

1. Calculate the refractive index and dispersion of the fundamental mode in a vacuum-filled hollow waveguide with perfectly reflecting walls. How does this expression change if a gas is added to the core?
2. Derive the dispersion relation for the Bloch waves in an array of parallel single-mode waveguides spaced Λ apart on a flat plane.
3. Derive the dispersion surfaces at fixed optical frequency for a graphene-like lattice of coupled single-mode guiding cores, assuming a nearest-neighbour coupling constant of κ .
4. A single-mode waveguide spirals around an axis at 2 turns per mm, keeping a constant distance of $6\ \mu\text{m}$ from the axis. By what factor will the group and phase velocities change compared to the case when the waveguide is on-axis?
5. A mode propagating in a photonic crystal fibre core has phase index, along the axis, of $n_m = 1.34$. What is the transverse effective wavelength at $\lambda = 1.55\ \mu\text{m}$ in (a) the glass ($n_g = 1.45$) and (b) the air?
6. A straight PCF has three cores in a ring around the axis. Calculate the guided eigenmodes of the structure, assuming nearest neighbour coupling. How do these relate to modes that carry orbital angular momentum?