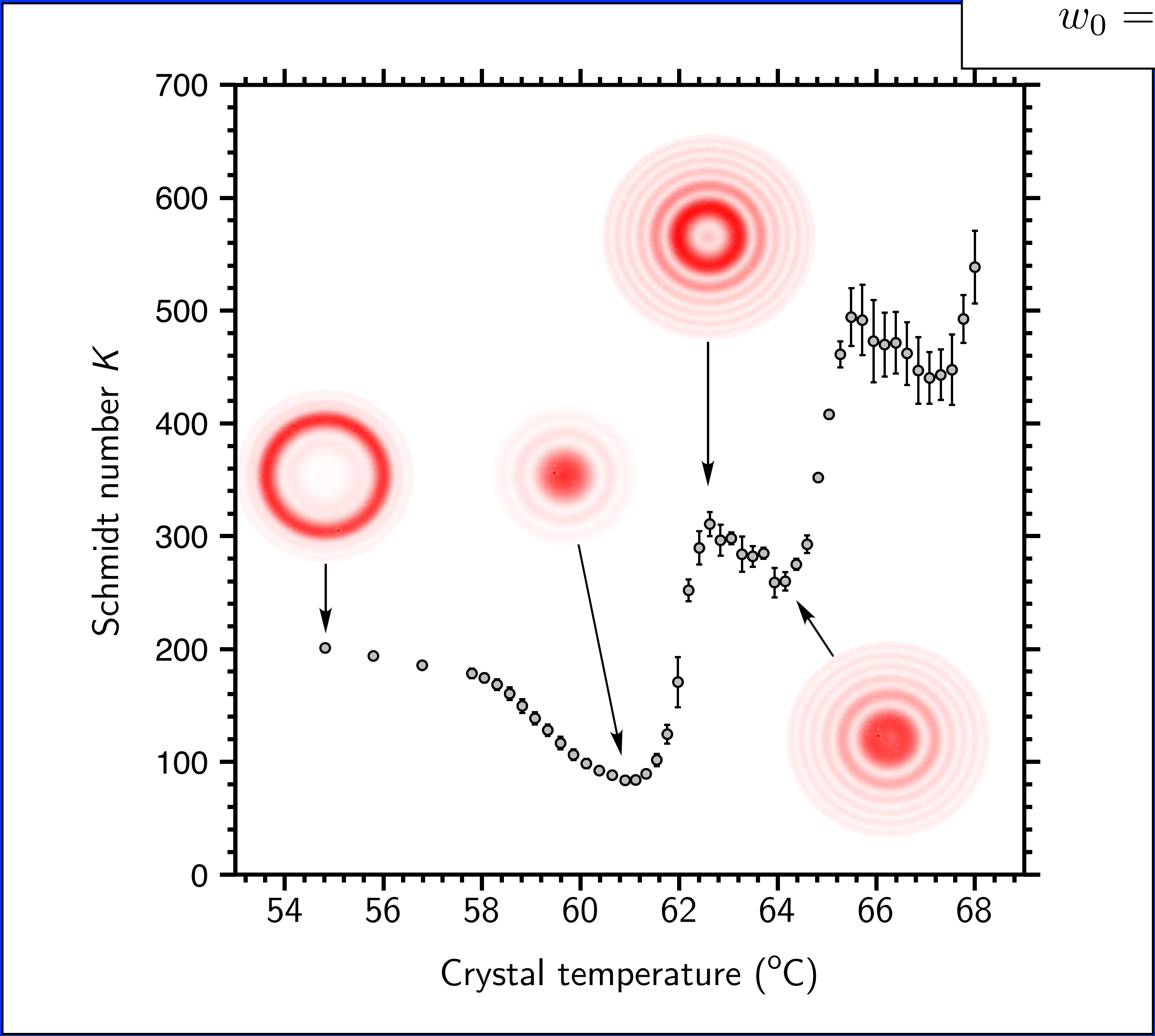
Experimental setup



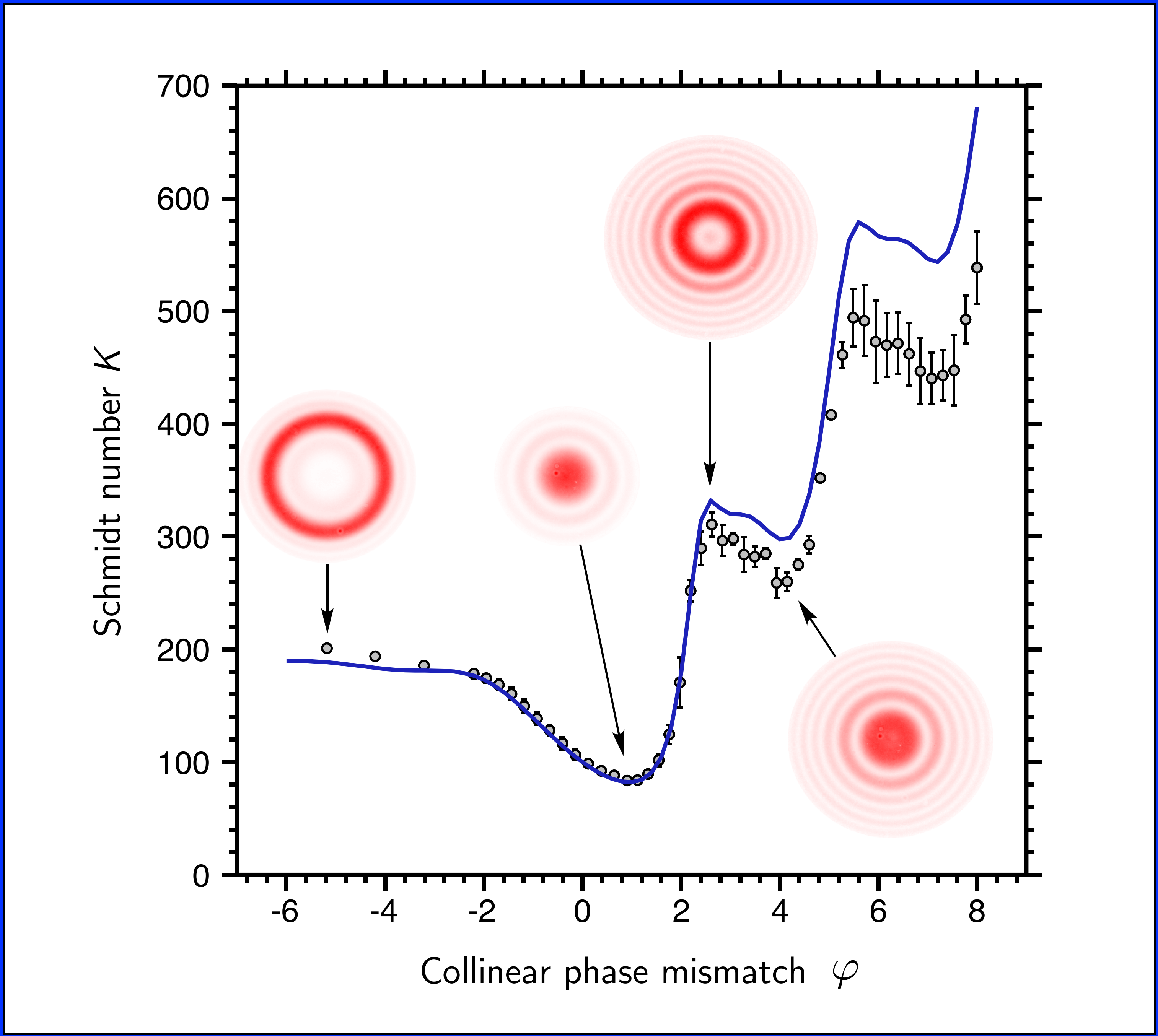
Fourier transform (far field)

Experimental results

$$L_{\text{cryst.}} = 5.06\text{mm}$$
$$w_0 = 161\mu\text{m}$$



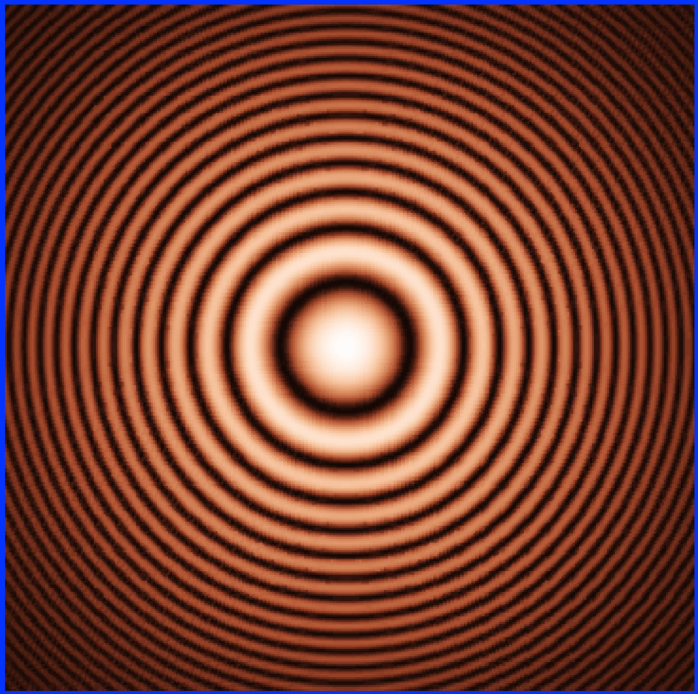
Experimental results + theoretical predictions



Theoretical predictions

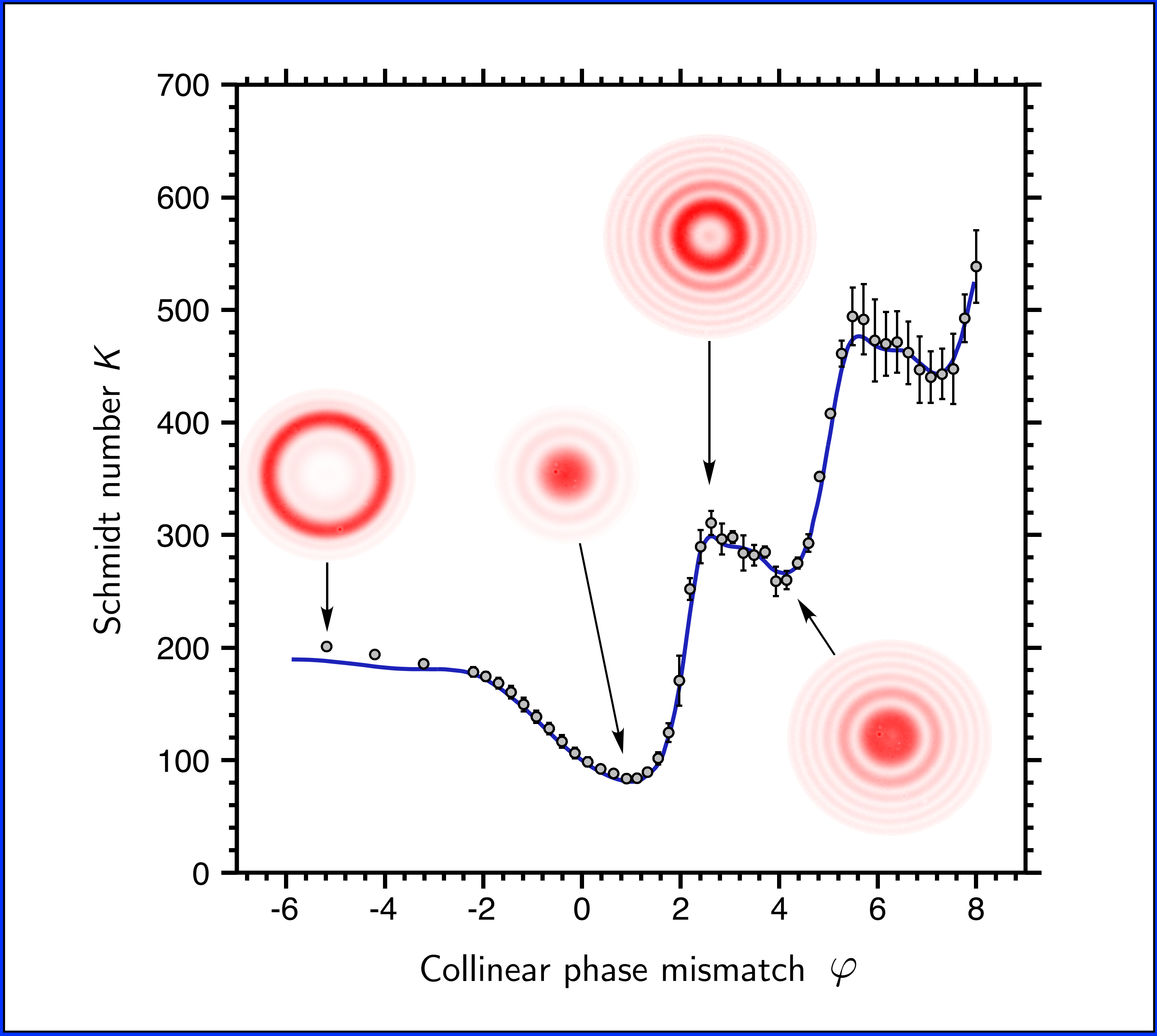
Correction for the finite detection aperture

$\varphi < 0$



$\varphi > 0$

Experimental results + theoretical predictions



Conclusions

The Schmidt number of two-photon states entangled in transverse modes is identical to the inverse of the overall degree of coherence of the reduced one-photon states.

If the reduced (one-photon) states describe quasi-homogeneous light sources, the overall degree of coherence can be easily measured, providing a direct measurement of the Schmidt number.

Phys. Rev. A 80, 022307 (2009)