Semiclassical approximation - Bloch equations

Go to interaction picture corresponding to

$$\hat{H}_{0R} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2} + \frac{1}{2}\hat{\sigma}_3\right) \text{ (rotating frame), and set } \hat{a}_I \to \alpha$$

(slowly-varying envelope of classical field):

$$\hat{H}_R = \frac{\hbar\delta}{2}\hat{\sigma}_3 + \frac{\hbar\Omega_0}{2}\left(\hat{\sigma}_{+,I}\alpha + \hat{\sigma}_{-,I}\alpha^*\right) = \boxed{\frac{\hbar}{2}\vec{\hat{\sigma}}_I \cdot \vec{\Omega},}$$

Spin-precession equation

where $\vec{\Omega} = (V_1, V_2, \delta)$, with $\Omega_0\alpha \equiv V \equiv V_1 - iV_2$.

Atomic dynamics may be described in terms of the precession of a pseudo-spin around a pseudo-magnetic field $\vec{\Omega}$.

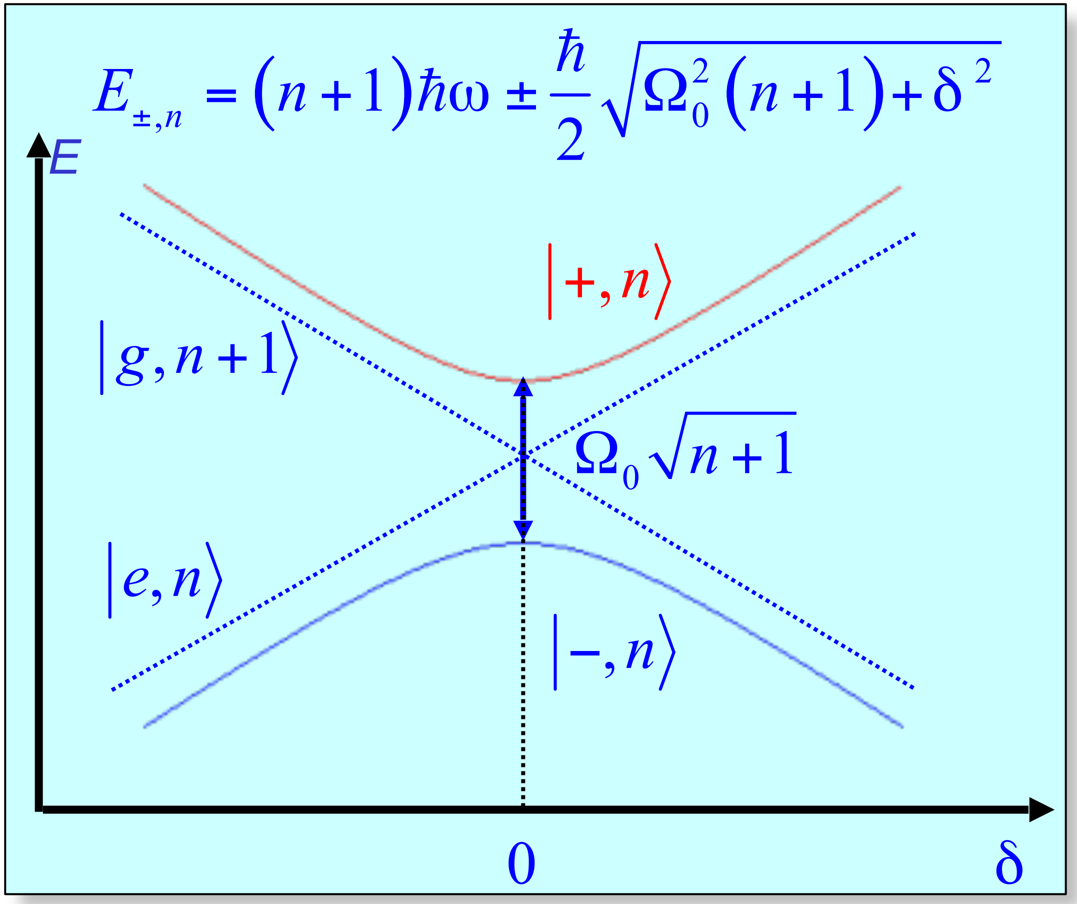
Set $r_1 \equiv \langle\hat{\sigma}_{1,I}\rangle, r_2 \equiv \langle\hat{\sigma}_{2,I}\rangle, r_3 \equiv \langle\hat{\sigma}_{3,I}\rangle$, then

$$\boxed{\frac{d\vec{r}}{dt} = \vec{\Omega} \times \vec{r}}$$

Bloch vector

SHOW THAT!

The dressed atom and the dispersive limit



Exercise: Show that, for $|\delta| \gg \Omega_0\sqrt{n+1}$,

AC Stark shift $\Rightarrow$

$$\Delta E_{e,n} \approx \hbar(\Omega_0^2/4\delta)(n+1)\,,$$
$$\Delta E_{g,n} \approx -\hbar(\Omega_0^2/4\delta)n\,.$$

QUIZZ

Where has the coherence gone? Can you describe what happens with the environment?

Why are coherent states more stable than superpositions?

Does this answer Einstein's question?