Imperial College London

FIBRE BASED NONLINEARITY

J.R. Taylor

jr.taylor@ic.ac.uk

Femtosecond Optics Group, Imperial College London

www.femto.ph.ic.ac.uk

Outline

- Introduction
- Basic non linear processes
- Supercontinuum generation – development
- Controlling the nonlinear processes
- Improving the technology – fibres and lasers
- Extending the palette
- Limitations and Alternatives
- Summary , conclusions

Imperial College London

Brief Background History

- **1960 Invention of the laser**

Maiman, Brit. Comm. & Elect. Sept, 674

Nature 187, 493

- **1962 Q-switching of the laser**

McClung and Hellwarth, J. App. Phys. 33, 828

- **1965/1966 Mode locking of laser**

Mocker and Collins, App. Phys. Lett. 7, 270

DeMaria et al., App. Phys. Lett. 8, 22

Nanosecond-picosecond regime

1μJ,1ps, 5μm spot ~ $4x10^{12}Wcm^{-2}$ ~ 10^{6} Vcm^{-1}

The birth of nonlinear optics

Polarization

$$P = \varepsilon_0 \chi^{(1)} E$$

ε_0 is the permittivity of free space and χ is the susceptibility

$$n_0 = \sqrt{1 + \chi^{(1)}}$$

For large field strengths:-

$$P = \varepsilon_0 \left(\chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots.. \right)$$

In materials with a centre of symmetry $\chi^{(2)}$ is zero

$\chi^{(3)}$ dominates

$$n = n_0 + n_2 I$$

Nonlinear Optics before the laser

Kerr Effect 1875

Isotropic media

Birefringence induced by a DC field $\alpha\, E^2$

Described by third order nonlinear susceptibility

$$P \,\alpha\, \chi^{(3)} E^3$$

But only one field is the optical field

Linear Pockels Effect 1895

Piezoelectric crystal

Birefringence induced by external DC field

Linear function of E_{DC} $\quad Pol \,\alpha\, E_{Opt} E_{DC}$

For a field at ω_M $\quad \omega-\omega_M, \omega+\omega_M$

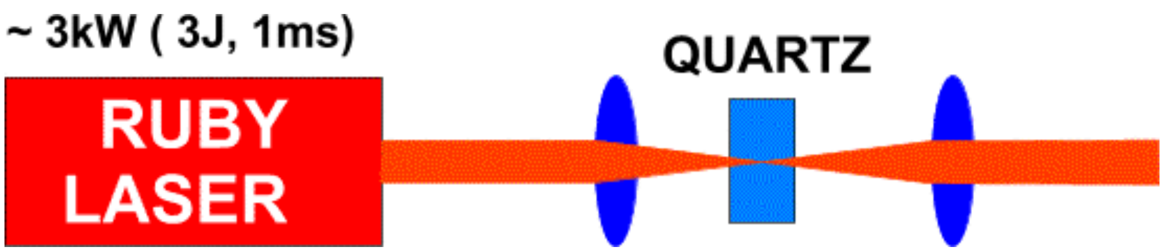
First reported nonlinear optical process

Second harmonic generation

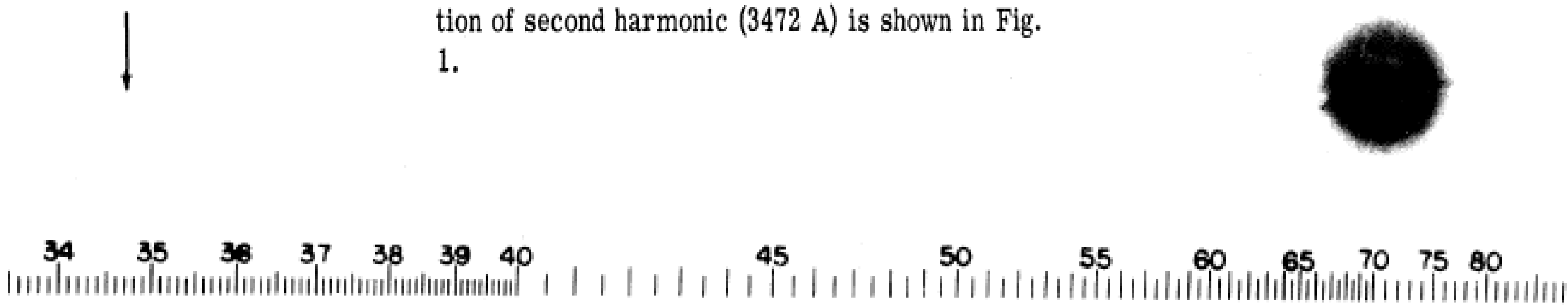
VOLUME 7, NUMBER 4 PHYSICAL REVIEW LETTERS AUGUST 15, 1961

GENERATION OF OPTICAL HARMONICS*

P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich
The Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan
(Received July 21, 1961)



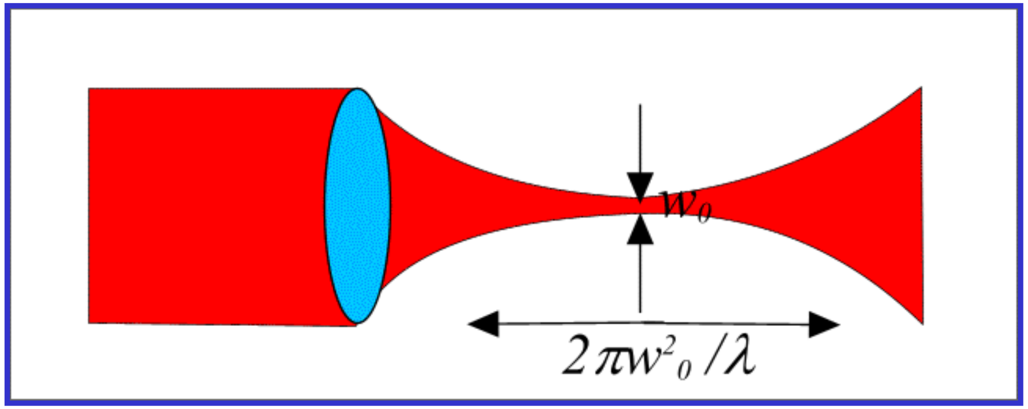
A reproduction of the first plate in which there was an unambiguous indication of second harmonic (3472 A) is shown in Fig. 1.



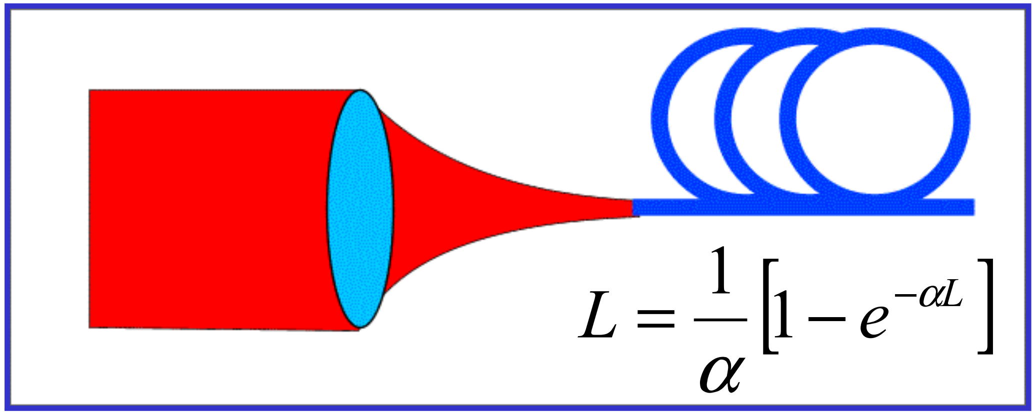
The advantage of fibre

Nonlinearity – Power X Length process

BULK



FIBRE



Enhancement 10^6 - 10^7

What about dispersion ?

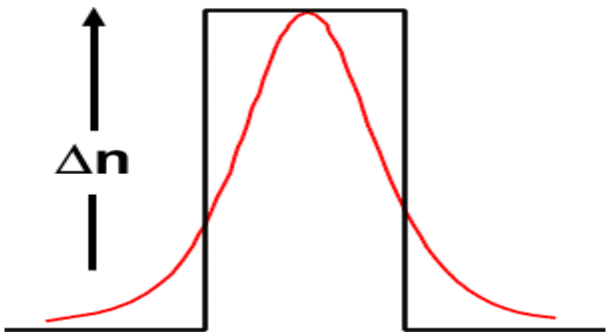
Frequency dependence of phase velocity gives rise to dispersion

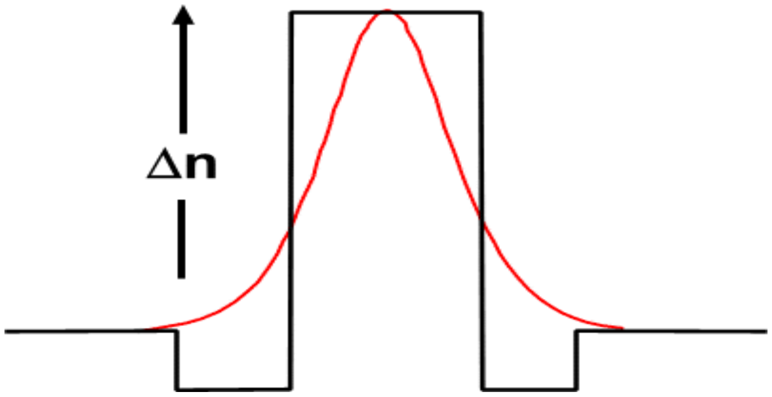
In fibre dispersion is sum of:-

Material Dispersion

Waveguide dispersion

Modal dispersion – ignored here –single mode only





Dispersion

$$k(\omega) = k_0 + \frac{\partial k}{\partial \omega}(\omega - \omega_0) + \frac{1}{2}\frac{\partial^2 k}{\partial \omega^2}(\omega - \omega_0)^2 + \dots.$$

$$= k_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \dots.$$

First order term results in overall delay on pulse without affecting pulse shape $\beta_1 = 1/v_g$

Second order term, results in intra-pulse dispersion or group velocity dispersion

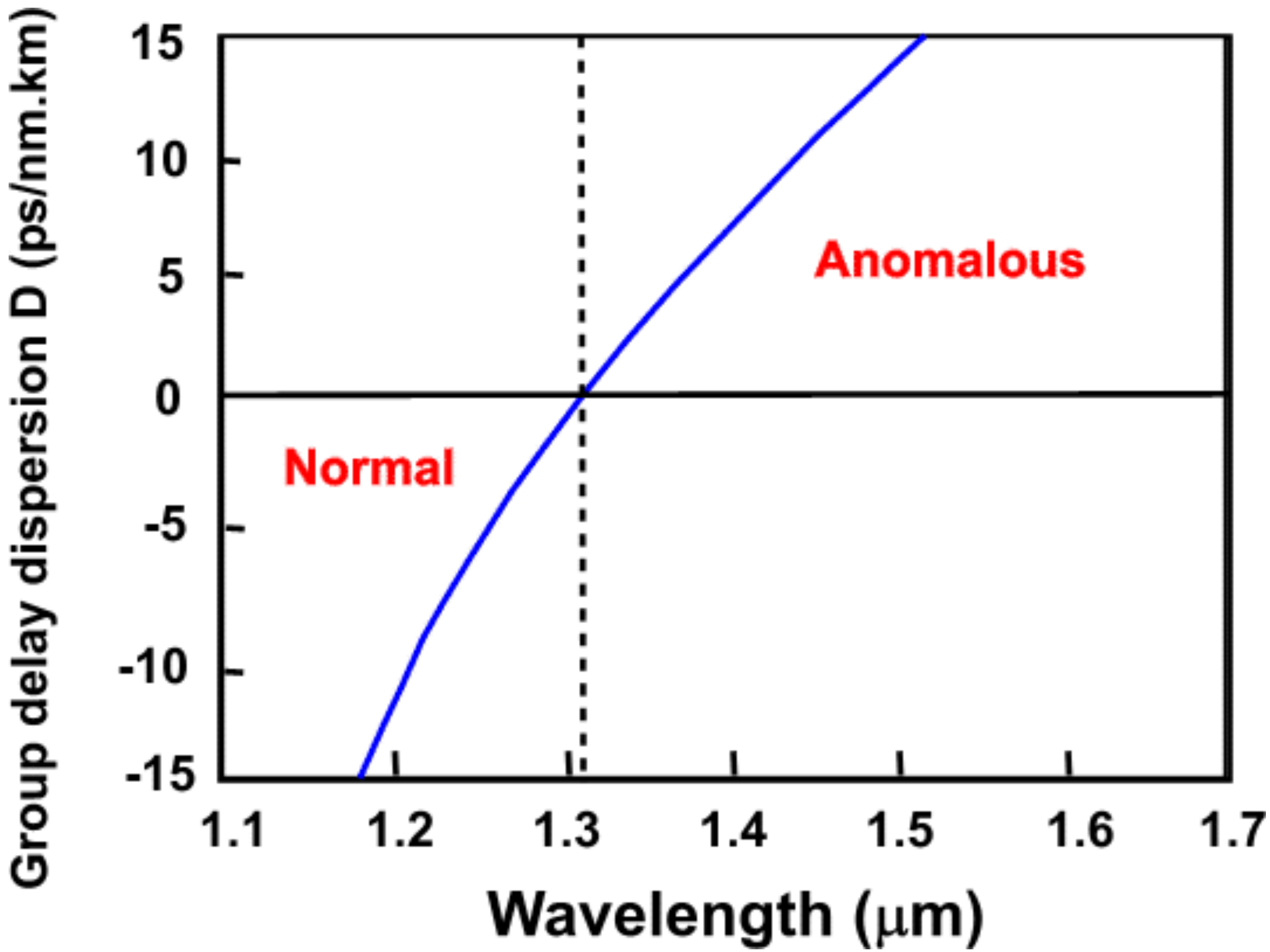
$$D = -\frac{\lambda}{c}\frac{d^2 n}{d\lambda^2} = -\frac{2\pi c}{\lambda^2}\beta_2$$

Units of D ps. nm^{-1}. km^{-1},

$\beta_2 > 0$ (D<0)	**Normal dispersion**
$\beta_2 < 0$ (D>0)	**Anomalous dispersion**

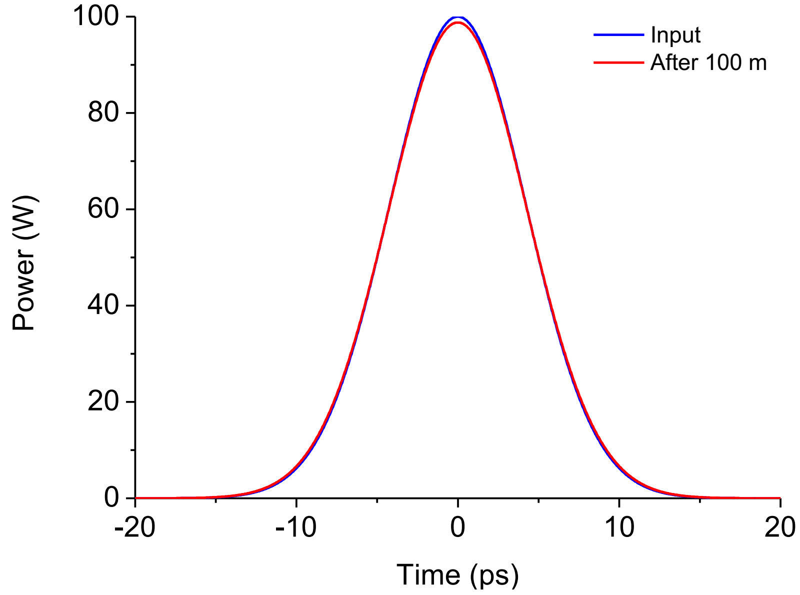
Imperial College London

Dispersion of a conventional silica fibre



For silica based fibres with conventional structures, Minimum achievable zero dispersion is 1.27 μm

Dispersion of picosecond pulses



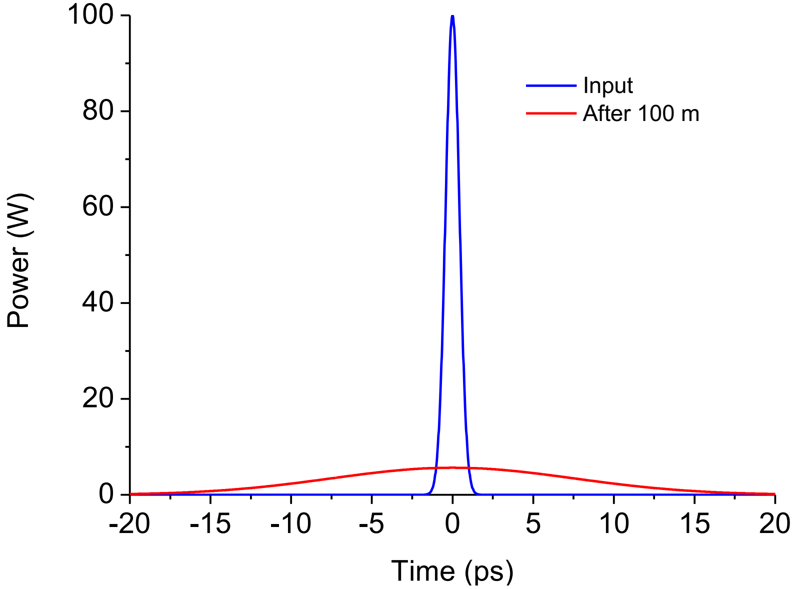
Input : 100 ps transform limited

$\Delta\lambda$ = 0.0165nm

Fibre length 100m

Dispersion 50 ps/(nm.km)

Broadening ~ 0.0825 ps



Input : 1 ps transform limited

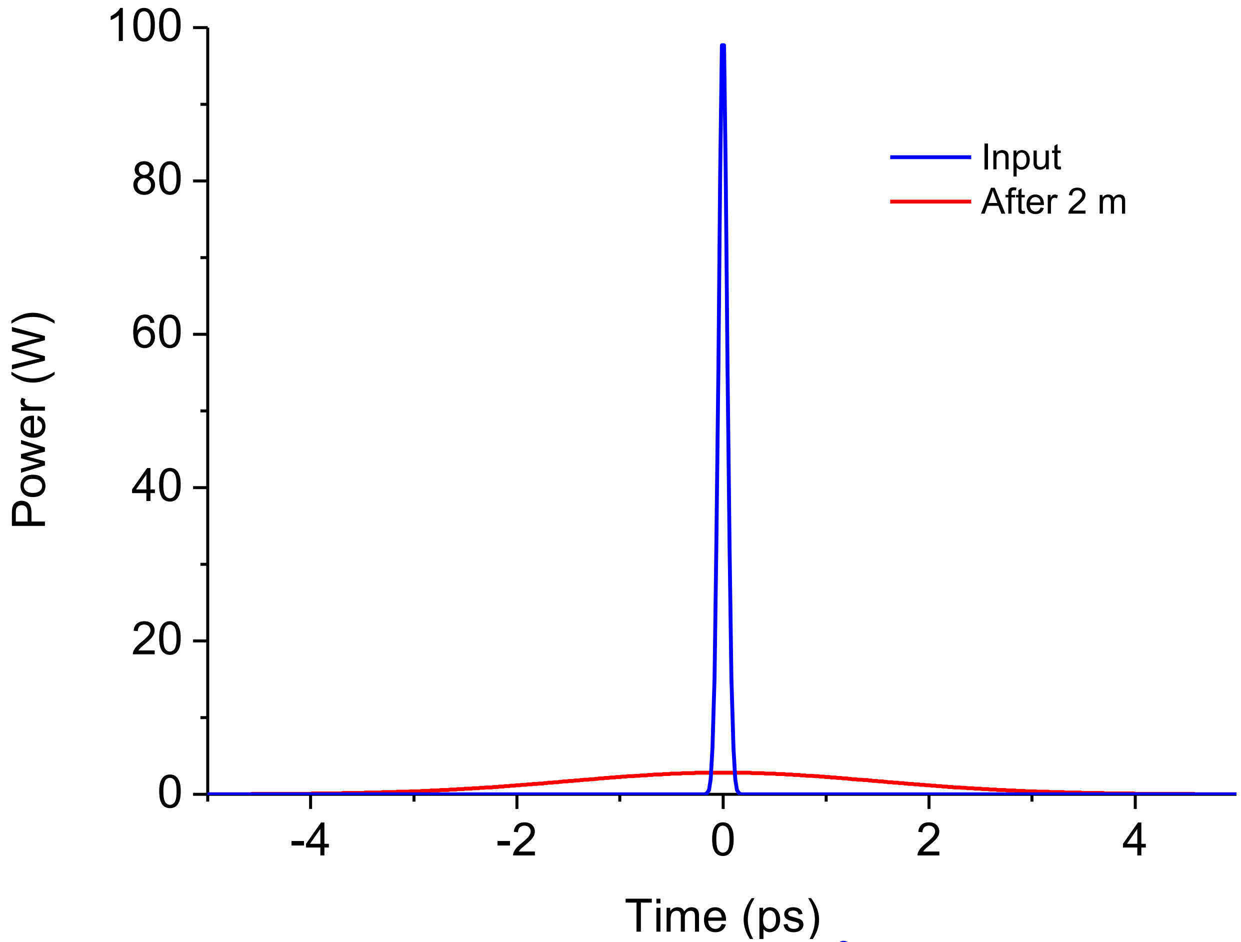
$\Delta\lambda$ = 1.65nm

Fibre length 100m

Dispersion 50 ps/(nm.km)

Broadening ~ 8.25 ps

Dispersion of femtosecond pulses



Input : 100 fs transform limited pulse ($\Delta\lambda$= 10.65nm) at 1060nm

Output : After 2 m at 50ps/(nm.km) broadening is 1.06 ps

Nonlinearity – Self phase modulation

Shimizu 1967 – Phys. Rev. Lett. 19, 1097

Arising from the intensity dependent refractive index

$$n = n_0 + n_2 I$$

For silica n_2= 3.2 x 10^{-20} m^2/W

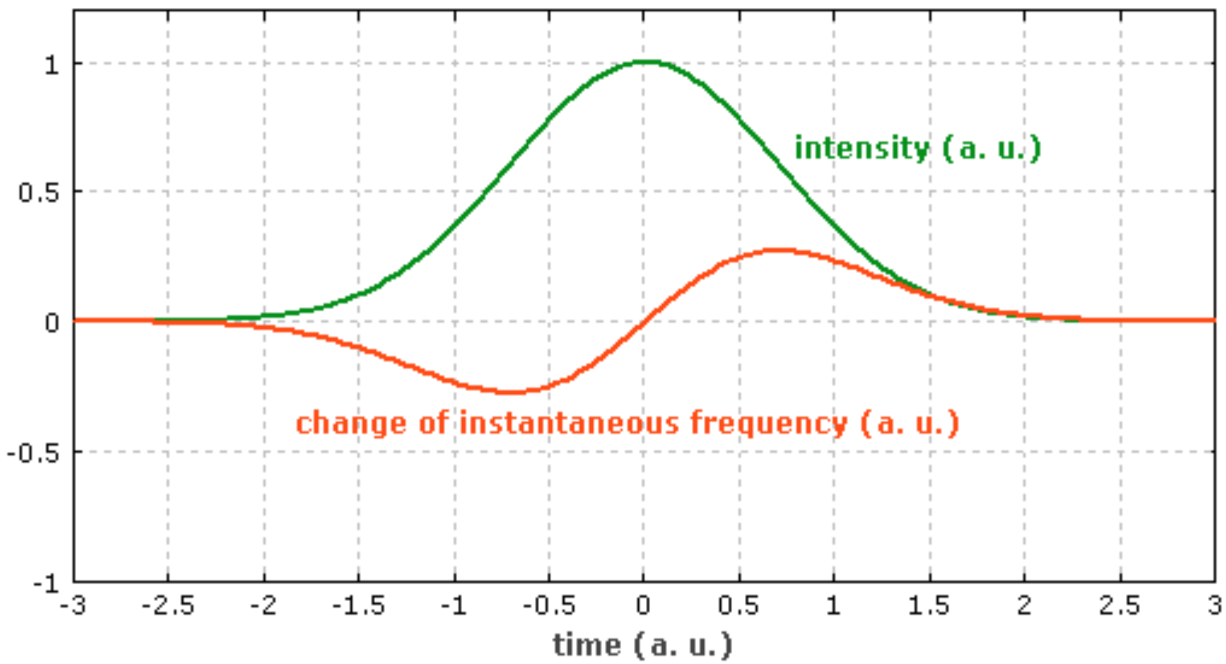
The phase change over a fibre length L is:-

$$\Delta\phi(t) = dn.k.L = n_2 I(t) k.L$$

$$\Delta\omega(t) = -\frac{d(\Delta\varphi(t))}{dt} = -n_2 kL \frac{dI(t)}{dt}$$

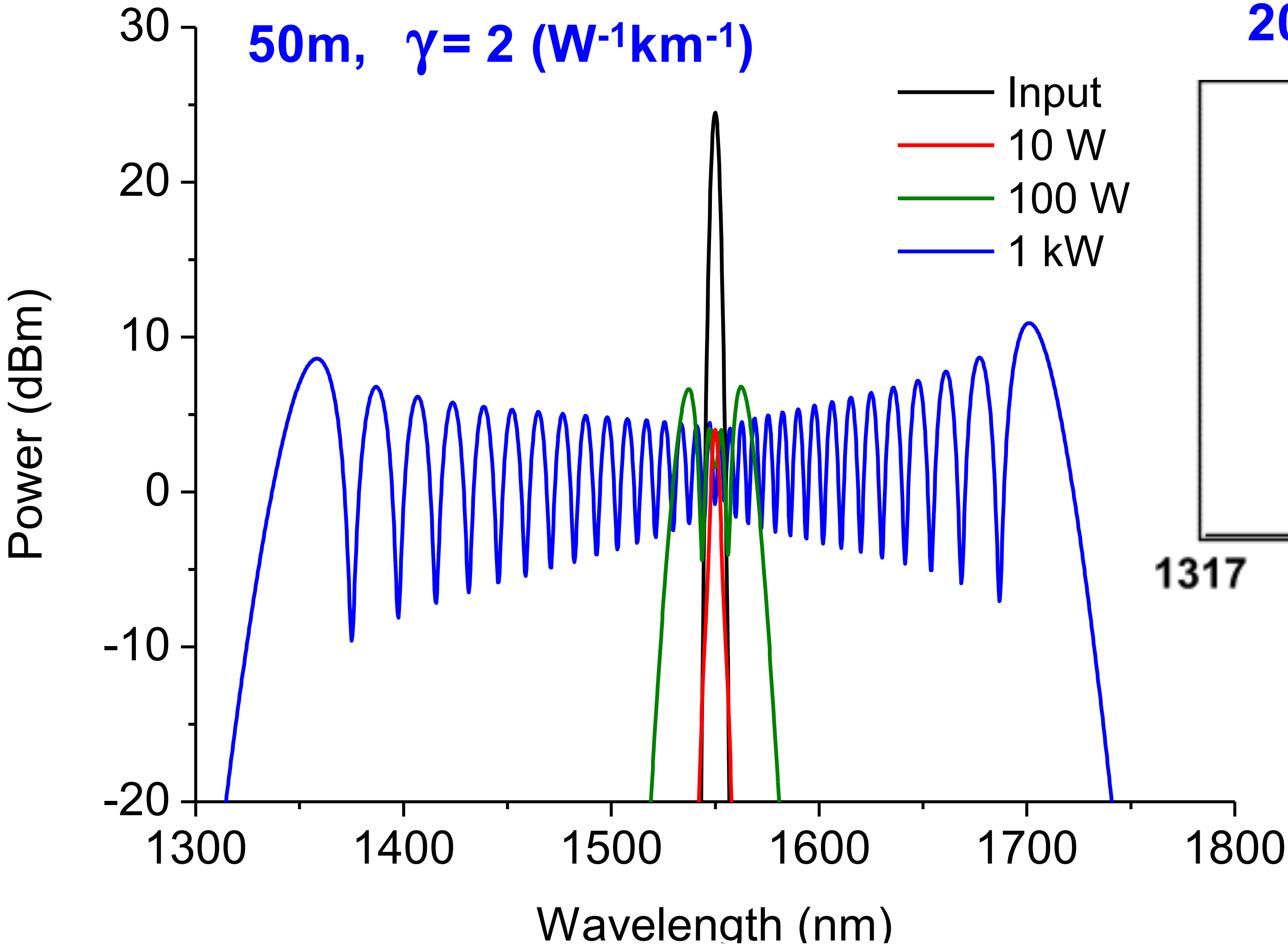
Self phase modulation

Assumes an instantaneous response of the medium

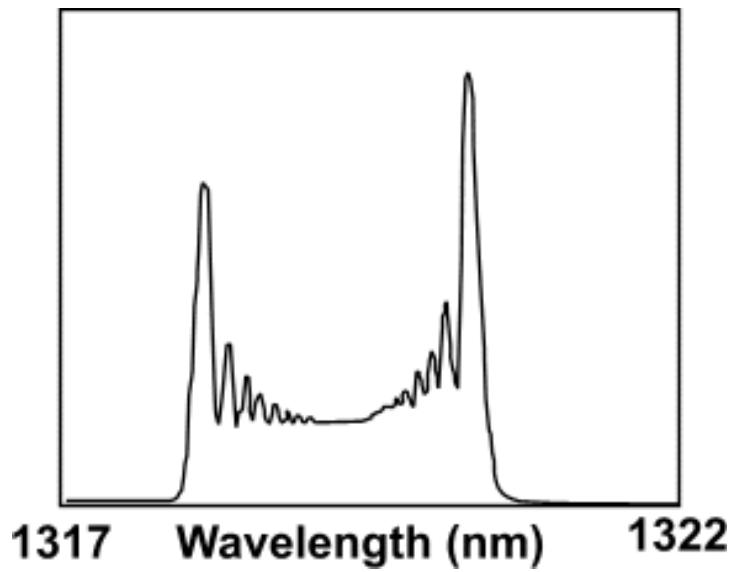


Observation in fibre:_
Ippen et al. (1974) App. Phys. Lett. 24, 190
Stolen and Lin (1978) Phys. Rev. A 17, 1448

SPM Theory and Experiment

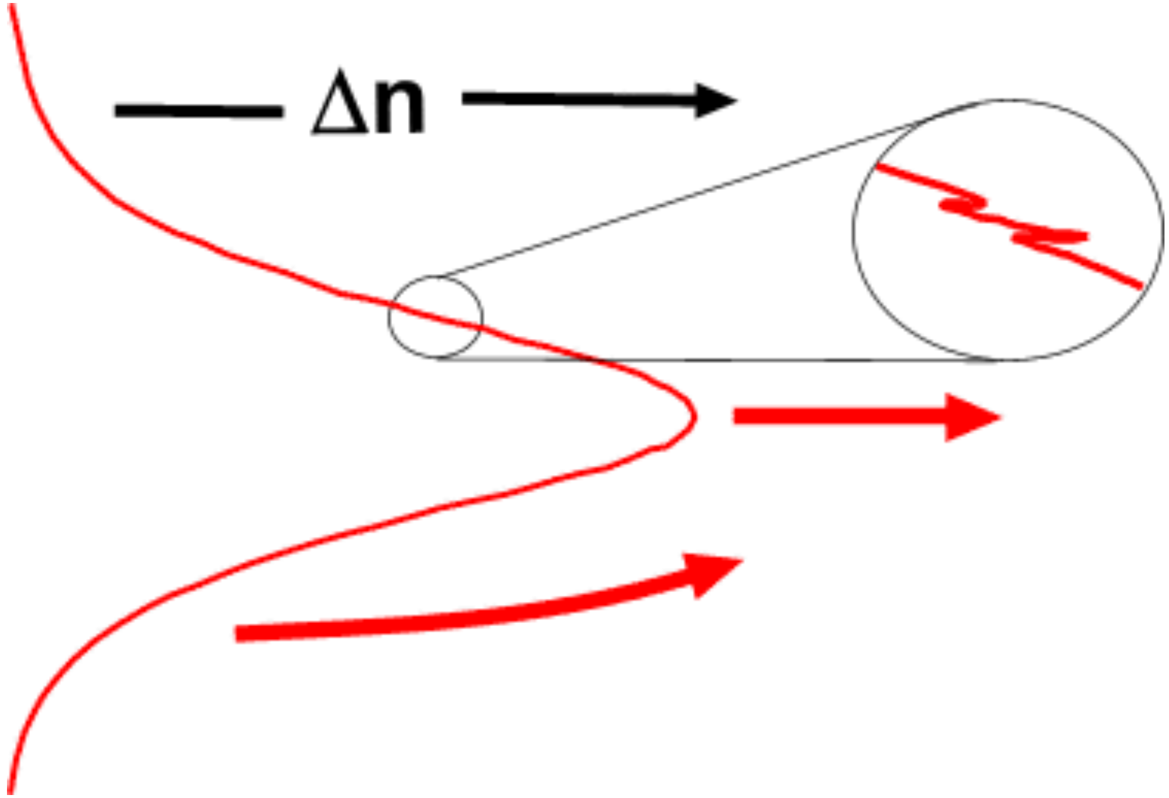


200m, 90ps, 50W



Self focussing and filamentation

In early bulk based systems



Nonlinearity – Four wave mixing

Hill et al. 1978 J. App. Phys. 49, 5098

Most general case of four co-propagating waves at ω_1, ω_2, ω_3, ω_4
From the conservation of energy

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

From the conservation of momentum

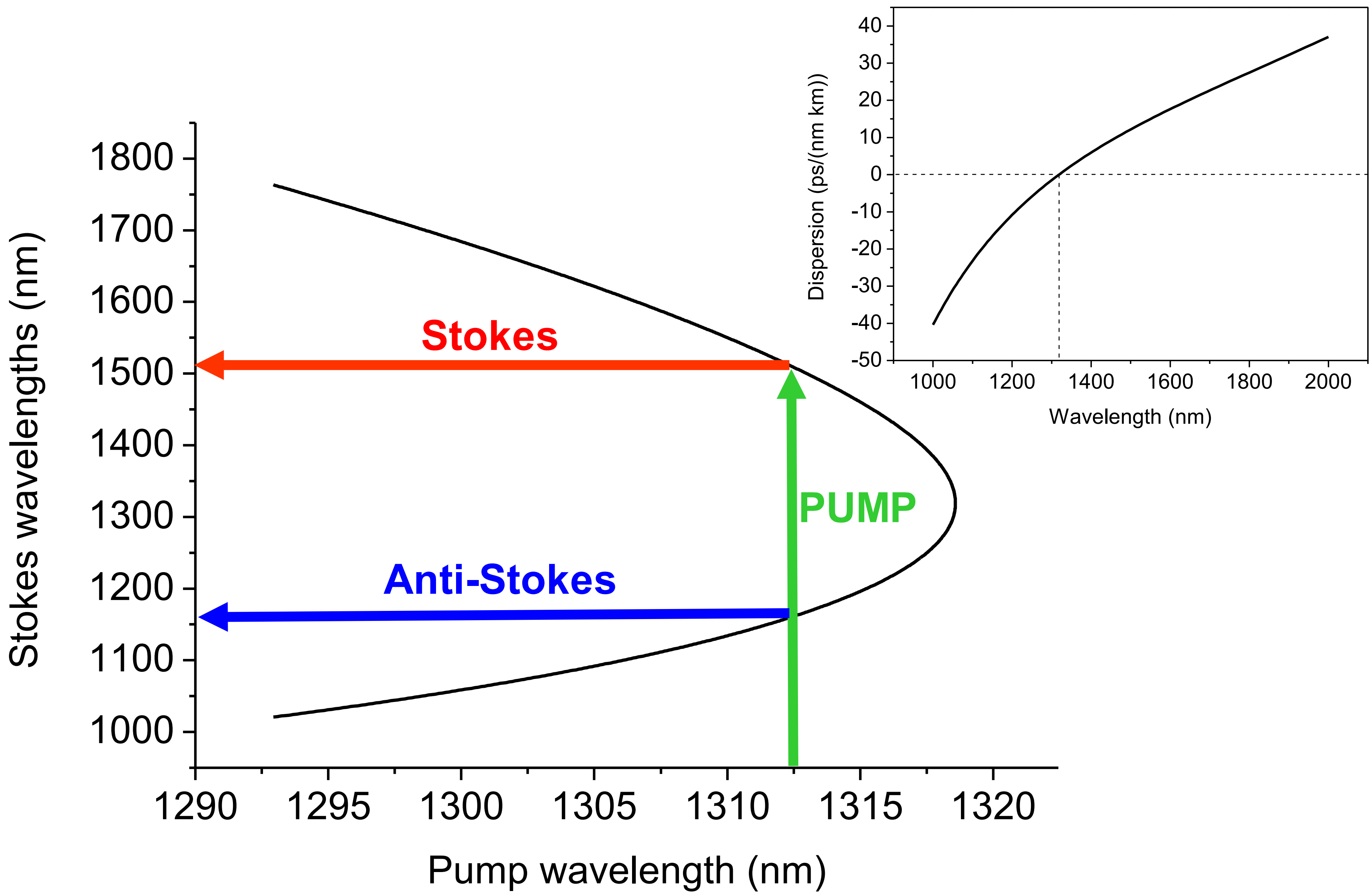
$$k(\omega_1) + k(\omega_2) - k(\omega_3) - k(\omega_4) = \Delta k$$

$$\Delta k = \Delta k_M + \Delta k_W$$

Most generally $\omega_3 = \omega_4 = \omega_P$
In single mode fibres normal dispersion precludes phase matching

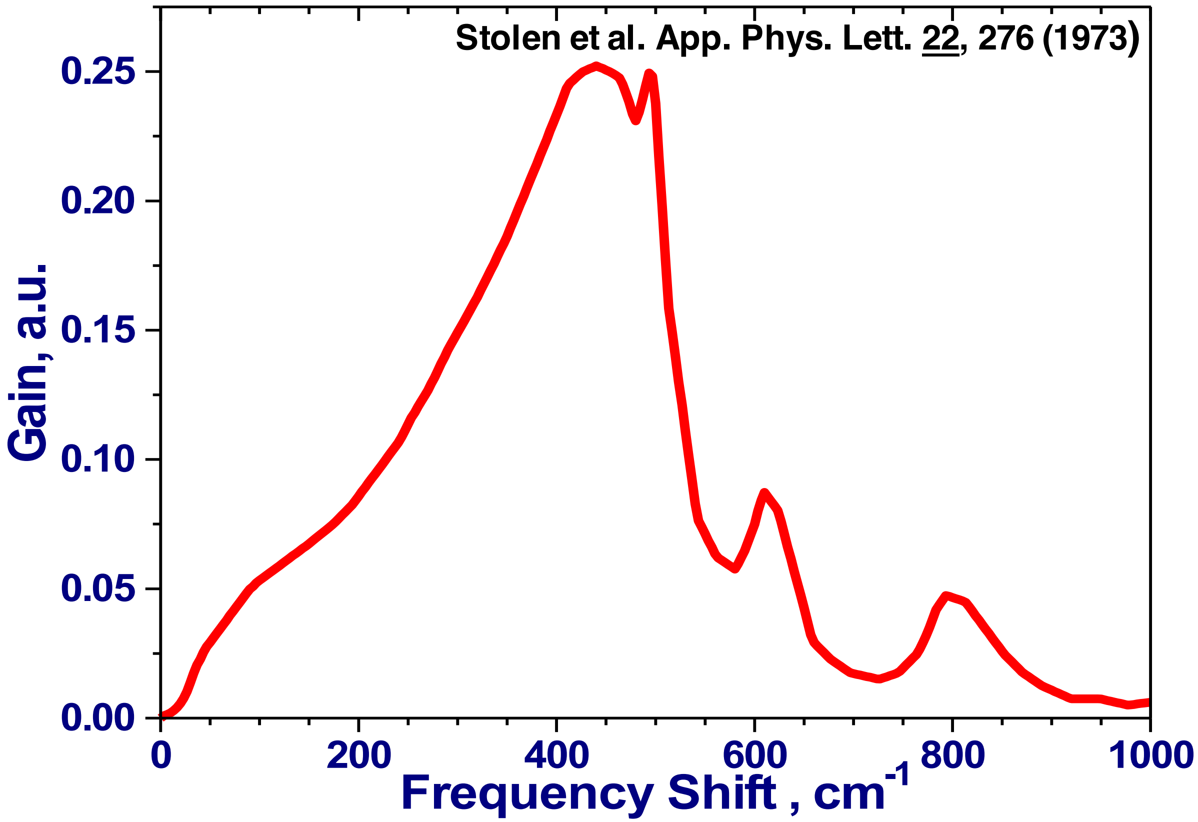
- Use multi-mode operation
- Operate around the zero dispersion
- Remember intensity dependent refractive index

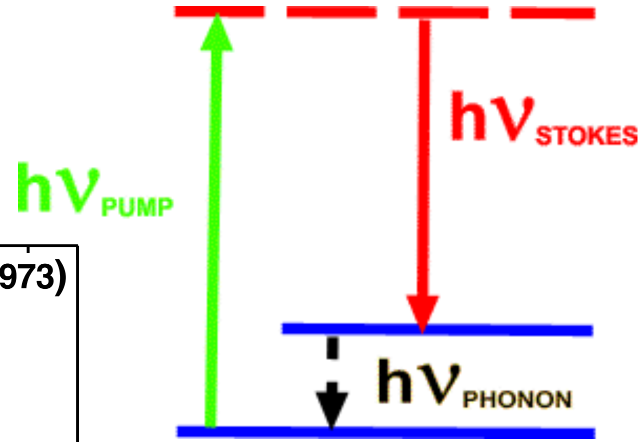
Nonlinearity – Four wave mixing



Nonlinearity – Stimulated Raman Scattering

Woodbury & Ng 1970, Proc IRE 50, 2367
Ippen 1970, App. Phys. Lett. 16, 303
Stolen et al. 1972, App. Phys. Lett. 20, 62





Stimulated Raman Scattering

Present in all fibres, non elastic process, energy lost

Coupling via optical phonons

Ultrafast response (~ few femtoseconds)

Gain can be at any wavelength – pump dependent

In silica, peak gain ~ 10^{-13} m/W

Gain can be cascaded, generated Raman component acts as pump

In silica maximum of gain shifted by ~ 13THz (440 cm^{-1})

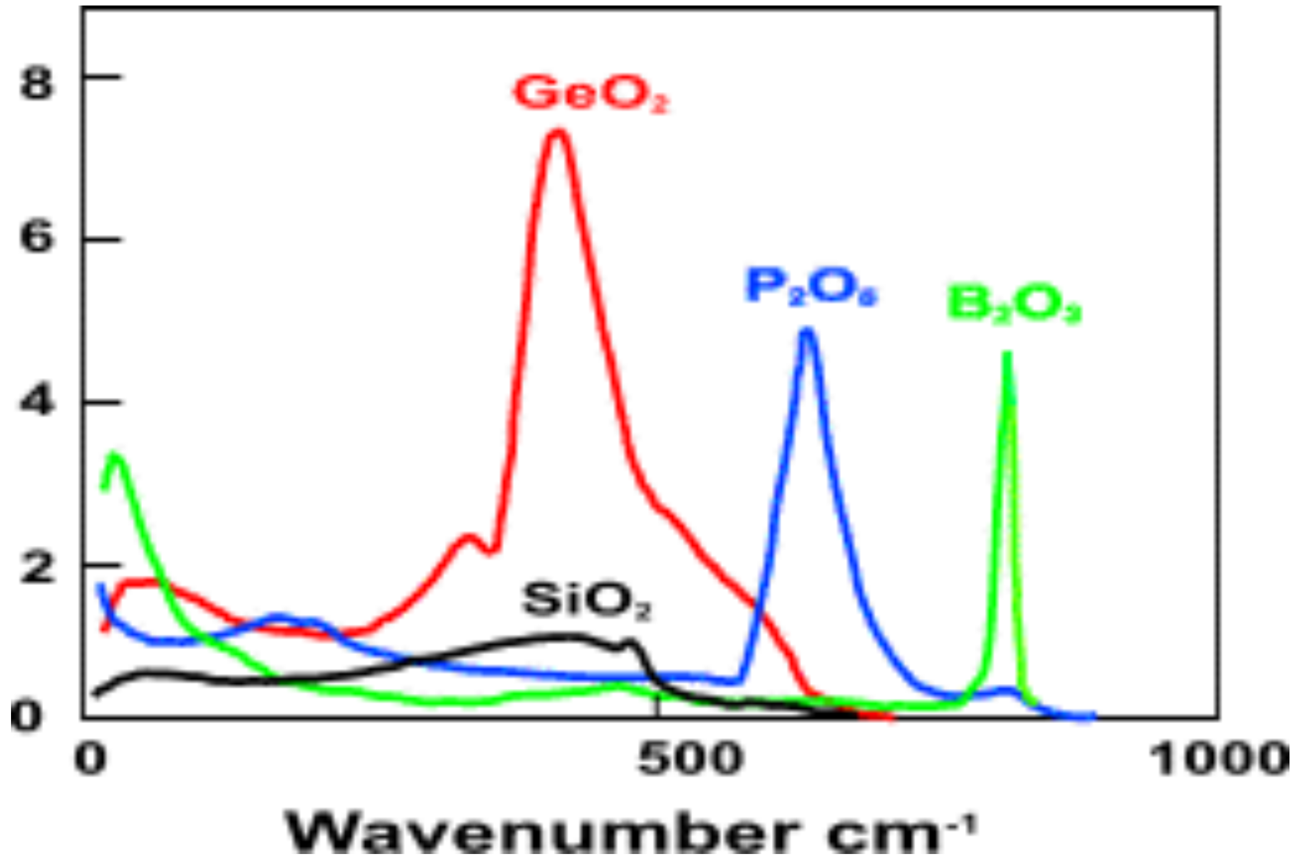
Broad gain bandwidth in silica > 40 THz

Gain is polarization dependent

Gain is material dependent

Stimulated Raman Scattering

Material dependence



$$P_{cr} \cong 16 \frac{A}{g_R L_{eff}}$$

Stimulated Raman Scattering Gain

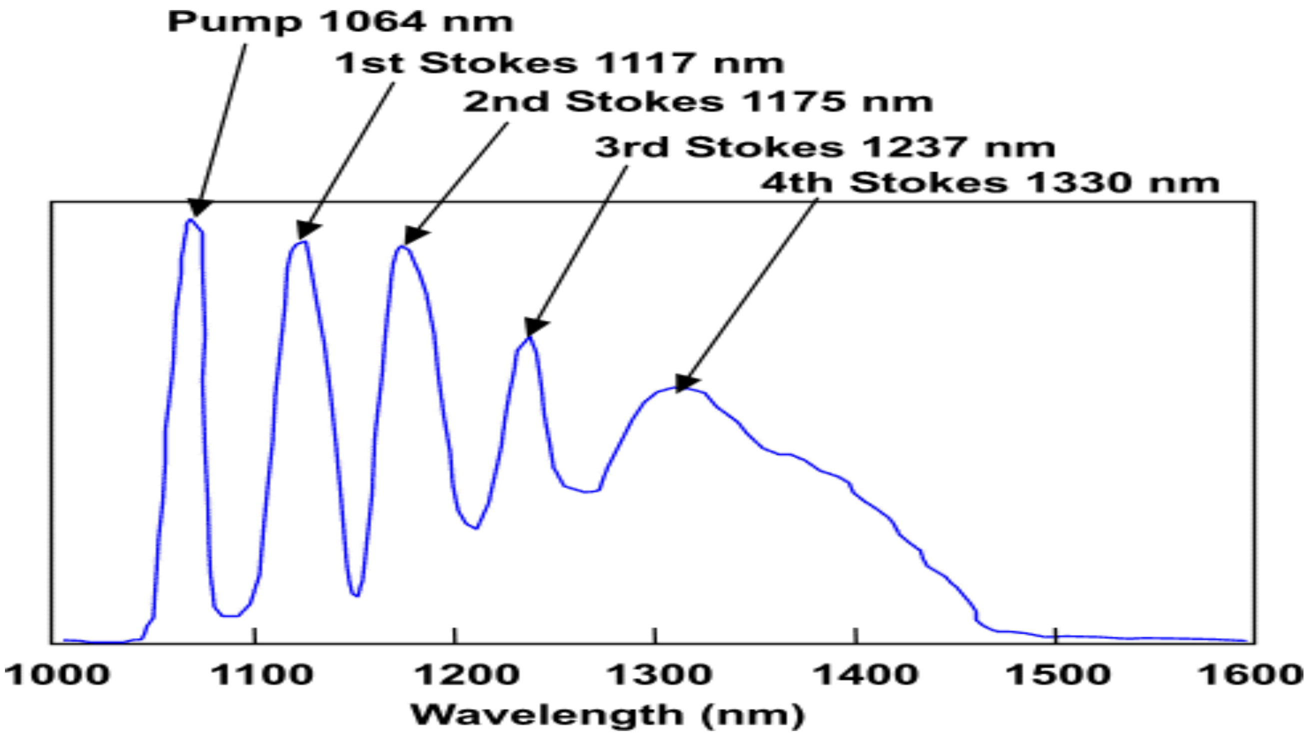
$$G(\lambda) = \exp\left[\frac{g(\lambda)}{A_{eff}(\lambda)} P_P L_{eff} - \alpha_S L\right]$$

$$L_{eff} = \frac{1}{\alpha_P}\left[1 - e^{-\alpha_P L}\right]$$

Pump at 1420 nm, loss 0.25dB/km over 20km
L_{eff} ~ 12.1 km,
For 20 dB gain ($g(\lambda)/A_{eff}$ ~1.5 $W^{-1}km^{-1}$ for DSF)
REQUIRE P ~305 mW (x2 if depolarized pump)

Imperial College London

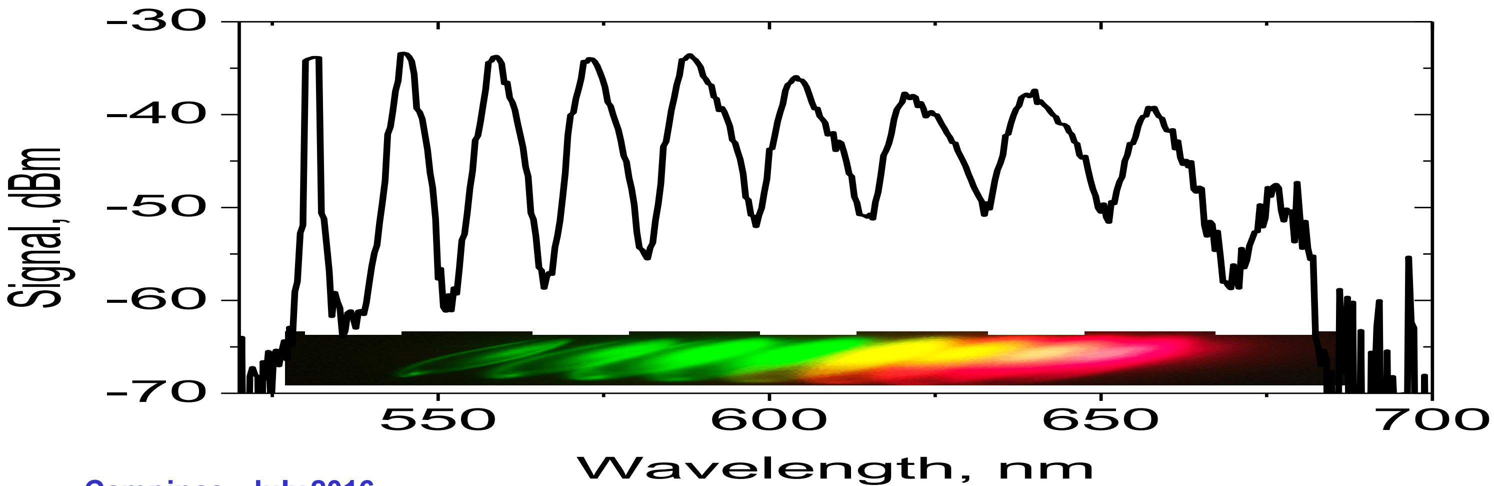
Stimulated Raman Scattering



1km STF

P_{av}~ 1.8W

P_{peak}~ 100W

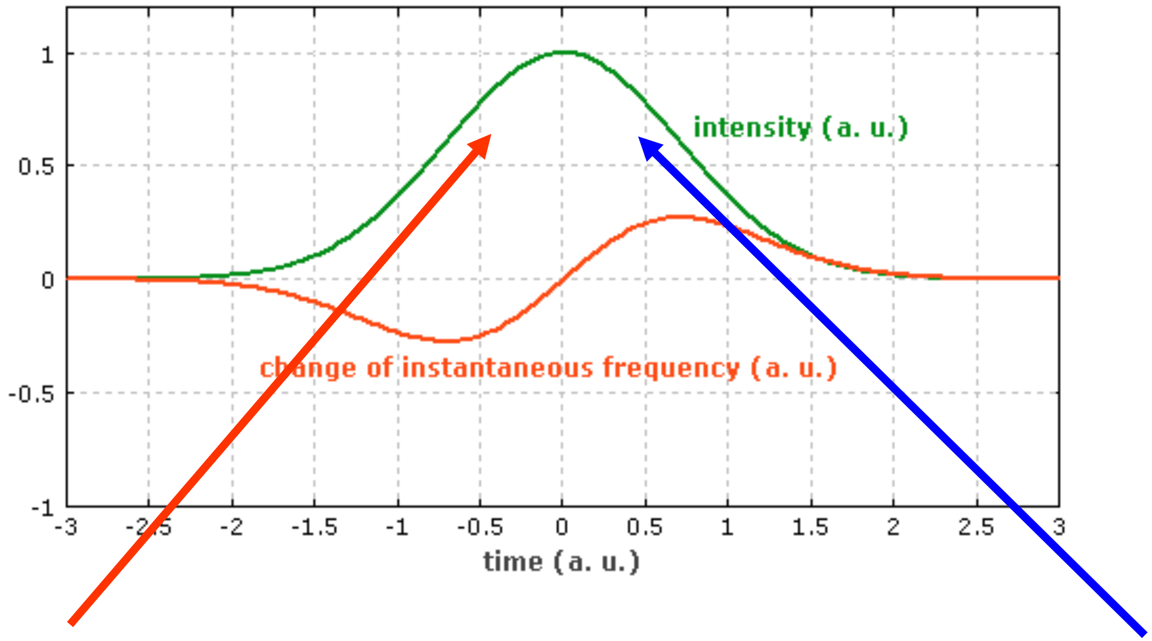


Pump 530 nm

1^{st} S 542 nm

Imperial College London

Effect of dispersion and nonlinearity



Front of pulse – downshift - redder

Back of pulse – downshift - bluer

In normal dispersion

pulse will broaden, intensity reduced, nonlinearity reduced

In anomalous dispersion

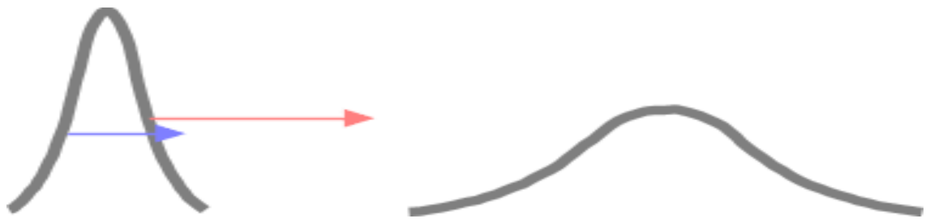
pulse will compress, intensity increased, nonlinearity increased

Imperial College London

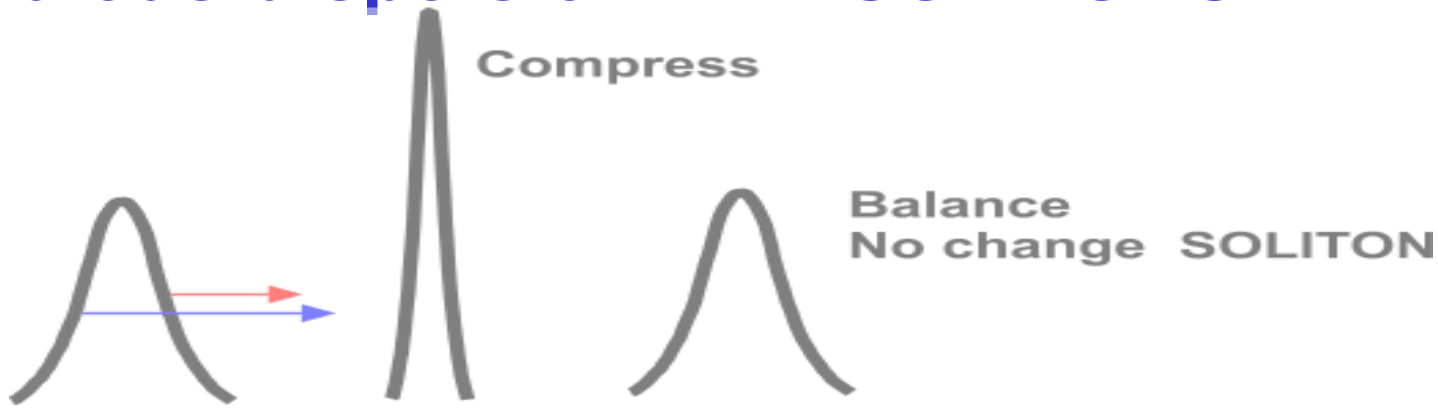
Optical Solitons

Balance between dispersion and nonlinearity (SPM)

SPM + normal dispersion BROADENING



SPM + anomalous dispersion SOLITONS



In conventional silica fibre $\lambda > 1270$ nm

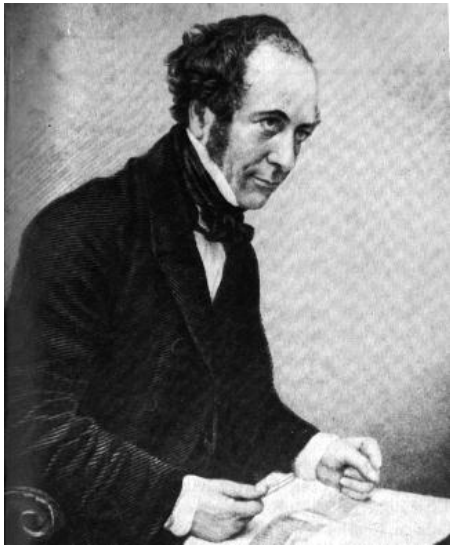
Soliton Power $P \propto D/\tau^2$ or $\tau \propto D/E$

Imperial College London

Optical solitons

Hasegawa and Tappert 1973, App. Phys. Lett. 23, 142
Mollenauer et al. 1980, Phys. Rev. Lett. 45, 1095

Solitons in water waves :
Scott Russell 1844, 14th British Assoc Advancement of Science, York

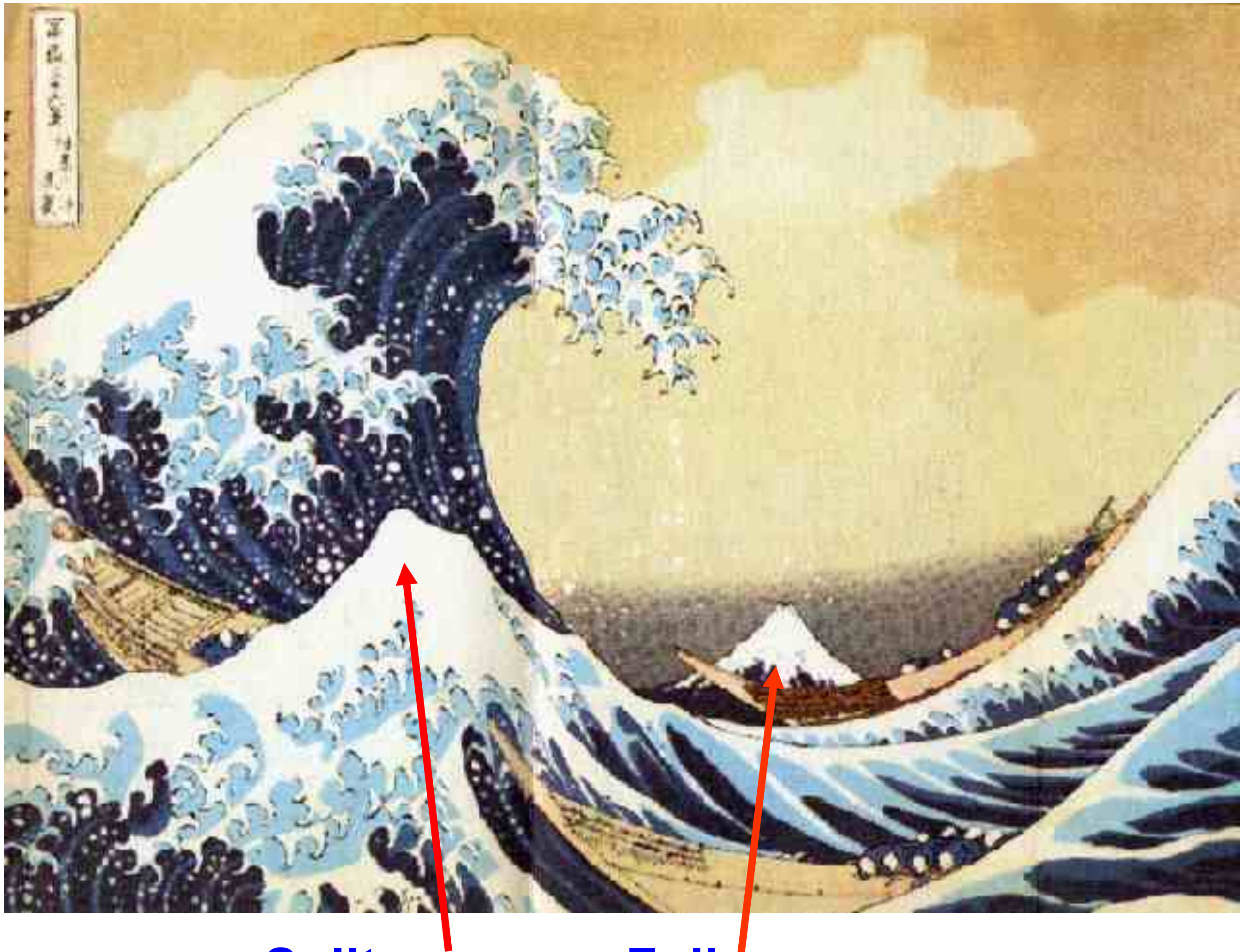


``I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation''

Before that – First experimental recording?

Solitons

Hokusai (1760-1849) " The Great Wave"
"The Breaking Wave off Kanagawa" – 36 views of Fuji



Solitons in optical fibre - Theory

Hasegawa & Tappert , App Phys Lett 23, 142 (1973)

Balance of dispersion (anomalous) and nonlinearity (SPM)

$$-i\frac{\partial U}{\partial z} = \frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} + |U|^2 U$$

"If the absorption is small and the nonlinear term can be made comparable to the dispersion term…."

$$U(z,t) = N \sec h(t) \exp(iz/2)$$

$$N = \sqrt{\frac{L_D}{L_{NL}}} = \sqrt{\frac{\gamma P \tau^2}{\beta_2}}$$

Fundamental optical soliton - Hasgawa

$$P = \left(\frac{1.763}{2\pi}\right)^2 \frac{A_{eff}\lambda^3}{n_2 c} \frac{D}{\tau^2}$$

At 1550 nm, D= 17ps/nm/km, τ = 1ps, 100 MHz, STF
Soliton period ~20 m   Soliton power ~ 30W   ie ~ 3 mW average power

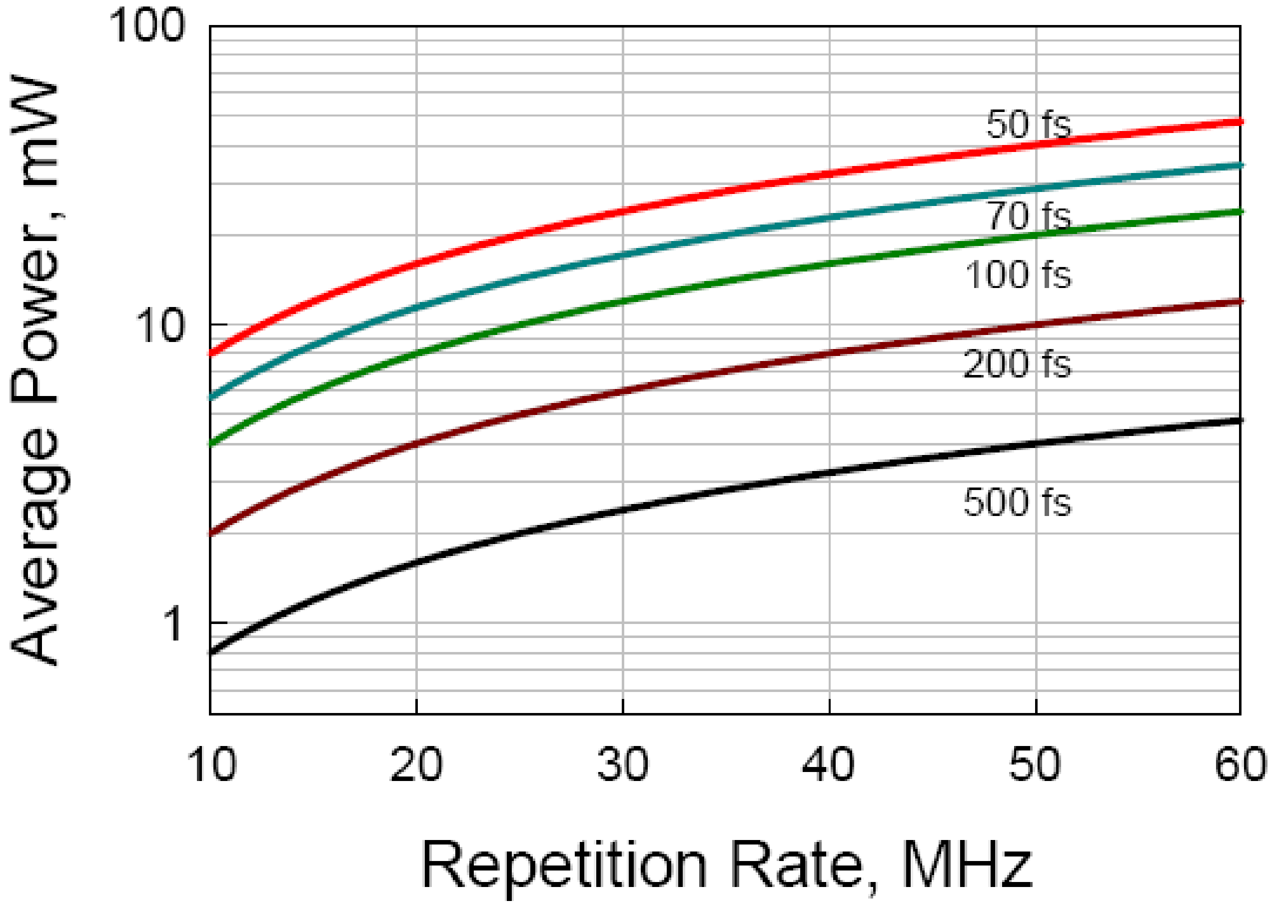
Hasegawa had predicted 90mW :-
"……a rather modest power and well within the capabilities of available lasers."

Only two minor factors stood in the way :-

- **No picosecond sources above 1.27 μm**
- **Low loss fibres above 1.27 μm did not exist**

  ~ 17 dB/km Maurer, Schultz and Keck   Corning 1970

Power requirements for soliton



At repetition rates from a conventional fibre laser, for pulse durations in the 500fs-1ps regime only a few mw average power is required

Dark optical solitons

Hasegawa & Tappert , App Phys Lett 23, 171 (1973)

Normal dispersion

Antisymmetric function of time with an abrupt π phase shift and zero intensity at the centre of the "pulse"

Experimentally realized

Emplit et al. Optics Commun 63, 374 (1987)

Imperial College London

High order optical solitons

Satsuma & Yajima, Prog Theor Phys Suppl 55, 284 (1974)
Invesitgated input function

$$U(z=0,t)=A\sec h(t)$$

solitons plus some dispersive radiation

$$0.5\leq A\leq 1.5$$ **1 soliton**

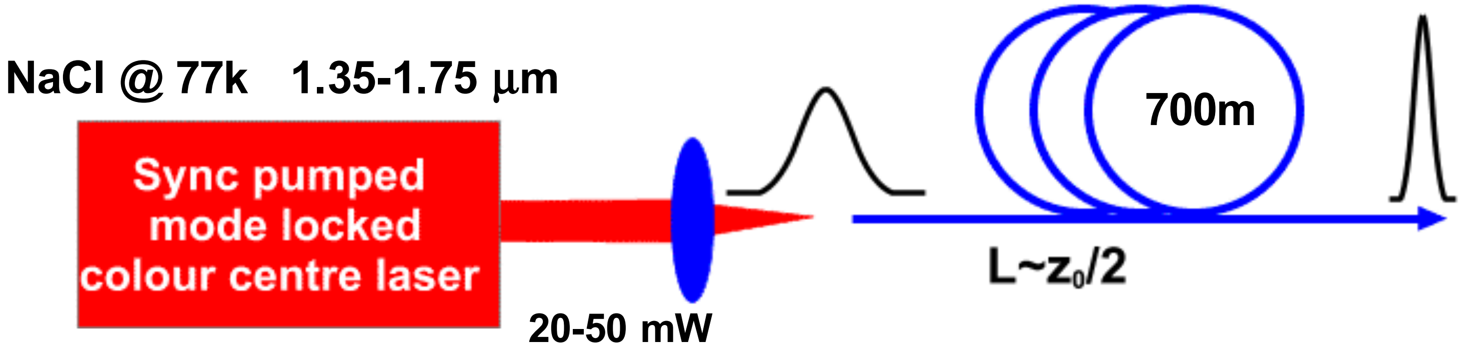
$$1.5\leq A\leq 2.5$$ **2 solitons etc**

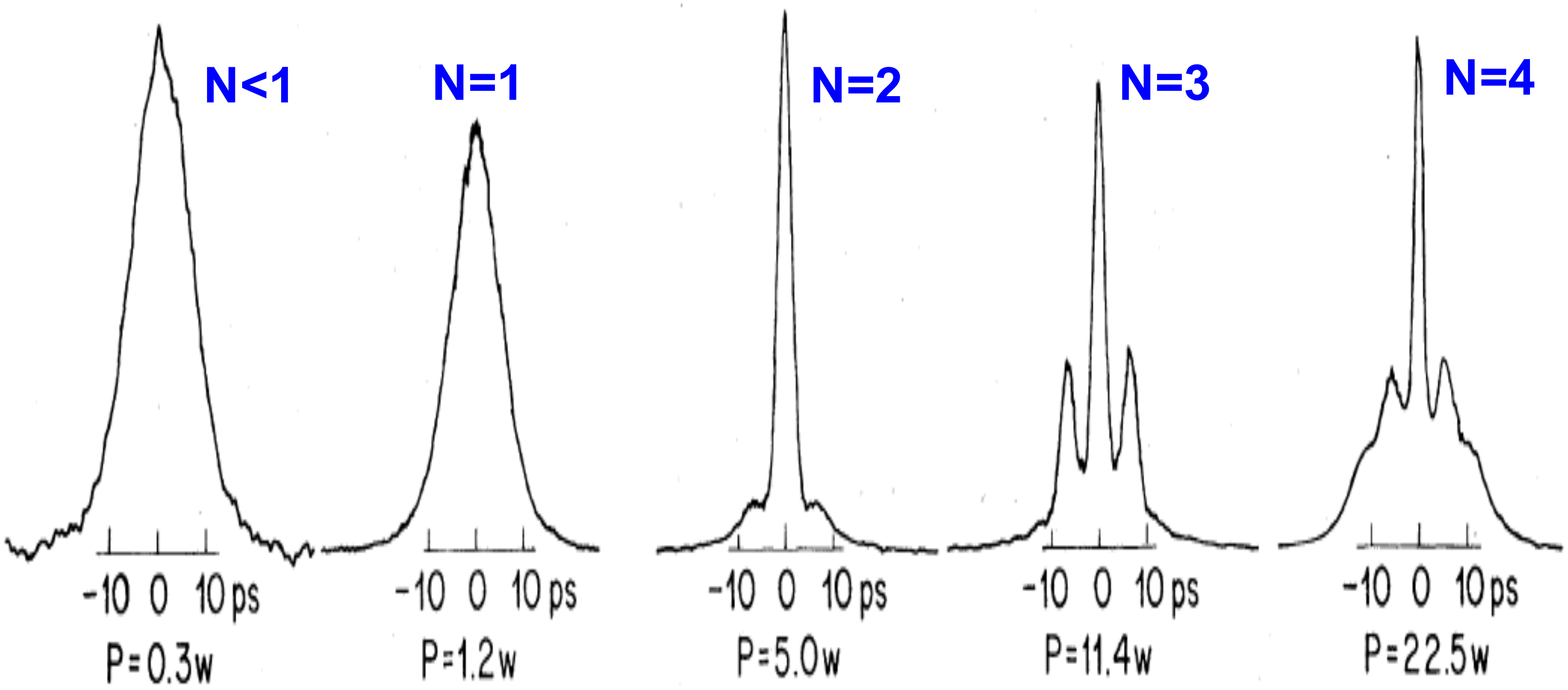
A=N **pulse exhibits a periodic narrowing and splitting, takin place with a period in "practical" units**

$$z_0=\frac{1}{(1.763)^2}\frac{\pi^2 c\,\tau^2}{\lambda^2 D}$$

$P_N = N^2P_1$ ***Compression ~ 4.1N***

Optical Solitons Experimental Realization





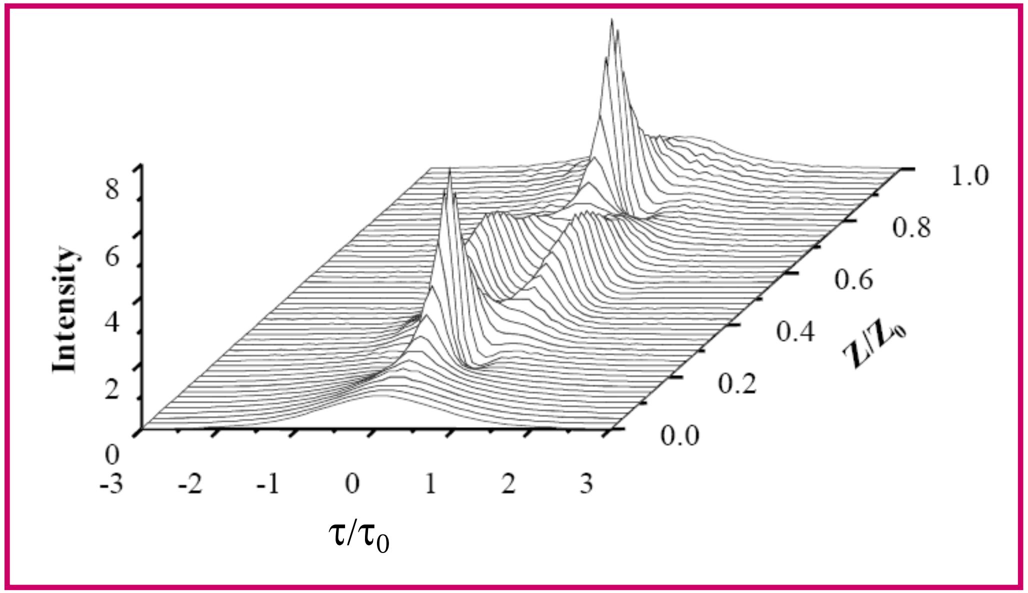
Mollenauer, Stolen and Gordon Phys Rev Letters 45, 1095 (1980)

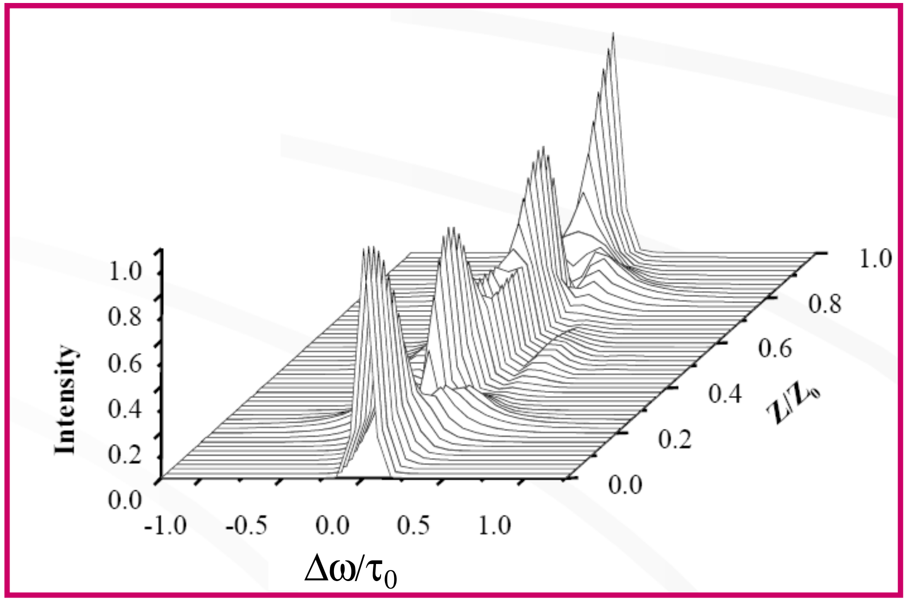
High Order Solitons

- **For N=2,3… soliton power increases as a square of the fundamental soliton.**
- **Higher order solitons "breathe" reproducing their temporal and spectral profile after propagating a soliton period**
- **Use process for extreme pulse compression 4.1 N**

$$Z_0 = Z_D\pi/2 \sim \tau_0^2/(2D),$$

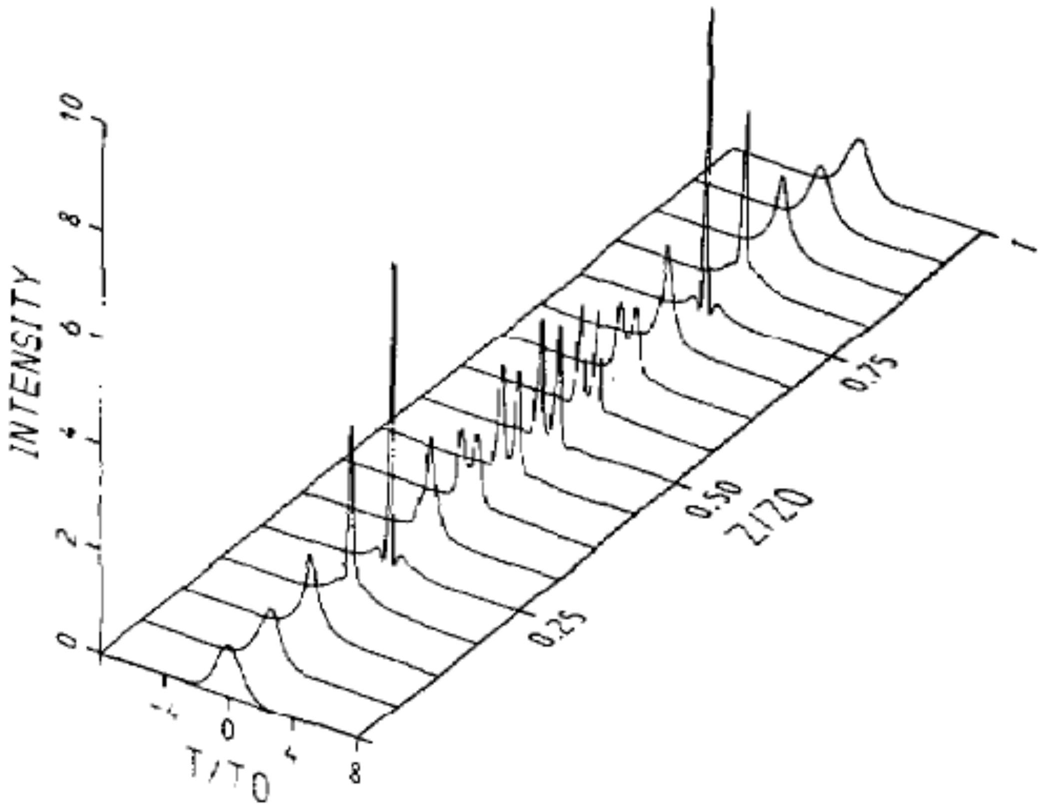
N=3

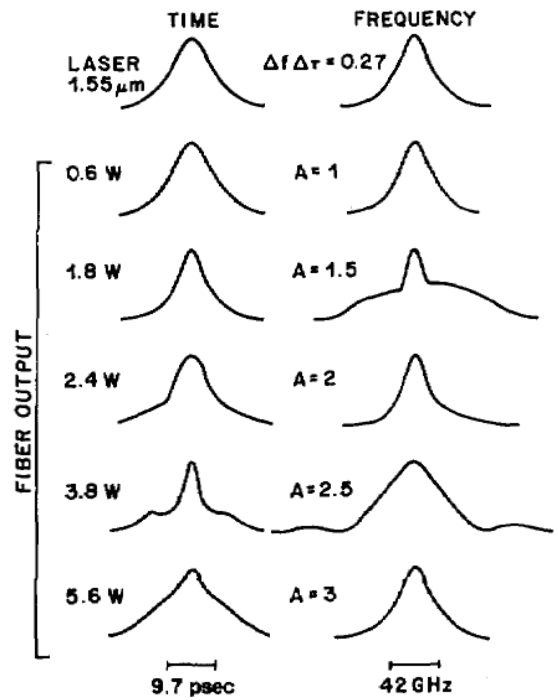




Mollenauer Soliton Experiments

Observation of pulse restoration at the soliton period in optical fibers
Optics Letters 8, 186 (1983)
N= 1,2 &3 solitons

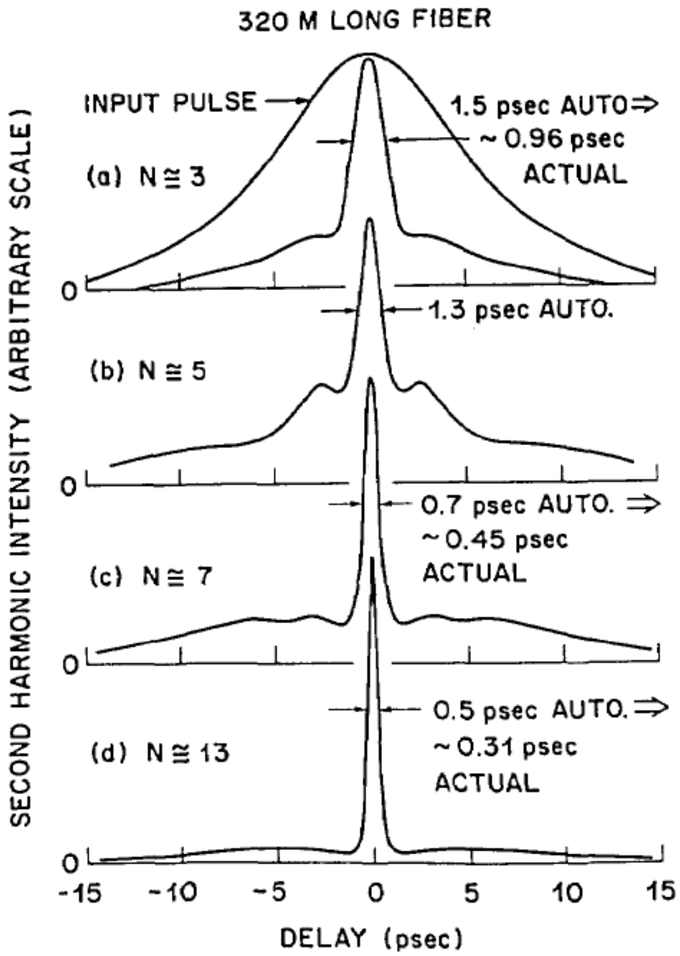




Imperial College London

Mollenauer Soliton Experiments

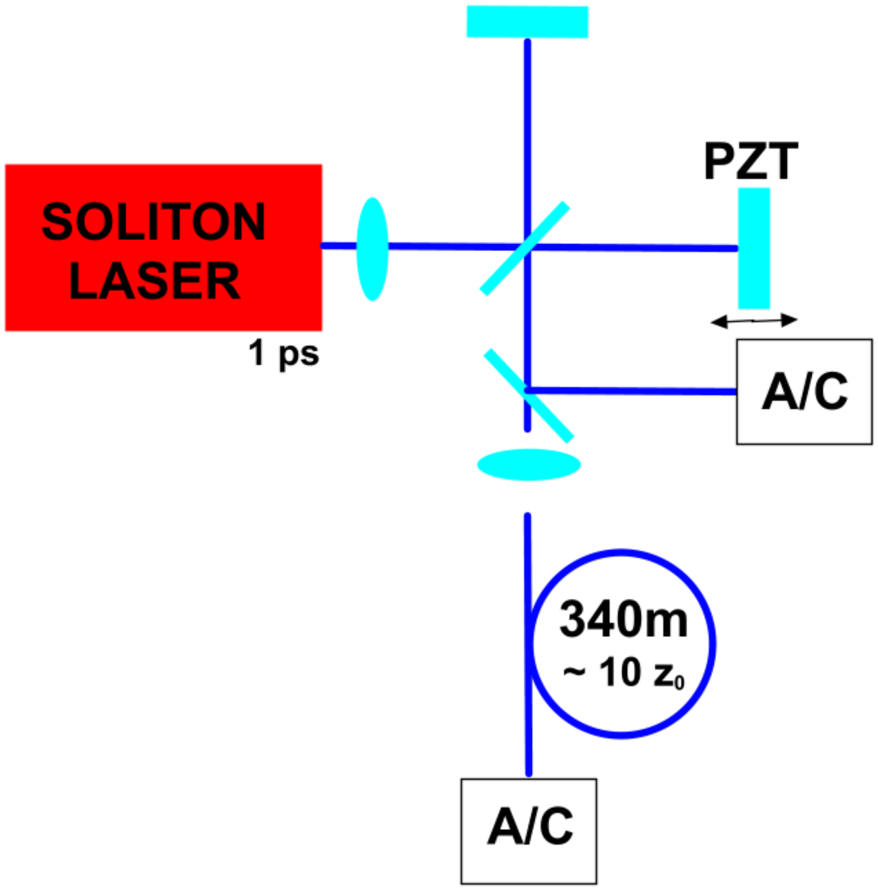
Extreme picosecond pulse narrowing by means of soliton effect in single-mode optical fibers
Optics Letters 8, 289 (1983)
7 psec to 260 fsec

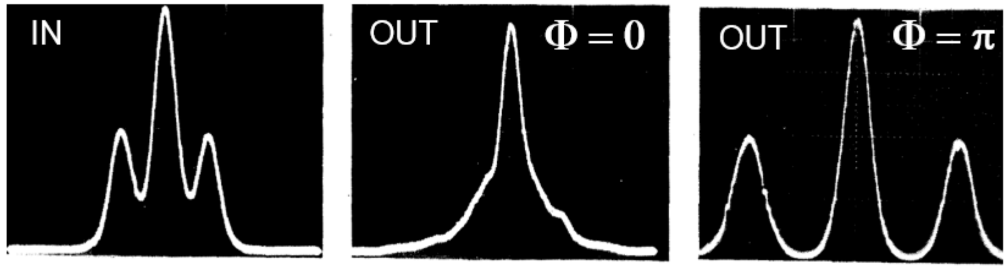


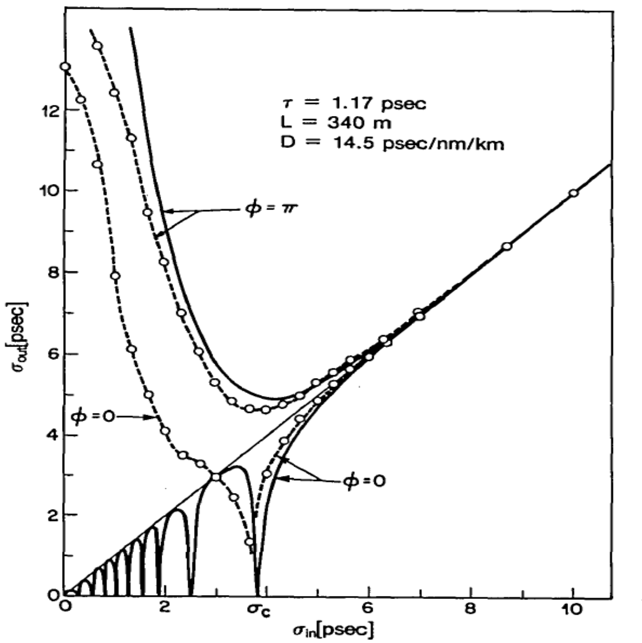
Mollenauer Soliton Experiments

Experimental observation of the interaction forces between solitons in optical fibers
Optics Letters 12, 355 (1987)

Karpman and Solov'ev Physica 3D, 487 (1981) - Prediction
Gordon Opt. Lett. 8, 596 (1983)







Properties of Fundamental Solitons

- **Robust**
  An input pulse of power P does not have to be an exact soliton, it will readjust itself into an N=1 soliton, shedding off a dispersive non soliton component if $0.25 < P/P_0 < 2.25$

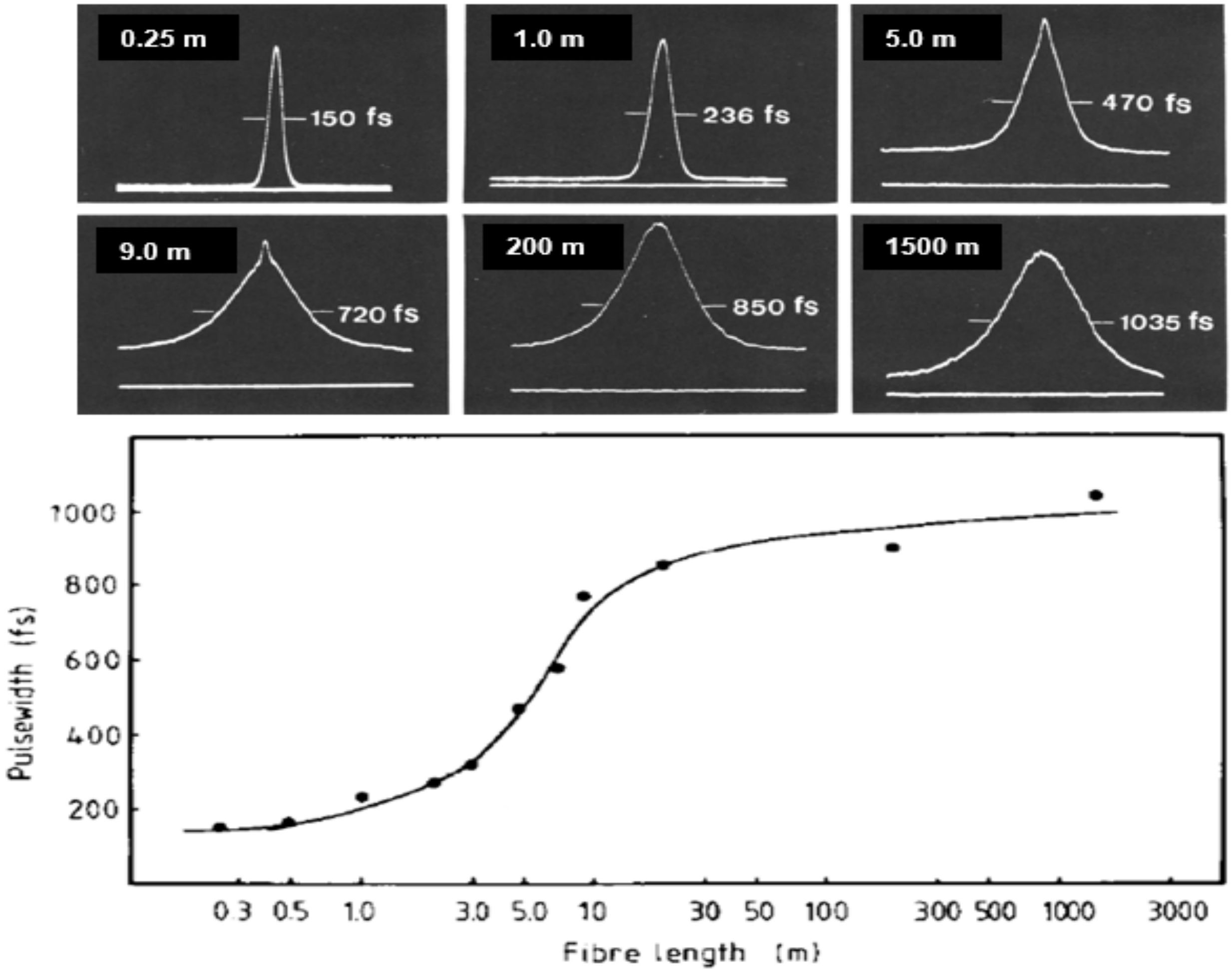
- **Insensitive to fast perturbations**
  Variations to P or system parameters (A, D) provided the length scale of the perturbation << Z_0
  "Average", "guiding centre" soliton dynamics
  Mollenauer et al. 1991 J. Lightwave Tech, 9, 194
  Hasegawa et al. 1991 Phys Rev Lett 66, 161
  Kelly et al. 1991 Opt. Lett. 16, 1337

- Soliton energy ("area") is conserved

$$Z_0 = \frac{1}{(1.763)^2}\frac{2\pi c}{\lambda^2}\frac{\tau^2}{D}$$ **at 1550 nm good approximation** $$= 0.25\tau^2/D$$

Soliton relaunch

Effects of chirp and phase noise

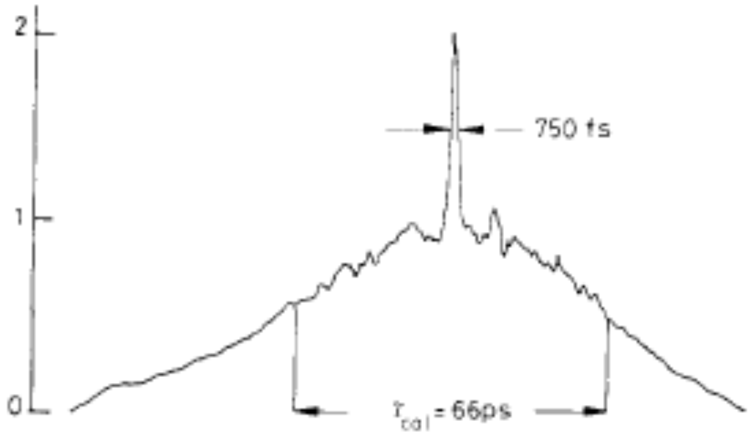


Solitons from noise

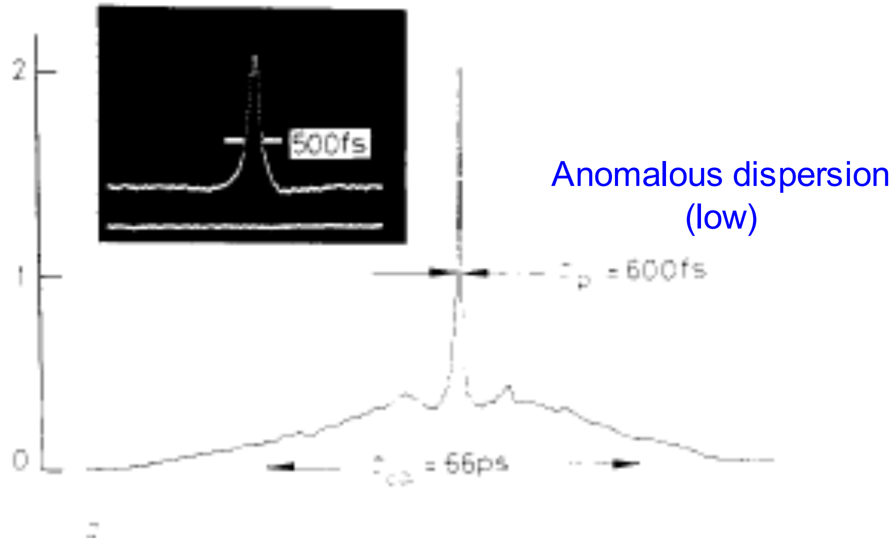
Gouveia-Neto & Taylor, Electronics Lett. 25, 736 (1989)

Autocorrelation trace

INPUT



OUTPUT



Soliton Amplification

"Amplification and reshaping of optical solitons in a glass fiber I"
Hasegawa & Kodama **Opt. Lett. 7, 285 (1982)**

"Amplification and reshaping of optical solitons in a glass fiber IV: Use of the stimulated Raman process"
Hasegawa **Opt. Lett. 8, 650 (1983)**

"Numerical study of optical soliton tranmsmission amplified periodically by the stimulated Raman process"
Hasegawa **App. Opt. 23, 3302 (1984)**

"Demonstration of soliton transmission over more than 4000 km in fiber with los periodically compensated by Raman gain"
Mollenauer & Smith **Opt. Lett. 13, 675 (1988)**

"10 Gbit/s soliton data transmission over one million kilometres"
Nakazawa et al. , **Elect. Lett. 27, 1270 (1991)**

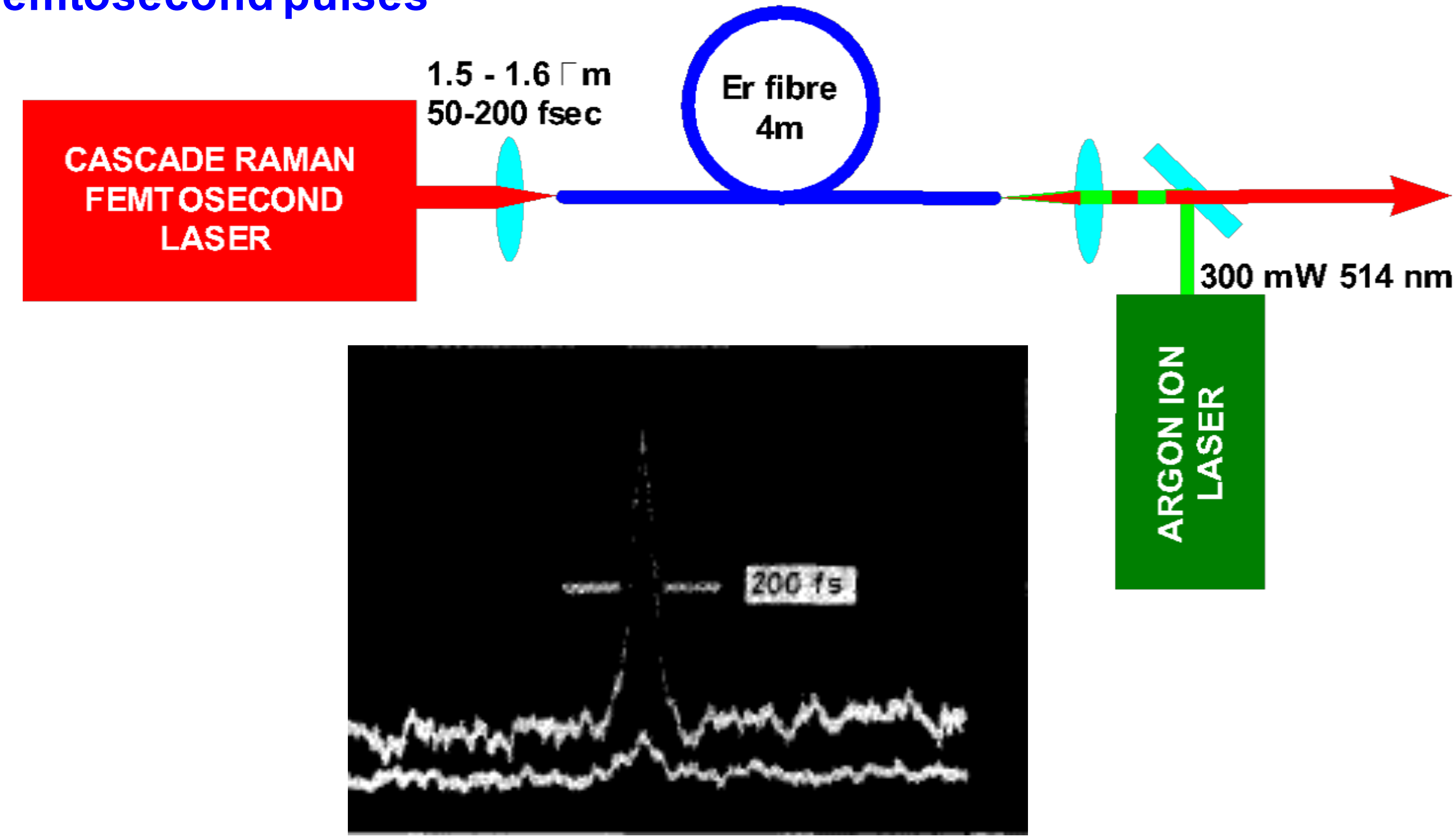
THz bandwidth soliton amplification in Er

Nakazawa, Kimura & Suzuki **Elect. Lett. 25, 199 (1989)**

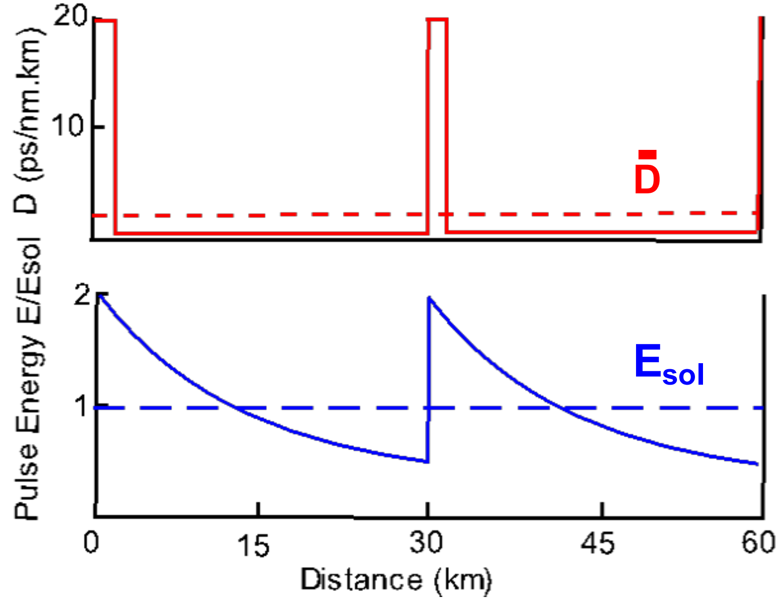
9 picosecond pulses

Ainslie et al. **Elect. Lett. 26, 186 (1990)**

200 femtosecond pulses

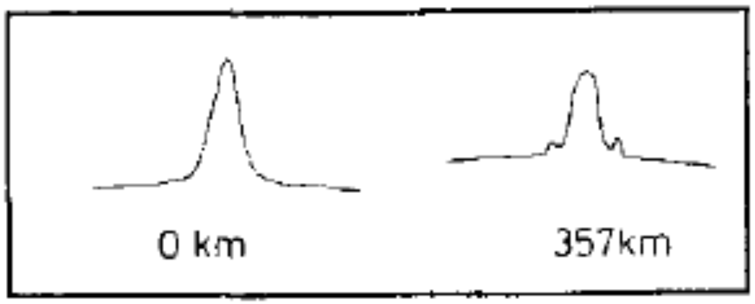


Path Average Solitons



Want

$Z_0 >> L_{AMP}$ or $L_{Perturb}$



$Z_0 \sim$ 300 km,

$L_{AMP} \sim$71 km

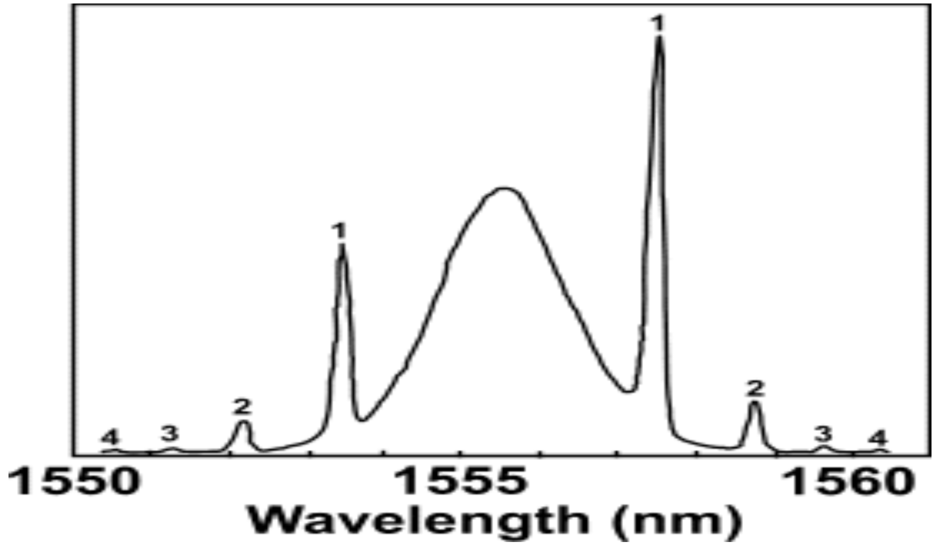
Ellis et al. **Elect. Lett 27, 878 (1991)**

Soliton instability – spectral sidebands

$nK_A = K_{SOL} + K_{DISP}$ **gives** $$\frac{2\pi n}{Z_A} = \frac{1}{2} + \frac{\Delta\omega^2}{2}$$

Rearranging $$\Delta\lambda = \frac{\lambda^2}{2\pi c\tau}\sqrt{\frac{8nZ_0}{Z_A} - 1}$$

Smith, Blow & Andonovic, J. Lightwave Tech 10, 1329 (1992)



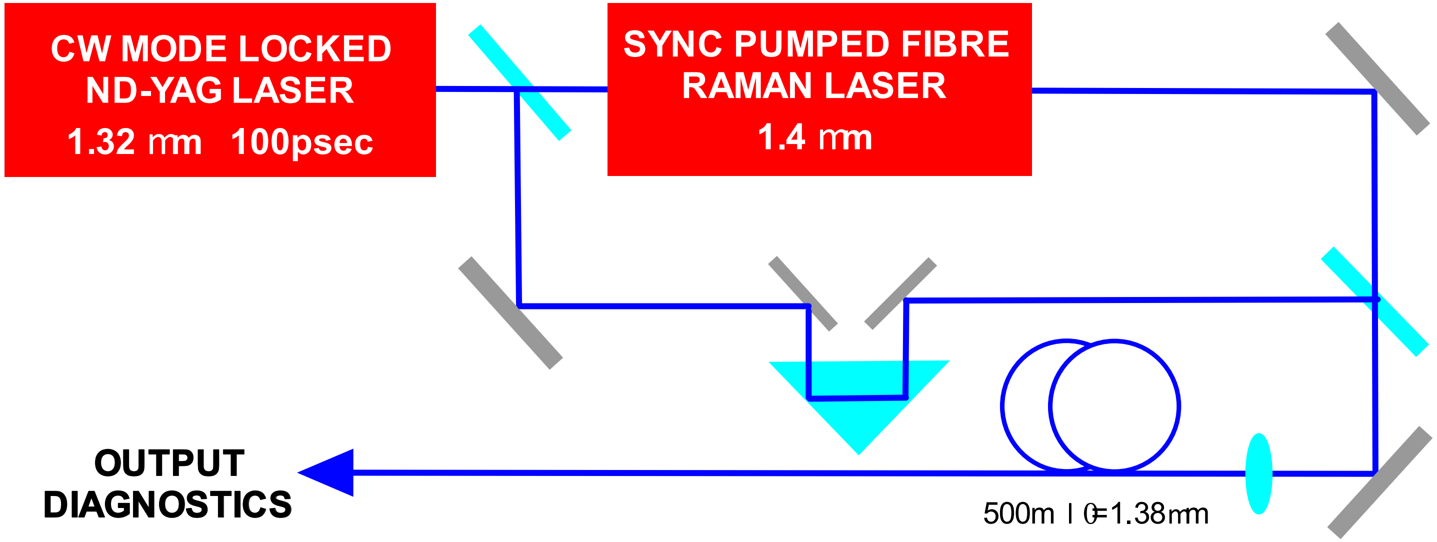
Sidebands:-

- Independent of power
- Non uniform distribution
- Determine "average" D
- Eliminate by filtering

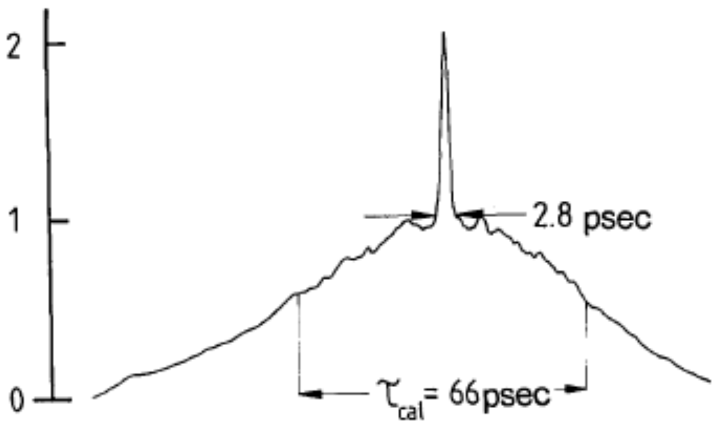
Pandit, Noske, Kelly & Taylor Elect. Lett. 28, 455 (1992)
Noske, Pandit & Taylor Opt. Lett. 17, 1515 (1992)

Solitons from amplified noise

Gouveia-Neto, Wigley & Taylor, Opt. Lett. <u>14</u>, 1122 (1989)

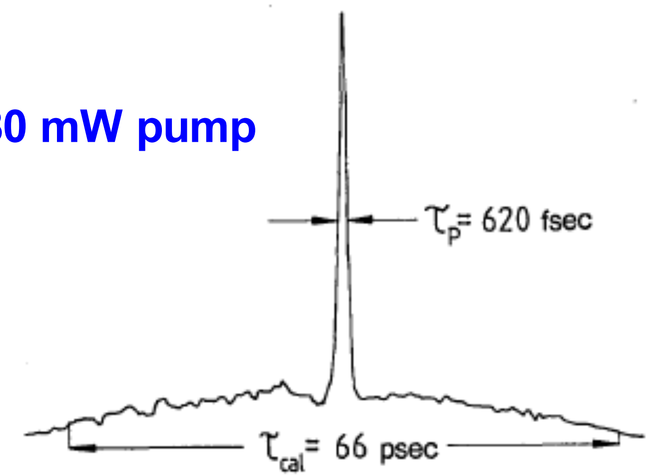


Input



Output

80 mW pump

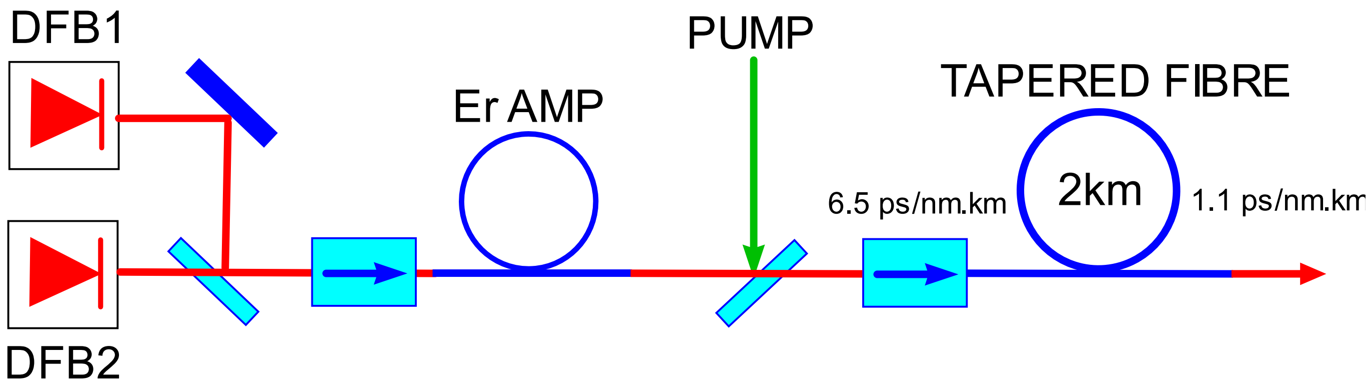


Imperial College London

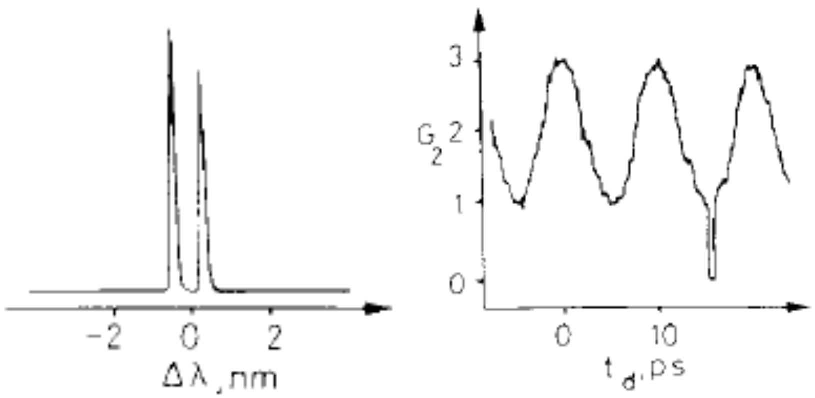
Adiabatic soliton amplification

Soliton duration

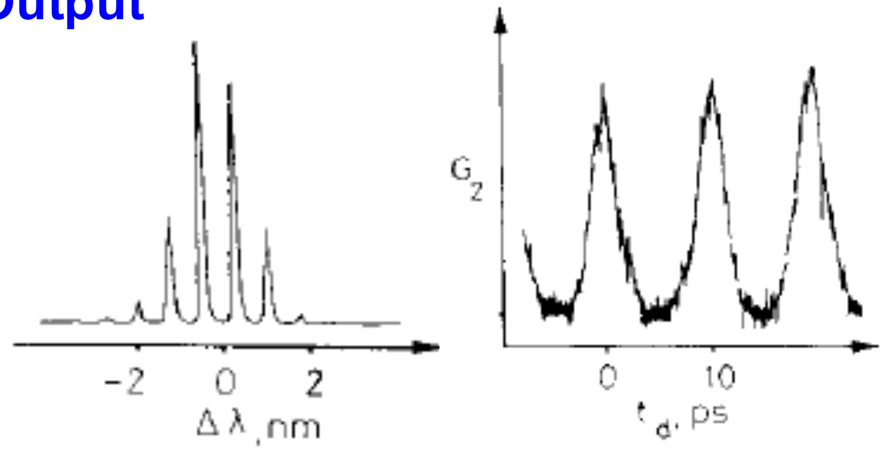
$$\tau_0 = \frac{2|\beta_2|}{\gamma E_s}$$



Input



Output



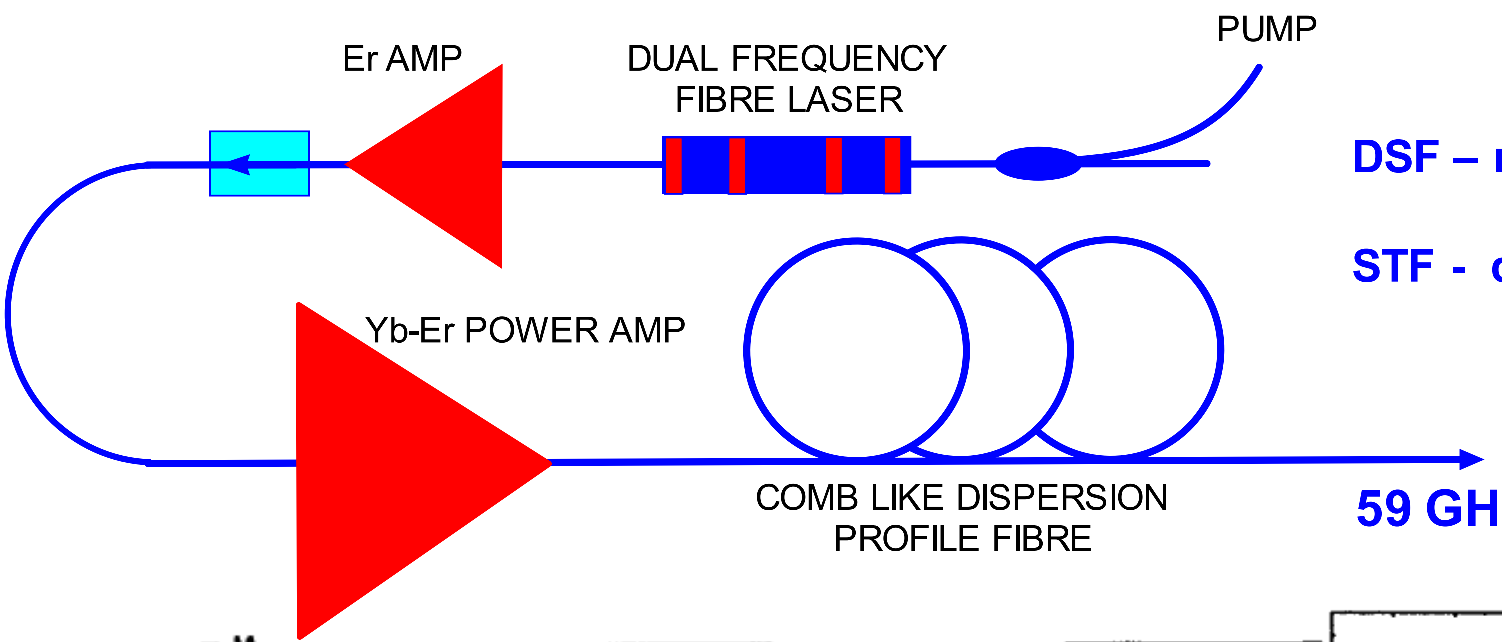
Chernikov, Mamyshev, Dianov & Taylor Elect. Lett. 28, 931 (1992)

Imperial College London

Alternatives to tapered fibres

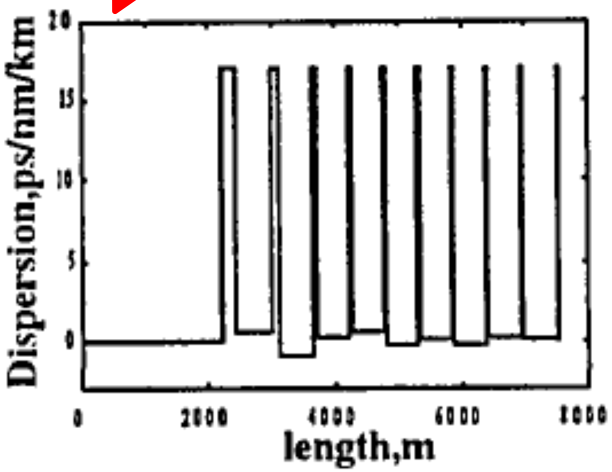
Comb-like dispersion profiled fibre assemblies

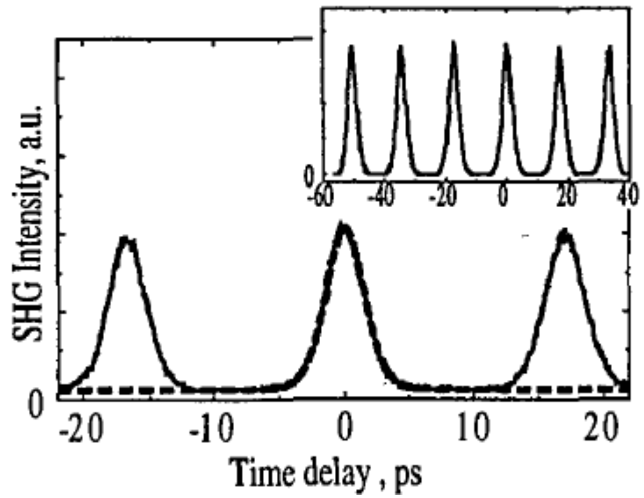
Chernikov, Kashyap & Taylor Opt. Lett. 19, 539 (1994)



DSF – nonlinearity
2.2 km, ~500/m
STF - dispersion
220 m – 30 m

59 GHz, 2.2 psec



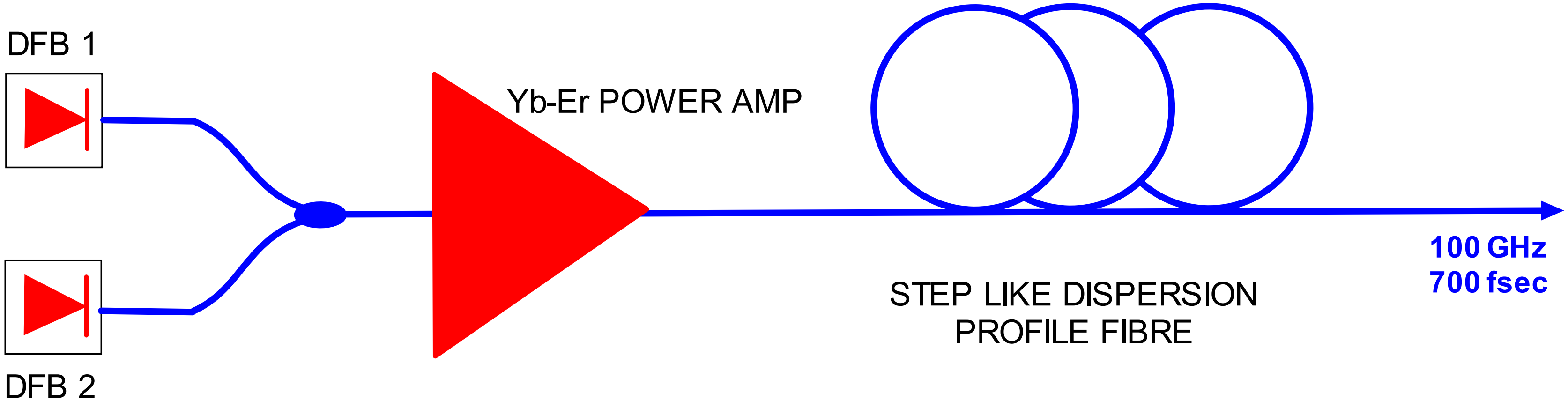


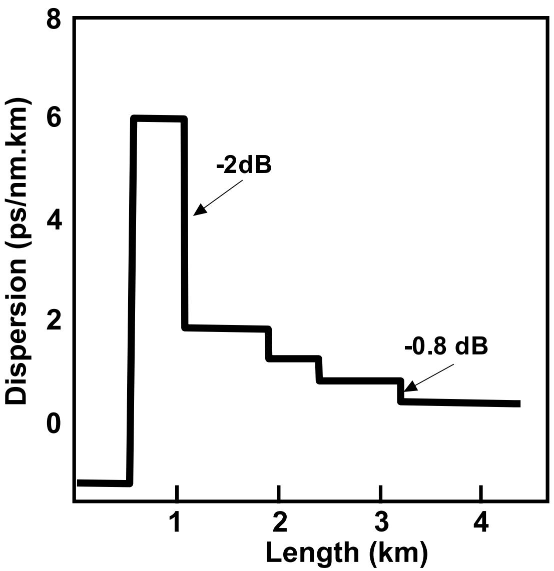
Imperial College London

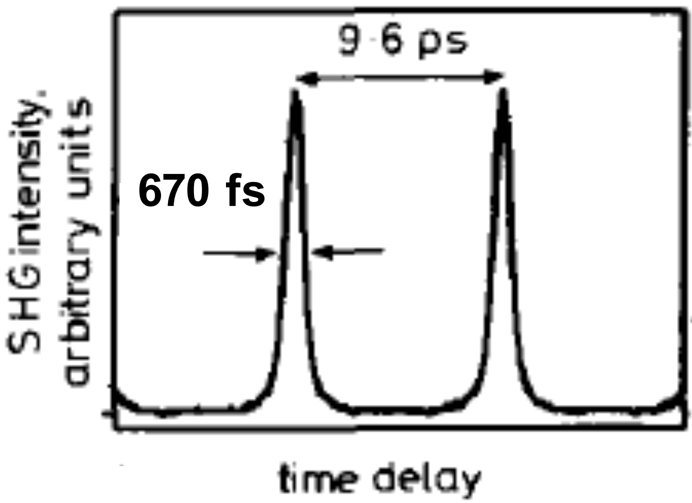
Alternatives to tapered fibres

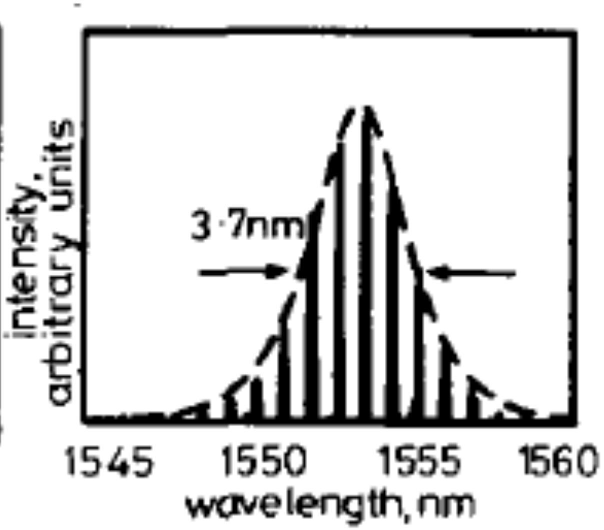
Step-like dispersion profiled fibre assemblies

Chernikov, Kashyap & Taylor   Elect. Lett. 19, 539 (1994)









Soliton compression – adiabatic Raman gain

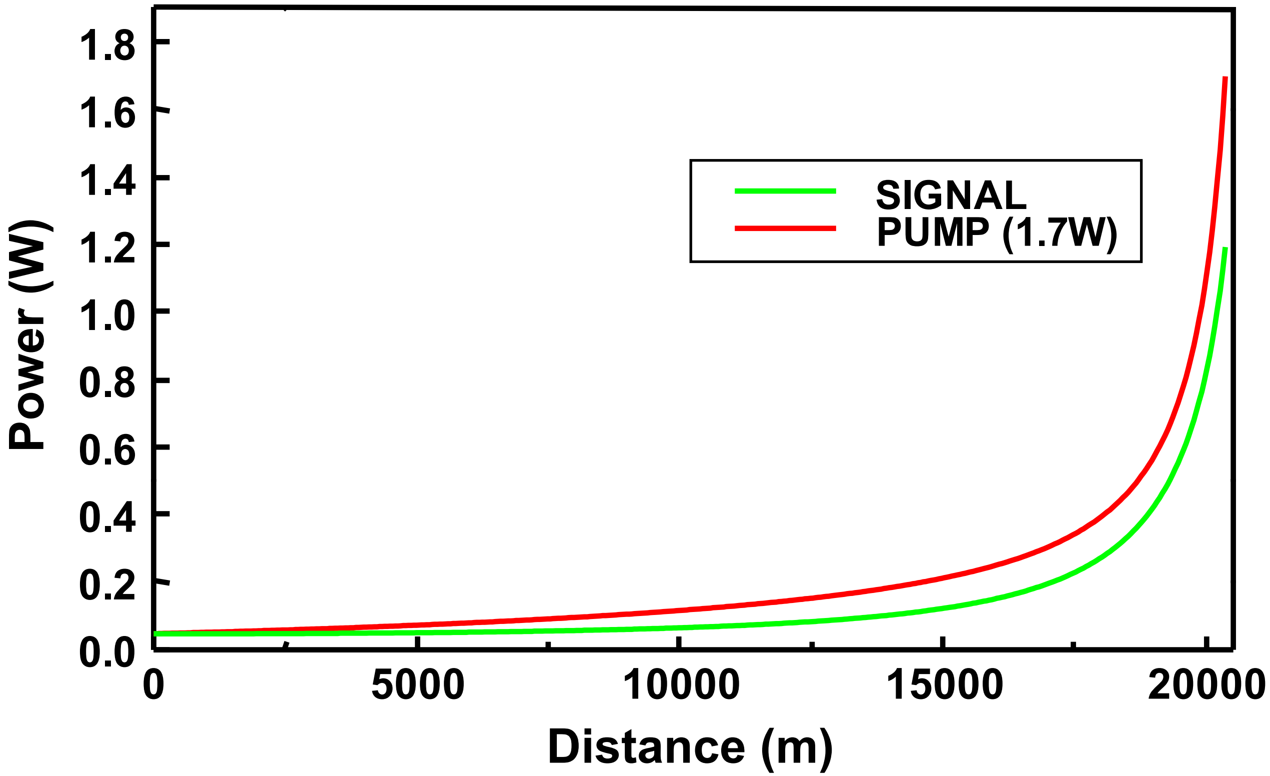
Soliton amplification and compression

Blow, Doran & Wood JOSA B 5, 381 (1988)

- Soliton pulse duration $\tau \propto D/E$
- Soliton period $Z \propto \tau^2$
- Require pulse compression <10% over Z

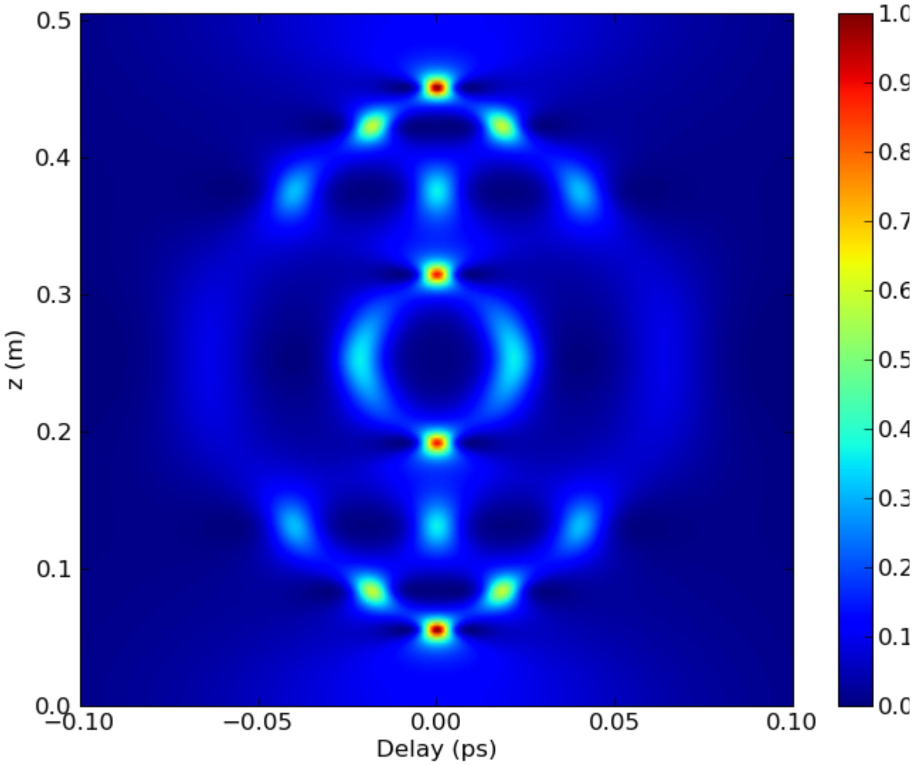
Input 10 ps soliton period 2.48km gain ~ 1.7%

Output 1ps soliton period 24.8 m gain ~ 3.4%

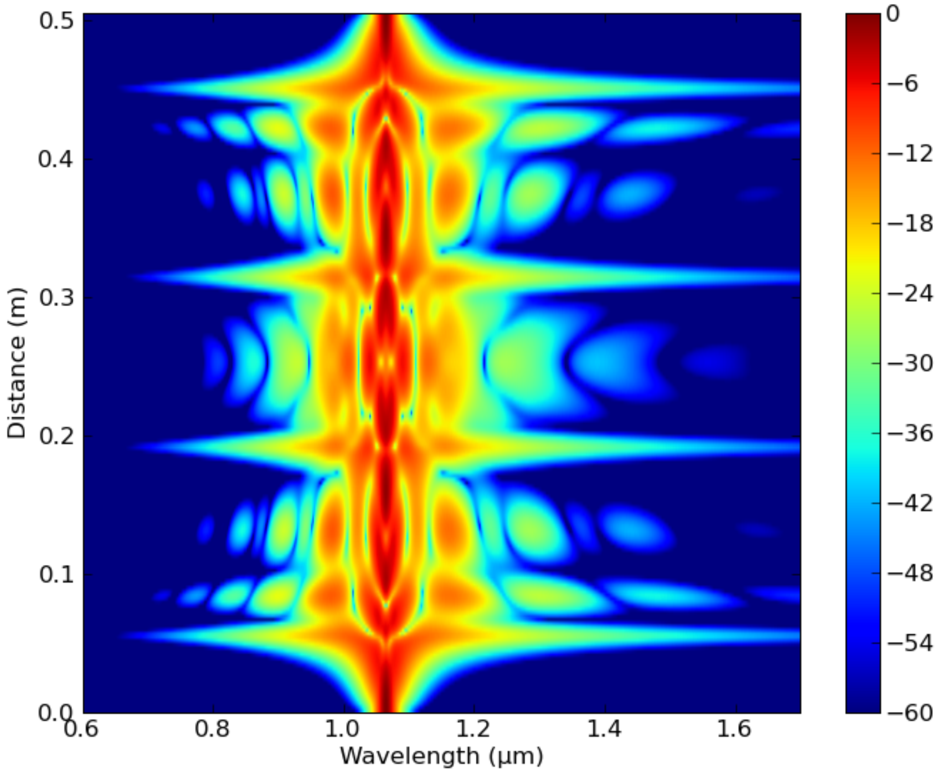


100 fs FWHM, N=5 Soliton evolution

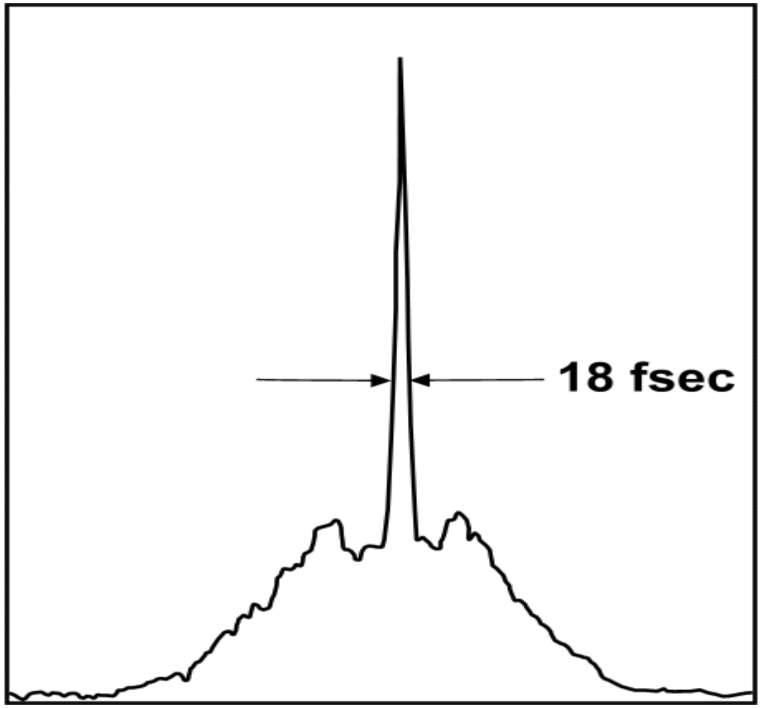
Temporal

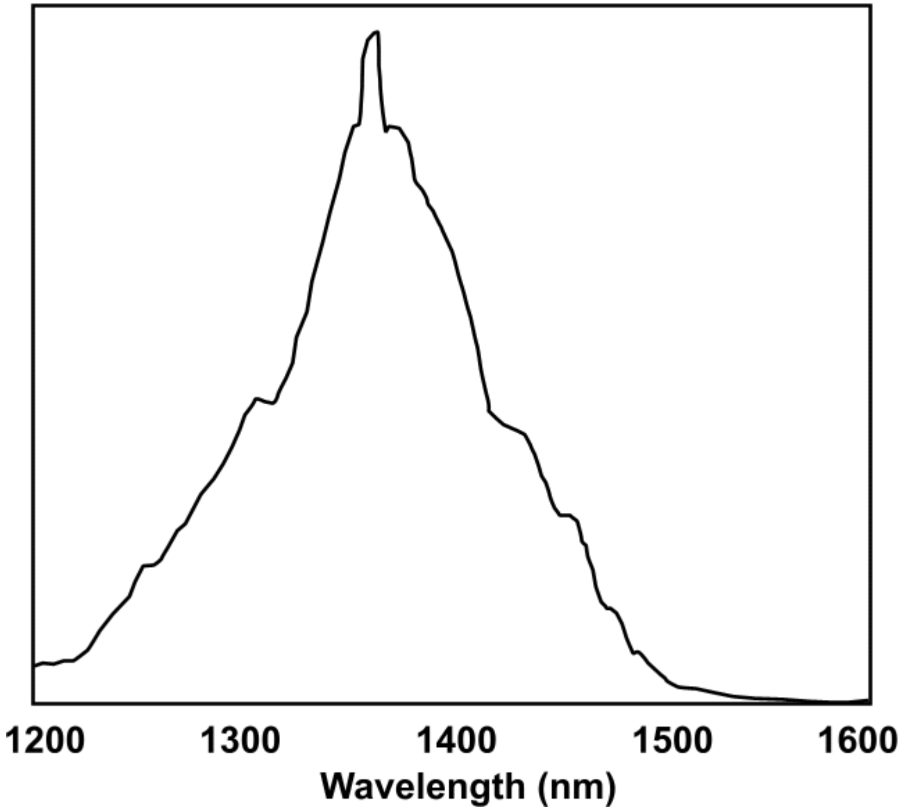


Spectral



High order solitons –pulse compression , spectral broadening





N=14 soliton (P_0 = 8.2W) – compression factor 57
Input 1.1 psec output 18fsec (four optical cycles)
Soliton length 132 m, optimized compression length 11m
Gouveia-Neto et al. 1988 J. Mod Opt. 35, 7

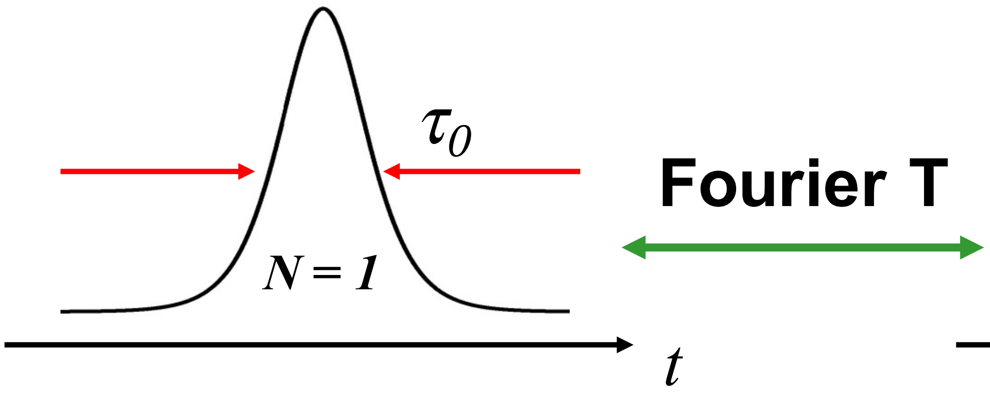
Imperial College London

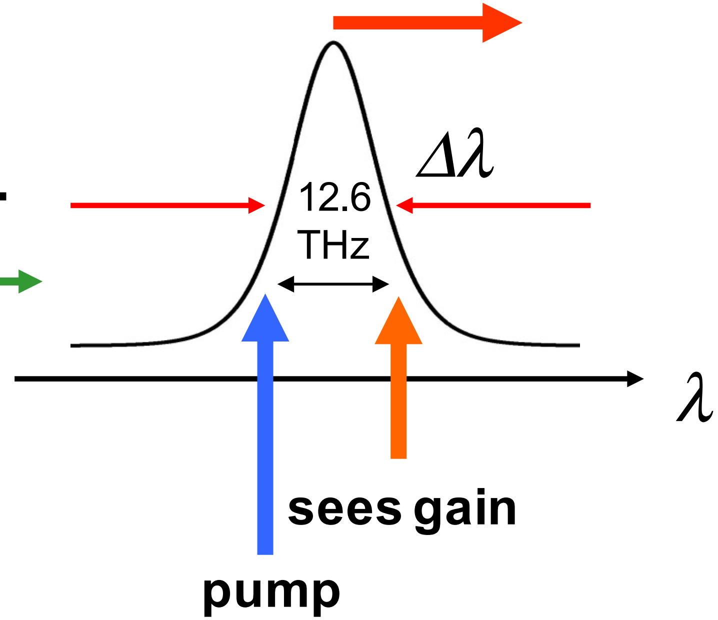
Instabilities and nonlinearity

Soliton self interaction Dianov et al. 1985 JETP Lett. 41, 294

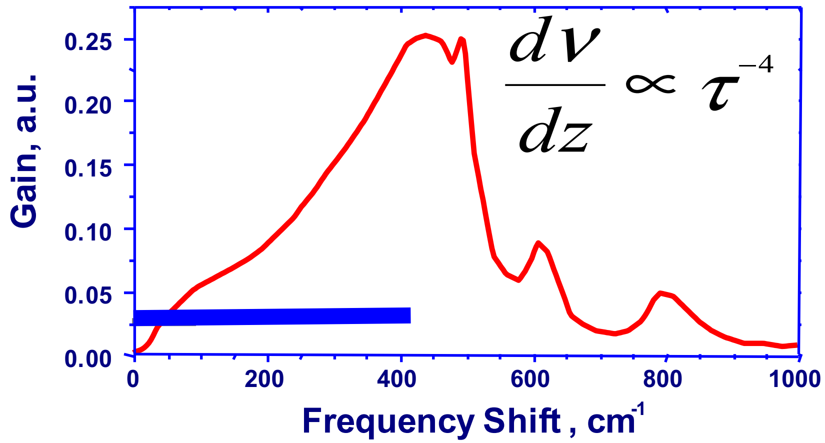
Soliton self frequency shift Mitschke & Mollenauer 1986, Opt. Lett. 11, 659

Gordon 1986, Opt. Lett. 11, 662





25 fs FWHM ~ 12.6 THz bandwidth

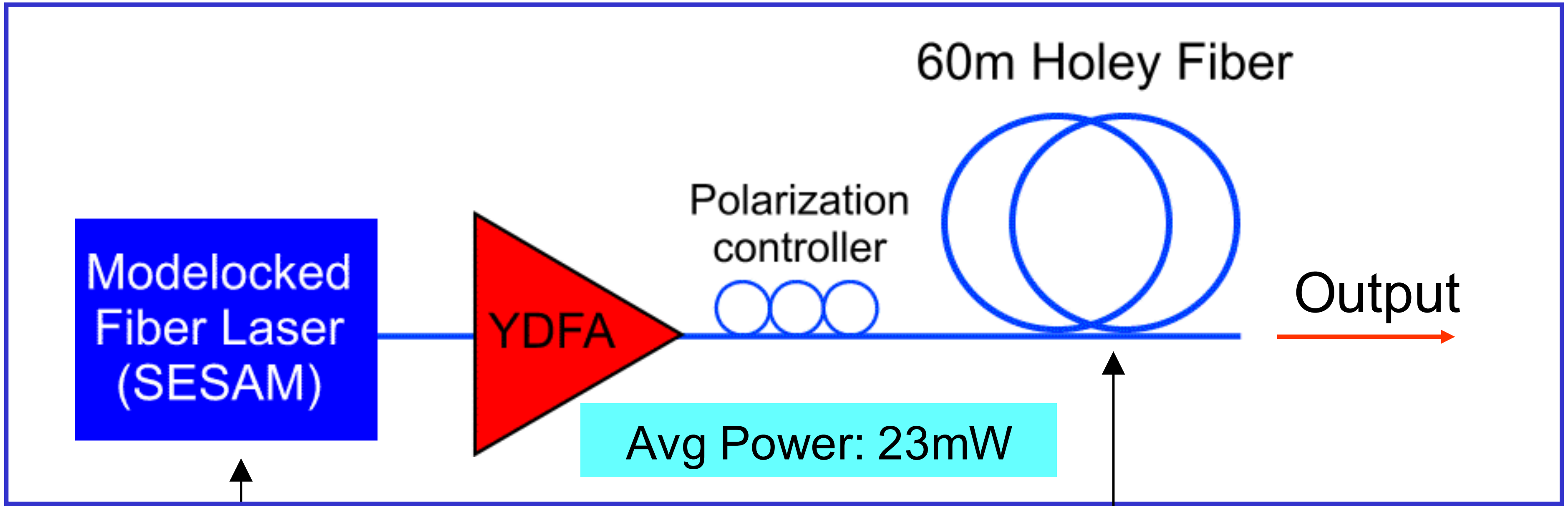


$$\frac{d\nu}{dz} \propto \tau^{-4}$$

$$\Delta\omega_{SSFS} = -\frac{8|\beta_2|T_R}{15\tau_0^4} z$$

$$P_0 \propto \frac{D\lambda^3}{\tau^2}$$

Self frequency shifted source



Pulse Duration: 4ps

Repetition Rate: 50MHz

Average Power: 4.5mW

Center Wavelength: 1063nm

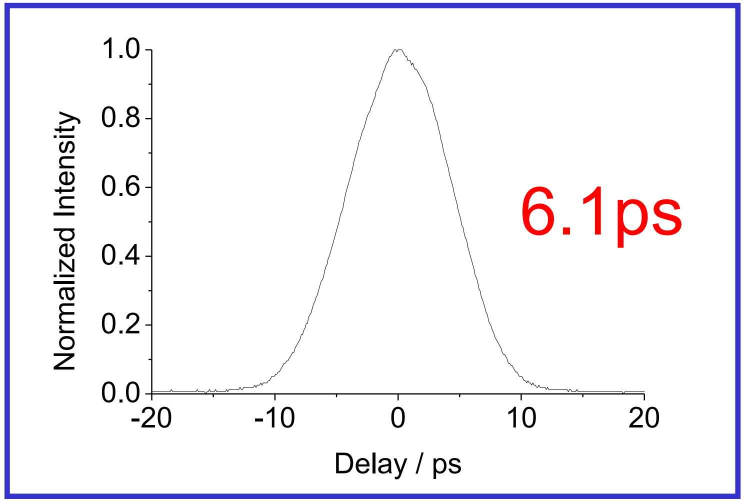
Holey Fiber Parameters

Zero Dispersion @ 1040nm

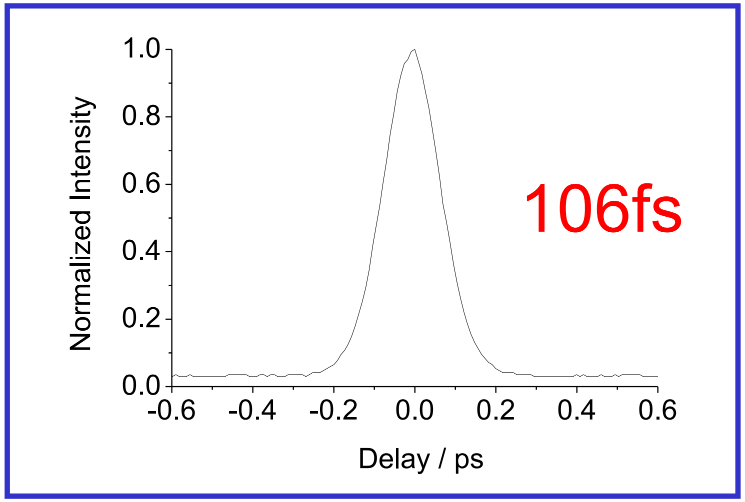
Nonlinear Coefficient: $17W^{-1}km^{-1}$

Core Size: 5µm

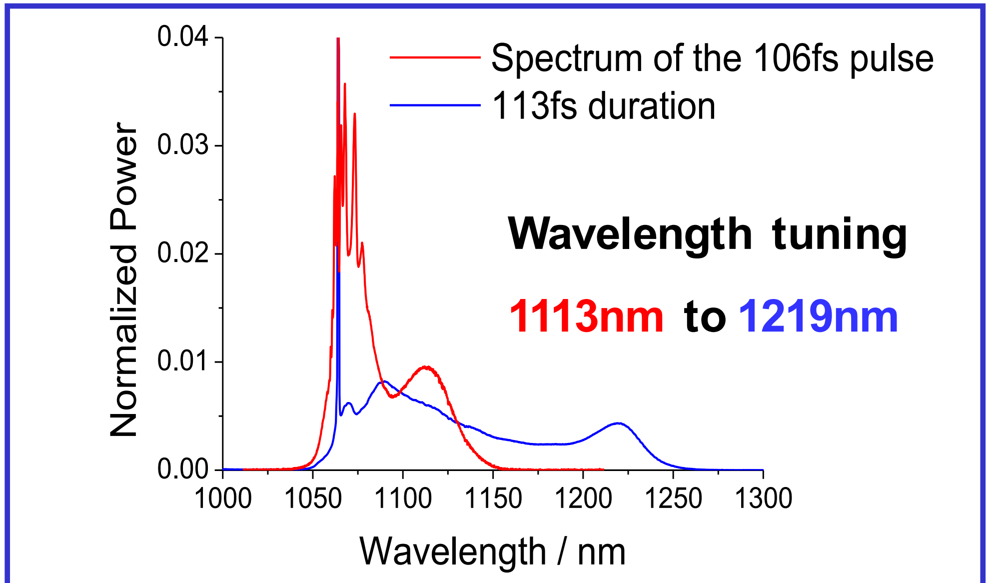
SSFS femtosecond soliton tunability



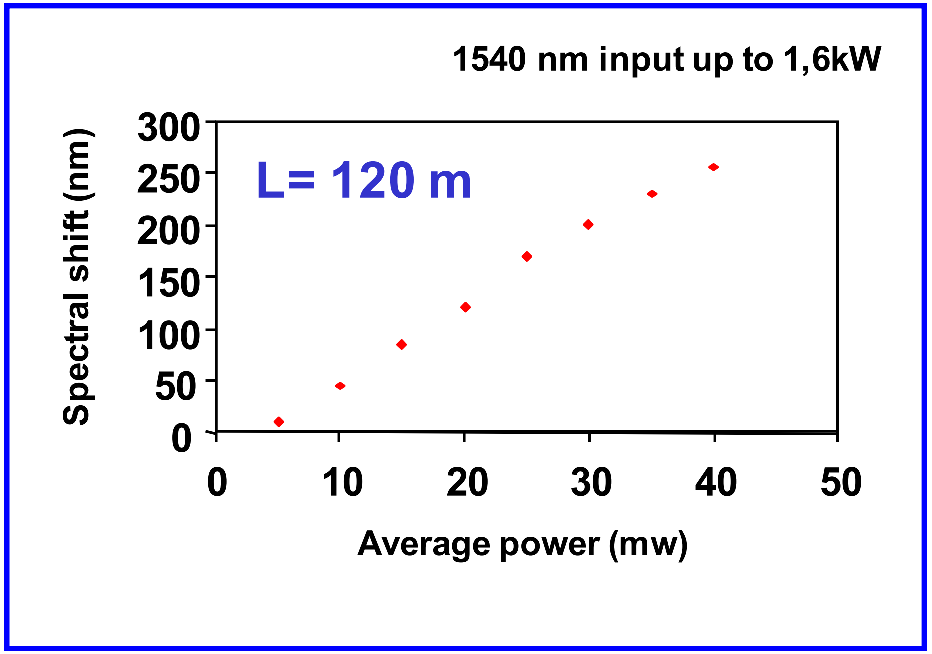
Autocorrelation after Amplification



Autocorrelation after Compression



Spectra after Compression



Instabilities and nonlinearity
Modulational instability

Hasegawa and Brinkman 1980, IEEE J. Quant. Elect. QE16, 694
Tai et al. 1986, Phys. Rev. Lett. 56, 135
Itoh et al. 1989, Opt Lett. 14, 1368

Many systems exhibit instability that leads to modulation of the steady state as a result of interplay of dispersion and nonlinearity

In optical fibre – requirement – anomalous dispersion - solitons

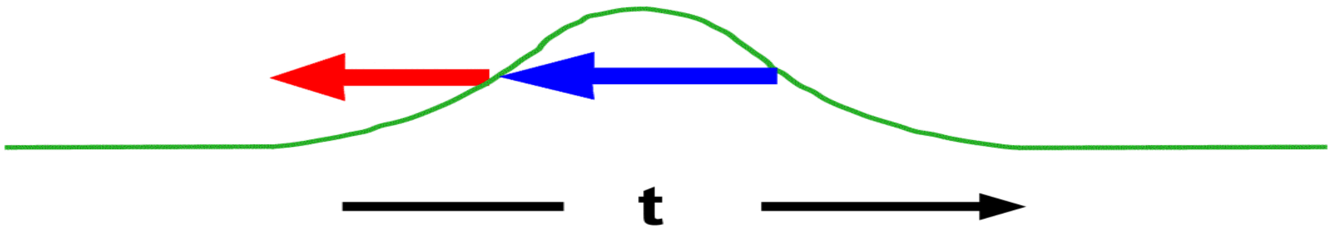
$$i\frac{\partial A}{dz} = \frac{\beta_2}{2}\frac{\partial^2 A}{\partial^2 T^2} - \gamma \lfloor A \rfloor^2 A$$

$$\overline{A} = \sqrt{P_0}\exp(i\phi_{NL}) \qquad \phi_{NL} = \gamma P_0 z$$

Introduce an amplitude perturbation a

$$A = \left(\sqrt{P_0 + a}\right)\exp(i\phi_{NL})$$

Modulational instability



The perturbations grow exponentially when their frequency Ω

$$|\Omega| < \Omega_C = \left(\frac{4\gamma P_0}{|\beta_2|}\right)^{1/2}$$

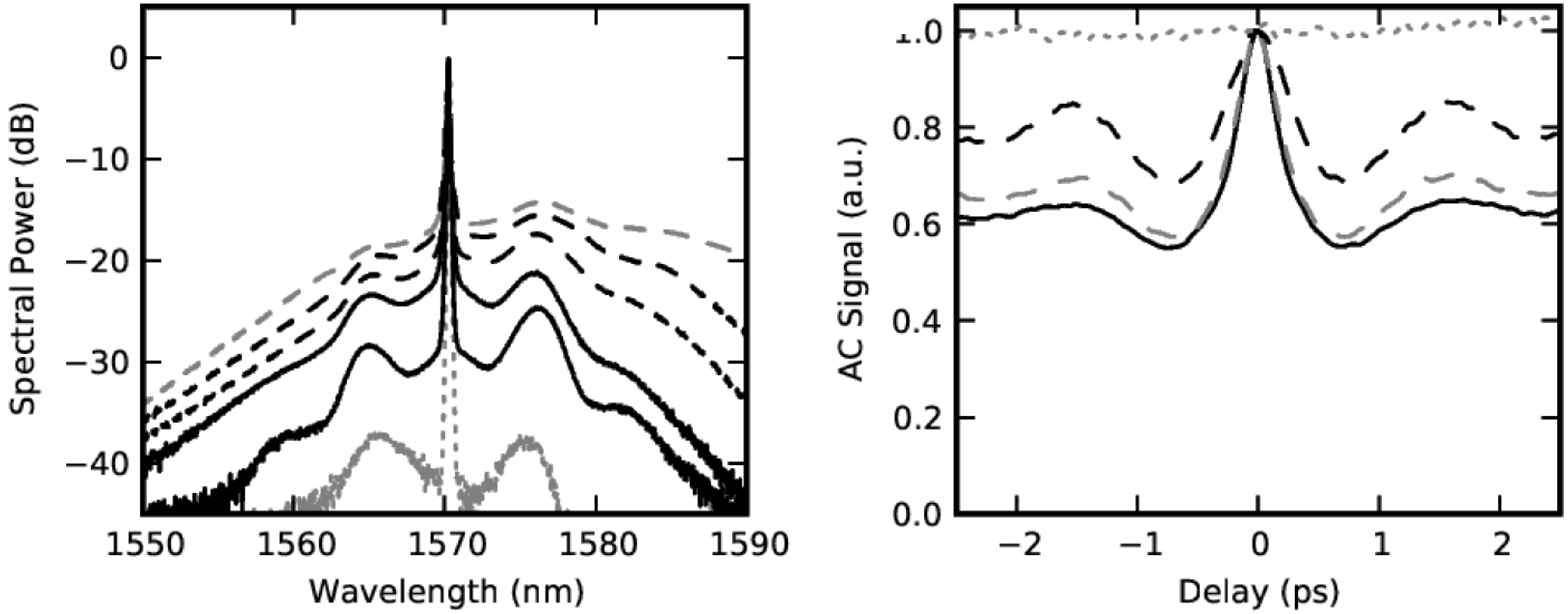
The maximum gain g_{max} = 2γP_0 occurs at

$$\Omega_{max} = \frac{\Omega_C}{\sqrt{2}} = \pm\left(\frac{2\gamma P_0}{|\beta_2|}\right)^{1/2}$$

For 10W in a STF with 3ps/nm.km at 1.55 µm Ω_{max}~ 2.5 THz

Modulational instability

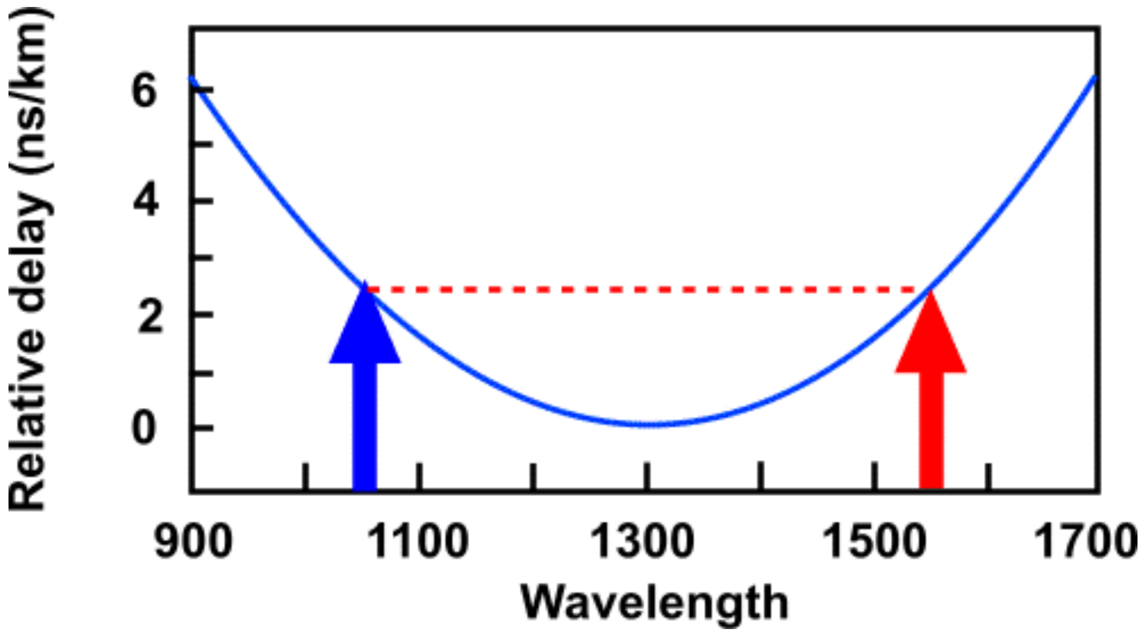
Experimental measurement



$$\Omega_{\max} = \frac{\Omega_C}{\sqrt{2}} = \pm\left(\frac{2\gamma P_0}{|\beta_2|}\right)^{1/2}$$

Nonlinearity and dispersion
Cross –phase modulation

The effective refractive index depends on the intensity of all co-propagating beams



Intense signal in the normal dispersion regime can induce modulational instability on a weak signal in anomalous dispersion Region

Inherent in the Raman process

Imperial College London

Cross phase modulation

Experimental:-
Pump - cw mode locked Nd:YAG at 1064nm 100 ps pulses
Signal - cw Nd:YAG at 1319 nm