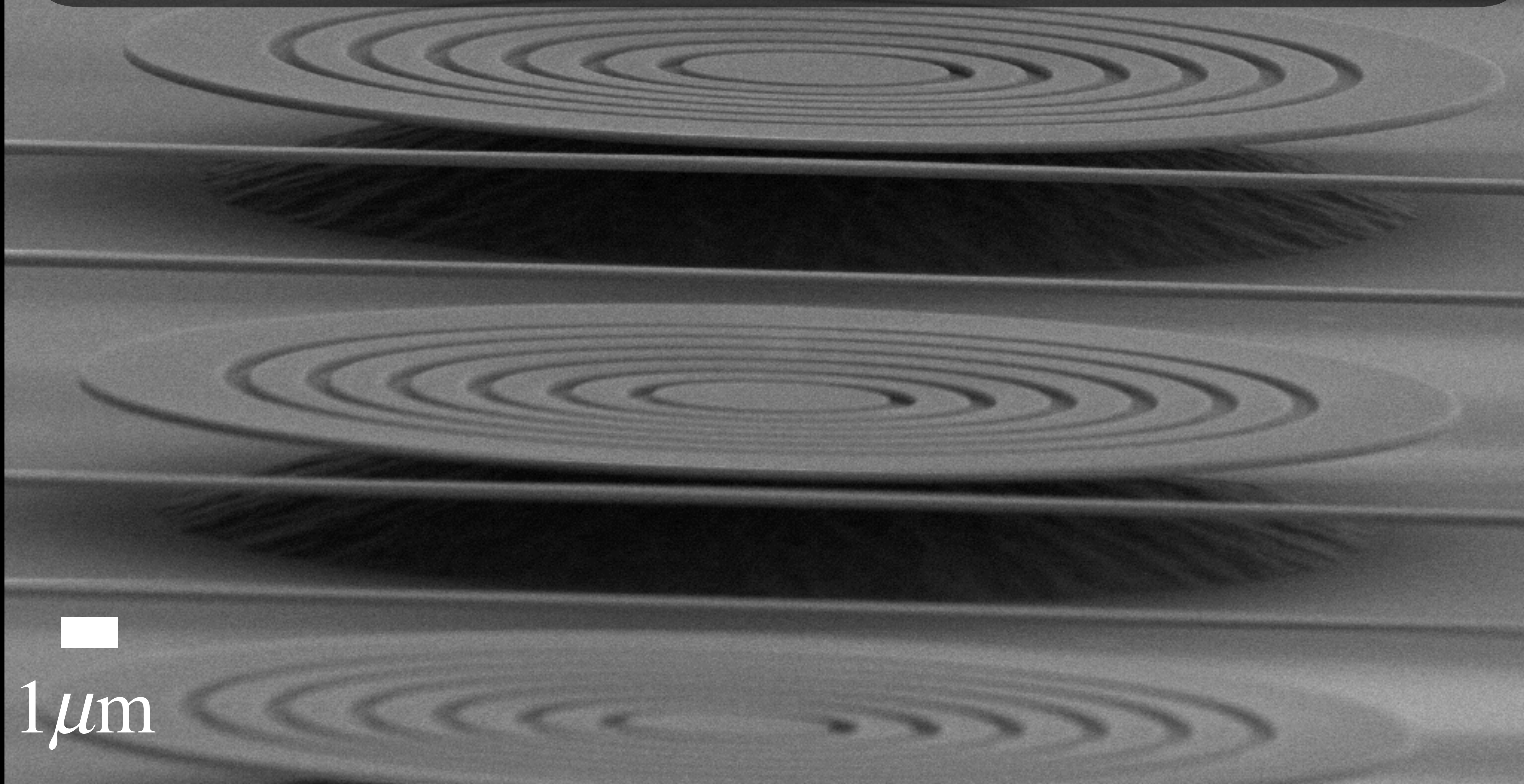
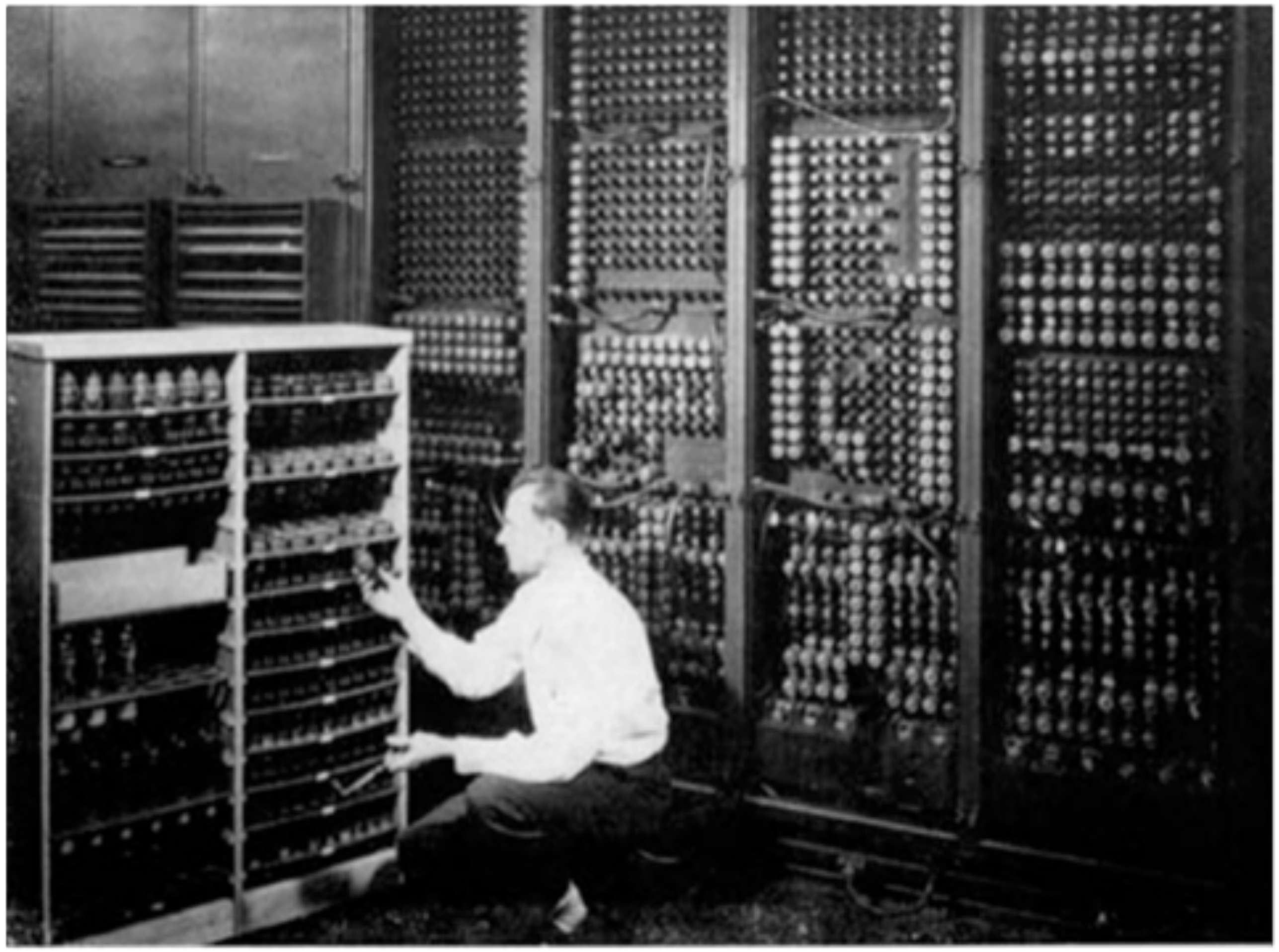
Cavity optomechanics: keeping light and sound under control

Gustavo Wiederhecker
University of Campinas



Silicon Nanophotonics



Z2 (Germany, 1939)

- **1.2 flops**
- 300 kg
- 1 kW

ENIAC (USA, 1946)

- **300 flops**
- 27,000 kg
- 150 kW

Konrad Zuze Internet Archive (http://zuse.zib.de/z2)
Wikipedia contributors. "ENIAC." Wikipedia, The Free Encyclopedia.

Silicon Nanophotonics

Z2 (Germany, 1939)

- **1.2 flops**
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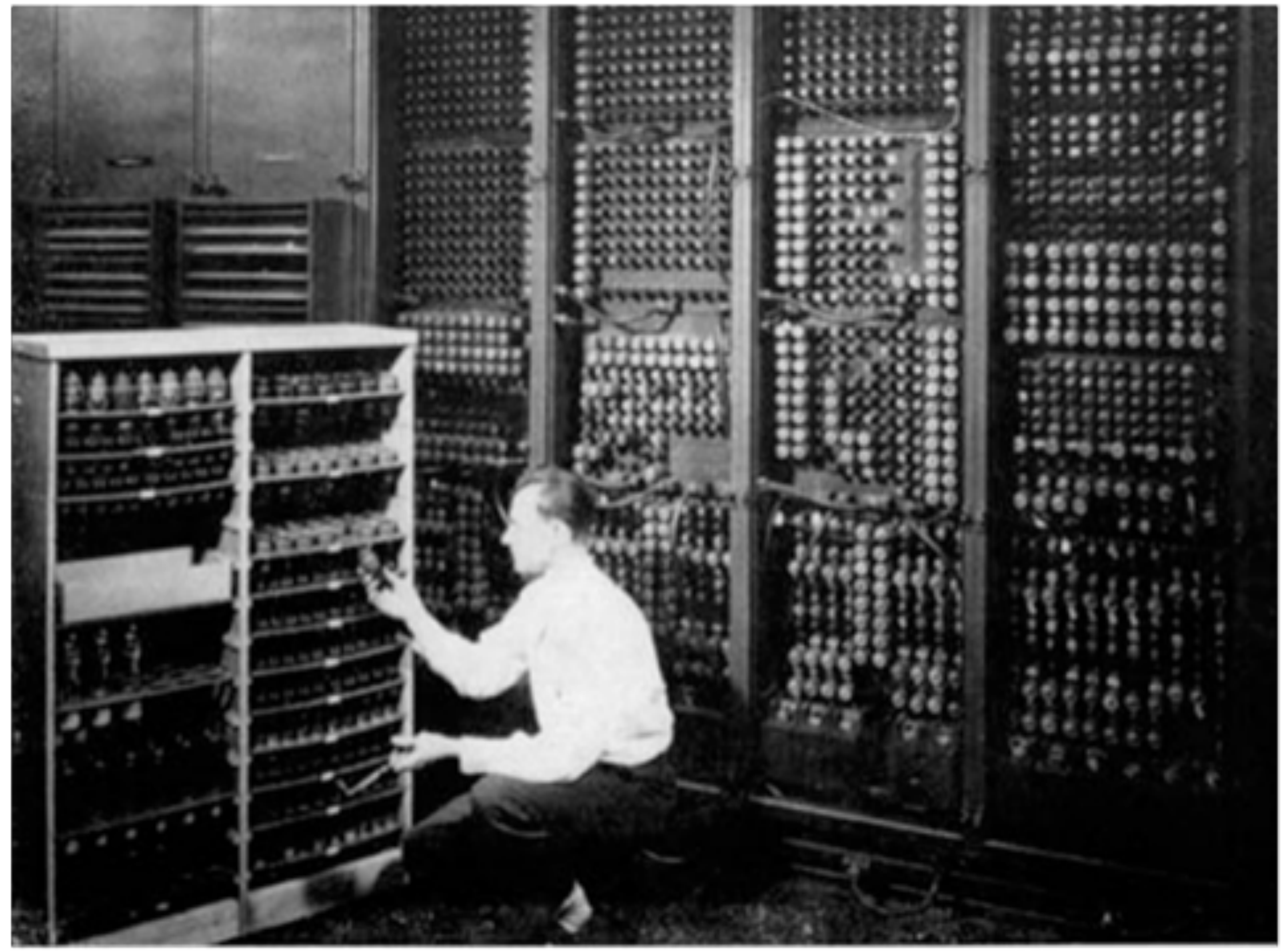

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Iphone 6 (2014)

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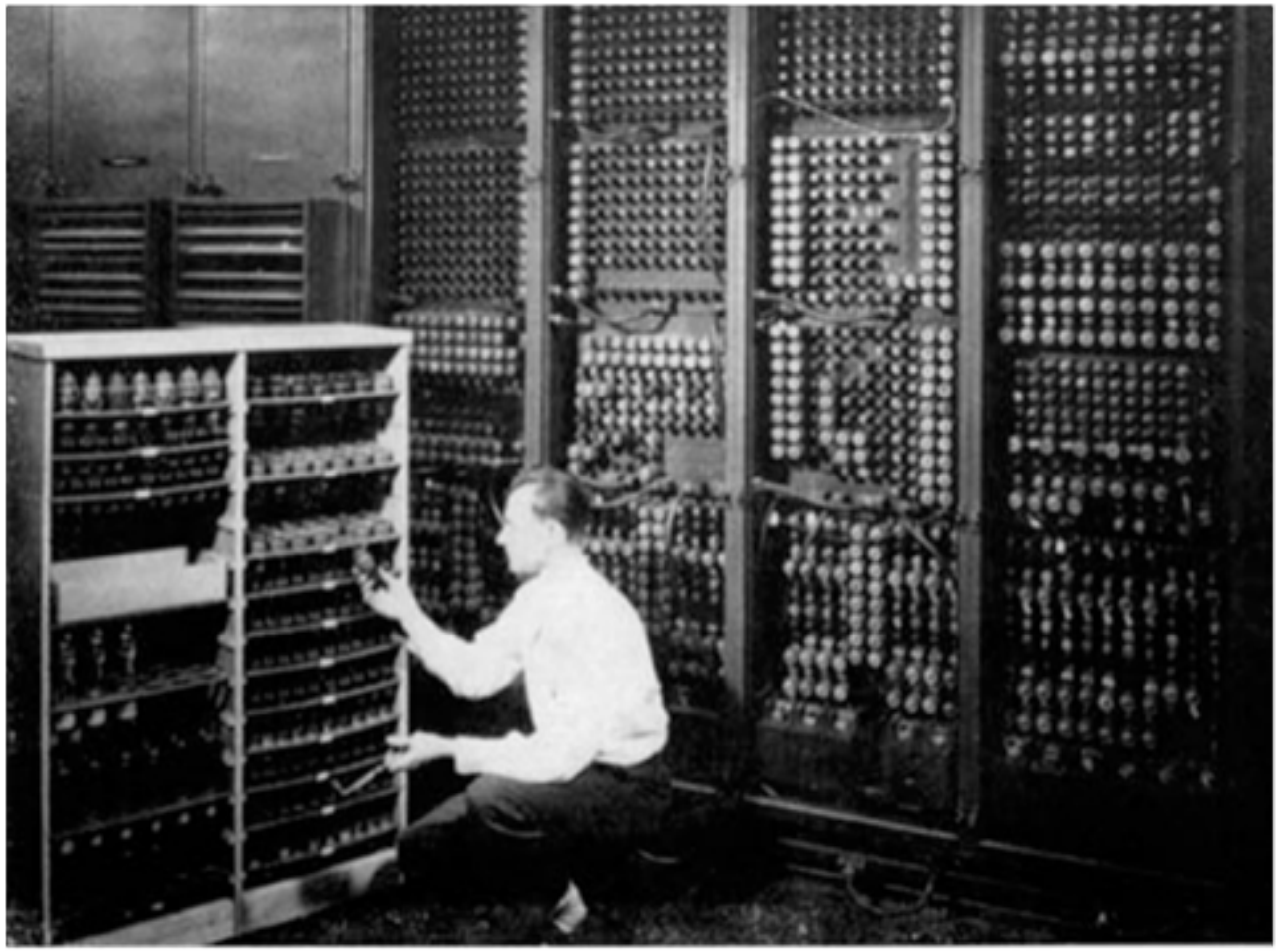
Silicon Nanophotonics



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- **1.2 flops**
- 300 kg
- 1 kW

ENIAC (USA, 1946)

- **300 flops**
- 27,000 kg
- 150 kW

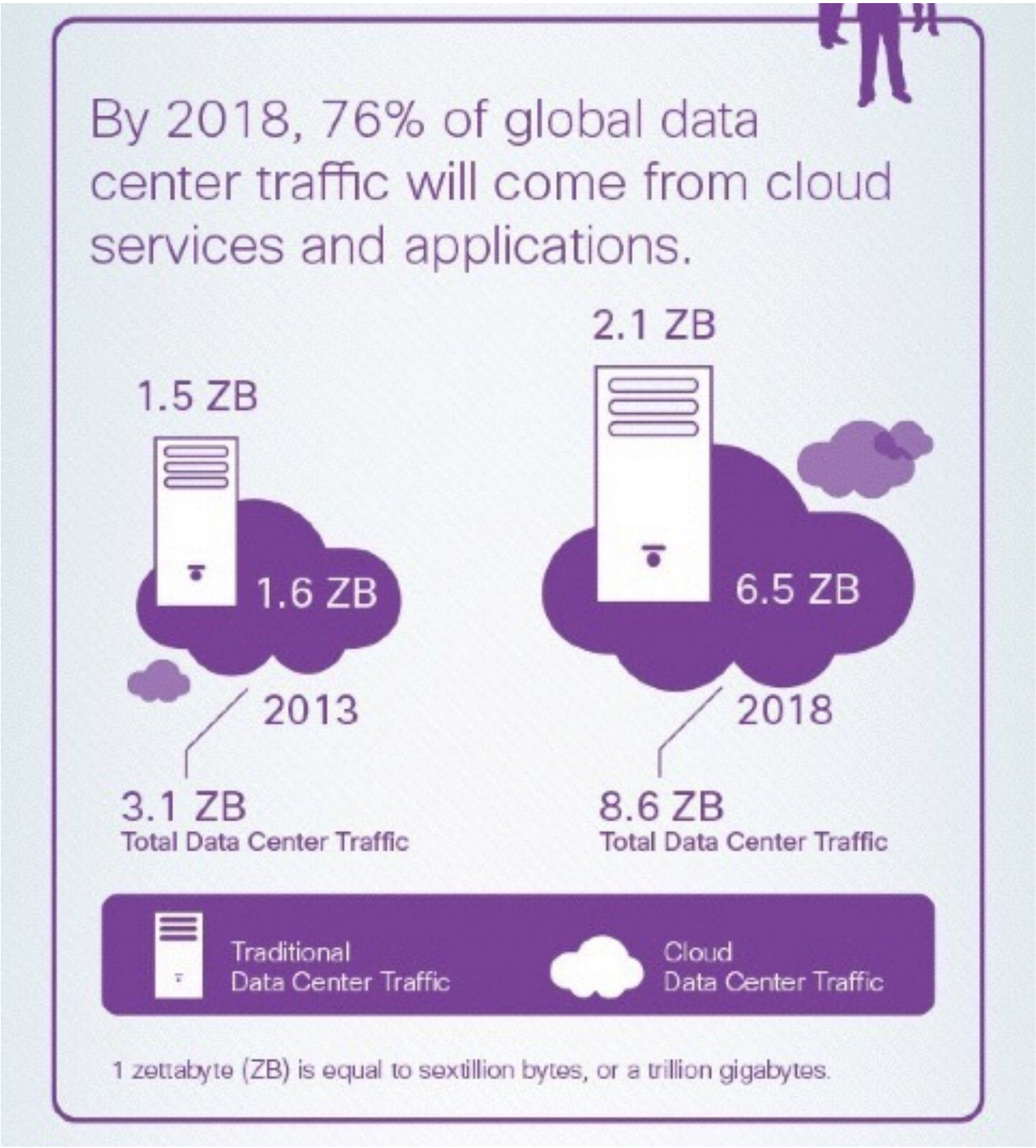
Iphone 6 (2014)

- **150 Gflops**
- 130 g
- battery

Konrad Zuze Internet Archive (http://zuse.zib.de/z2)
Wikipedia contributors. "ENIAC." Wikipedia, The Free Encyclopedia.

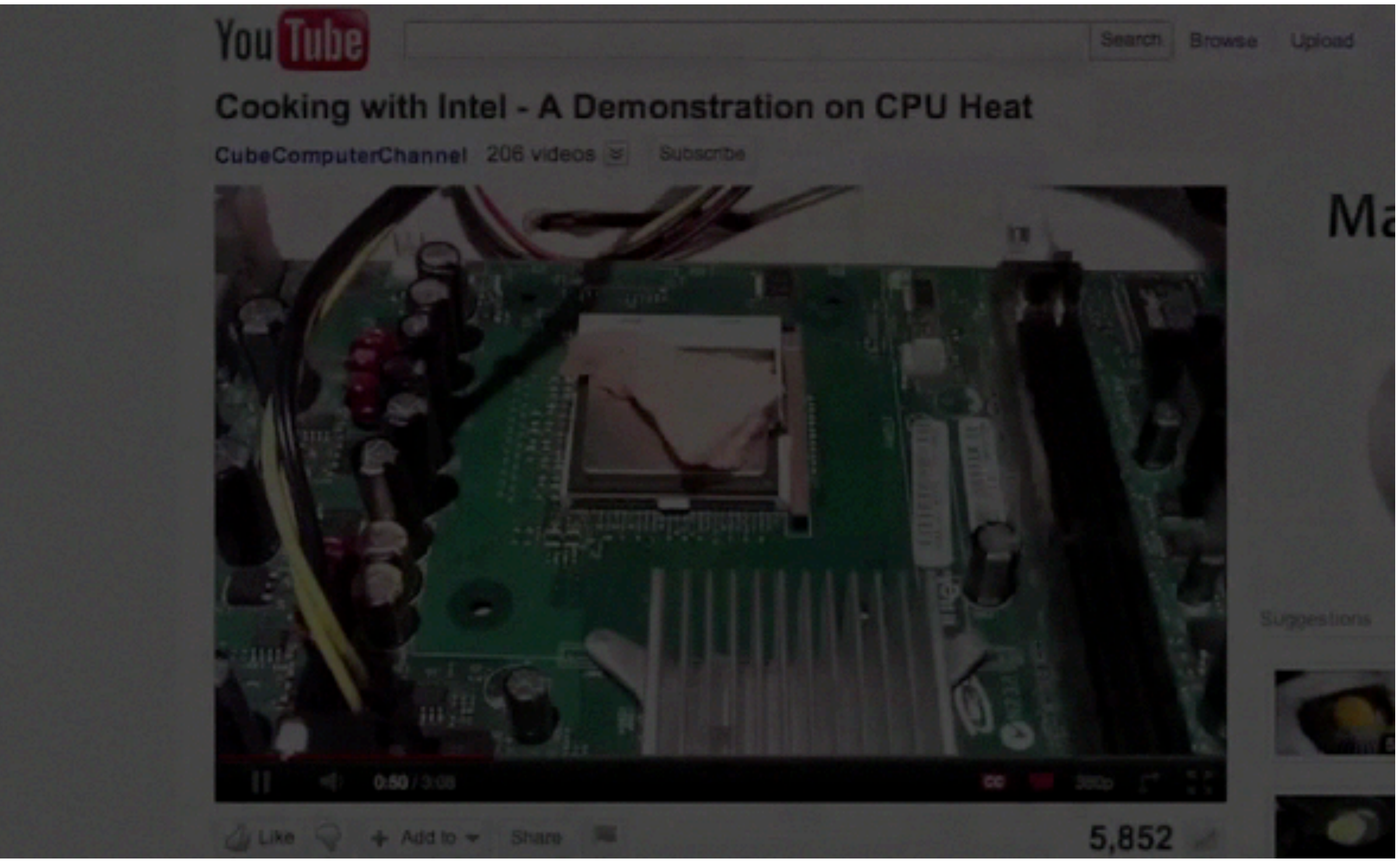
Silicon Nanophotonics

Datacenter bottleneck

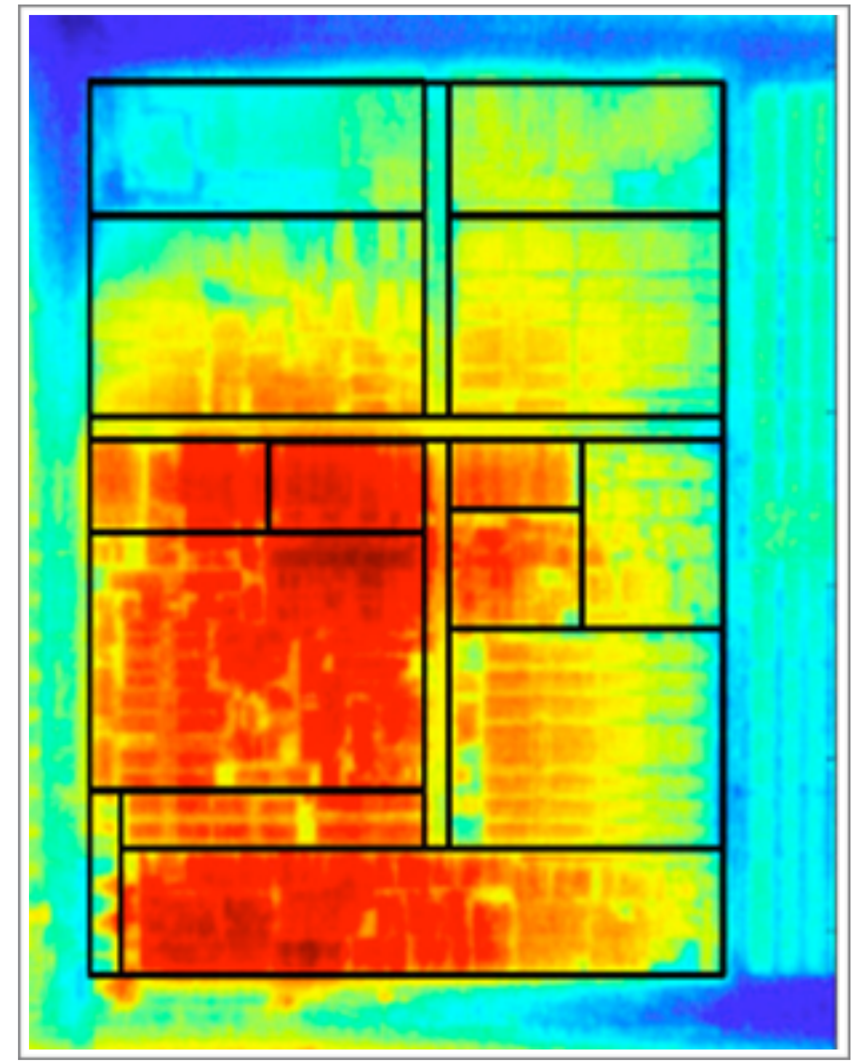


Cisco Global Cloud Index

Temperature Issues



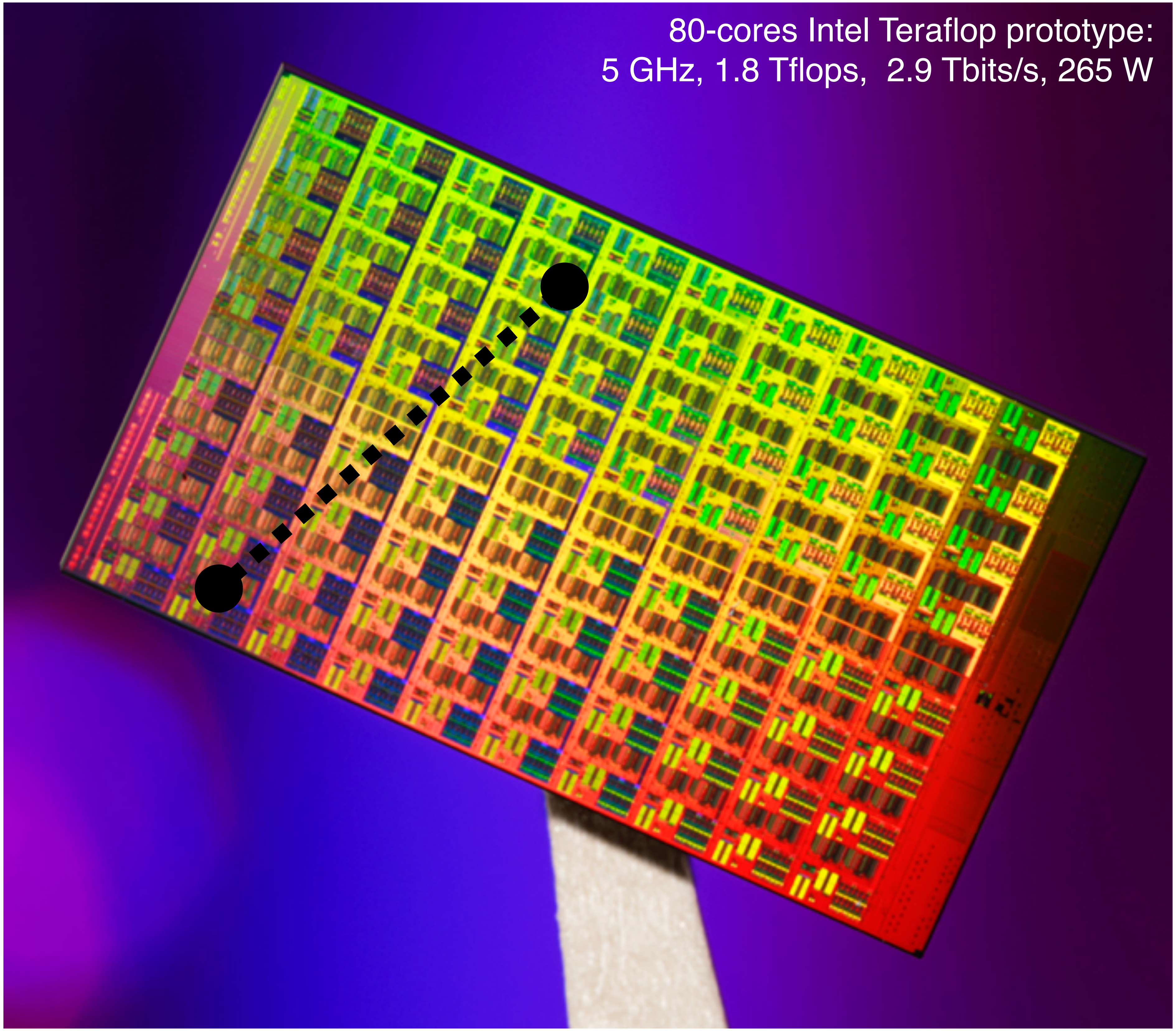
Bacon “on-chip”



≈mm

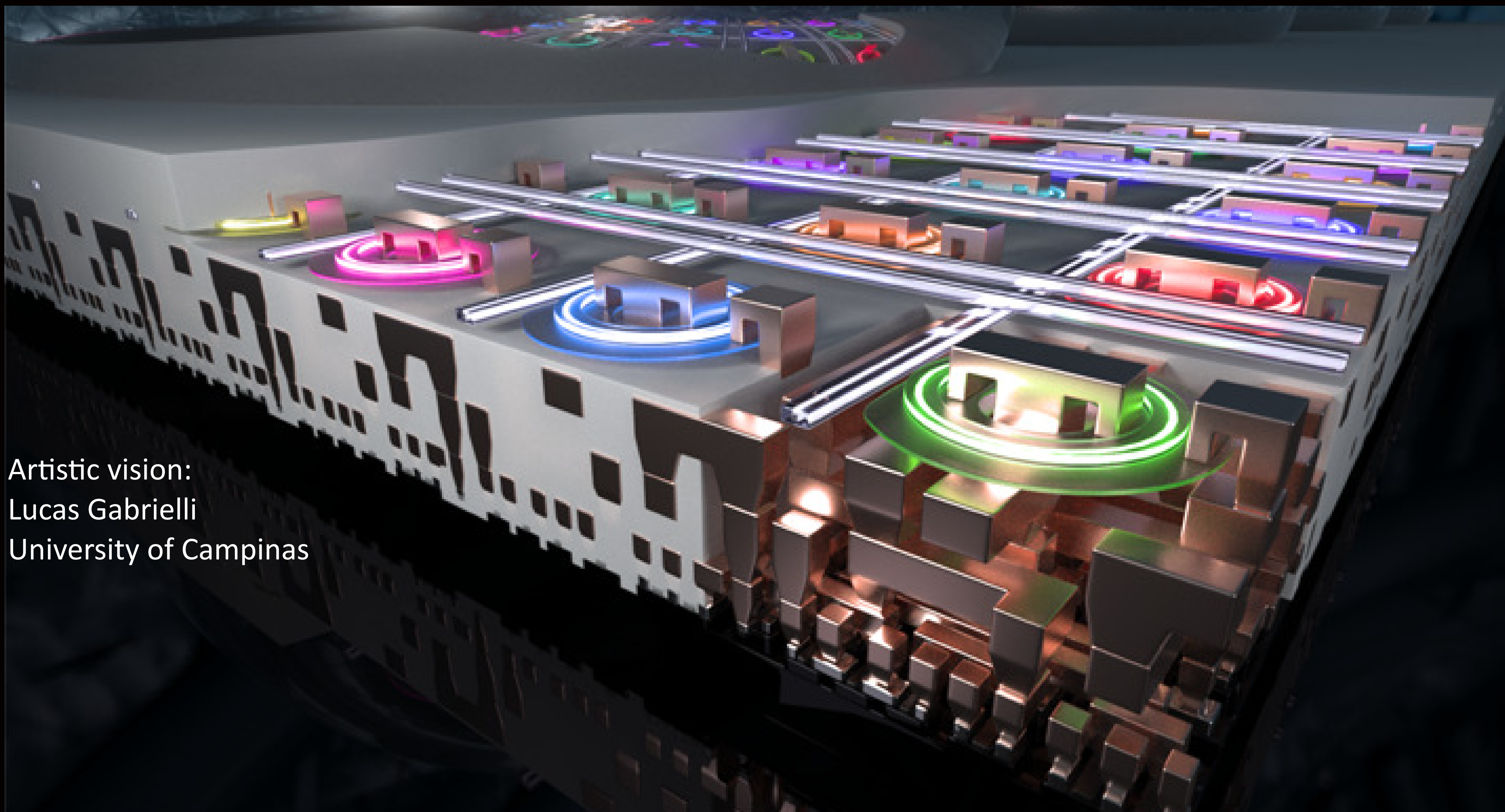
Temperature map
ΔT≈80 K

The multi-core solution



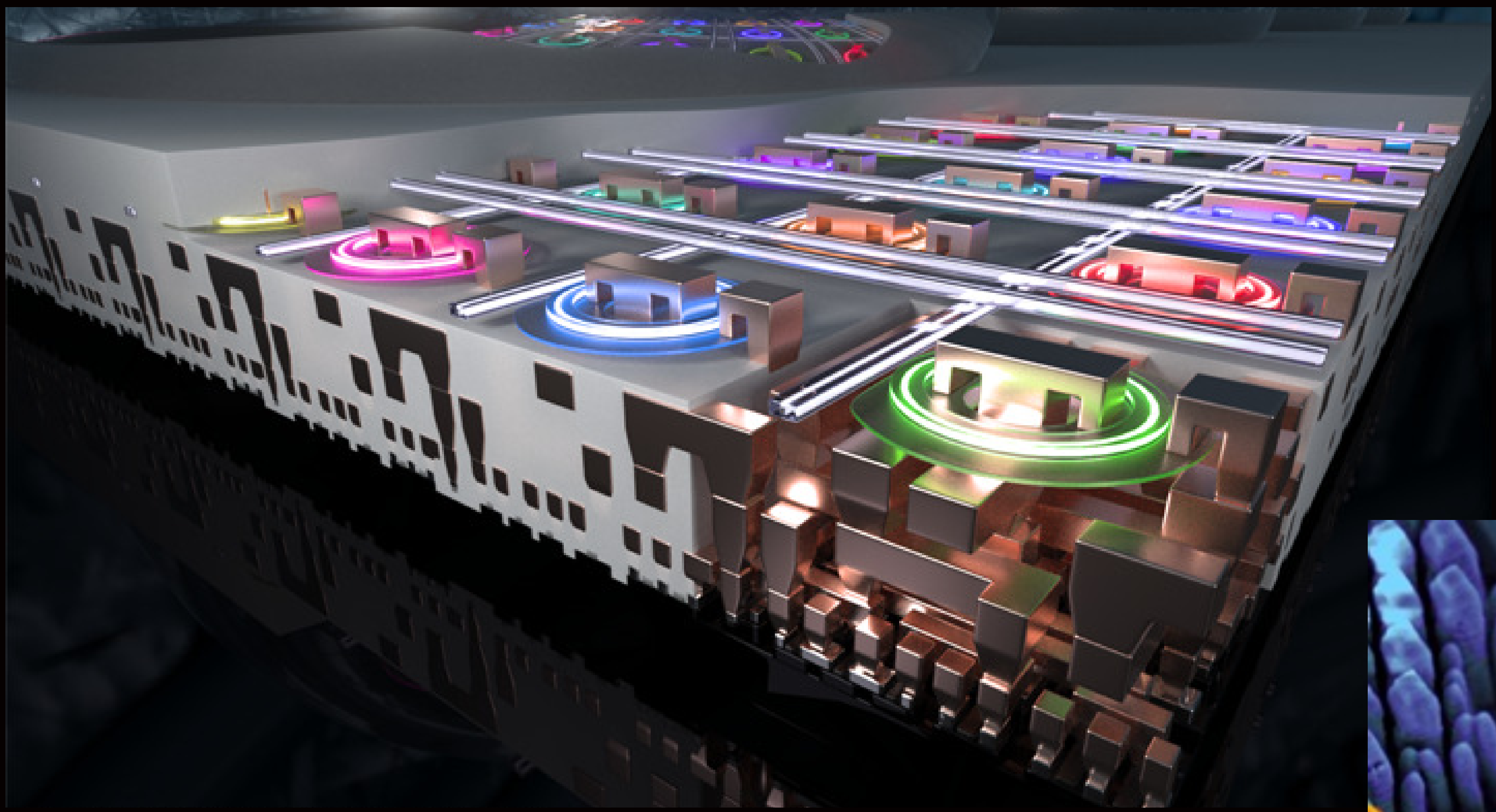
http://www.intel.com/pressroom/kits/teraflops/

Si Nanophotonics

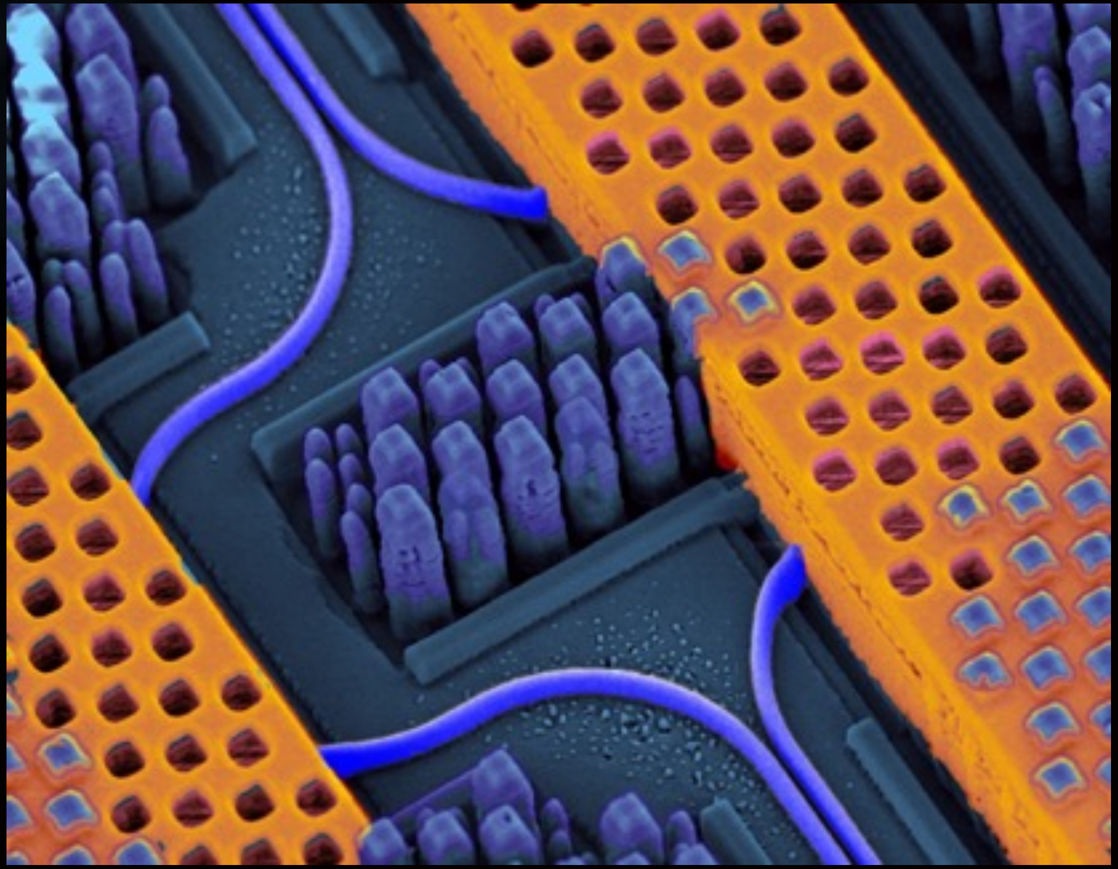


Artistic vision:
Lucas Gabrielli
University of Campinas

Si Nanophotonics



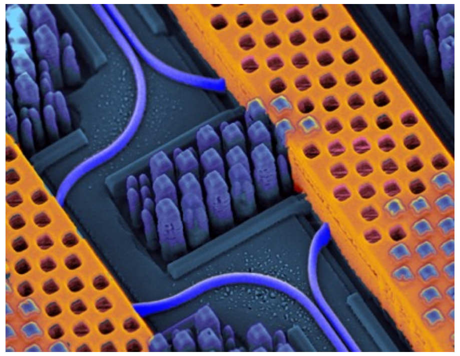
What kind of nanophotonic building blocks we need here?



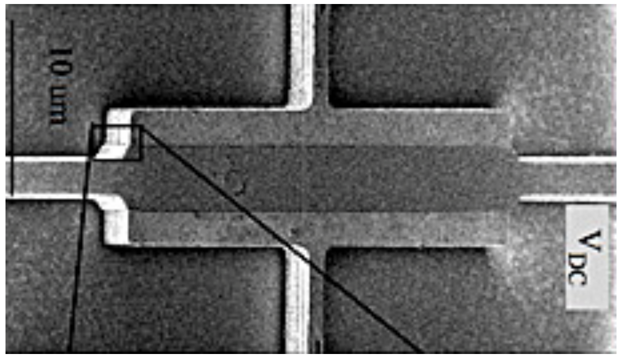
IBM 90-nm BEOL integration (2013)

Vlasov. Y, Presented at SPIE Photonics West 2015

Technological viewpoint: Si Nanophotonics Building Blocks

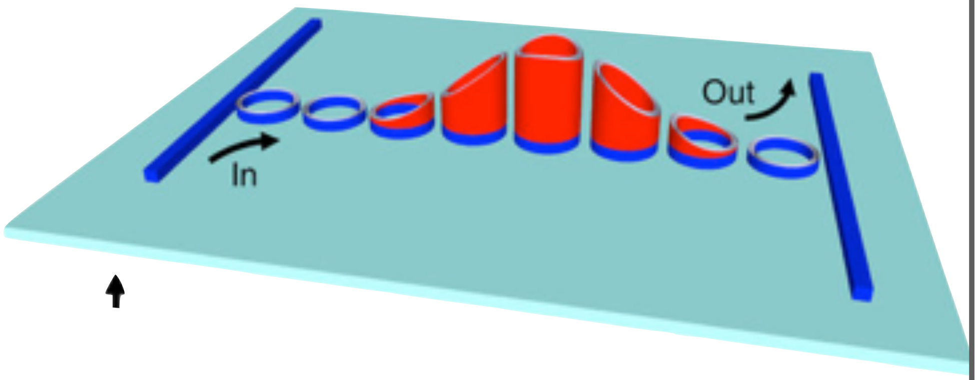


Silicon MEMS oscillators



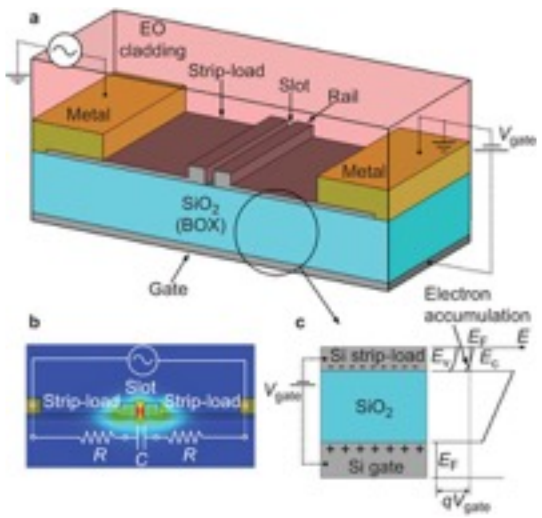
Bhave's group (Cornell)
Journal of MEMS (2009)

Wavelength Conversion



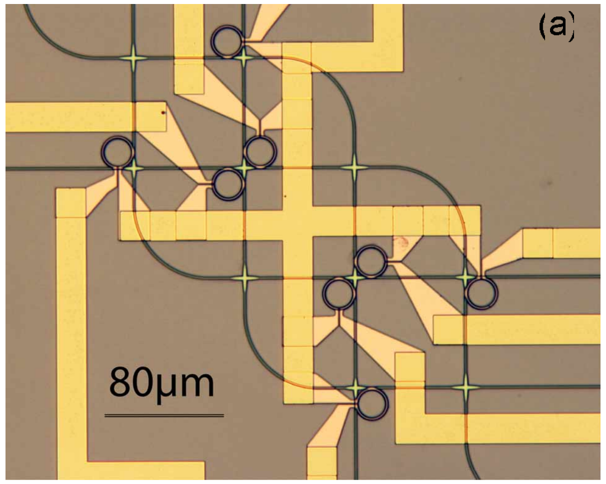
Melloni Group (Milano)
Nature Comm. 2011

Modulators



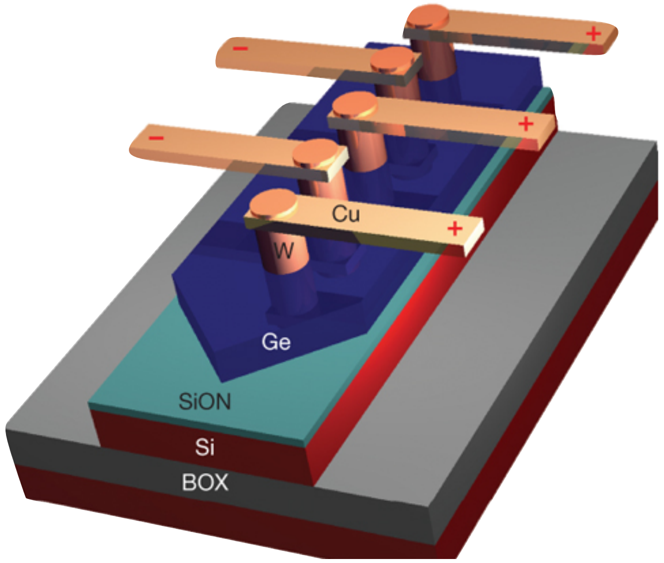
Leuthold group (KIT)
Nature 2005

Routers



Lipson's group (Cornell)
Opt. Express 2008

Photodetectors

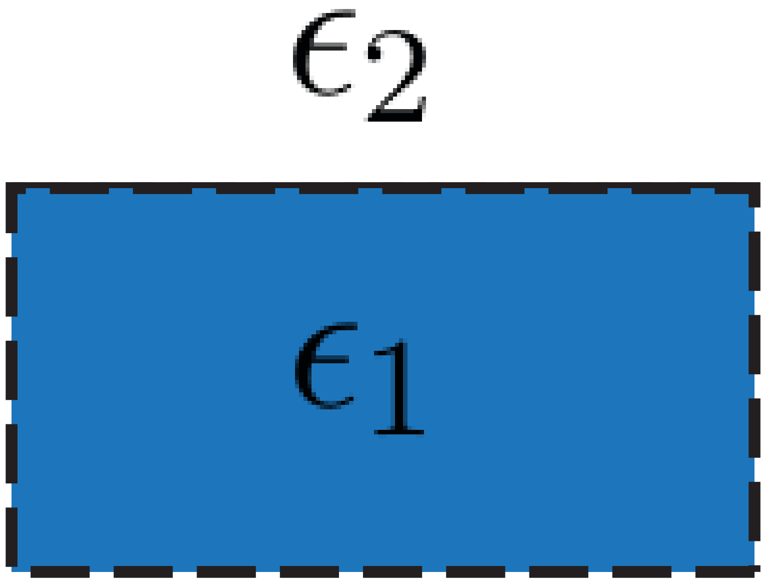


Vlasov's group (IBM)
Nature 2010

Outline

- **Optical and acoustic mode interaction**
- Optical force actuation
- Dynamical back-action
- Optomechanical clocks
- Bullseye - a case study
- Outlook

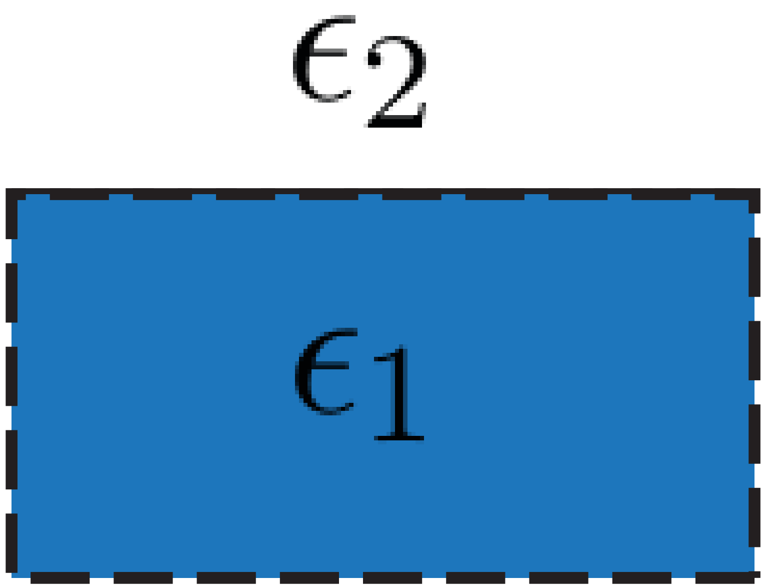
Light-sound interaction



Dielectric
waveguide

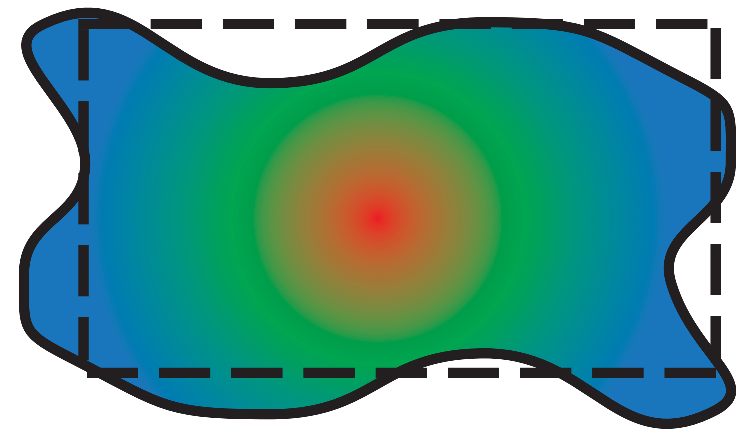
E. P. Ippen and R. H. Stolen, "Stimulated Brillouin scattering in optical fibers," Appl. Phys. Lett., vol. 21, pp. 539–541, Dec. 1972.
R. H. Stolen - "The Early Years of Fiber Nonlinear Optics". JLT, VOL. 26, NO. 9, MAY 1, 2008

Light-sound interaction

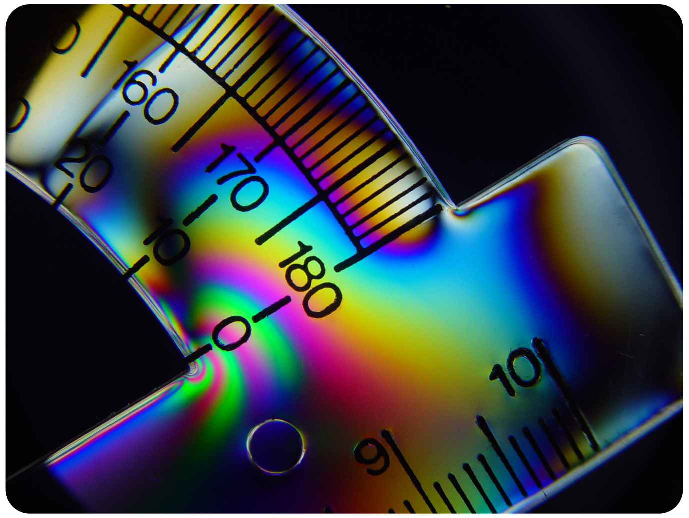


Dielectric waveguide

Mechanical mode Deformation



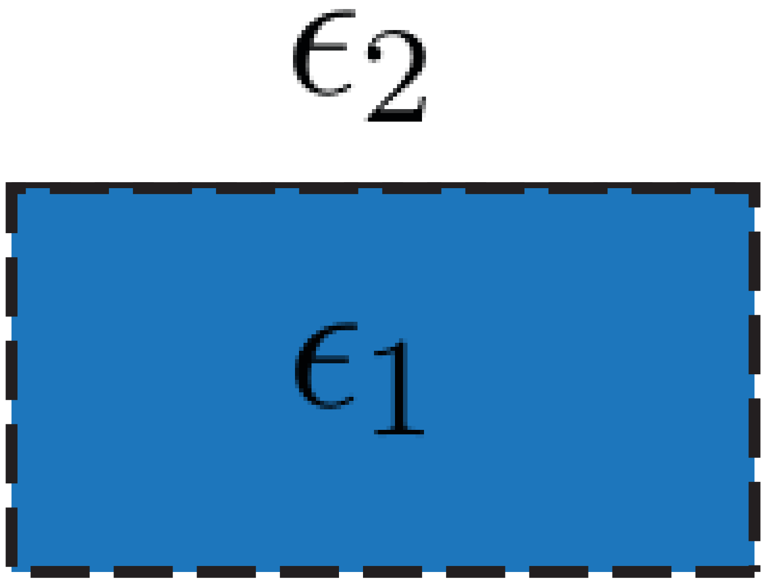
- boundary distortion
- strain



strain-optic effect

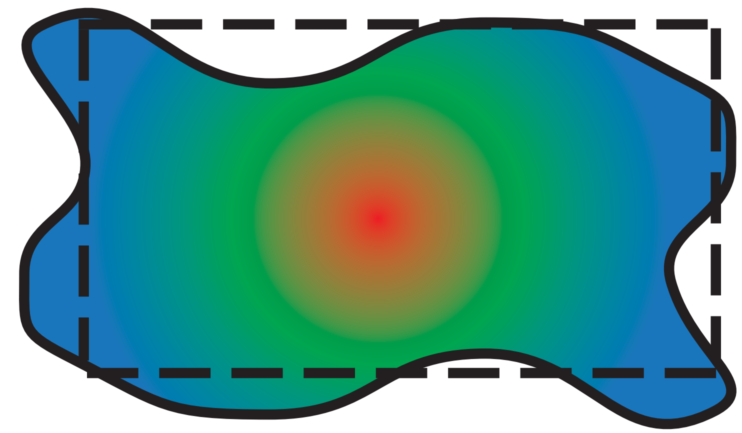
S. G. Johnson et al., Physical Review E **65**, (2002)

Light-sound interaction



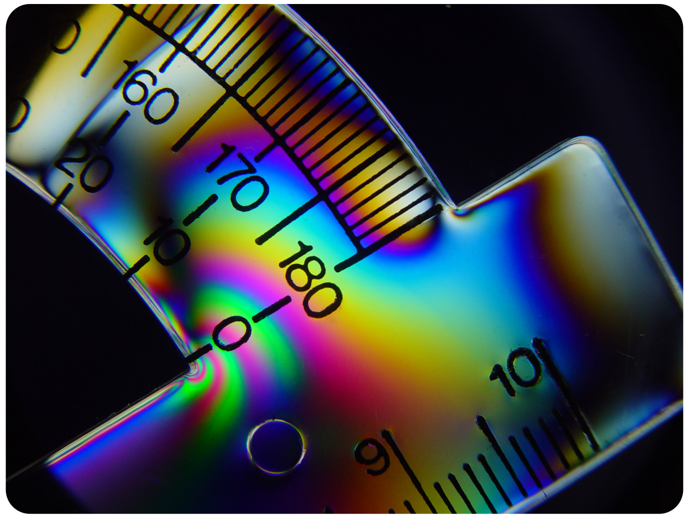
Dielectric waveguide

Mechanical mode Deformation



- boundary distortion
- strain

refractive index modulation



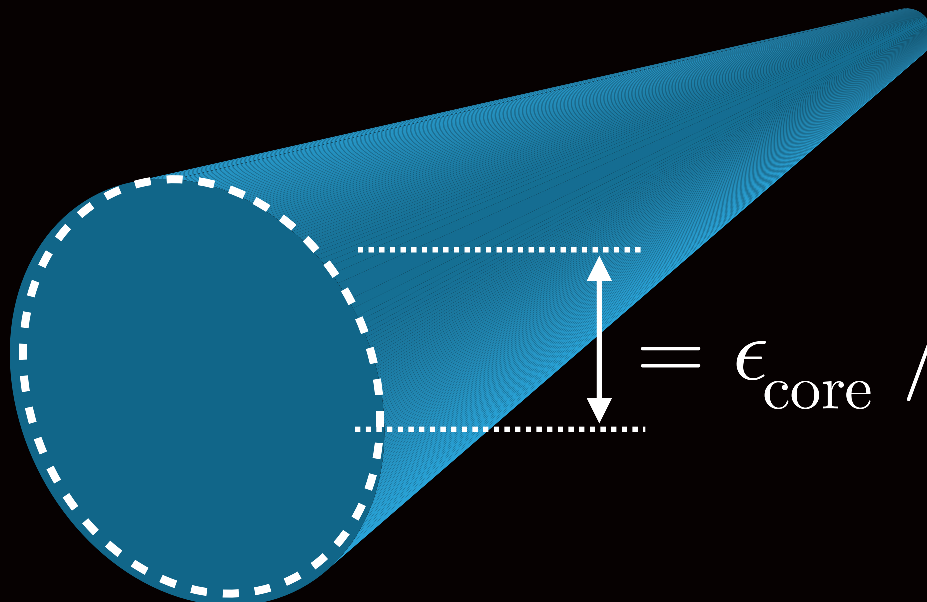
strain-optic effect

S. G. Johnson et al., Physical Review E **65**, (2002)

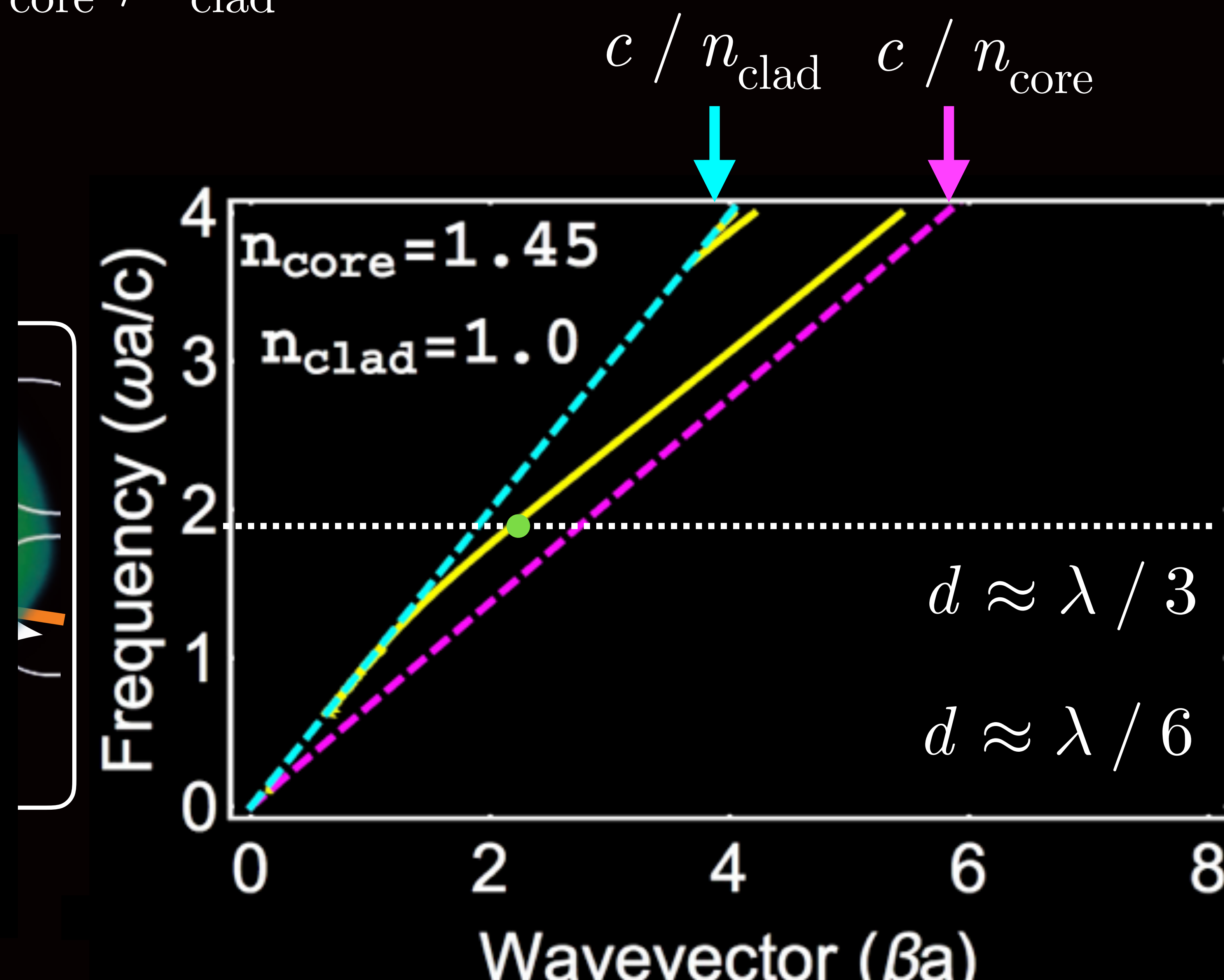
Sub-wavelength confinement

$n_{\mathrm{Si}} \approx 3.5$

$n_{\mathrm{SiO_2}} \approx 1.45$

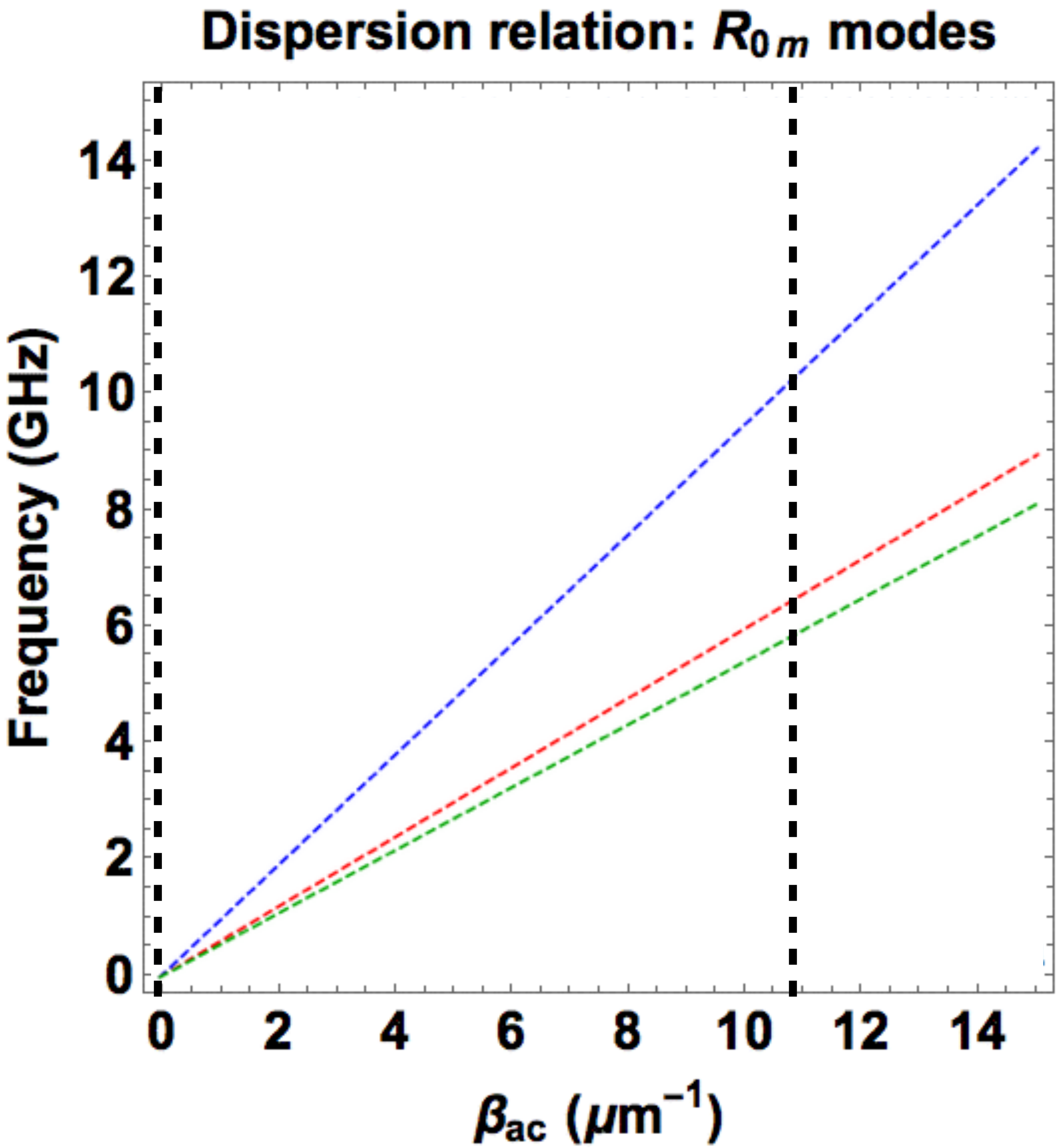


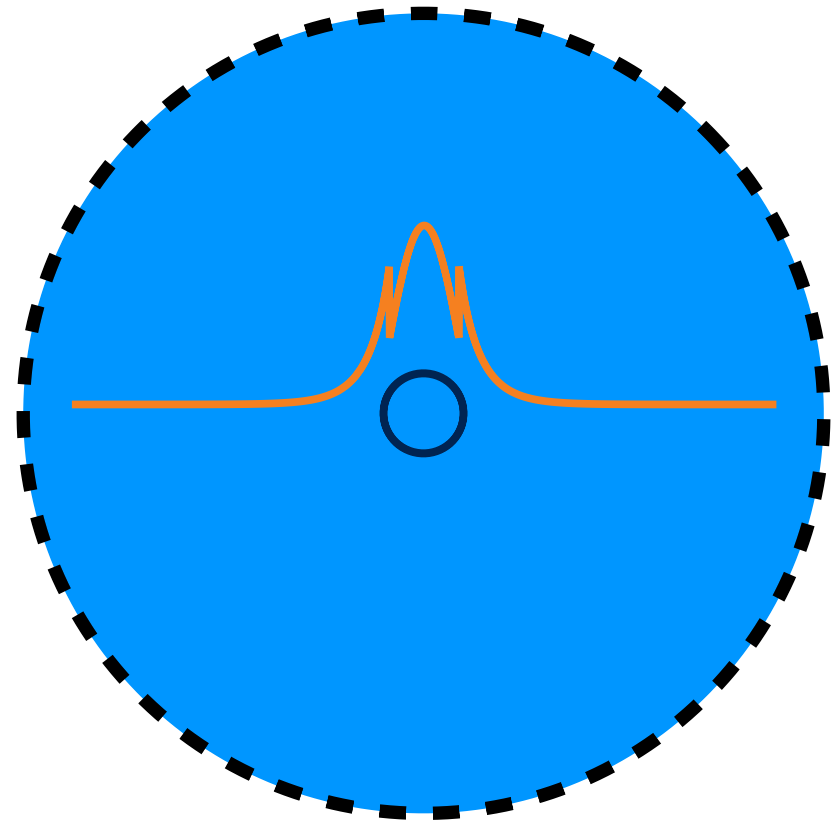
$= \epsilon_{\mathrm{core}} / \epsilon_{\mathrm{clad}}$

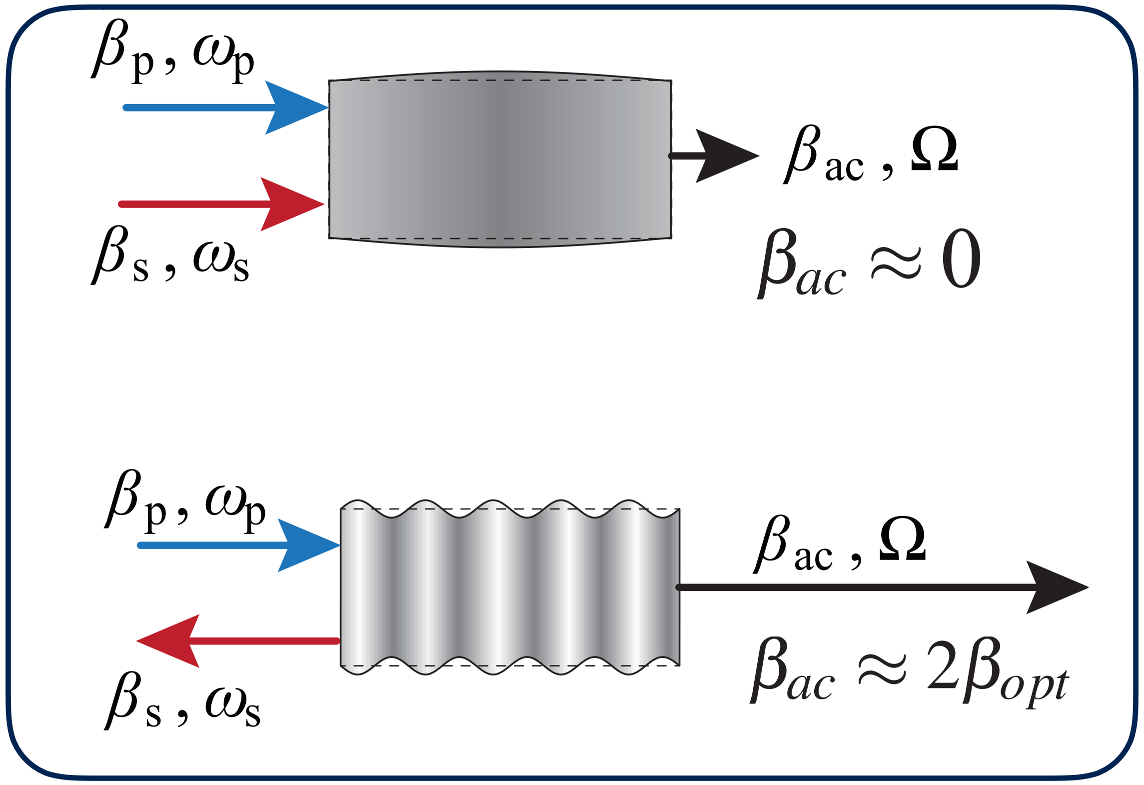


Wiederhecker, G. S. et al.
Nature Photonics 1, 115 (2007).

Light-sound interaction: Brillouin scattering

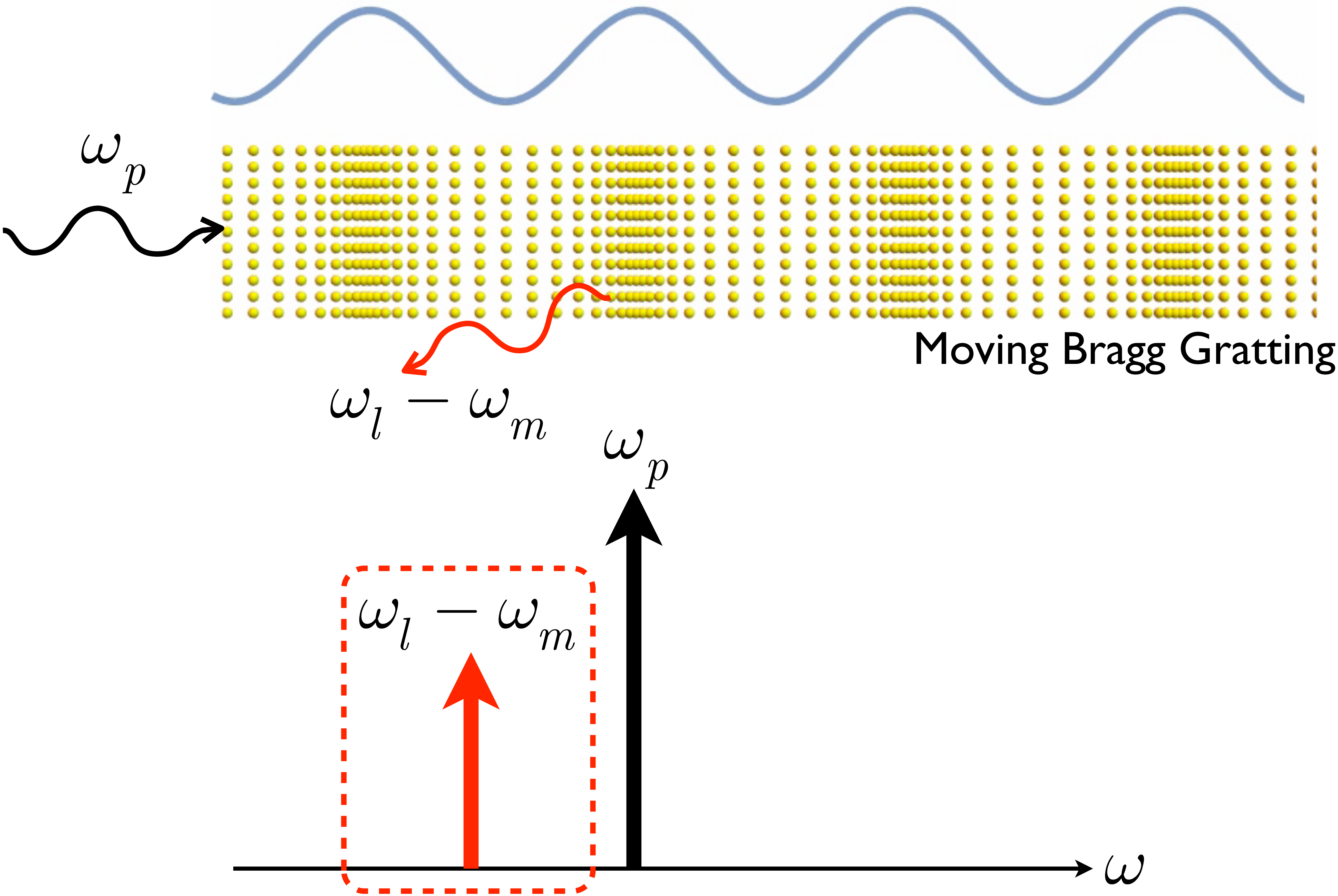




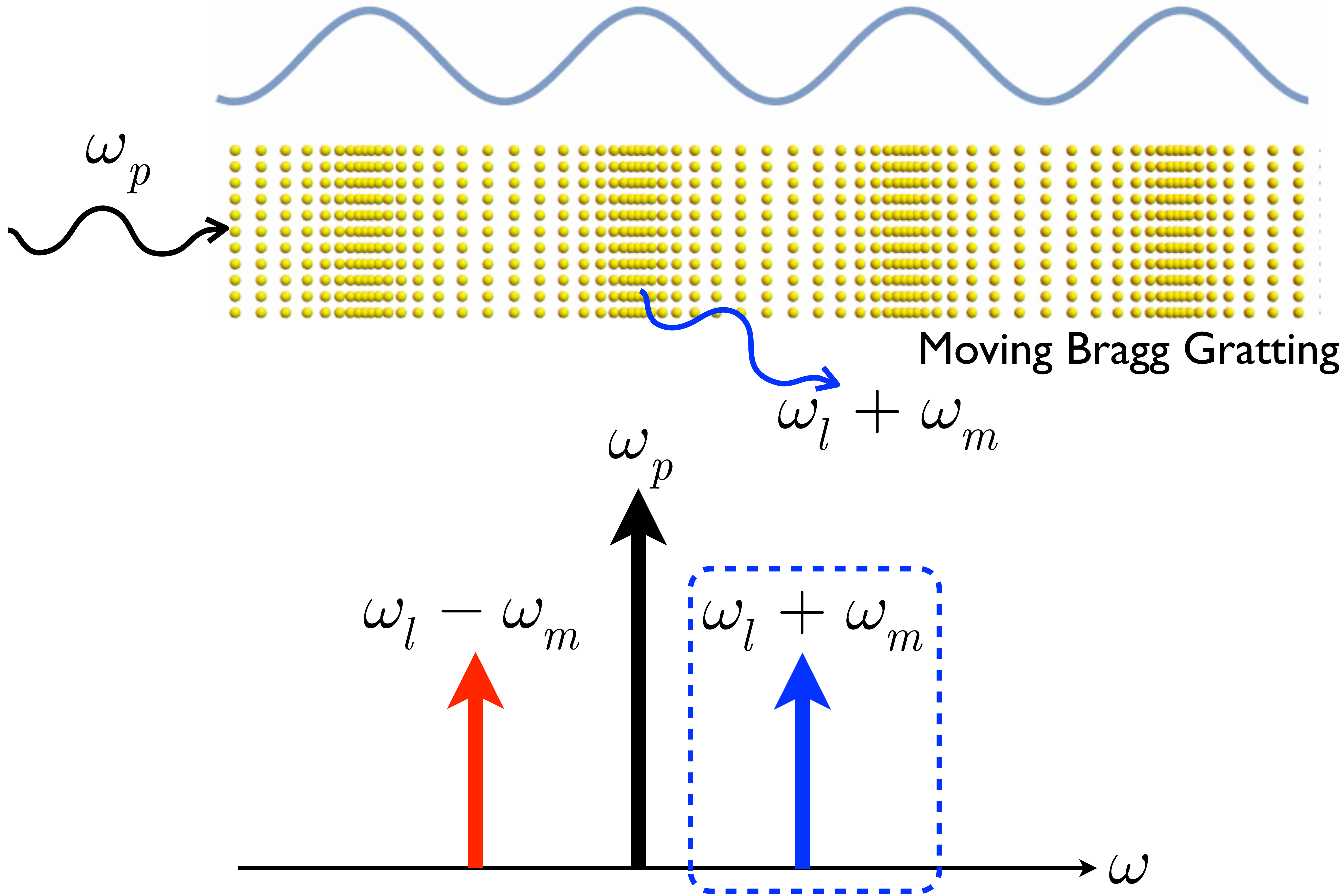


Phase matching for Stokes scattering

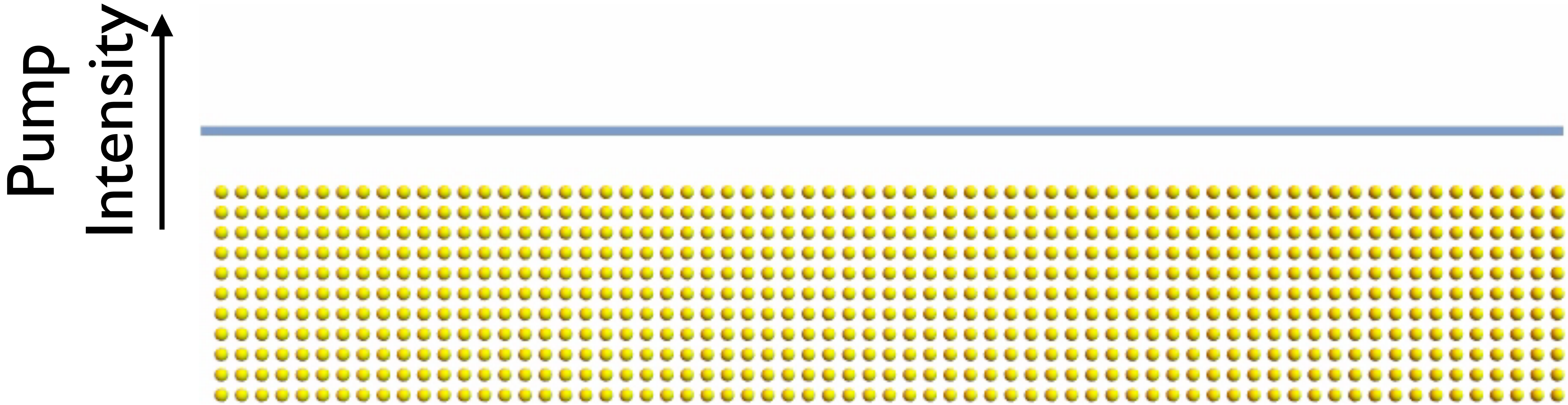
Photoelastic: Doppler Shift



Photoelastic: Doppler Shift



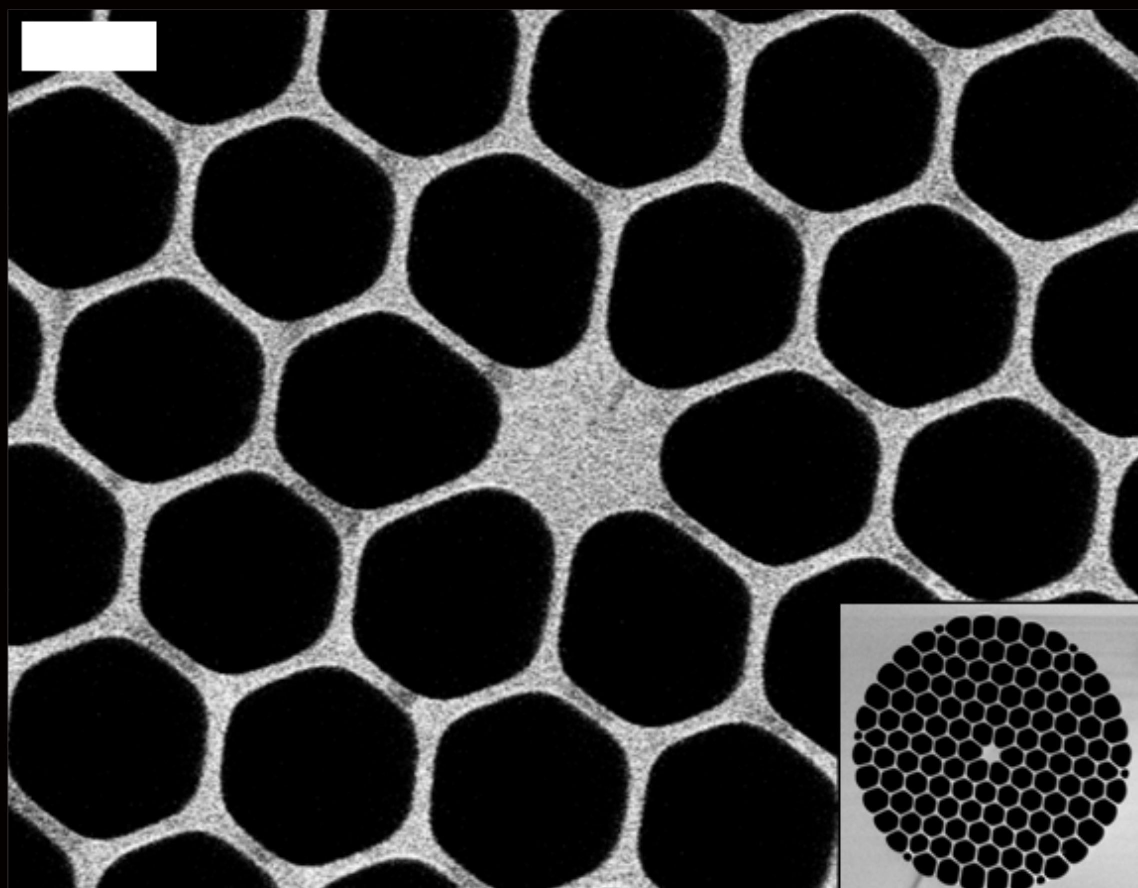
Light-sound interaction: Brillouin scattering

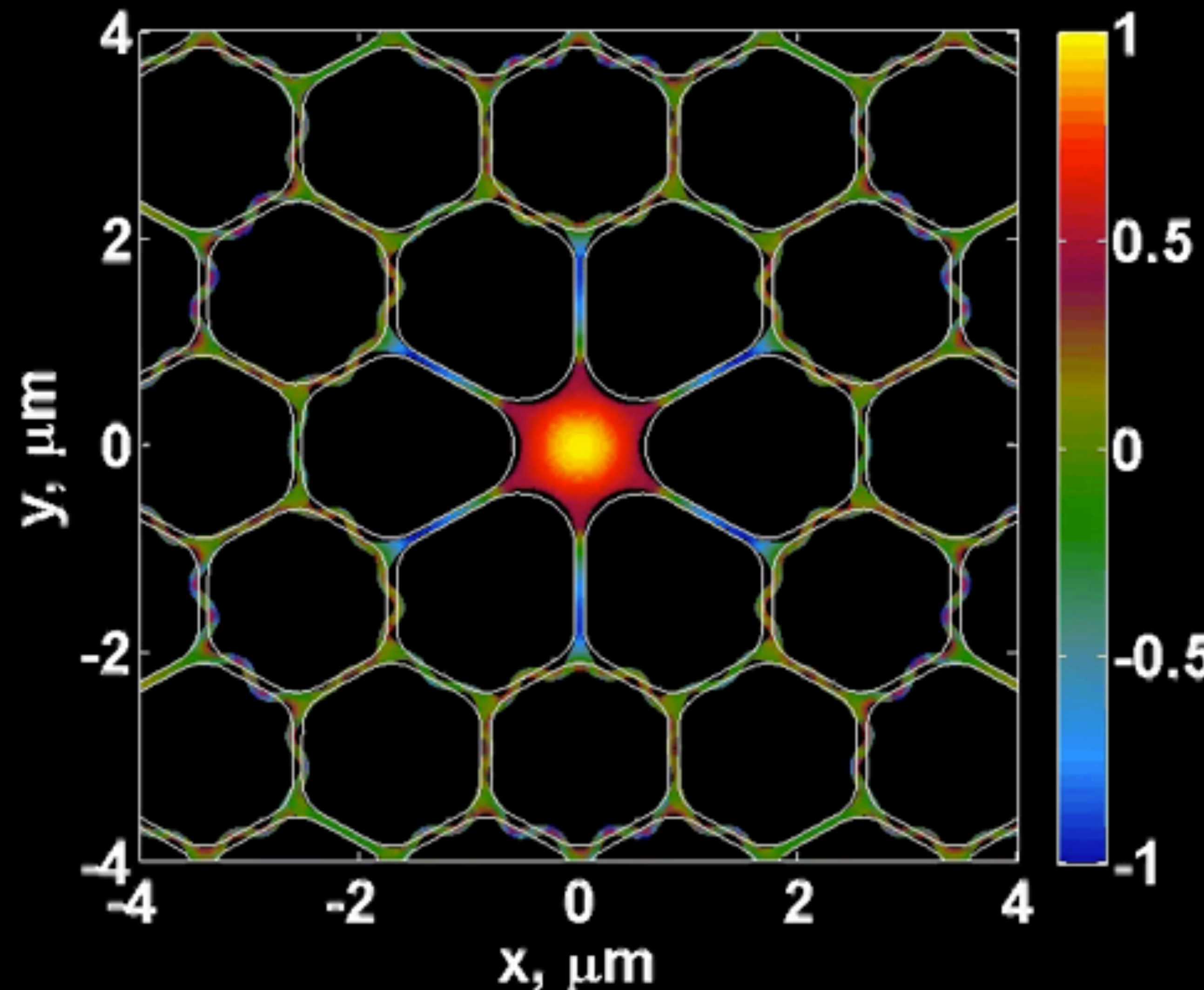


Steady Bragg Gratting

Brillouin Scattering in microw

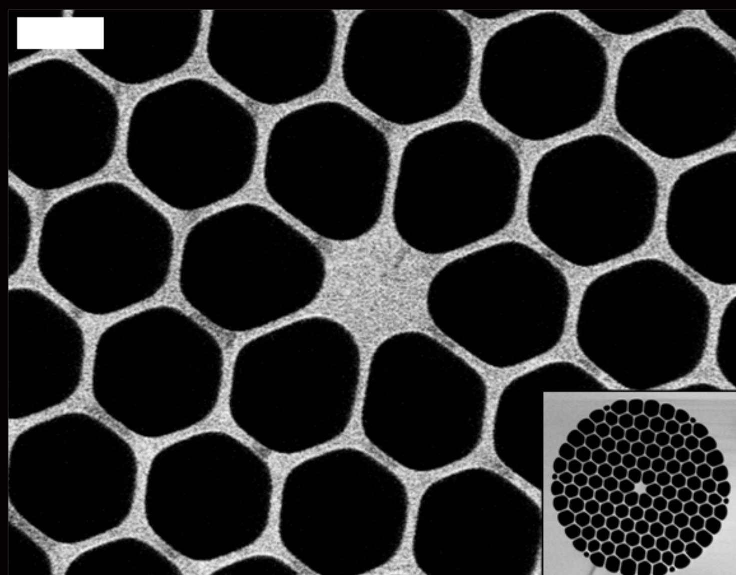
a.

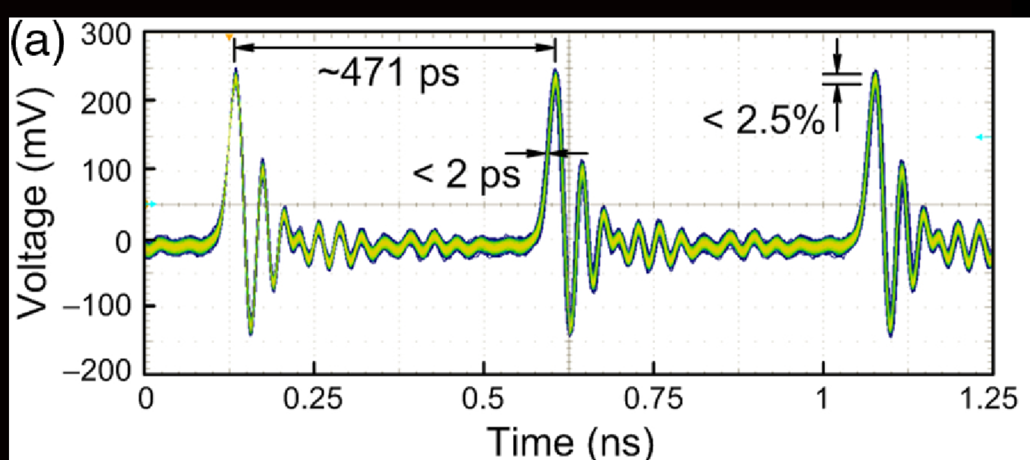




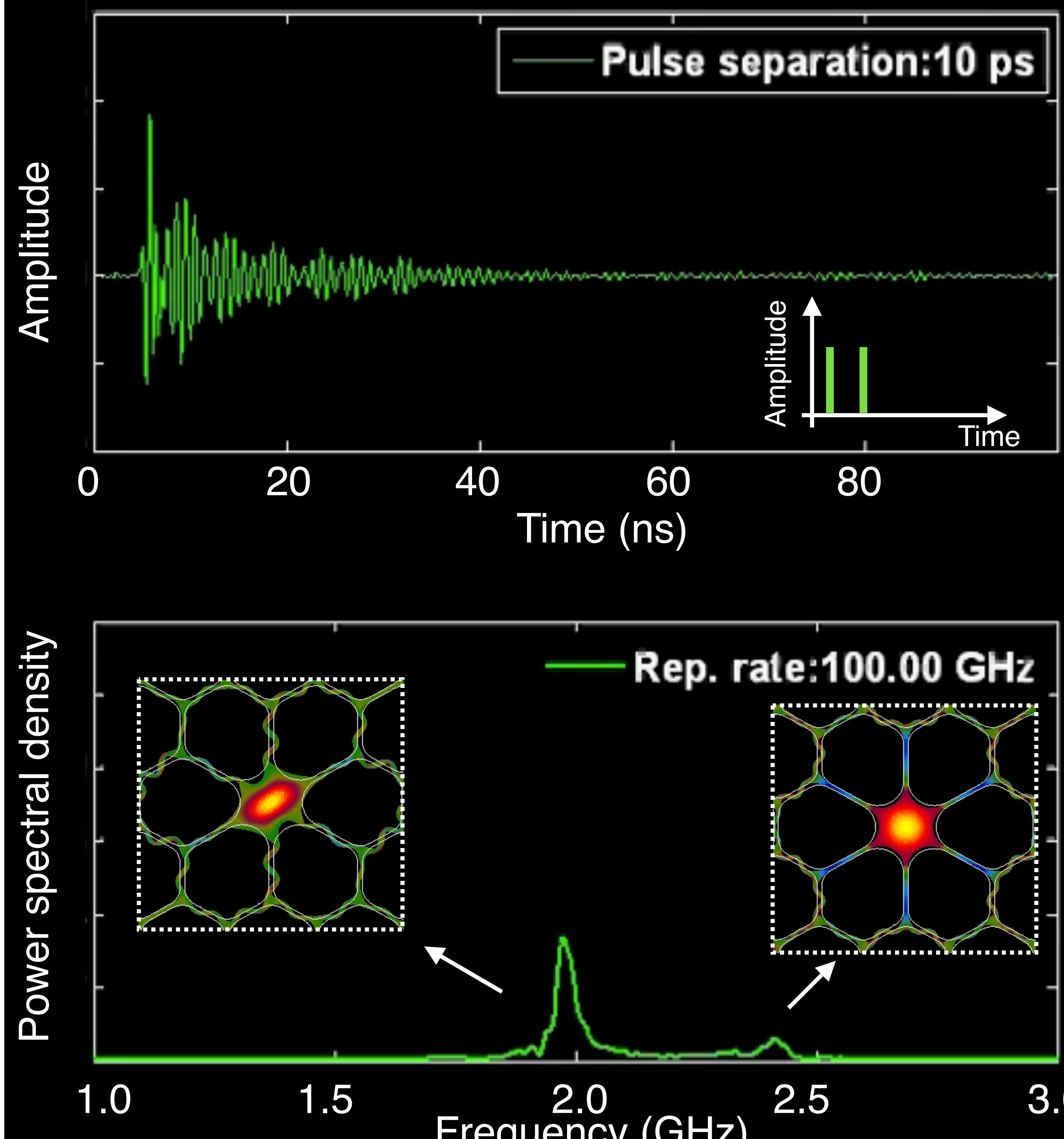
Dainese, P., et al. (2006). Nature Physics, 2(6), 388.
Dainese, P., et al (2006). Optics Express, 14(9), 4141–4150
Wiederhecker, G. S., et al. (2008). PRL, 100(20), 203903.
Kang, M., et al (2008). Applied Physics Letters, 93, 131110.
Brenn, A., et al (2009). Josa B, 26(8), 1641–1648.

Brillouin Scattering in microwires



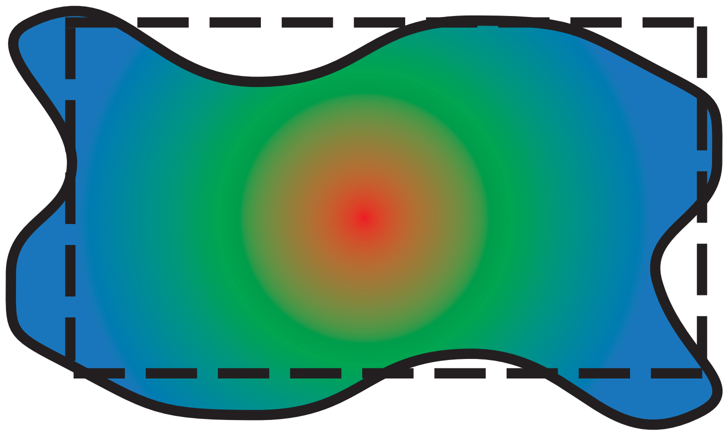


W. He, et al, Optics Express, 23(19), 24945-24954 (2015)
M. Pang, Optica, Vol. 2, Issue 4, pp. 339-342 (2015)



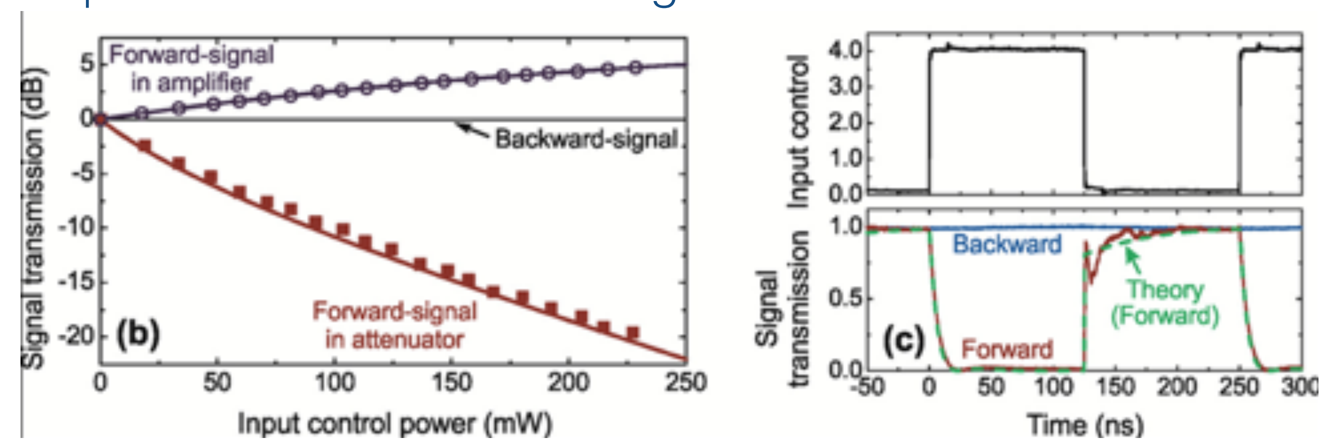
Wiederhecker, G. S., et al. (2008). PRL, 100(20), 203903.

Light-sound interaction: Brillouin scattering



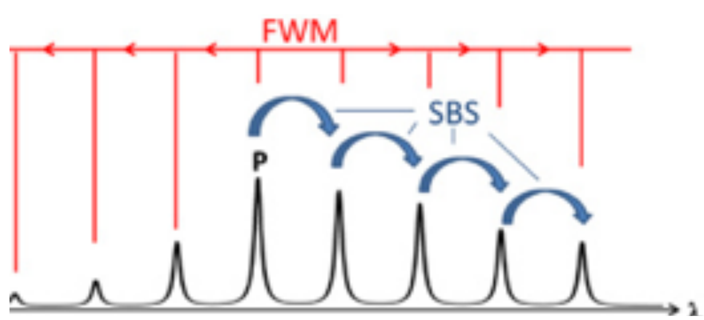
S. G. Johnson et al., Physical Review E **65**, (2002)

Optical isolator and switching



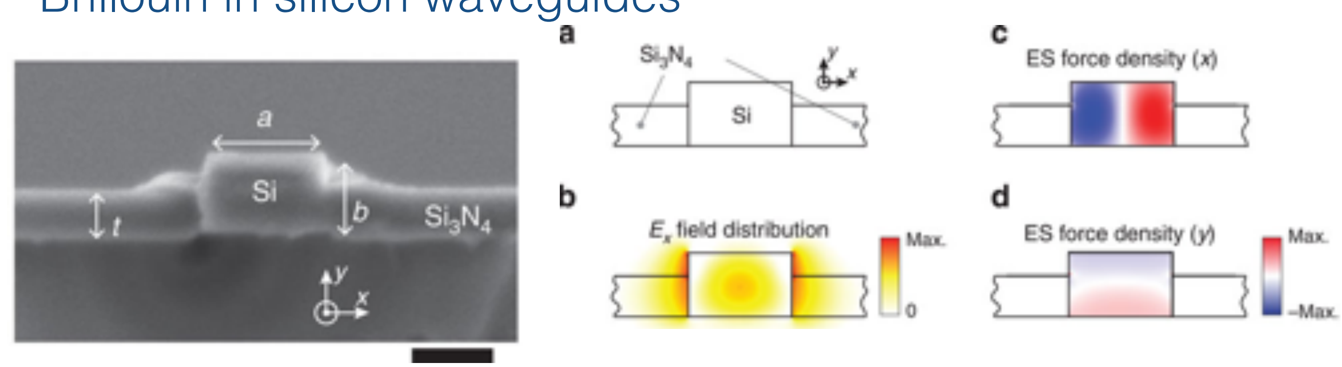
M. Kang et al. Nature Photonics **5**, 549–553 (2011).

Cascaded Brillouin Scattering



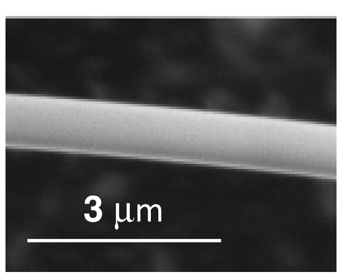
Büttner T.F.S. et al.,Optica 1, 311-314 (2014)

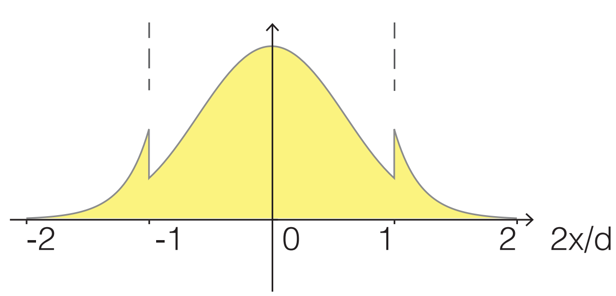
Brillouin in silicon waveguides



Shin, H., et al. *Nature Communications*, *4*. (2013)

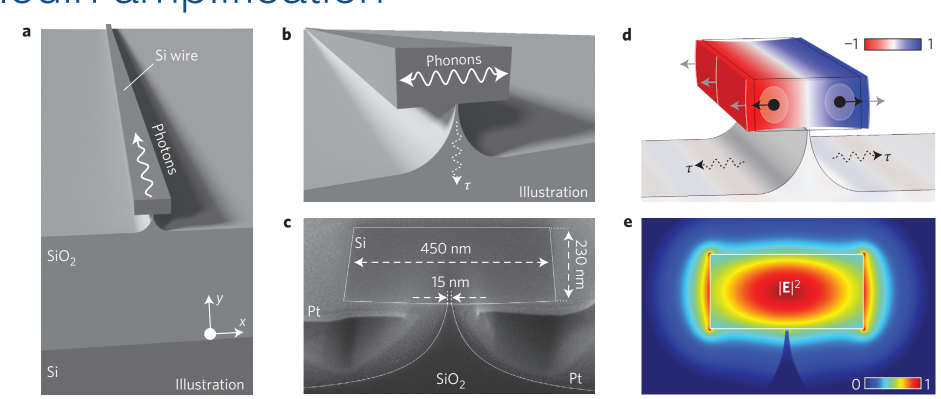
Brillouin Self-Cancellation





Florez, O., et al. Nature Communications, 7, 11759.

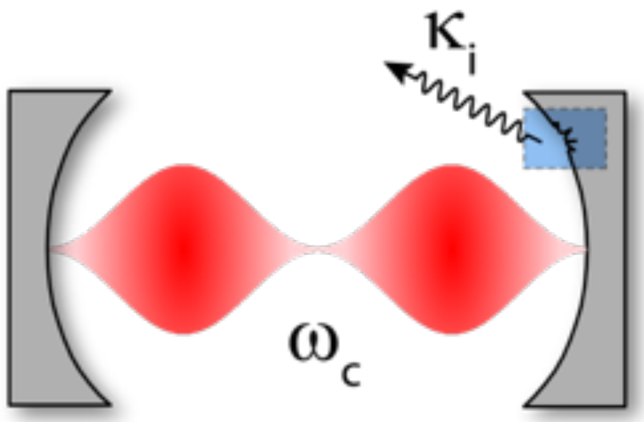
Brillouin amplification



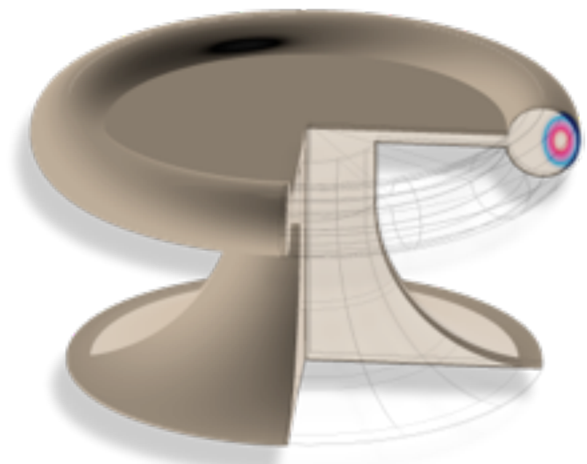
R. Van Laer et al, Nature Photonics **9**, 199–203 (2015)

Examples of optical cavities

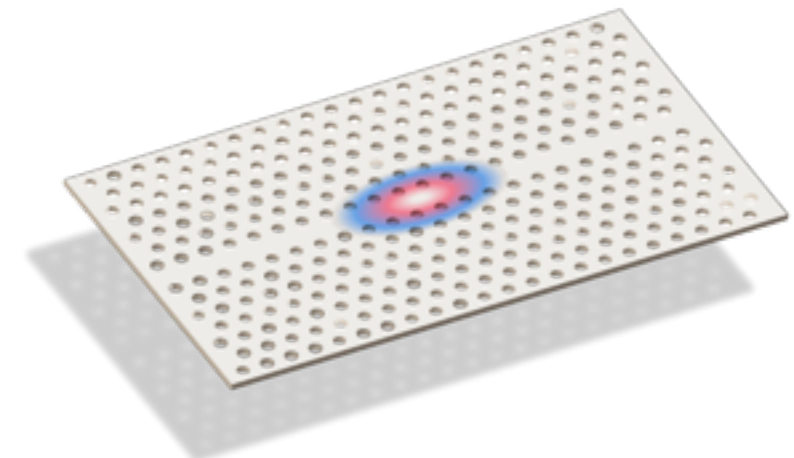
Fabry-Perot

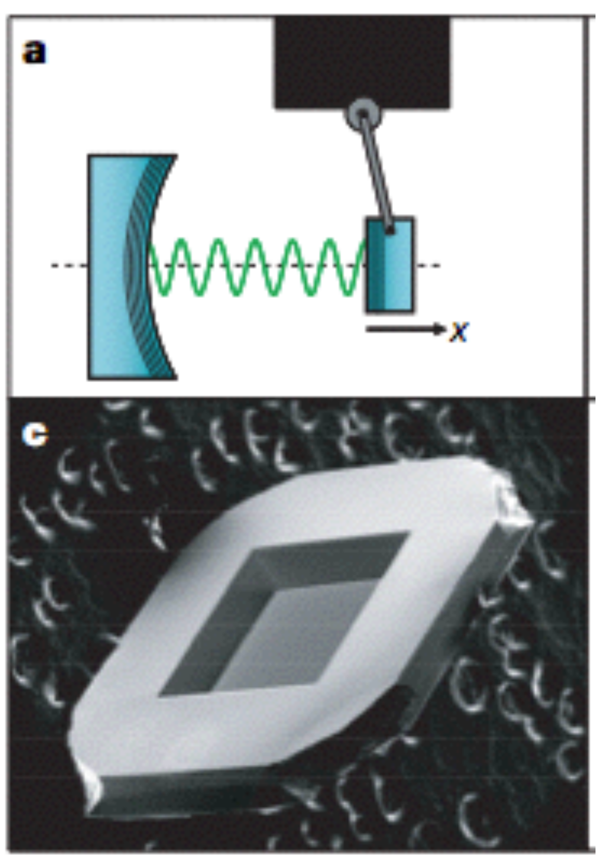


Ring and disk resonator

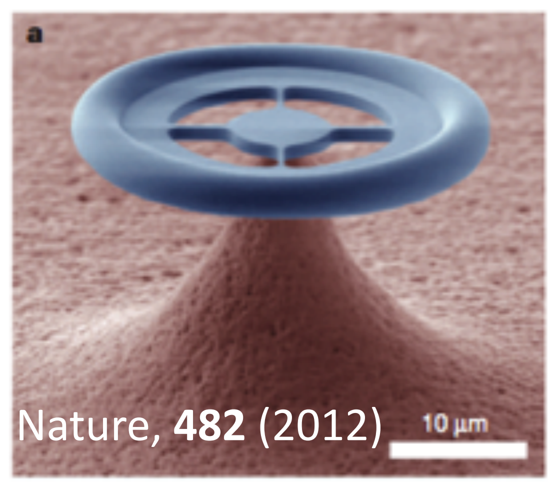


PhC

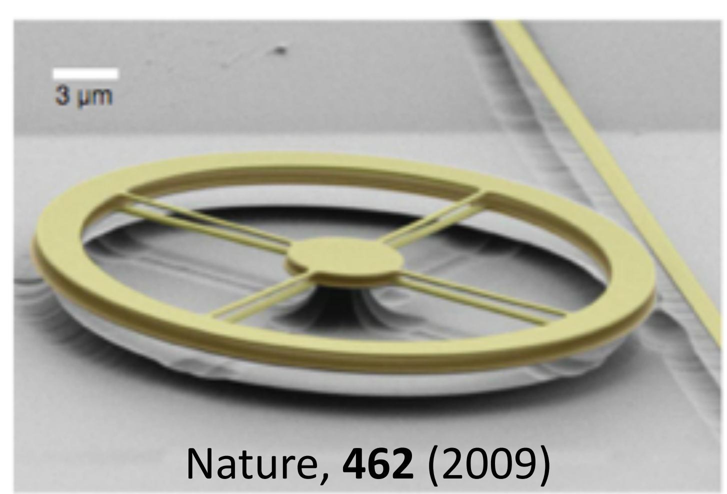




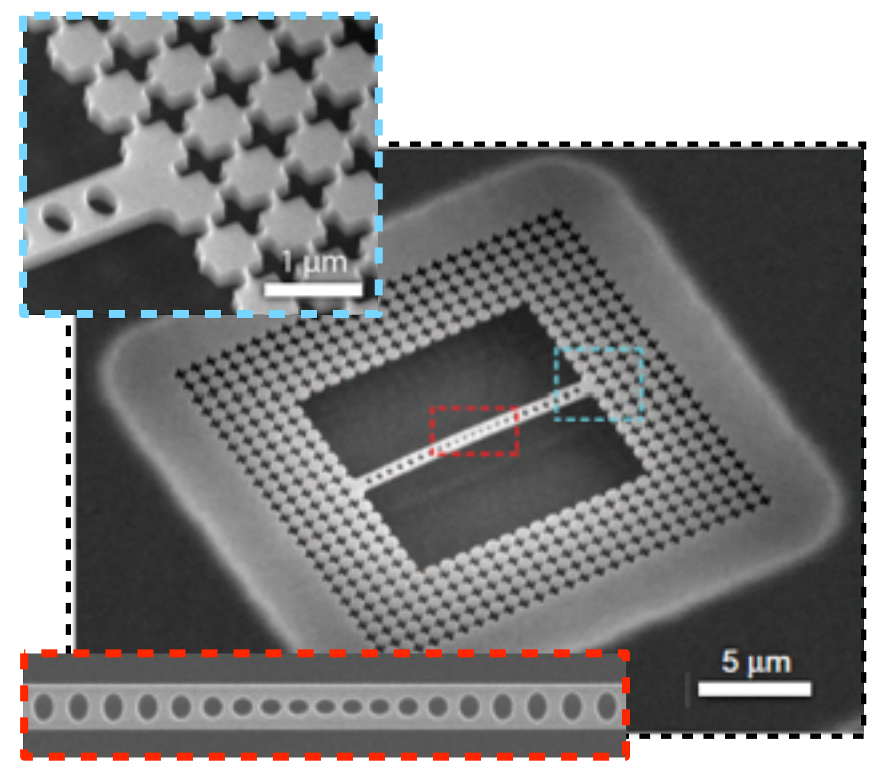
Nature, **452** (2008)



Nature, **482** (2012)

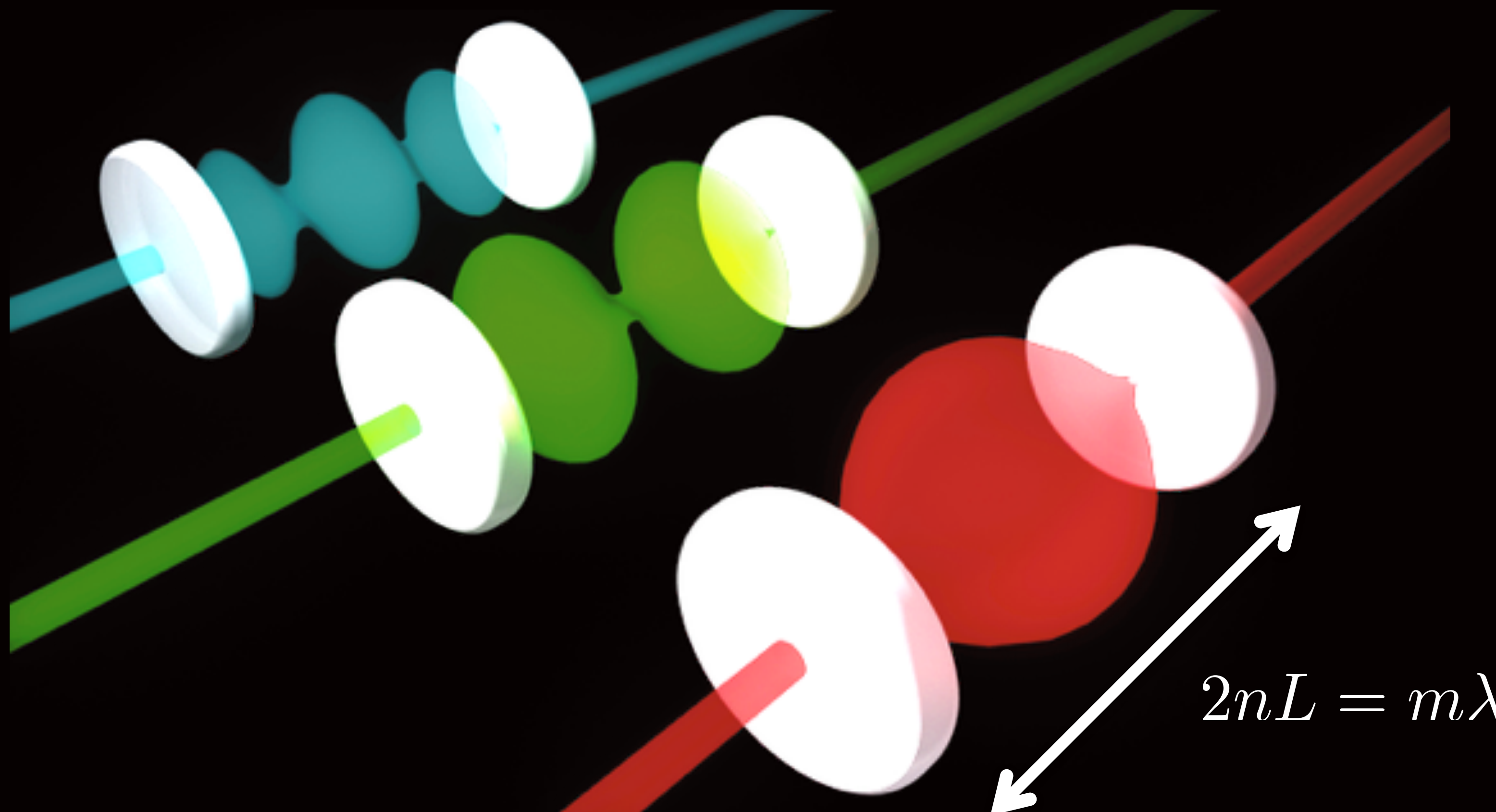


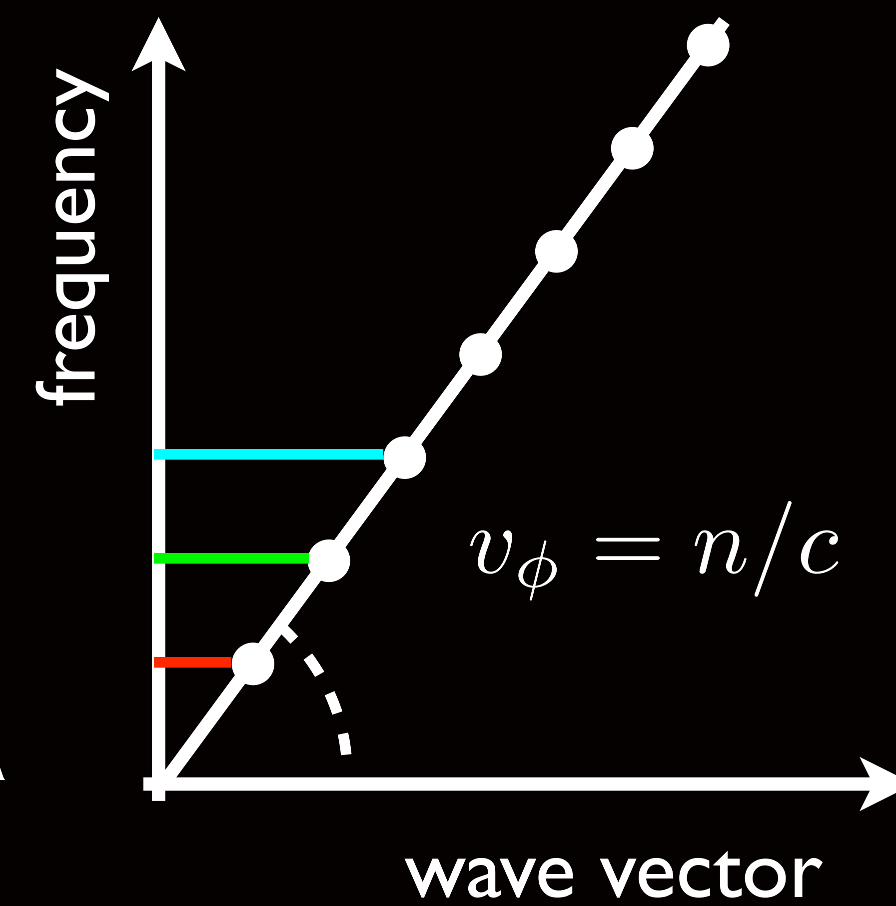
Nature, **462** (2009)

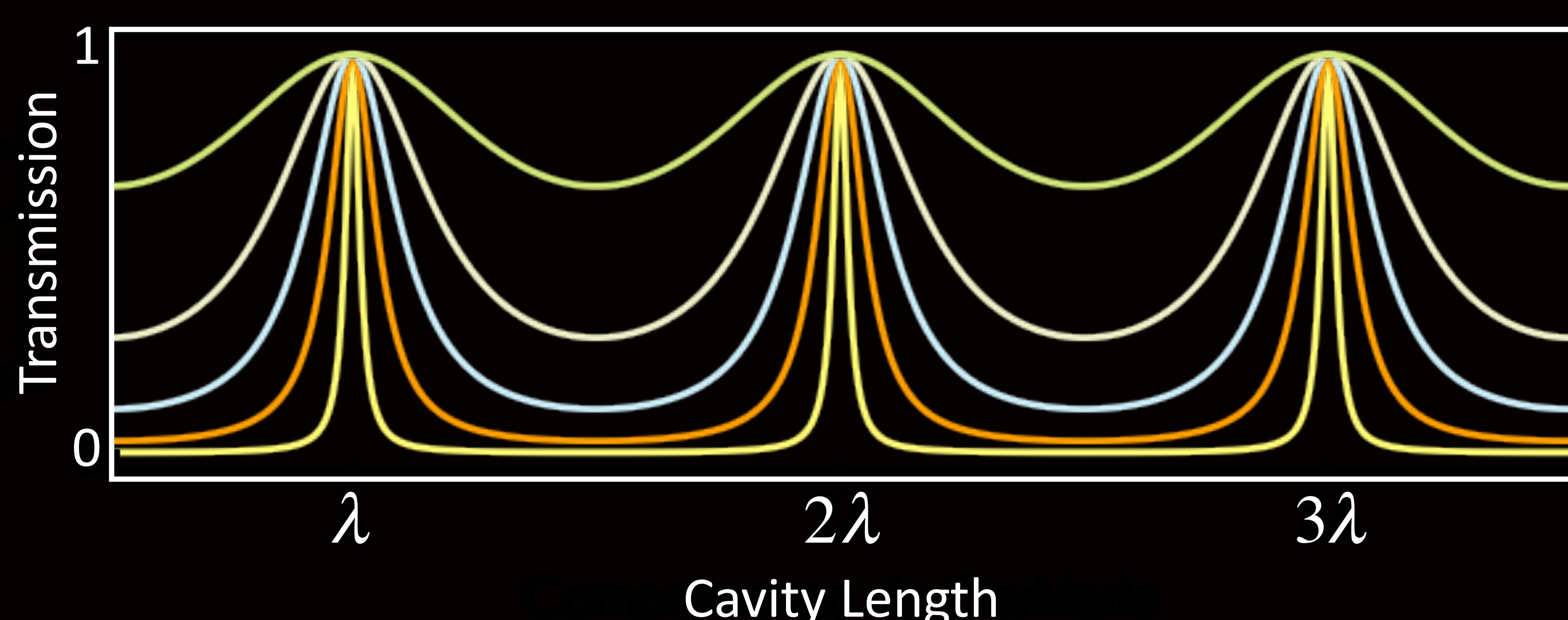


Nature, **472** (2011)

Optical Cavities

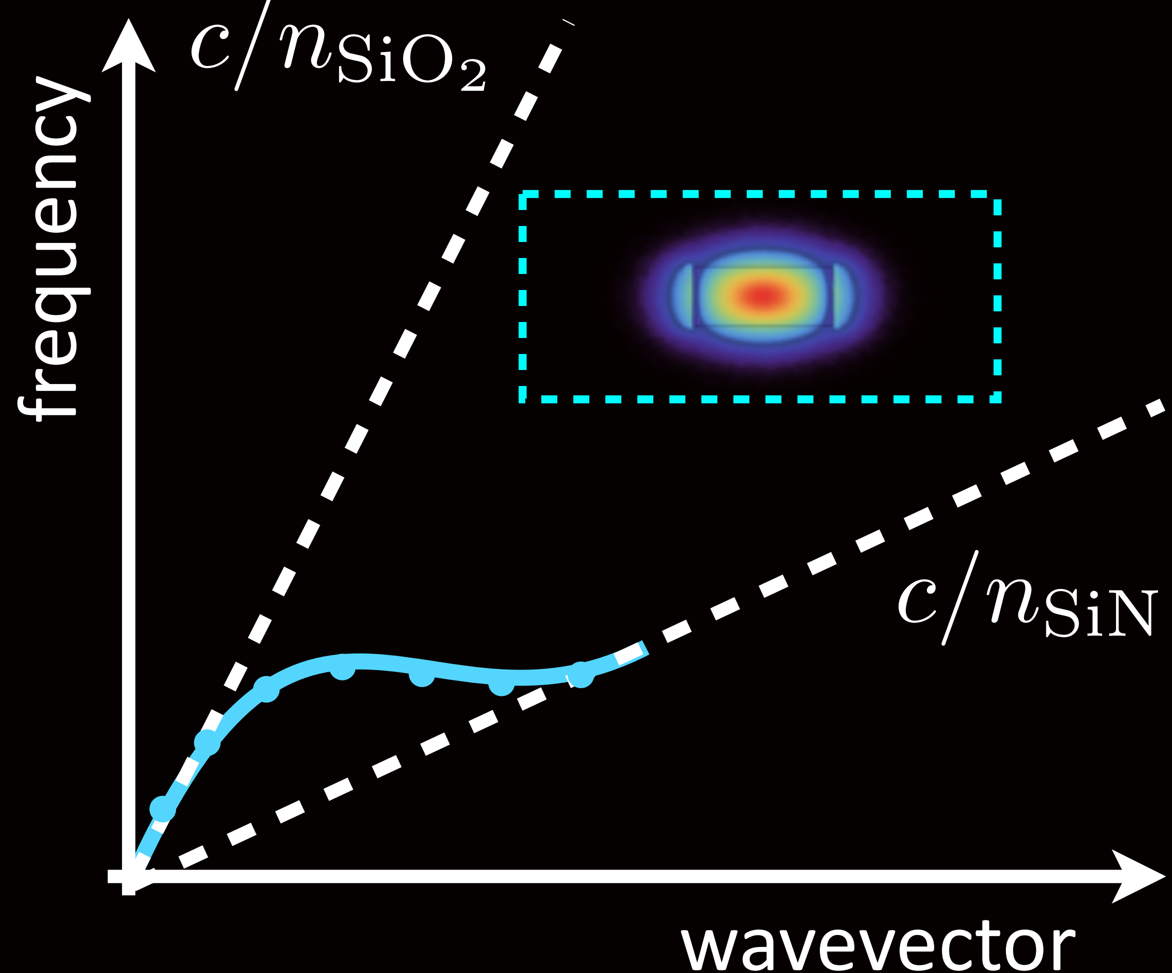




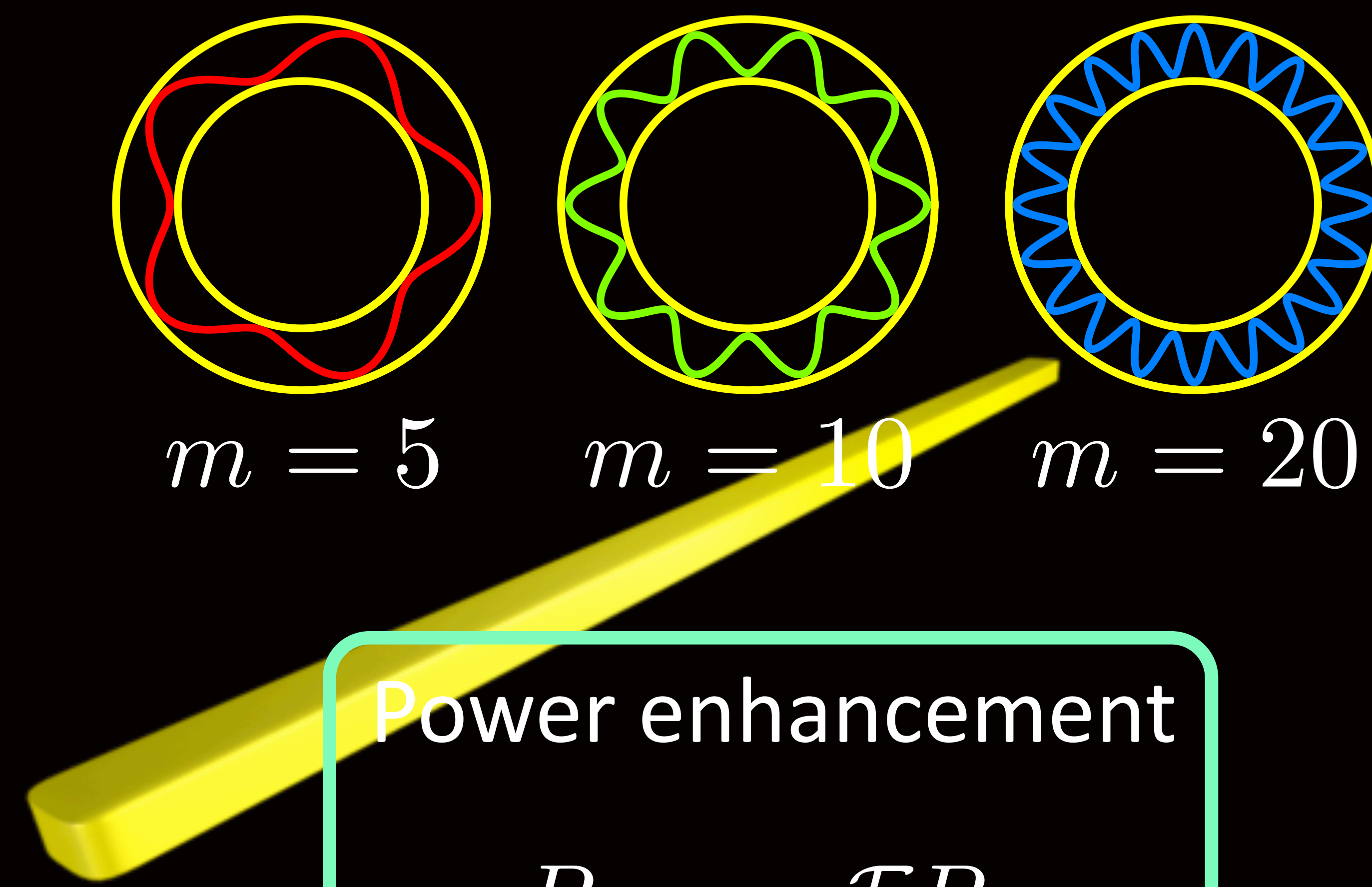


$$T = \left| \sum_{m=0}^{\infty} R^m e^{im\delta} \right|^2$$

Ring cavities



$$\omega(k) = k_z \frac{c}{n}$$



Power enhancement

$$P_{\mathrm{circ}} \propto \mathcal{F} P_0$$

$R = 15\ \mu\mathrm{m}$

$Q = 10^6\ (\tau \approx 1\ \mathrm{ns})$

$\mathcal{F} \approx 50 \times 10^3$

A brief story of micro cavities

Volume 137, number 7,8 PHYSICS LETTERS A

QUALITY-FACTOR AND NONLINEAR PROPERTIES OF OPTICAL WHISPERING-GALLERY MODES

V.B. BRAGINSKY, M.L. GORODETSKY and V.S. ILCHENKO
Department of Physics, Moscow University, 119899 Moscow, USSR

Received 10 March 1989; accepted for publication 21 March 1989
Communicated by V.M. Agranovich

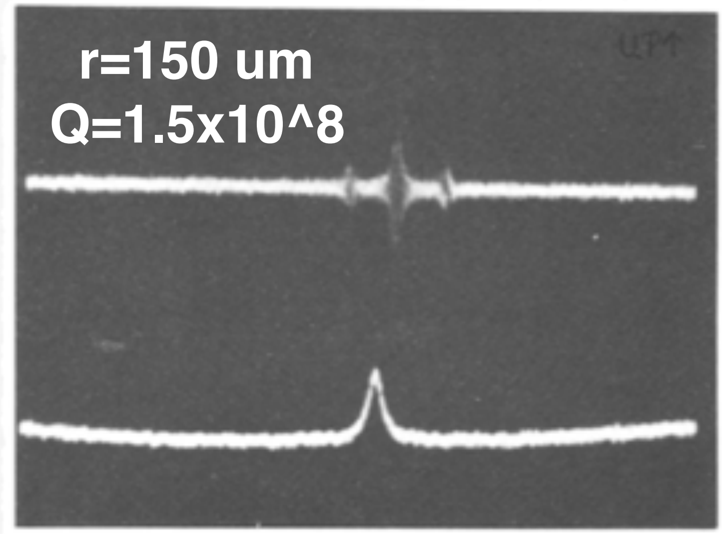
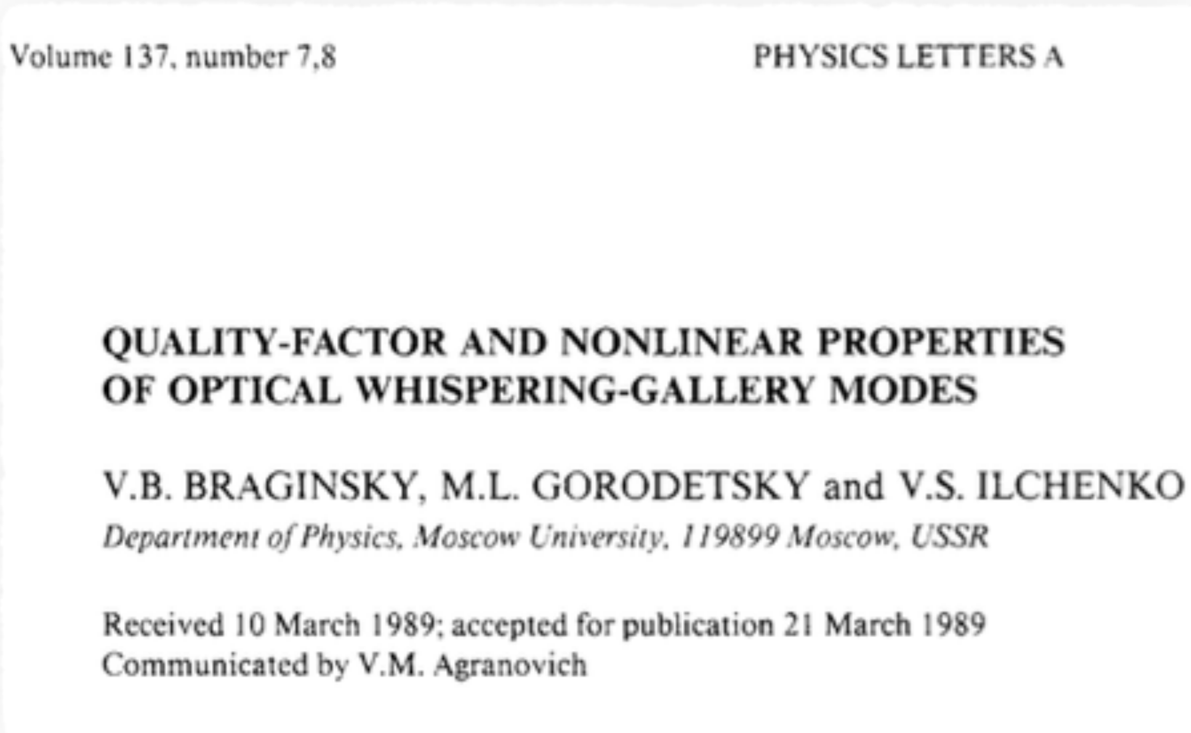


Fig. 2. Detailed resonant curve of the mode; second trace is the precise marks ±7 MHz. Bandwidth of the mode ≈3 MHz, quality-factor 1.5×10^8 (the whispering-gallery microresonator 150 μm in diameter).

Braginsky, V. B., et al. Physics Letters A, 137(7-8), 393–397. (1989)
Collot, L., Europhys. Lett. 23, 327-334 (1993).
M. L. Gorodetsky et al, Opt. Commun.113, 133 (1994).
J. C. Knight, et al. Opt. Lett. 20, 1515-1517 (1995)
D. W. Vernooy and H. J. Kimble, Phys. Rev. A 55, 1239 (1997).

A brief story of micro cavities



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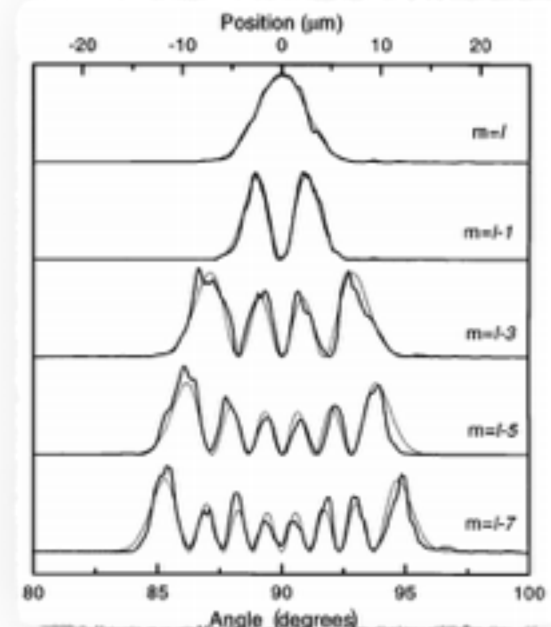
Received 10 March 1989; accepted for publication 21 March 1989
Communicated by V.M. Agranovich



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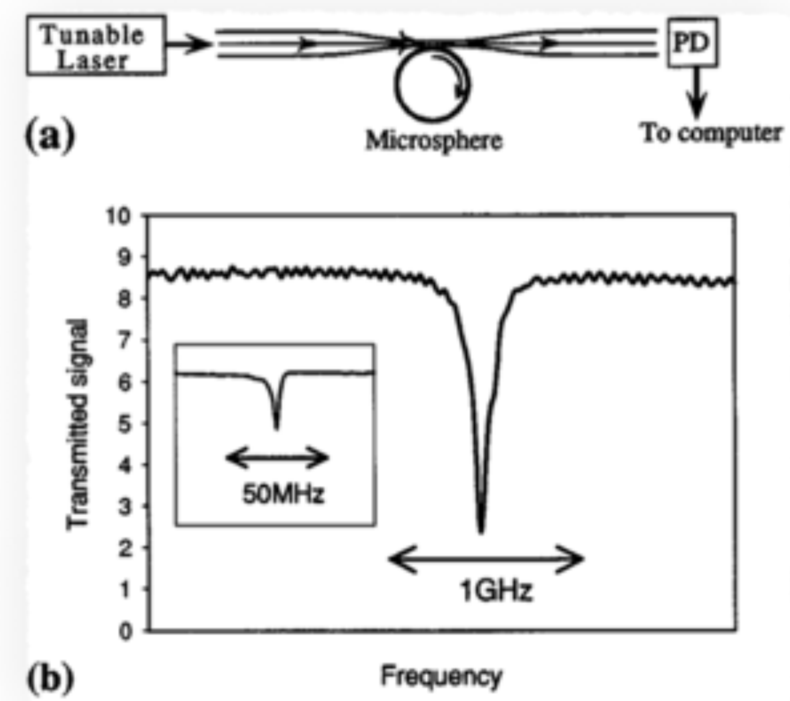

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J. C. Knight, et al. Opt. Lett. 20, 1515-1517 (1995)
D. W. Vernooy and H. J. Kimble, Phys. Rev. A 55, 1239 (1997).

A brief story of micro cavities

Nature 415, 621-623 (2002).
Silica Raman Laser

Nature 421, 925-928 (2003)
Ultra-high Q thoroids

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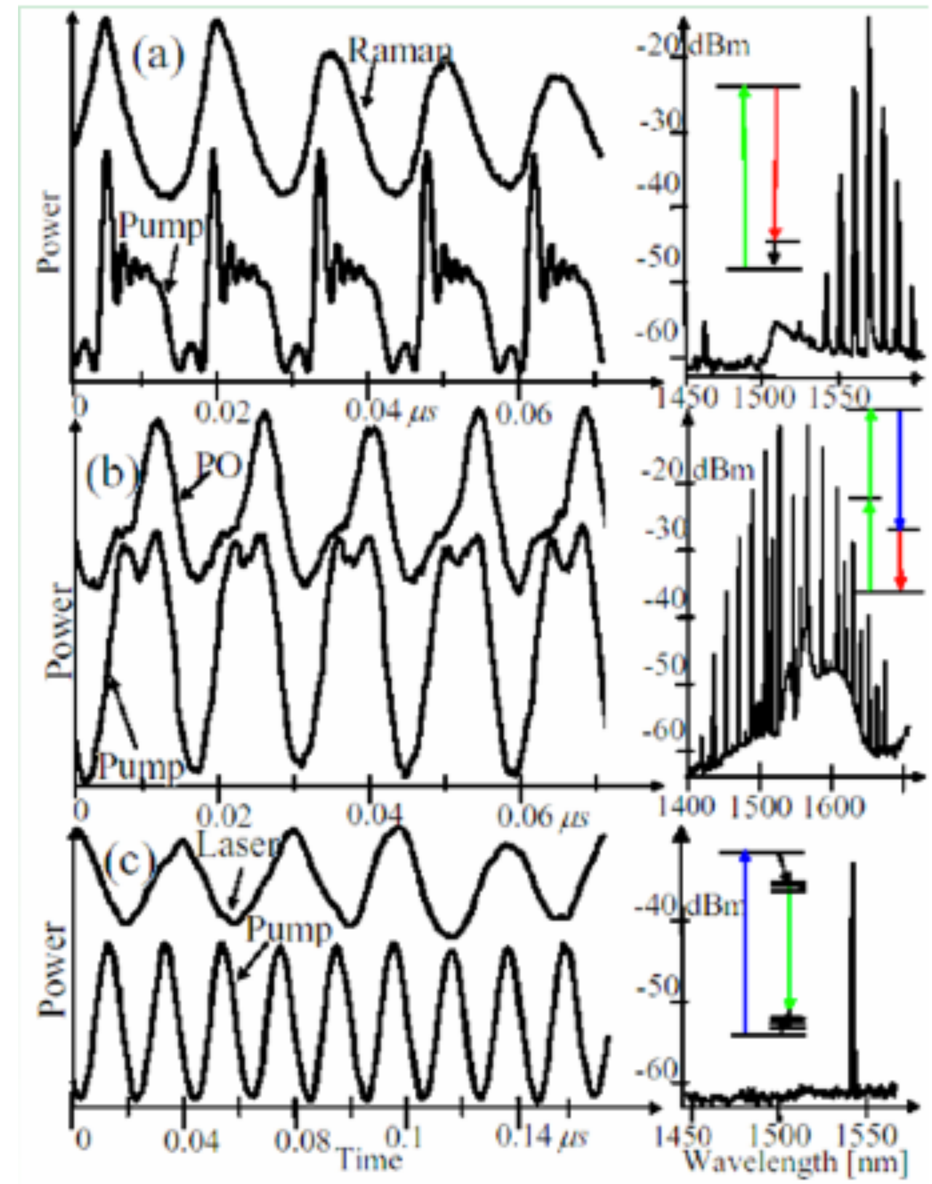

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Carmon, T., et al.
Physical Review Letters, 94(22), 223902. (2005)

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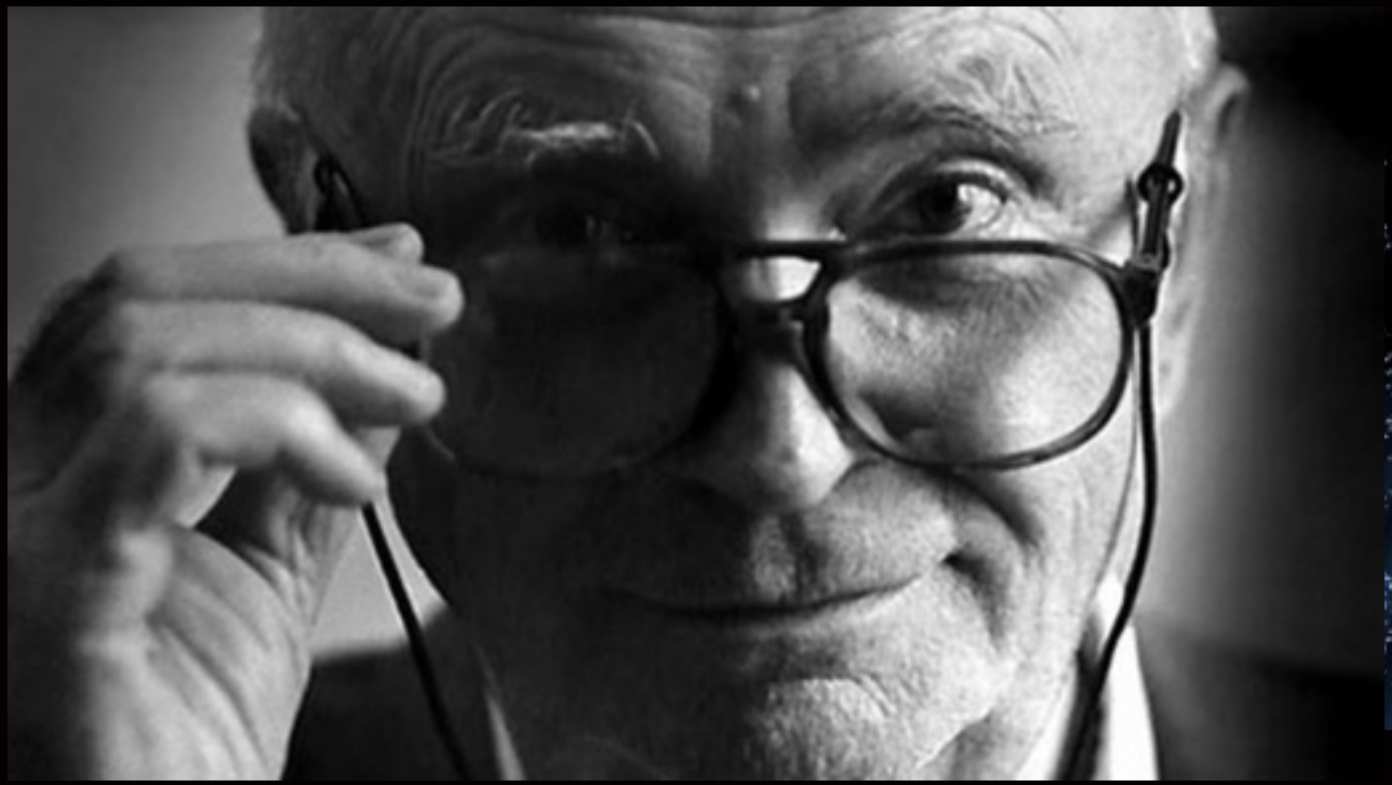
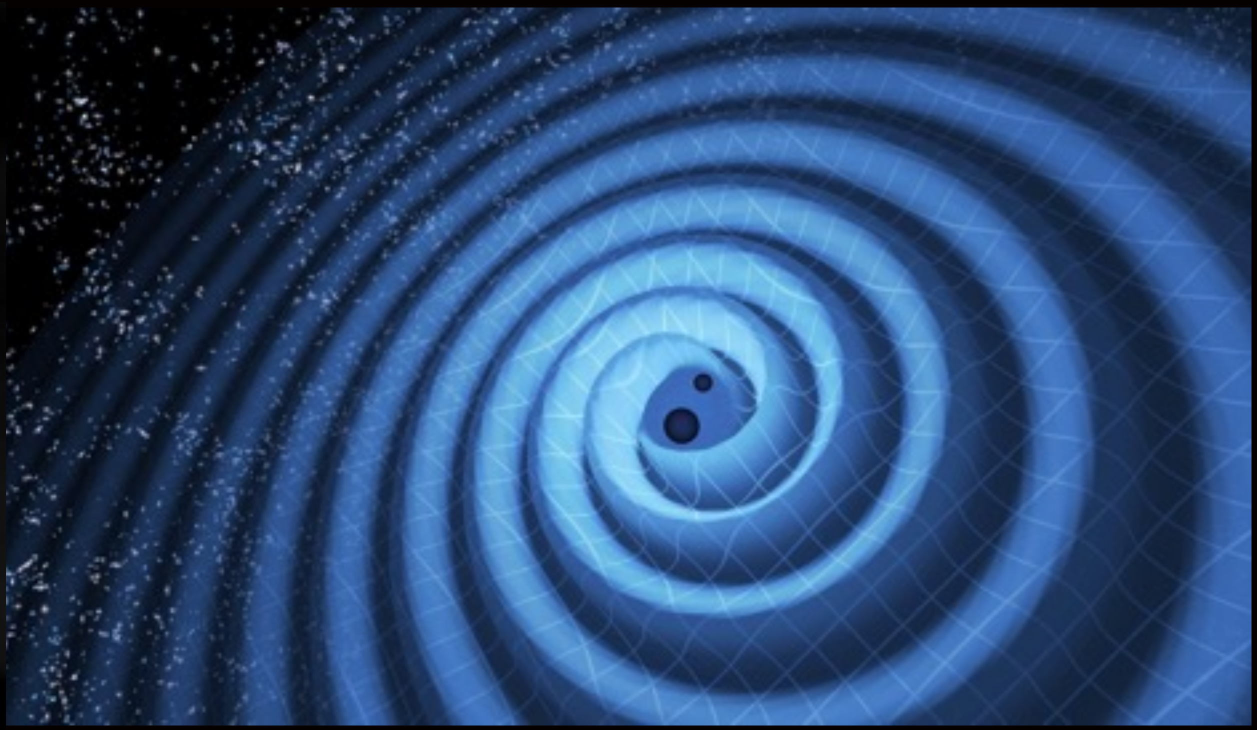
D. K. Armani, et al., Nature 421, 925-928 (2003)

Standing on the shoulder of giants



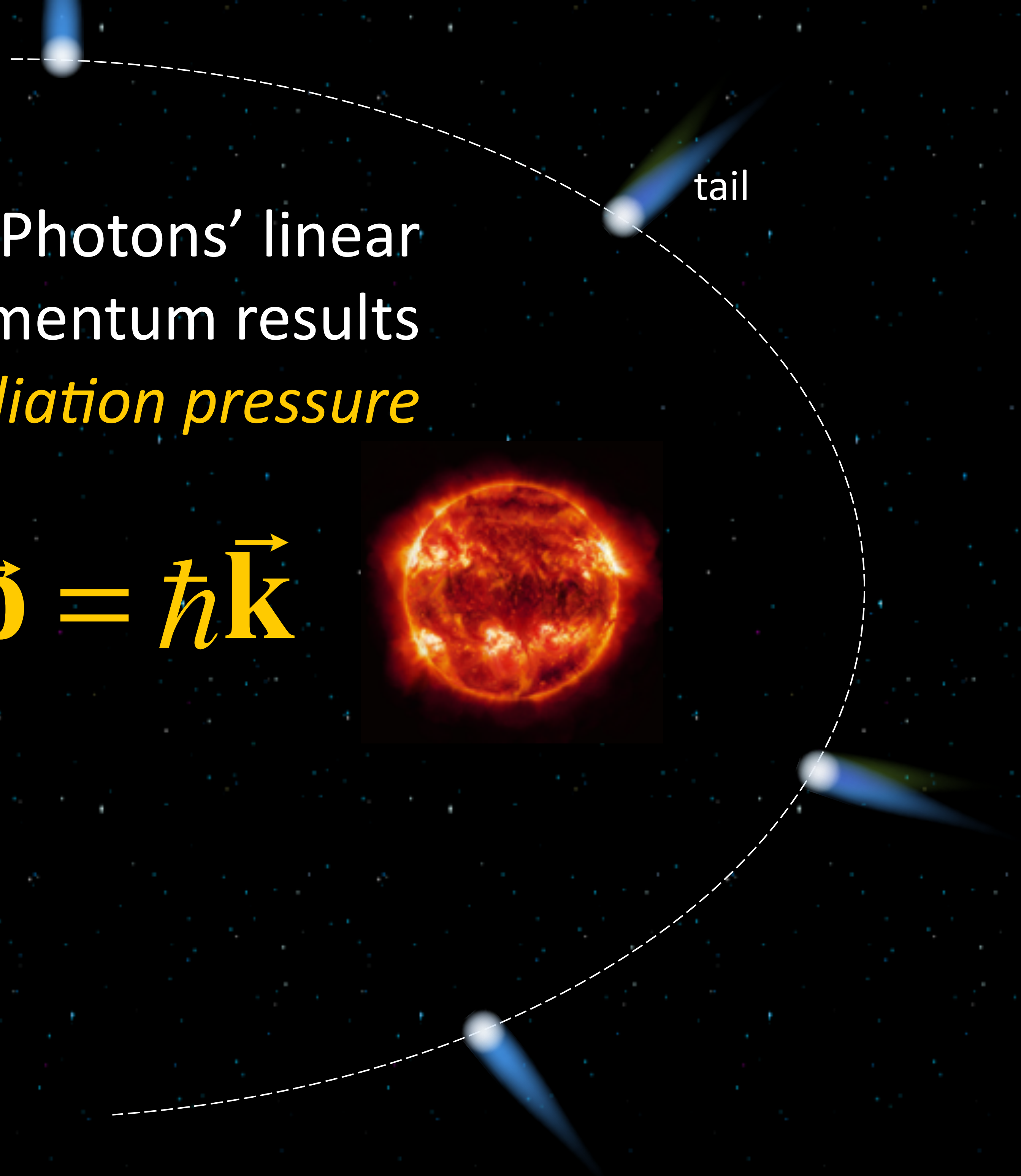
Vladimir Braginsky, 1931-2016



Credit: LIGO website

V.B. Braginsky, Y.I. Vorontsov, K.S. Thome: Science **209**, 547 (1980)
V.B. Braginsky, S.E. Strigin, S.P. Vyatchanin, Parametric oscillatory instability in Fabry–Perot interferometer, Physics Letters A, Volume 287, Issues 5–6, 3 September 2001, Pages 331-338,

Mechanical effects of light



Photons' linear momentum results in *radiation pressure*

$$\vec{\mathbf{p}} = \hbar\vec{\mathbf{k}}$$

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Fulle, M. (2004). Motion of cometary dust. Comets II, 565–575.

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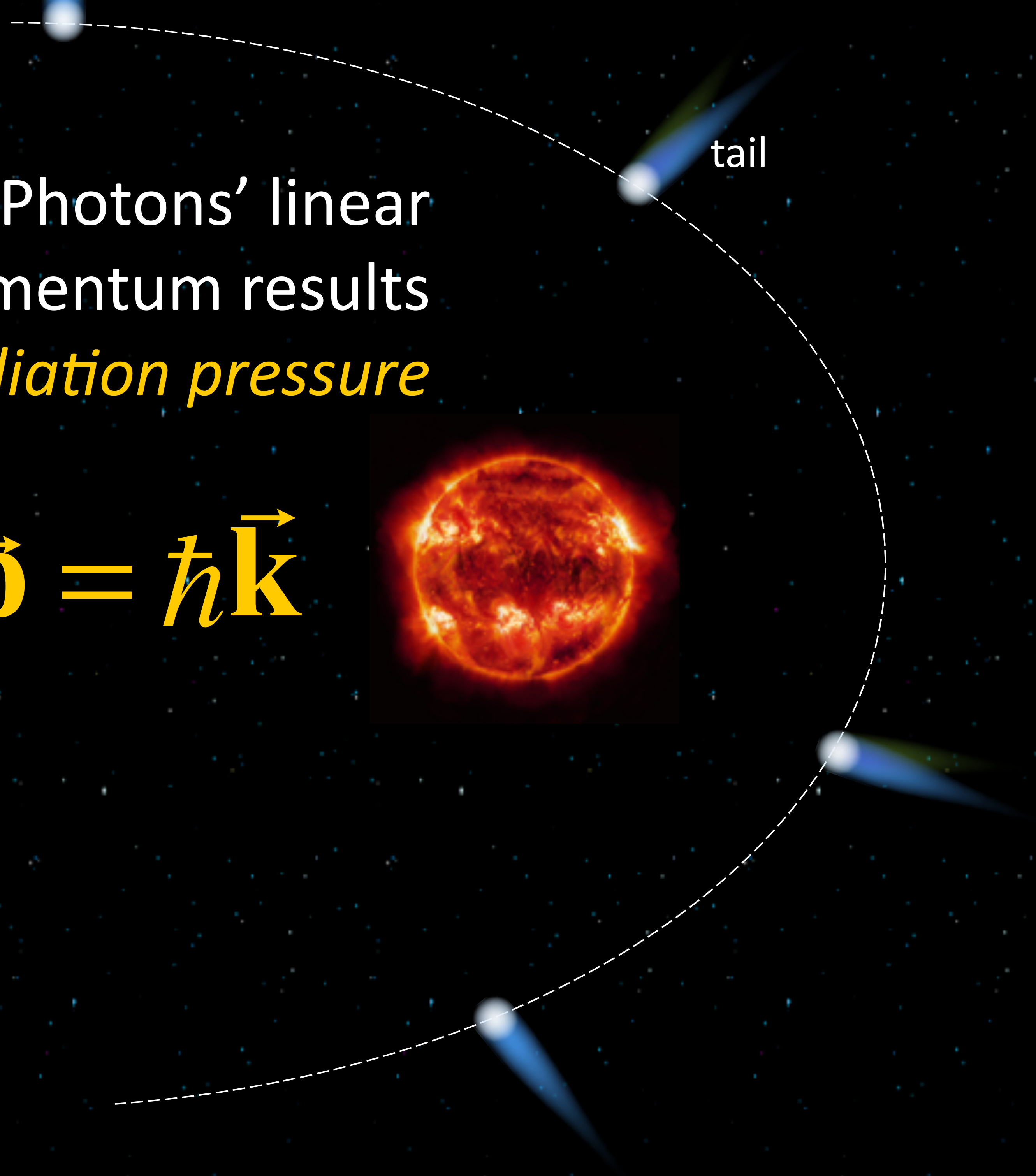

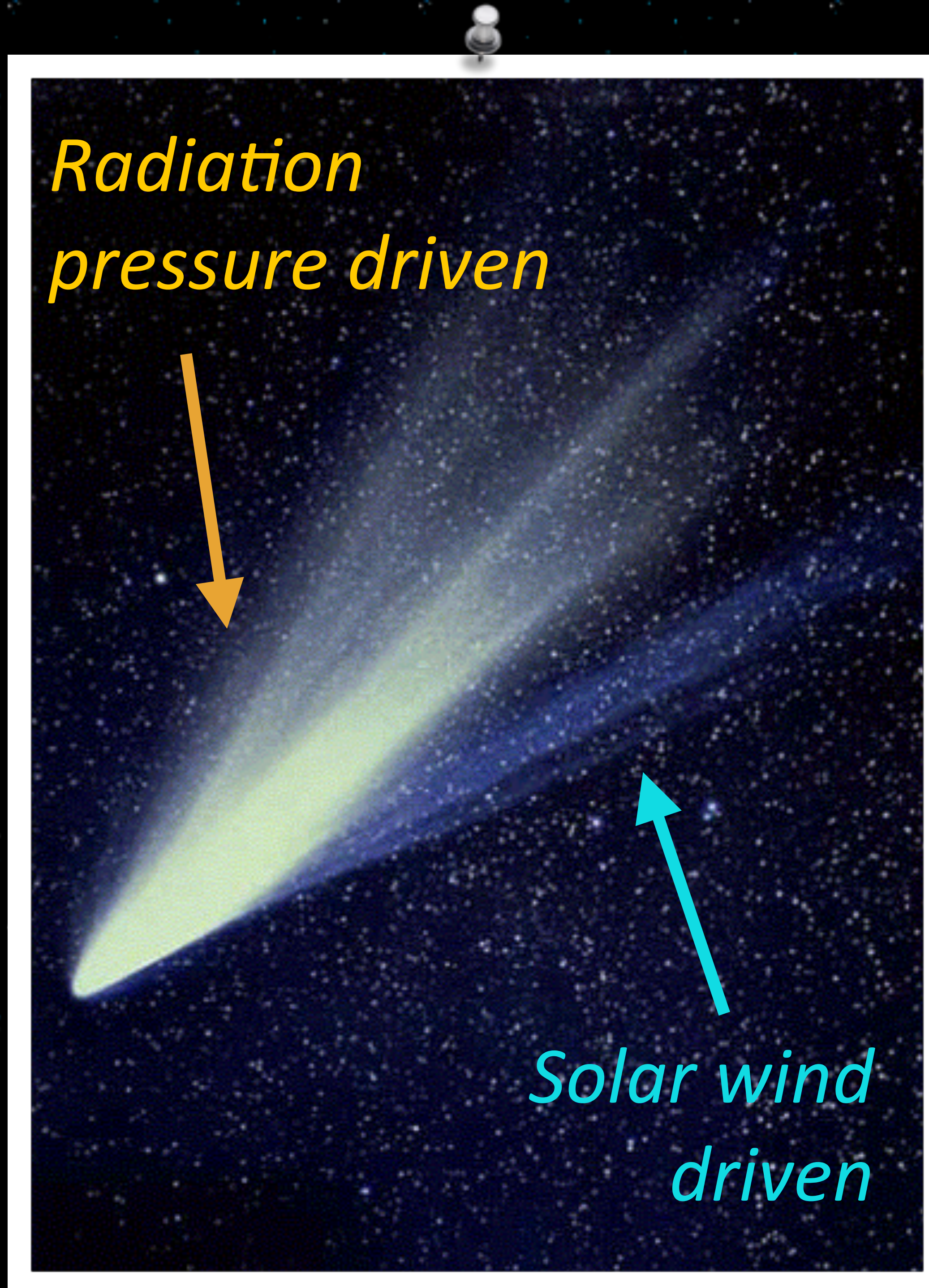

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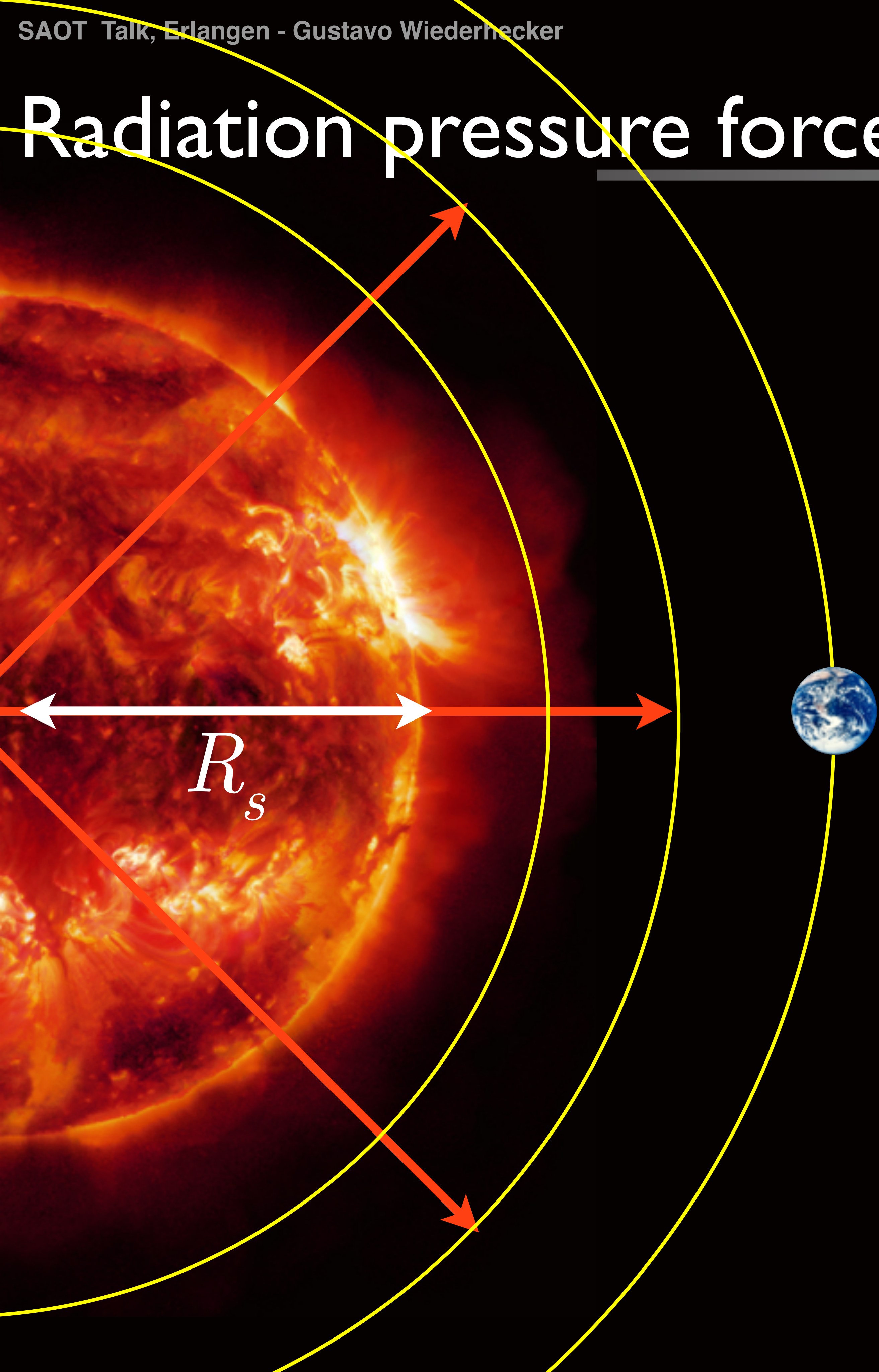
Fulle, M. (2004). Motion of cometary dust. Comets II, 565–575.

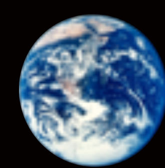
Radiation pressure force

$$P_{rad} = (4\pi R_s^2)\sigma T^4$$



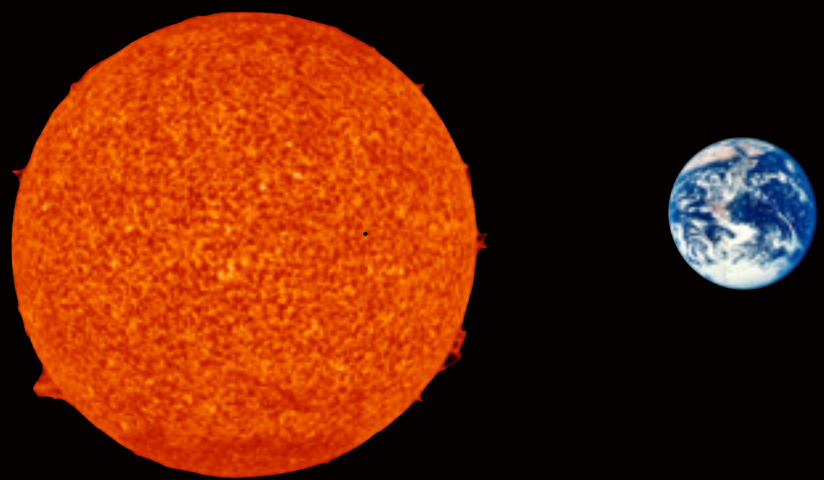
Sun vs laser pointer

Sun



$$F \approx \frac{P}{c} \approx 0.6 \times 10^{9}\ \mathrm{N}$$

Sun vs laser pointer



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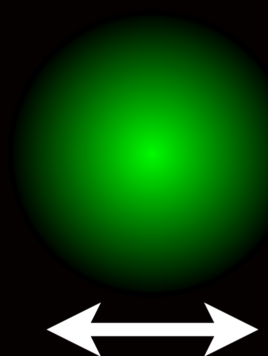
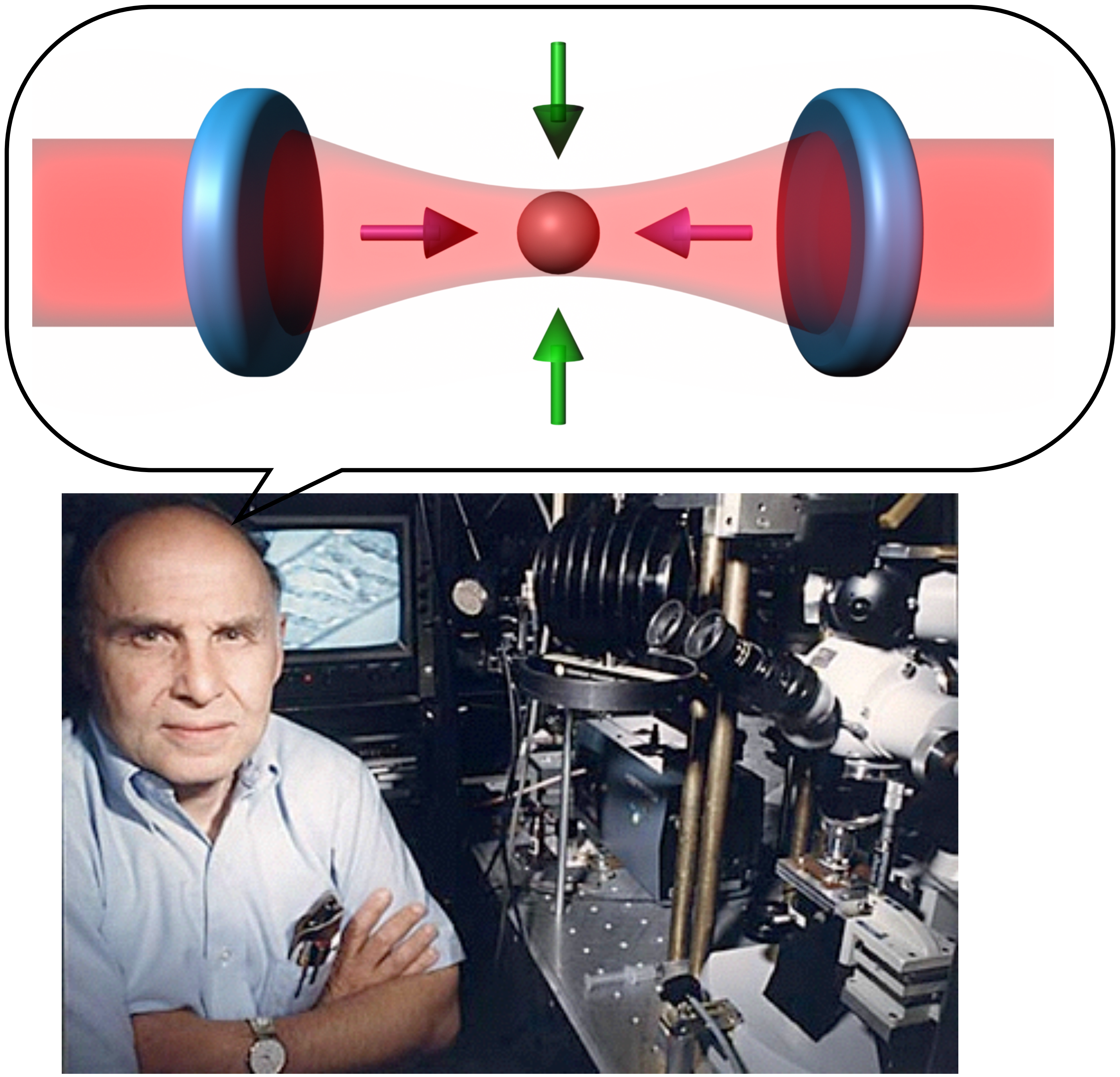


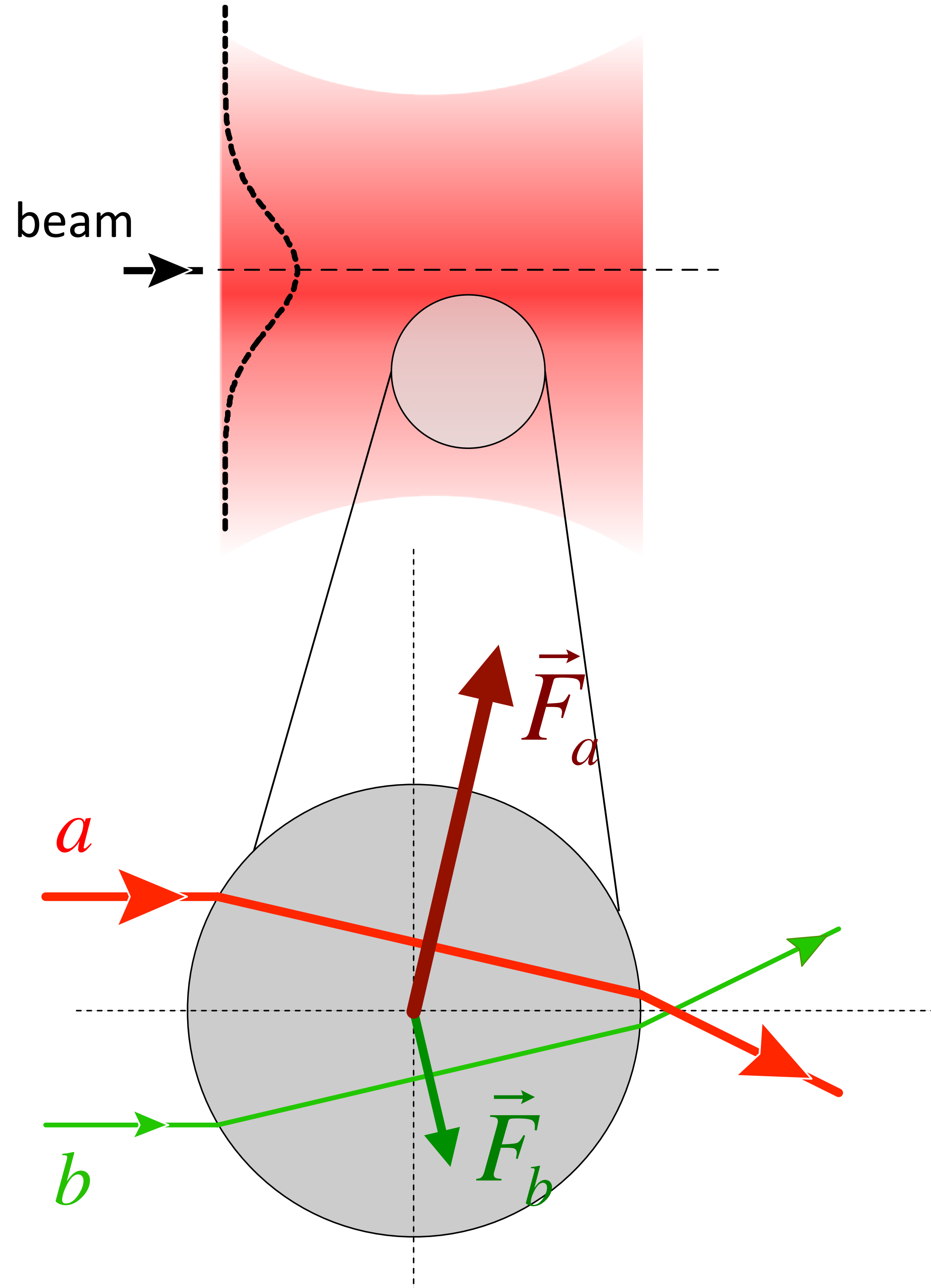
$$F \approx 15 \times 10^{-12} \text{ N}$$

What can we do with this force?

Optical Forces in the Microscopic World



A. Ashkin et al. Science (1987)



A. Ashkin. IEEE JSTE, 6(6):841–856, 2000.
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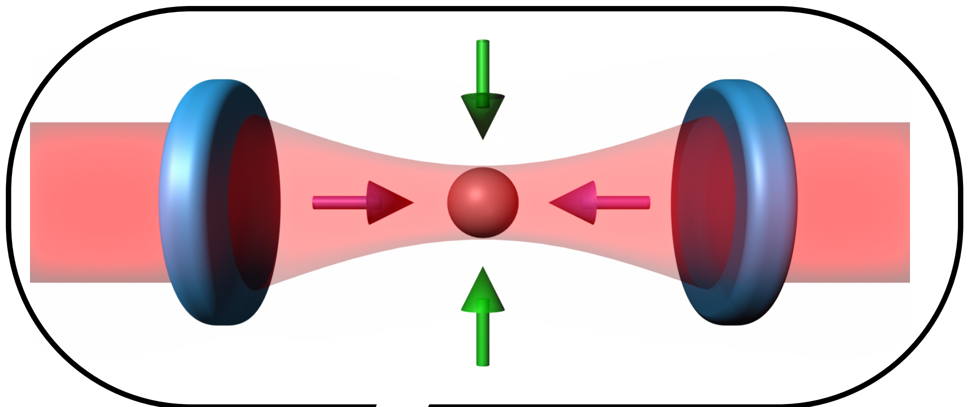

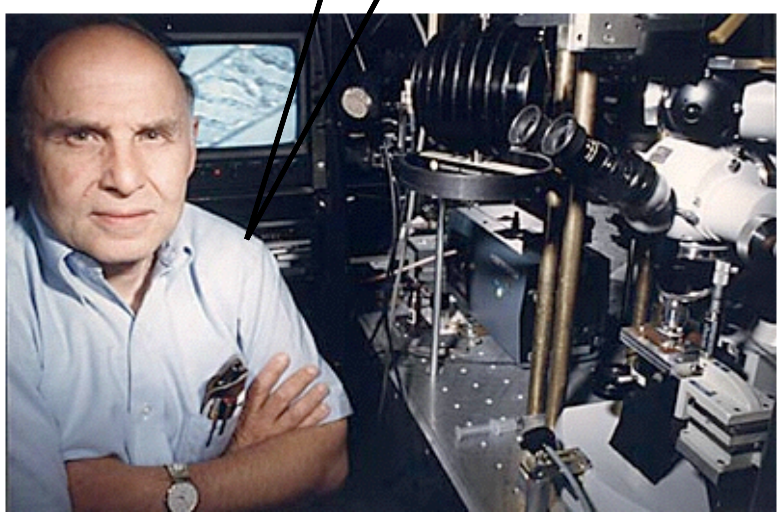

A. Ashkin et al. Science (1987)

$$\vec{F} = \hbar n_{photons} \frac{\Delta \vec{k}}{\Delta t}$$

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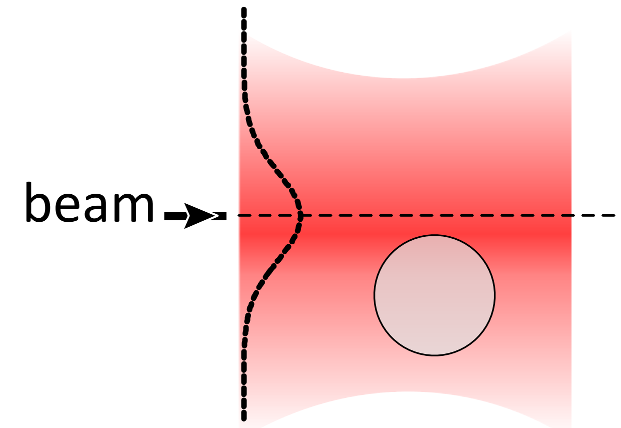

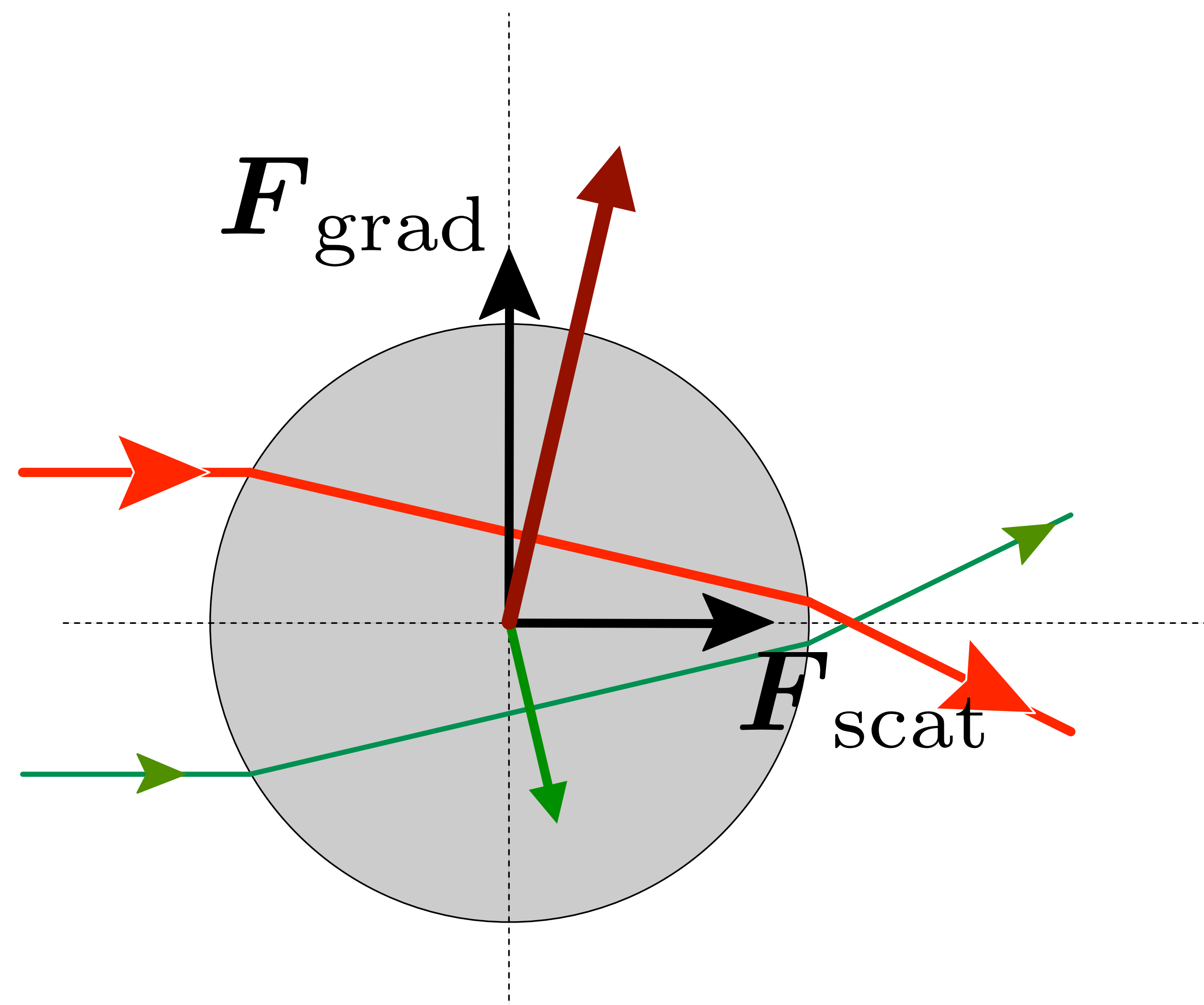

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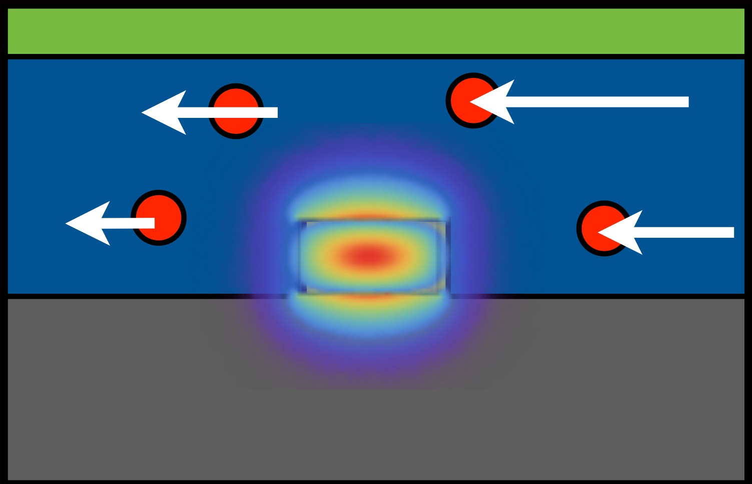
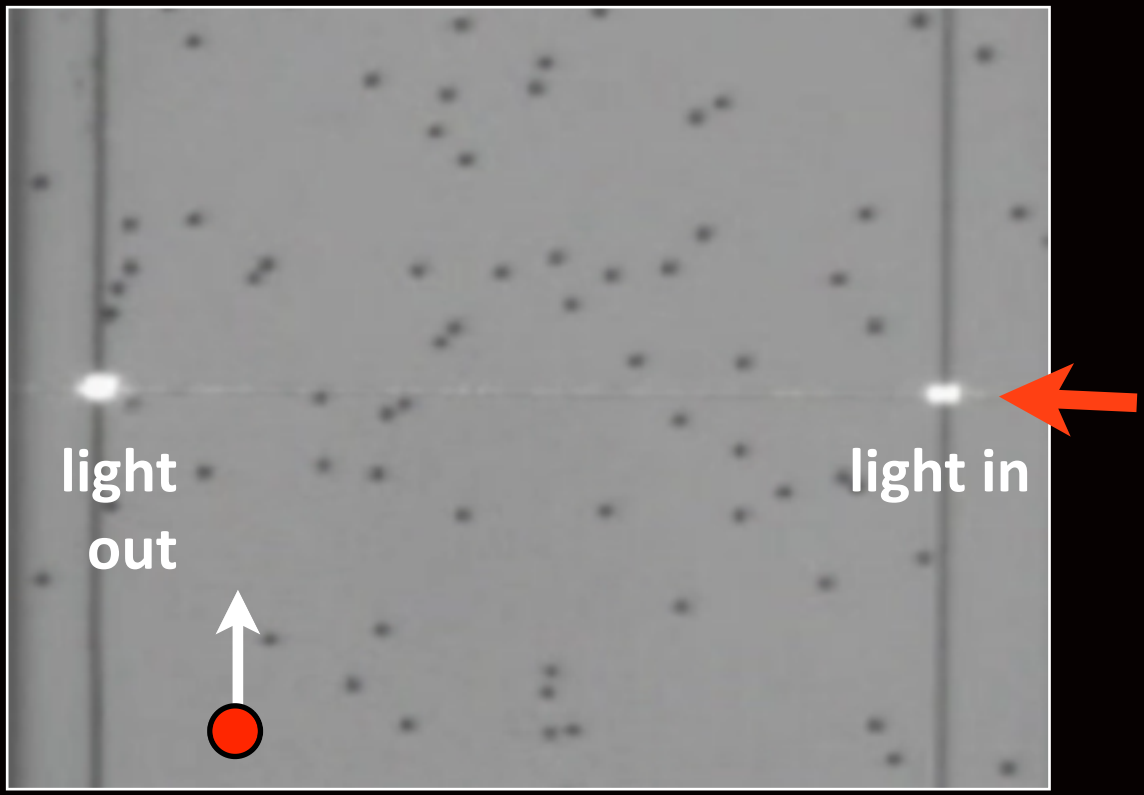
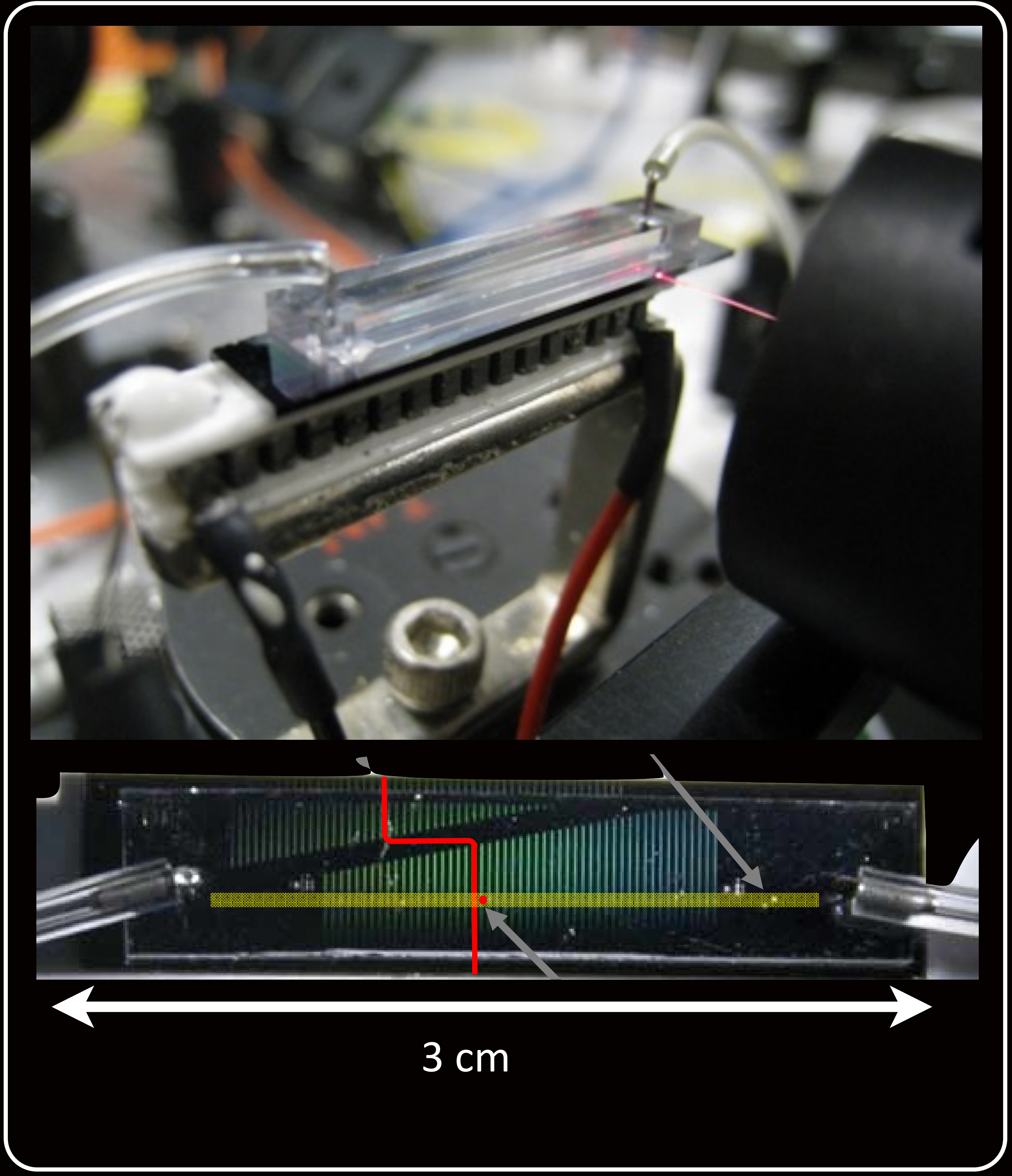


waveguide cross-section

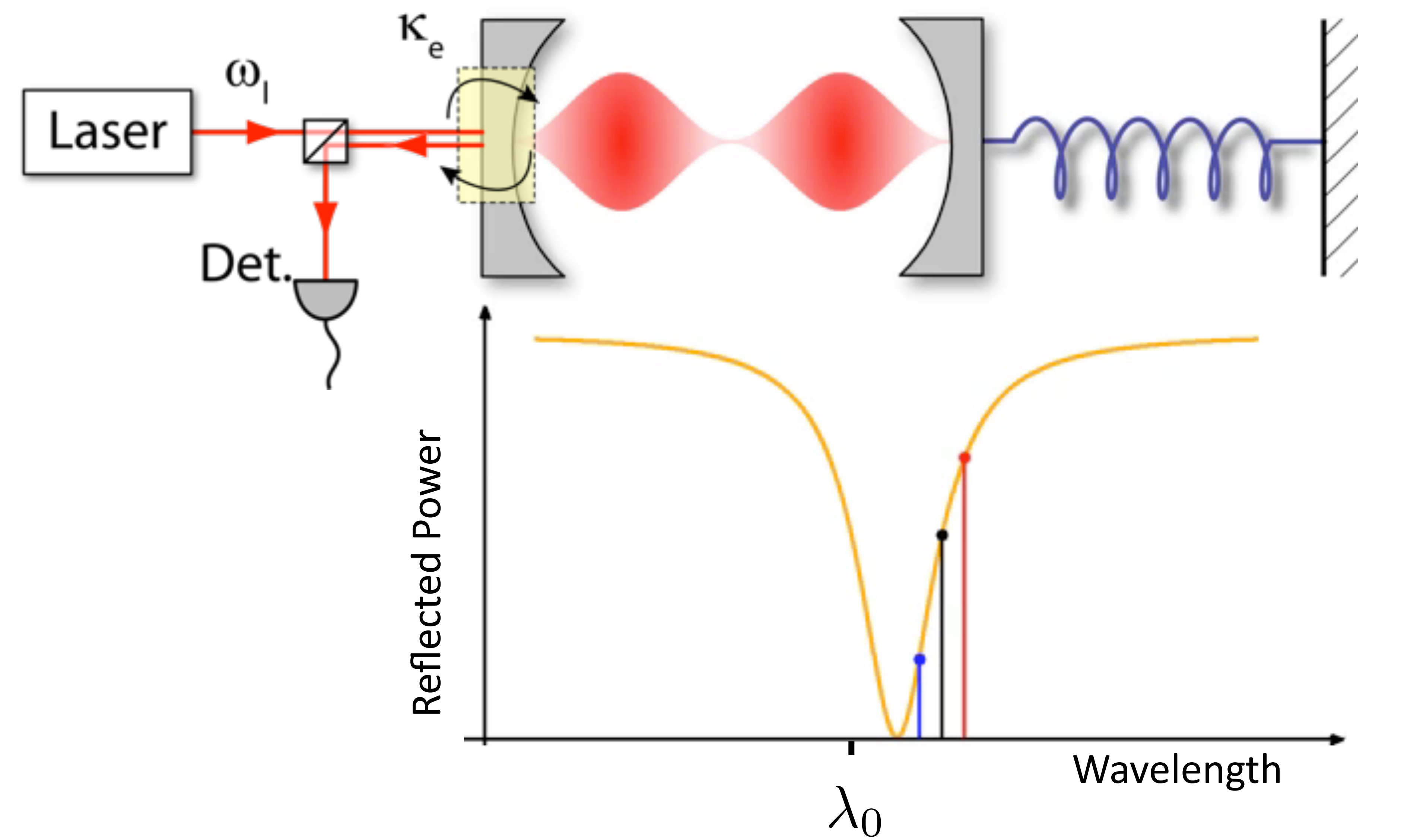


A. Nitkowski et al, Optics Letters 2009



Integrated particle trapping setup at Cornell (Lipson group)

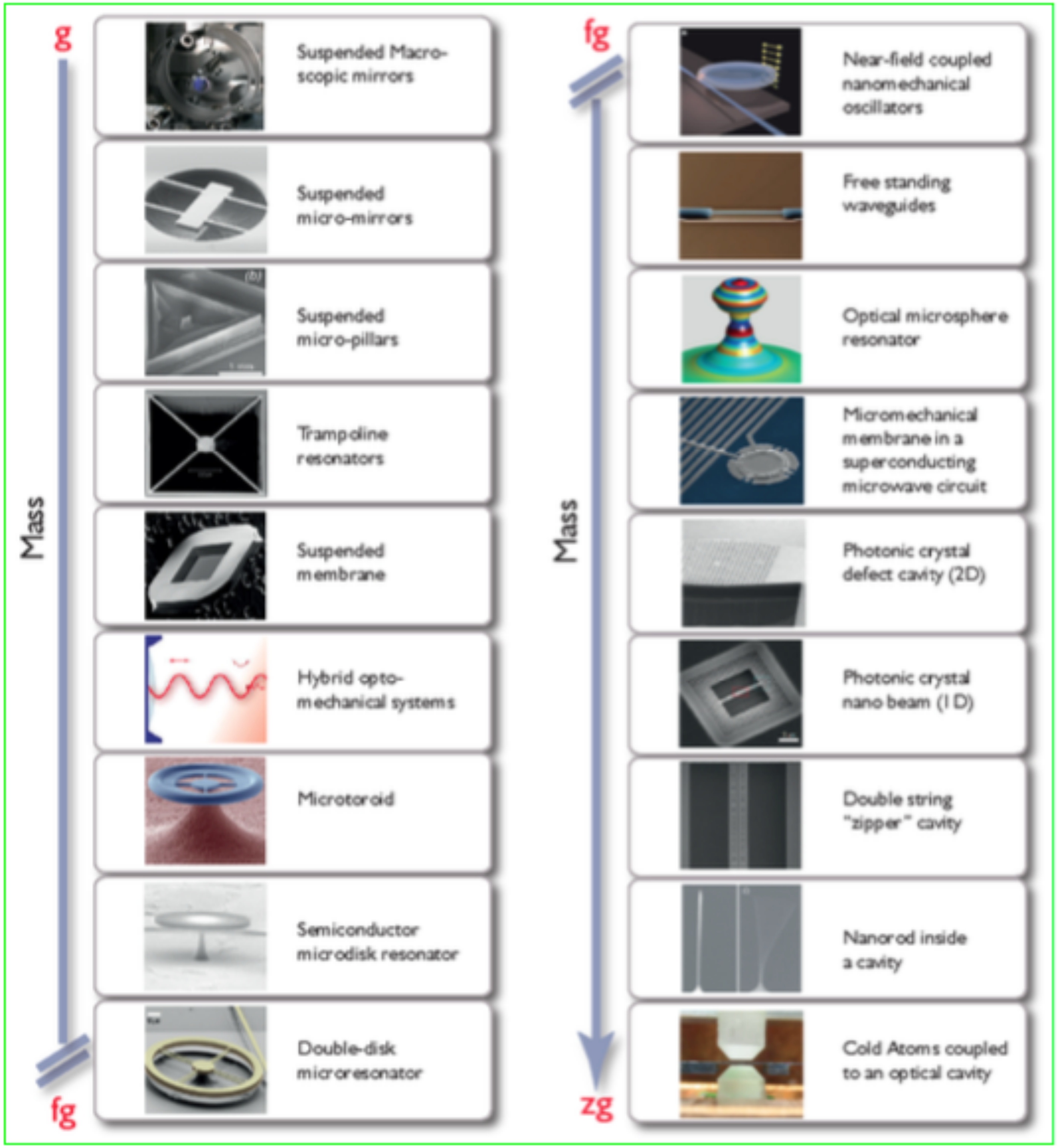
Mechanically Susceptible Cavity



$$\phi_{\mathrm{RT}} = m(2\pi)$$
$$= \frac{\omega_l}{c}(2L)$$
$$\Rightarrow \omega_m = m\frac{\pi c}{L}$$

$$L = L_0 + x(t) \;\; \Rightarrow \omega_m(t) = m\frac{\pi c}{L_0 + x(t)} \approx \omega_m - \underbrace{\left(\frac{\omega_m}{L}\right)}_{g_{\mathrm{OM}}} x(t)$$

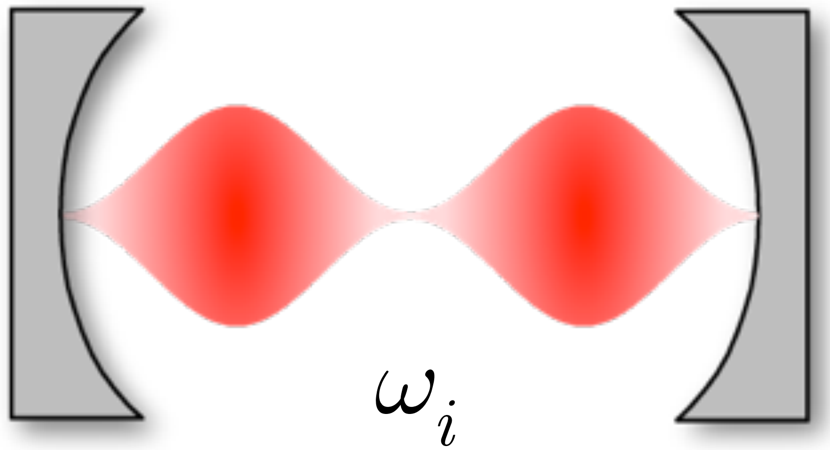
Mass-spring Fabry-Perot systems?



Aspelmeyer, M., Kippenberg, T. J., & Marquardt, F. (2014). Cavity optomechanics. *Reviews of Modern Physics*, *86*(4), 1391–1452.

Modal field expansion

Maxwell's equations
+
boundary conditions



Optical Modes

$$\mathbf{E}_j(\mathbf{r},t) = \mathcal{E}_j(\mathbf{r})e^{-i\omega_j t}$$

$$\mathbf{B}_j(\mathbf{r},t) = \mathcal{H}_j(\mathbf{r})e^{-i\omega_j t}$$

H. A. Haus. Waves and fields in optoelectronics. Prentice-Hall, 1984.
Haus, H. A., & Huang, W. (1991). Proceedings of the IEEE, 79(10), 1505–1518.

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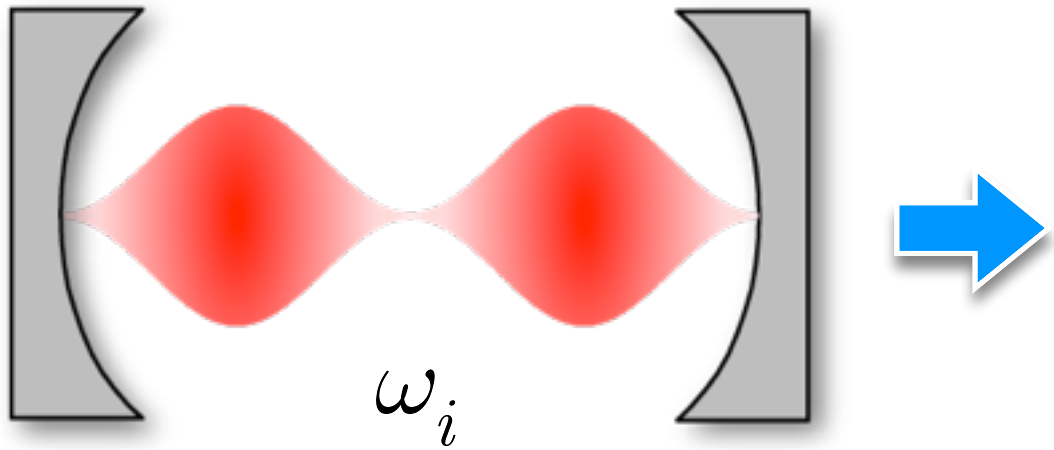

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Orthogonality relation

$$\int \frac{1}{\mu}\left(\mathcal{H}_i^* \cdot \mathcal{H}_j\right) dV = \delta_{ij}$$

$$\int \epsilon\left(\mathcal{E}_i^* \cdot \mathcal{E}_j\right) dV = \delta_{ij}$$

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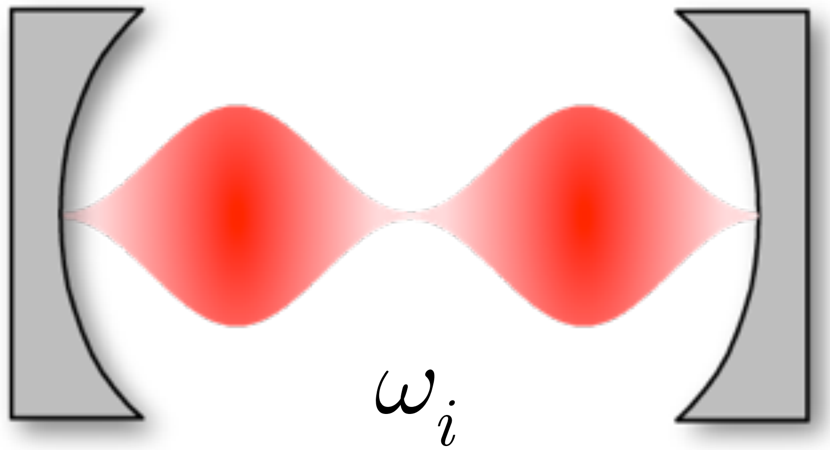

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Spatial solution

$$\nabla \times \mathcal{H}_j = -\mathrm{i}\omega_j \epsilon_0 \mathcal{E}_j$$

$$\nabla \times \mathcal{E}_j = \mathrm{i}\mu_0 \omega_j \mathcal{H}_j$$

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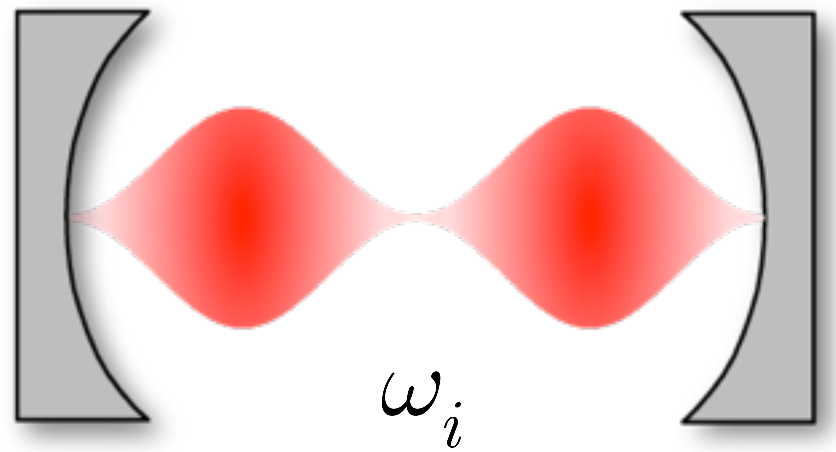

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Expand total optical field as linear superposition of the optical modes

$$\mathbf{E}\left(\mathbf{r},t\right) = \sum a_j(t)\mathcal{E}_j(\mathbf{r})$$

$$\mathbf{H}\left(\mathbf{r},t\right) = \sum a_j(t)\mathcal{H}_j(\mathbf{r})$$

Spatial solution

$$\nabla \times \mathcal{H}_j = -\mathrm{i}\omega_j \epsilon_0 \mathcal{E}_j$$

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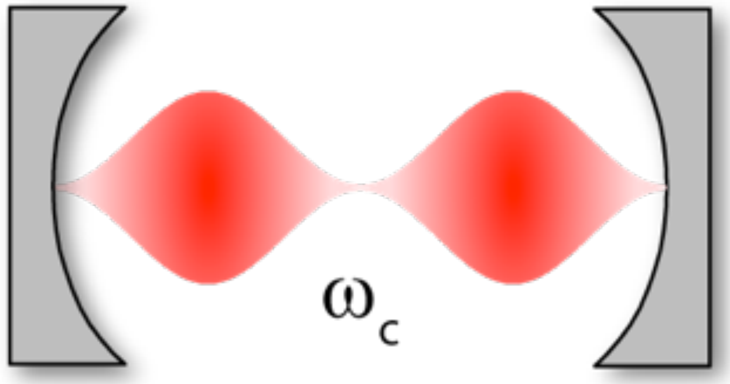
Derivation of field amplitude lumped model



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$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\mu_0\, \partial_t \mathbf{H}(\mathbf{r},t)$$
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Spatial solution

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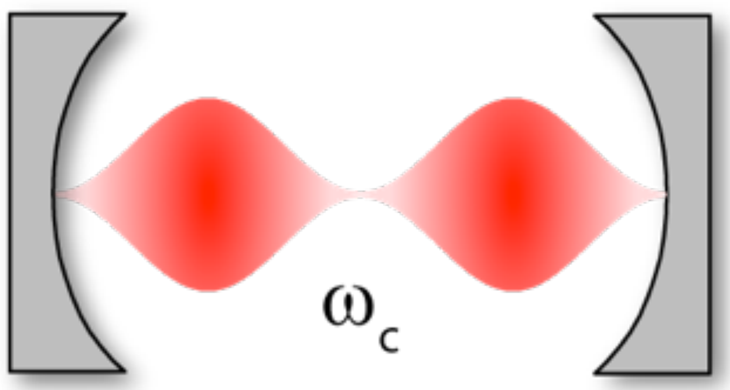

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$$\nabla \times \mathbf{H}(\mathbf{r},t) = \epsilon\, \partial_t \mathbf{E}(\mathbf{r},t)$$

To account for cavity loss we assume that:

$$\epsilon = \epsilon_r + i\epsilon_i$$

$$\epsilon_r \gg \epsilon_i \rightarrow \epsilon \approx \epsilon_r$$

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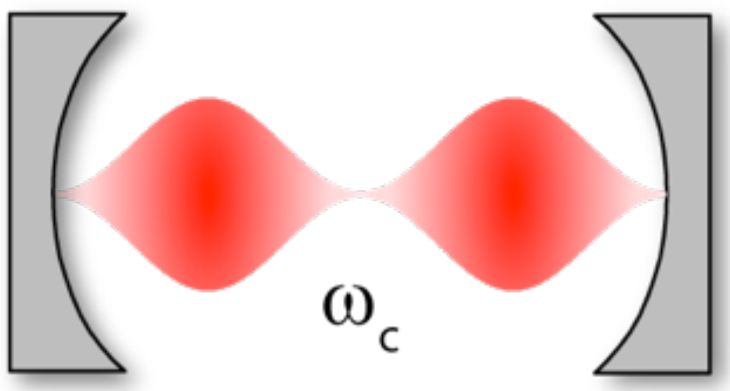
Derivation of field amplitude lumped model



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Therefore:

$$\nabla \times \left(\sum a_j(t)\mathcal{H}_j(\mathbf{r})\right) = (\epsilon_r + i\epsilon_i)\, \partial_t \left(\sum a_j(t)\mathcal{E}_j(\mathbf{r})\right)$$

$$\sum a_j(t)\left(\nabla \times \mathcal{H}_j(\mathbf{r})\right) = \sum (\epsilon_r + i\epsilon_i)\mathcal{E}_j(\mathbf{r})\dot{a}_j(t)$$

Spatial solution

$$\nabla \times \mathcal{H}_j = -\mathrm{i}\omega_j\epsilon_0\mathcal{E}_j$$
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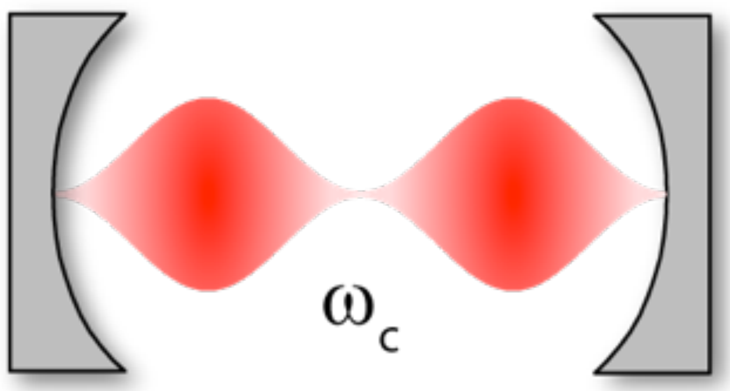
Derivation of field amplitude lumped model



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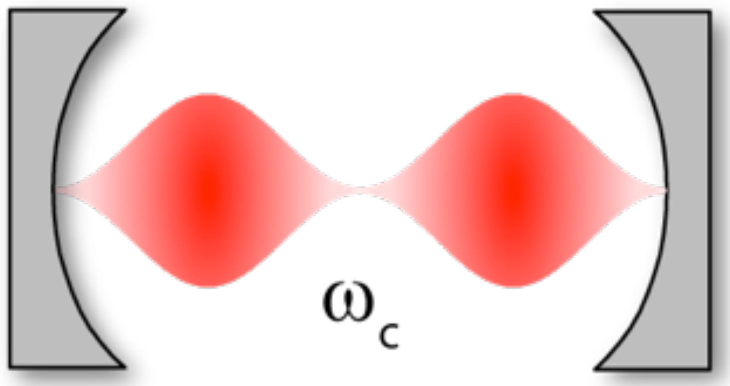

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$$\int\left[\sum \epsilon\,\mathcal{E}_j(\mathbf{r})\left[\left(1 + i\frac{\epsilon_i}{\epsilon}\right)\dot{a}_j(t) + i\omega_j a_j(t)\right] = 0\right]\mathcal{E}_n^*(\mathbf{r})dV$$

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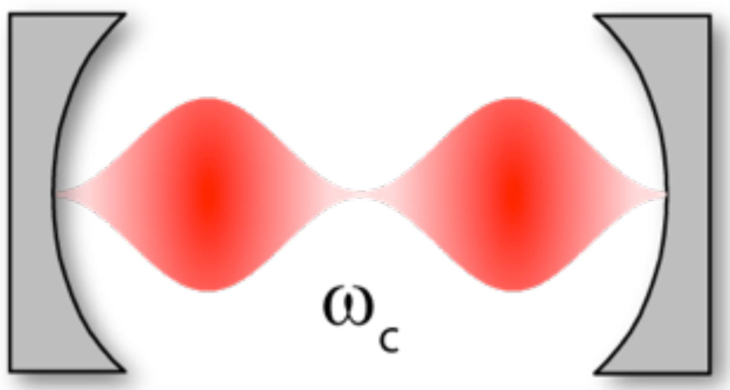

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$$\dot{a}_n(t) = -i\omega_n a_n(t) - \left(\omega_n \frac{\epsilon_i}{\epsilon}\right) a_n(t)$$

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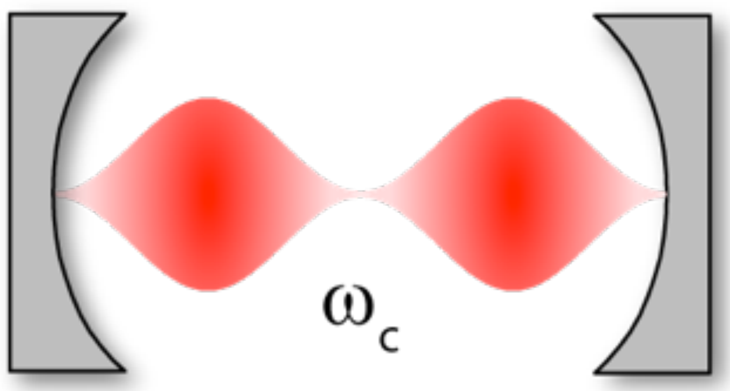
Derivation of field amplitude lumped model



$$\mathbf{E}(\mathbf{r},t) = \sum a_j(t)\mathcal{E}_j(\mathbf{r})$$
$$\mathbf{H}(\mathbf{r},t) = \sum a_j(t)\mathcal{H}_j(\mathbf{r})$$

Orthogonality relation

$$\int \frac{1}{\mu}\left(\mathcal{H}_i^* \cdot \mathcal{H}_j\right) dV = \delta_{ij}$$
$$\int \epsilon\left(\mathcal{E}_i^* \cdot \mathcal{E}_j\right) dV = \delta_{ij}$$

Spatial solution

$$\nabla \times \mathcal{H}_j = -\mathrm{i}\omega_j \epsilon_0 \mathcal{E}_j$$
$$\nabla \times \mathcal{E}_j = \mathrm{i}\mu_0 \omega_j \mathcal{H}_j$$

Therefore:

$$\mathbf{E}(\mathbf{r},t) = \sum a_j(t)\mathcal{E}_j(\mathbf{r})$$
$$\mathbf{H}(\mathbf{r},t) = \sum a_j(t)\mathcal{H}_j(\mathbf{r})$$

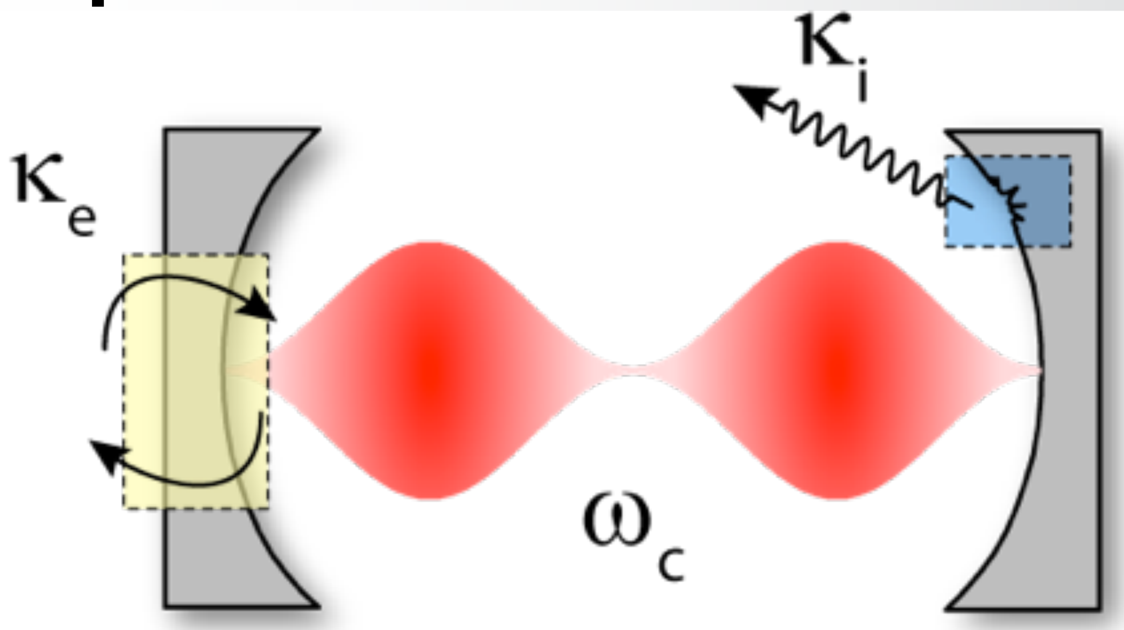
$$\dot{a}_n(t) = -i\omega_n a_n(t) - \left(\omega_n \frac{\epsilon_i}{\epsilon}\right) a_n(t)$$

Lumped Model

$$\dot{a}(t) = -i\omega_c a(t) - \frac{\kappa}{2} a(t)$$

H. A. Haus. Waves and fields in optoelectronics. Prentice-Hall, 1984.

Equation for the field amplitude

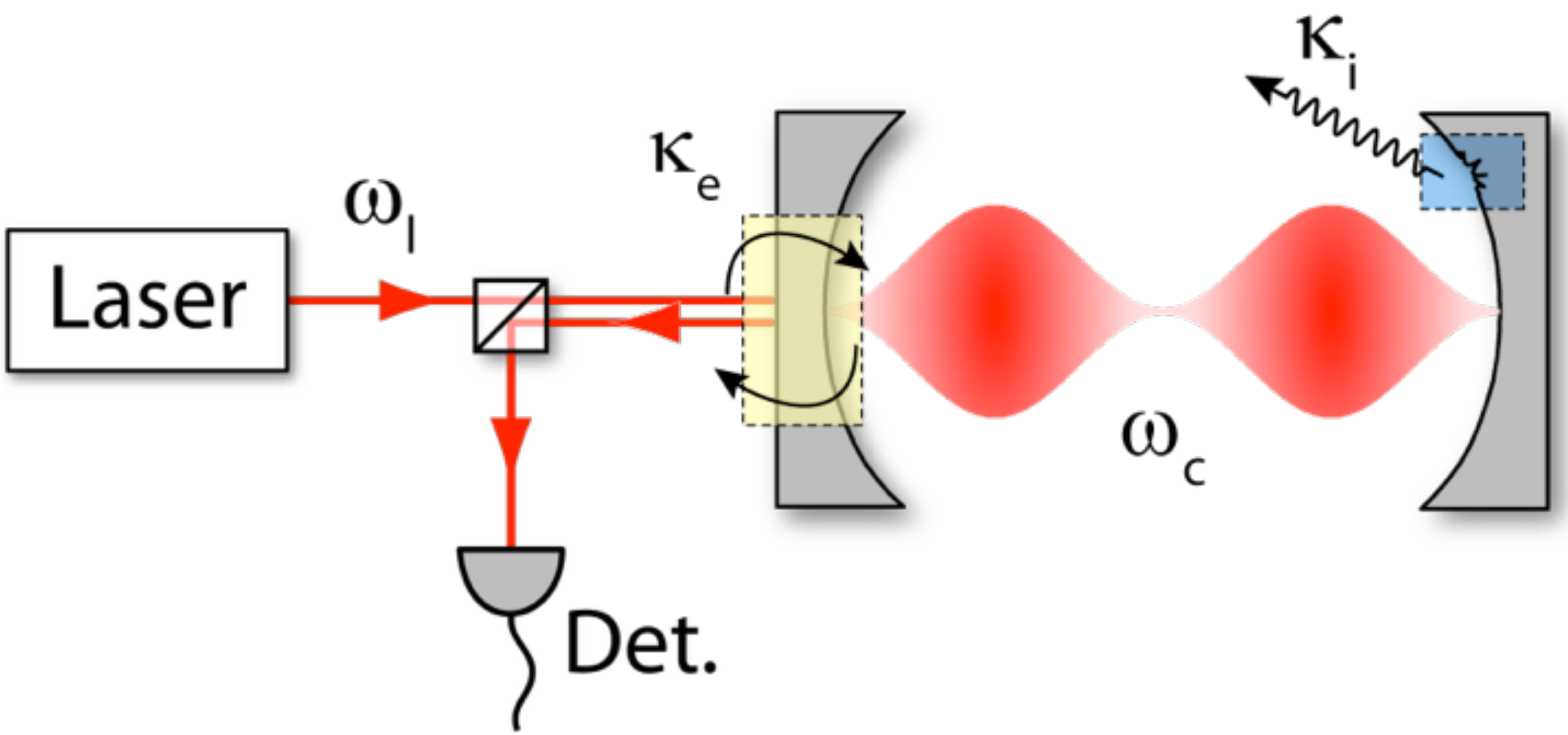


$$\kappa = \kappa_e + \kappa_i$$

$$\dot{a}(t) = -\mathrm{i}\omega_c a(t) - \frac{\kappa}{2} a(t) \quad + \sqrt{\kappa_e}\alpha_{\mathrm{in}}\, \mathrm{e}^{-\mathrm{i}\omega_l t}$$

H. A. Haus. Waves and fields in optoelectronics. Prentice-Hall, 1984.

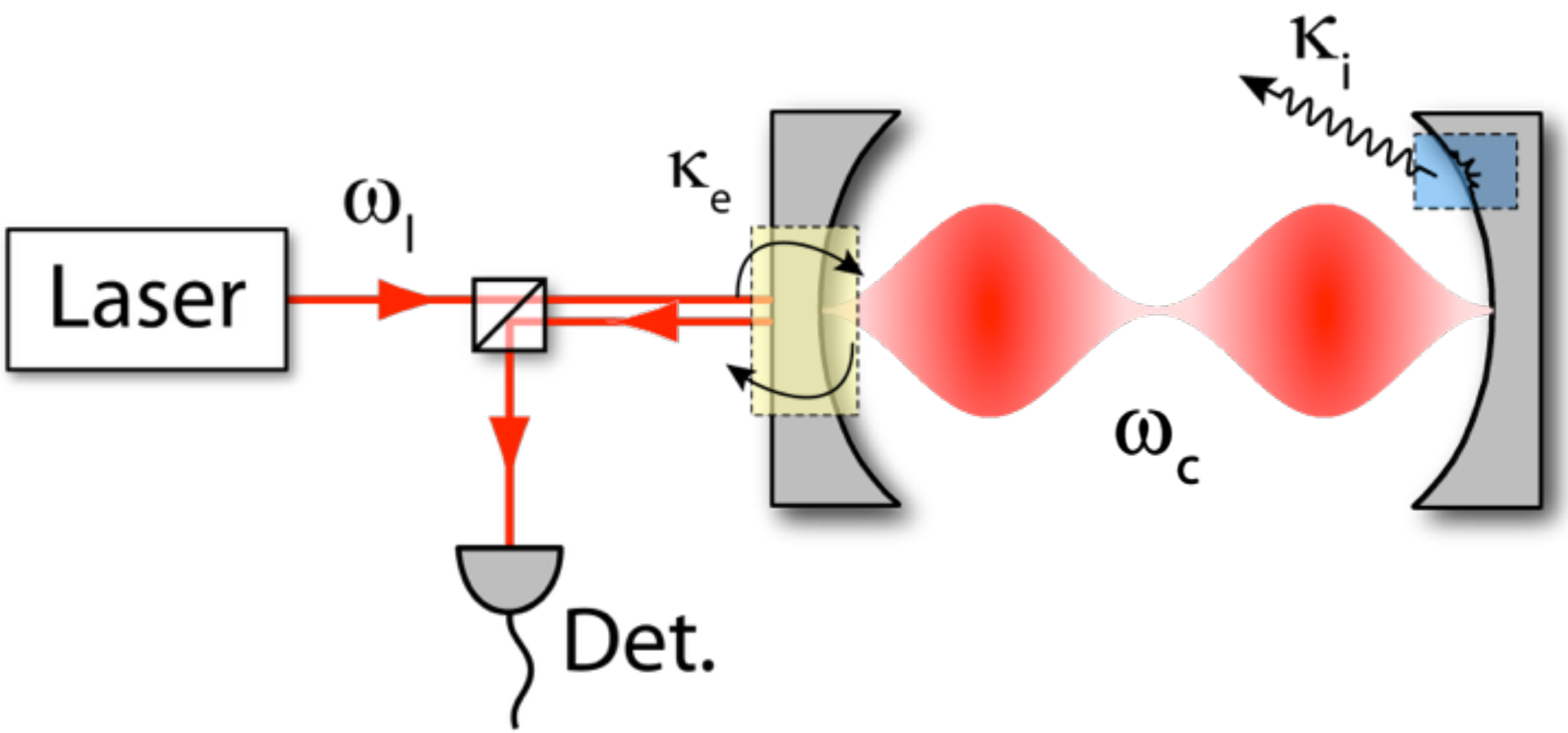
Optical Time Domain Equation



$$\dot{a}(t) = -i\omega_c a(t) - \frac{\kappa}{2} a(t) + \sqrt{\kappa_e}\, \alpha_{in}\, e^{-i\omega_l t}$$

α_{in} = Optical Pump Field Rate

Optical Time Domain Equation



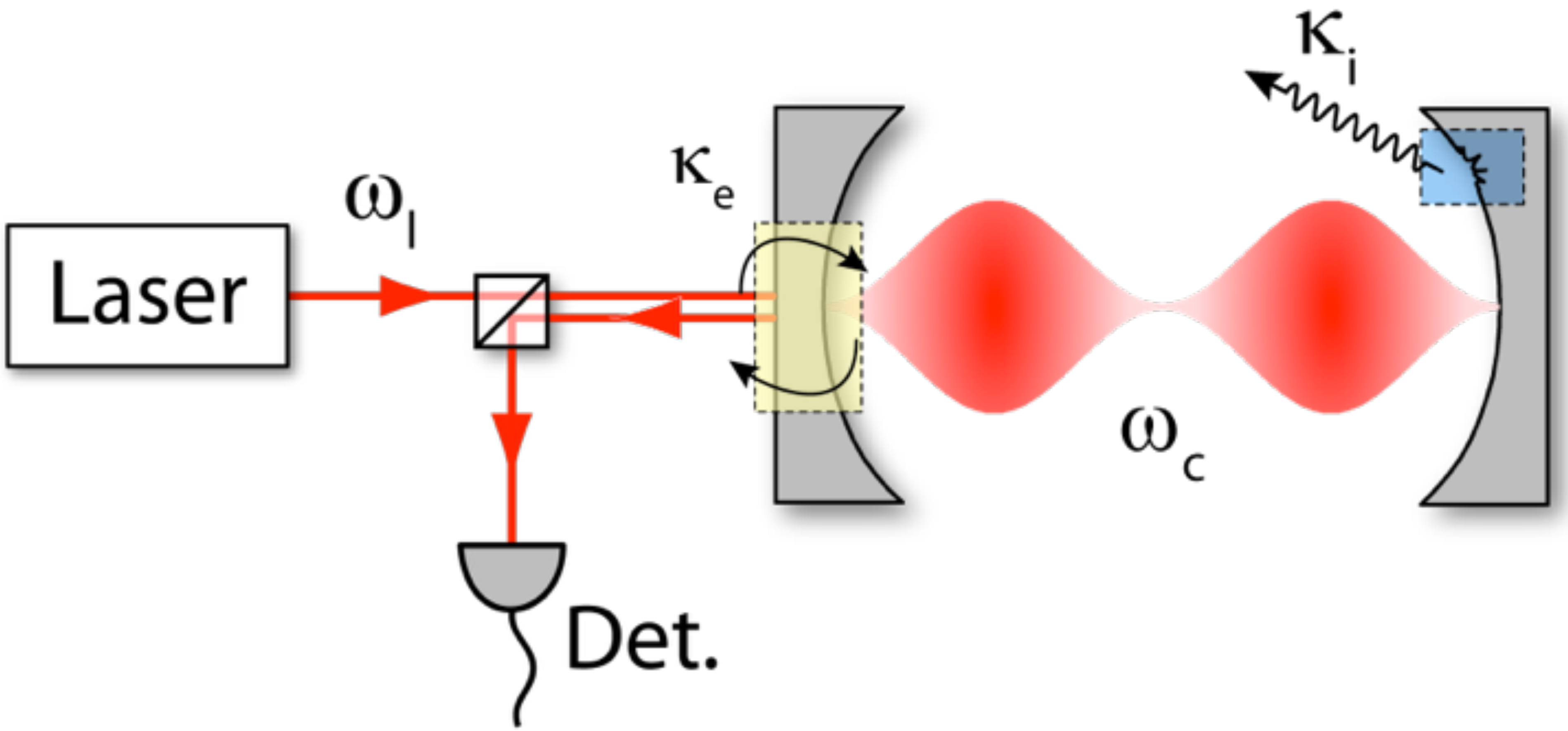
$$\dot{a}(t) = -\mathrm{i}\omega_c a(t) - \frac{\kappa}{2} a(t) + \sqrt{\kappa_e}\,\alpha_{\mathrm{in}}\, \mathrm{e}^{-\mathrm{i}\omega_l t}$$

Normalization

$$\hbar\omega_l \mid \alpha_{\mathrm{in}} \mid^2 = P_{in}$$

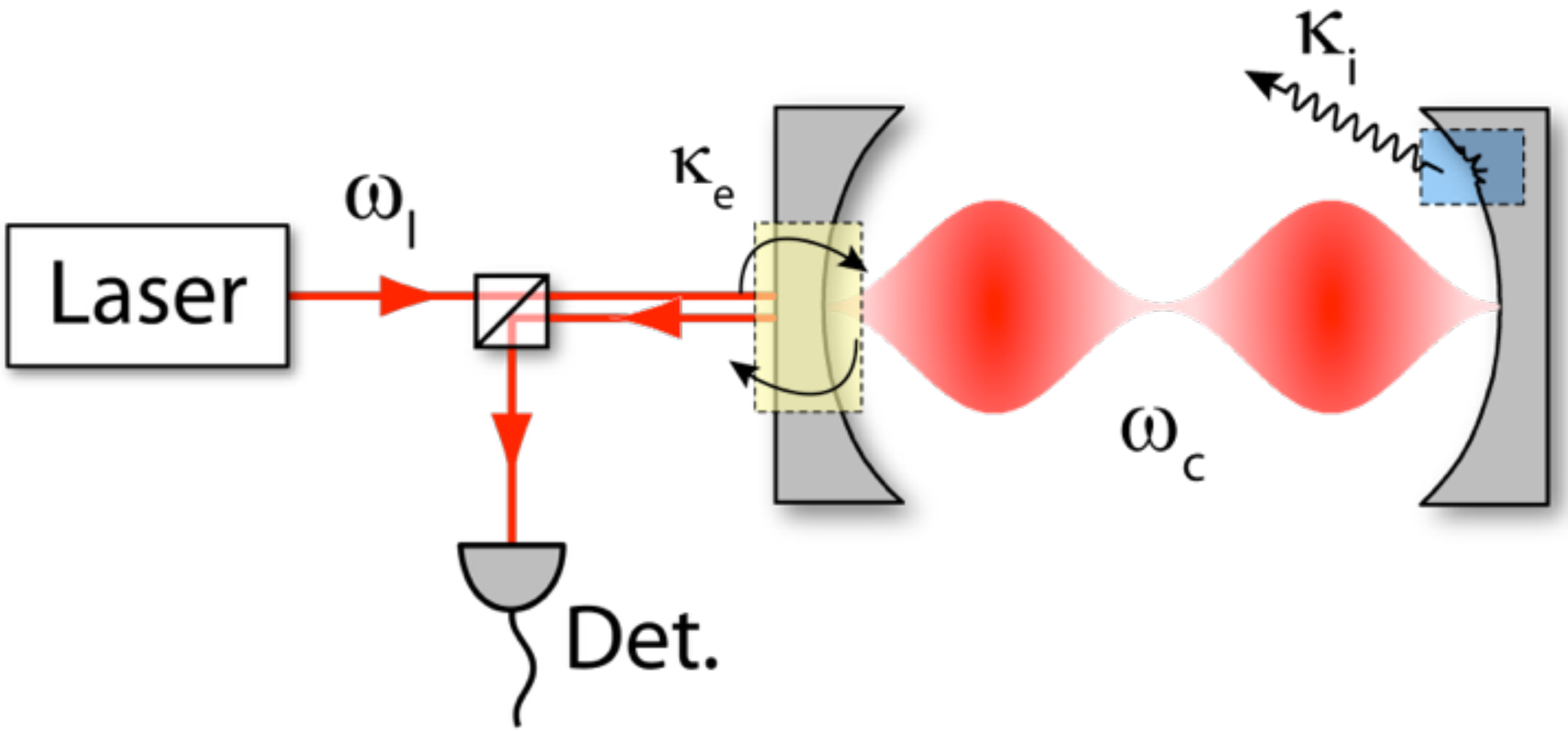
α_{in} = Optical Pump Field Rate

Rotating wave approximation



$$\dot{a}(t) = -\mathrm{i}\omega_c a(t) - \frac{\kappa}{2} a(t) + \sqrt{\kappa_e}\alpha_{\mathrm{in}} \boxed{\mathrm{e}^{-\mathrm{i}\omega_l t}}$$

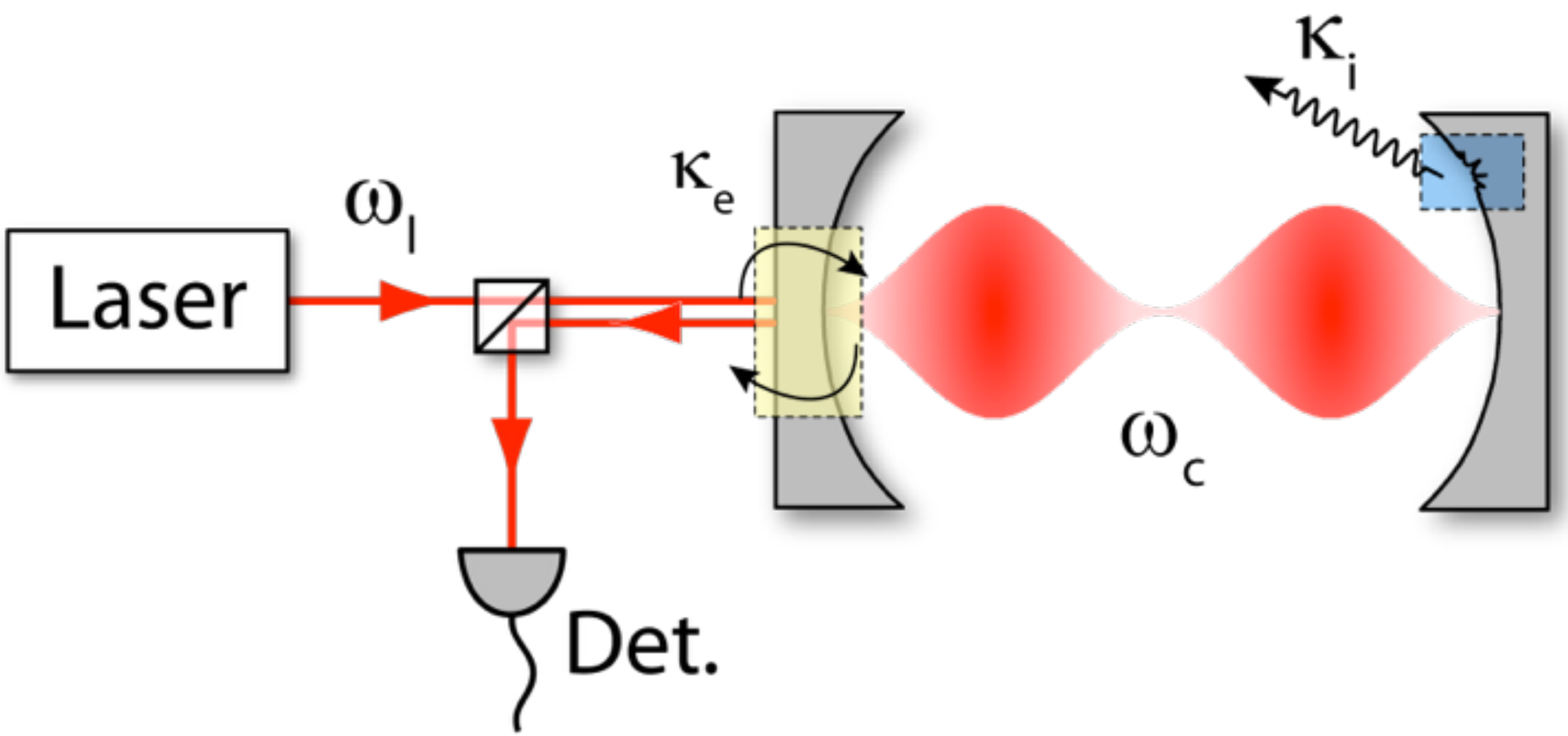
Rotating wave approximation



$$\dot{a}(t) = -\mathrm{i}\omega_c a(t) - \frac{\kappa}{2} a(t) + \sqrt{\kappa_e}\alpha_{\mathrm{in}}\, \mathrm{e}^{-\mathrm{i}\omega_l t}$$

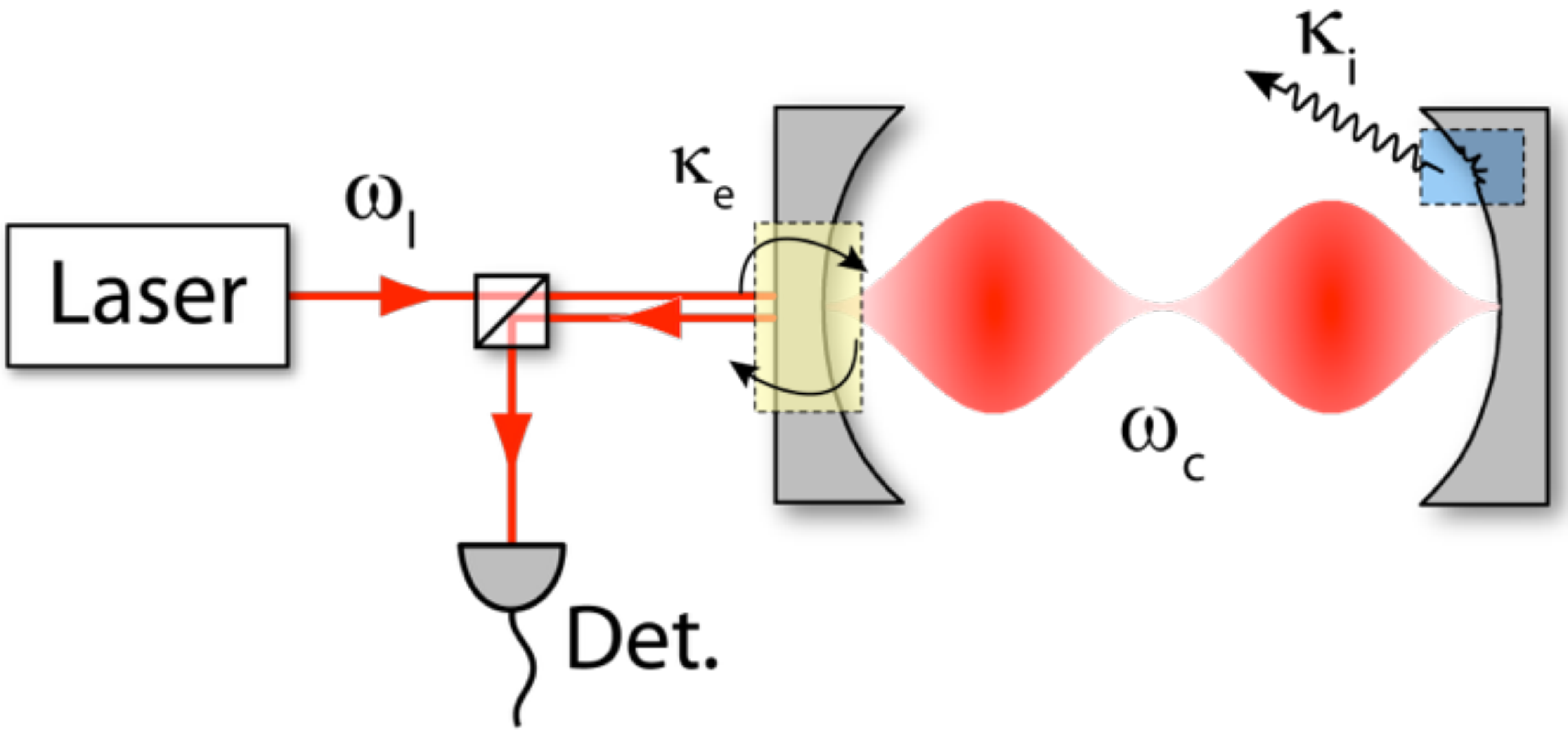
$$a_{\mathrm{old}}(t) = a(t) e^{-i\omega_l t}$$

Rotating wave approximation



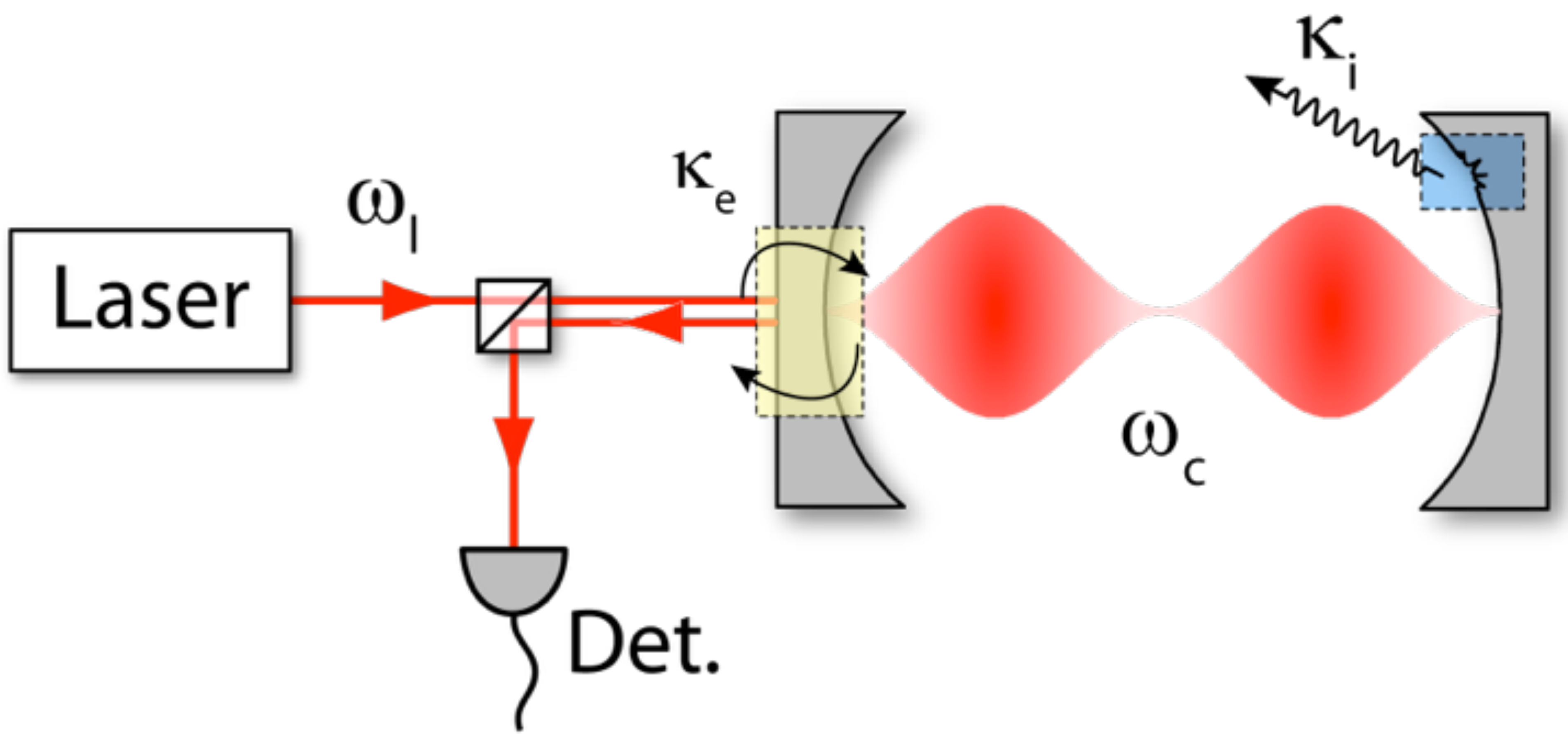
$$\dot{a}(t) = \boxed{-\mathrm{i}\omega_c} a(t) - \frac{\kappa}{2} a(t) + \sqrt{\kappa_e}\alpha_{\mathrm{in}} \boxed{\mathrm{e}^{-\mathrm{i}\omega_l t}}$$

Optical Time Domain Equation



$$\dot{a}(t) = \mathrm{i}(\omega_l - \omega_c)a - \frac{\kappa}{2}a + \sqrt{\kappa_e}\alpha_{\mathrm{in}}$$

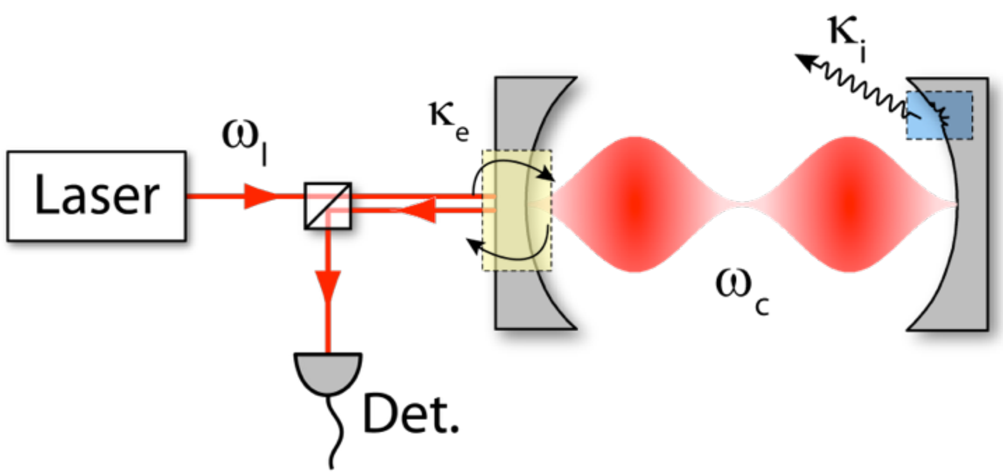
Optical Time Domain Equation

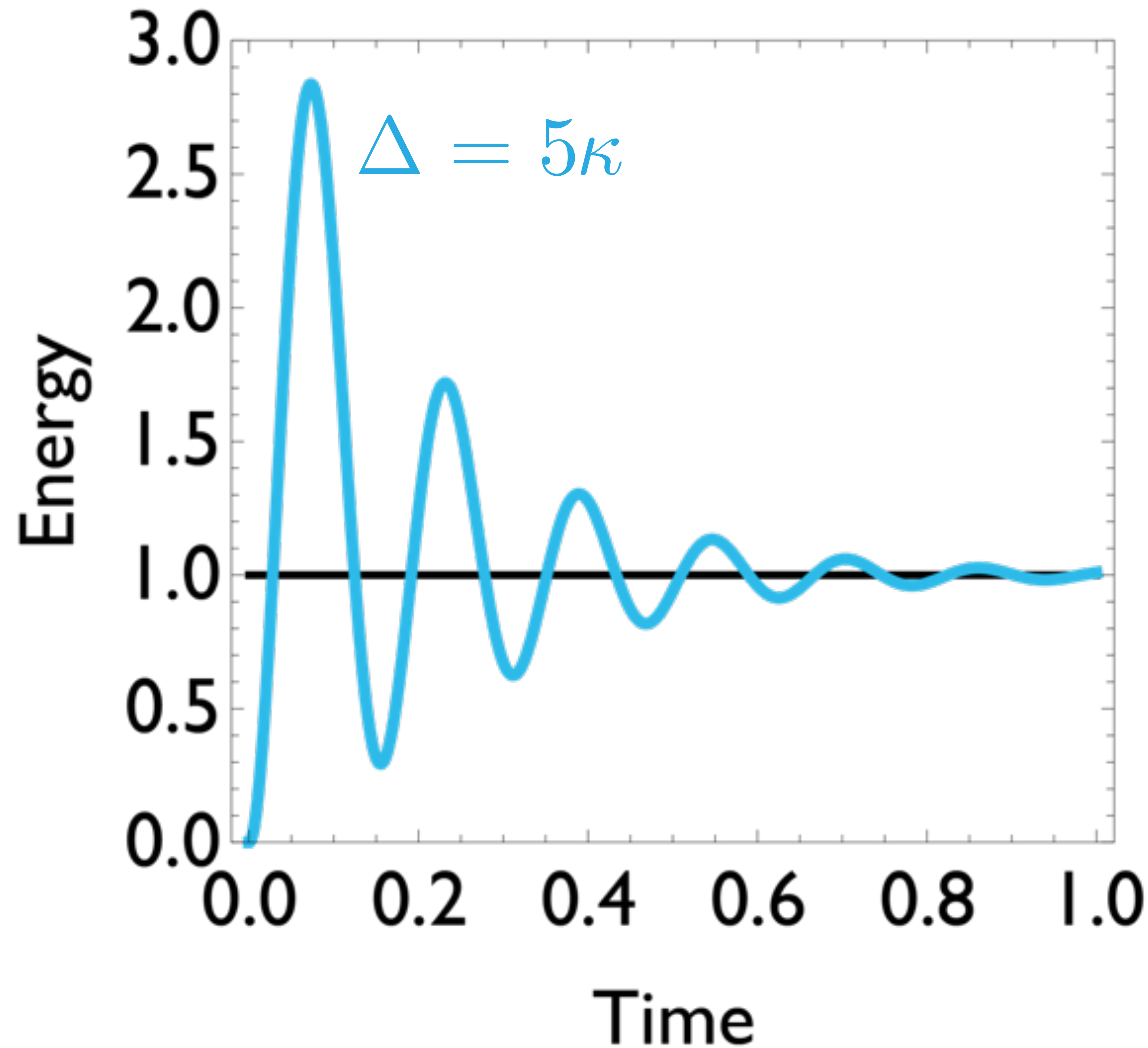


$$\dot{a}(t) = \mathrm{i}\Delta a - \frac{\kappa}{2} a + \sqrt{\kappa_e}\alpha_{\mathrm{in}}$$

Optical Amplitude Equation

Optical Time Domain Equation



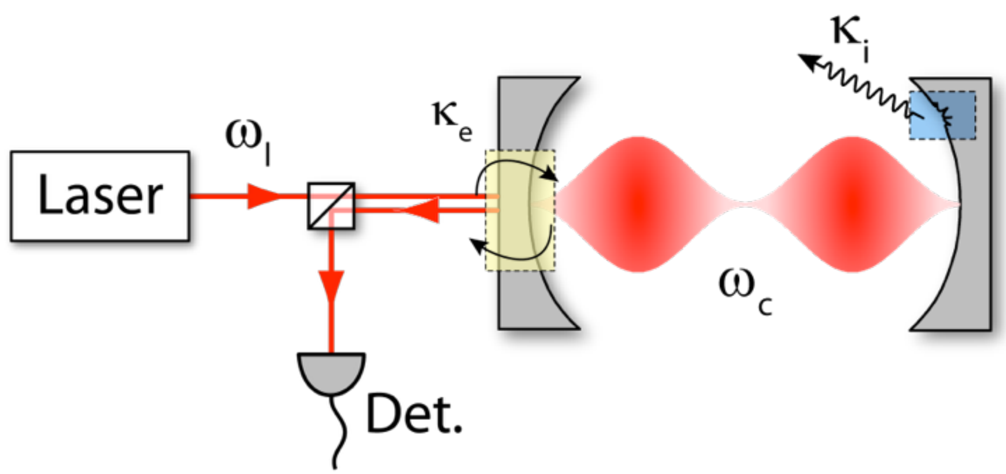


Solution

$$\dot{a}(t) = \mathrm{i}\Delta a - \frac{\kappa}{2} a + \sqrt{\kappa_e}\alpha_{\mathrm{in}}$$

$$|a(t)|^2 \propto \text{Energy}$$

Optical Time Domain Equation



Solution

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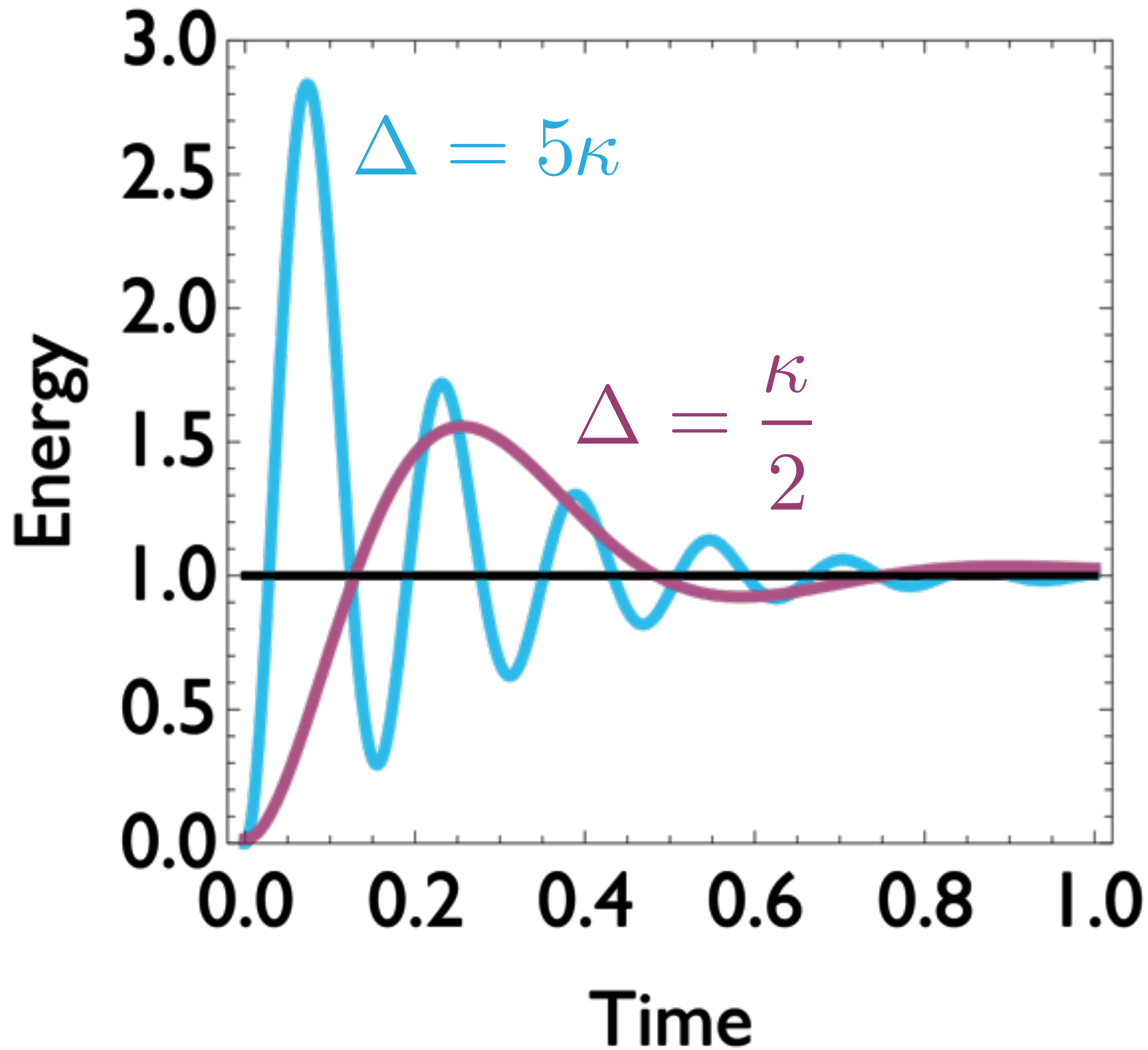
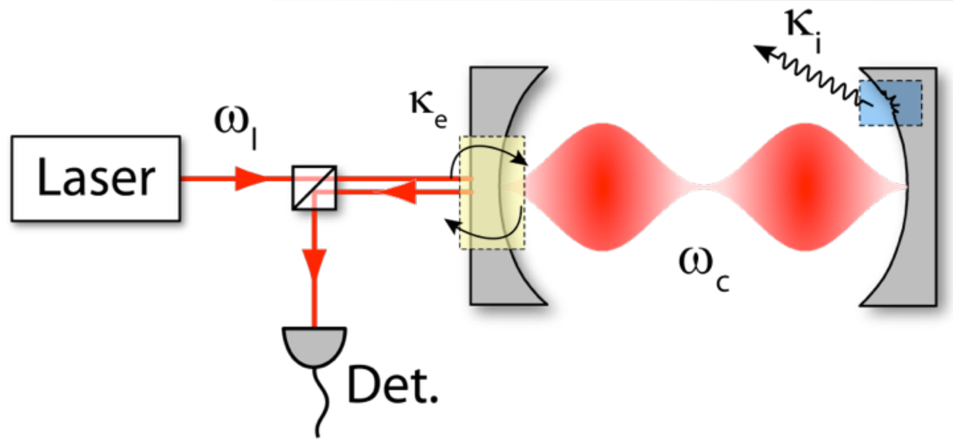
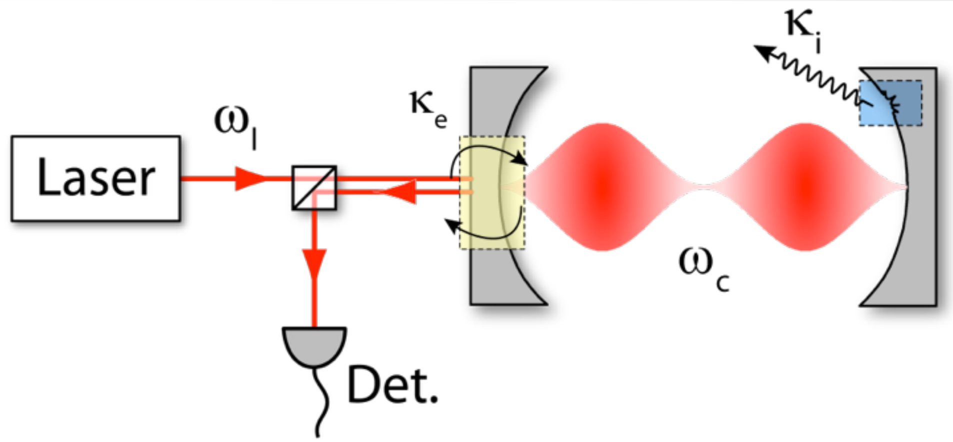


Optical - Frequency Domain



$$\dot{a}(t) = i\Delta a - \frac{\kappa}{2}a + \sqrt{\kappa_e}\alpha_{\text{in}}$$

Optical - Frequency Domain



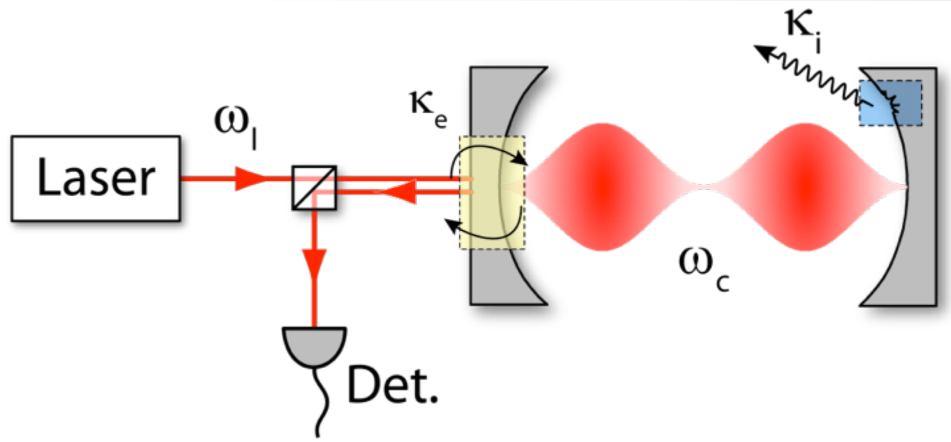
$$\dot{a}(t) = i\Delta a - \frac{\kappa}{2}a + \sqrt{\kappa_e}\alpha_{\text{in}}$$

Steady state

$$\dot{a}(t) = 0$$

$$\langle a \rangle = \frac{\sqrt{\kappa_e}\alpha_{\text{in}}}{\frac{\kappa}{2} - i\Delta}$$

Optical - Frequency Domain



$$\dot{a}(t) = i\Delta a - \frac{\kappa}{2} a + \sqrt{\kappa_e}\alpha_{\text{in}}$$

Steady state

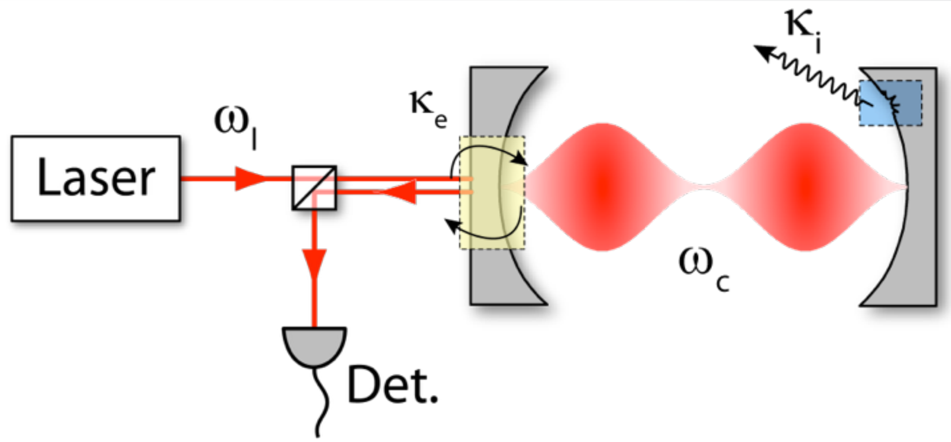
$$\dot{a}(t) = 0$$

$$\langle a \rangle = \frac{\sqrt{\kappa_e}\alpha_{\text{in}}}{\frac{\kappa}{2} - i\Delta}$$

Input-output relation

$$\alpha_{\text{out}} = \alpha_{\text{in}} - \sqrt{\kappa_e} a$$

Optical - Frequency Domain



$$\dot{a}(t) = i\Delta a - \frac{\kappa}{2}a + \sqrt{\kappa_e}\alpha_{\text{in}}$$

Input-output relation

$$\alpha_{\text{out}} = \alpha_{\text{in}} - \sqrt{\kappa_e}a$$

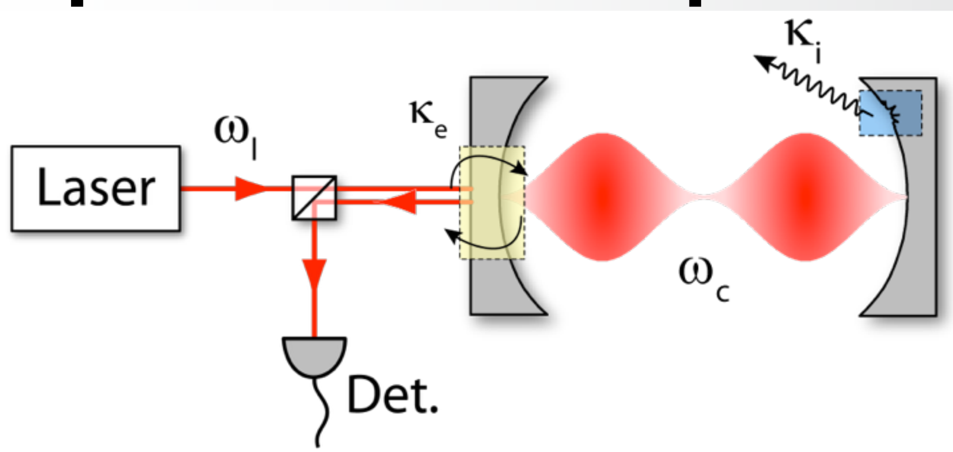
Steady state

$$\dot{a}(t) = 0$$

$$\langle a \rangle = \frac{\sqrt{\kappa_e}\alpha_{\text{in}}}{\frac{\kappa}{2} - i\Delta}$$

$$R = \frac{\alpha_{\text{out}}}{\alpha_{in}} = \frac{(\kappa_i - \kappa_e)\,/\,2 - i\Delta}{(\kappa_i + \kappa_e)\,/\,2 - i\Delta}$$

Optical - Frequency Domain



$$\dot{a}(t) = i\Delta a - \frac{\kappa}{2}a + \sqrt{\kappa_e}\alpha_{\text{in}}$$

Steady state

$$\dot{a}(t) = 0$$

$$\langle a \rangle = \frac{\sqrt{\kappa_e}\alpha_{\text{in}}}{\frac{\kappa}{2} - i\Delta}$$

Input-output relation

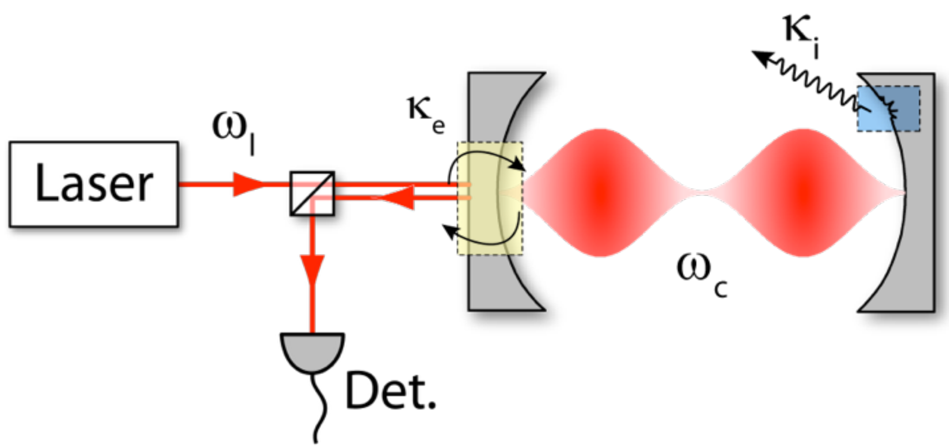
$$\alpha_{\text{out}} = \alpha_{\text{in}} - \sqrt{\kappa_e}a$$

$$R = \frac{\alpha_{\text{out}}}{\alpha_{in}} = \frac{(\kappa_i - \kappa_e)/2 - i\Delta}{(\kappa_i + \kappa_e)/2 - i\Delta}$$

Optical Spectra (Probability)

$$\left|R(\omega)\right|^2 = \frac{(\kappa_i - \kappa_e)^2 + 4\Delta^2}{(\kappa_i + \kappa_e)^2 + 4\Delta^2}$$

Optical - Frequency Domain



Steady state

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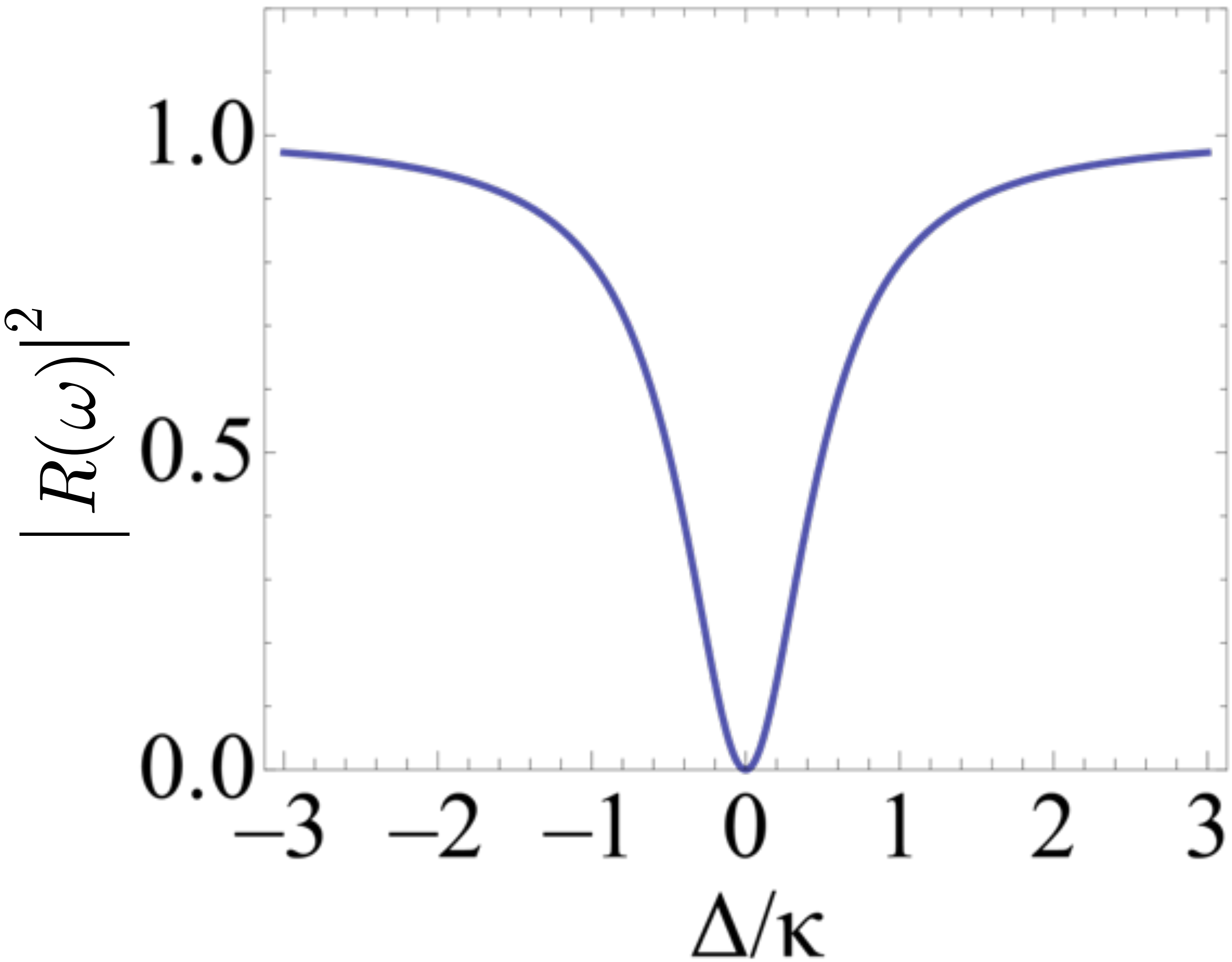
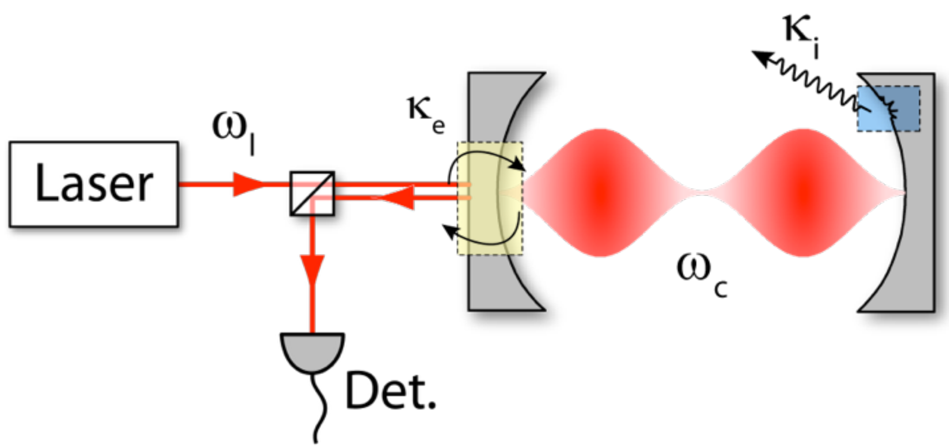
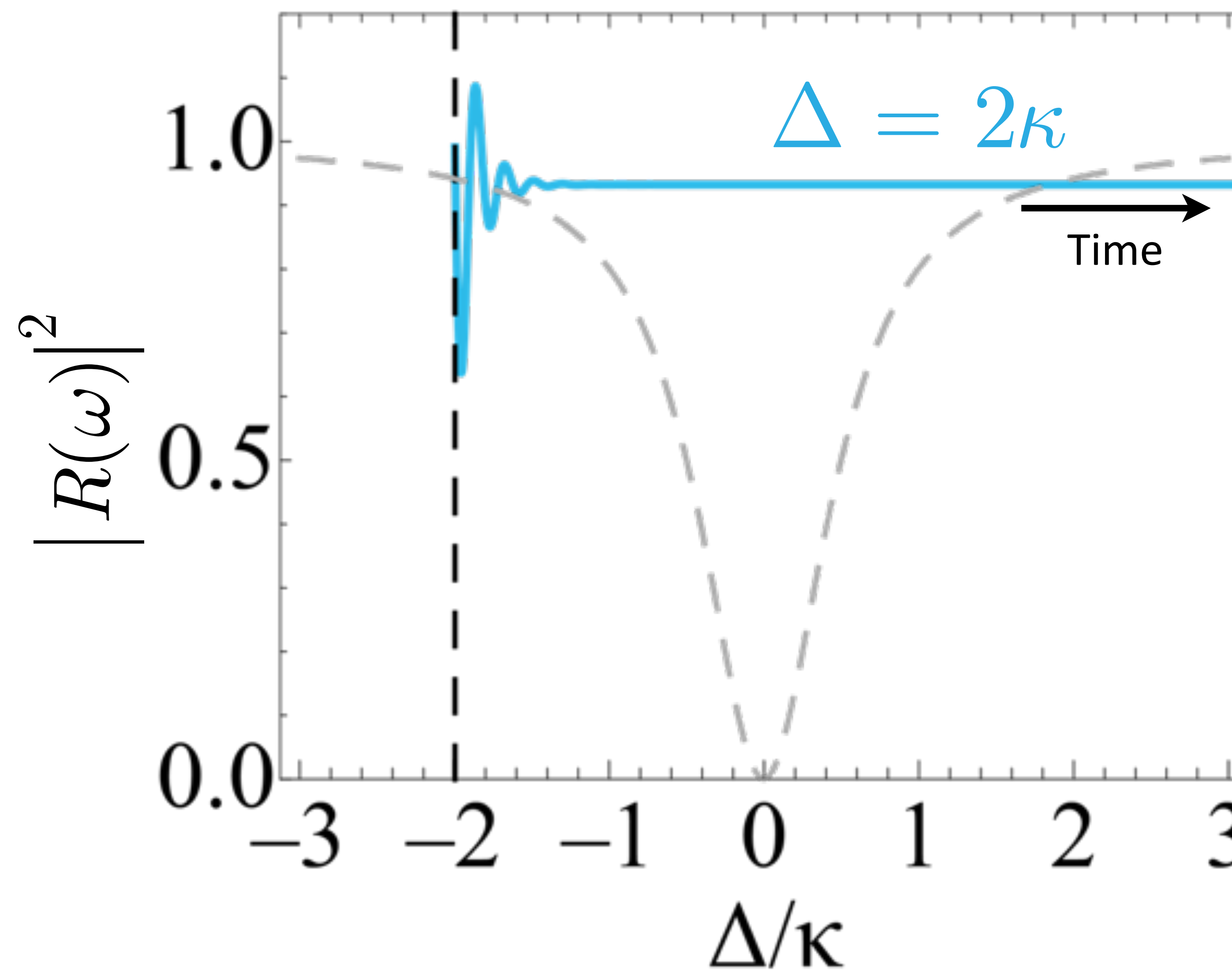


Optical - Frequency Domain

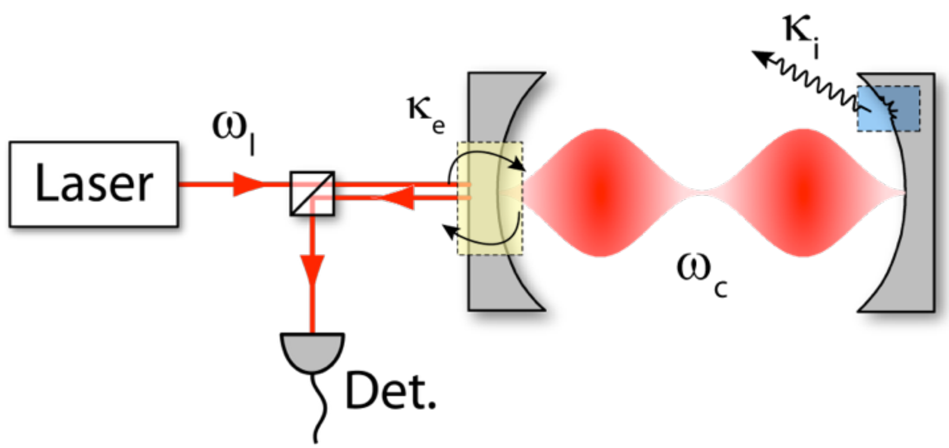


Steady state

$$\left|R(\omega)\right|^2 = \frac{(\kappa_i - \kappa_e)^2 + 4\Delta^2}{(\kappa_i + \kappa_e)^2 + 4\Delta^2}$$



Optical - Frequency Domain



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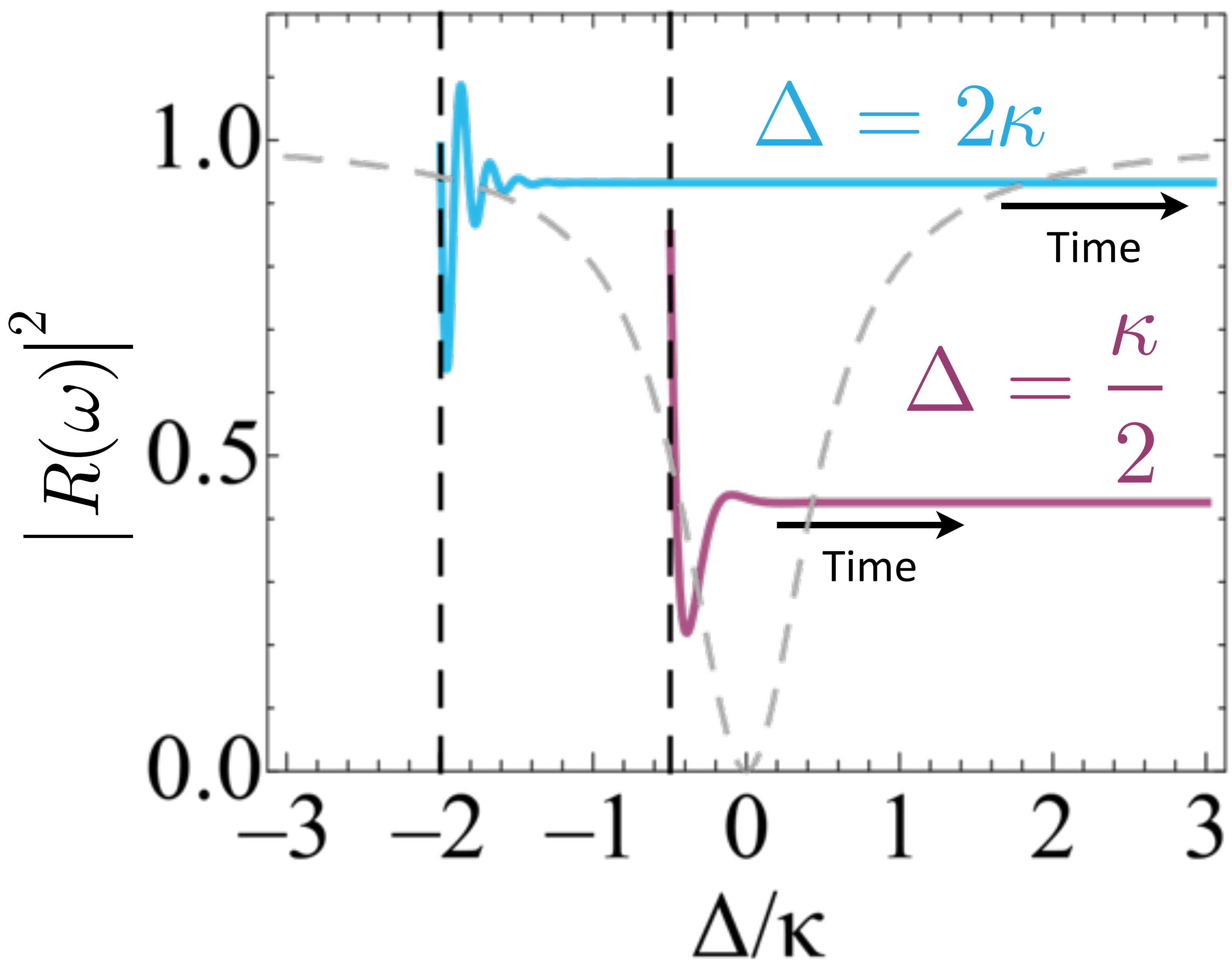
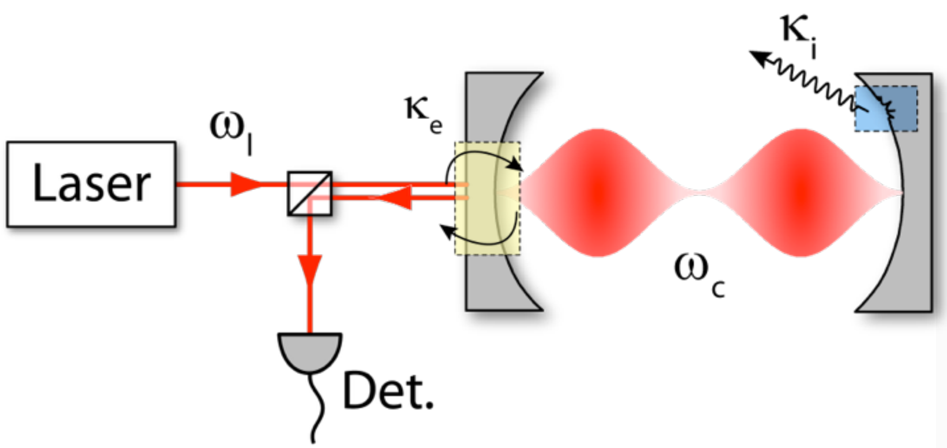


Optical - Frequency Domain



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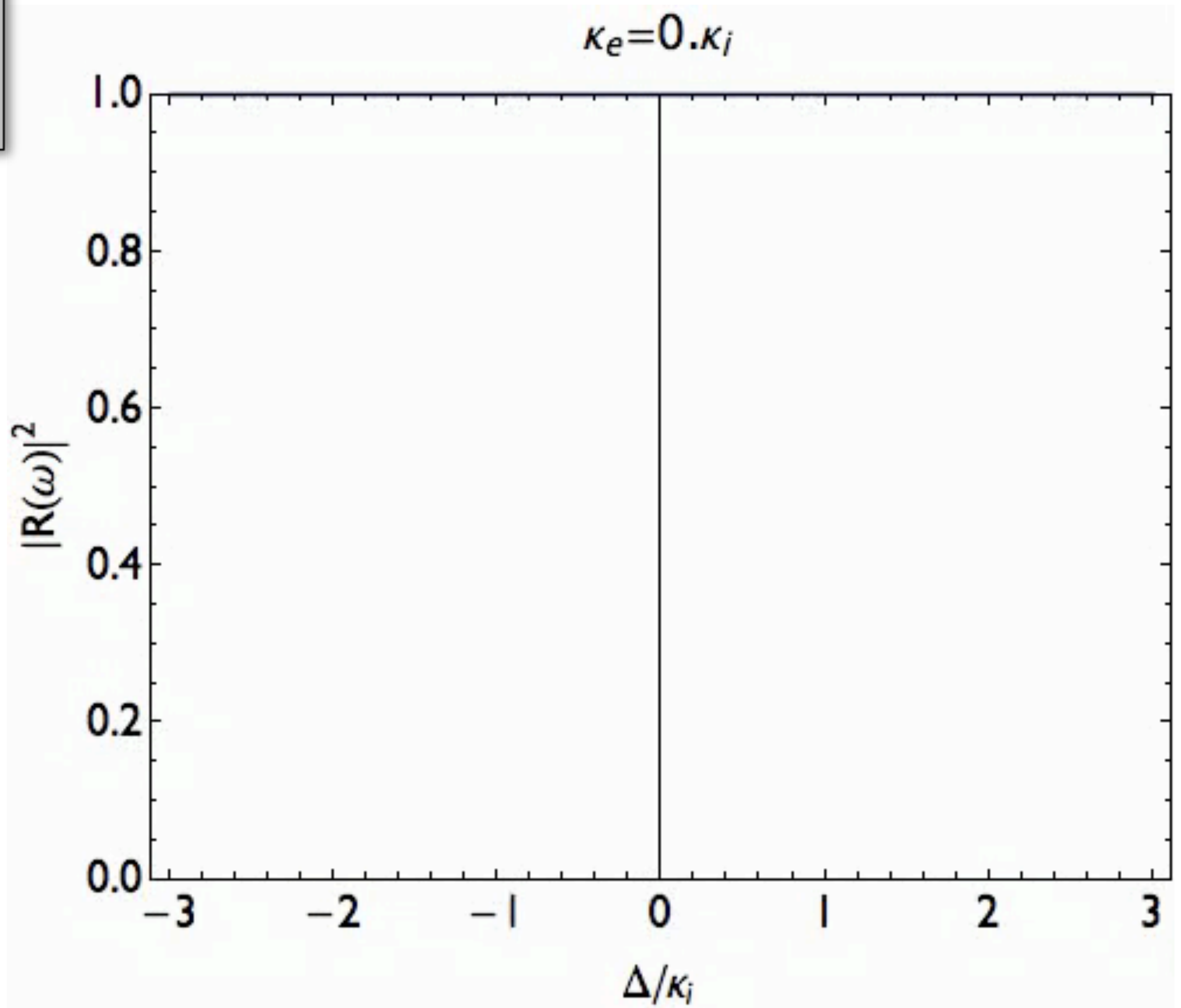
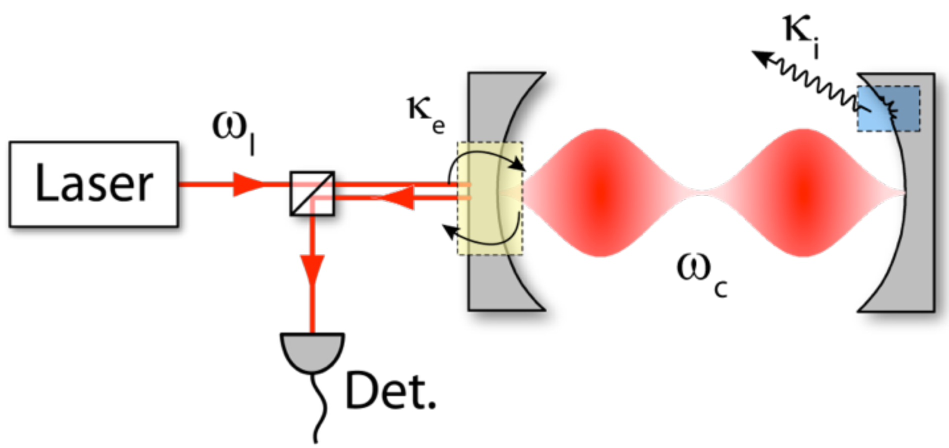


Optical - Frequency Domain



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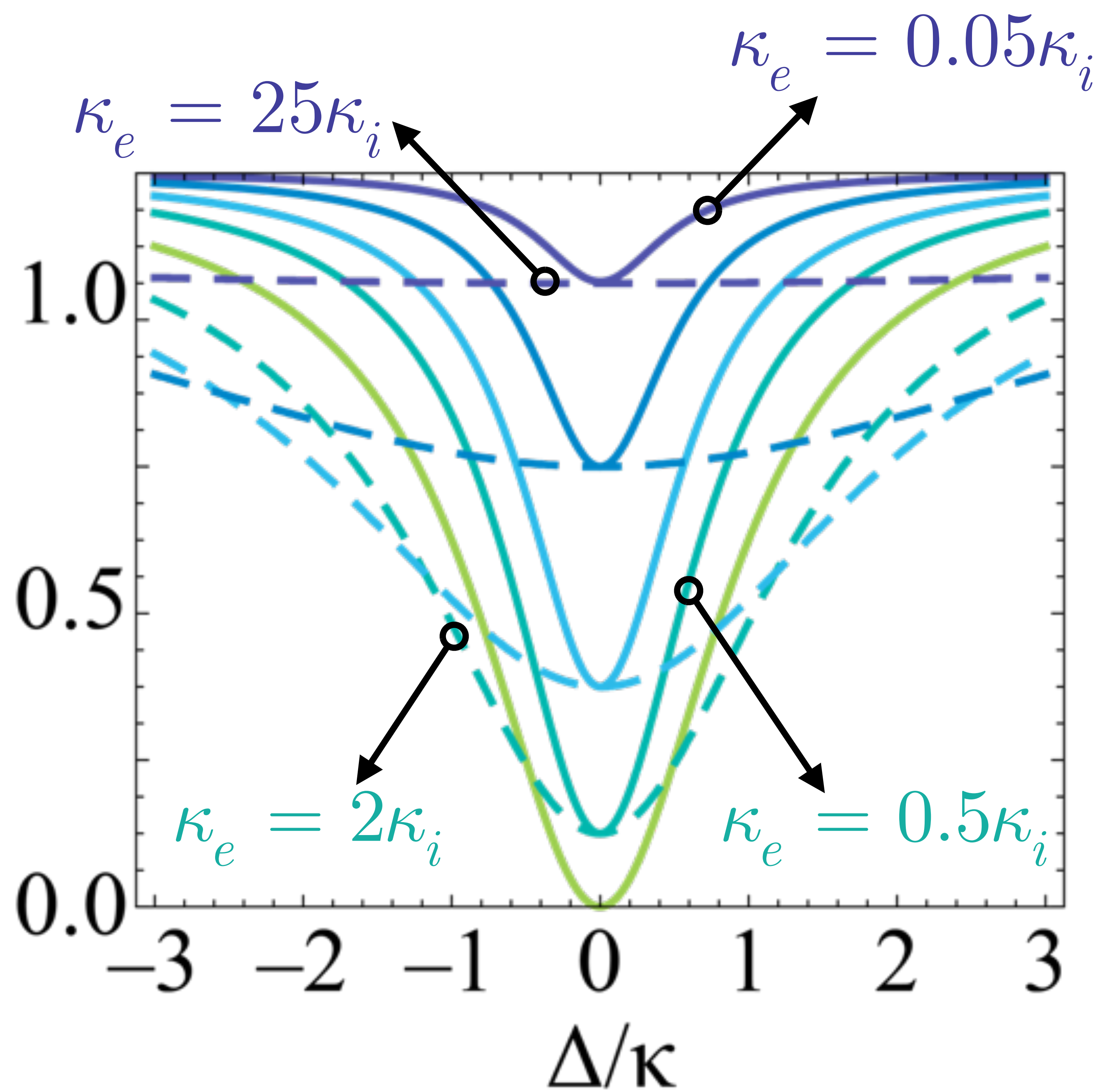
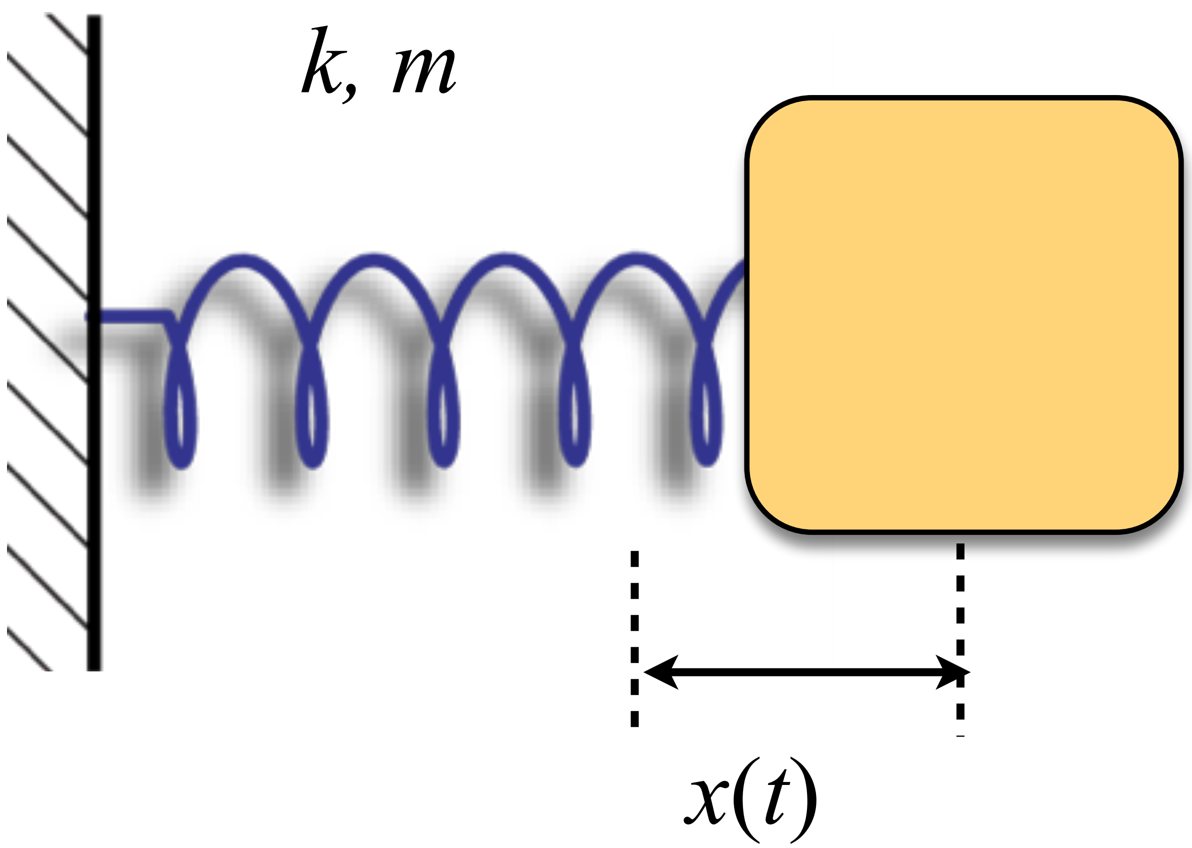
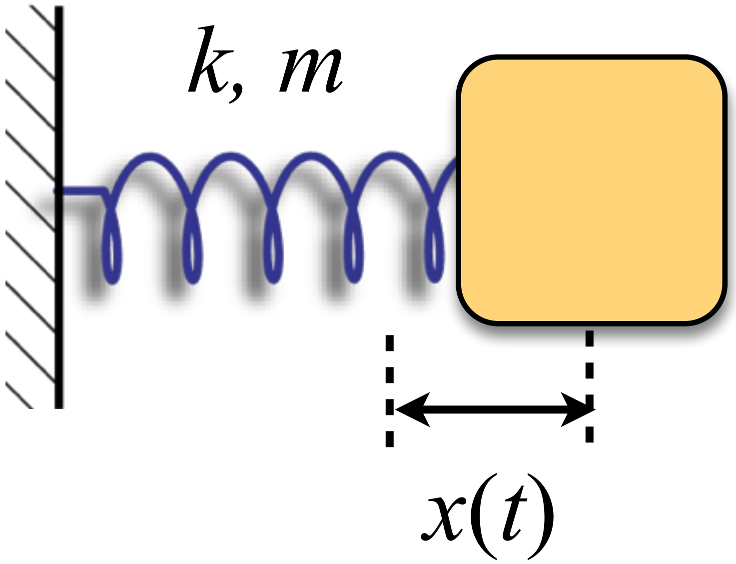


Mechanical modes



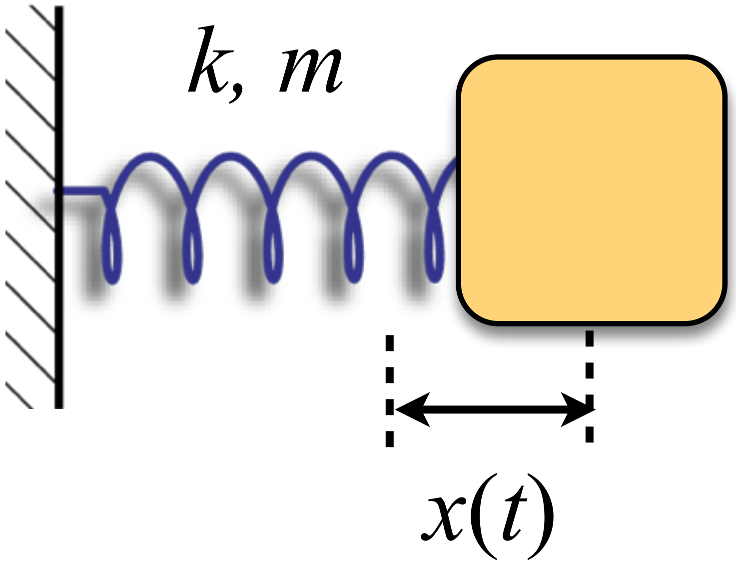
Mechanics of continuous

Single particle



Mechanics of continuous

Single particle

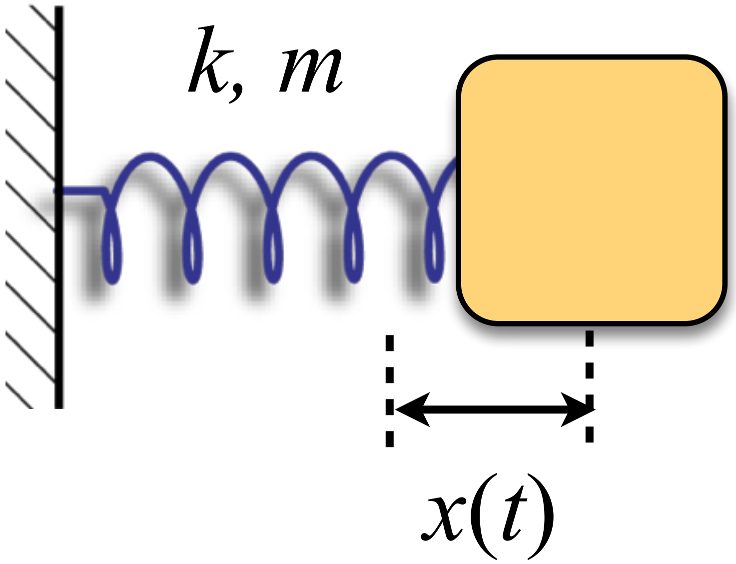


Newton's Law

$$F_x = m\frac{d^2x}{dt^2}$$

Mechanics of continuous

Single particle



Newton's Law

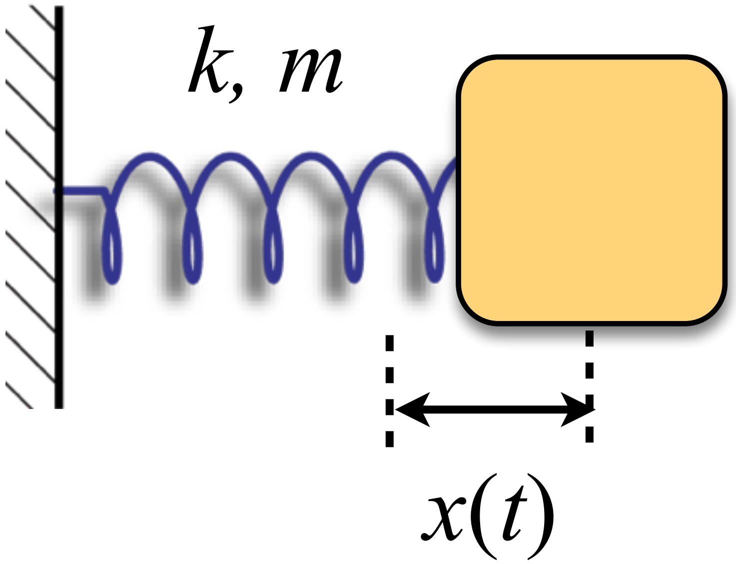
$$F_x = m\frac{d^2x}{dt^2}$$

Hooke's Law

$$F_x = -kx$$

Mechanics of continuous

Single particle



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Hooke's Law

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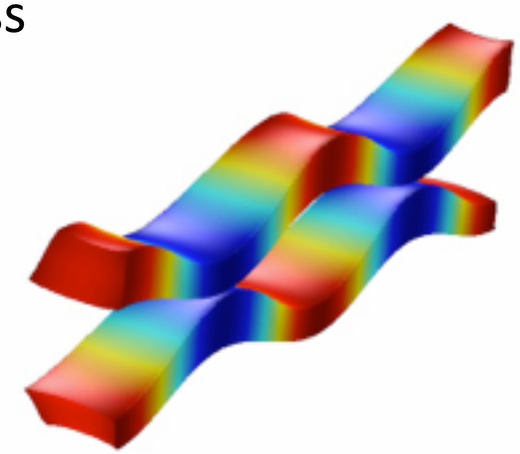
Solid mechanics

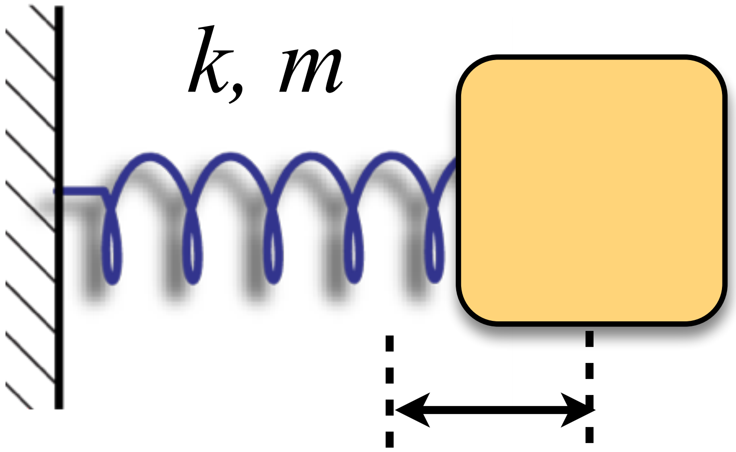
$\mathbf{c}$: elastic stiffness

ρ : density



Mechanics of continuous

Single particle



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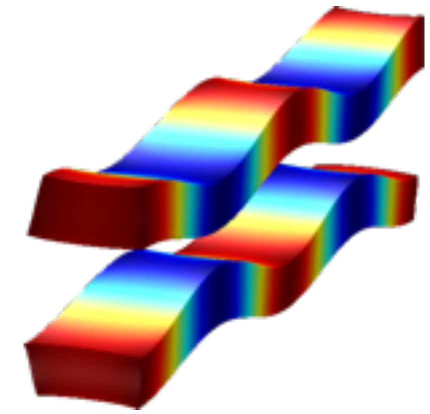


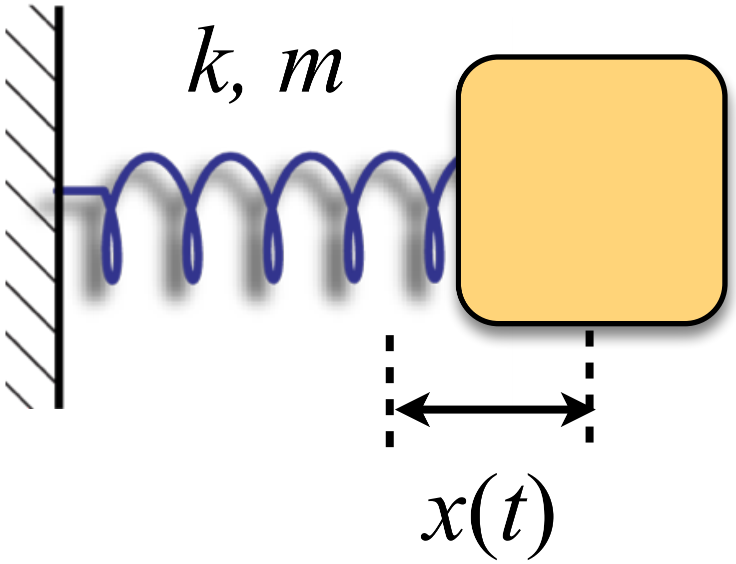
$\mathbf{u}(\vec{r},t)$

Newton's Law (solid)

$$\nabla \cdot \mathbf{T} = \rho\frac{d^2\mathbf{u}(\vec{r},t)}{dt^2}$$

Mechanics of continuous

Single particle



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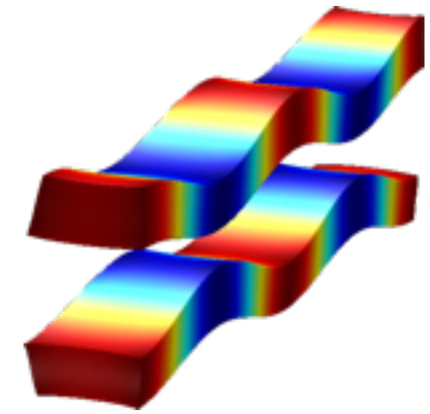


$\mathbf{u}(\vec{r},t)$

Newton's Law (solid)

$$\nabla\cdot\mathbf{T} = \rho\frac{d^2\mathbf{u}(\vec{r},t)}{dt^2}$$

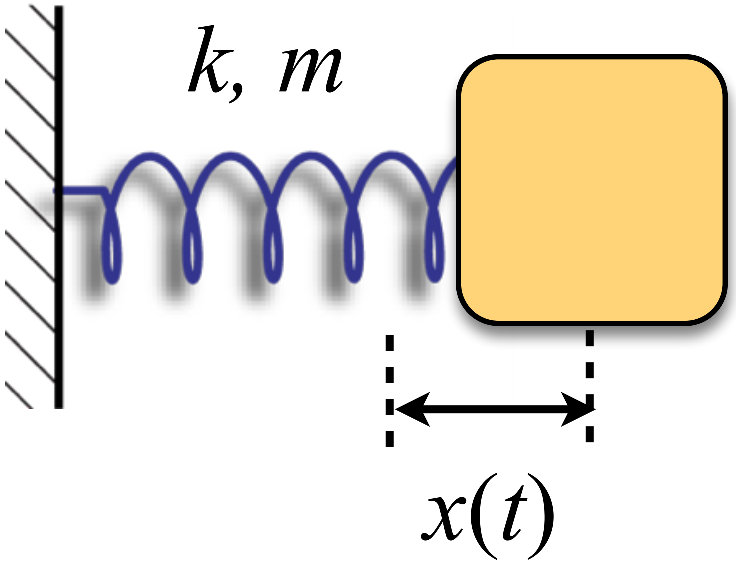
Hooke's Law

$$\mathbf{T} = \mathbf{c} : \mathbf{S}$$

$$\left(\underset{\text{strain}}{\mathbf{S}} = \nabla_s\mathbf{u}\right)$$

Mechanics of continuous

Single particle



Newton's Law

$$F_{\text{ext}} + F_x = m\frac{d^2x}{dt^2}$$

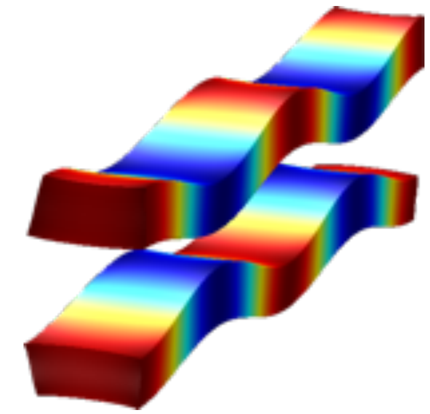
Hooke's Law

$$F_x = -kx$$

Solid mechanics

$\mathbf{c}$: elastic stiffness

ρ : density



$\mathbf{u}(\vec{r},t)$

Newton's Law (solid)

$$\nabla\cdot\mathbf{T}_{\text{ext}} + \nabla\cdot\mathbf{T} = \rho\frac{d^2\mathbf{u}(\vec{r},t)}{dt^2}$$

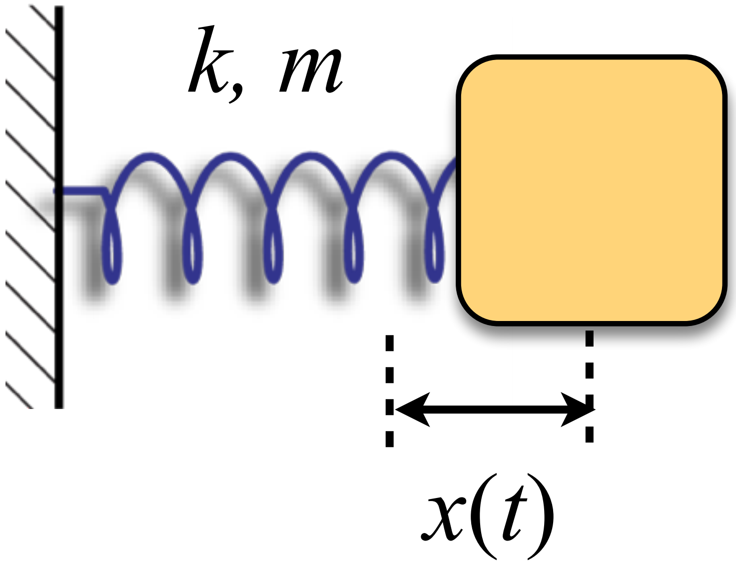
Hooke's Law

$$\nabla\cdot\mathbf{T} = \nabla\cdot\left(\mathbf{c}:\mathbf{S}\right)$$

$$\left(\underset{\text{strain}}{\mathbf{S} = \nabla_s\mathbf{u}}\right)$$

Mechanics of continuous

Single particle



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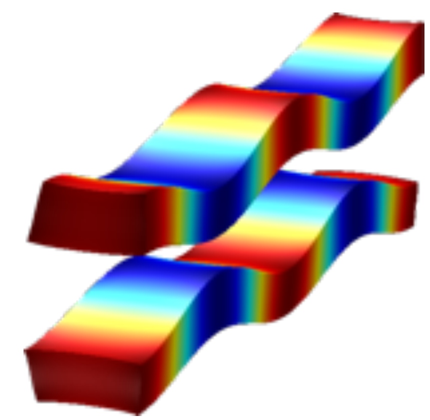


$\mathbf{u}(\vec{r},t)$

Newton's Law

$$\nabla\cdot\mathbf{T}_{\text{ext}} + \nabla\cdot(\mathbf{c}:\mathbf{S}) = \rho\frac{d^2\mathbf{u}(\vec{r},t)}{dt^2}$$

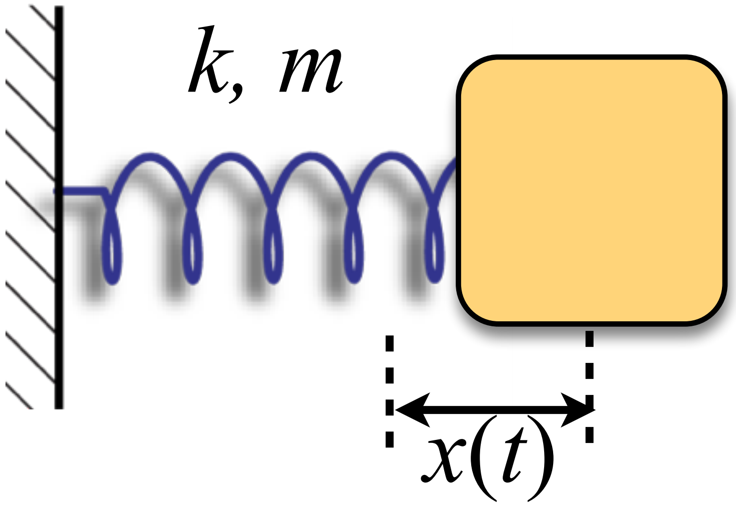
Hooke's Law

$$\nabla\cdot\mathbf{T} = \nabla\cdot\left(\mathbf{c}:\mathbf{S}\right)$$

$$\underset{\text{strain}}{\left(\mathbf{S} = \nabla_s\mathbf{u}\right)}$$

Mechanics of continuous

Single particle



Newton's Law

$$F_{\text{ext}} + F_x = m\frac{d^2x}{dt^2}$$

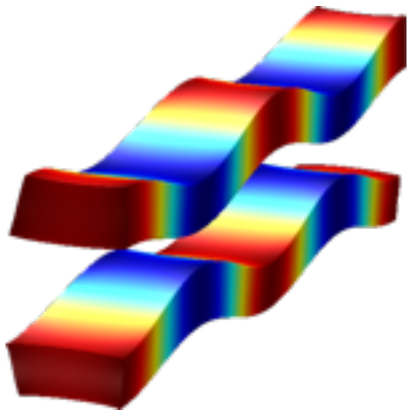
Hooke's Law
(single mode)

$$F_x = -m\left(\frac{k}{m}\right)x = -m\Omega_j^2 x$$

Solid mechanics

$\mathbf{c}$: elastic stiffness

ρ : density



$\mathbf{u}(\vec{r},t)$

Newton's Law

$$\nabla\cdot\mathbf{T}_{\text{ext}} + \nabla\cdot(\mathbf{c}:\mathbf{S}) = \rho\frac{d^2\mathbf{u}(\vec{r},t)}{dt^2}$$

Hooke's Law
(single mode)

$$S_j(\vec{r}) = \nabla_s U_j(\vec{r})$$

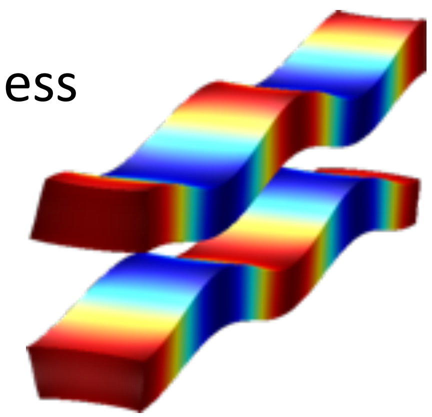
$$\nabla\cdot\left(\mathbf{c}:S_j\right) = -\rho\Omega_j^2 U_j$$

Mechanics of continuous

Solid mechanics

$\mathbf{c}$: elastic stiffness

ρ : density



$$\mathbf{u}(\vec{r}, t)$$

Newton's Law

$$\nabla \cdot \mathbf{T}_{\text{ext}} + \nabla \cdot \mathbf{T} = \rho \frac{d^2 \mathbf{u}(\vec{r}, t)}{dt^2}$$

Hooke's Law
(single mode)

$$S_j(\vec{r}) = \nabla_s U_j(\vec{r})$$

$$\nabla \cdot \left(\mathbf{c} : S_j \right) = -\rho \Omega_j^2 U_j$$

Orthogonality relation

$$\int \rho \left(U_i^* U_j \right) dV = \delta_{ij} m_{\text{eff}}$$

Mechanics of continuous

Solid mechanics

$\mathbf{c}$: elastic stiffness

ρ : density

$$\mathbf{u}(\vec{r},t)$$

Mechanical Field

$$\mathbf{u}(\vec{r},t) = \sum b_j(t) U_j(\vec{r})$$

$$\mathbf{S}(\vec{r},t) = \sum b_j(t) S_j(\vec{r})$$

Newton's Law

$$\nabla \cdot \mathbf{T}_{\text{ext}} + \nabla \cdot \mathbf{T} = \rho \frac{d^2 \mathbf{u}(\vec{r},t)}{dt^2}$$

Hooke's Law
(single mode)

$$S_j(\vec{r}) = \nabla_s U_j(\vec{r})$$

$$\nabla \cdot \left(\mathbf{c} : S_j \right) = -\rho \Omega_j^2 U_j$$

Orthogonality relation

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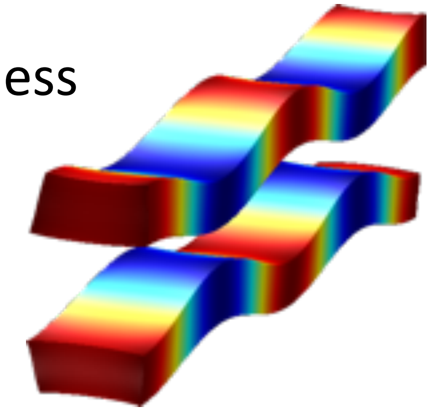

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$$\mathbf{f}_{\text{ext}} + \nabla \cdot \left(\mathbf{c} : \sum b_j(t) S_j\right) = \rho \sum \frac{d^2}{dt^2}\left(b_j(t) U_j\right)$$

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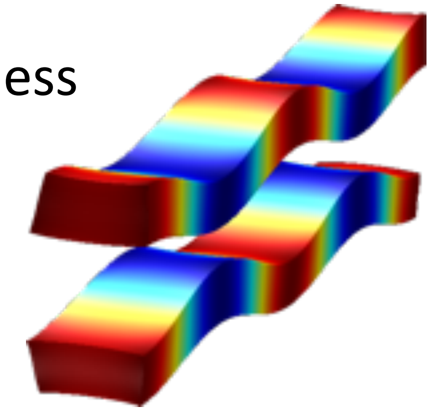

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$$\mathbf{f}_{\text{ext}}+\nabla\cdot\left(\mathbf{c}:\sum b_j(t)S_j\right)=\rho\sum\frac{d^2}{dt^2}\left(b_j(t)U_j\right)$$

$$\int\left[\mathbf{f}_{\text{ext}}-\sum\Omega_j^2 b_j(t)\rho U_j=\sum\ddot{b}_j(t)\rho U_j\right]U_i^* dV$$

Orthogonality relation

$$\int\rho\left(U_i^* U_j\right)dV=\delta_{ij}m_{\text{eff}}$$

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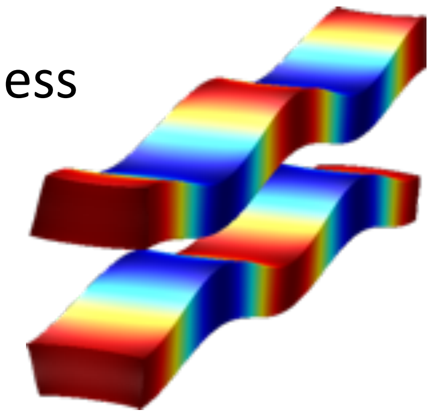

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$$\int \left[\mathbf{f}_{\text{ext}} - \sum \Omega_j^2 b_j(t) \rho U_j = \sum \ddot{b}_j(t) \rho U_j \right] U_i^* dV$$

Lumped Model

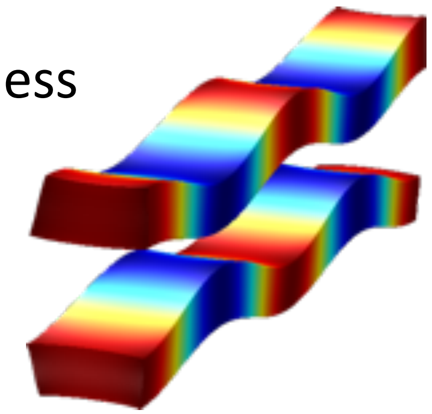
$$\ddot{b}_j(t) + \Omega_i^2 b_i(t) = \frac{\mathbf{F}_{i,\text{ext}}}{m_{\text{eff}}}$$

Lumped Model - Frequency Domain

Solid mechanics

$\mathbf{c}$: elastic stiffness

ρ : density



$\mathbf{u}(\vec{r}, t)$

Mechanical Loss Channel

$$\ddot{b}(t) + \gamma_m \dot{b}(t) + \Omega_m^2 b(t) = \frac{\mathbf{F}_{\text{ext}}}{m_{\text{eff}}}$$

Lumped Model - Frequency Domain

Solid mechanics

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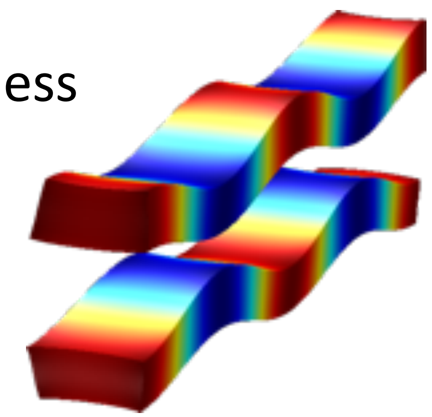

$\mathbf{u}(\vec{r}, t)$

$$\tilde{b}(\Omega) = \int_{-\infty}^{+\infty} dt \;\; e^{i\Omega t} b(t)$$

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$$\tilde{b}(\Omega) = \int_{-\infty}^{+\infty} dt \;\; e^{i\Omega t} b(t)$$

$$-\Omega^2 \tilde{b}(\Omega) - i\Omega\gamma_m \tilde{b}(\Omega) + \Omega_m^2 \tilde{b}(\Omega) = \frac{\tilde{\mathbf{F}}_{\text{ext}}(\Omega)}{m_{\text{eff}}}$$

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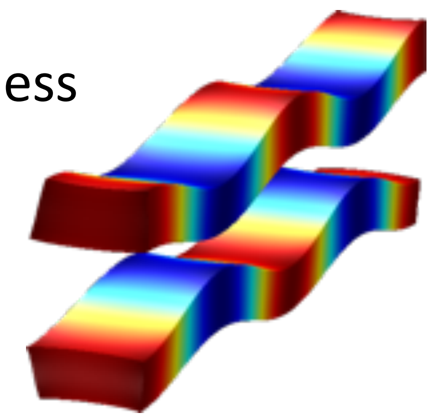

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$$-\Omega^2 \tilde{b}(\Omega) - i\Omega\gamma_m \tilde{b}(\Omega) + \Omega_m^2 \tilde{b}(\Omega) = \frac{\tilde{\mathbf{F}}_{\text{ext}}(\Omega)}{m_{\text{eff}}}$$

Mechanical Frequency Response

$$\tilde{b}(\Omega) = \chi_{bb}(\Omega) \tilde{\mathbf{F}}_{\text{ext}}(\Omega)$$

Mechanical Susceptibility

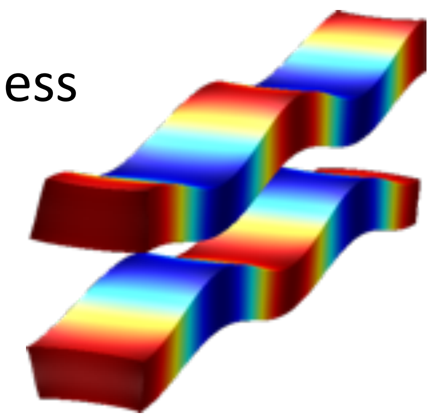
$$\chi_{bb}(\Omega) = \left[m_{\text{eff}} \left(\Omega_m^2 - \Omega^2 - i\Omega\gamma_m \right) \right]^{-1}$$

Lumped Model - Frequency Domain

Solid mechanics

$\mathbf{c}$: elastic stiffness

ρ : density



$\mathbf{u}(\vec{r}, t)$

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Mechanical Susceptibility

$$\chi_{bb}(\Omega) = \left[m_{\text{eff}}\left(\Omega_m^2 - \Omega^2 - i\Omega\gamma_m\right)\right]^{-1}$$

DC response

$\Omega \approx 0$ ⇒

$$\chi_{bb}(0) = \left[m_{\text{eff}}\Omega_m^2\right]^{-1} = 1 / k$$

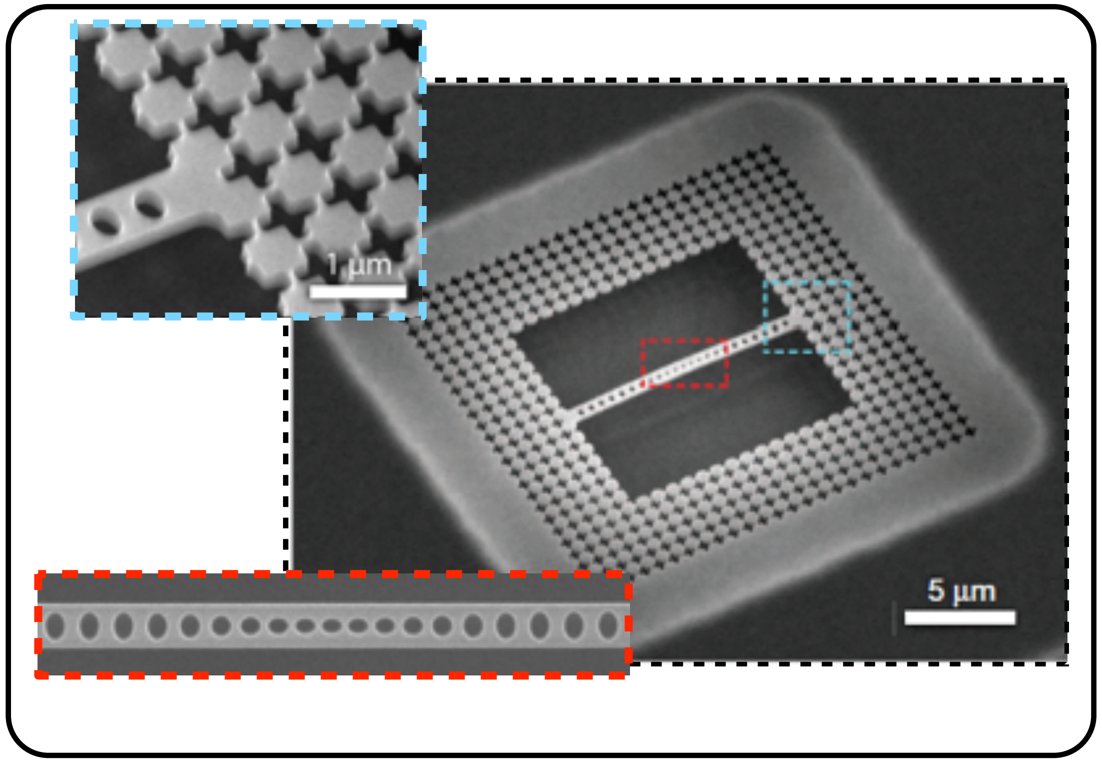
Lorentzian shape

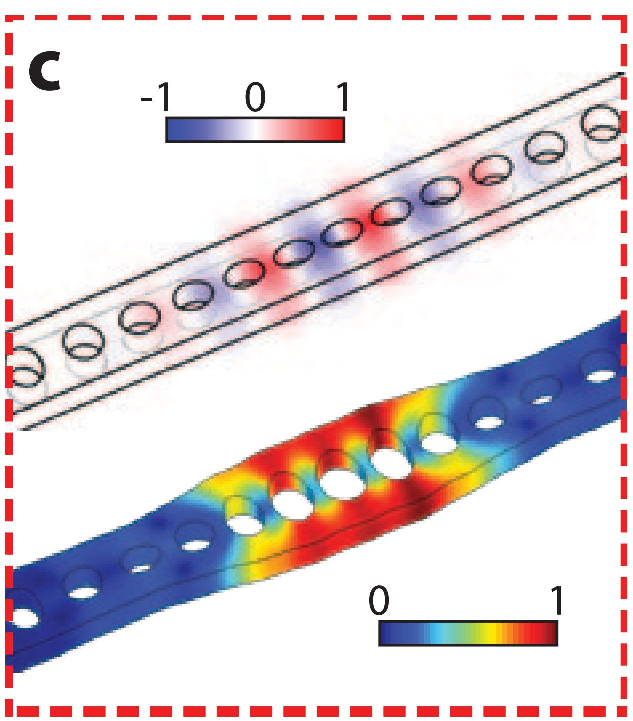
$\Omega \approx \Omega_m$ ⇒

$$\chi_{bb}(\Omega) \approx \left[2m_{\text{eff}}\Omega_m\left((\Omega_m - \Omega) - i\,\gamma_m/2\right)\right]^{-1}$$

Lumped Model - Frequency Domain

Optomechanical Crystal





Mechanical Frequency Response

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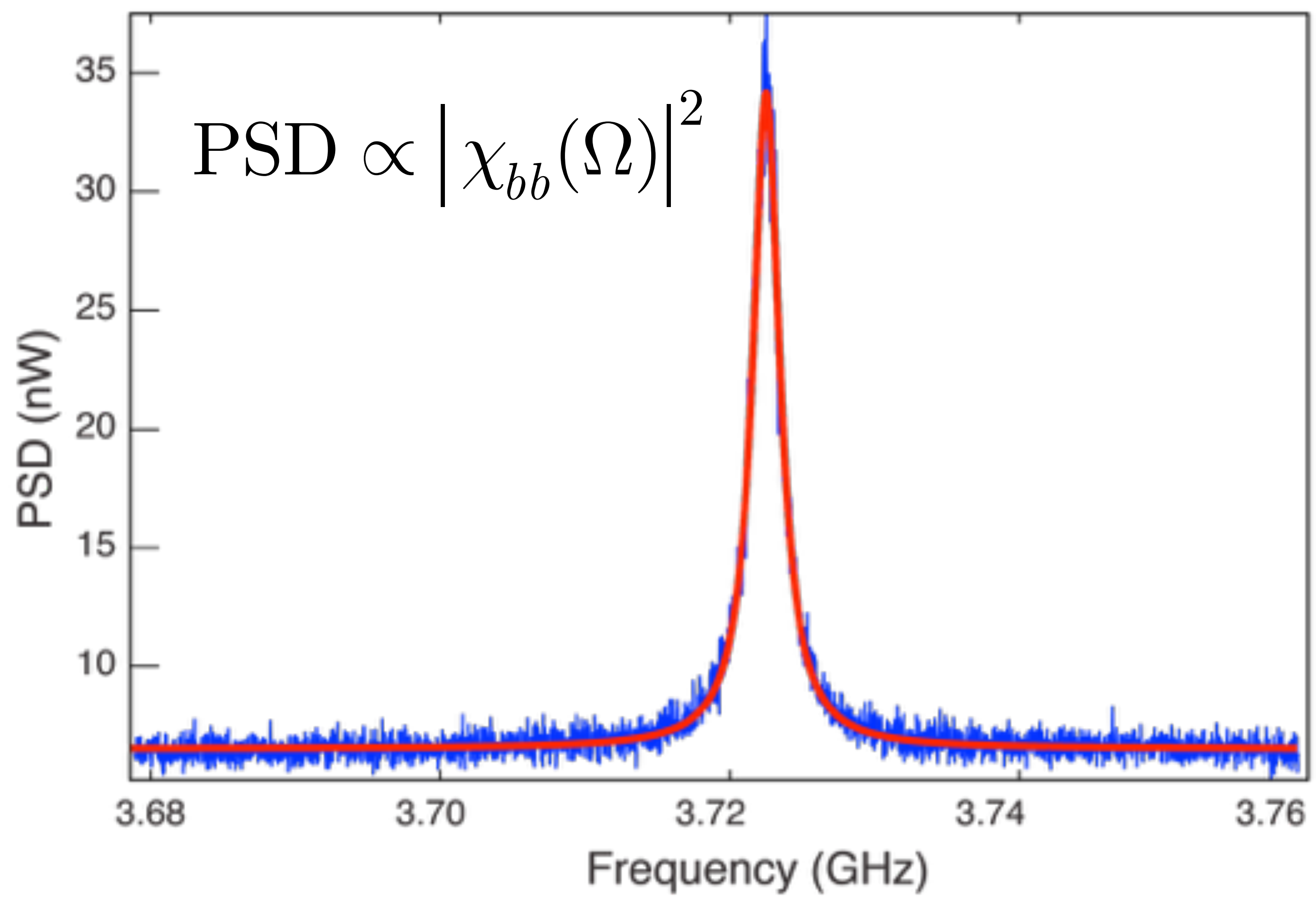
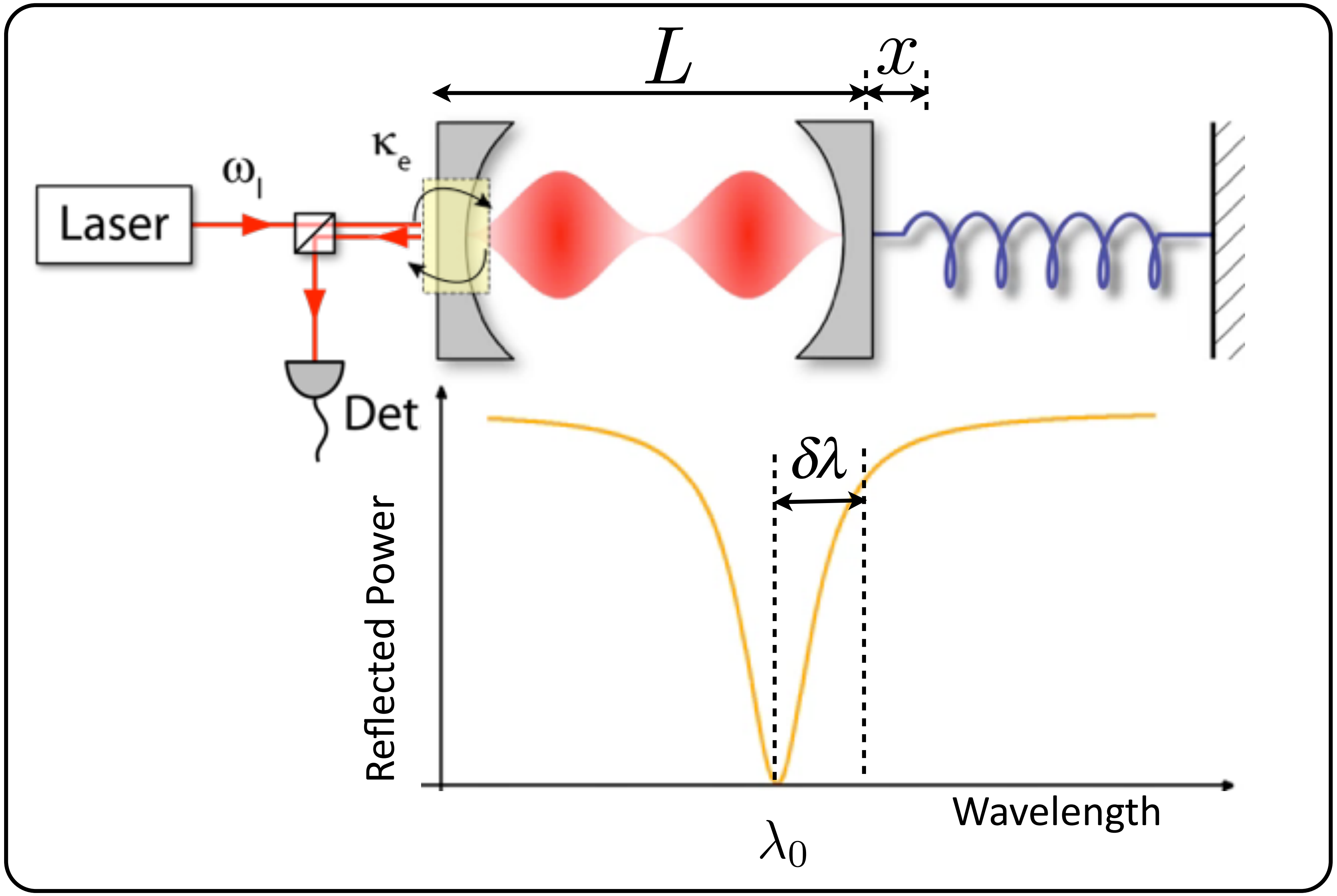
$$\chi_{bb}(\Omega) \approx \left[2m_{\text{eff}}\Omega_m\left((\Omega_m - \Omega) - i\gamma_m/2\right)\right]^{-1}$$

Lorentzian shape

$$\left|\chi_{bb}(\Omega)\right|^2 \approx \frac{1}{4m_{\text{eff}}^2\Omega_m^2\left((\Omega_m - \Omega)^2 - (\gamma_m/2)^2\right)}$$



Optomechanical cavity toy-model



$$\omega_c = n\frac{\pi c}{L}$$

$$\omega_c(x) \approx \omega_c + \frac{\partial \omega}{\partial x}x = \omega_c - \left(\frac{\omega}{L}\right)x$$

$\left(\frac{\omega}{L}\right) \rightarrow g_{\mathrm{om}}$

Outline

- Optical and acoustic mode interaction
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Optical Cavities: Harnessing Light Force

$$\omega_c(x) \approx \omega_c - g_{\mathrm{om}} x$$

$$g_{\mathrm{om}} = \omega / L$$

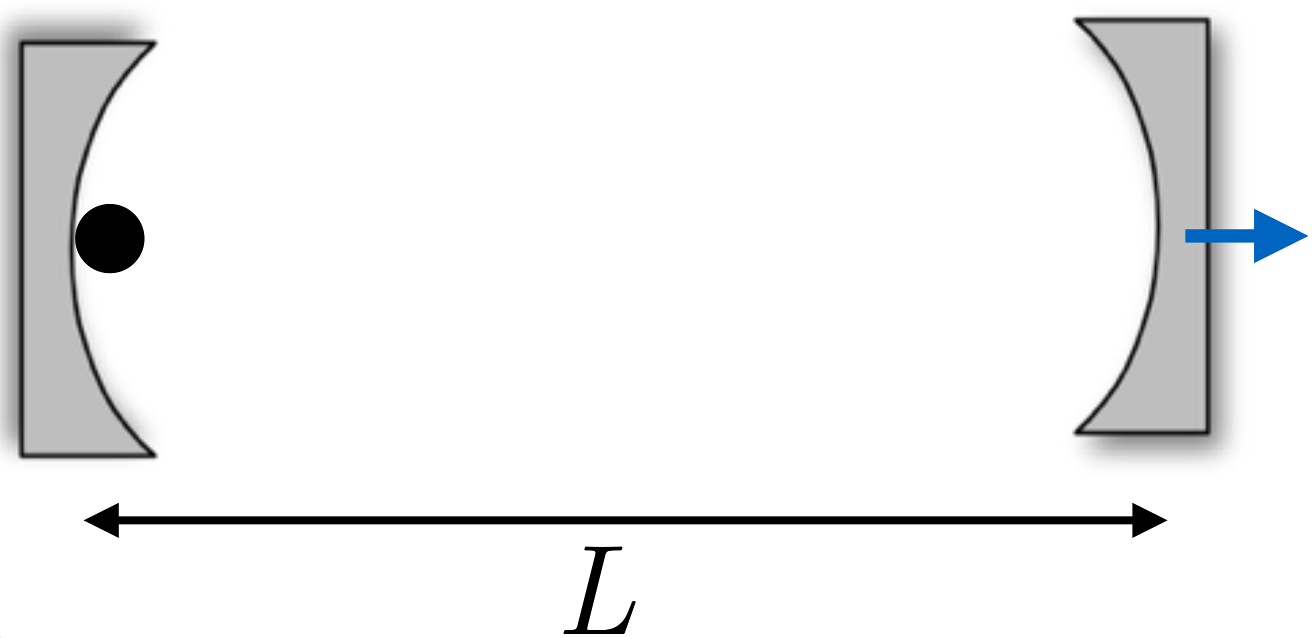
Frequency pull parameter

$$f = \frac{\Delta p}{\Delta t} = \frac{2\hbar k}{\Delta t} = 2\hbar k\left(\frac{c}{2L}\right) = \hbar\frac{\omega_c}{L} = \hbar g_{\mathrm{om}}$$

Single-photon force

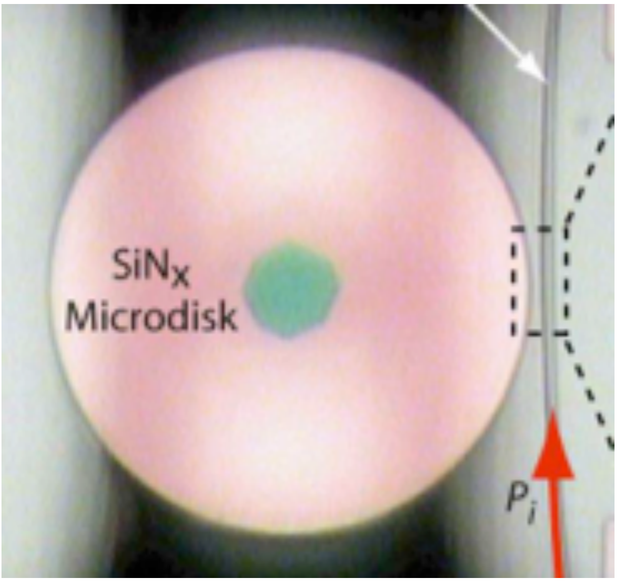
single-photon force

$$\approx 2 \text{ fN}$$

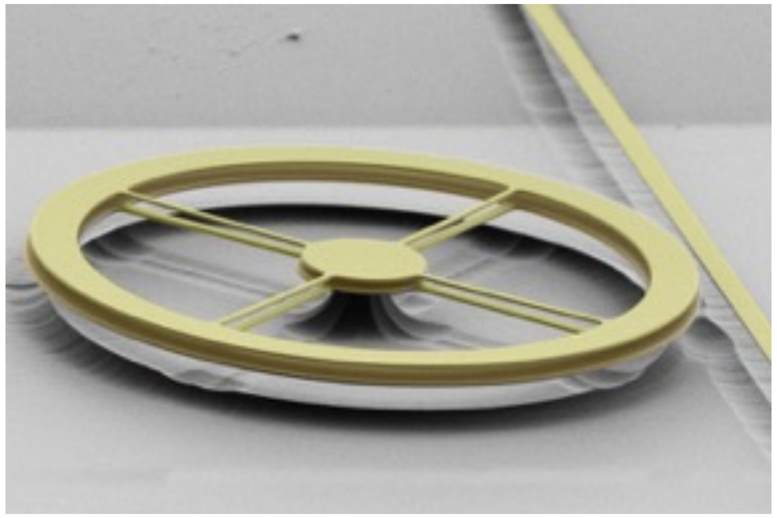


$$\Delta p = 2\hbar k$$

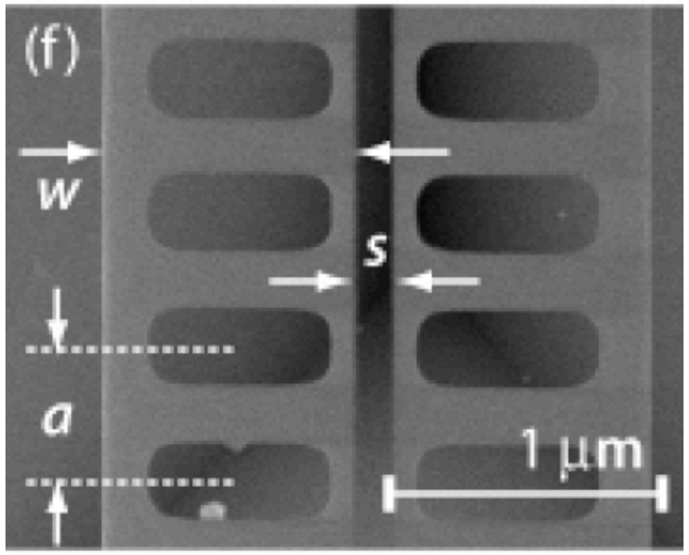
Optical Forces Among Guiding Structures



Eichenfield et al.
Nature Photonics (2007)



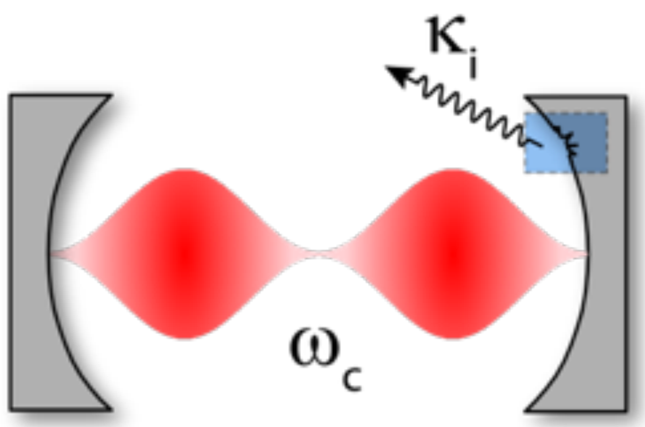
Wiederhecker et al.
Nature (2009)



Eichenfield et al.
Nature (2009)

Optical Cavities: Harnessing Light Force

Fabry-Perot



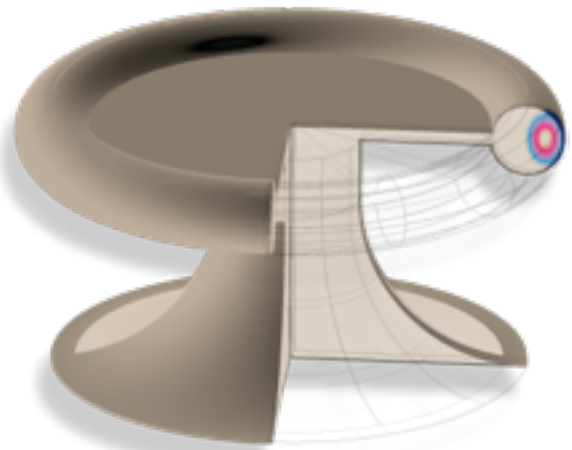
$L = 2.5\ \text{cm}$

$Q = 10^6\,(1\,\text{ns})$

$\mathcal{F} \approx 10$

$10^{-5}\ \text{pN}$

Ring resonator



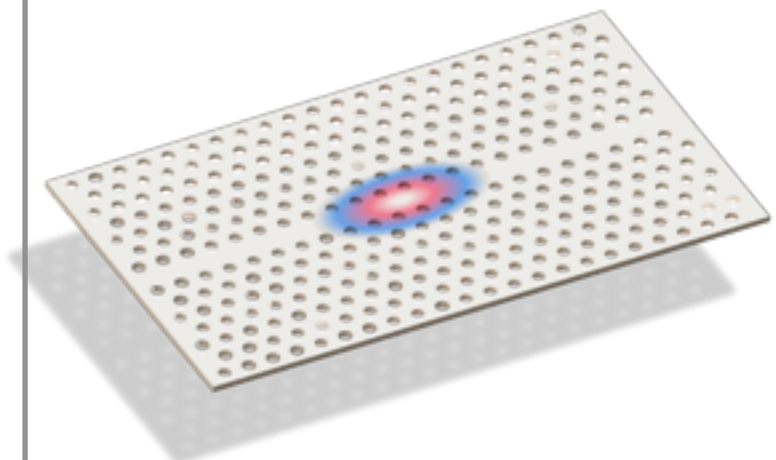
$L = 2\pi R \approx 60\ \mu m$

$Q = 10^6\,(1\,\text{ns})$

$\mathcal{F} \approx 4 \times 10^3$

$10^{-3}\,\text{pN}$

PhC



$L = 4\ \mu m$

$Q = 10^6\,(1\,\text{ns})$

$\mathcal{F} \approx 60 \times 10^3$

$10^{-2}\ \text{pN}$

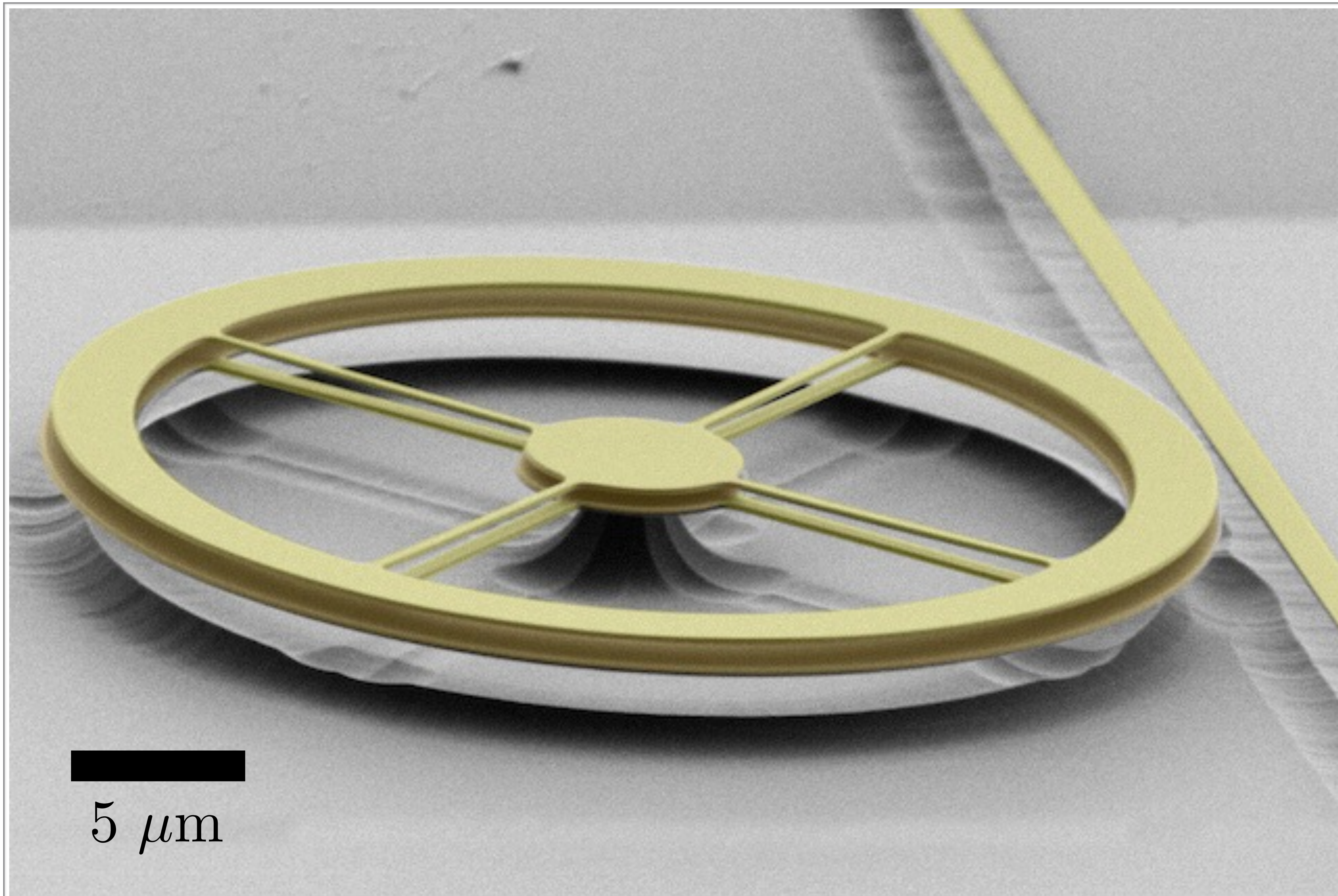
$$f = \hbar g_{om}$$

force due to single-photon

A standard laser pointer can load a cavity with as much as 10 million photons!

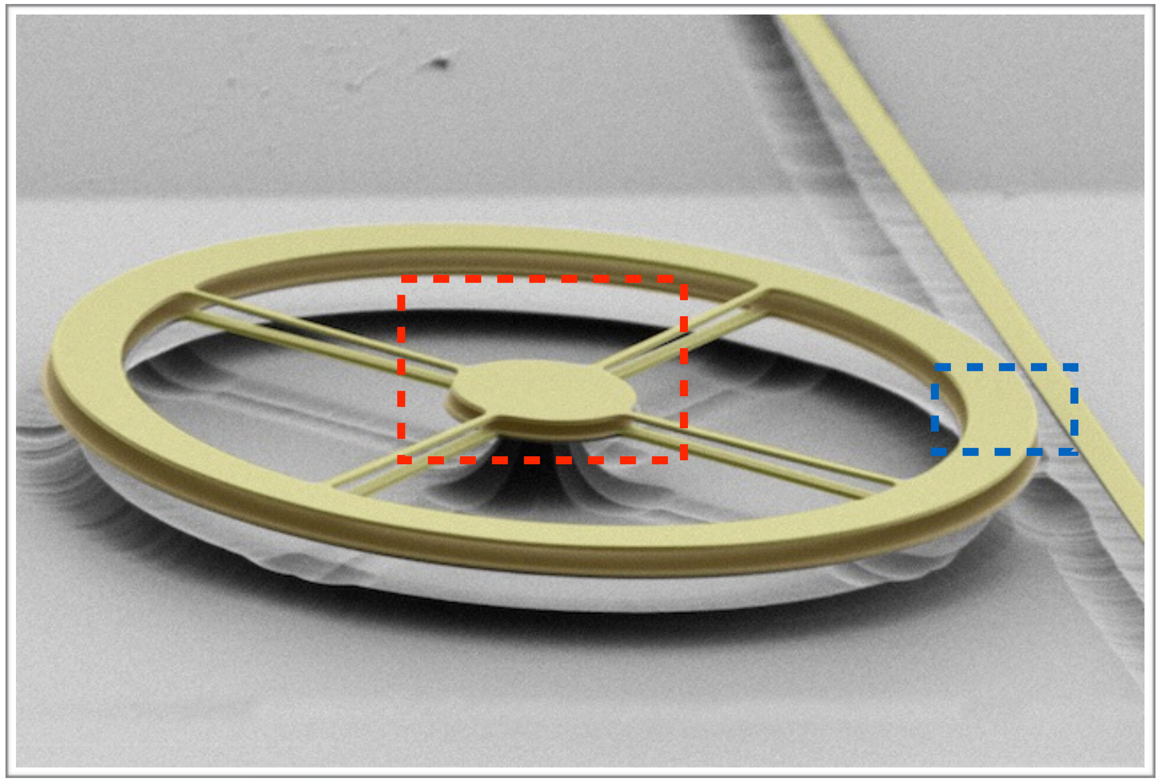
$$F = 10^6 \hbar g_{om} \approx \text{nN}$$

Controlling Cavities with Optical Forces



Wiederhecker et al, Nature 462 (2009)
Wiederhecker et al, OpEx 19, 2782 (2011)

Controlling Cavities with Optical Forces



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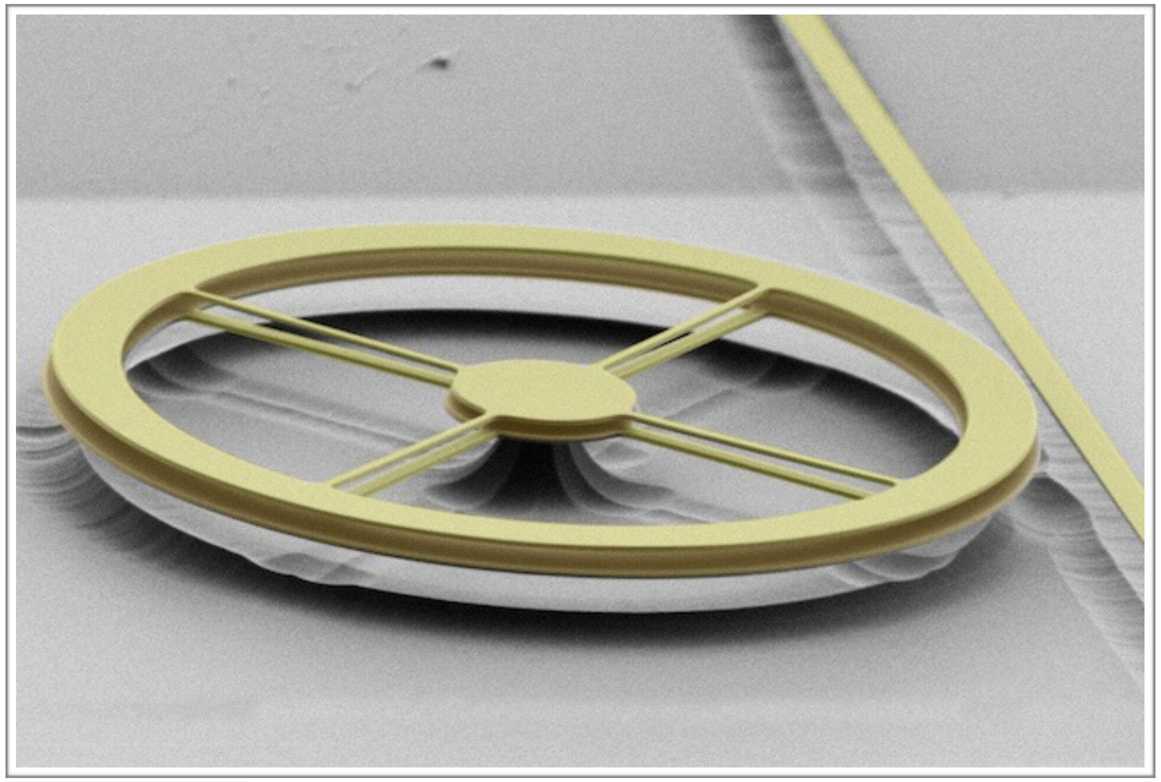
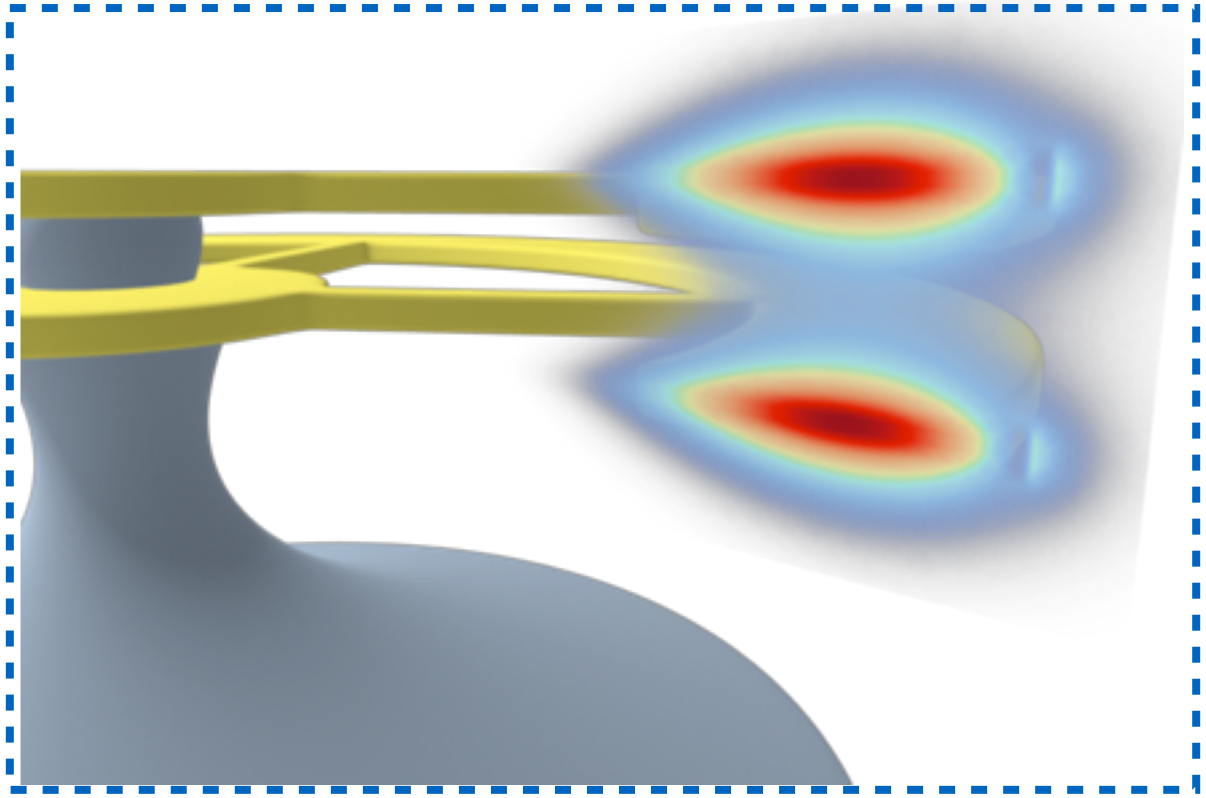
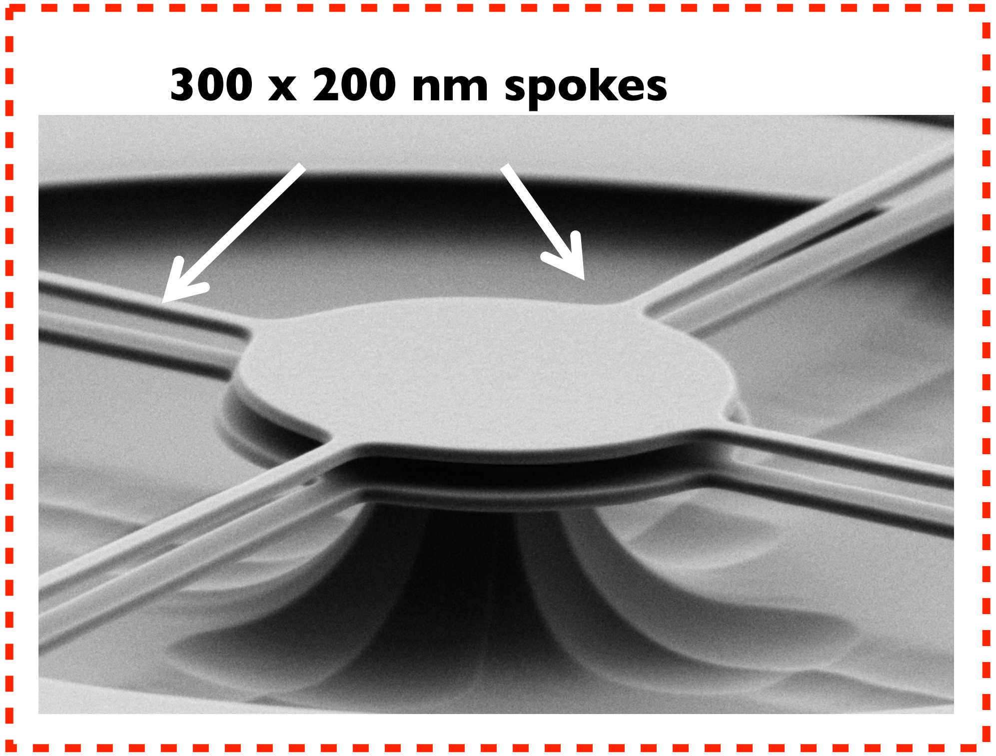
Controlling Cavities with Optical Forces







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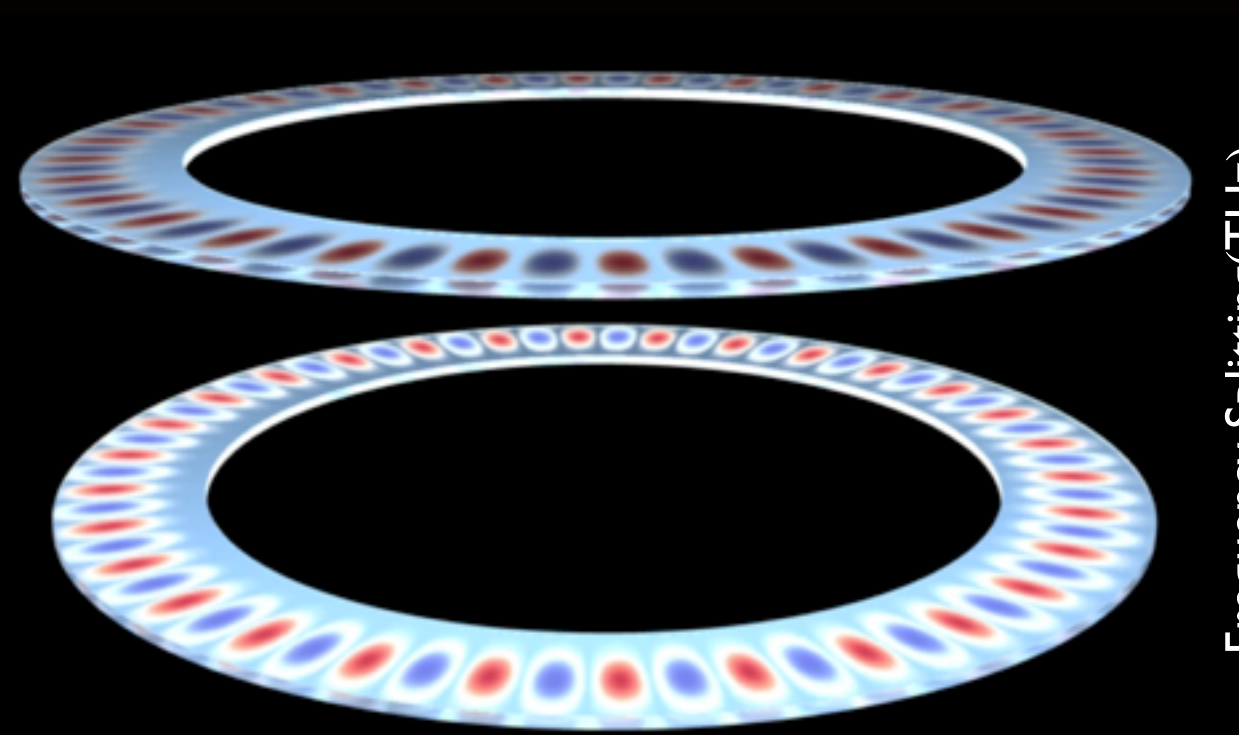
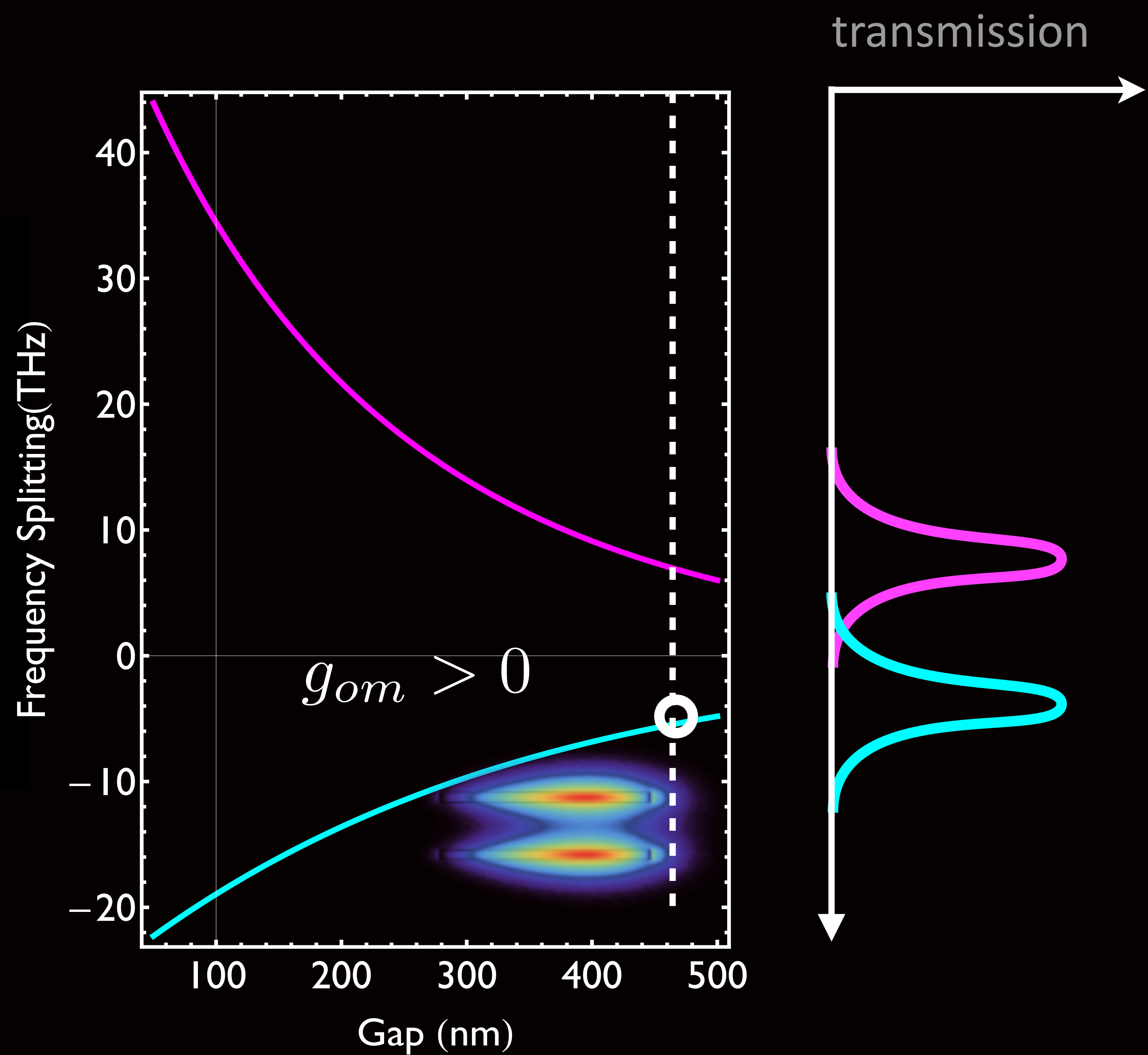
Attractive and Repulsive Forces



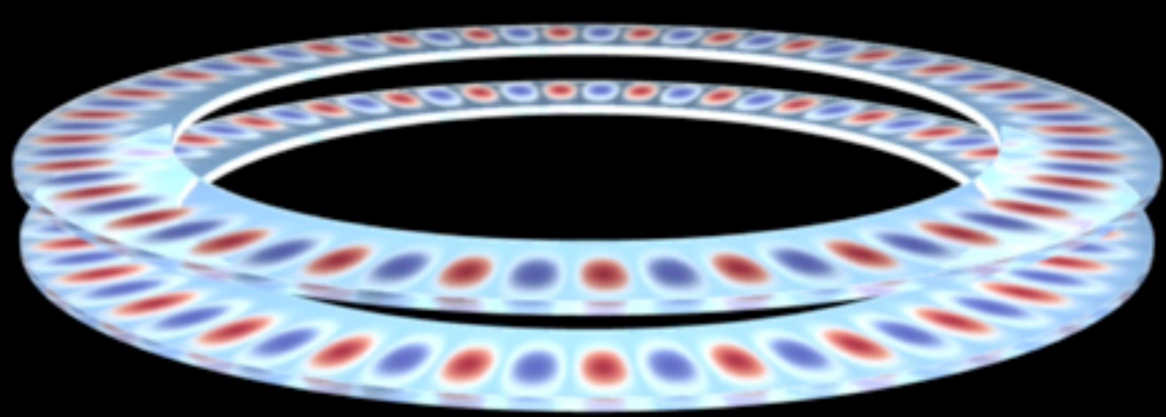


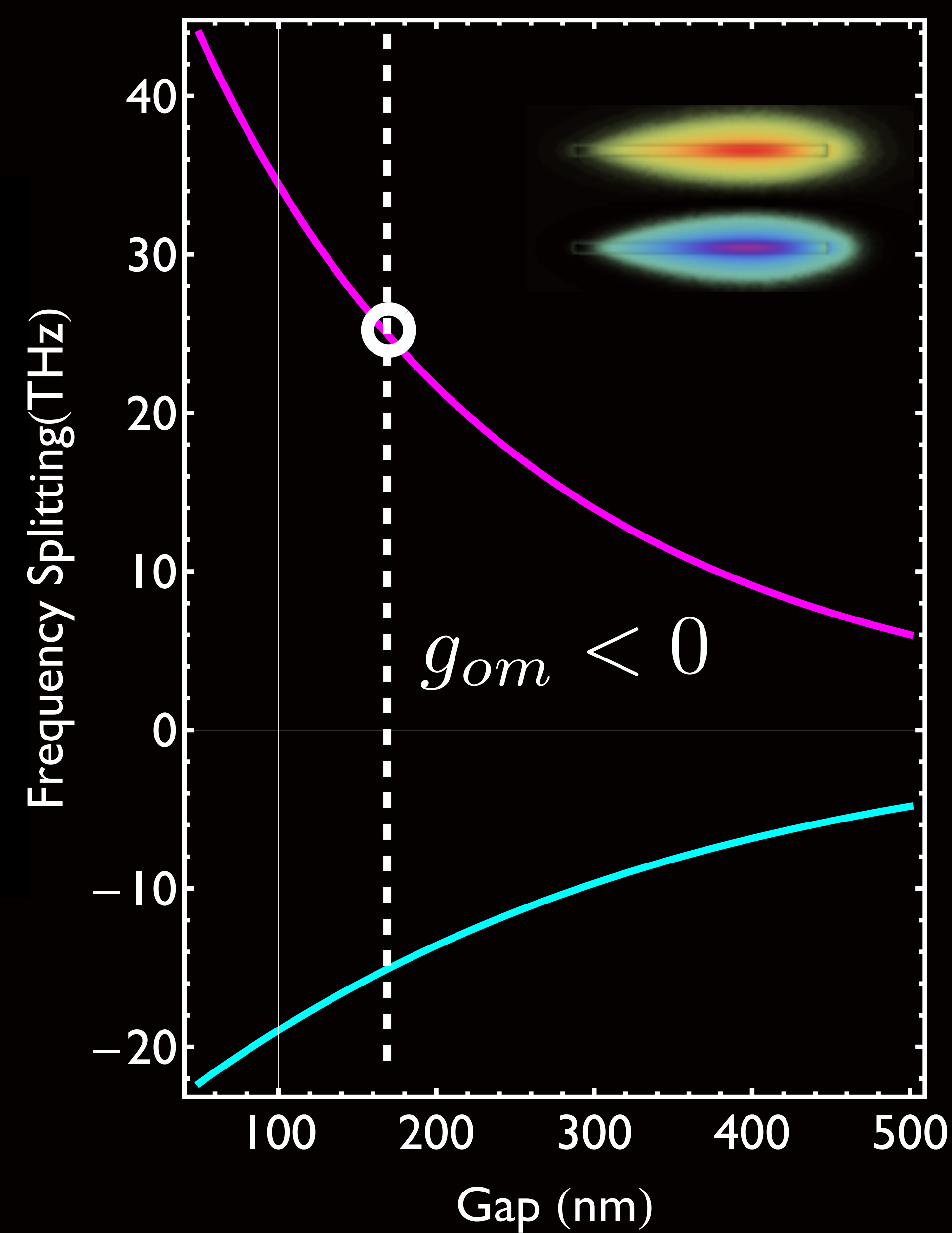
$$F \propto g_{\mathrm{om}}$$

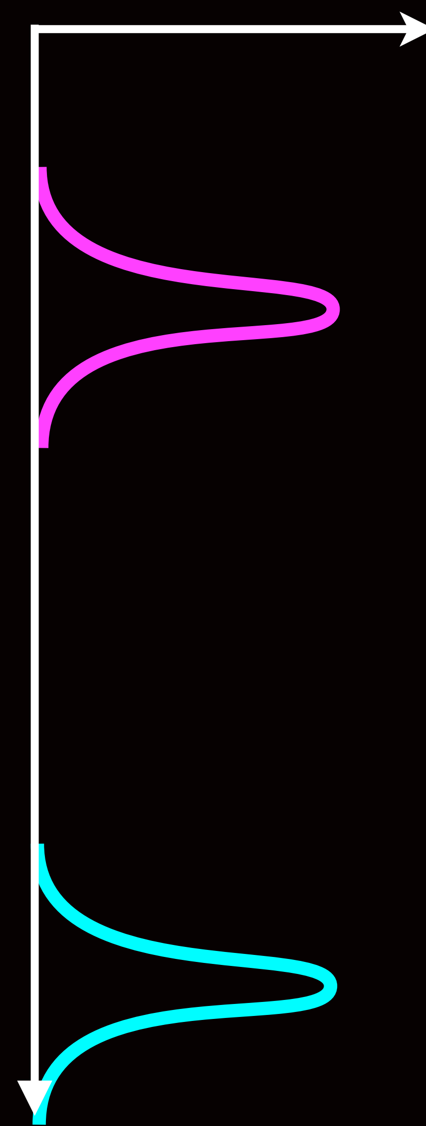
single-photon force

Wiederhecker et al, Nature 462 (2009)
Wiederhecker et al, OpEx 19, 2782 (2011)

Attractive and Repulsive Forces







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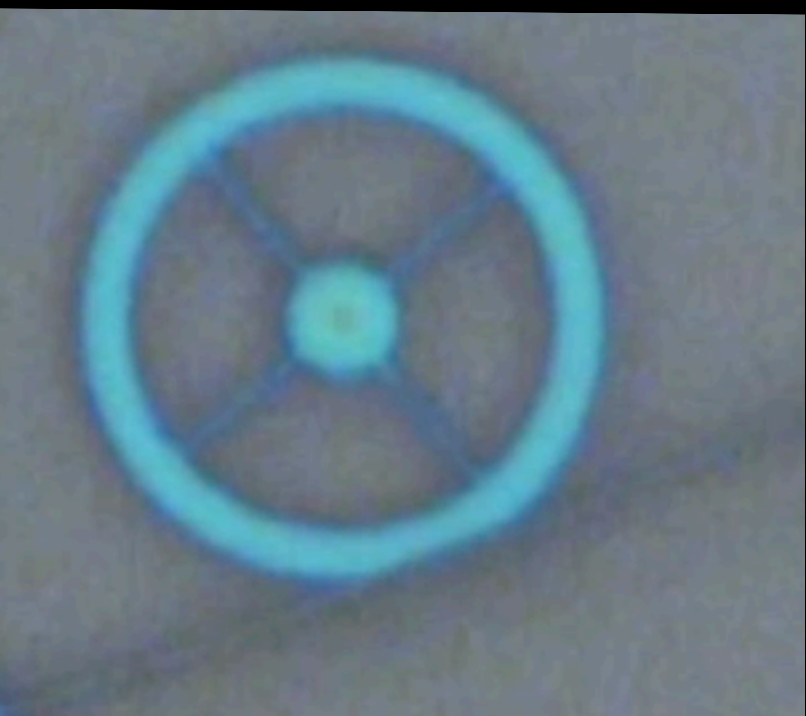
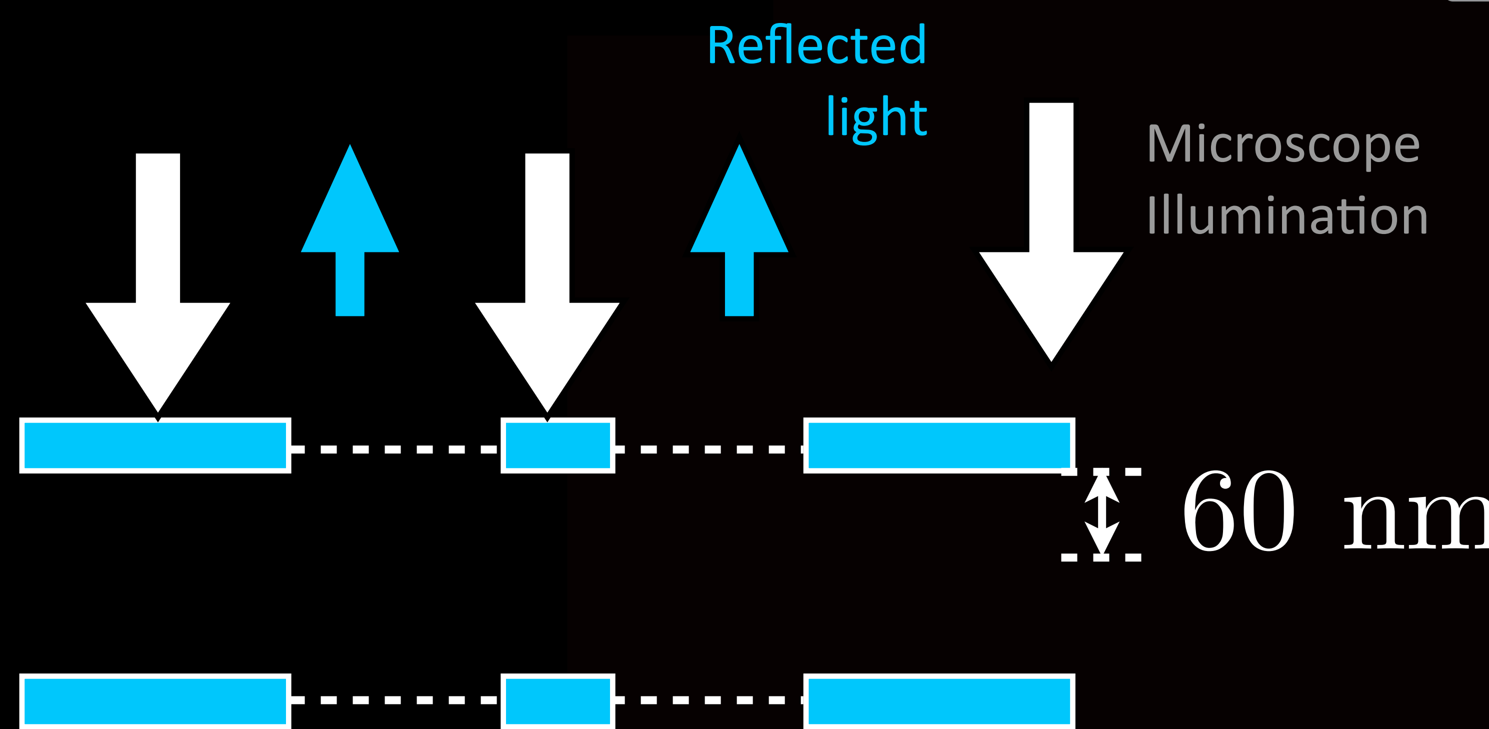
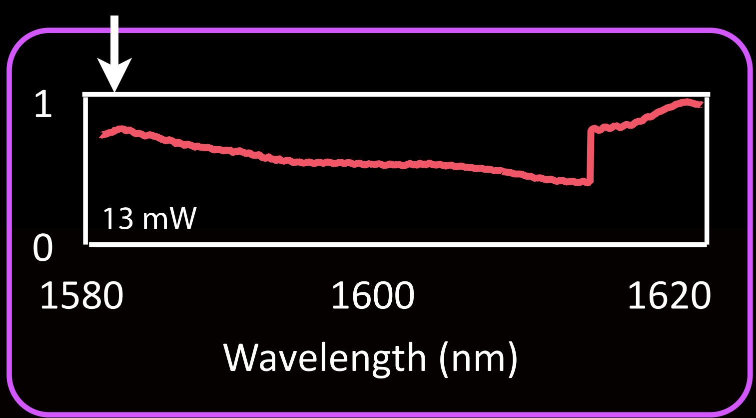


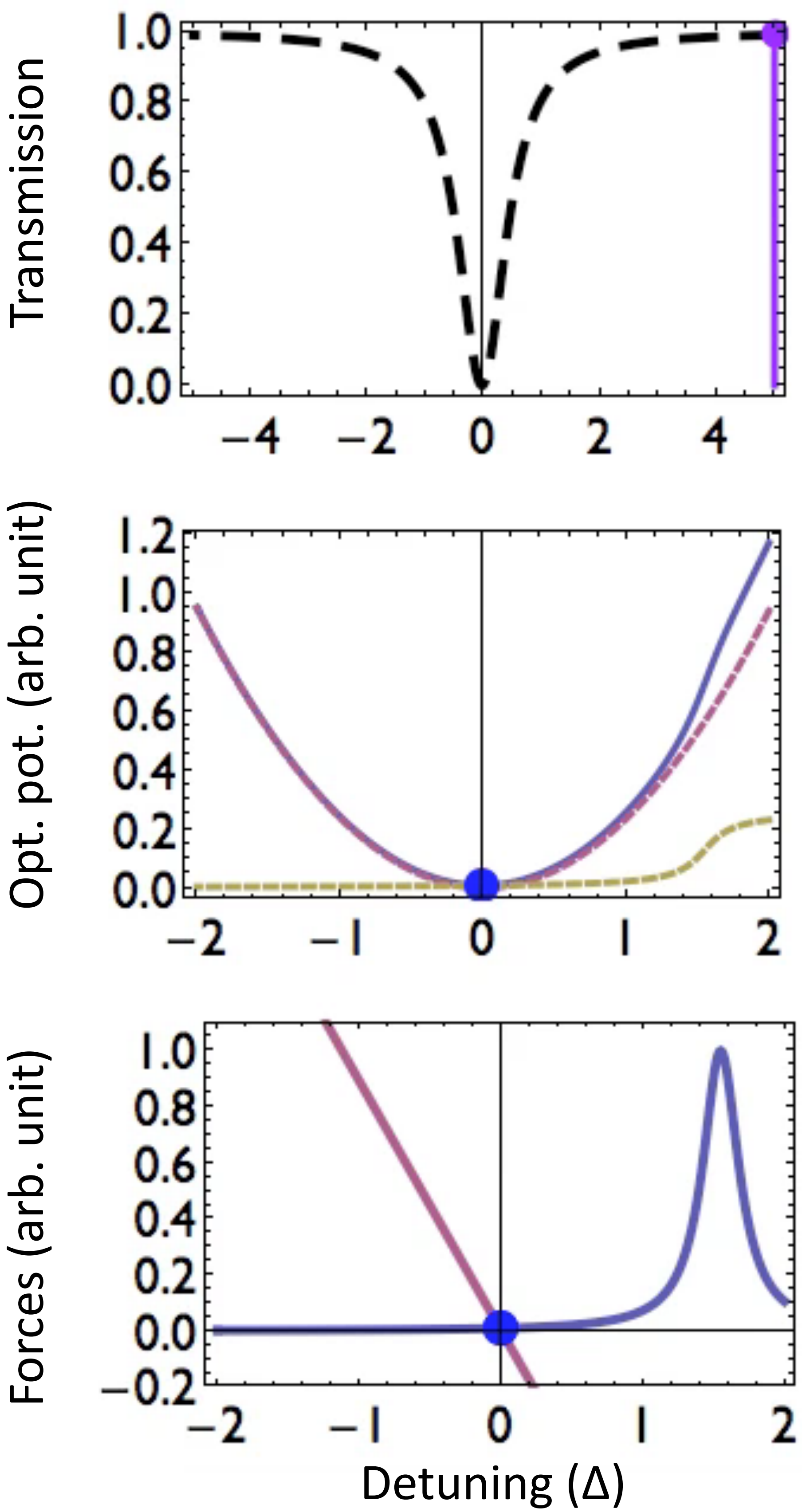




$$F_{opt} \approx 50 \text{ nN}$$

Wiederhecker et al, Nature 462 (2009)
Wiederhecker et al, OpEx 19, 2782 (2011)

Static Bistability



Solve for displacement: cubic equation

$$\frac{m_{\text{eff}}\omega_0}{g_{\text{OM}}} x_0 = \frac{\alpha_{in}^2 \kappa_e}{(\Delta - g_{\text{OM}} x_0)^2 + \kappa^2 / 4}$$

$$V \propto \int \frac{\alpha_{in}^2 \kappa_e}{(\Delta - g_{\text{OM}} x)^2 + \kappa^2 / 4} dx$$

Dorsel, A., J. D. McCullen, P. Meystre, E. Vignes, and H. Walther, 1983, Phys. Rev. Lett. 51, 1550.

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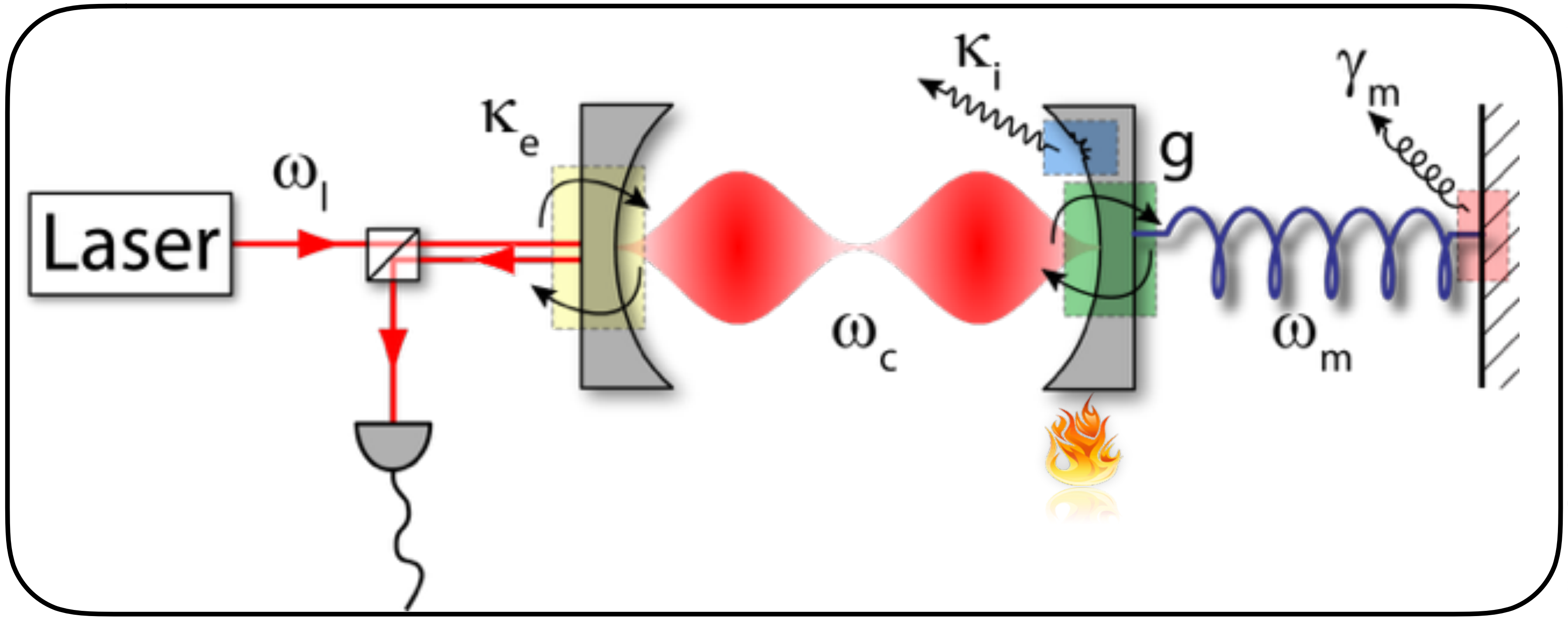
Dynamical Back-action



$m = 100$ pg

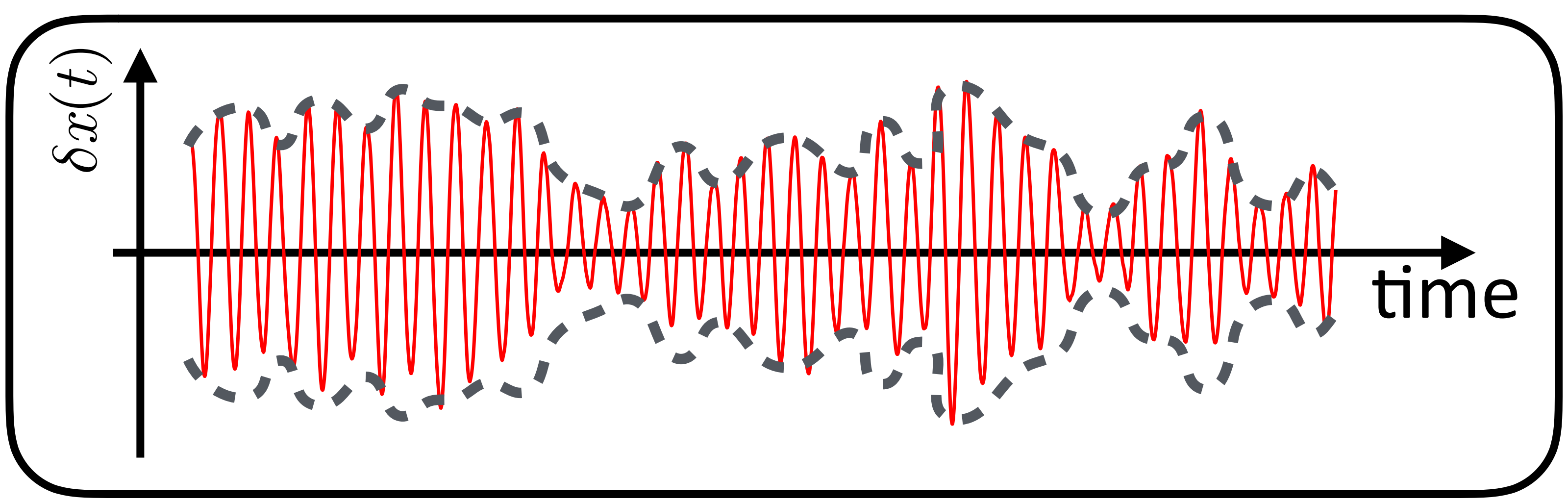
$\Omega \,/\, 2\pi = 1$ MHz

$\langle \delta x_T \rangle \approx 40$ pm

δF_{th} → $\delta x(t)$ →

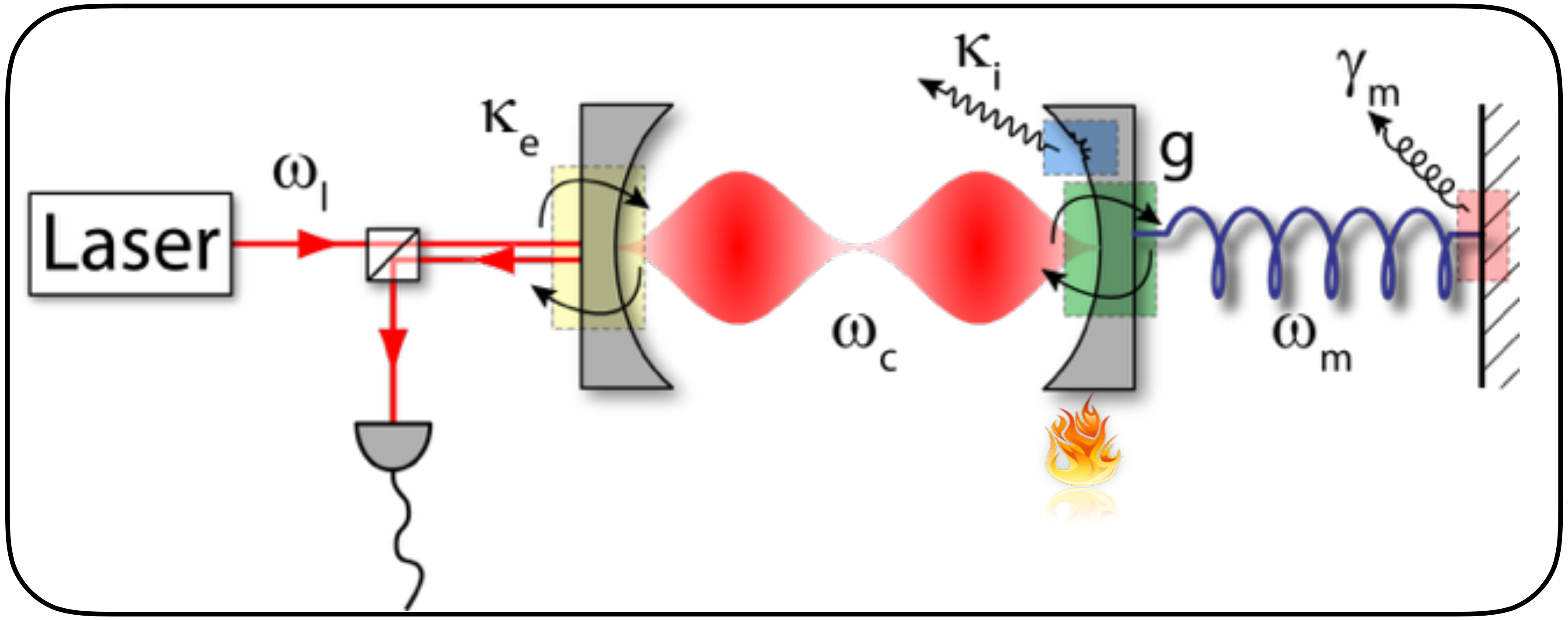
Thermal noise

Displacement noise



Aspelmeyer, et al (2014). RMP, 86(4), 1391–1452.

Dynamical Back-action



$$\langle \delta x_T \rangle \approx 40 \text{ pm}$$

$$g_{om} \,/\, 2\pi \approx 10^4 \text{ GHz/pm}$$

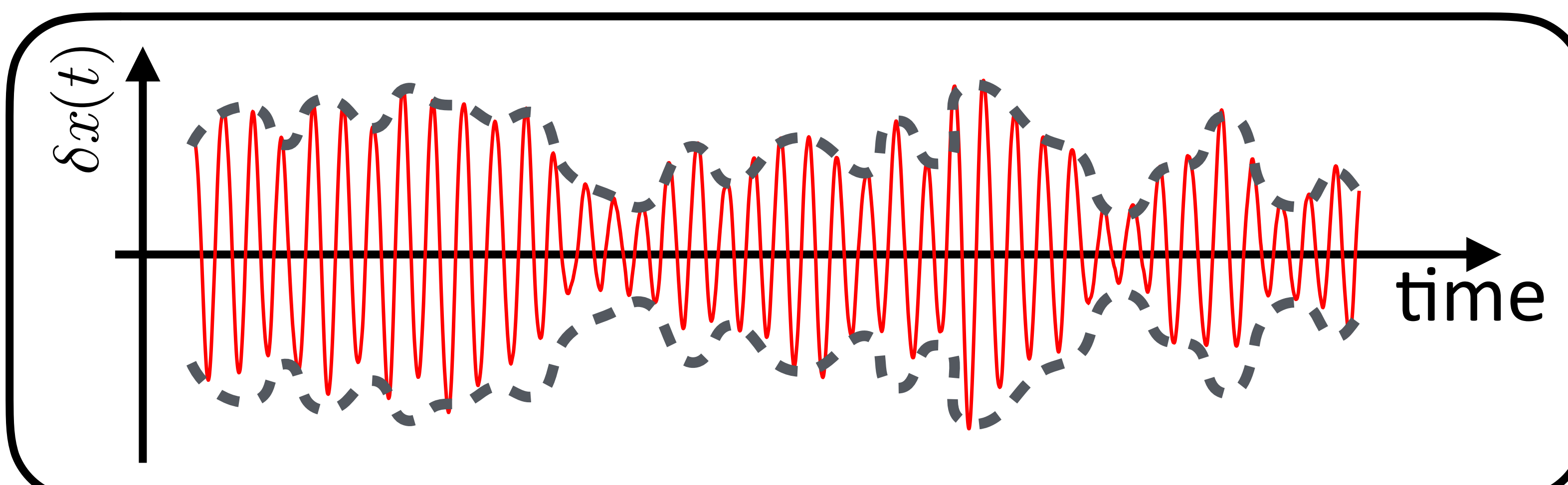
$$\langle \delta\omega \rangle \,/\, 2\pi \approx 400 \text{ MHz}$$

δF_{th} → $\delta x(t)$ → $\delta \omega(t)$ →

Thermal noise

Displacement noise

Frequency noise



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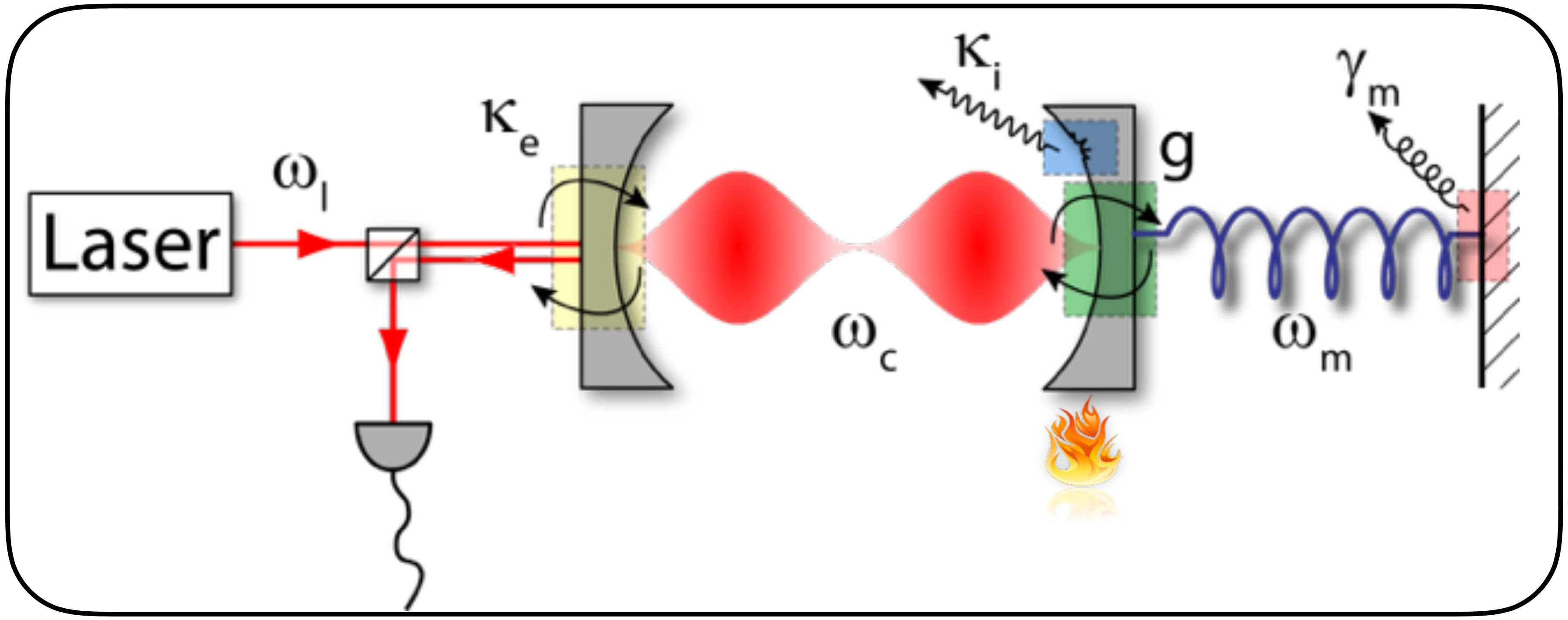
Dynamical Back-action



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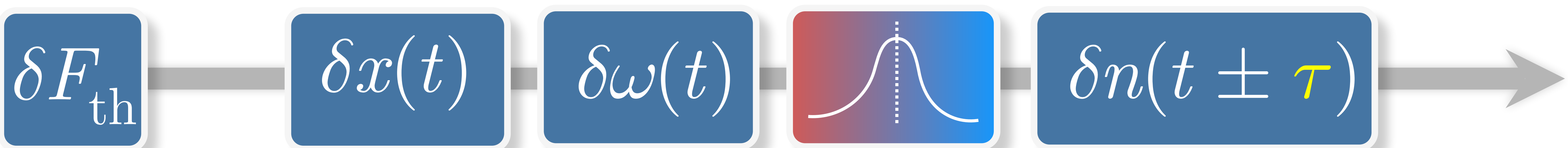
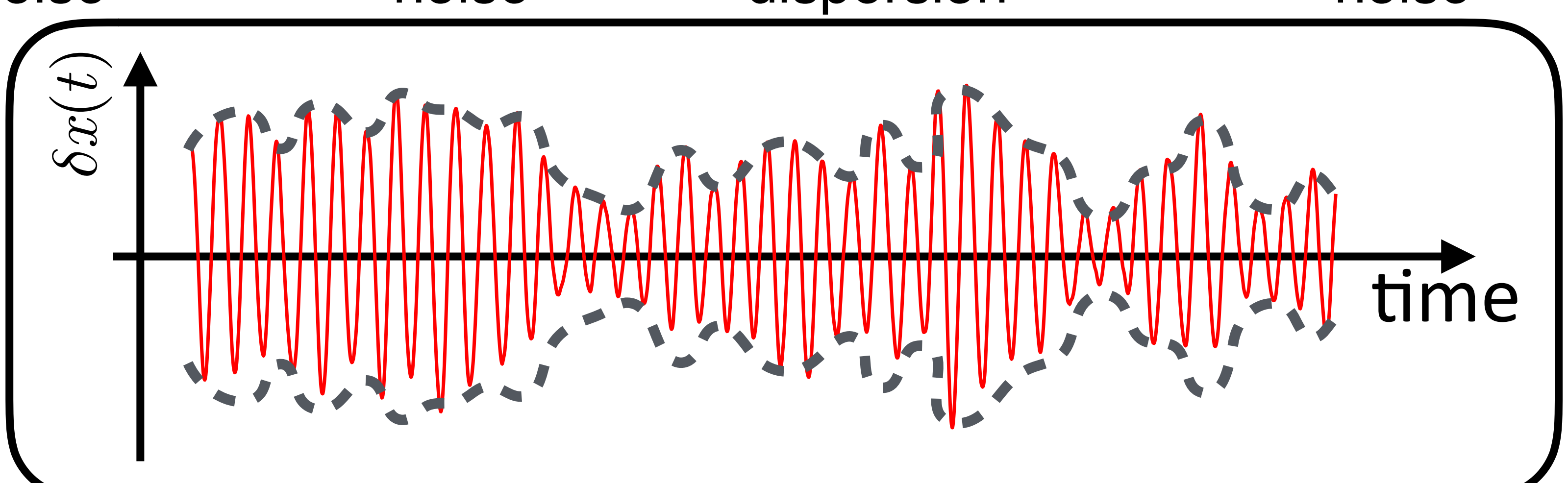


Displacement noise | Frequency noise | Cavity dispersion | Photon number noise

Thermal noise



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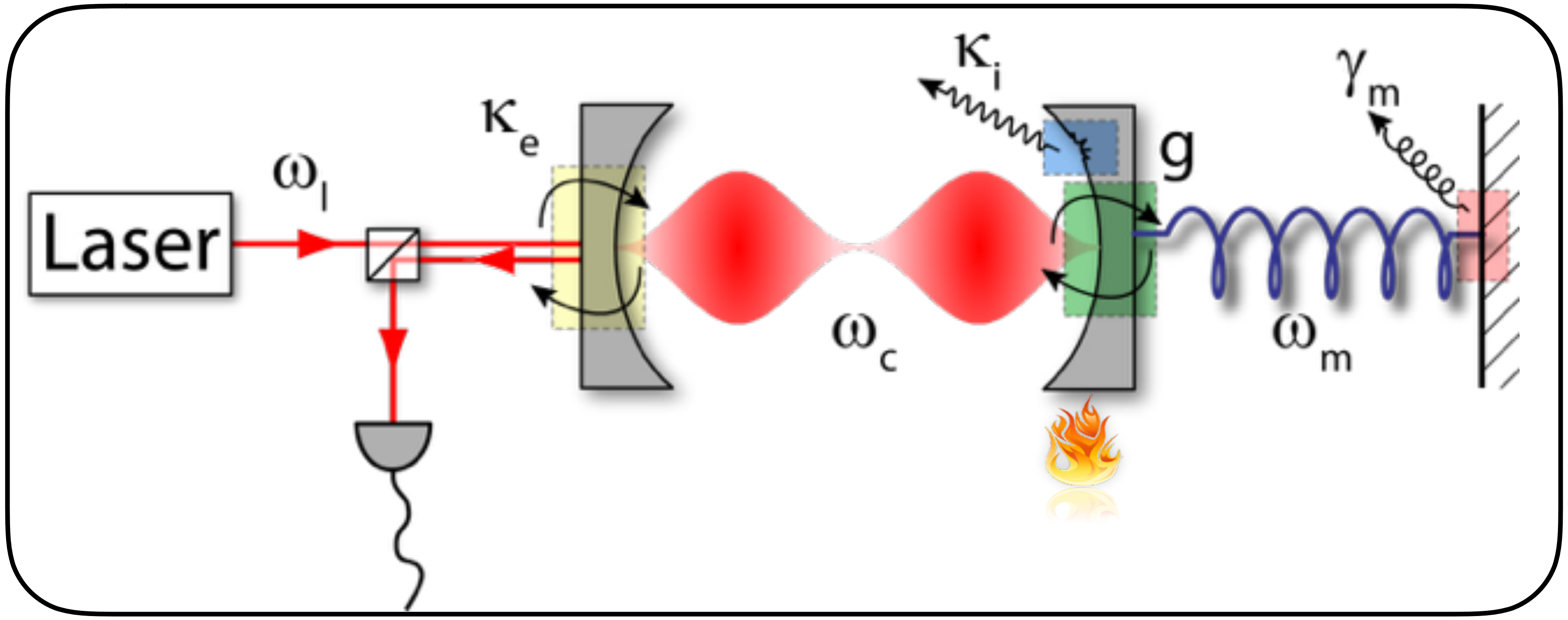
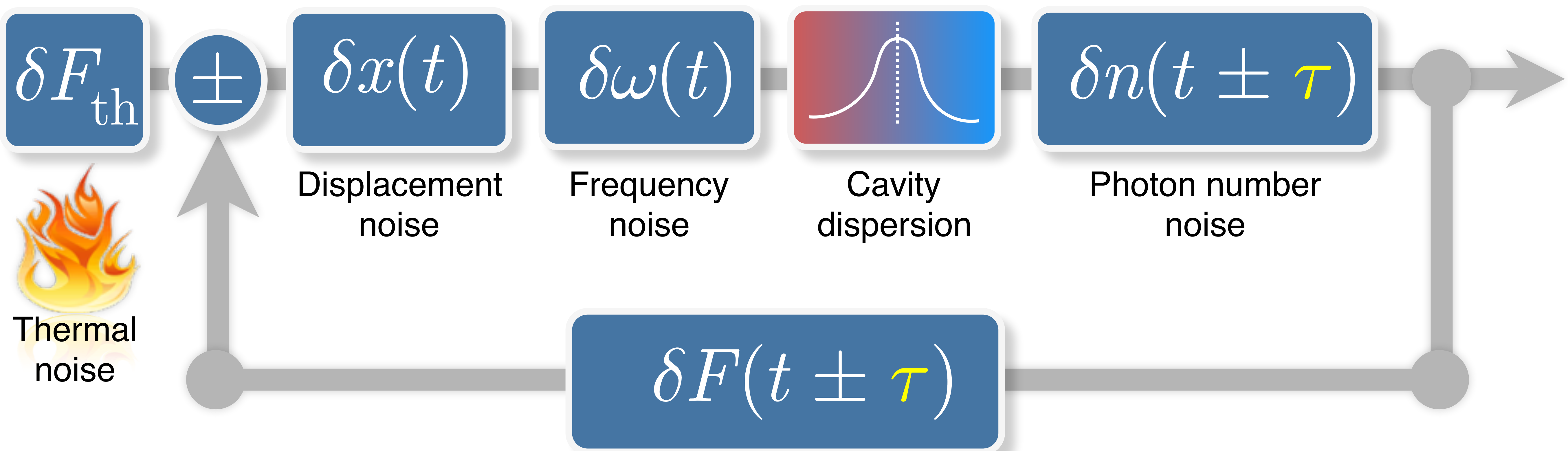
Dynamical Back-action



$$\langle \delta x_T \rangle \approx 40 \text{ pm}$$

$$g_{om} \,/\, 2\pi \approx 10^4 \text{ GHz/pm}$$

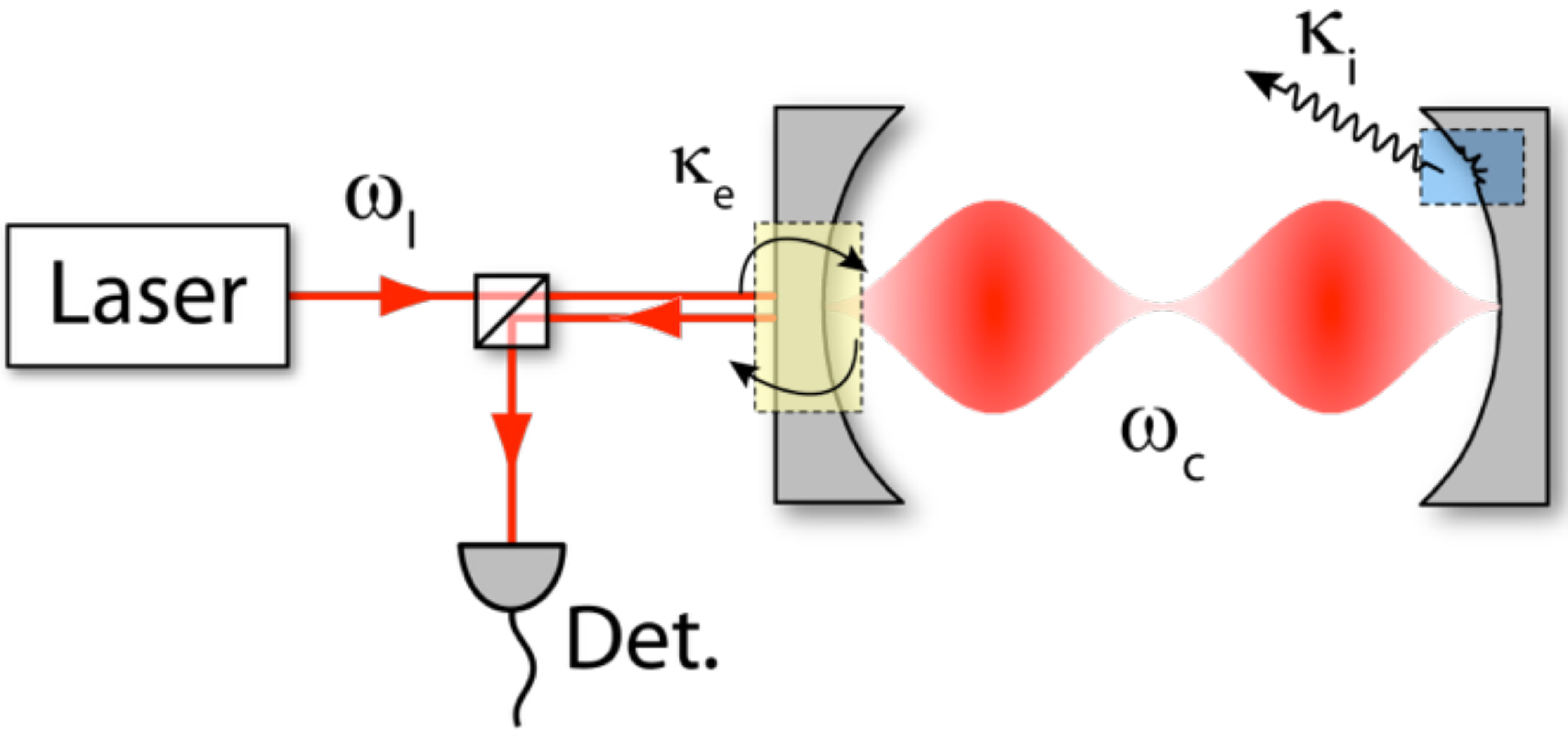
$$\langle \delta\omega \rangle \,/\, 2\pi \approx 400 \text{ MHz}$$



Optical force noise

Aspelmeyer, et al (2014).
RMP, 86(4), 1391–1452.

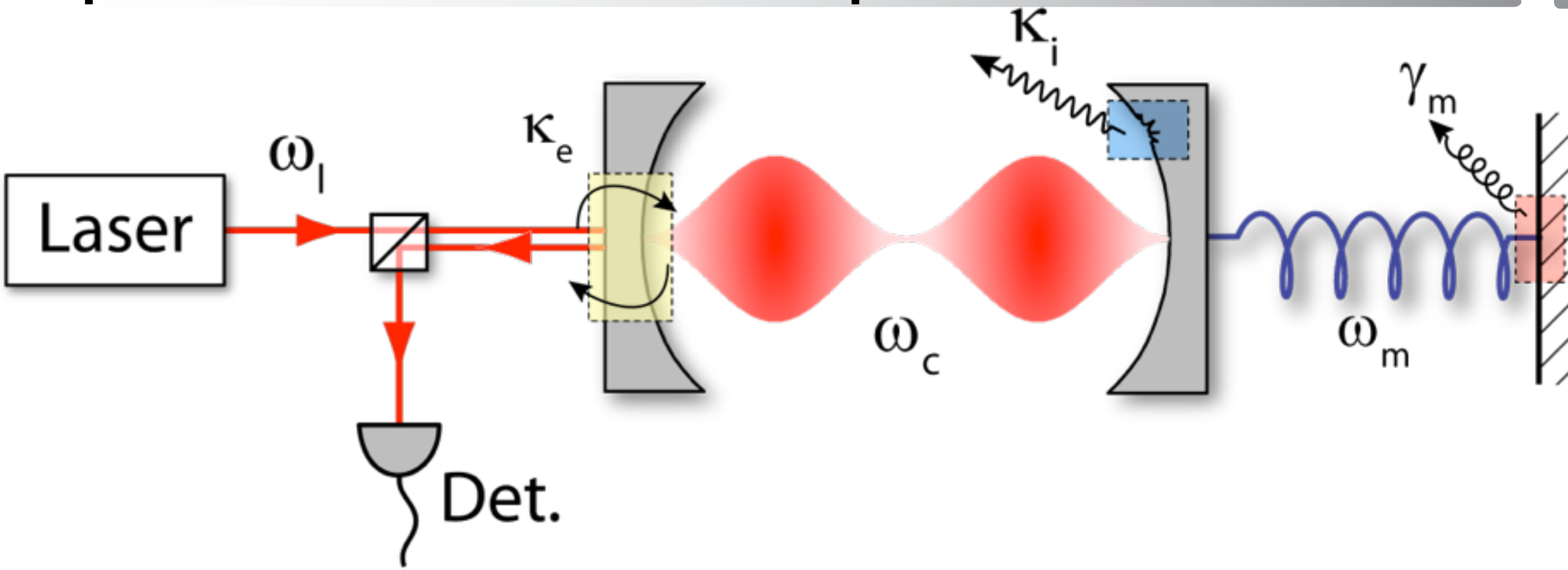
Optical Time Domain Equation



$$\dot{a}(t) = \mathrm{i}(\omega_l - \omega_c)a - \frac{\kappa}{2}a + \sqrt{\kappa_e}\alpha_{\mathrm{in}}$$

Optical Amplitude Equation

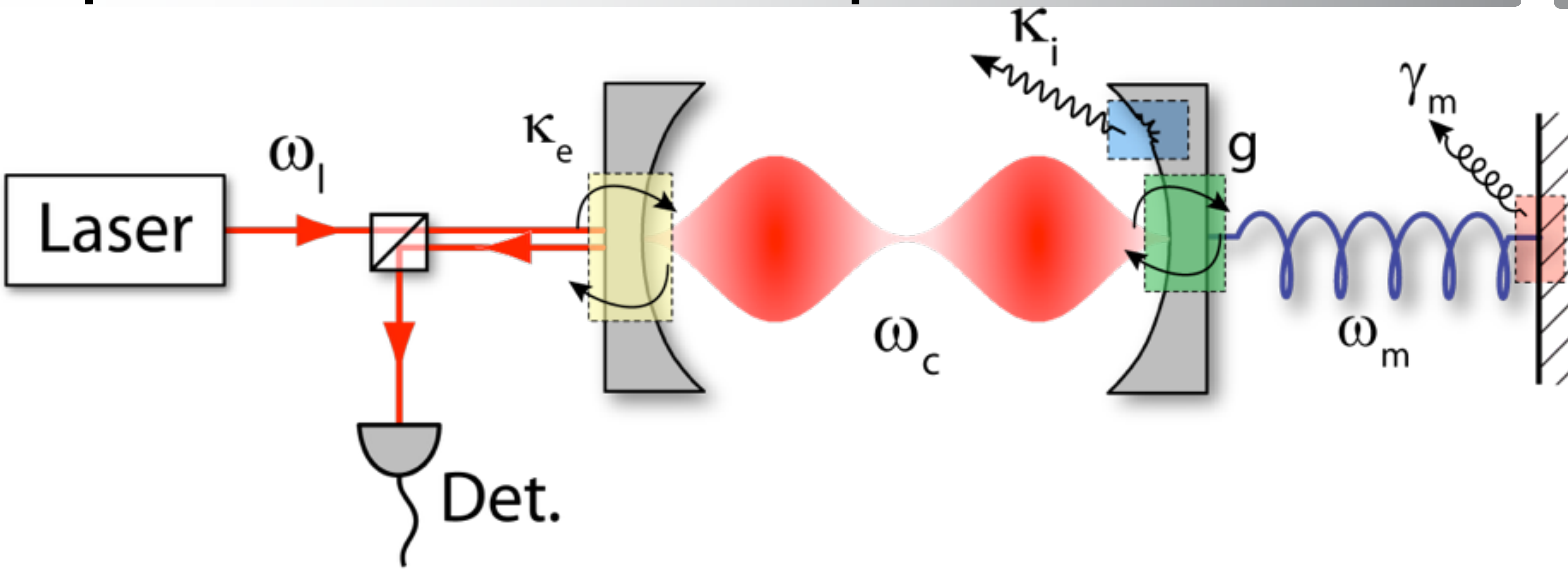
Optical Time Domain Equation



$$\dot{a}(t) = \left(\mathrm{i}(\omega_l - \omega_c) - \frac{\kappa}{2} \right) a + \sqrt{\kappa_e} \alpha_{\mathrm{in}}$$

$$\ddot{x} + \gamma_m \dot{x} + \omega_m^2 x = \frac{F_{\mathrm{opt}}}{m}$$

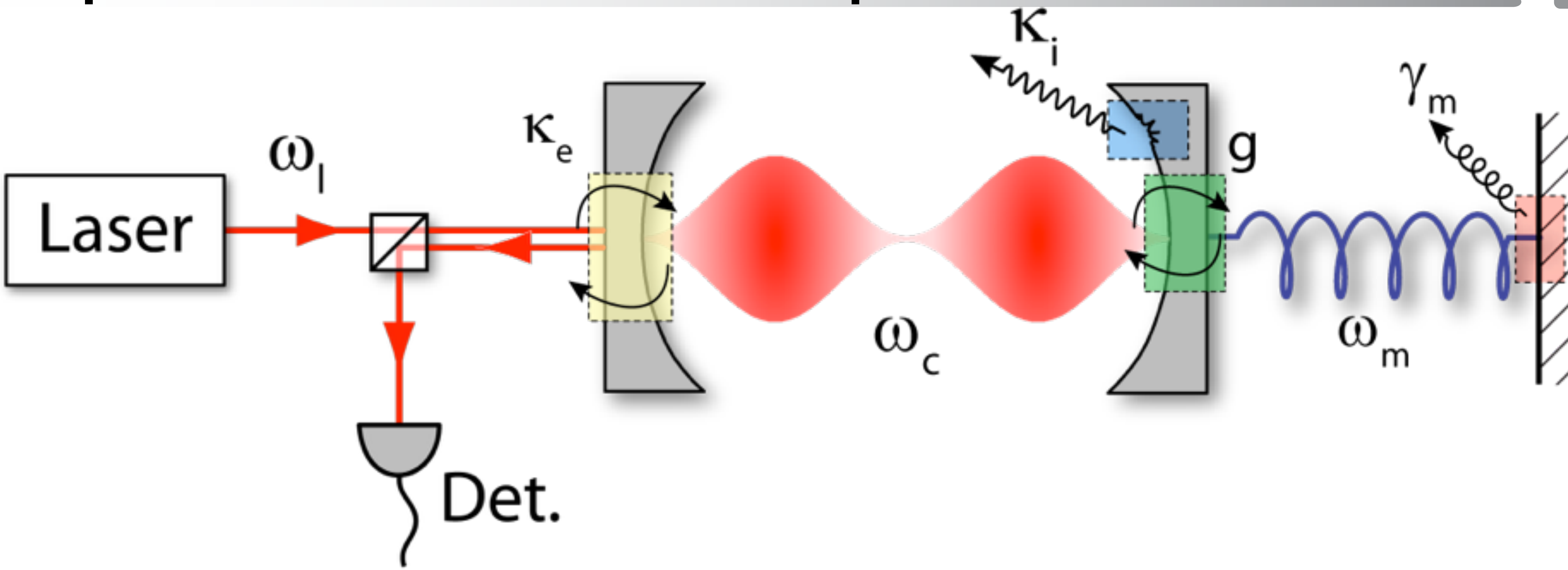
Optical Time Domain Equation



$$\dot{a}(t) = \left(\mathrm{i}(\omega_l - \omega_c(x)) - \frac{\kappa}{2} \right) a + \sqrt{\kappa_e}\alpha_{\mathrm{in}}$$

$$\ddot{x} + \gamma_m \dot{x} + \omega_m^2 x = \frac{F_{\mathrm{opt}}(a)}{m}$$

Optical Time Domain Equation



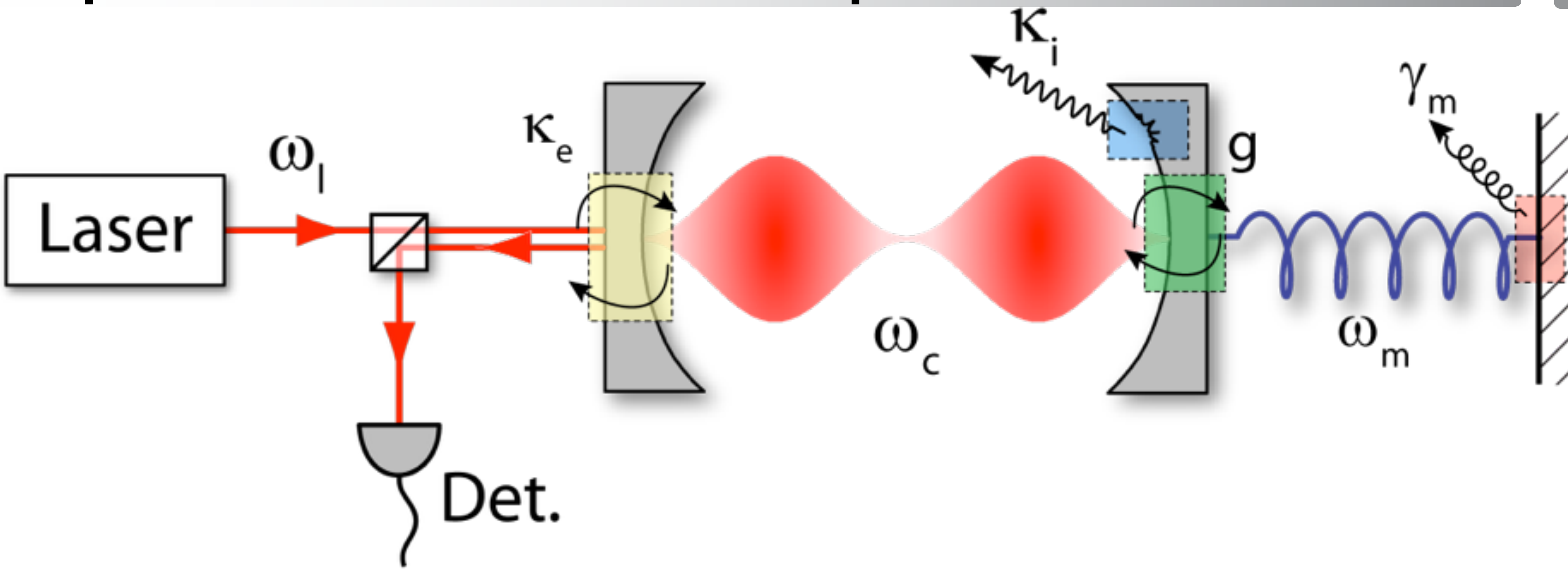
$$\dot{a}(t) = \left(\mathrm{i}(\Delta + g_{\mathrm{OM}} x(t)) - \frac{\kappa}{2} \right) a + \sqrt{\kappa_e} \alpha_{\mathrm{in}}$$

$$\ddot{x} + \gamma_m \dot{x} + \omega_m^2 x = \frac{g_{\mathrm{OM}}}{m \omega_0} \mid a \mid^2$$

$$\omega_c = \omega_o (1 - g_{\mathrm{OM}} x(t))$$

$$\Delta = \omega_l - \omega_o$$

Optical Time Domain Equation

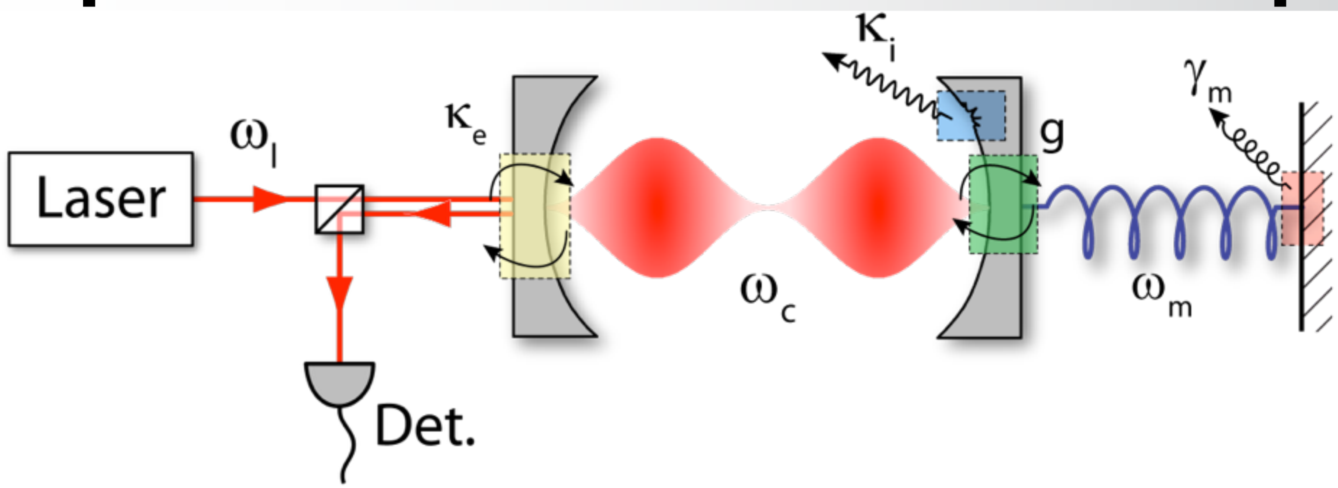


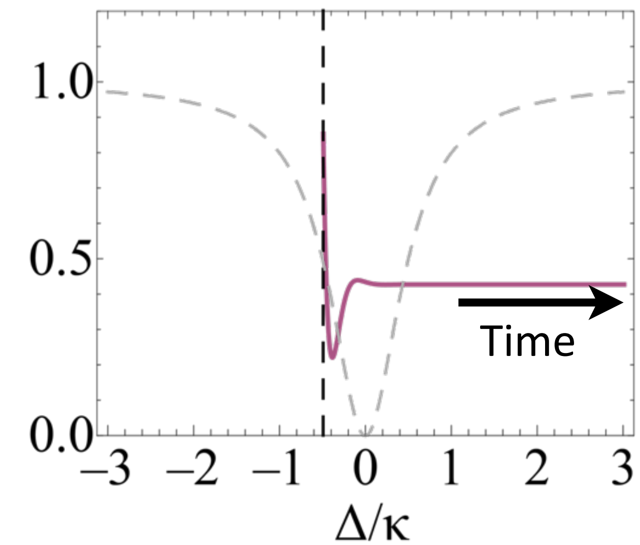
Coupled Equations

$$\dot{a}(t) = \left(\mathrm{i}\Delta - \frac{\kappa}{2} \right) a + \mathrm{i} g_{\mathrm{OM}} a x(t) + \sqrt{\kappa_e} \alpha_{\mathrm{in}}$$

$$\ddot{x} + \gamma_m \dot{x} + \omega_m^2 x = \frac{\left| a(t) \right|^2 g_{\mathrm{OM}}}{\omega_0 m}$$

Optical Time Domain Equation





$$\dot{a}(t) = \left(\mathrm{i}\Delta - \frac{\kappa}{2}\right)a + \mathrm{i}g_{\mathrm{OM}}ax(t) + \sqrt{\kappa_e}\alpha_{\mathrm{in}}$$

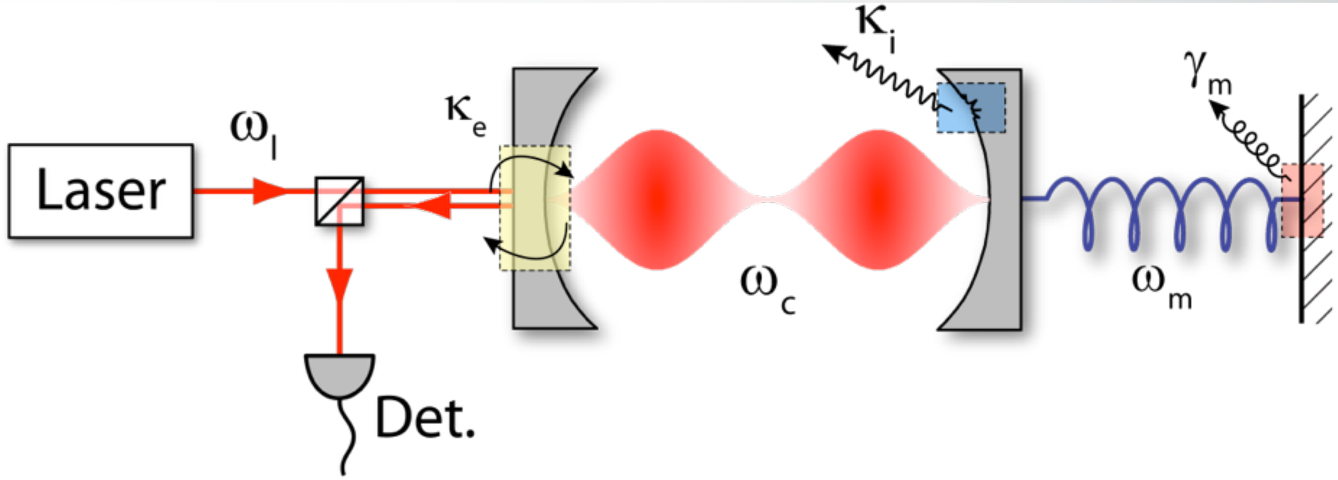
$$\ddot{x} + \gamma_m \dot{x} + \omega_m^2 x = \frac{g_{\mathrm{OM}}}{\omega_0 m} \mid a(t) \mid^2$$

Linearized dynamics:

$$a(t) \approx a_0 + \delta a(t)$$

$$x(t) \approx x_0 + \delta x(t)$$

Zeroth order equation: static shift

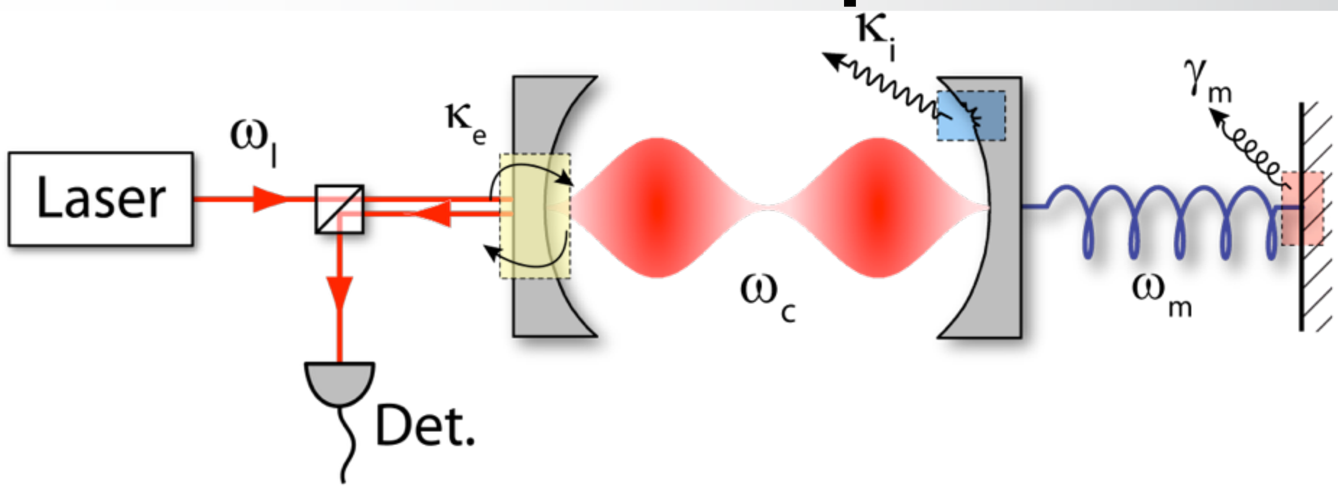


$$a(t) \approx a_0 + \delta a(t)$$
$$x(t) \approx x_0 + \delta x(t)$$

$$\dot{a}(t) = \left(\mathrm{i}\Delta - \frac{\kappa}{2}\right) a + \mathrm{i} g_{\mathrm{OM}} a x(t) + \sqrt{\kappa_e}\alpha_{\mathrm{in}} \qquad \ddot{x} + \gamma_m \dot{x} + \omega_m^2 x = \frac{\hbar g_{\mathrm{OM}}}{m_{\mathrm{eff}}} \left|a(t)\right|^2$$

The zeroth order solution is basically the exact static solution of the problem!

Zeroth order equation: static shift



$$a(t) \approx a_0 + \delta a(t)$$

$$x(t) \approx x_0 + \delta x(t)$$

$$\dot{a}(t) = \left(i\Delta - \frac{\kappa}{2} \right) a + i g_{OM} a x(t) + \sqrt{\kappa_e} \alpha_{in} \qquad \ddot{x} + \gamma_m \dot{x} + \omega_m^2 x = \frac{\hbar g_{OM}}{m_{eff}} |a(t)|^2$$

The zeroth order solution is basically the exact static solution of the problem!

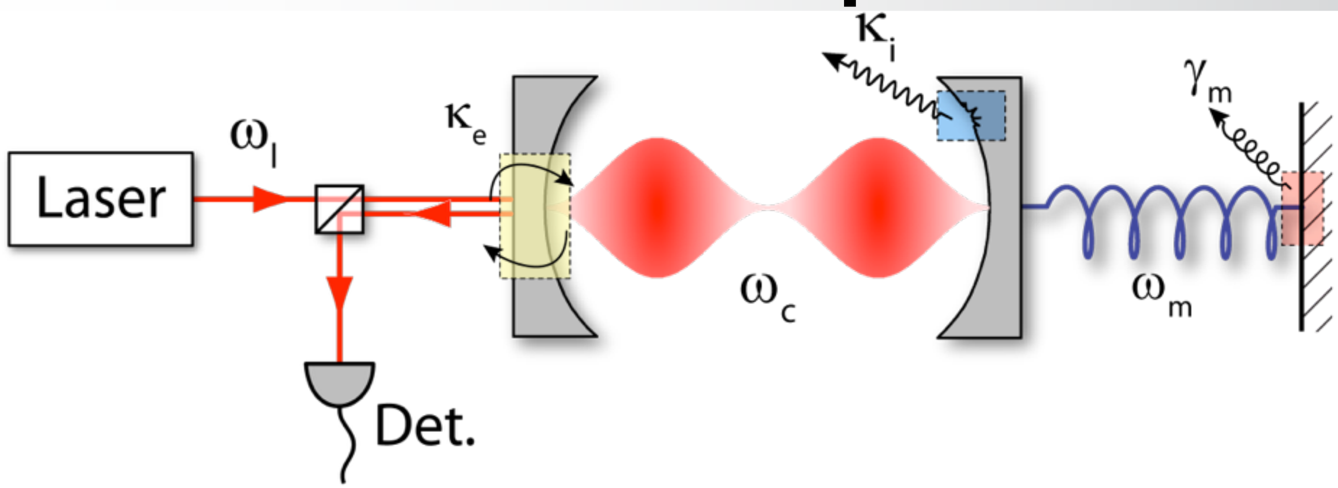
Average number of photons

$$U \equiv | a_0 |^2 = \frac{\alpha_{in}^2 \kappa_e}{\Delta'^2 + \kappa^2 / 4}$$

Static displacement

$$x_0 = \frac{g_{OM}}{\omega_0 m_{eff}} U$$

Zeroth order equation: static shift



$$a(t) \approx a_0 + \delta a(t)$$
$$x(t) \approx x_0 + \delta x(t)$$

Average number of photons

$$U \equiv |a_0|^2 = \frac{\alpha_{in}^2 \kappa_e}{\Delta'^2 + \kappa^2 / 4}$$

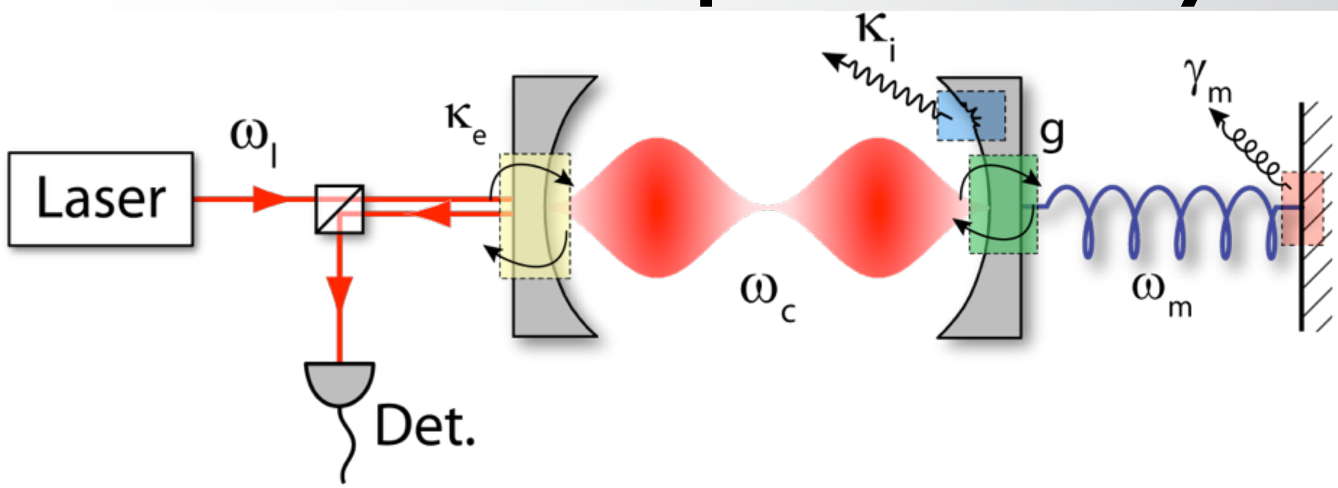
Static displacement

$$x_0 = \frac{g_{OM}}{\omega_0 m_{\text{eff}}} U$$

Not so easy buddy!
The detuning (Δ) is a function of position (x) !

$$\Delta' = \Delta - g_{\text{OM}} x_0$$

First order equation: dynamics

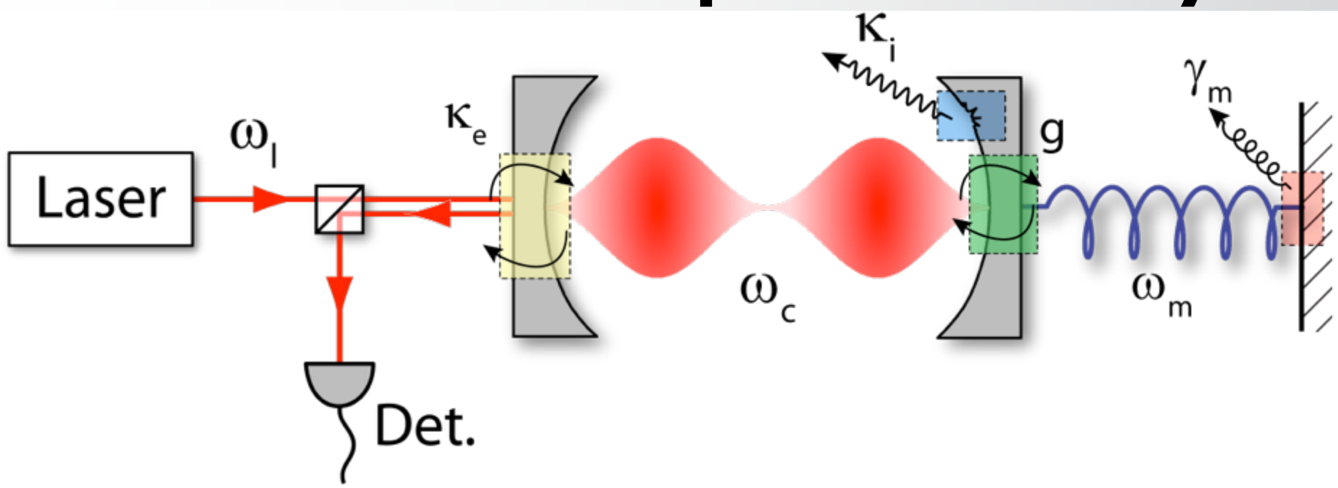


$$a(t) \approx a_0 + \delta a(t)$$

$$x(t) \approx x_0 + \delta x(t)$$

$$F(t) \propto |\, a_0 + \delta a \,|^2 = \frac{g_{\mathrm{OM}}}{\omega_0}\left(a_0{}^2 + \underbrace{a_0 \delta a^* + a_0{}^* \delta a}_{\delta F(t)} + \delta a^2\right)$$

First order equation: dynamics

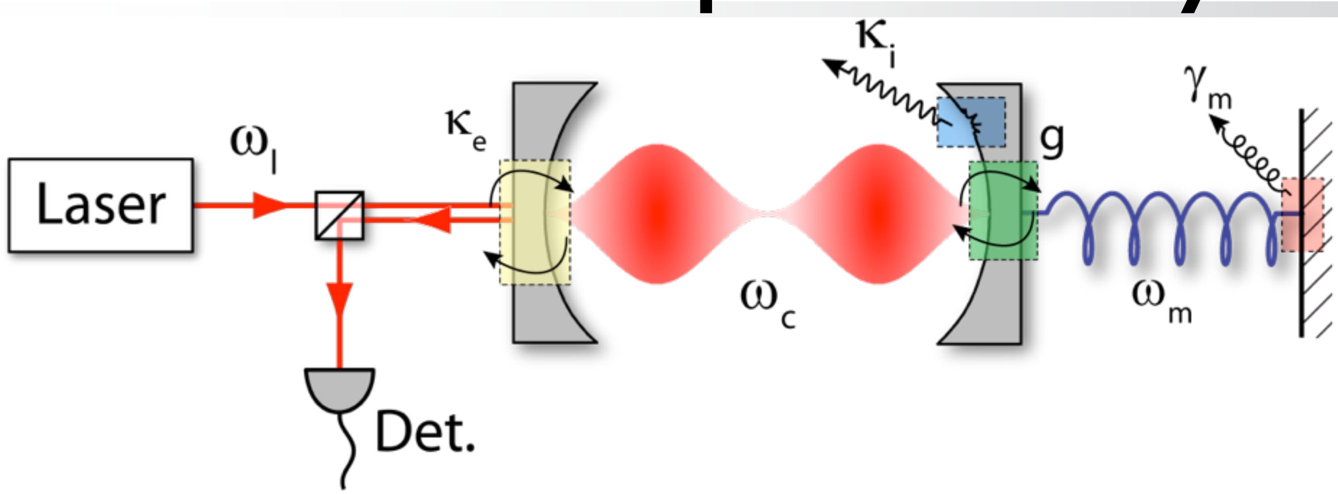


$$a(t) \approx a_0 + \delta a(t)$$
$$x(t) \approx x_0 + \delta x(t)$$

$$F(t) \propto | a_0 + \delta a |^2 = \frac{g_{\mathrm{OM}}}{\omega_0}\left(a_0{}^2 + \underbrace{a_0 \delta a^* + a_0{}^* \delta a}_{\delta F(t)} + \delta a^2\right)$$

$$\delta \ddot{x} = -\Omega^2 \delta x - \gamma \delta \dot{x} - \frac{g_{OM}}{\omega_0 m_{\mathrm{eff}}}\left(a_0 \delta a^* + a_0^* \delta a\right)$$

First order equation: dynamics



$$a(t) \approx a_0 + \delta a(t)$$
$$x(t) \approx x_0 + \delta x(t)$$

$$F(t) \propto |\, a_0 + \delta a \,|^2 = \frac{g_{\mathrm{OM}}}{\omega_0}\Big(a_0{}^2 + \underbrace{a_0\delta a^* + a_0{}^*\delta a}_{\delta F(t)} + \delta a^2\Big)$$

$$\delta\ddot{x} = -\Omega^2\delta x - \gamma\delta\dot{x} - \frac{g_{OM}}{\omega_0 m_{\mathrm{eff}}}\left(a_0\delta a^* + a_0^*\delta a\right)$$

$$\delta\ddot{x} = -\Omega^2\delta x - \gamma\delta\dot{x} + \delta F(t)\,/\,m_{\mathrm{eff}}$$

First order equation: dynamics

$$\delta\ddot{x} = -\Omega^2\delta x - \gamma\delta\dot{x} + \delta F(t)$$

$$\delta\dot{a} = (\mathrm{i}\Delta' - \frac{\kappa}{2})\delta a - i g_{\mathrm{OM}} a_0 \delta x$$

Fourier transform: $\tilde{f}[\omega] = \int_{-\infty}^{\infty} f(t) e^{i\omega t}\, dt$

$$m_{\mathrm{eff}}(\Omega^2 - \omega^2 + i\gamma\omega)\delta\tilde{x}[\omega] = \delta\tilde{F}[\omega]$$

$$\delta\tilde{a}(\omega) = \frac{-i g_{\mathrm{OM}} a_0}{-\omega - (\mathrm{i}\Delta - \frac{\kappa}{2})}\delta\tilde{x}(\omega)$$

First order equation: dynamics

$$\delta\tilde{a}(\omega) = \frac{-ig_{\mathrm{OM}}a_0}{-\omega - (\mathrm{i}\Delta - \frac{\kappa}{2})}\delta\tilde{x}(\omega)$$

$$m_{\mathrm{eff}}(\Omega^2 - \omega^2 + i\gamma\omega)\delta\tilde{x}[\omega] = \delta\tilde{F}[\omega]$$

$$\delta\tilde{F}[\omega] = \delta\tilde{x}\frac{g_{\mathrm{OM}}^2 U}{\omega_0}\left(\frac{1}{(\Delta' - \omega) - i\kappa / 2} + \frac{1}{(\Delta' + \omega) + i\kappa / 2}\right)$$

$$\equiv \Sigma(\omega)$$

First order equation: dynamics

$$\delta\tilde{F}[\omega] = \delta\tilde{x}\frac{g_{\mathrm{OM}}^2 U}{\omega_0}\left(\frac{1}{(\Delta' - \omega) - i\kappa/2} + \frac{1}{(\Delta' + \omega) + i\kappa/2}\right)$$

$$\equiv \Sigma(\omega)$$

$$m_{\mathrm{eff}}(\Omega^2 - \omega^2 + i\gamma\omega)\delta\tilde{x}[\omega] = 2g^2 m_{\mathrm{eff}}\Omega\Sigma(\omega)\delta\tilde{x}[\omega]$$

$$\equiv \chi_{xx}^{-1}(\omega) \qquad g \equiv (g_{\mathrm{OM}}x_{\mathrm{zpf}})\sqrt{n_c}$$

$$g_0\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger) \mapsto g_0\alpha(\delta\hat{a} + \delta\hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$

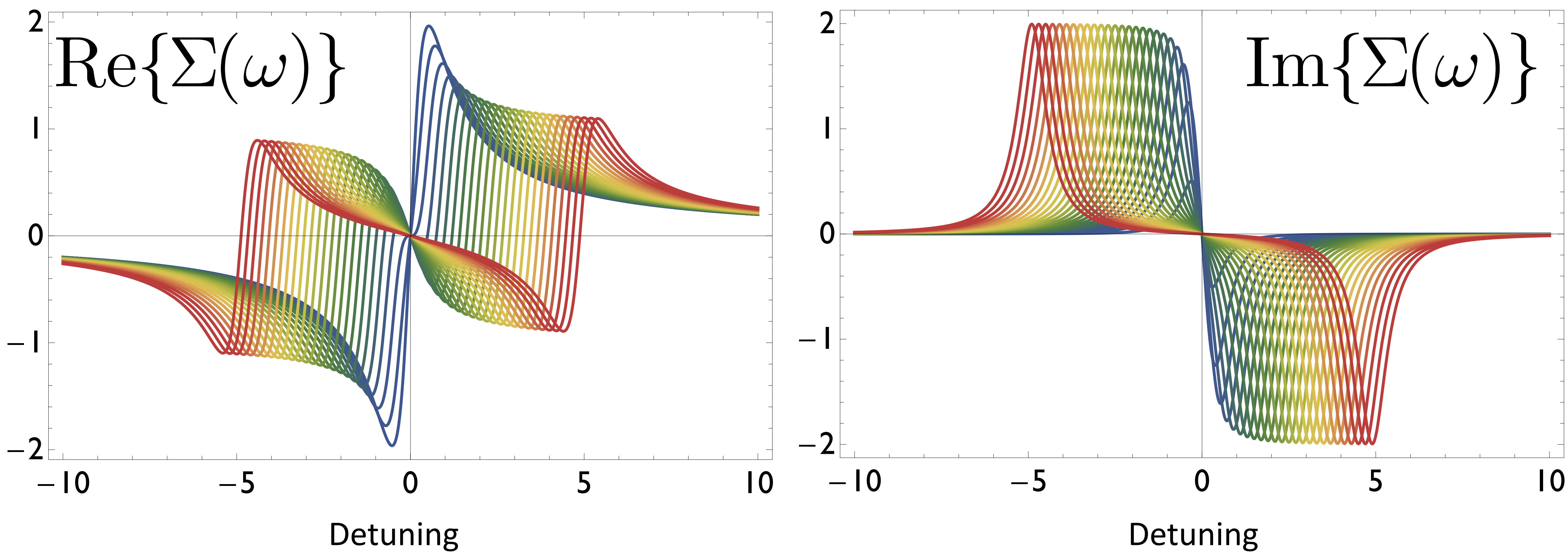
g (= $g_0\alpha$)

bilinear interaction
tunable coupling!

First order equation: dynamics

$$\delta\tilde{F}[\omega] = \delta\tilde{x}\,\frac{g_{\mathrm{OM}}^2 U}{\omega_0}\left(\frac{1}{(\Delta' - \omega) - i\kappa/2} + \frac{1}{(\Delta' + \omega) + i\kappa/2}\right)$$

$$\equiv \Sigma(\omega)$$



Dynamical Back-action

Optical damping

Optical spring effect

$$\ddot{x} + (\Gamma_m + \Gamma_{\mathrm{opt}})\dot{x} + (\Omega_m + \delta\Omega_{\mathrm{opt}})^2 x = 0$$

Dynamical Back-action

$$\ddot{x} + (\Gamma_m + \Gamma_{opt})\dot{x} + (\Omega_m + \delta\Omega_{opt})^2 x = 0$$

Optical damping

$$\Gamma_{opt} = 4g^2 n_c \left(\frac{\kappa}{\kappa^2 + 4(\Delta + \Omega_m)^2} - \frac{\kappa}{\kappa^2 + 4(\Delta - \Omega_m)^2} \right)$$

Optical spring effect

$$\delta\Omega_{opt} = 4g^2 n_c \left(\frac{\Delta - \Omega_m}{\kappa^2 + 4(\Delta + \Omega_m)^2} - \frac{\Delta + \Omega_m}{\kappa^2 + 4(\Delta - \Omega_m)^2} \right)$$

$$g = g_{om} x_{zpf}$$

Optomechanical coupling rate

x_{zpf} Zero-point fluctuation

n_c Intracavity photon number

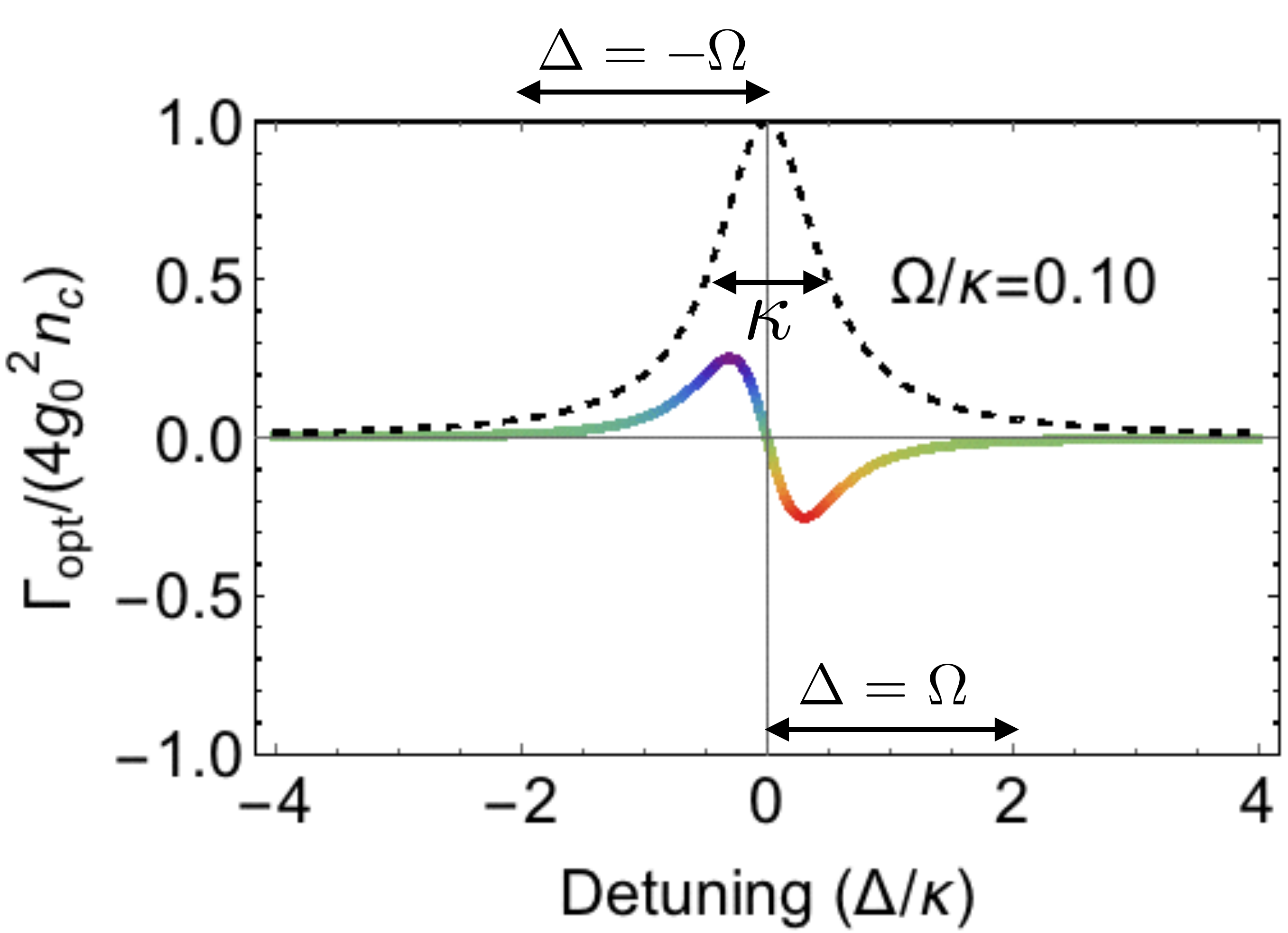
Dynamical Back-action

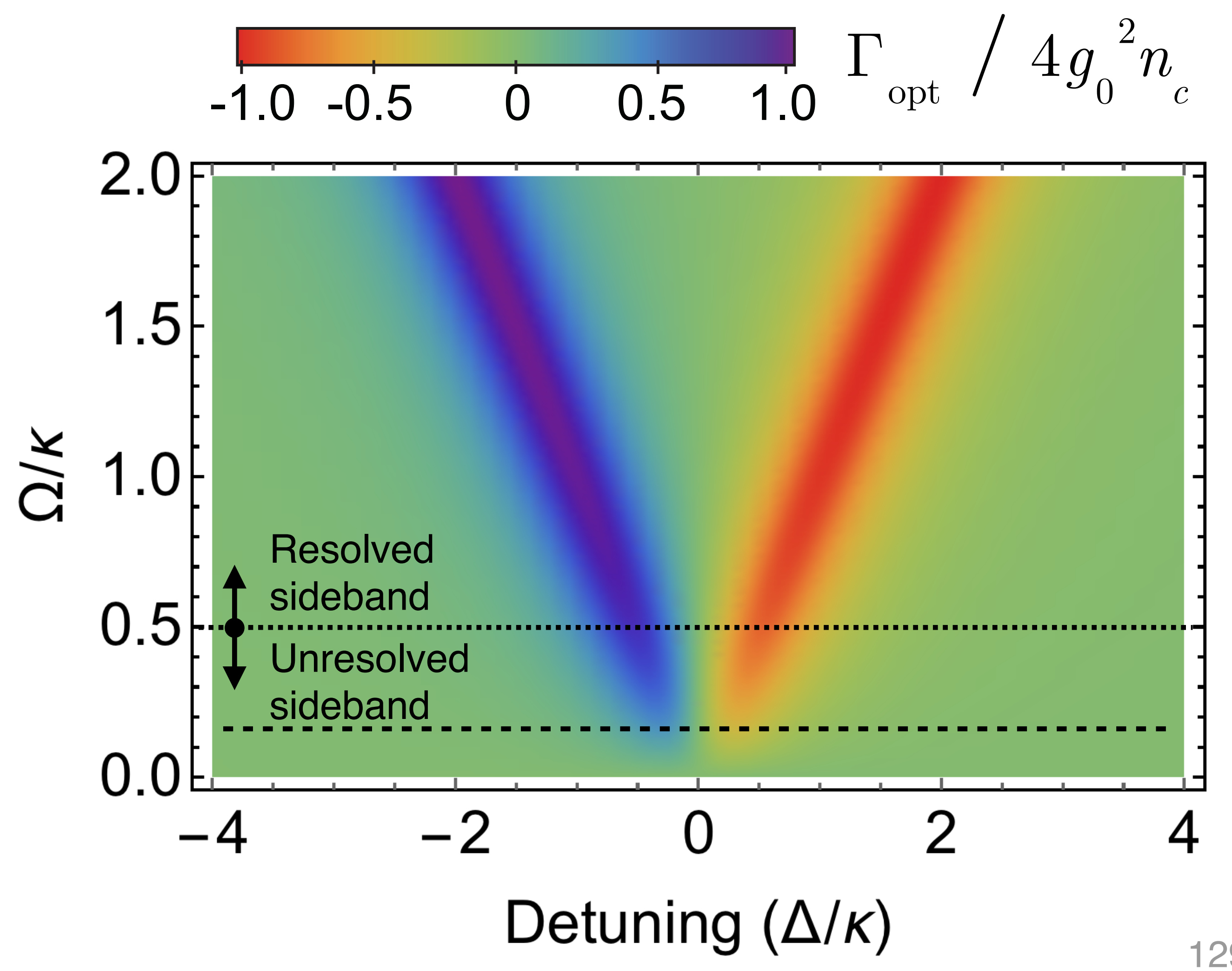
Effective damping

$$\Gamma_{\text{eff}} = (\Gamma_m + \Gamma_{\text{opt}})$$

Optical damping

$$\Gamma_{\text{opt}} = 4g_0^2 n_c \left(\frac{\kappa}{\kappa^2 + 4(\Delta + \Omega_m)^2} - \frac{\kappa}{\kappa^2 + 4(\Delta - \Omega_m)^2} \right)$$





Cooperativity

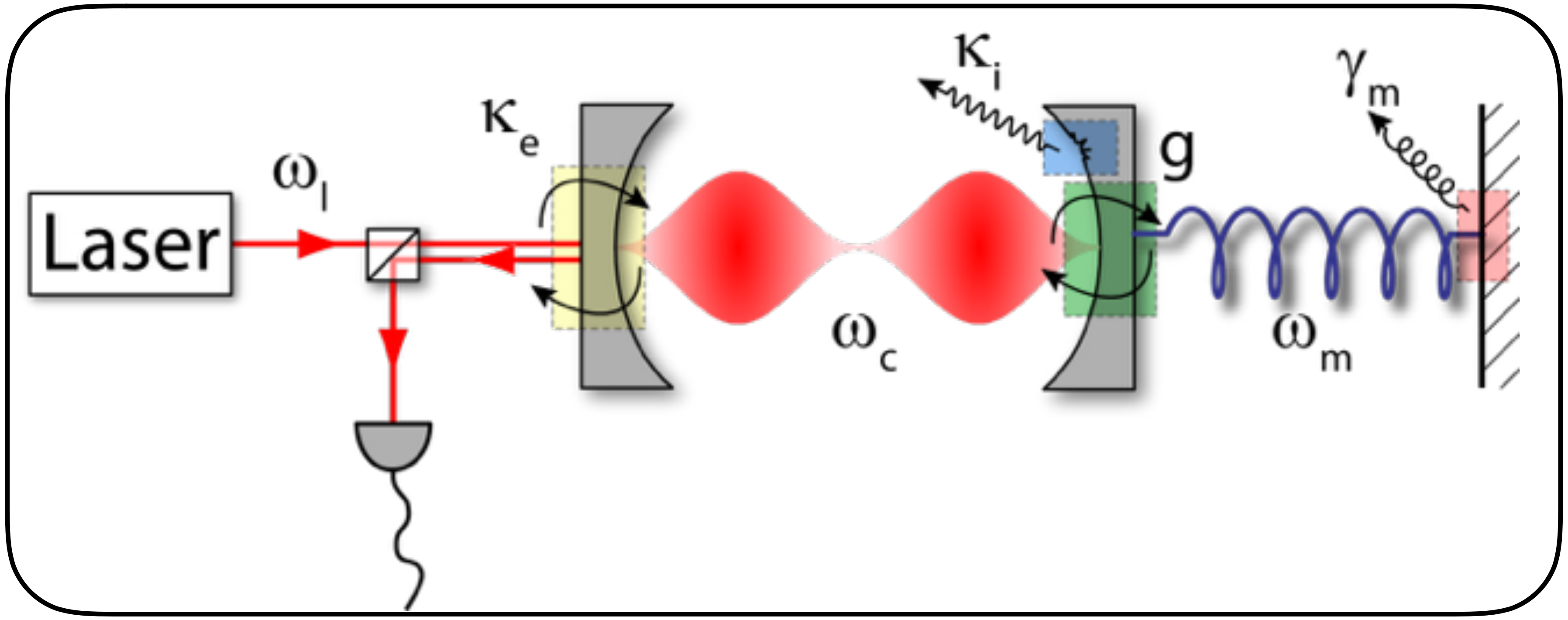


Figure of merit: cooperativity $(\Gamma_{\mathrm{opt}} = \pm\Gamma_m)$

$$C = \frac{4g^2 n_c}{\kappa \Gamma_m}$$

$$g = g_{\mathrm{om}} x_{\mathrm{zpf}}$$

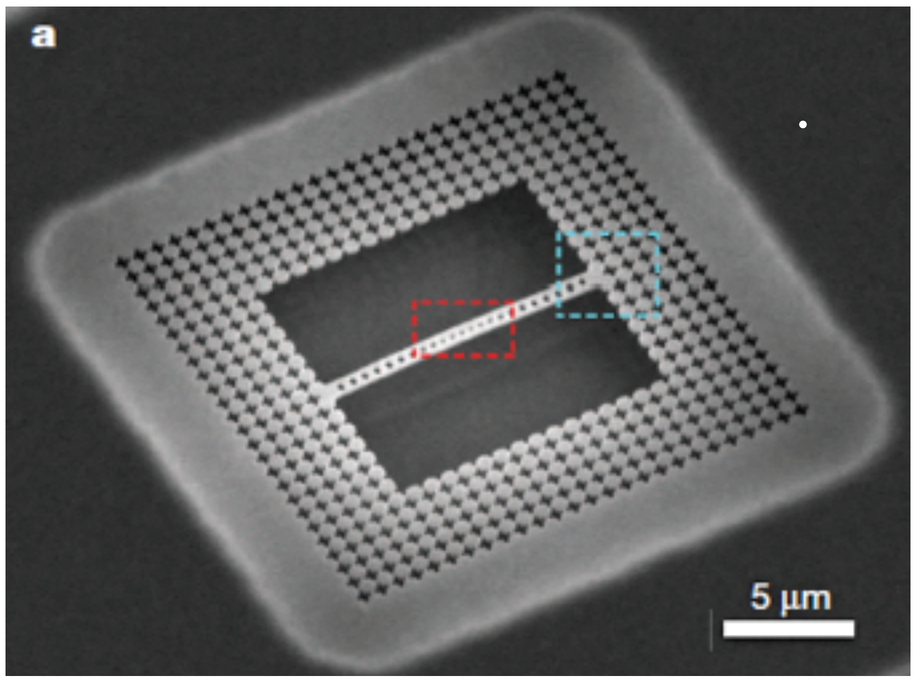
n_{c} Intracavity photon number

κ Optical mode linewidth

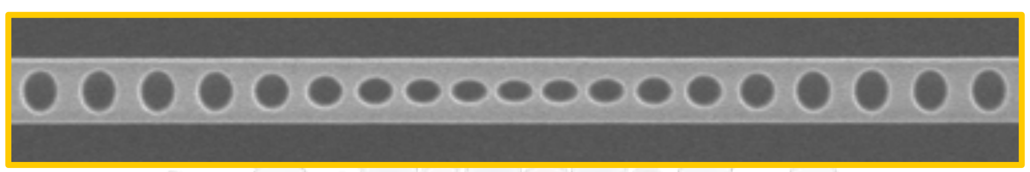
Γ_m Mechanical linewidth

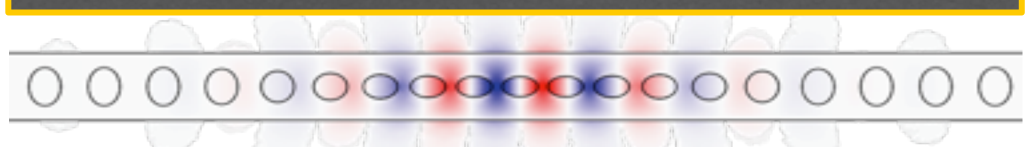
Real world devices

Non-linear optics and mechanical Optical Memories and Switches

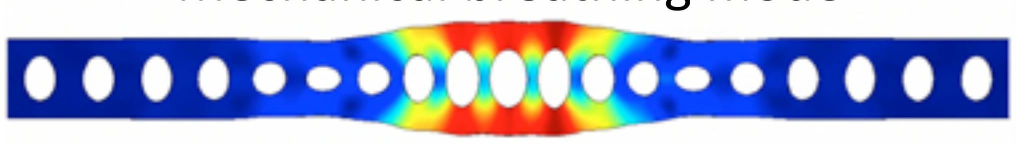


Confined photonic crystal optical Mode





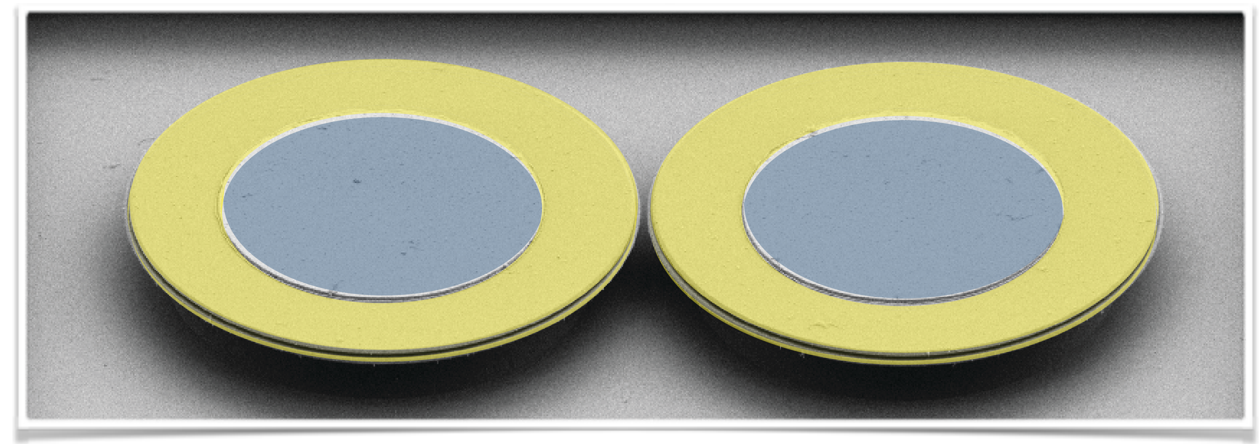
Mechanical breathing mode



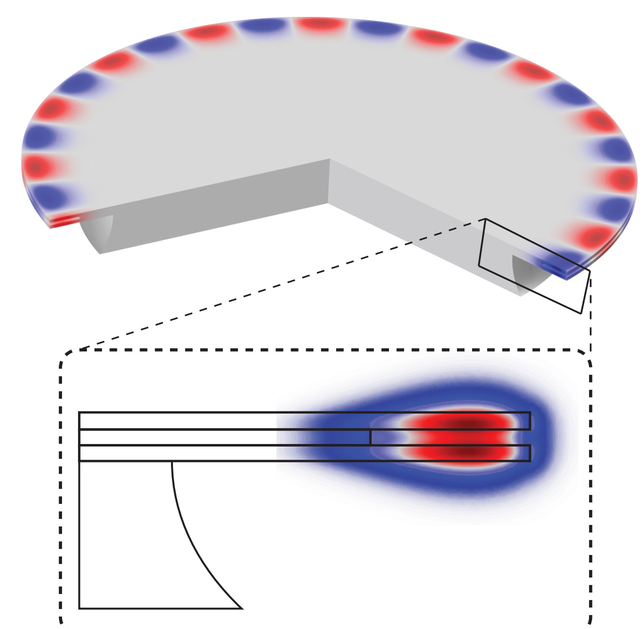
4 GHz

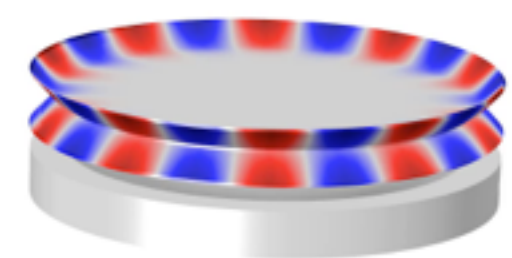
Alegre et. al, Nature **472**, 69 (2011)
J. Chan et. al, Nature **478**, 89 (2011)

Coupled Optomechanical Oscillators



Whispering Gallery Optical mode





Anti-symmetric mechanical mode

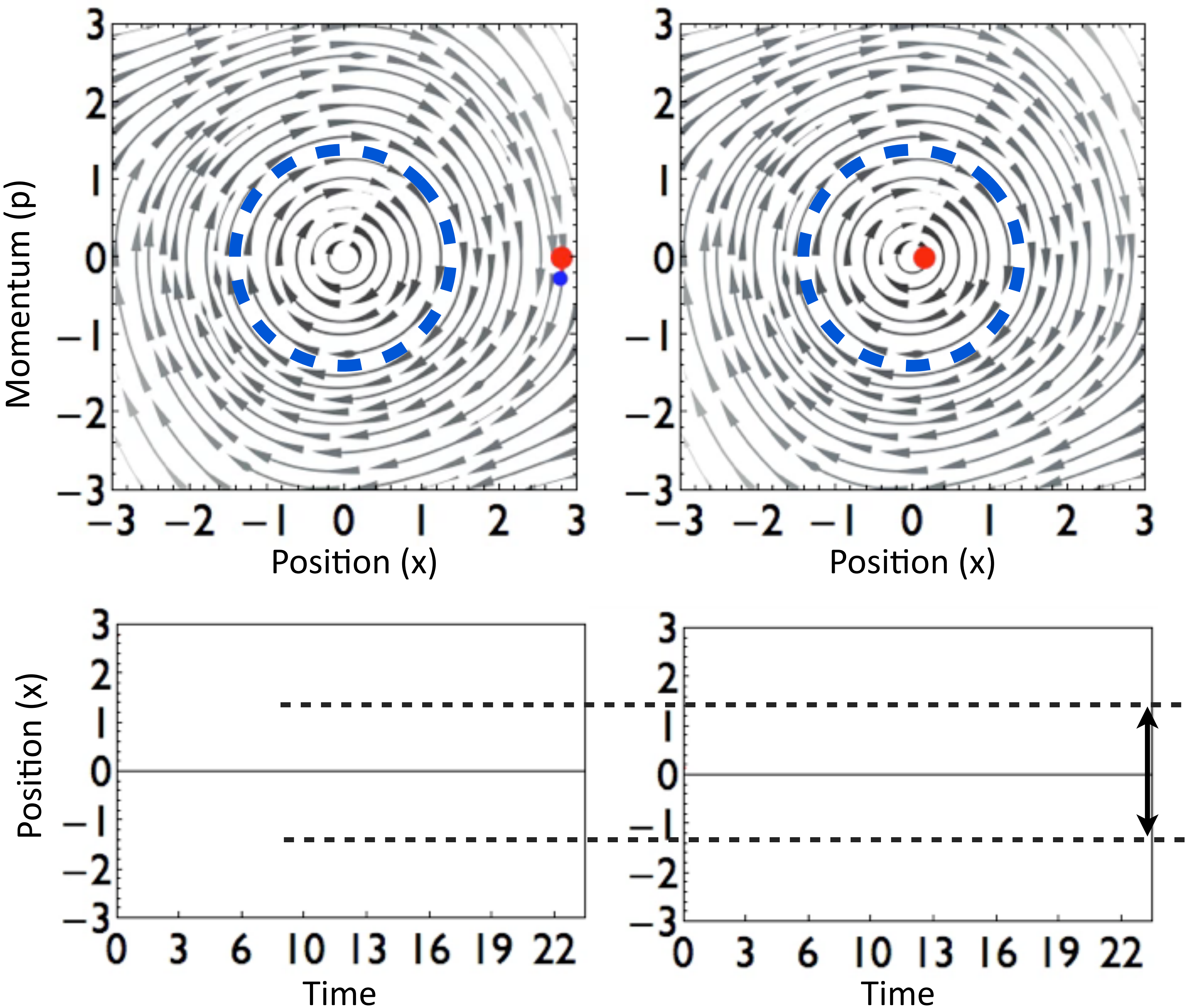
50 MHz

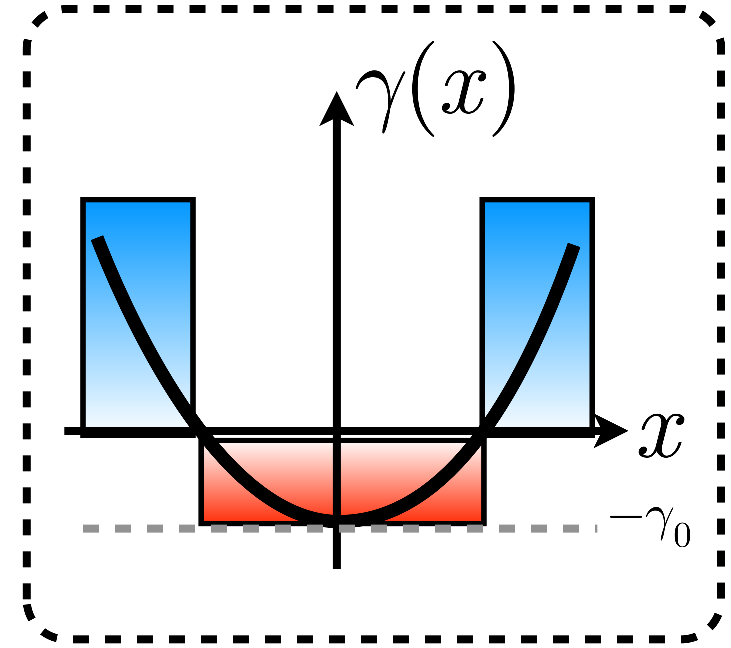
M. Zhang, G. S. Wiederhecker et al, PRL **109**, 233906 (2012)

Outline

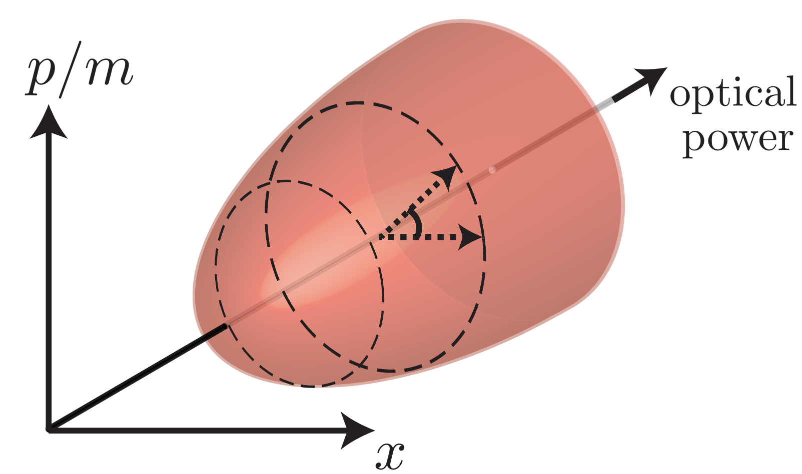
- **Optical and acoustic mode interaction**
- Optical force actuation
- Dynamical back-action
- **Optomechanical clocks**
- Bullseye - a case study
- Outlook

Self-sustaining oscillators

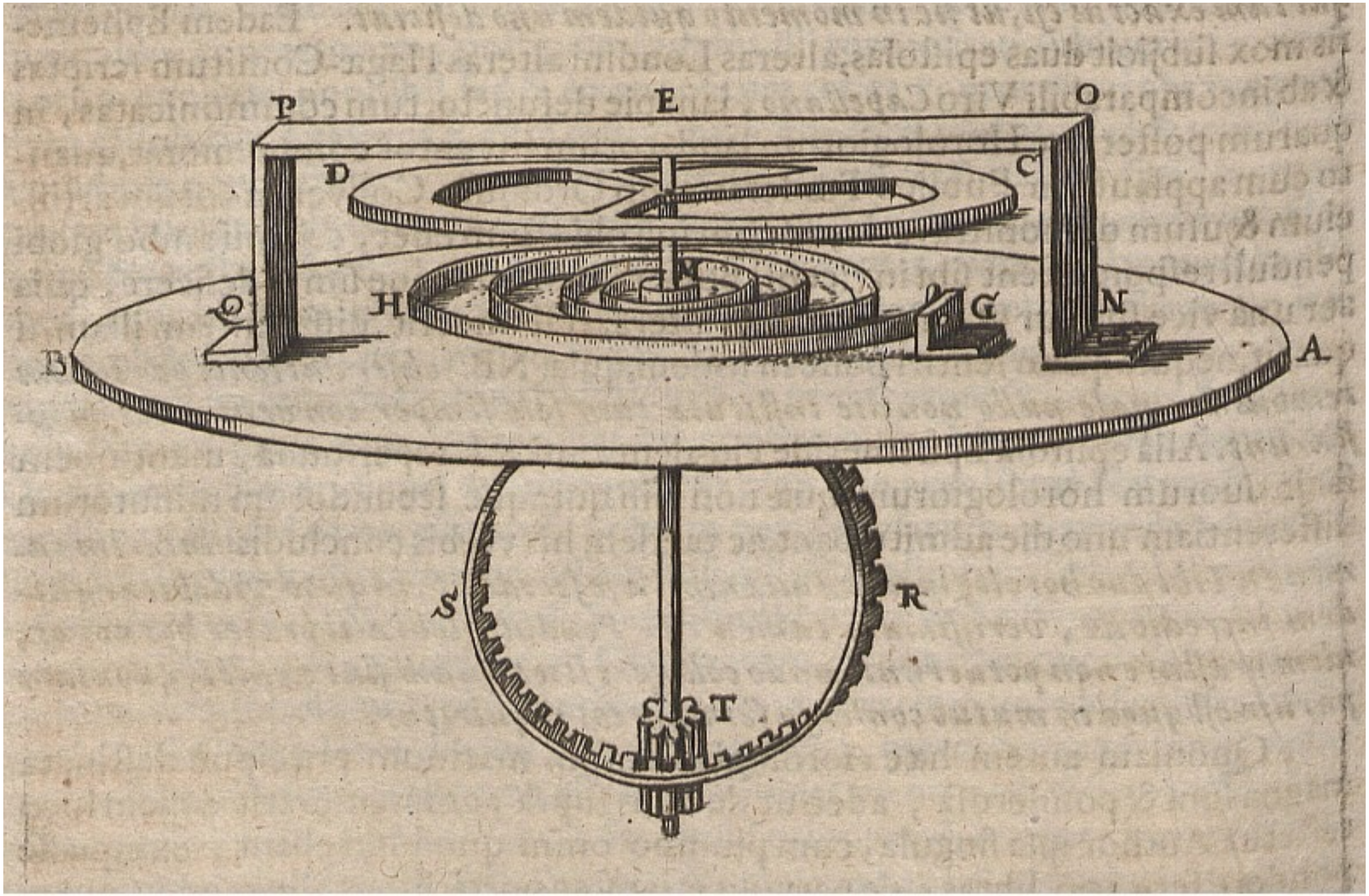




$\gamma(x) = \gamma_0(x^2 - 1)$



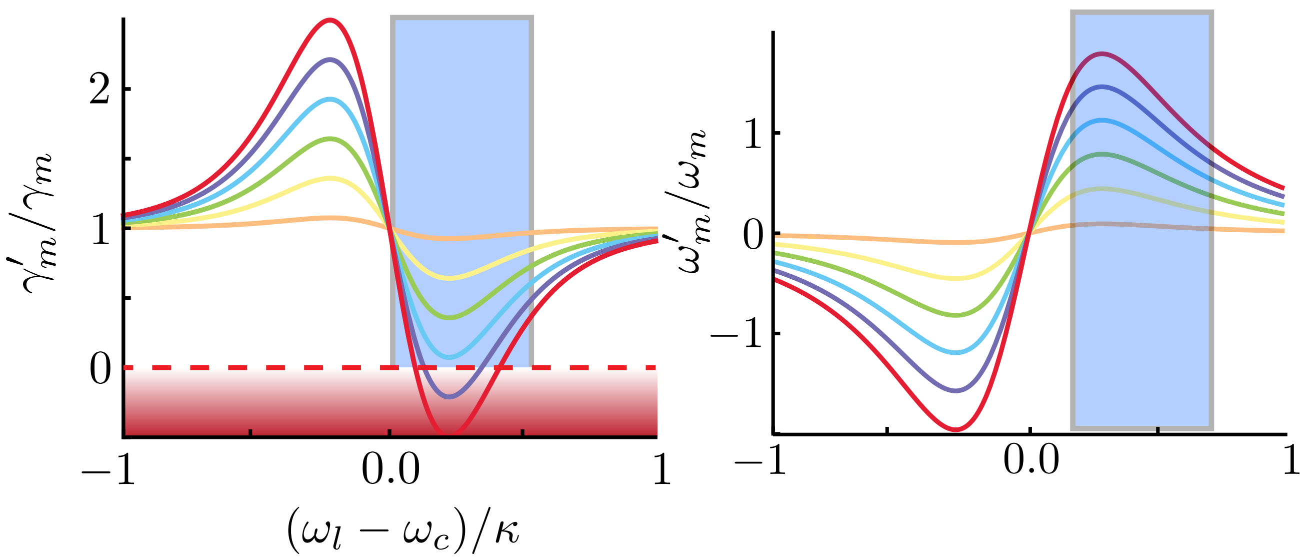
Building clocks

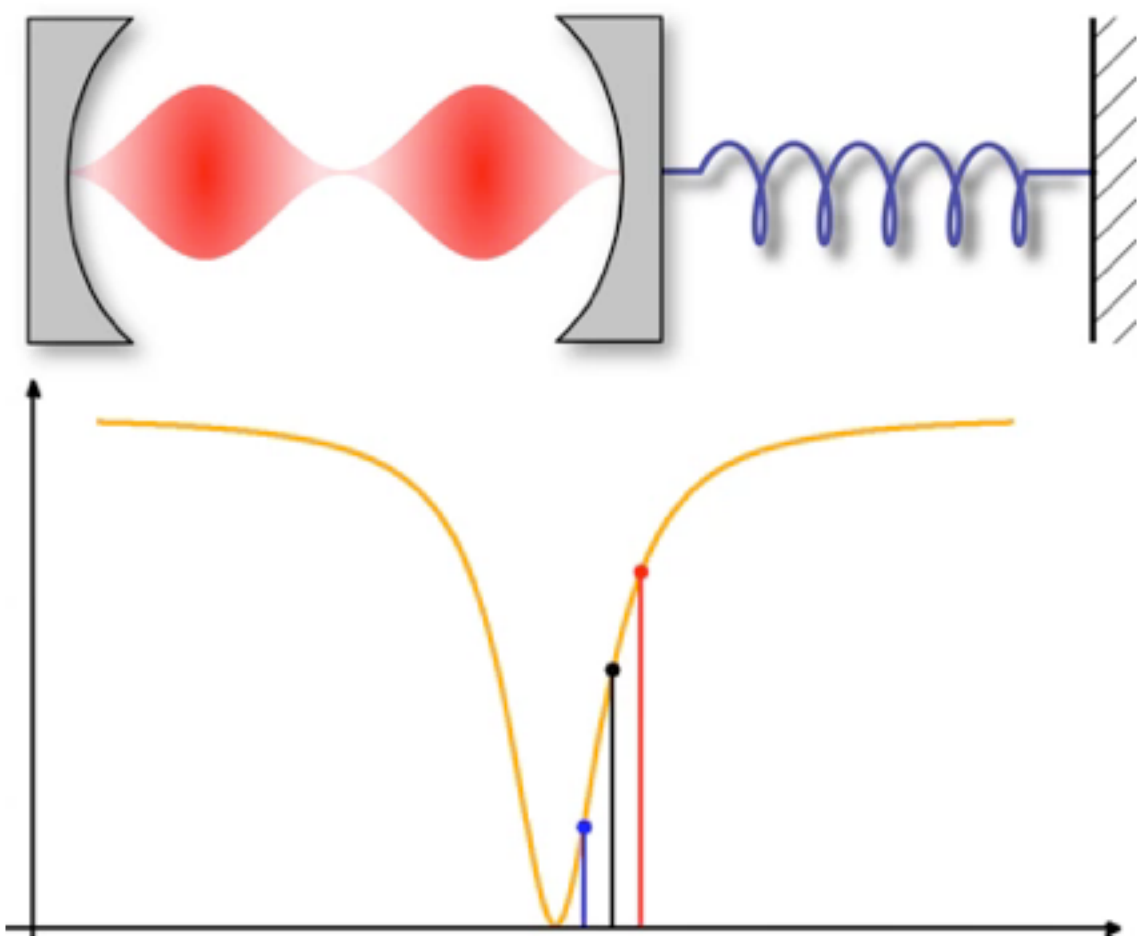


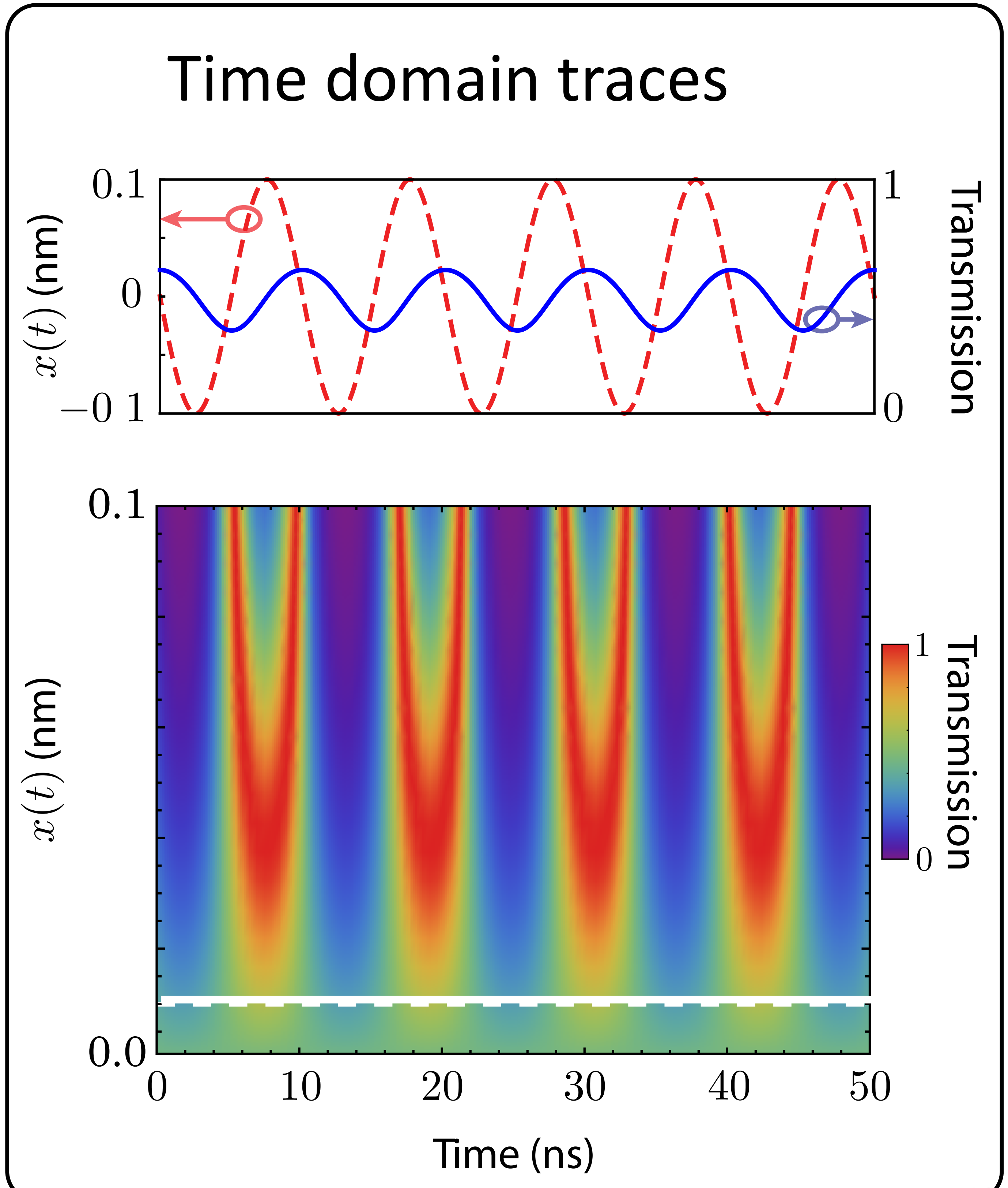
Quelle: Deutsche Fotothek

Christian Huygens, 1660

Optical Spring Effect, Cooling & Heating

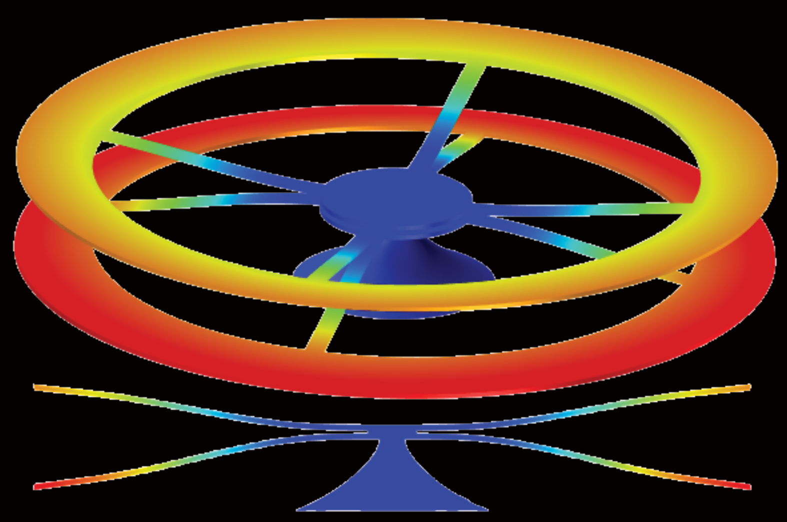




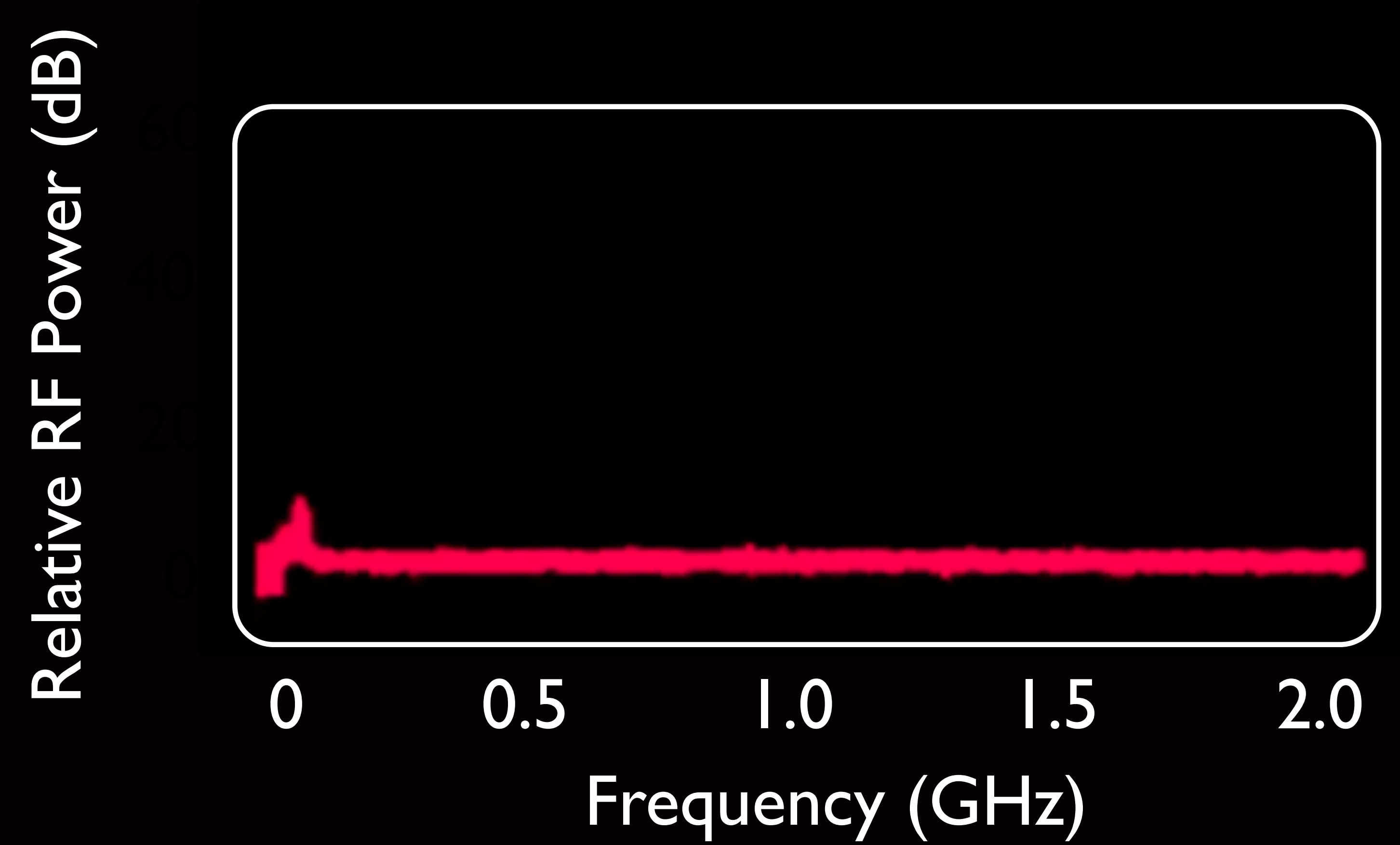


http://dx.doi.org/10.5196/physicae.10.1

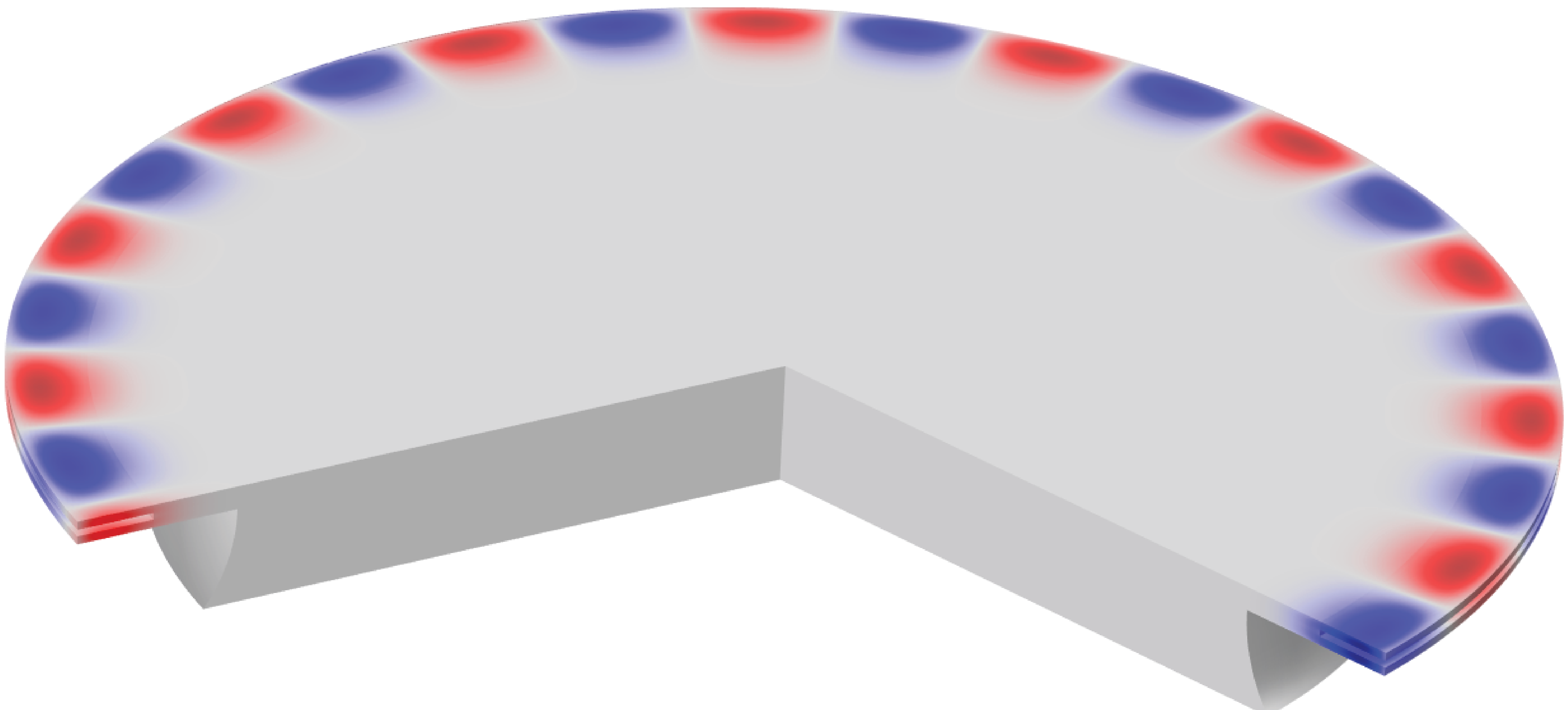
Limit cycle oscillations at the Lab.



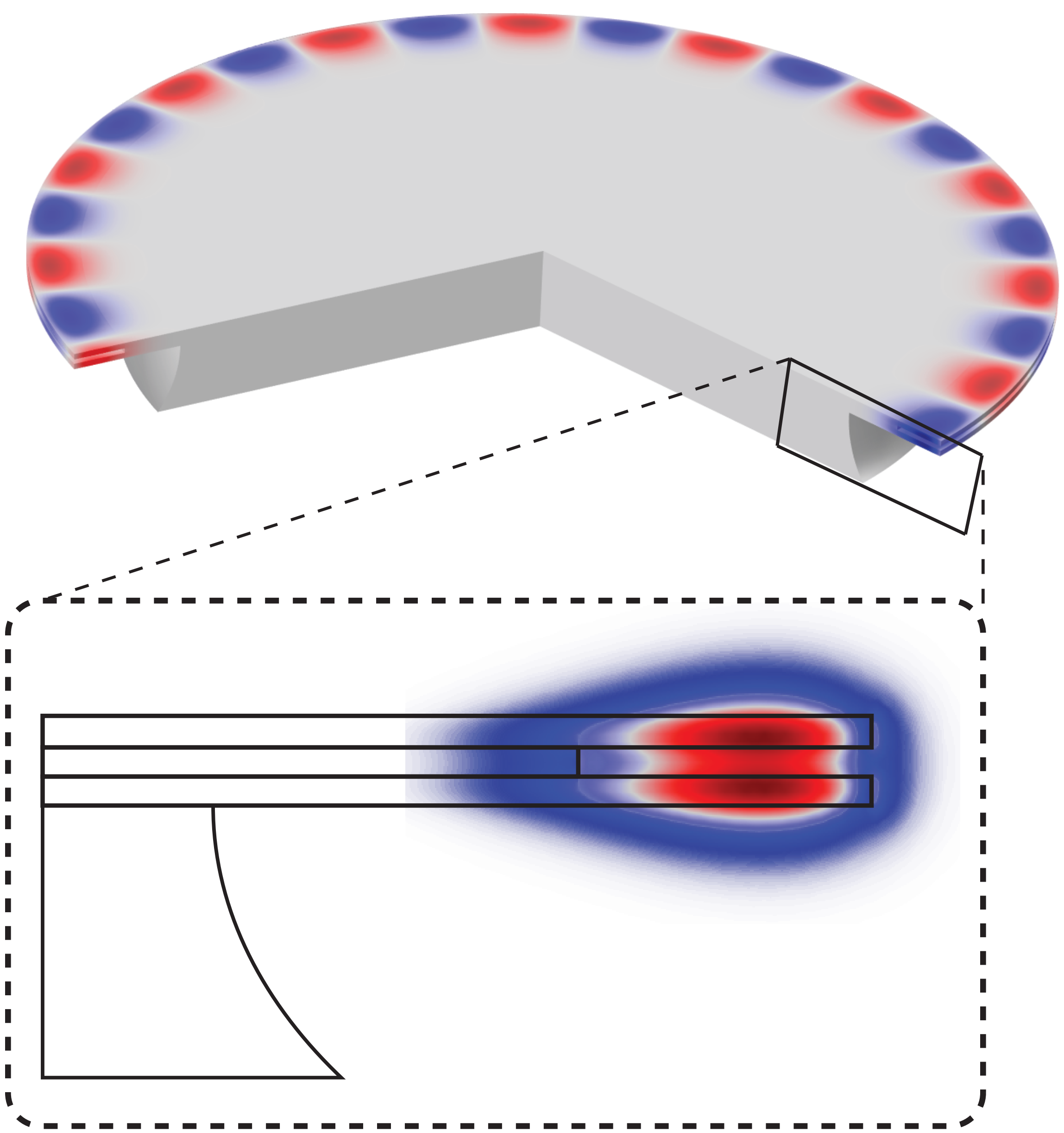
$$\Omega_0/2\pi \approx 650 \text{ KHz}$$



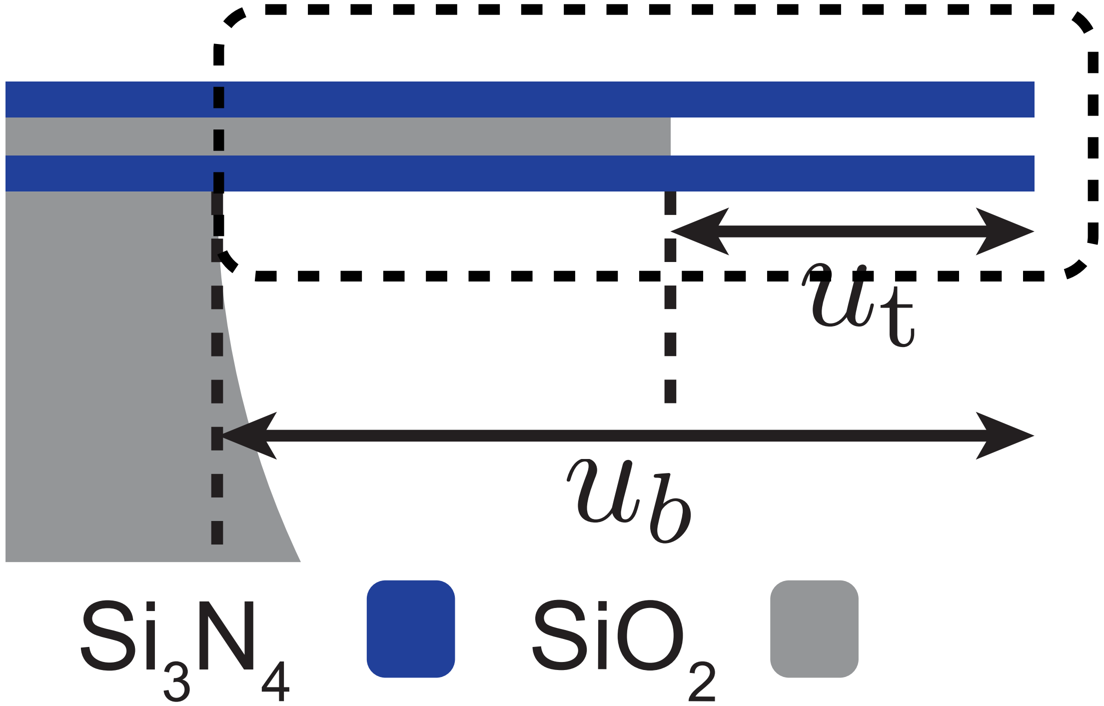
Double-disk Optomechanical Cavity



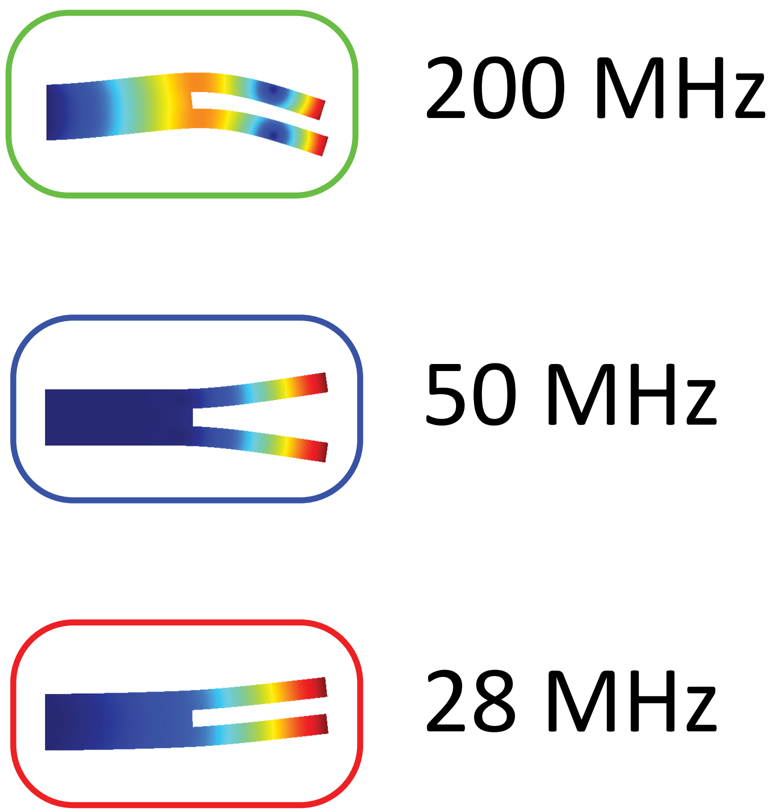
Single Optomechanical Cavity



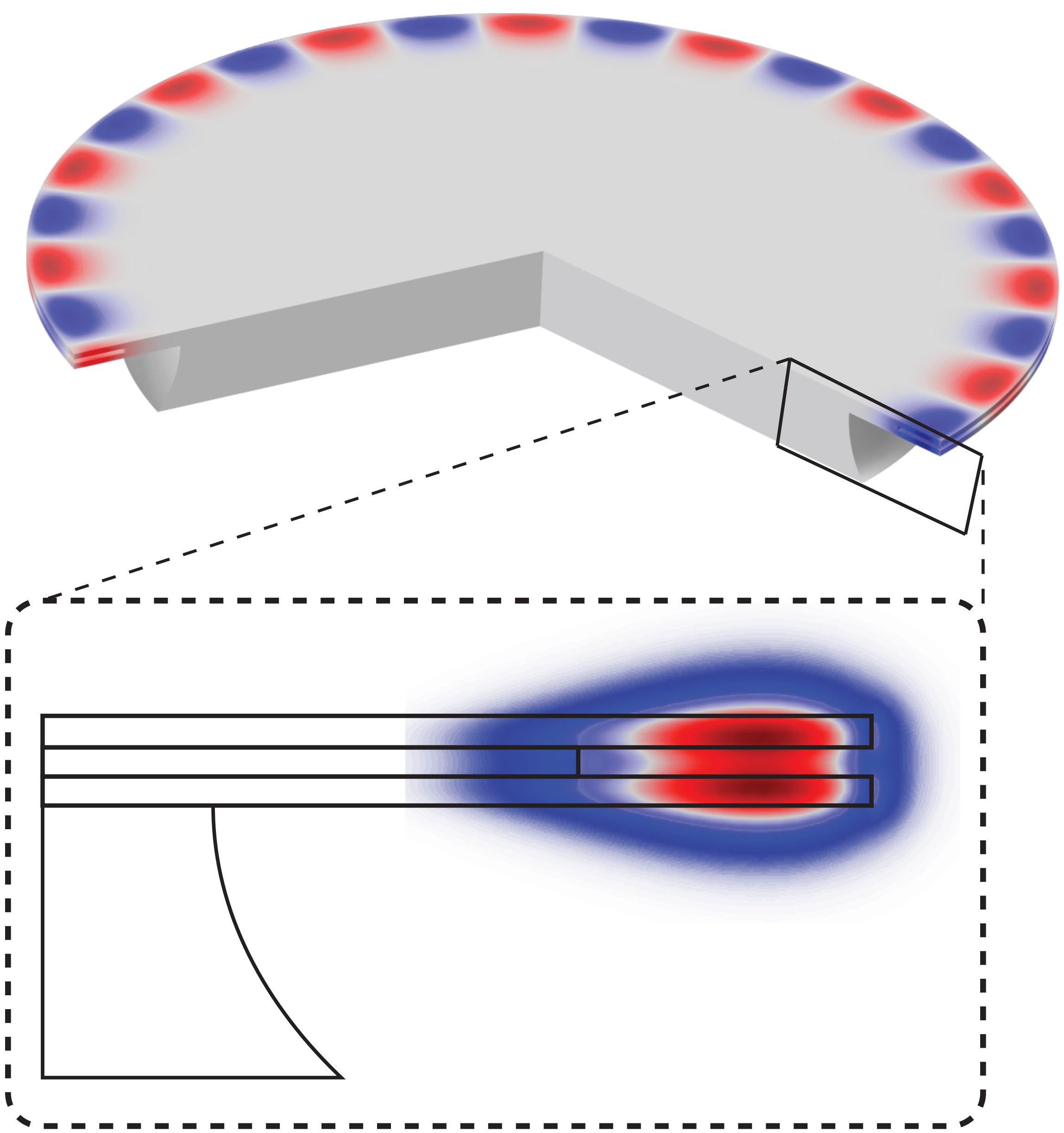
Whispering Gallery Optical mode



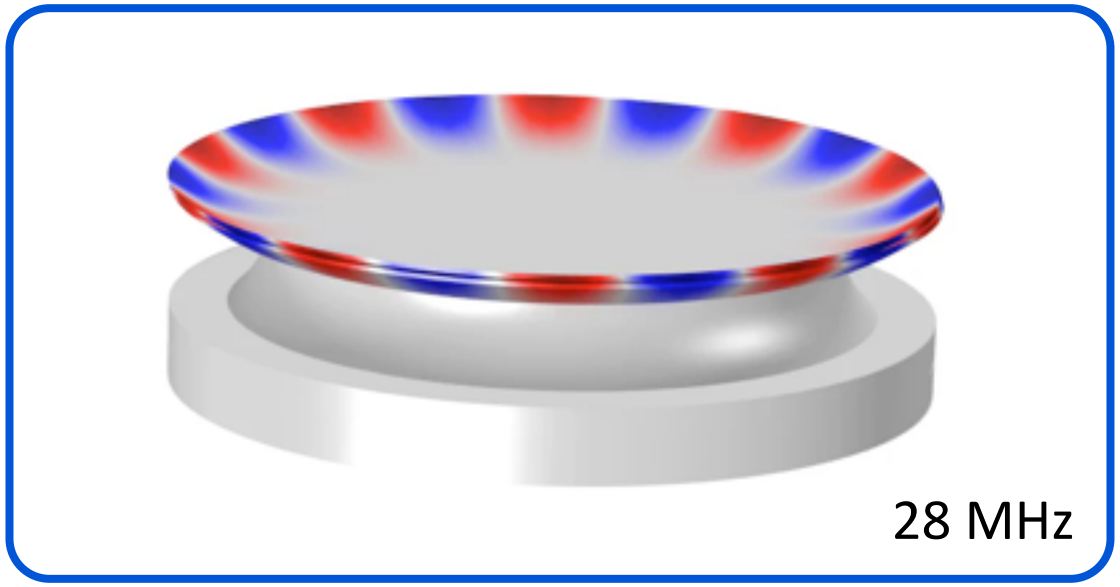
Mechanical Modes



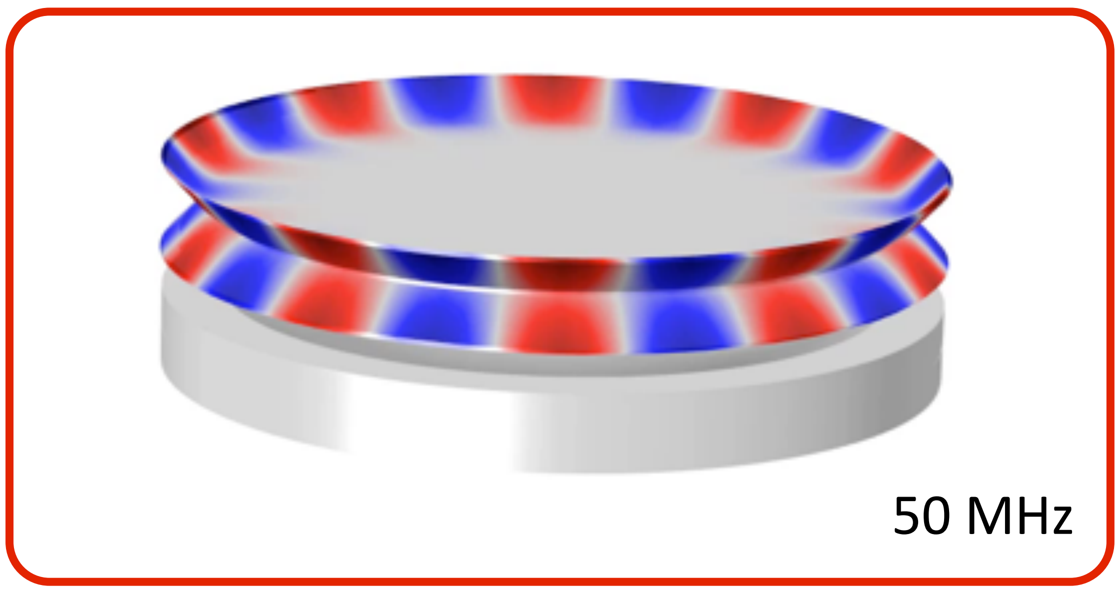
Double-disk Optomechanical Cavity



Whispering Gallery Optical mode

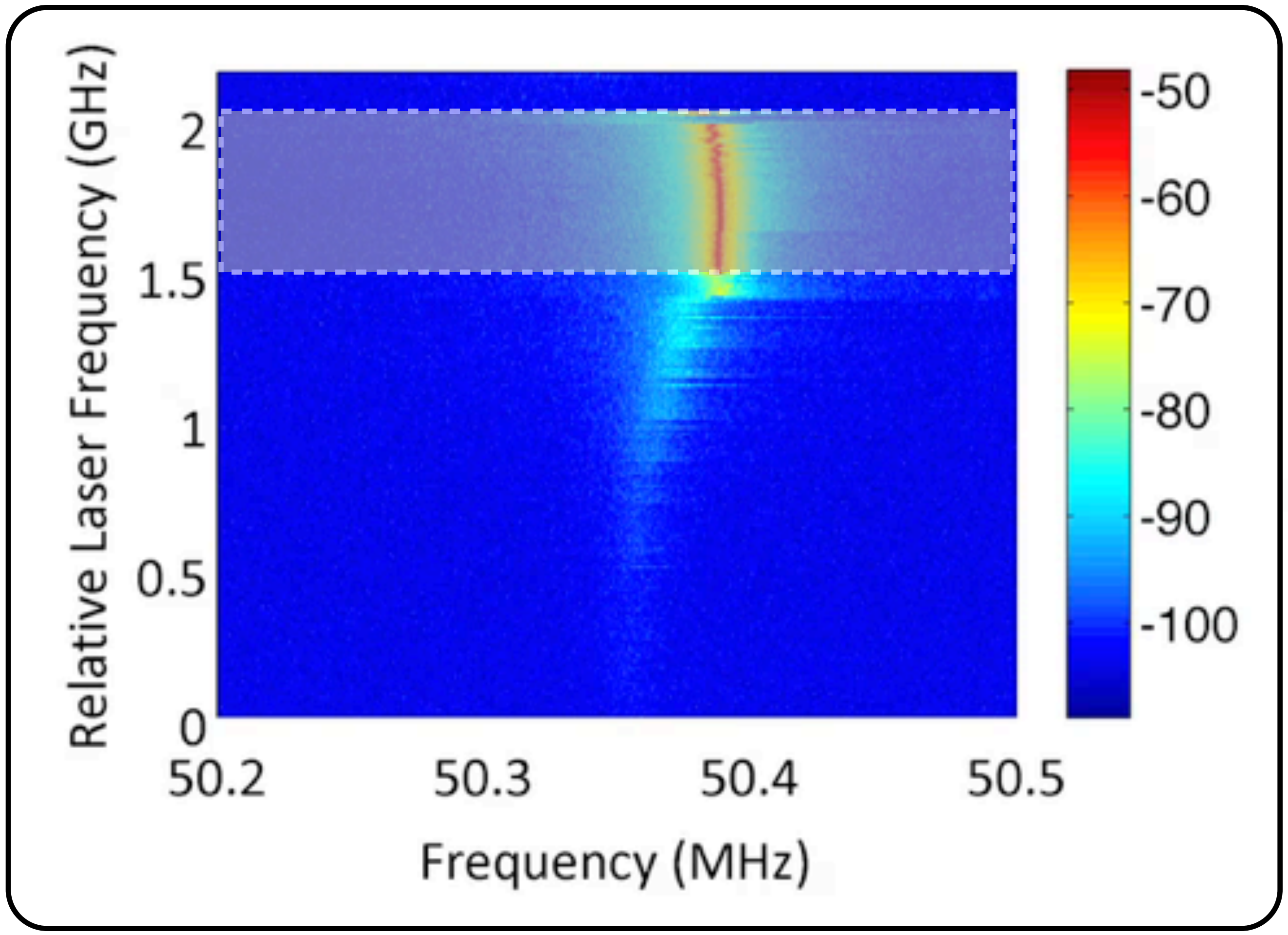


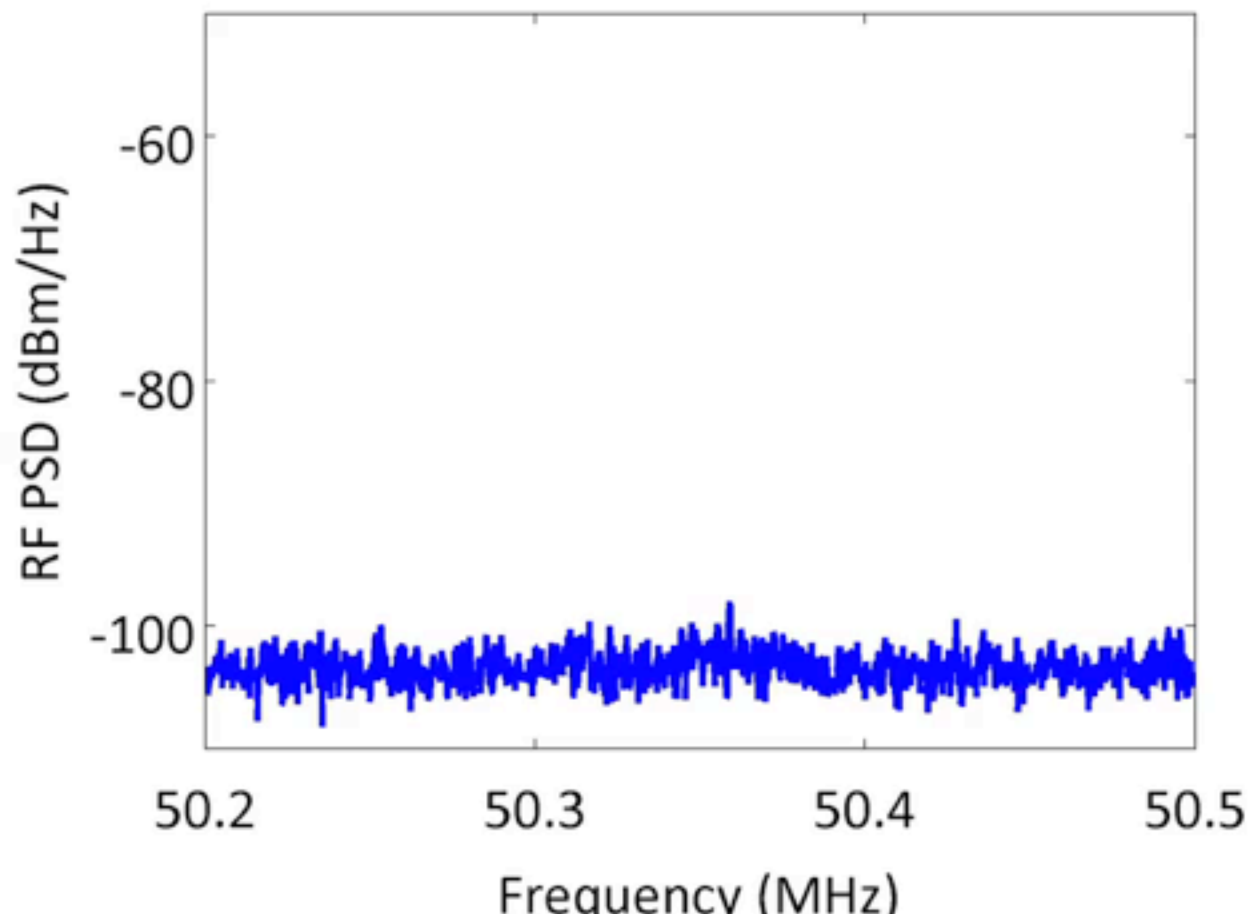
Symmetric mechanical mode

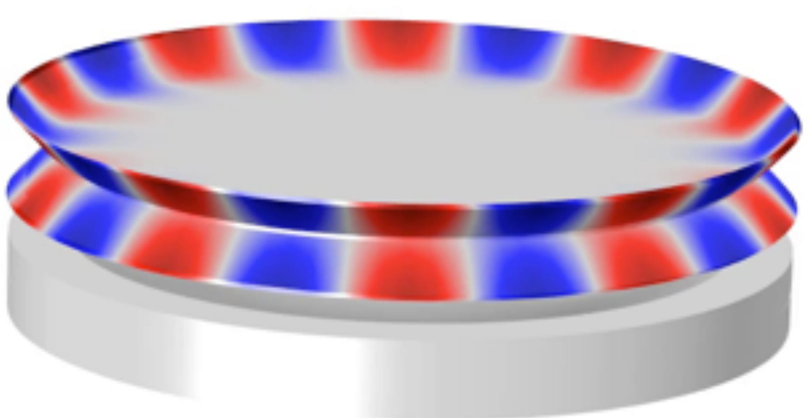


Anti-symmetric mechanical mode

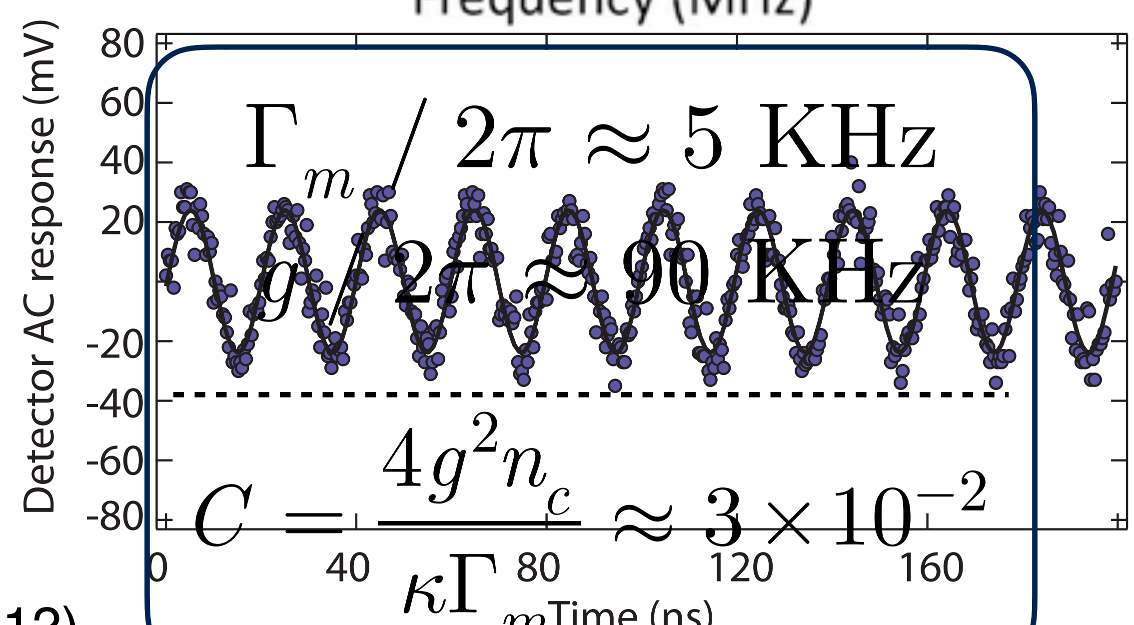
Double-disk Limit cycle







$$f = 50.35\ \mathrm{MHz}$$



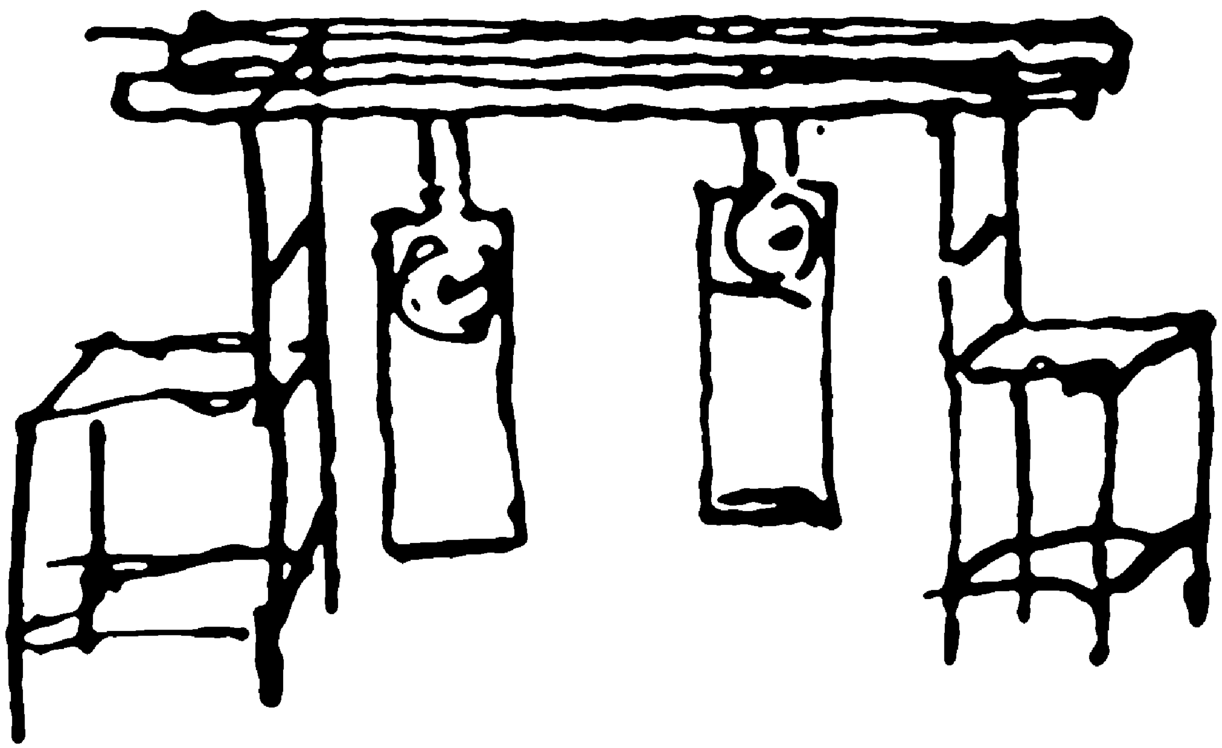
$$\Gamma_m / 2\pi \approx 5\ \mathrm{KHz}$$

$$g / 2\pi \approx 90\ \mathrm{KHz}$$

$$C = \frac{4g^2 n_c}{\kappa \Gamma_m} \approx 3 \times 10^{-2}$$

Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Synchronization of Oscillators



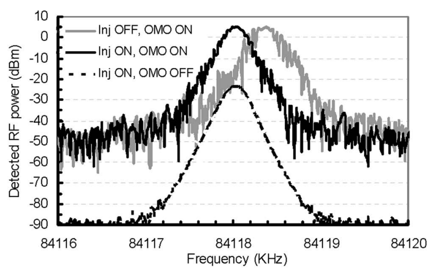
"It is quite worth noting that when we suspended two clocks so constructed from two hooks imbedded in the same wooden beam, the motions of each pendulum in opposite swings were so much in agreement that they never receded the least bit from each other and the sound of each was always heard simultaneously..."

"Further, if this agreement was disturbed by some interference, it reestablished itself in a short time."

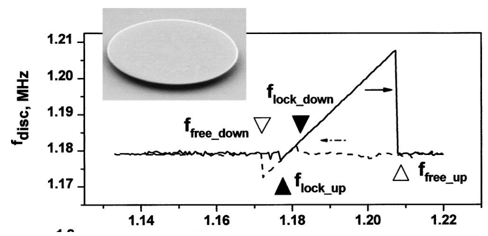
Christiaan Huygens, 26 February 1665.

1. Pikovsky, A., Rosenblum, M. & Kurths, J. *Synchronization.* (Cambridge University Press: 2003).

Synchronization at the nanoscale



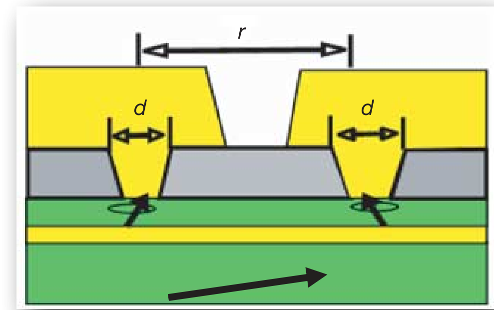
Hossein-Zadeh et al
APL 2008
Caltech



Zalalutdinov
APL 2003
Cornell

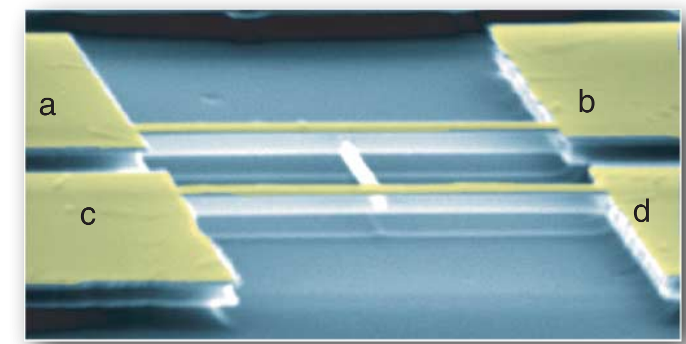
Entrainment by external force

Kaka, M. R. et al Nature 2005.



Spin-torque oscillators
(NIST)

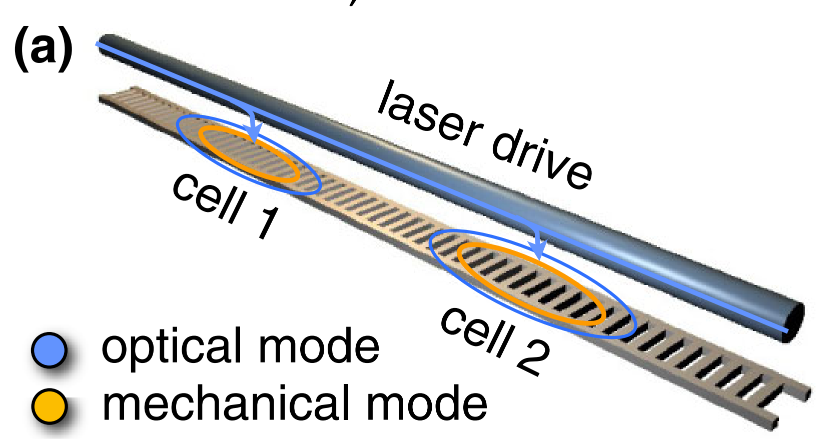
Shim, et al. Science 2007.



MEMS (U. Boston)

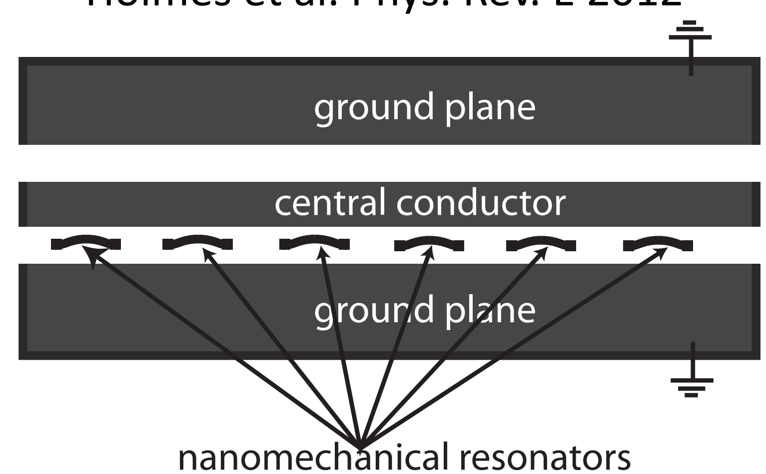
Mutual synchronization

G. Heinrich, et al. PRL 2011.



MPL

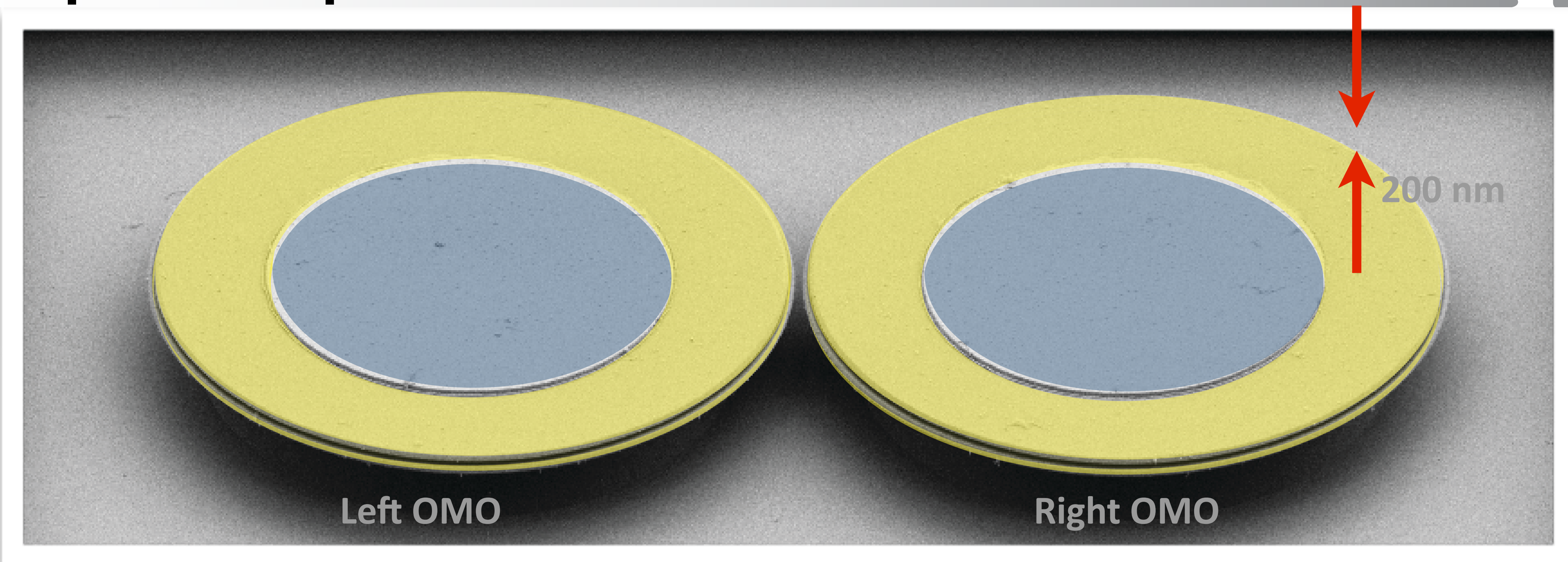
Holmes et al. Phys. Rev. E 2012

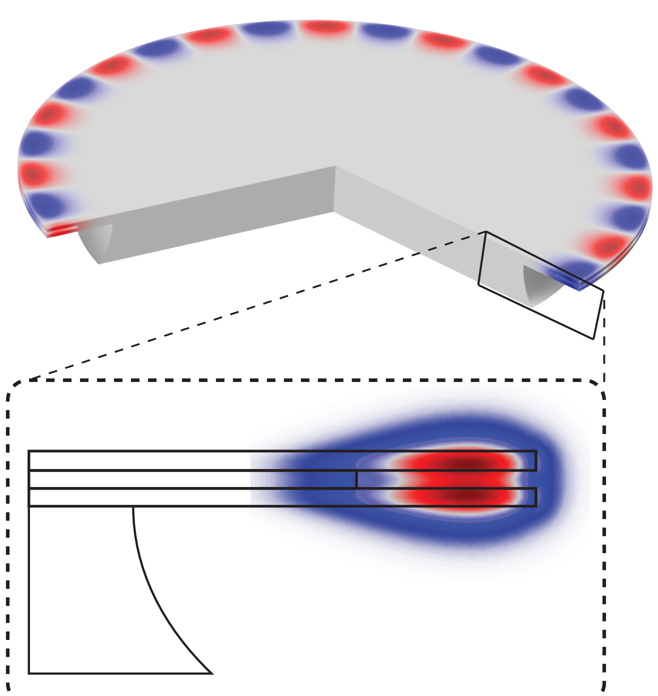


Queensland

Theory: Mutual synchronization with Nanomechanical oscillators

Coupled Optomechanical Oscillators

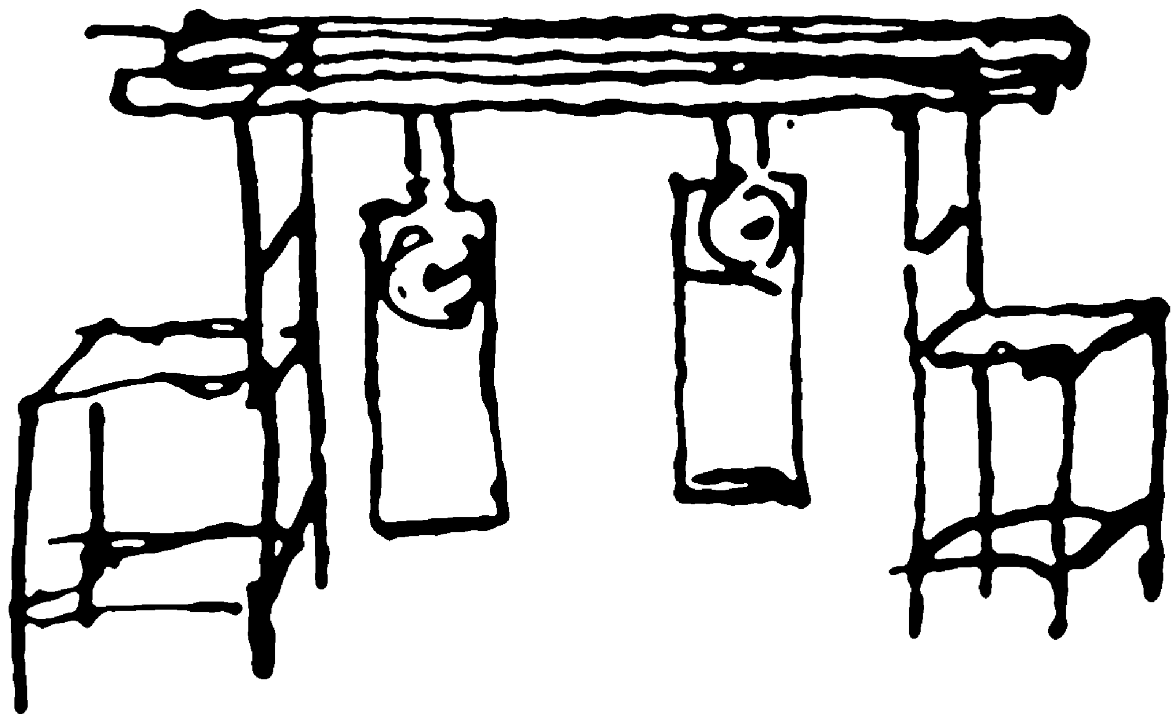


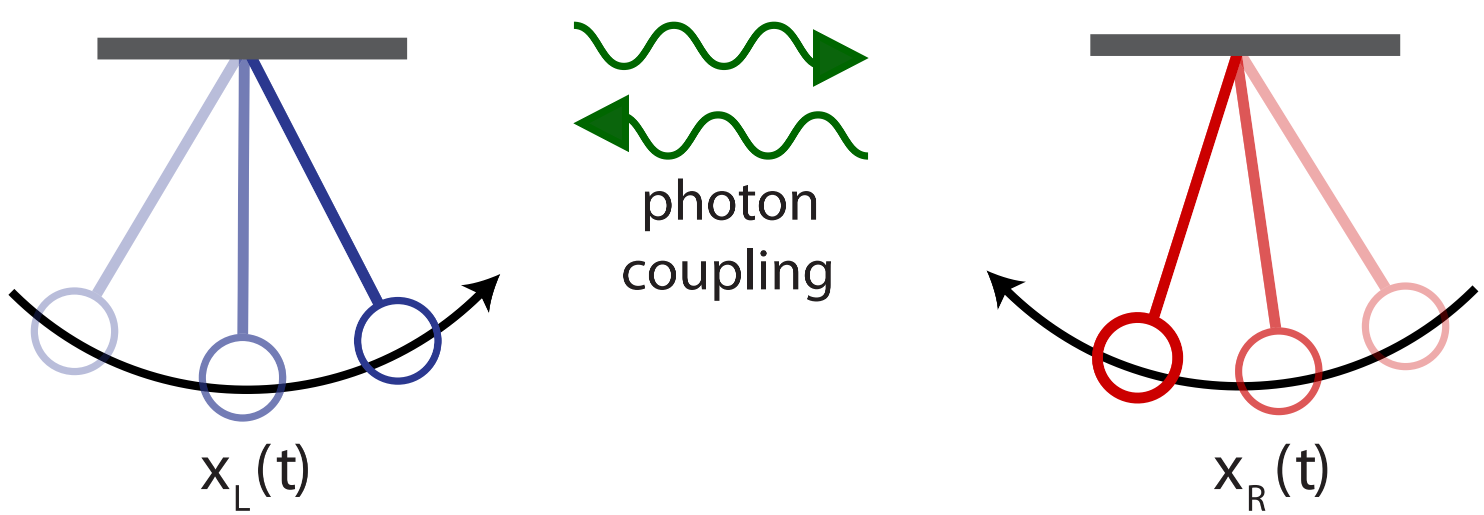


Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Synchronization of Oscillators

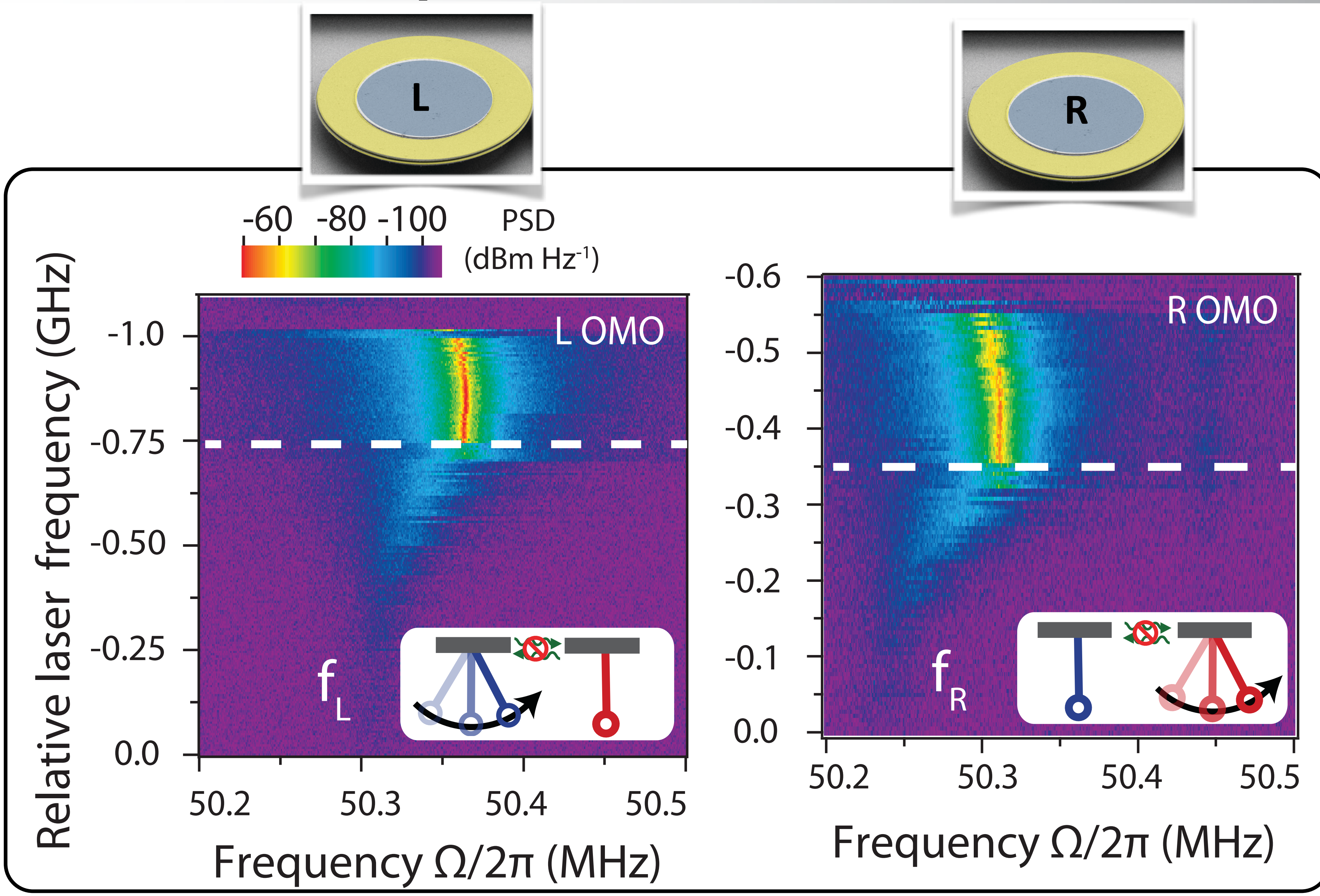
Huygens, 1665

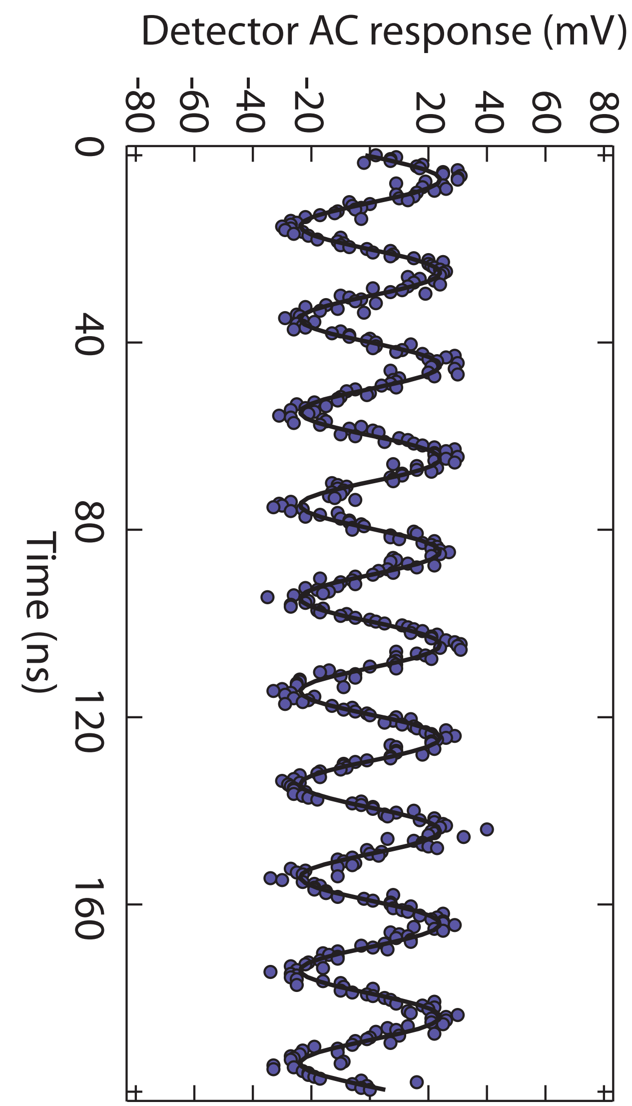




Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Individual Optomechanical Oscillations

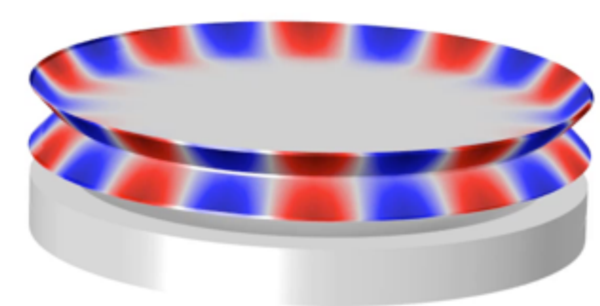




$f_L = 50.35$ MHz

$f_L = 50.30$ MHz

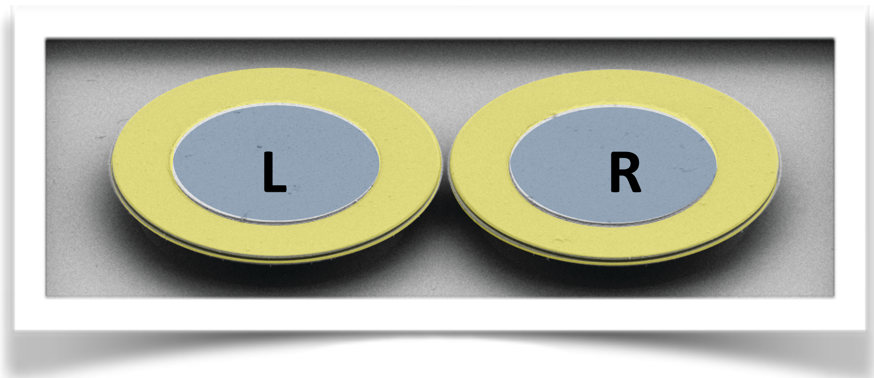
$\delta f \approx 50$ KHz

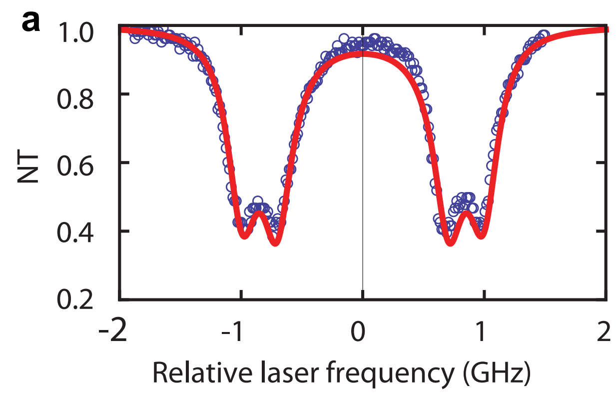


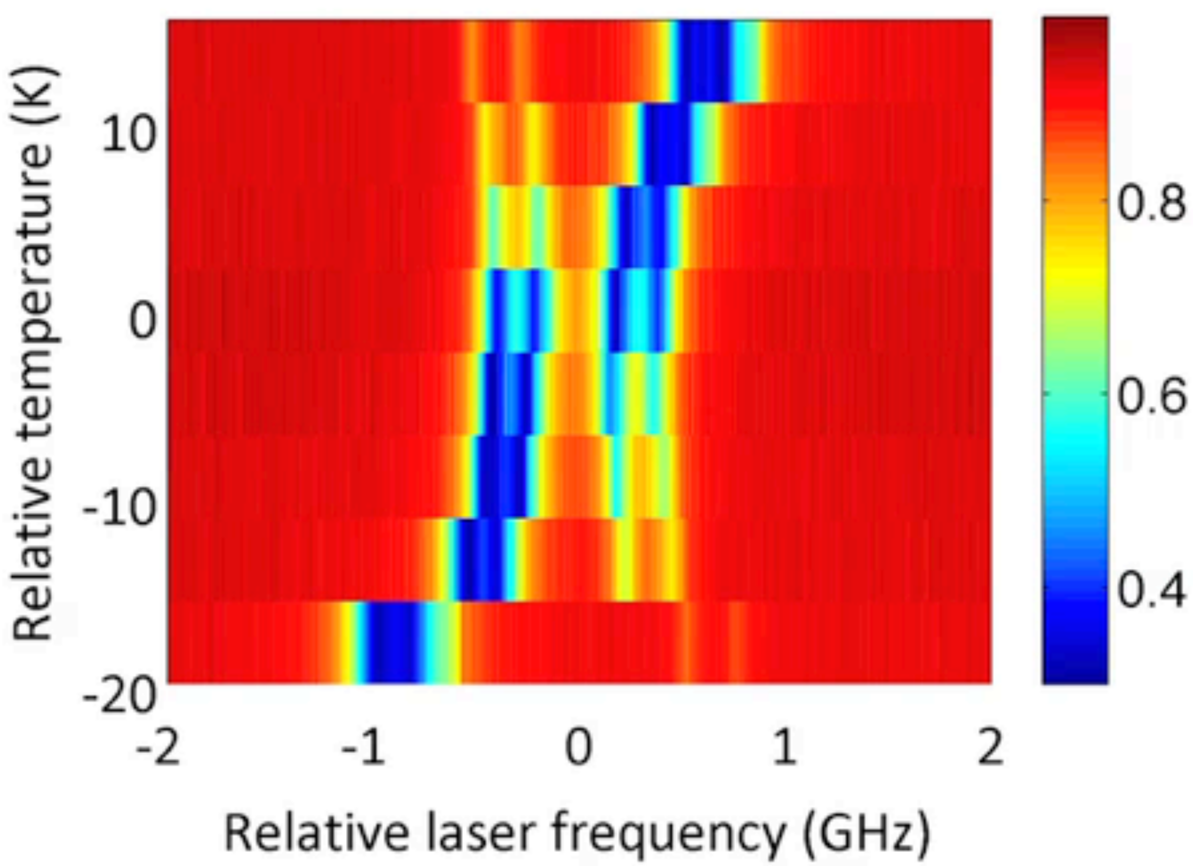
50 MHz mechanical mode

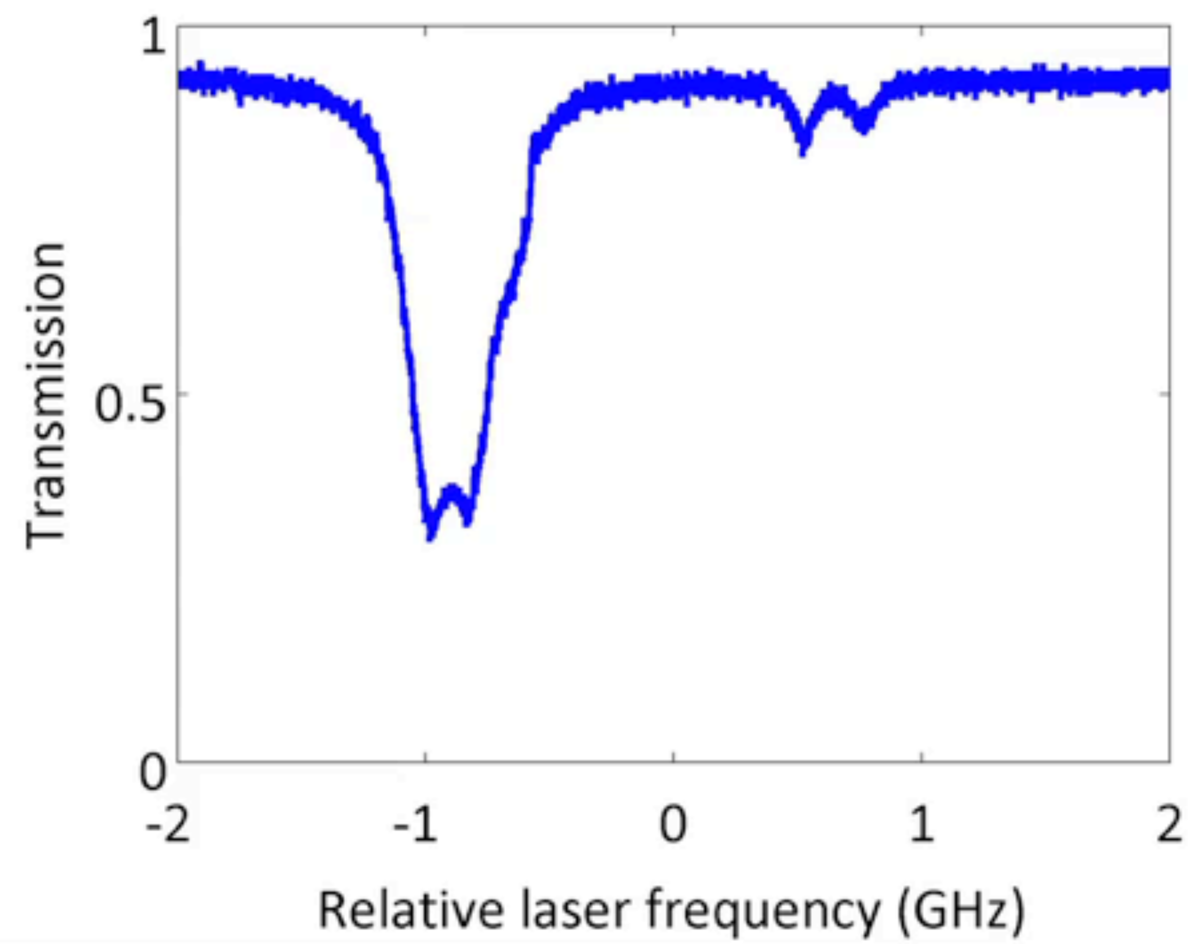
Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Tuning the Optical Coupling



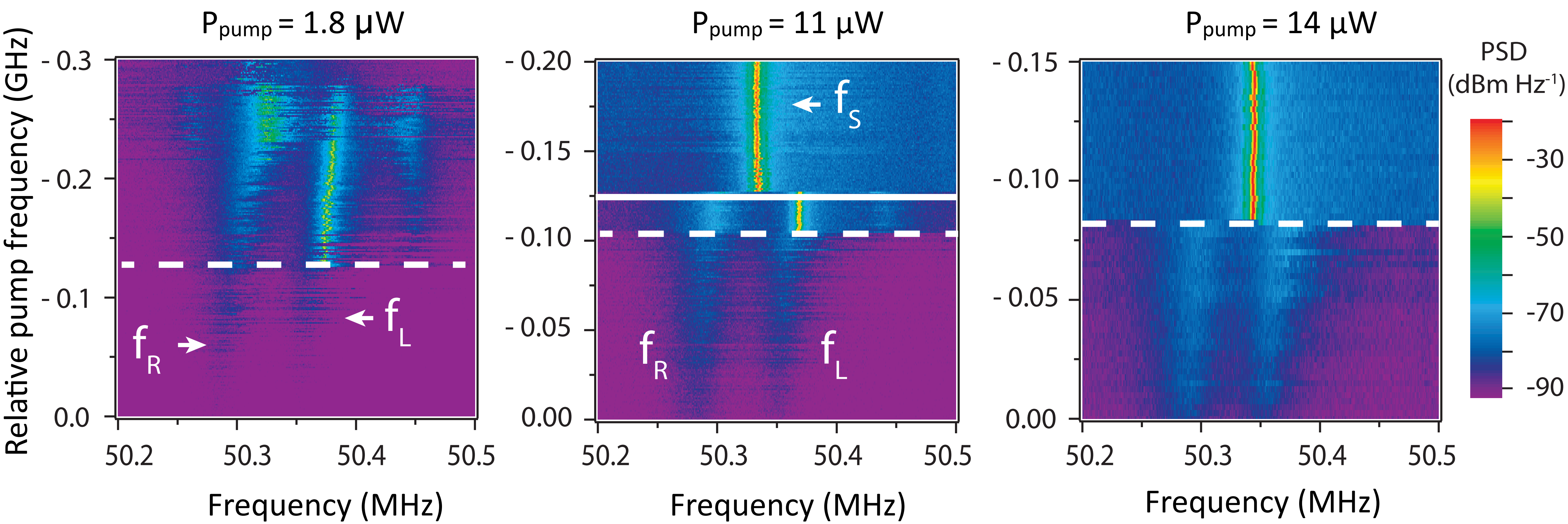


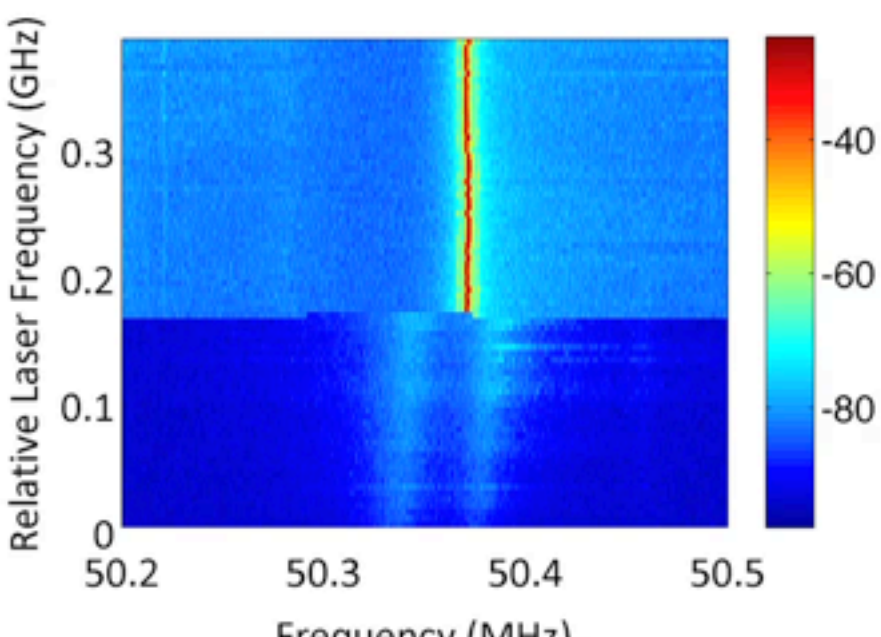


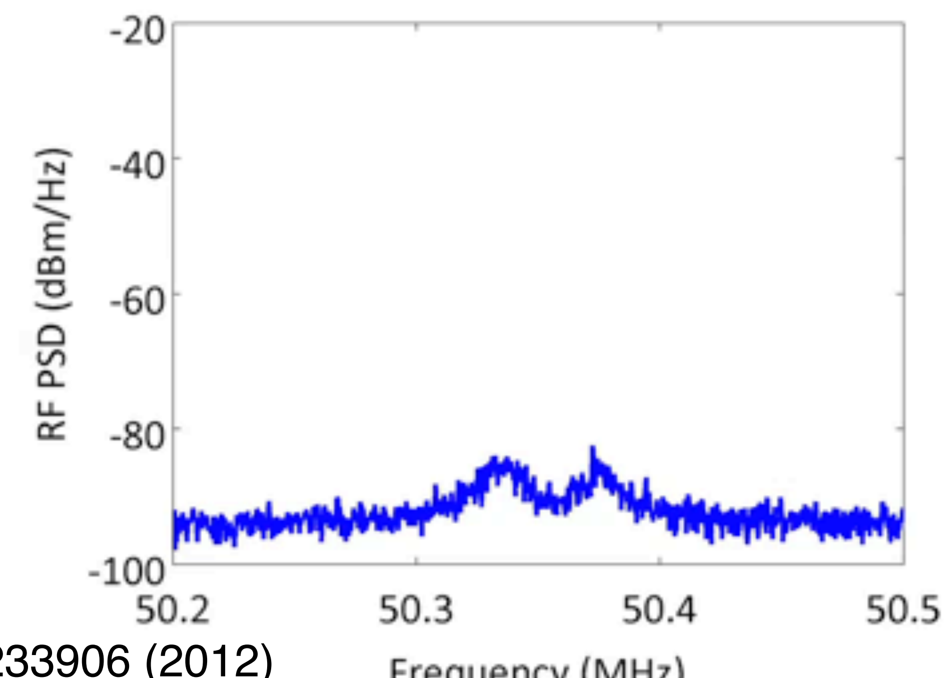


Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Synchronization







Mechanical synchronization through optical field

Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Optically-induced mechanical coupling

$$\ddot{x}_1 + \gamma(n_p, \Delta)\,\dot{x}_1 + \Omega^2(n_p, \Delta)\,x_1 = -k_{\text{in}} x_2 + k_{\text{quad}} \dot{x}_2$$

Optomechanical damping — Optical Spring

$$\ddot{x}_2 + \gamma(n_p, \Delta)\,\dot{x}_2 + \Omega^2(n_p, \Delta)\,x_2 = -k_{\text{in}} x_1 + k_{\text{quad}} \dot{x}_1$$

Optomechanical damping — Optical Spring

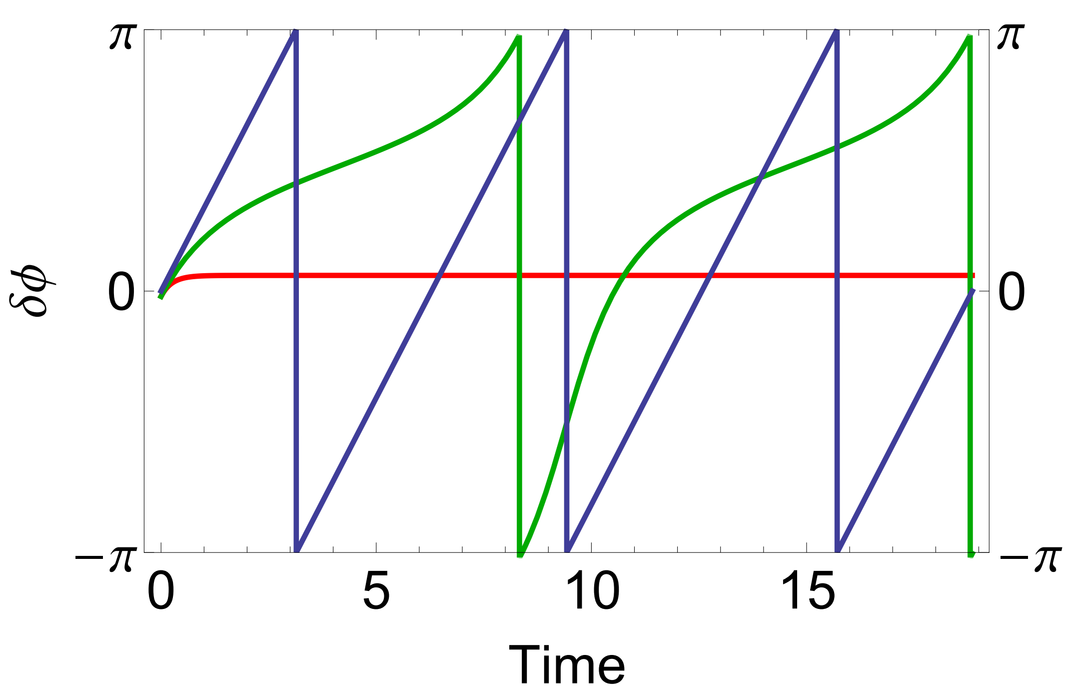
Linearized coupling

Kuramoto model

$$\Rightarrow \delta\dot{\phi} = \delta\Omega - 2k_{in} \sin(\delta\phi)$$

Periodic solutions $\delta\Omega > 2k_{in}$

Asymptotic solutions $\delta\Omega < 2k_{in}$

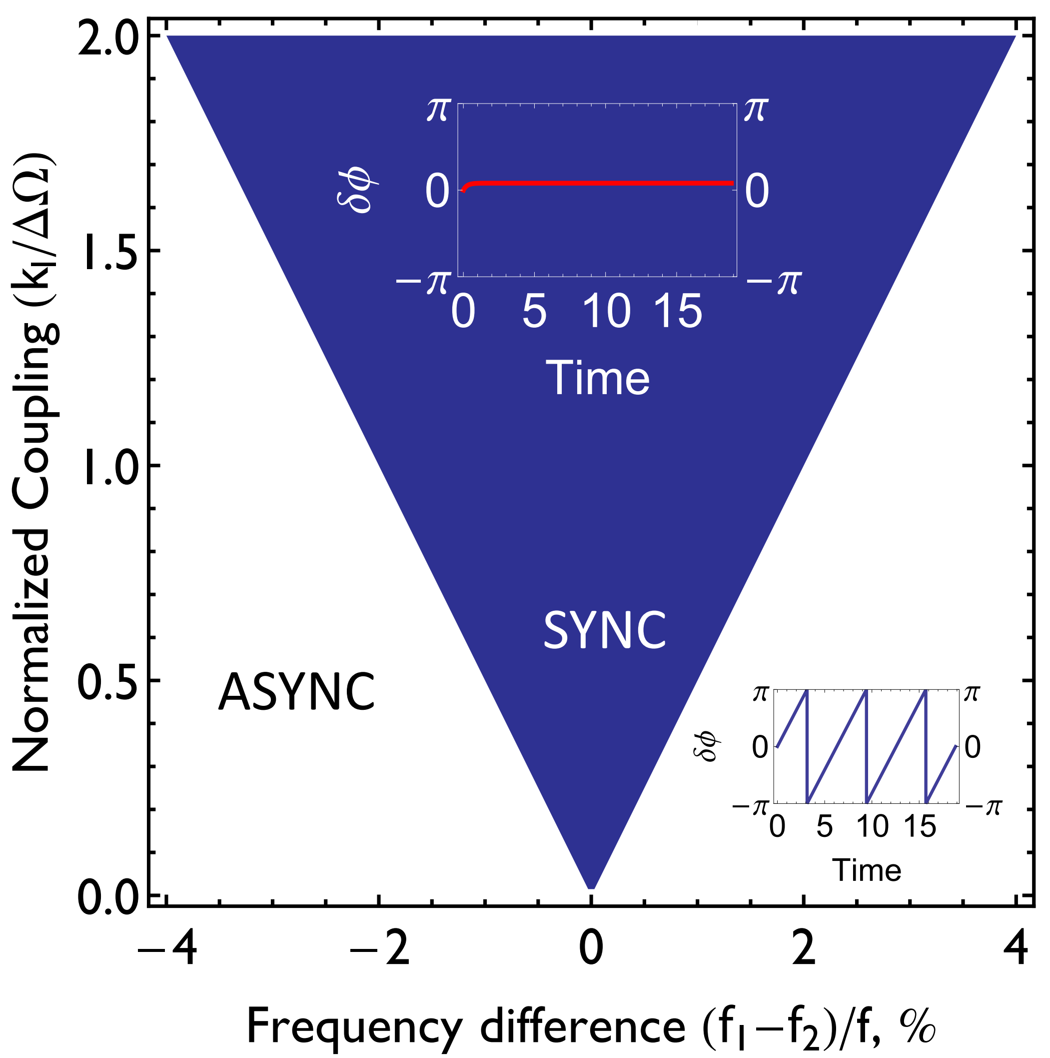


Optically-induced mechanical coupling

Kuramoto model

Periodic solutions	$\delta\Omega > 2k_{in}$
Asymptotic solutions	$\delta\Omega < 2k_{in}$

Arnold Tongue



Optically-induced mechanical coupling

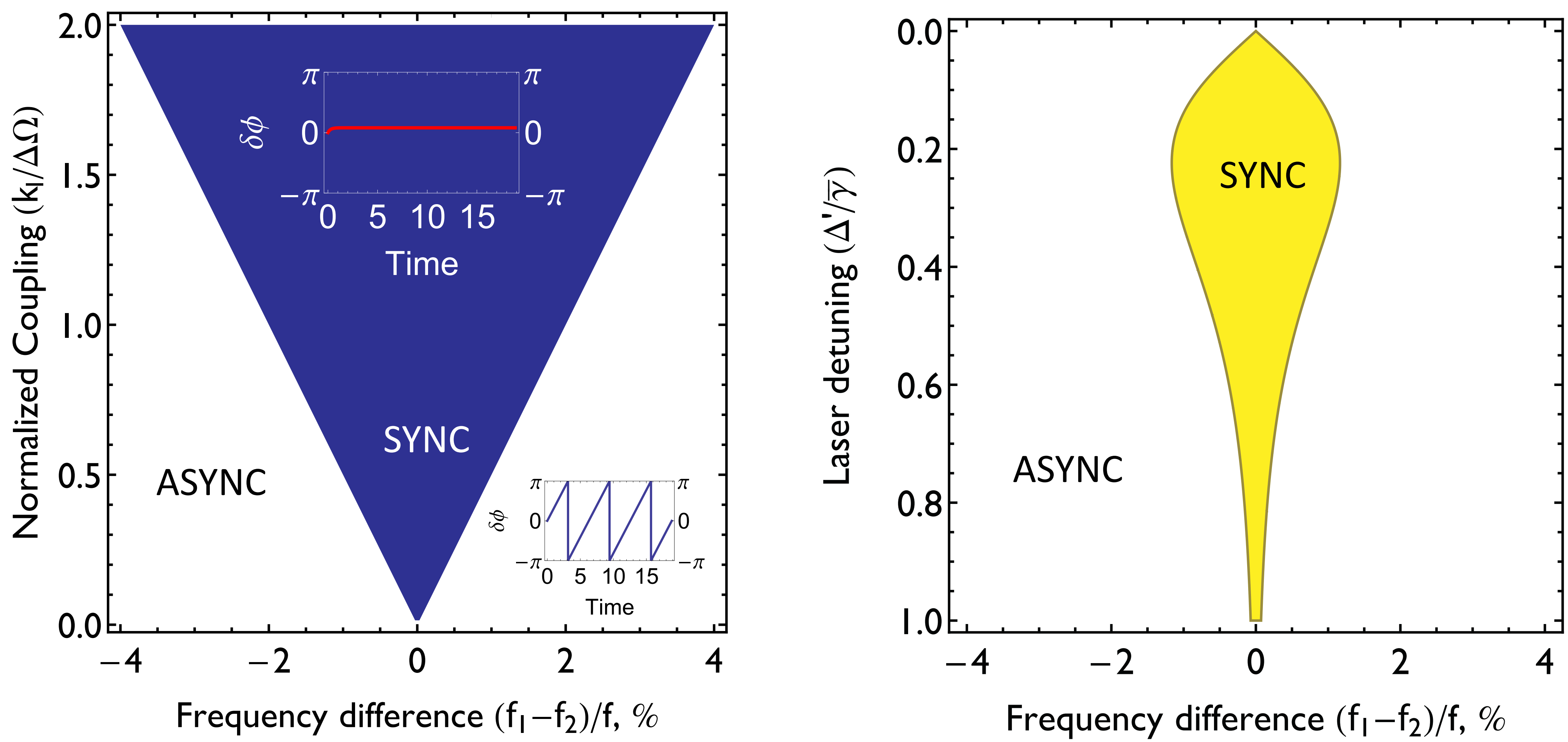
Kuramoto model

Periodic solutions $\delta\Omega > 2k_{in}$

Asymptotic solutions $\delta\Omega < 2k_{in}$

$$k_{in} \propto n_p \frac{\Delta}{((\kappa / 2)^2 + \Delta^2)^2}$$

Arnold Tongue



Optically-induced mechanical coupling

Kuramoto model

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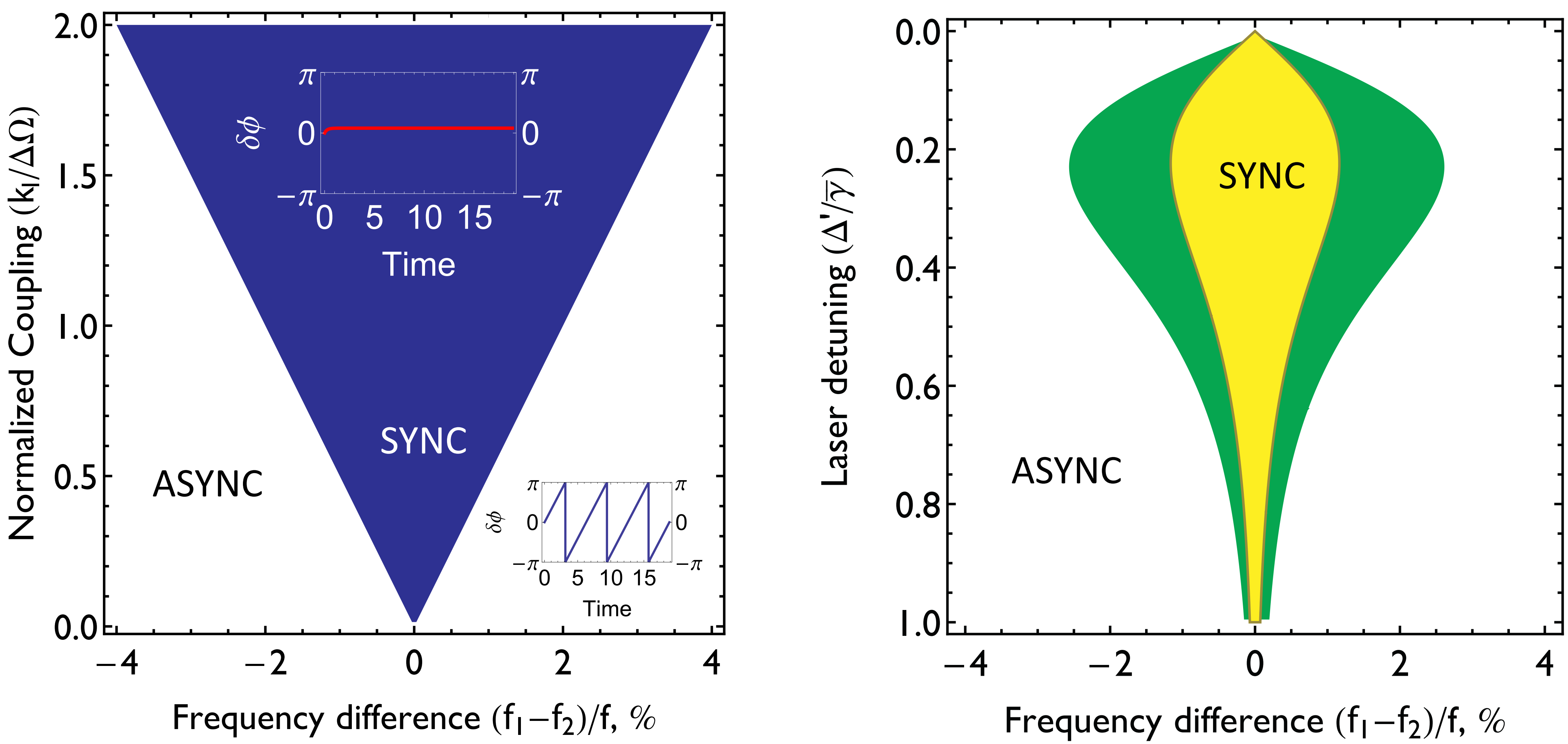

Optically-induced mechanical coupling

Kuramoto model

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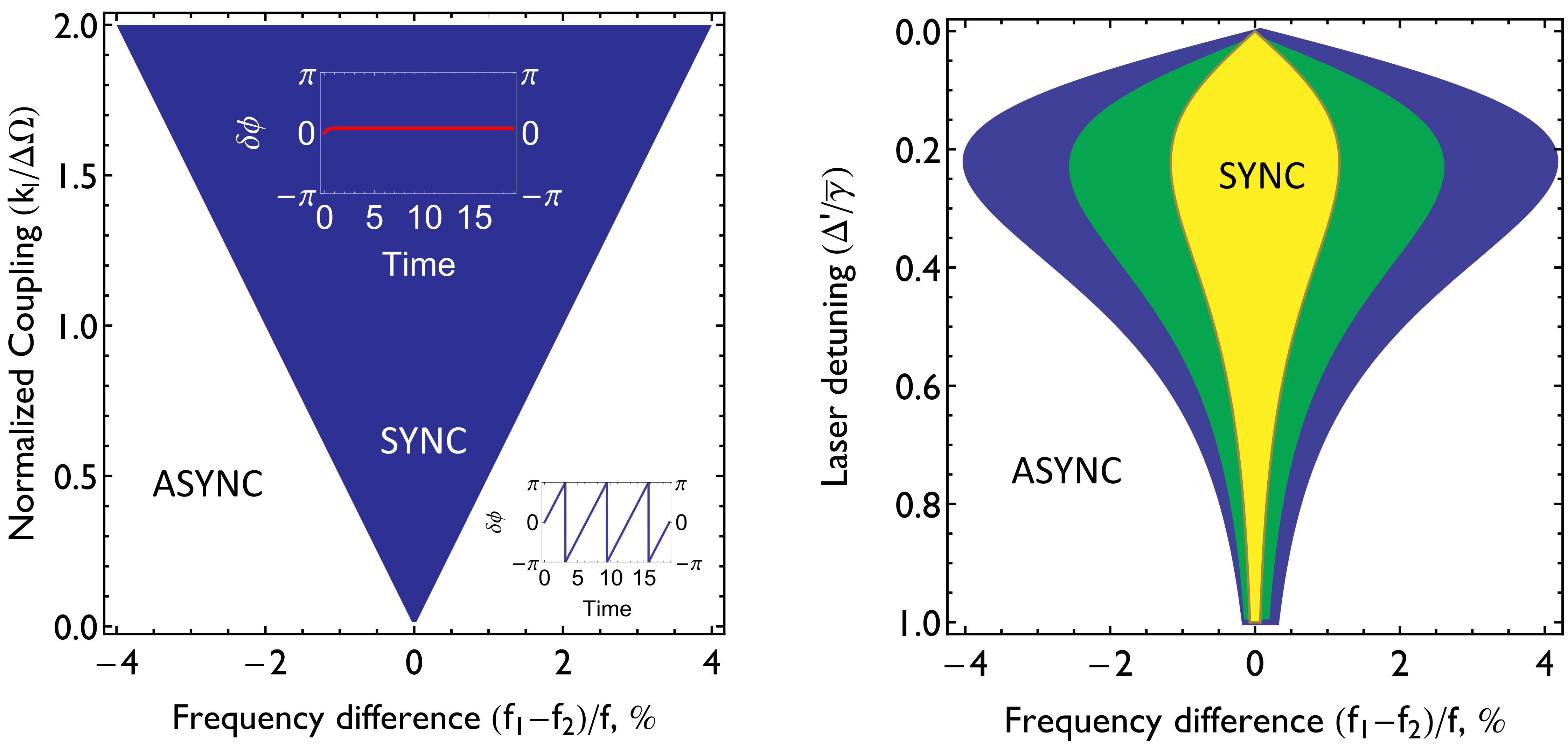
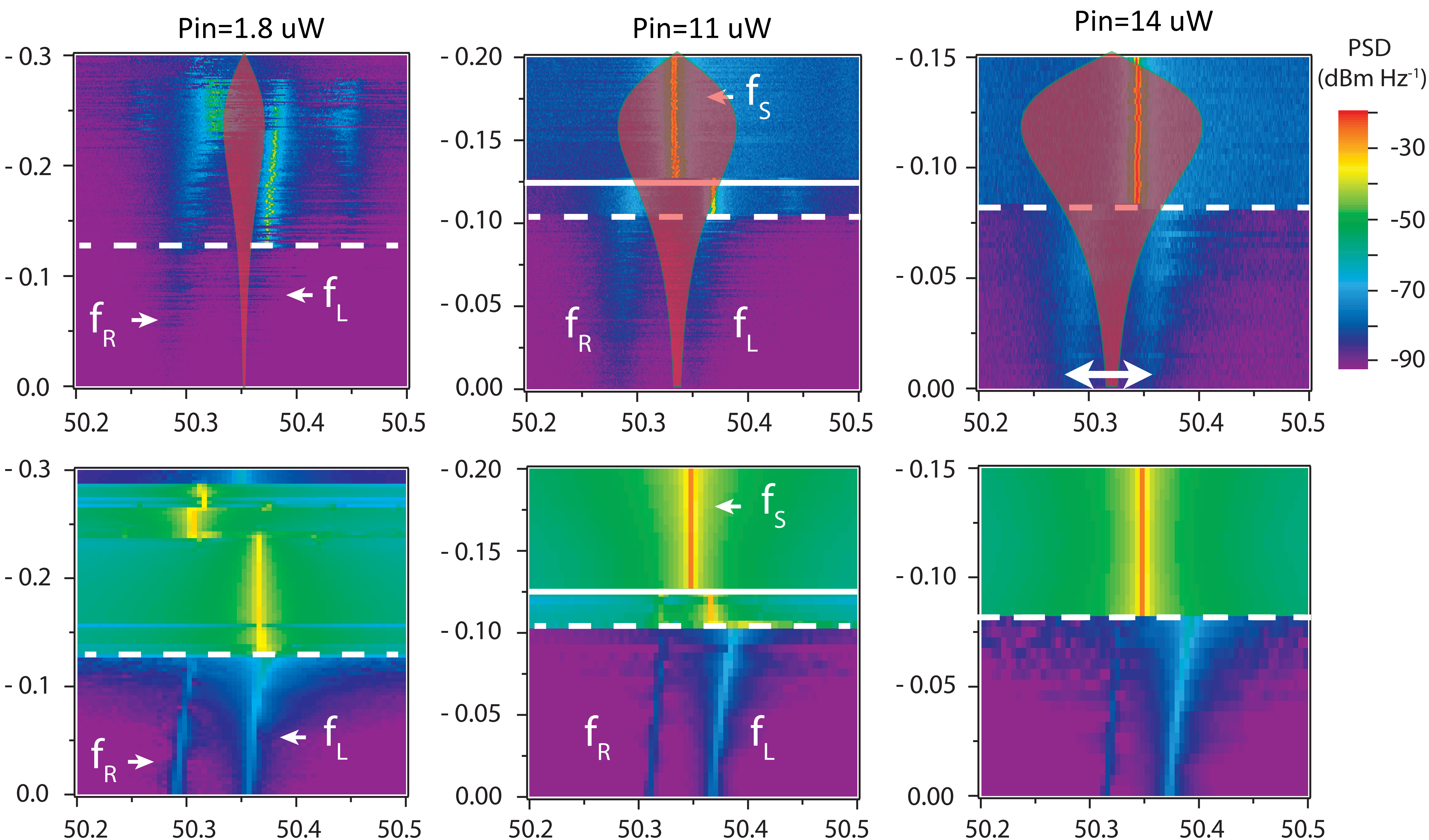
Arnold Tongue

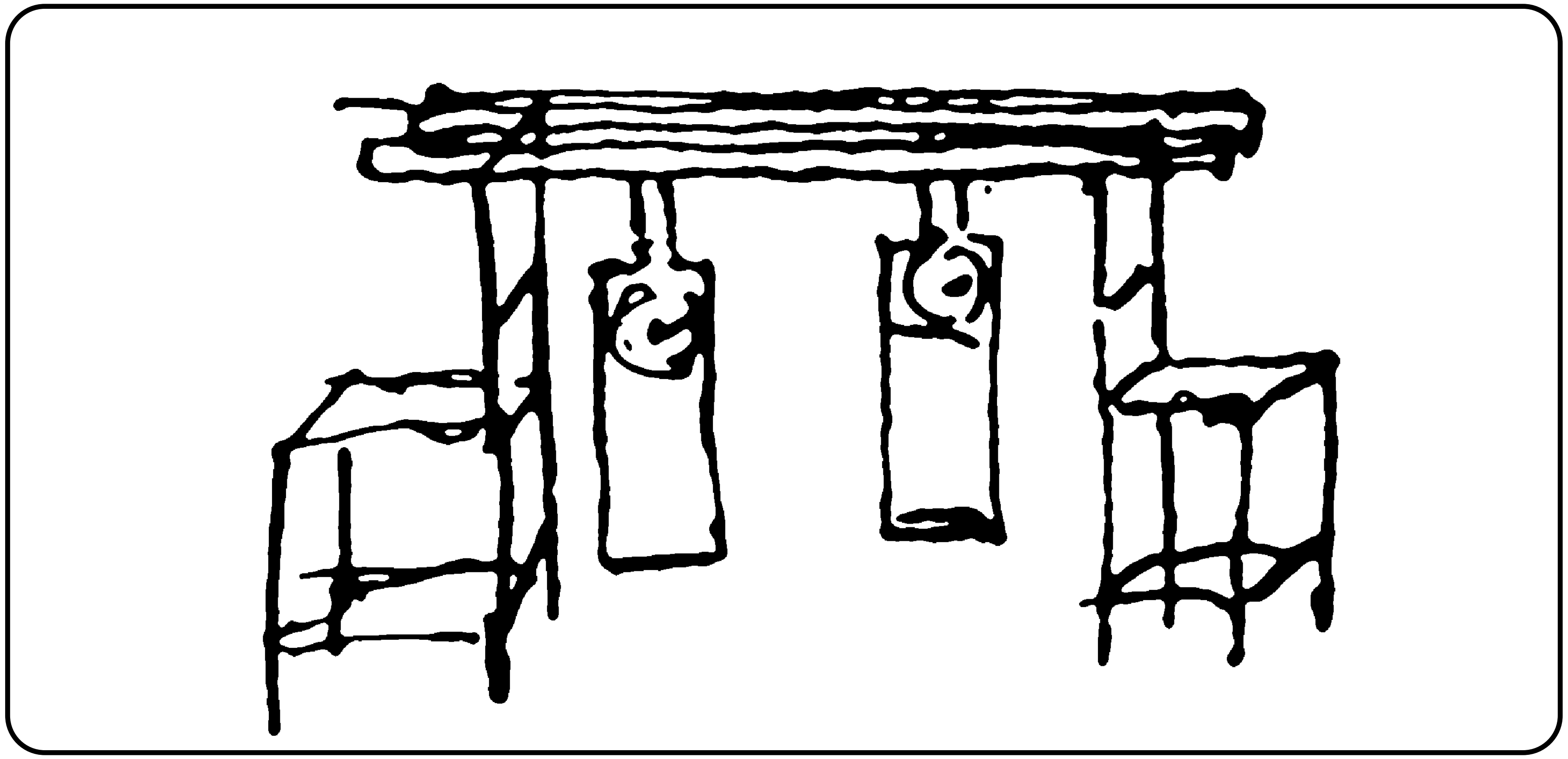


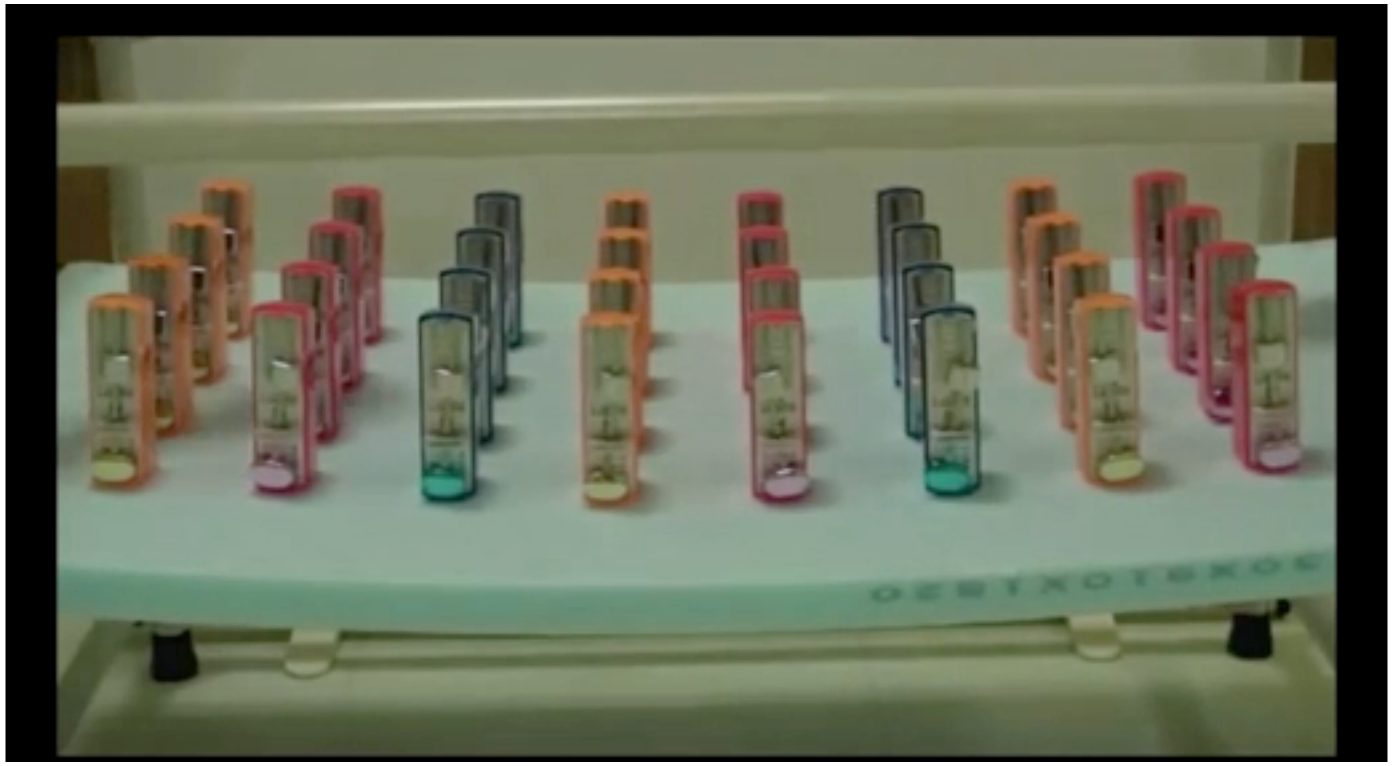
Synchronization



Synchronization of Oscillators

Huygens, 1665

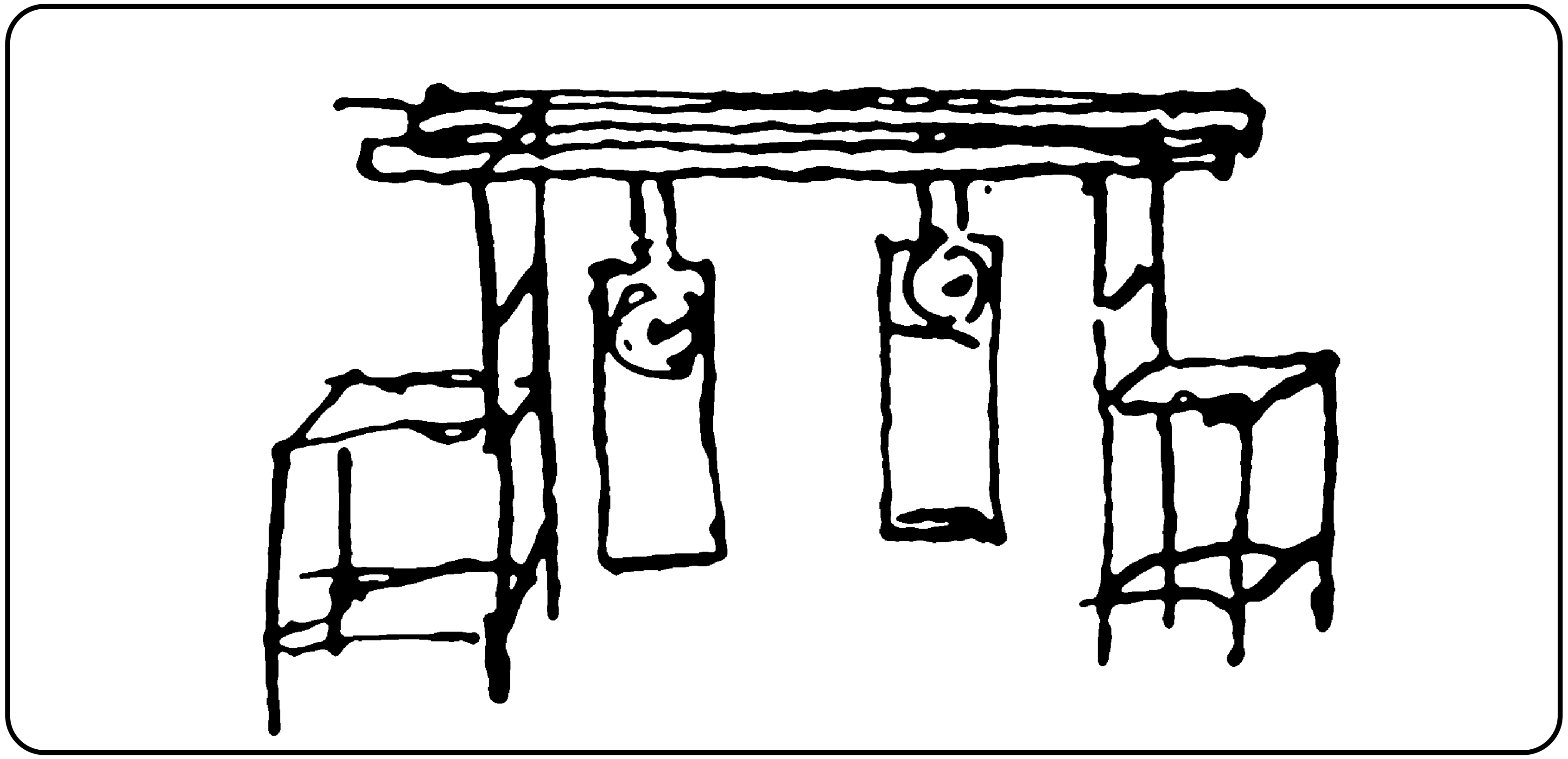


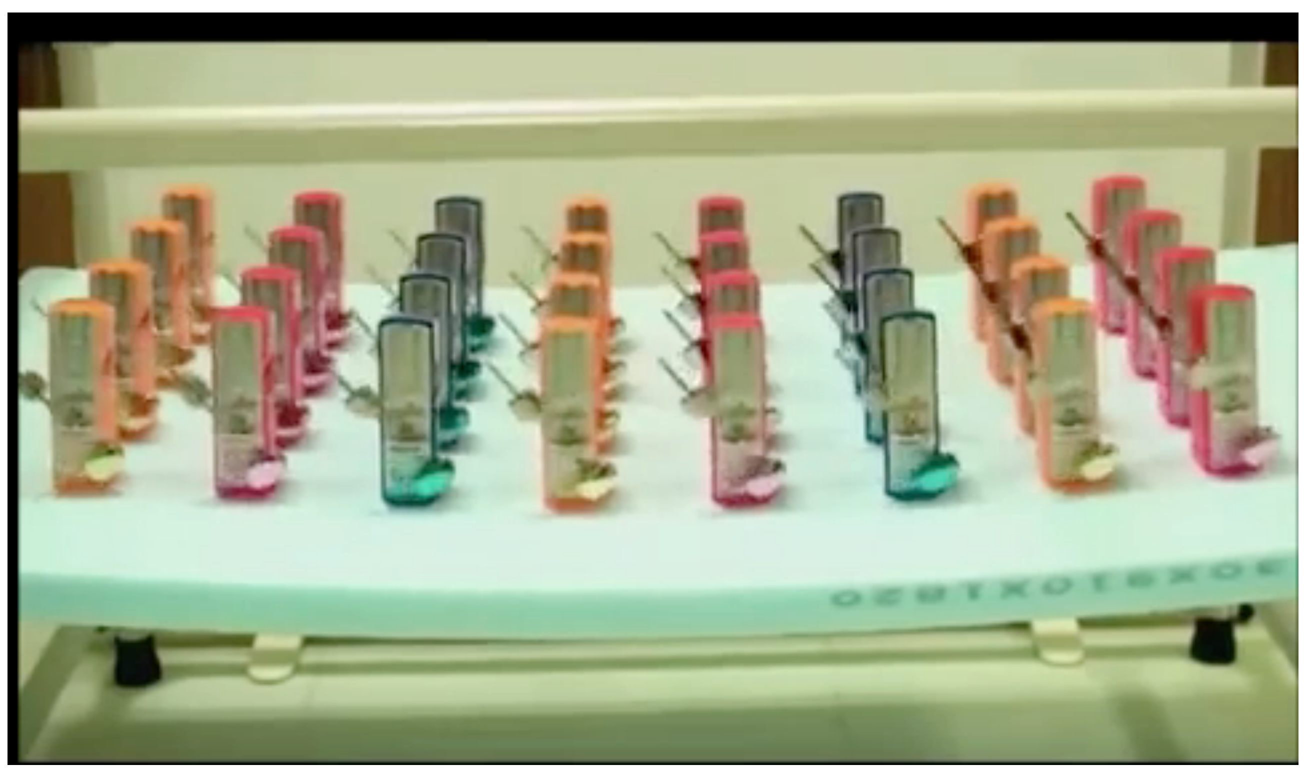


Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Synchronization of Oscillators

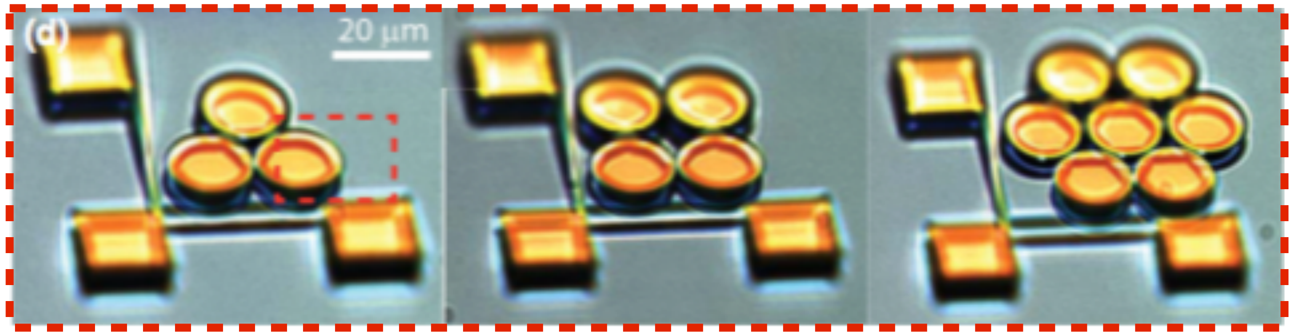
Huygens, 1665

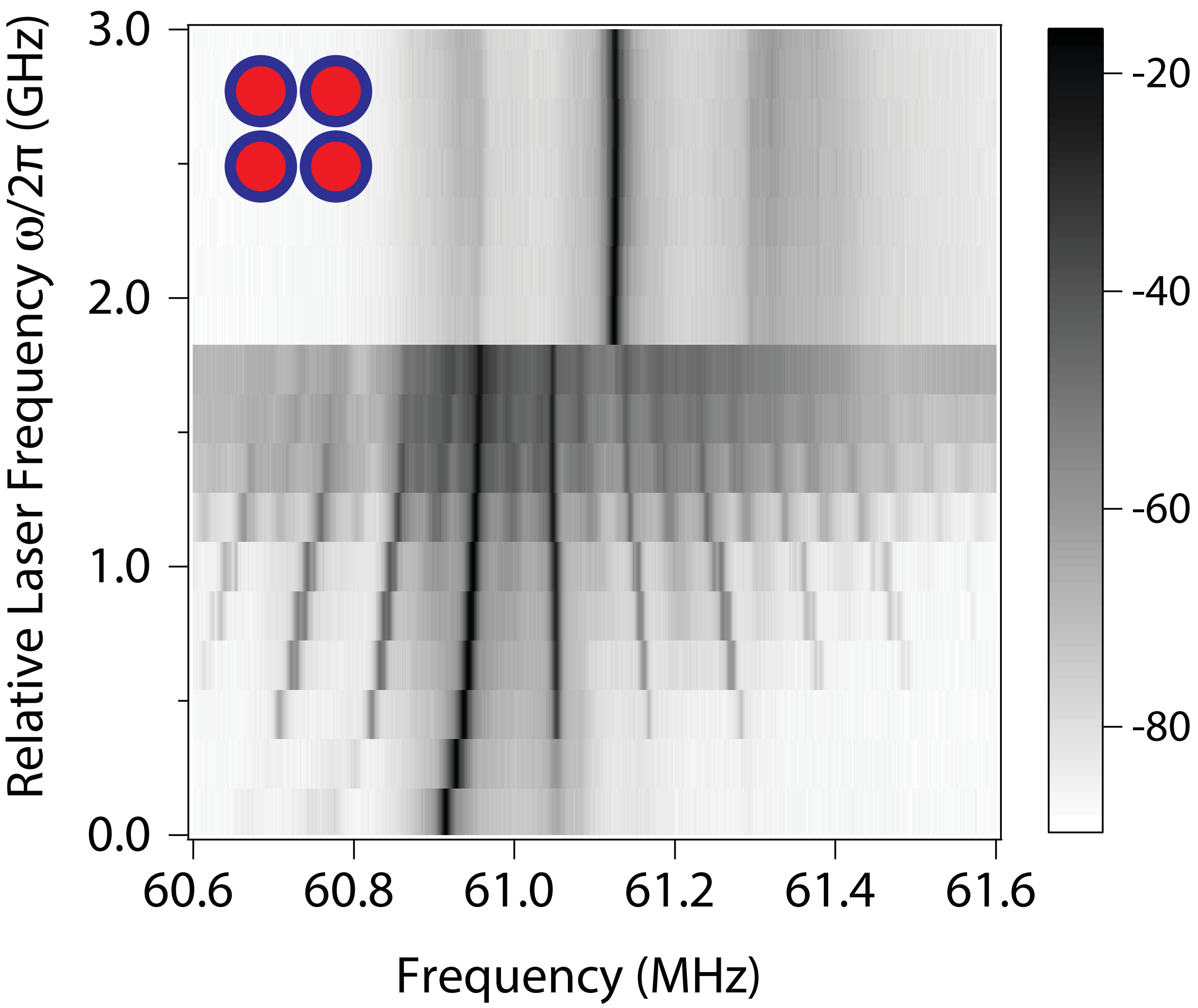


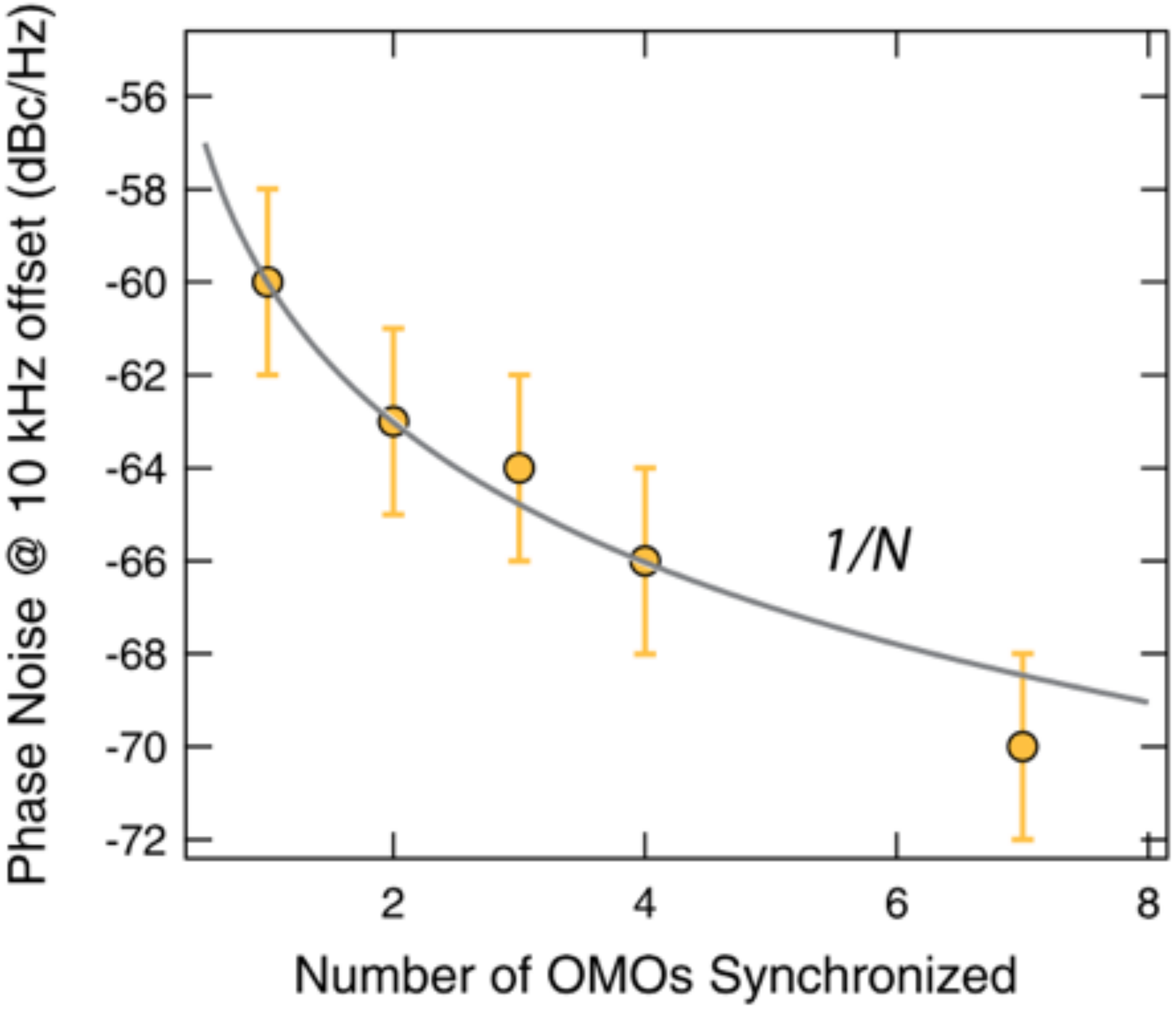


Zhang, Wiederhecker et al. Phys. Rev. Lett. 109, 233906 (2012)

Array Synchronization

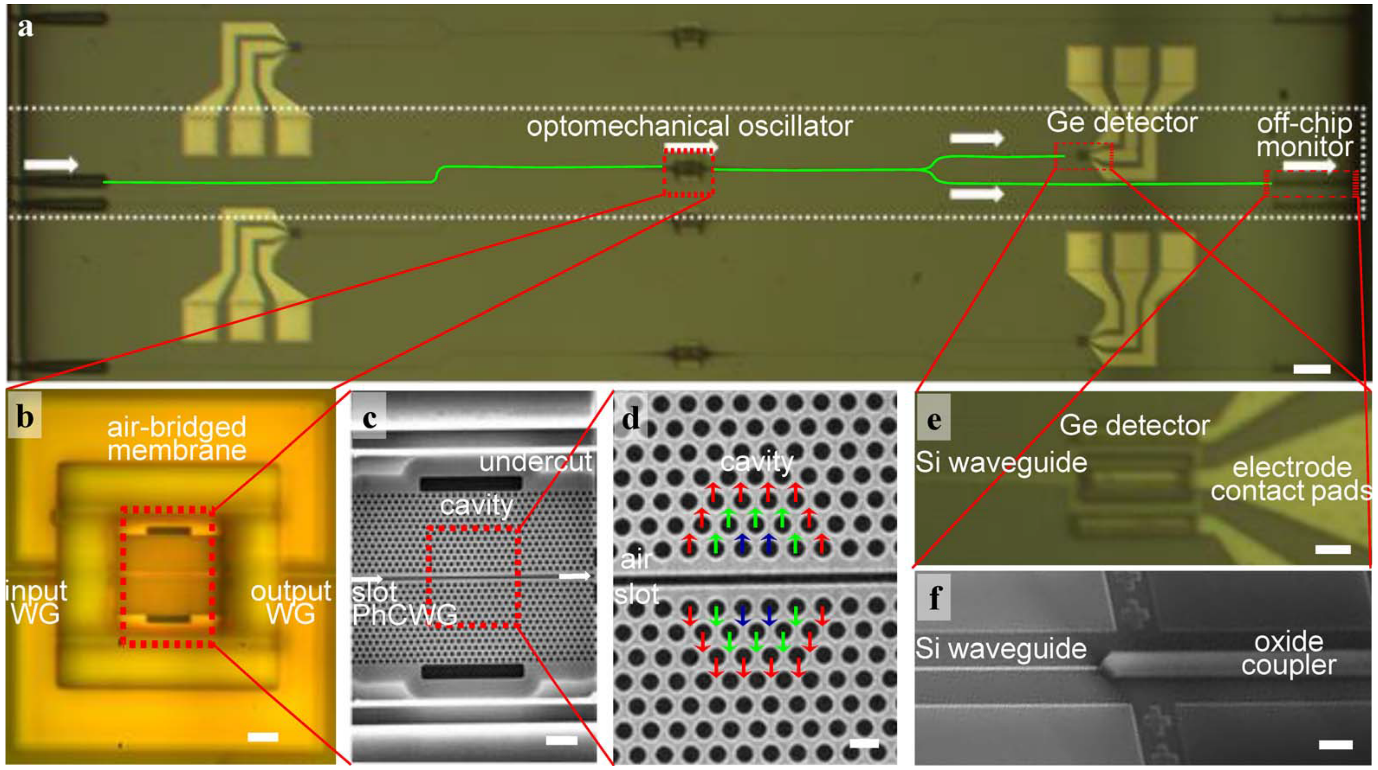






Zhang, M., et al (2015). PRL, 115(16), 163902.
Vahala, K. J. (2008). Physical Review A, 78(2), 023832.

Technological viewpoint: Si Nanophotonics Building Blocks

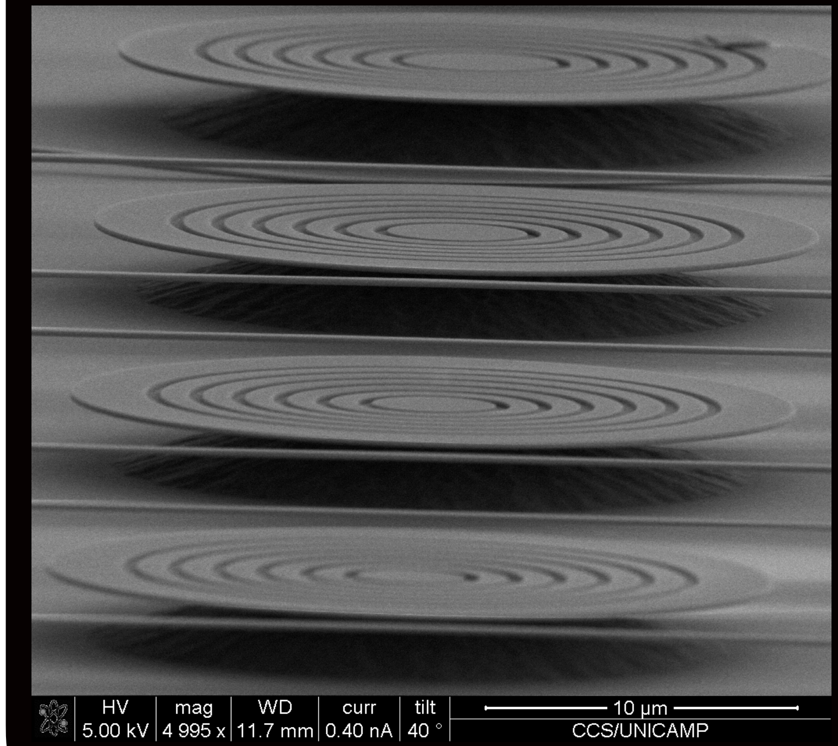


Integrated optomechanical oscillator chipset

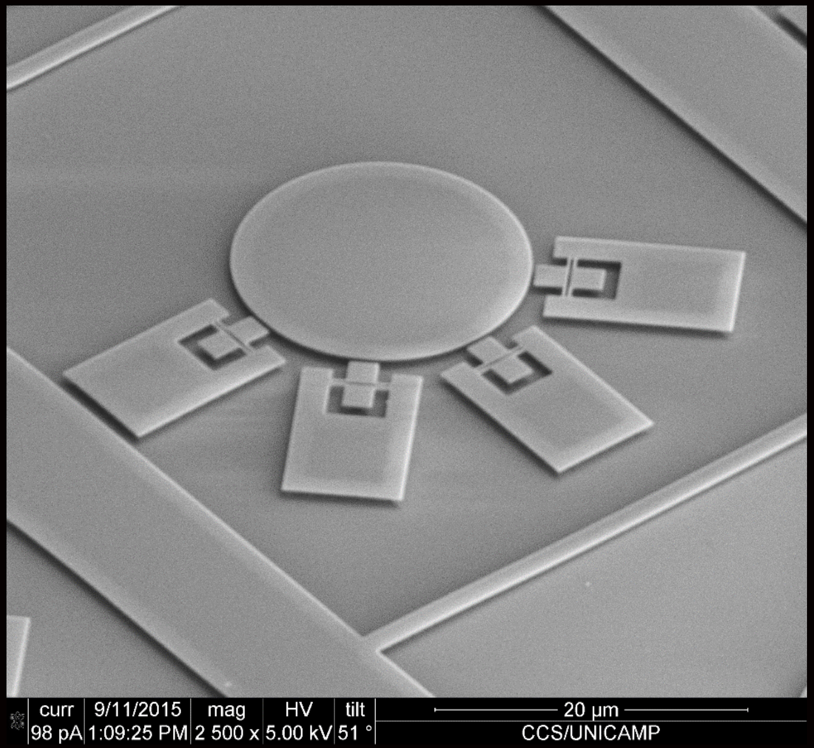
Chee Wei Wong et. al SCIENTIFIC REPORTS 4 : 6842 (2014)

Candidates for OM arrays

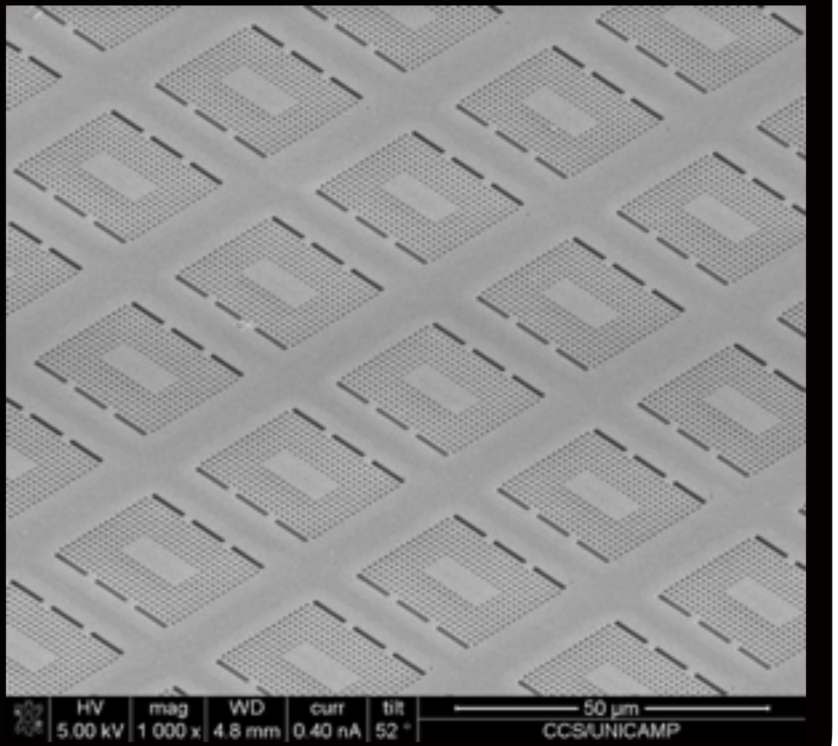
Bulls' eye cavity



Paddle resonators



Optomechanical crystals



Rodrigo Benevides

Gustavo Luiz

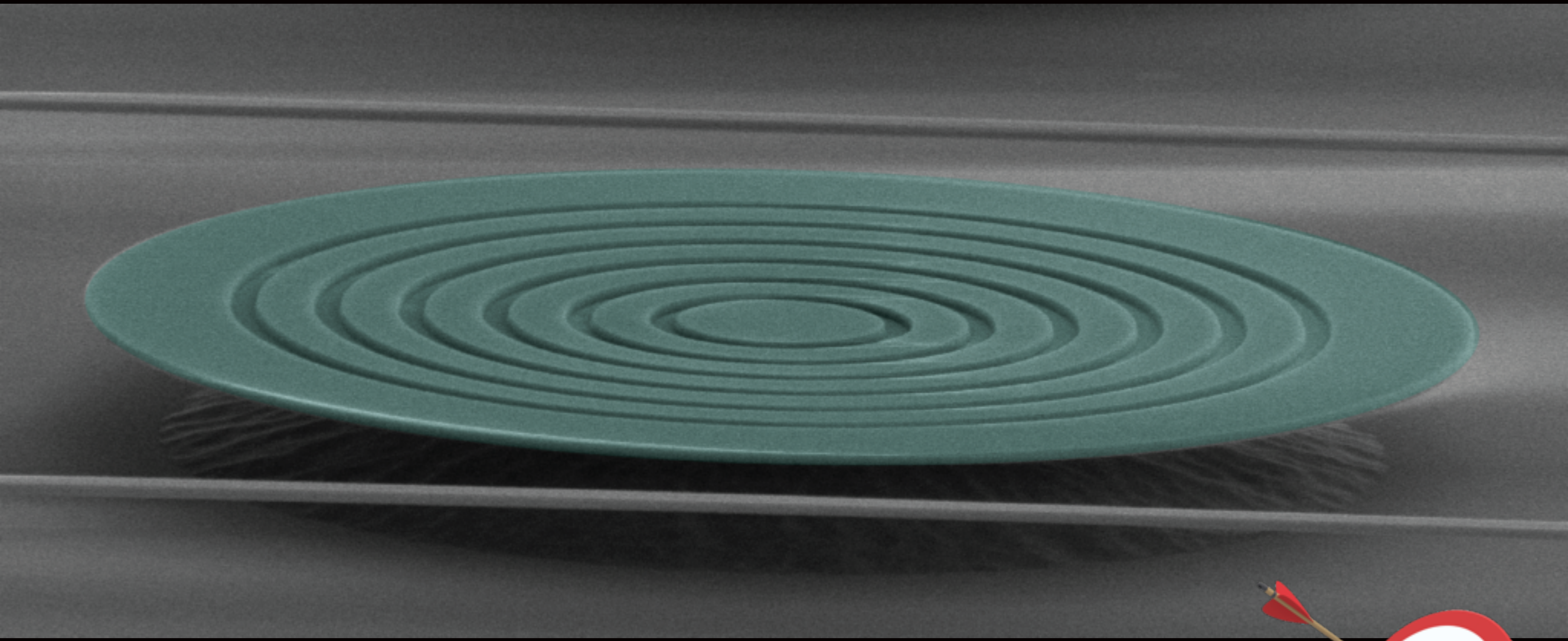
Felipe Santos

Benevides, et al. CLEO 2016
Luiz, et al CLEO 2106
Santos, F. G., et al. arXiv:1605.06318

Outline

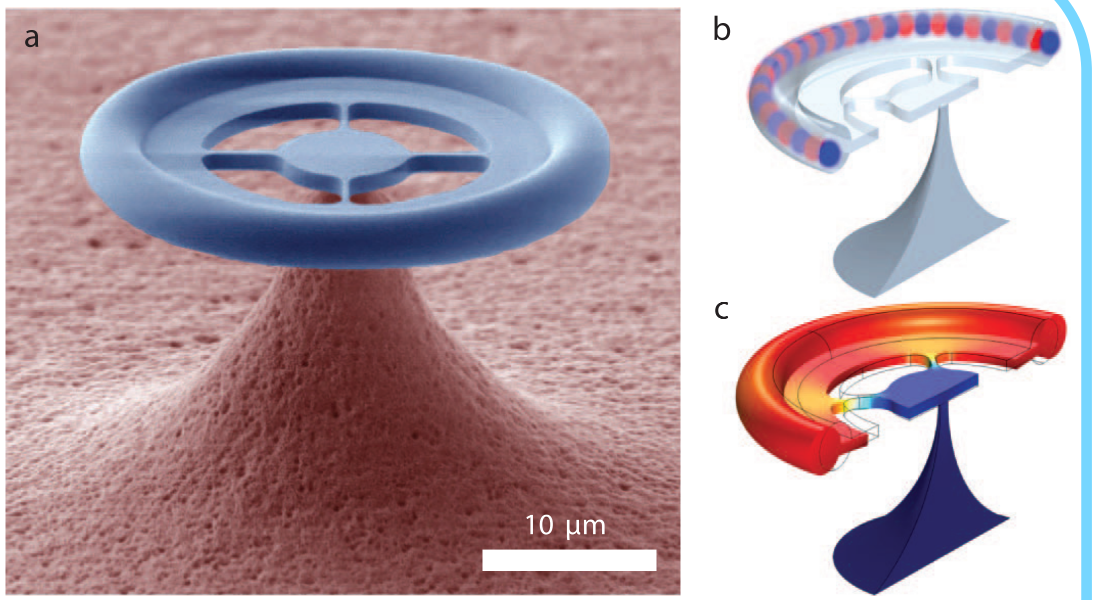
- Optical and acoustic mode interaction
- Optical force actuation
- Dynamical back-action
- Optomechanical clocks
- **Bullseye - a case study**
- Outlook

Bullseye



A new system for cavity optomechanics

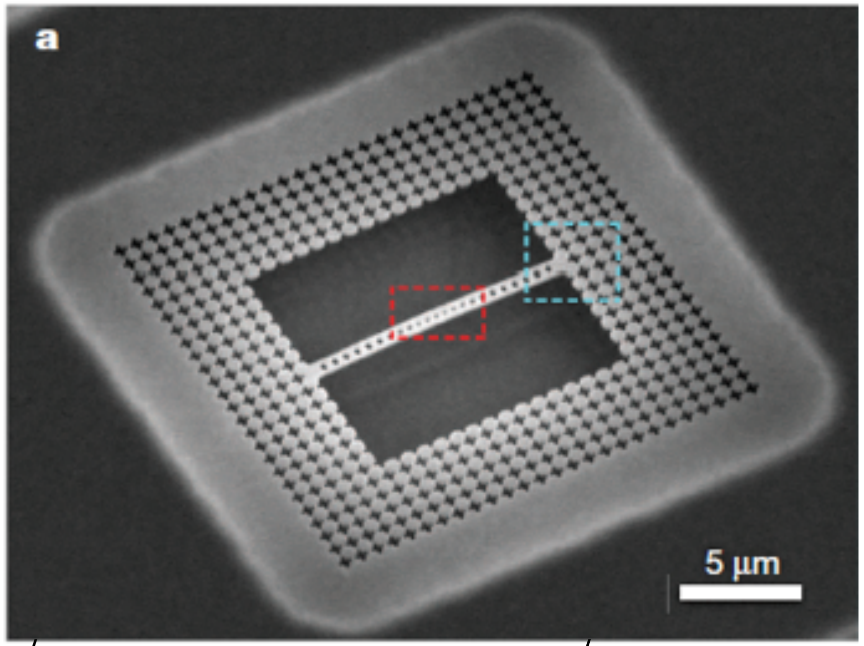
$\kappa \downarrow$



$\kappa_t / 2\pi \approx 10$ MHz $\omega_m / 2\pi \approx 80$ MHz

E. Verhagen et. al, Nature **482**, 63 (2012)

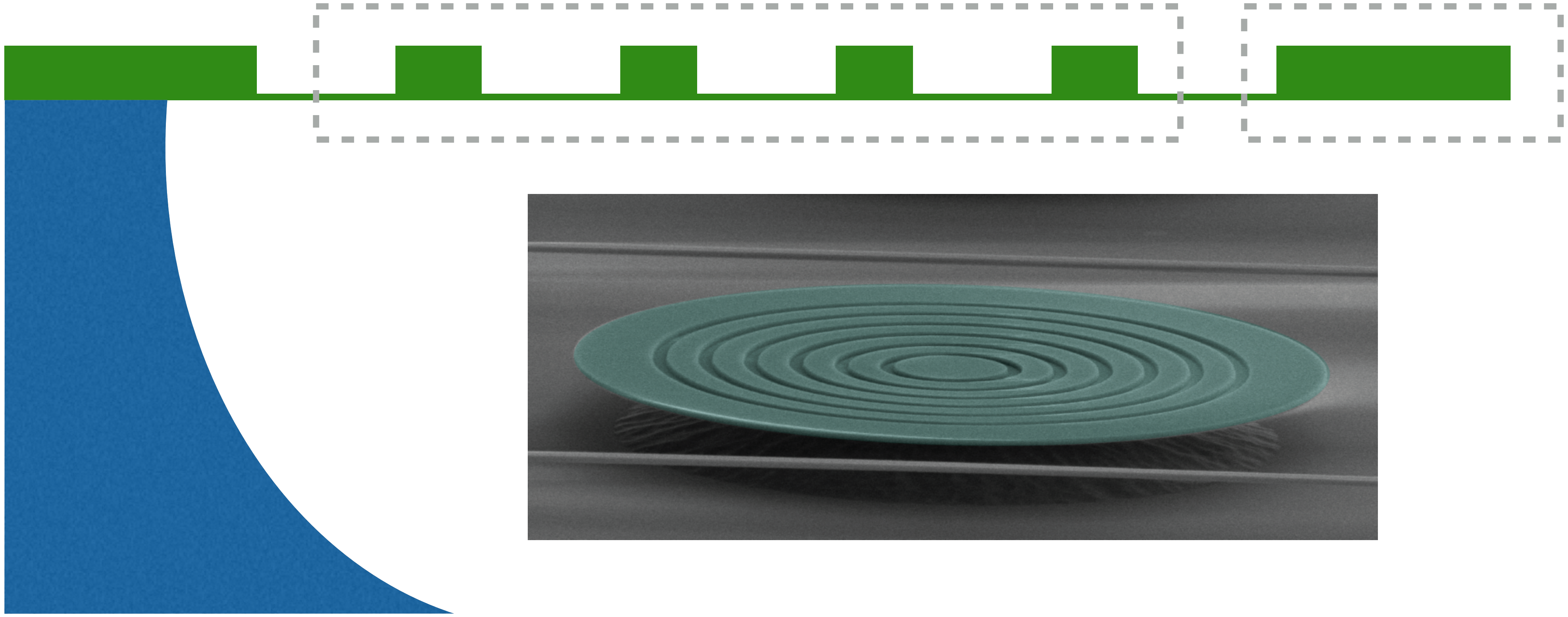
$\Omega_m \uparrow$



$\kappa_t / 2\pi \approx 500$ MHz $\omega_m / 2\pi \approx 4$ GHz

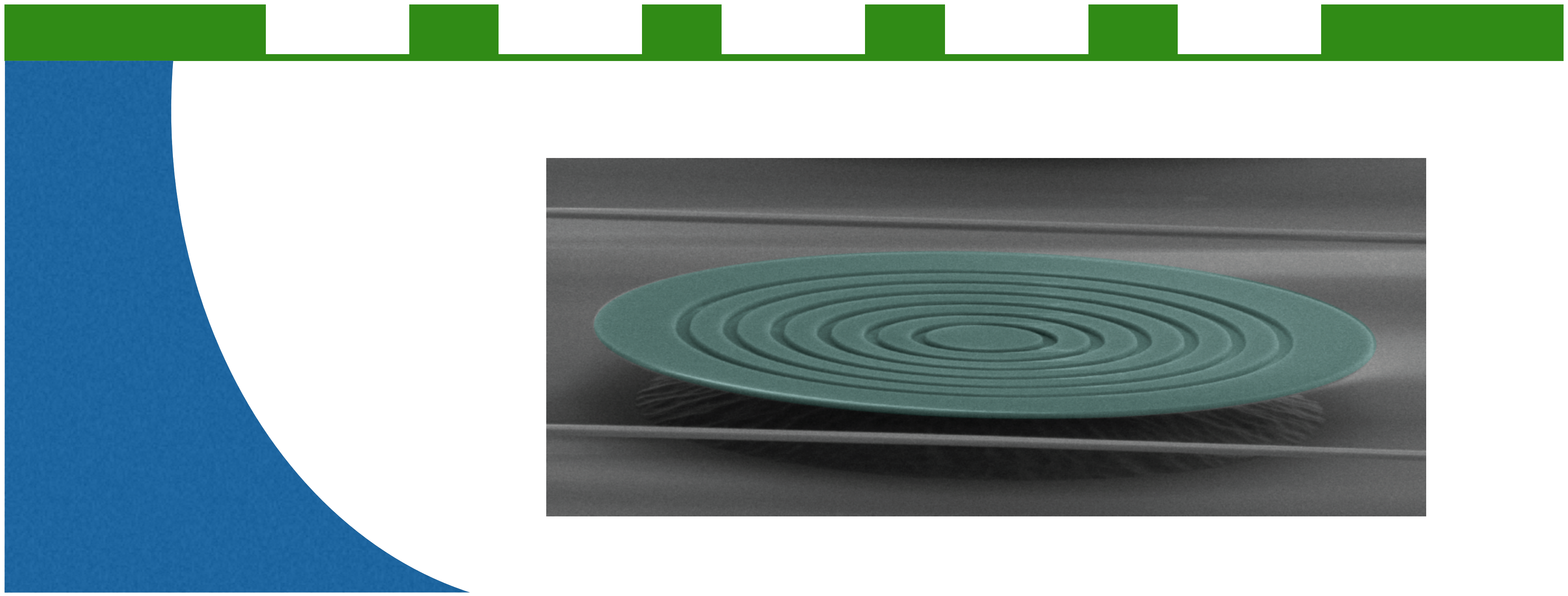
J. Chan et. al, Nature **478**, 89 (2011)

Optical Design

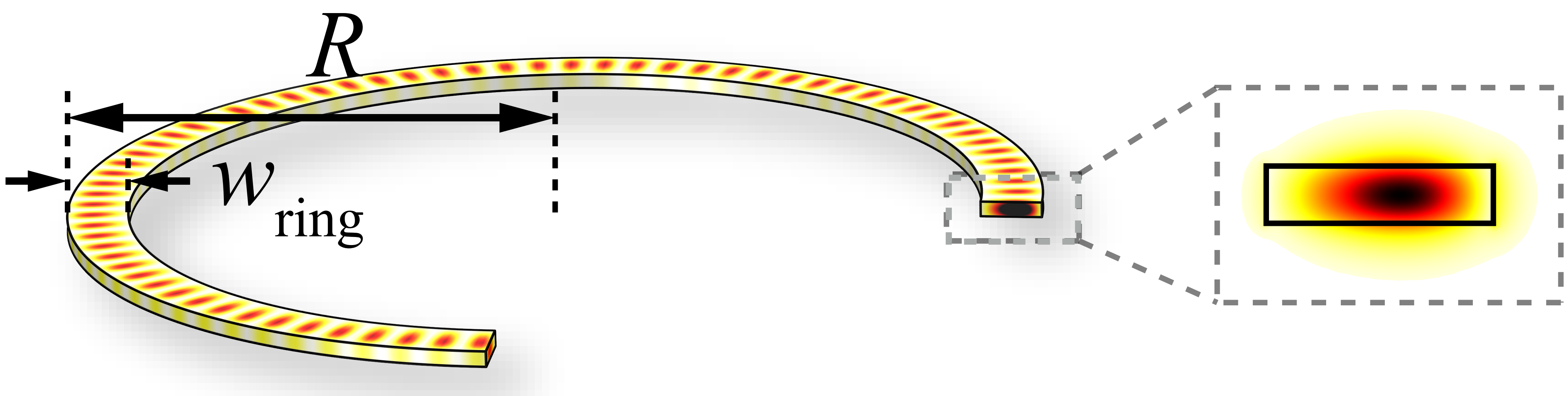


Santos, F. G., et al. arXiv:1605.06318

Optical Design

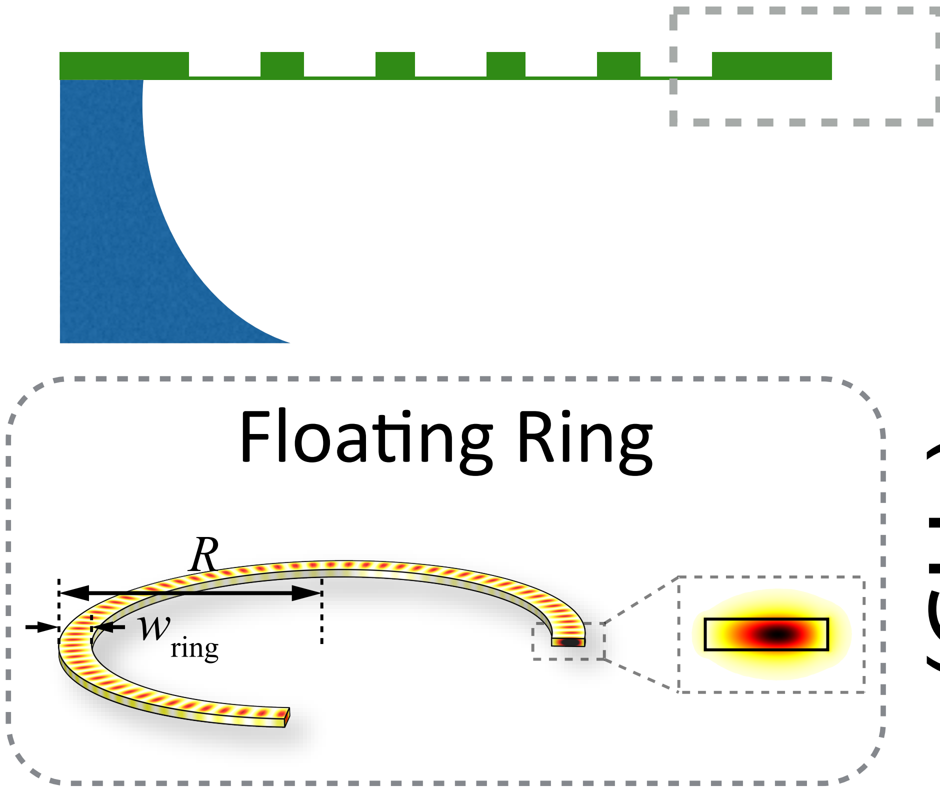


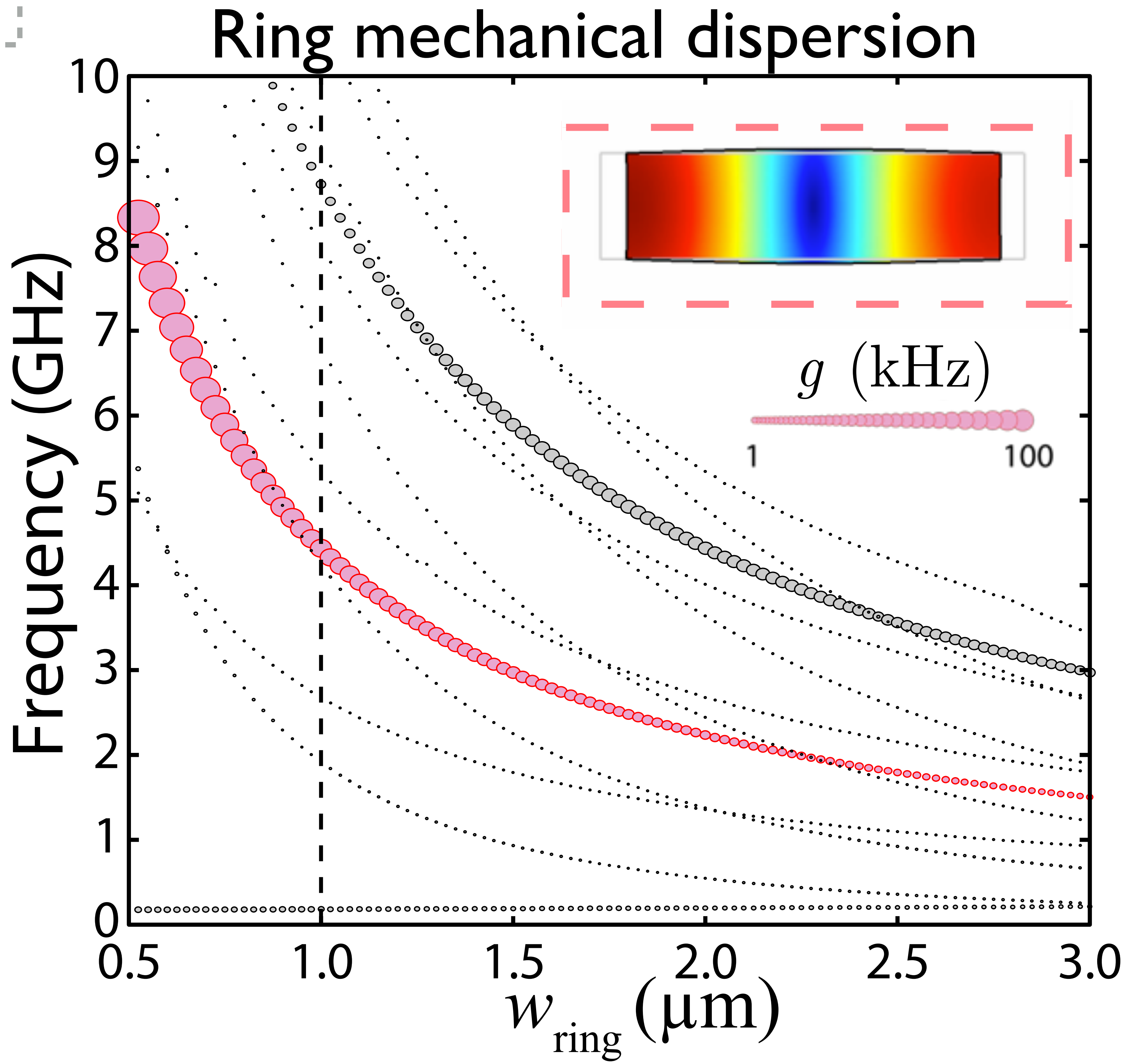
Floating ring resonator



Santos, F. G., et al. arXiv:1605.06318

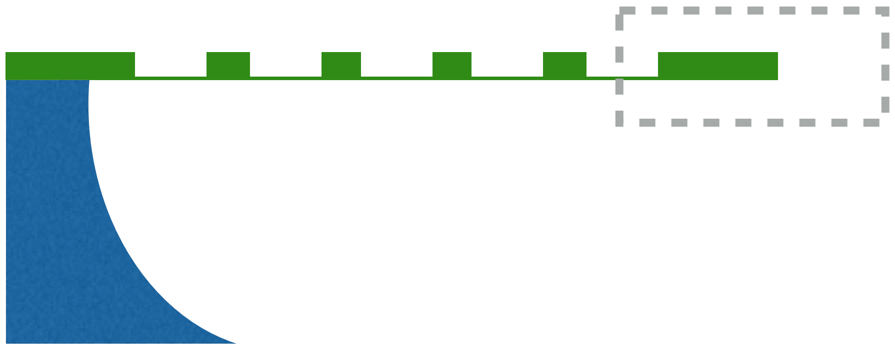
Mechanical Design



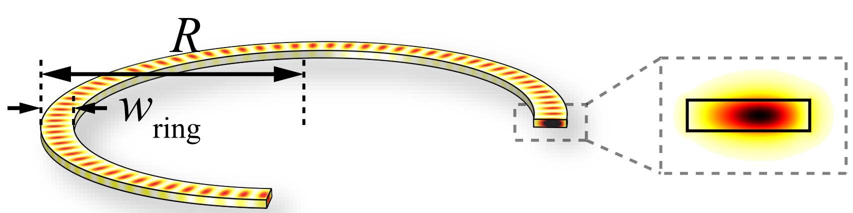


Santos, F. G., et al. arXiv:1605.06318

Mechanical Design

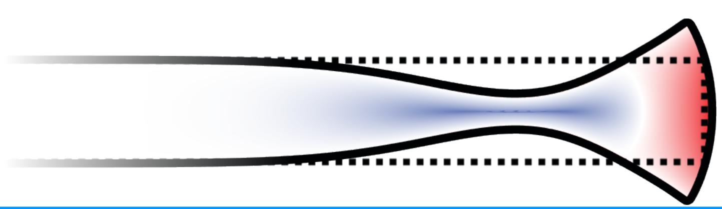


Floating Ring



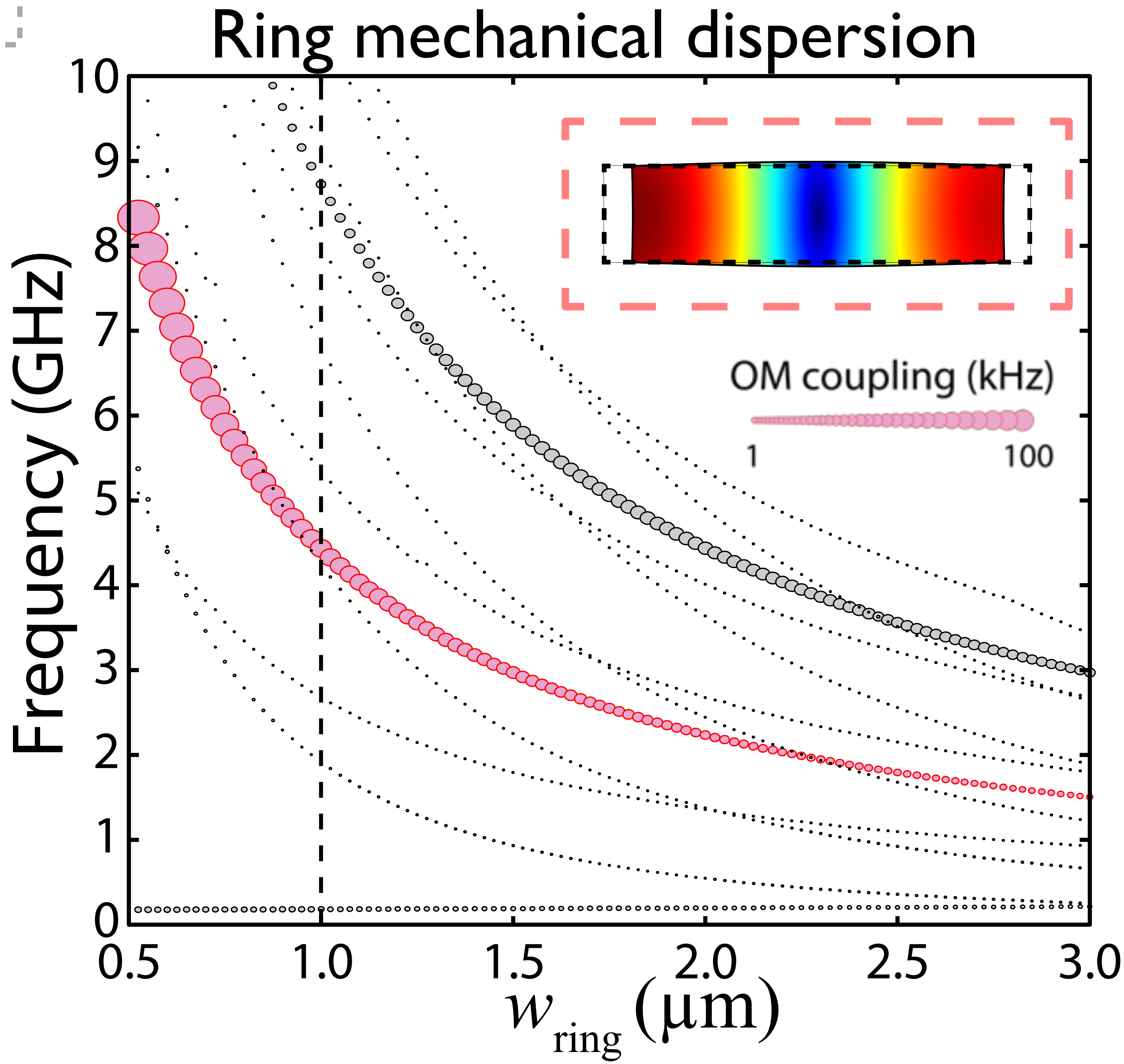
High optomechanical coupling

Photoelastic Effect

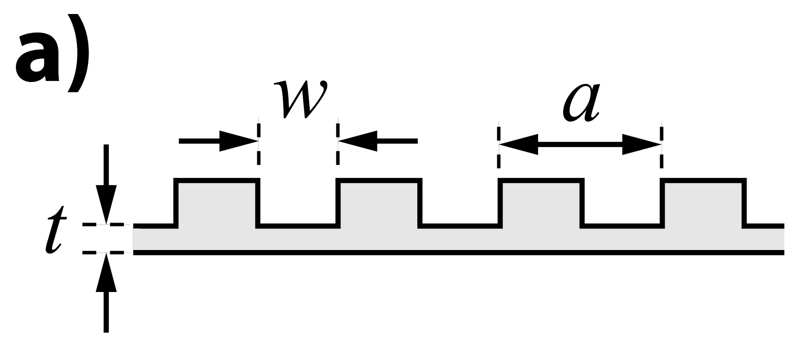


Tailorable mechanical frequency

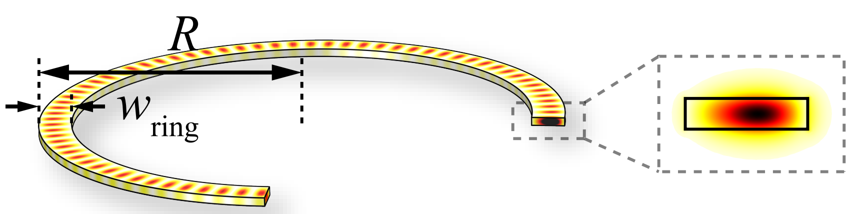
Santos, F. G., et al. arXiv:1605.06318



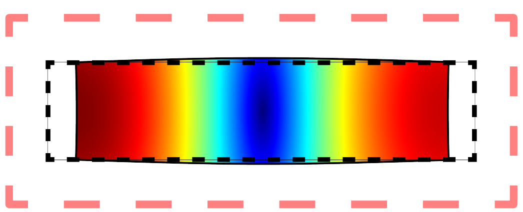
Mechanical Gratting



Floating Ring



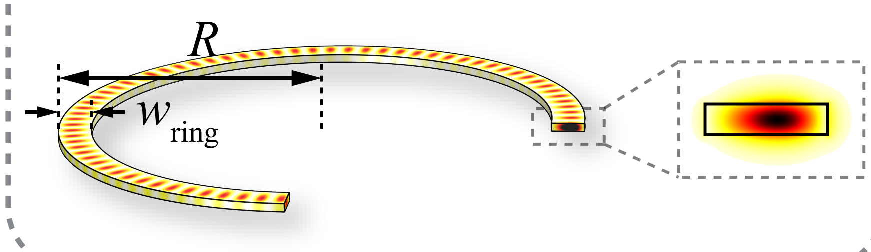
Tailorable Frequency



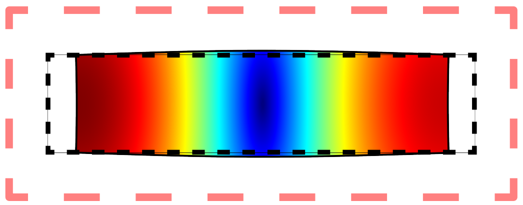
Santos, F. G., et al. arXiv:1605.06318

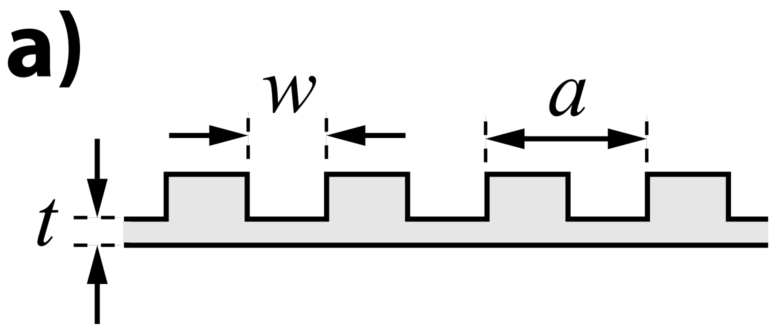
Mechanical Gratting

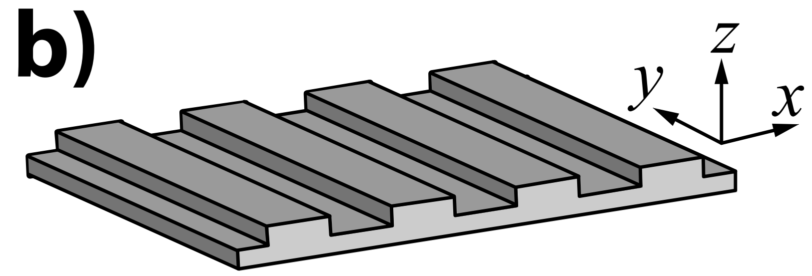
Floating Ring



Tuning Mechanical Frequency

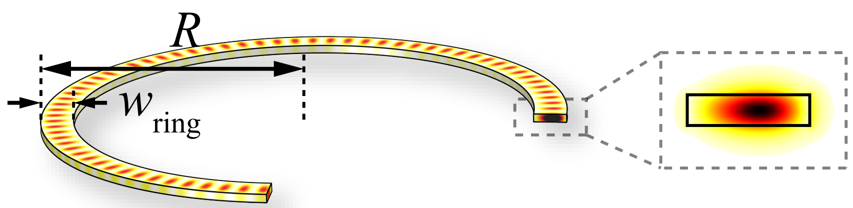


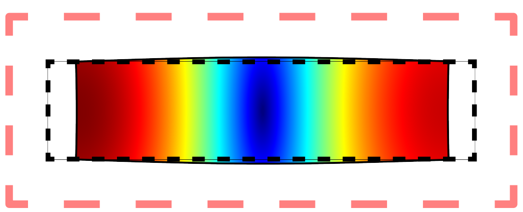


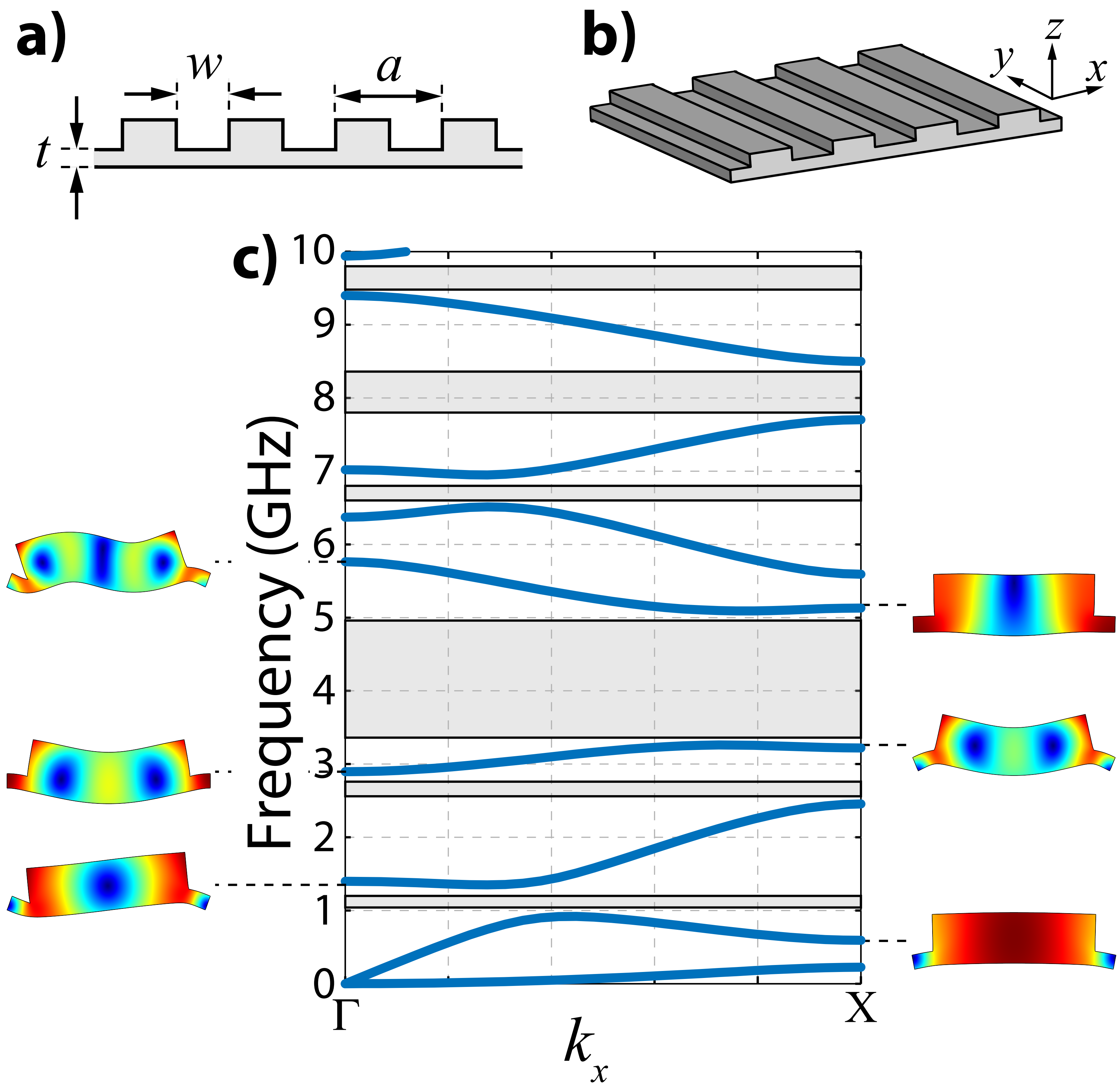


Santos, F. G., et al. arXiv:1605.06318

Mechanical Gratting



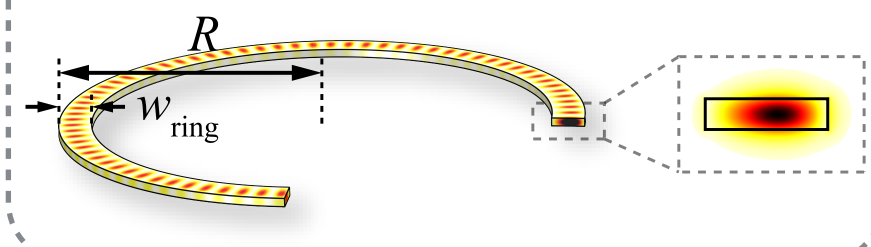




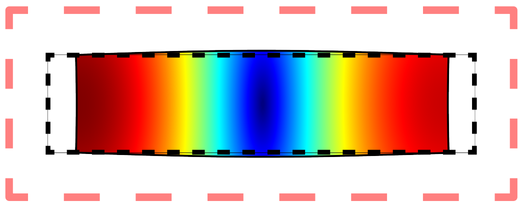
Santos, F. G., et al. arXiv:1605.06318

Mechanical Gratting

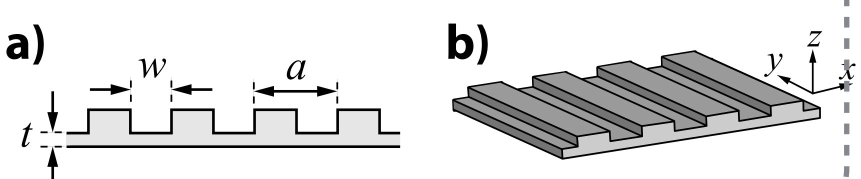
Floating Ring



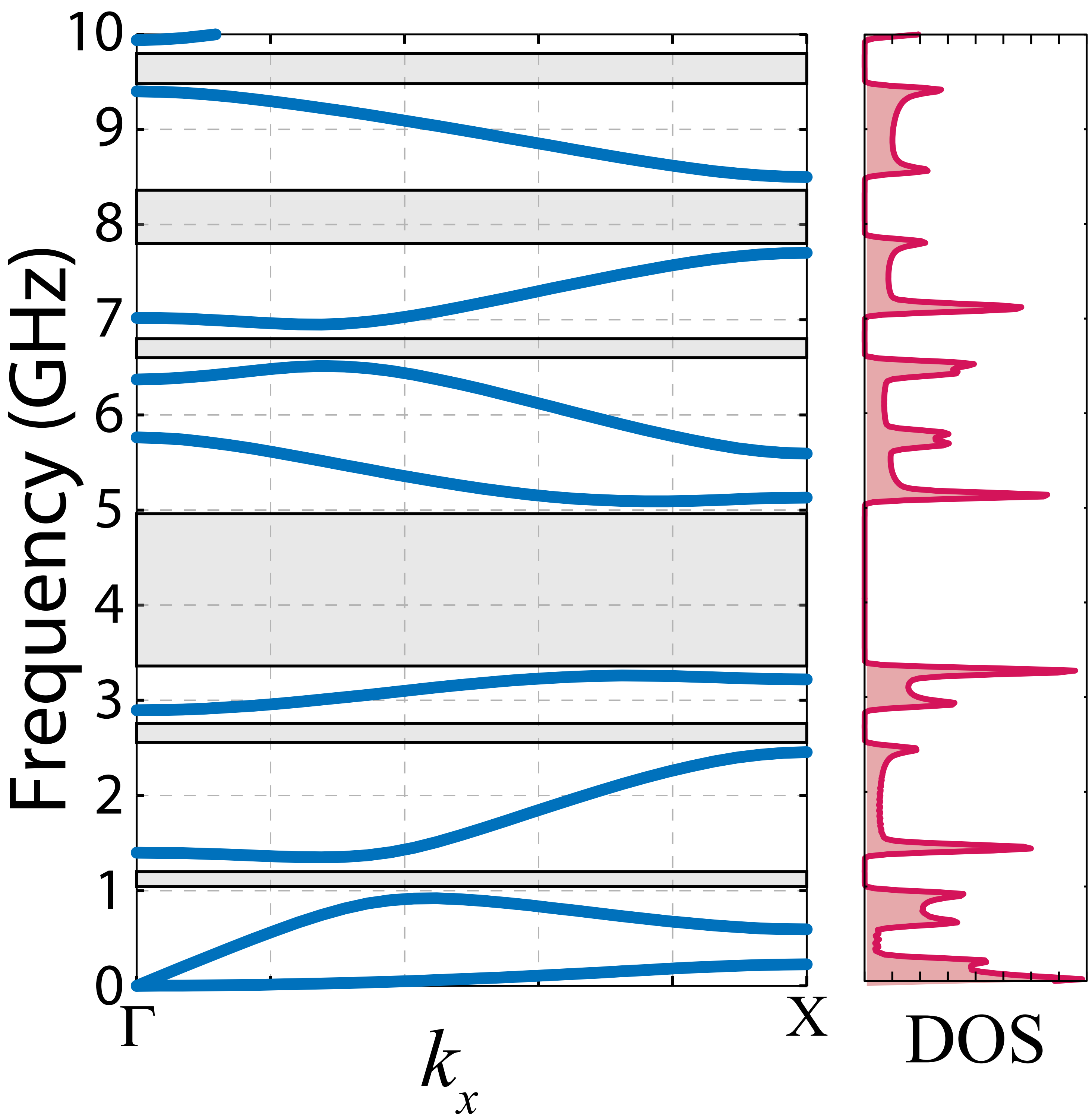
Tuning Mechanical Frequency



1D - Bragg

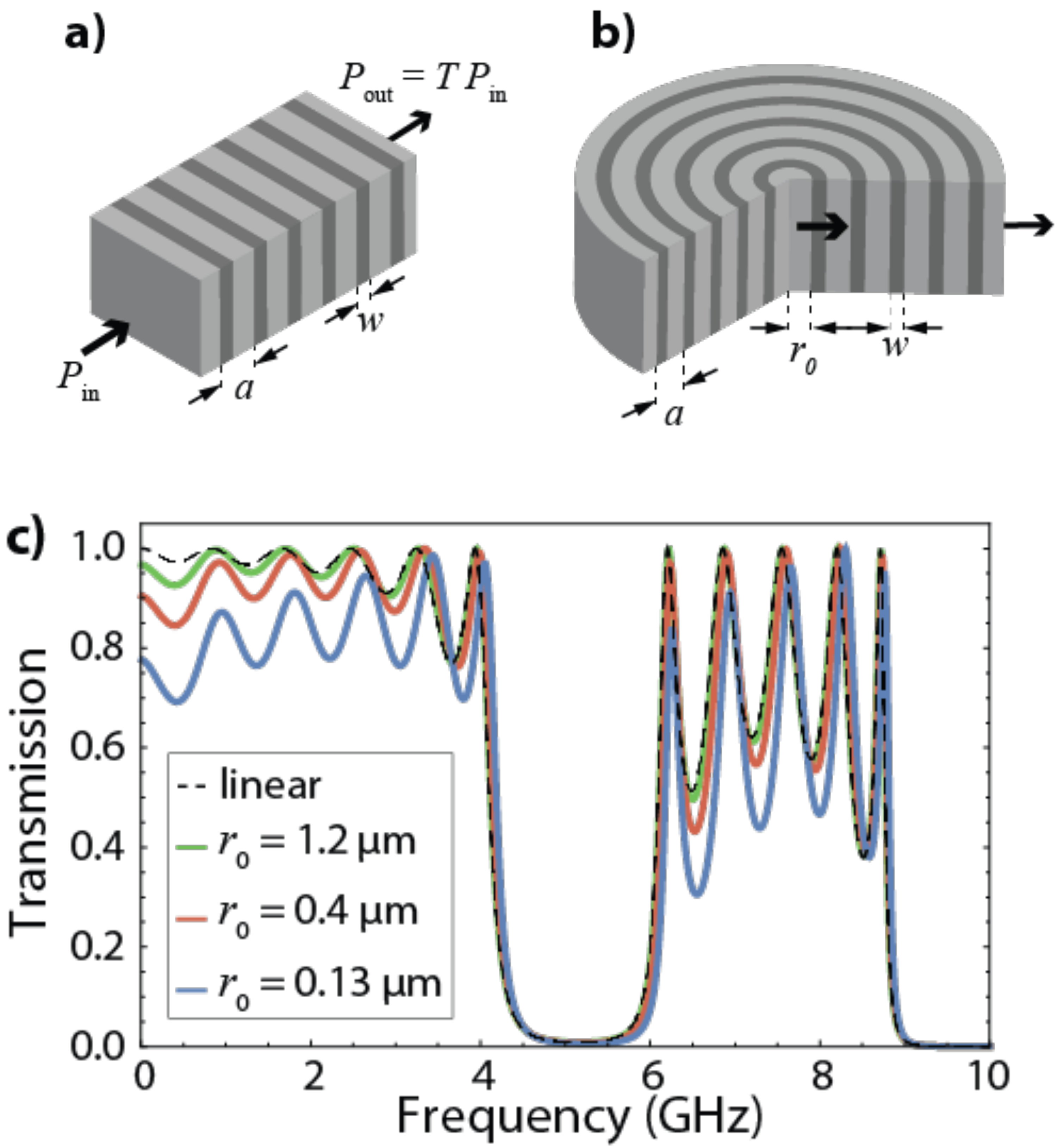


Bandgap vs lattice parameter



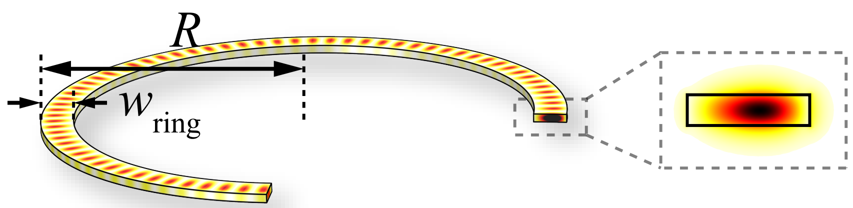
Santos, F. G., et al. arXiv:1605.06318

Acoustic linear grating?

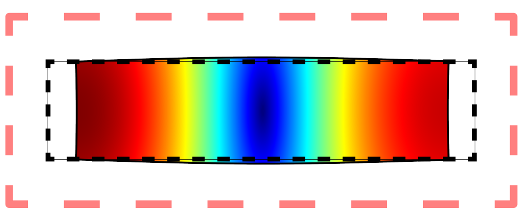


Mechanical Gratting

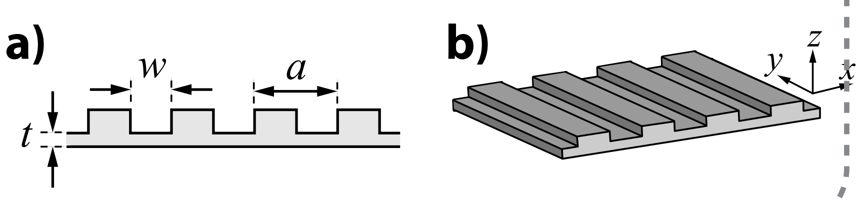
Floating Ring



Tuning Mechanical Frequency

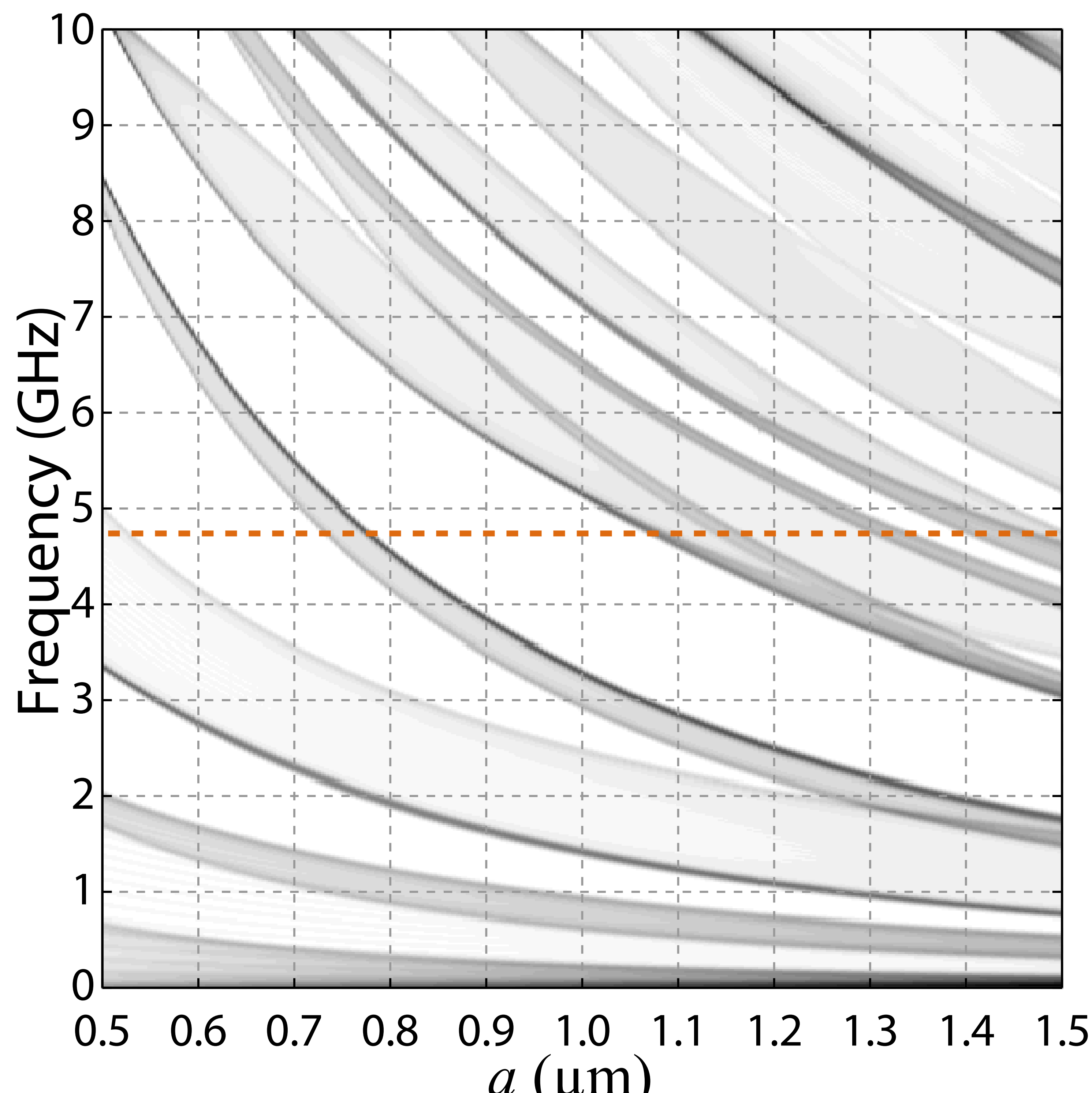


1D - Bragg



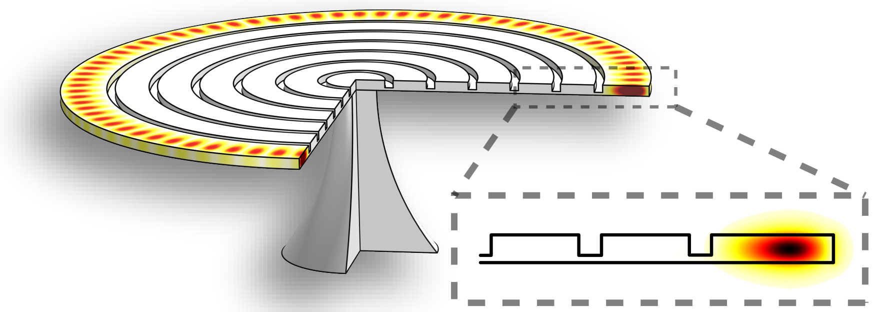
$$v_{ac} = 9600 \text{ m/s}$$
$$w_{ring} = 1\ \mu m$$
$$f_{ac} = \frac{v_{ac}}{2w_{ring}} \approx 4.8 \text{ GHz}$$

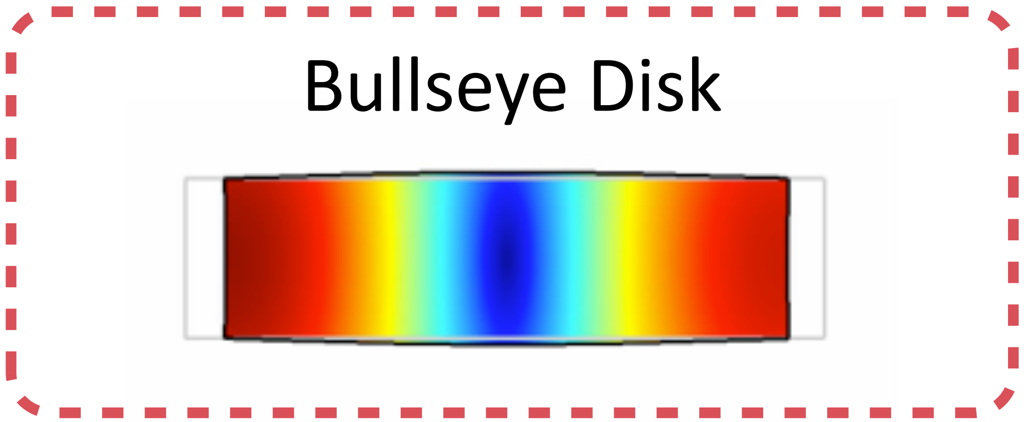
Bandgap DOS vs lattice parameter

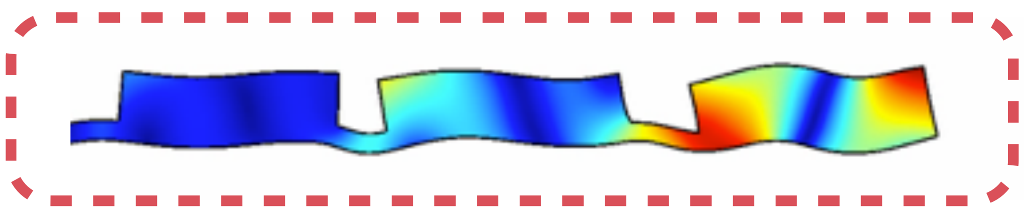


Mechanical Gratting

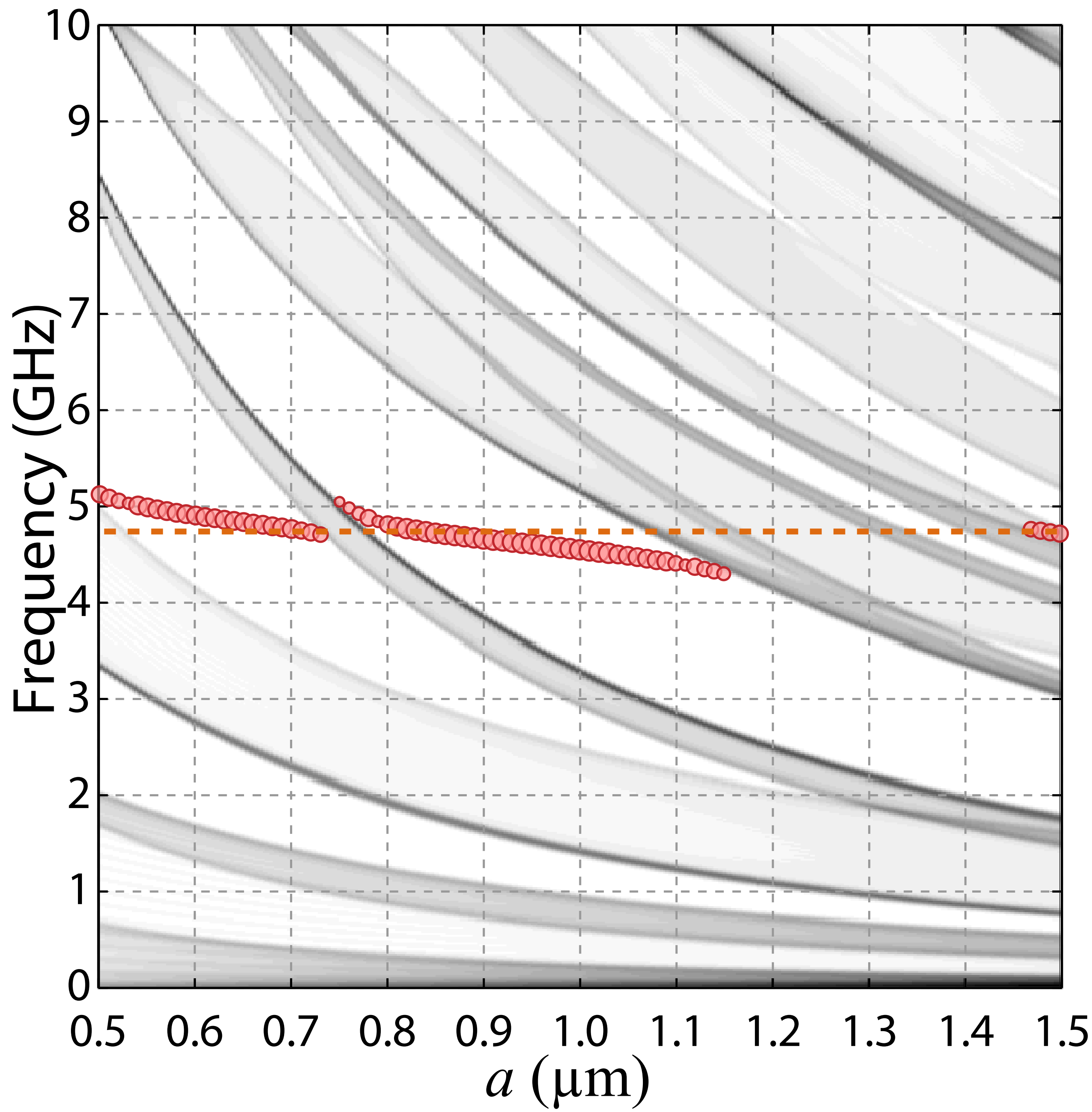
Bullseye Disk





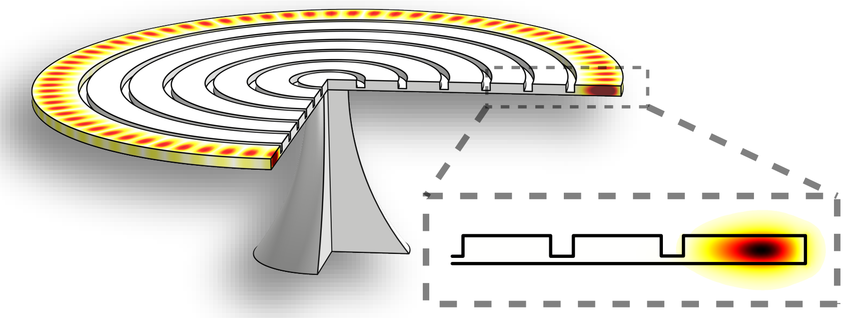


Bandgap DOS vs lattice parameter

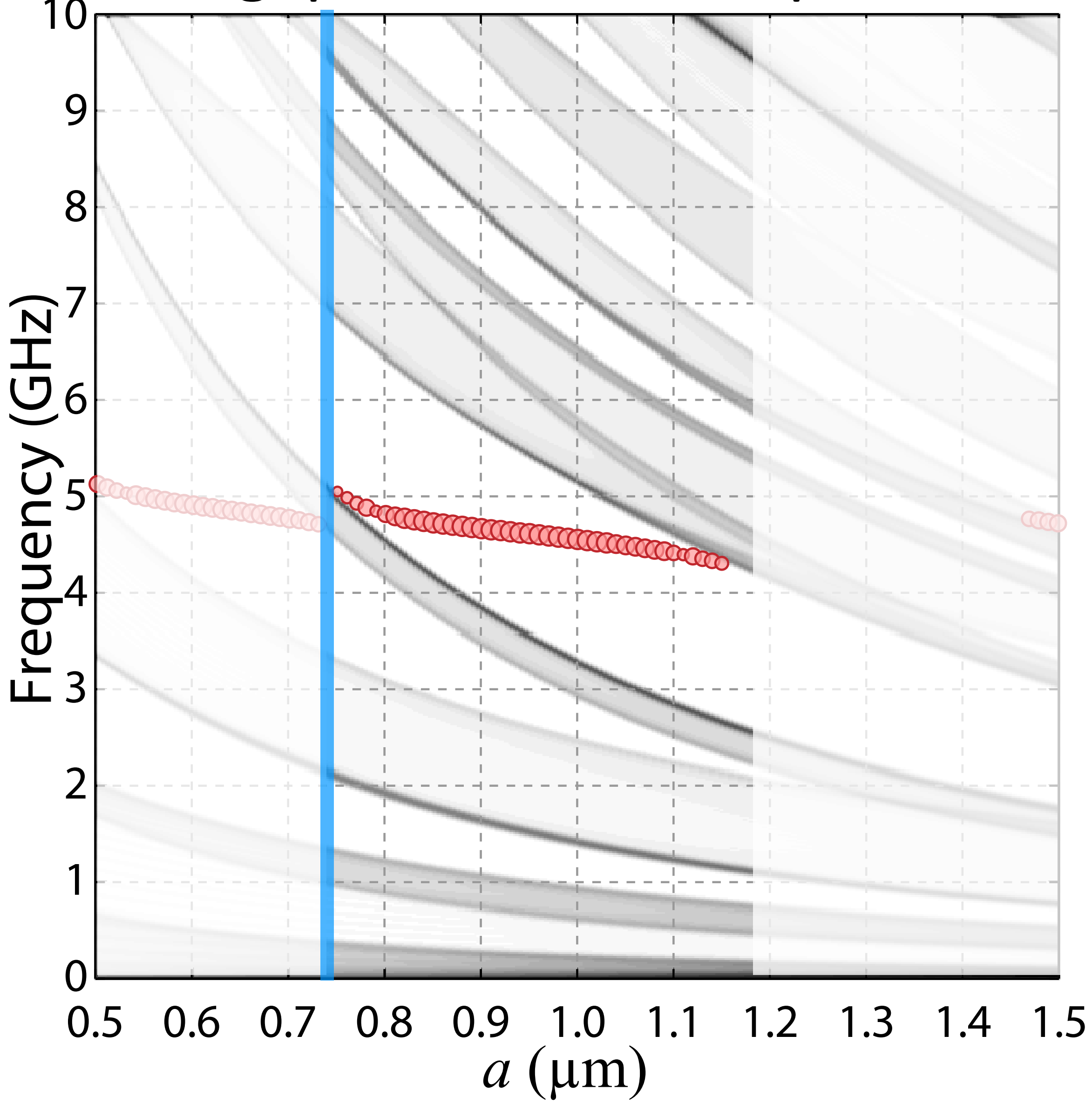


Mechanical Gratting

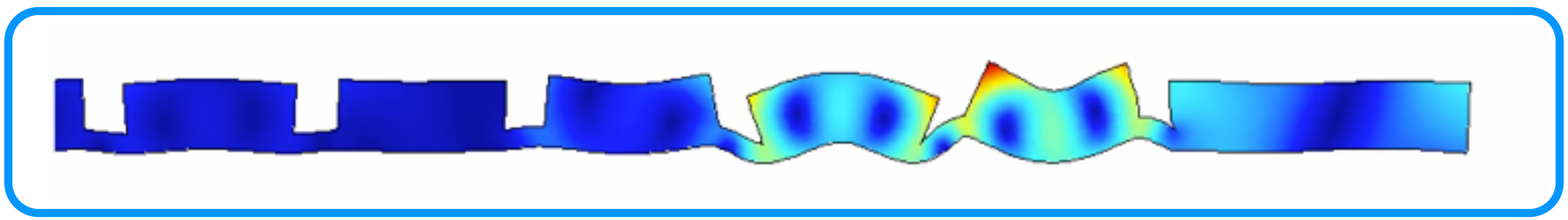
Bullseye Disk



Bandgap DOS vs lattice parameter



Simulated mode profile

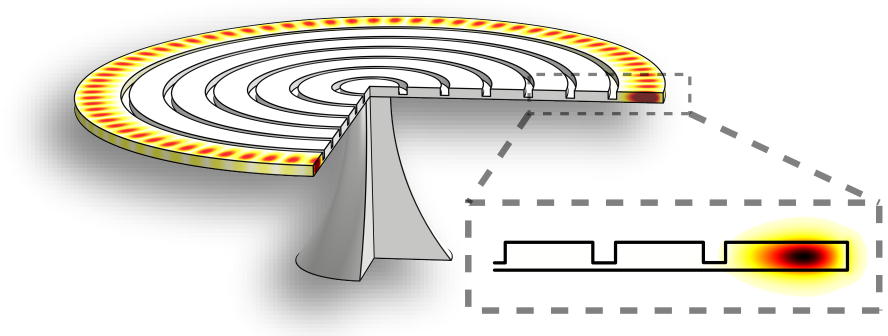


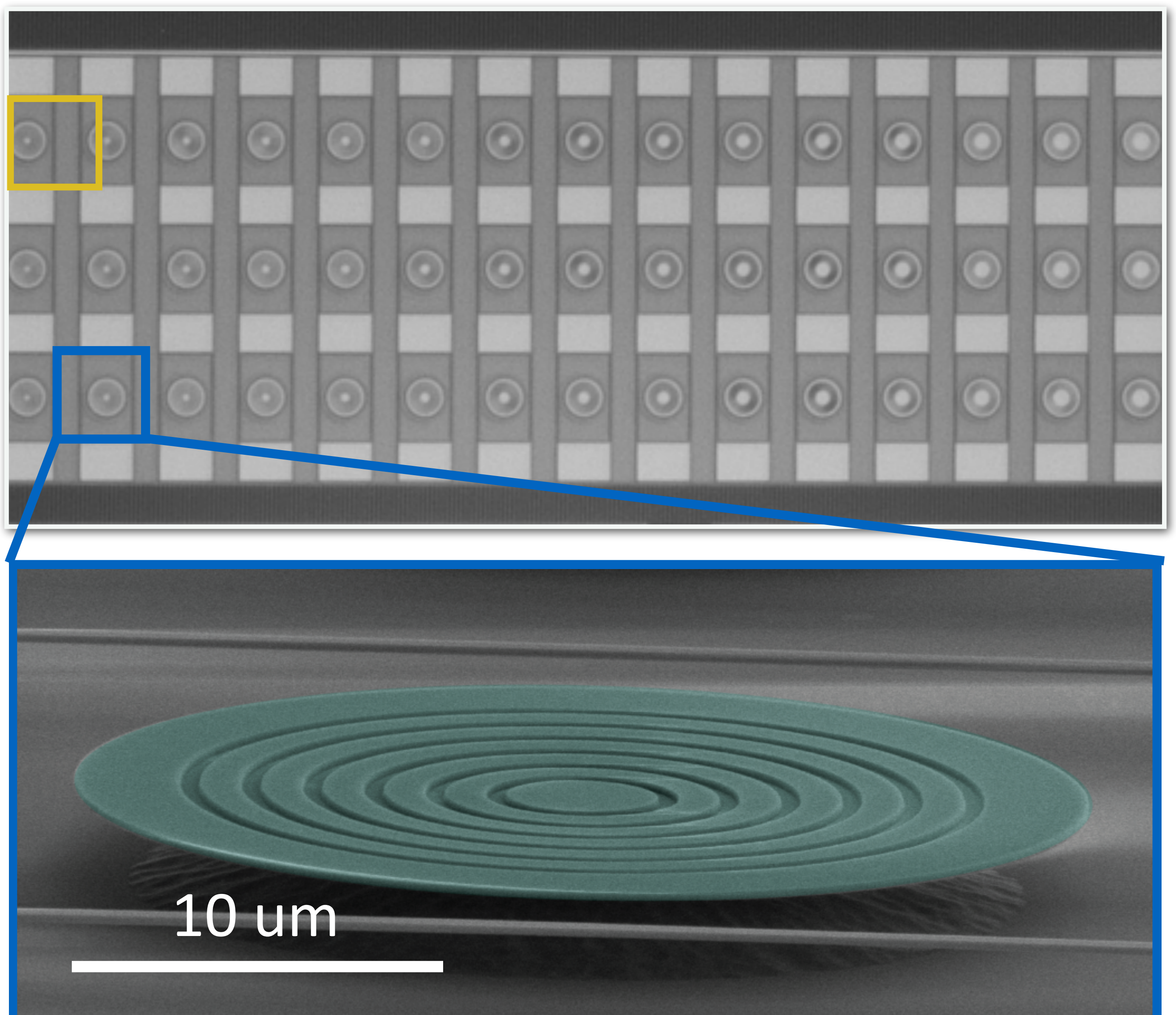
Bullseye fab: foundry based

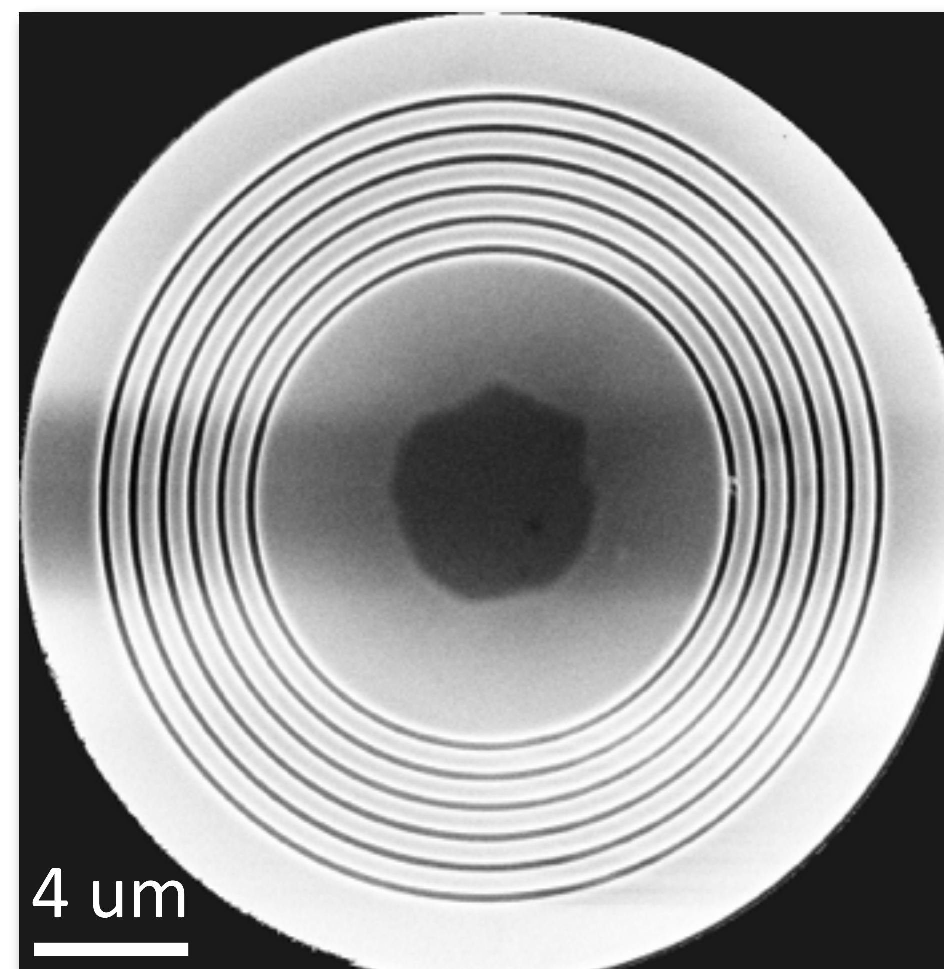
Integrated and scalable production

Passive SiPhotonics imec-ePIXfab

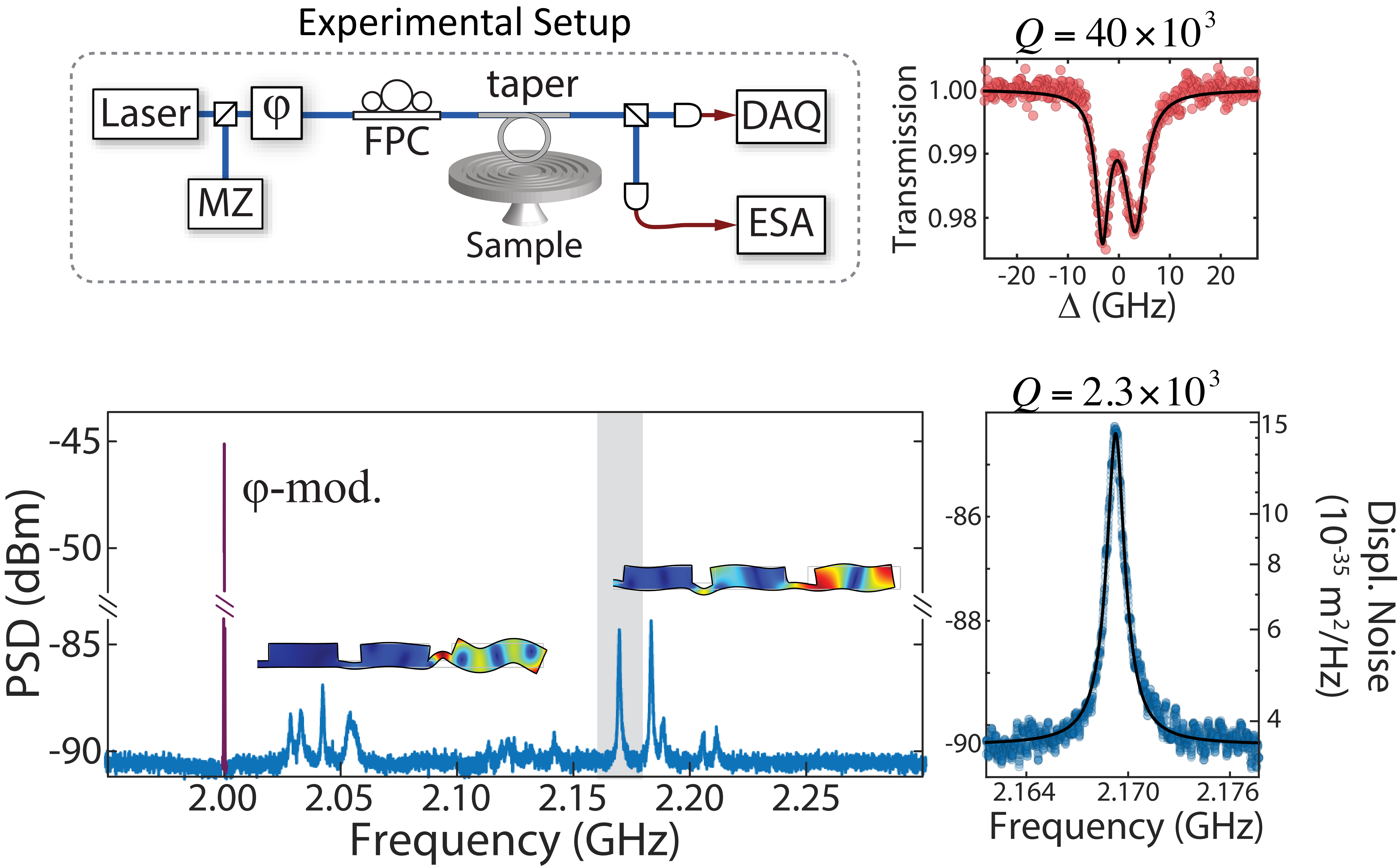
Bullseye Disk





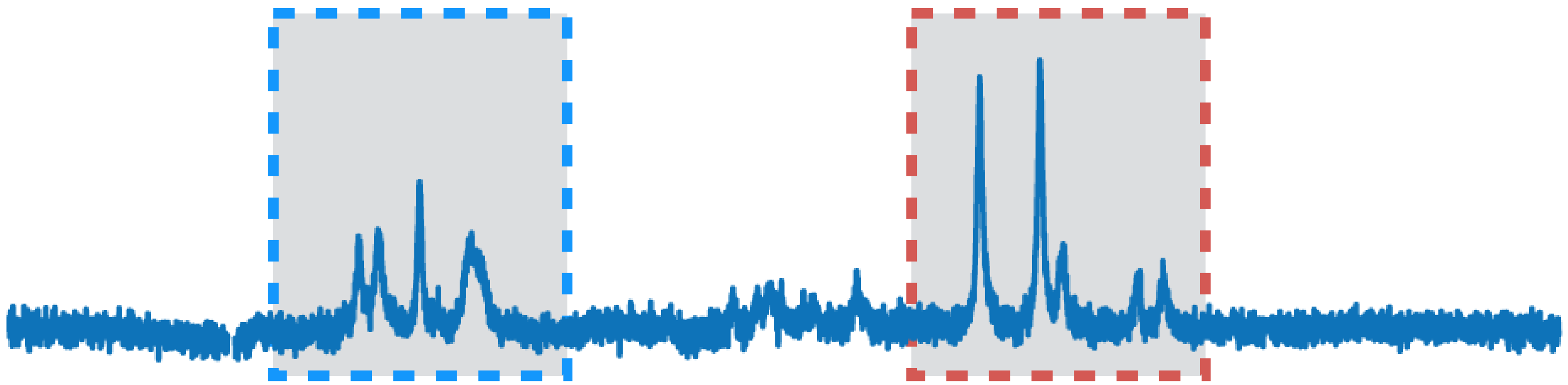


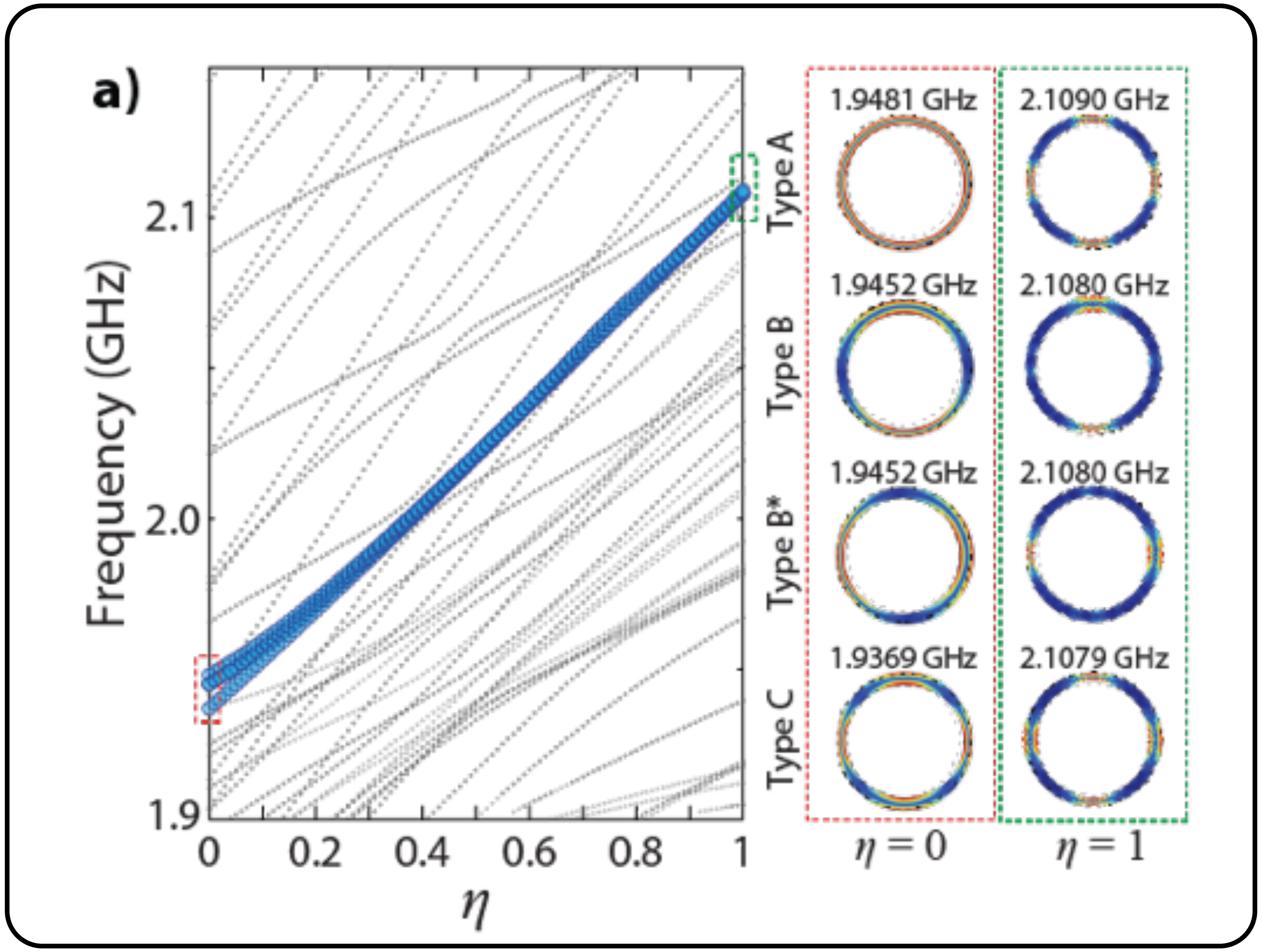
Mechanical mode testing



Santos, F. G., et al. arXiv:1605.06318

Multi-peak structure

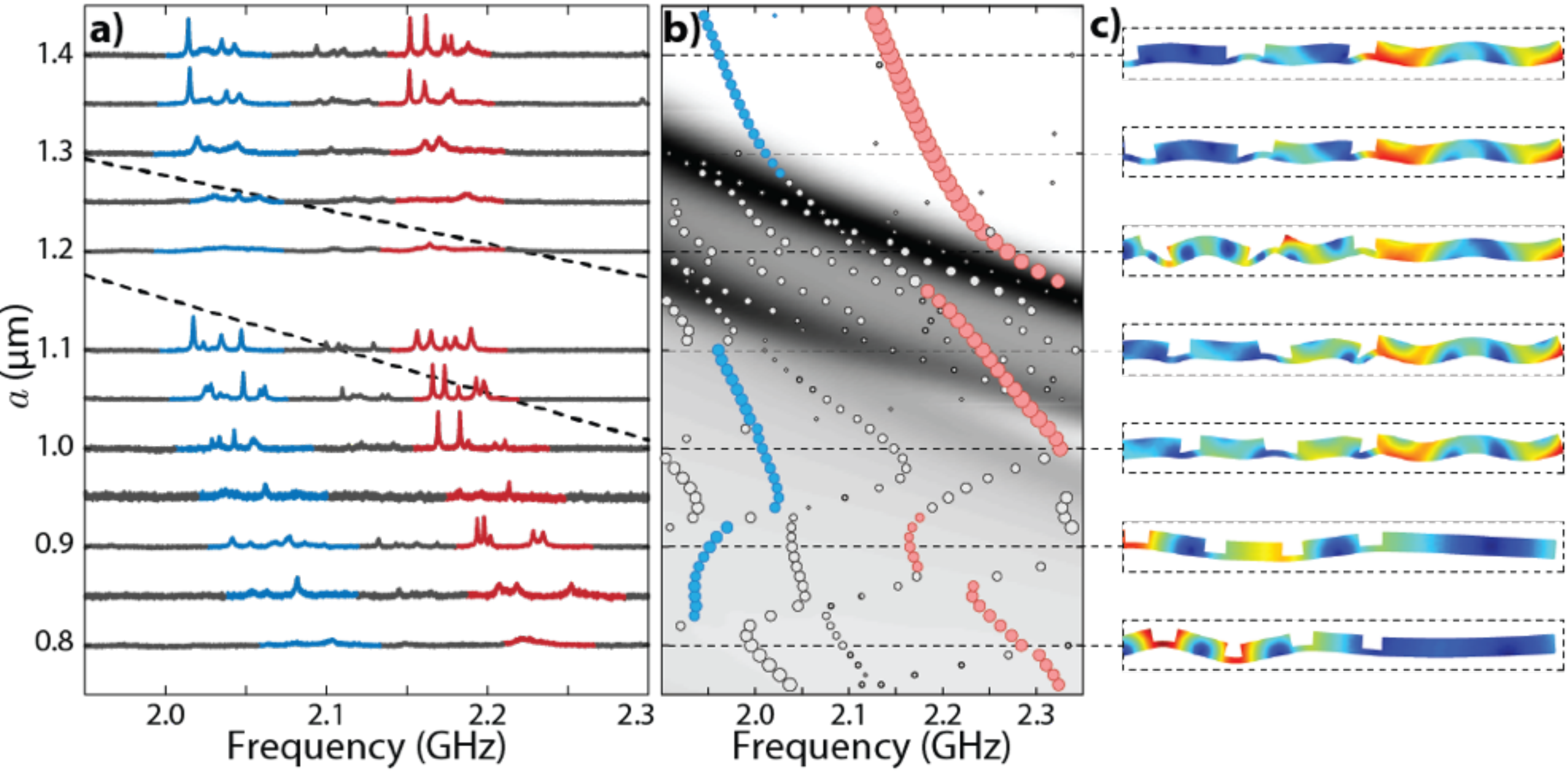




Si Anisotropy

Santos, F. G., et al. arXiv:1605.06318

Mechanical mode testing



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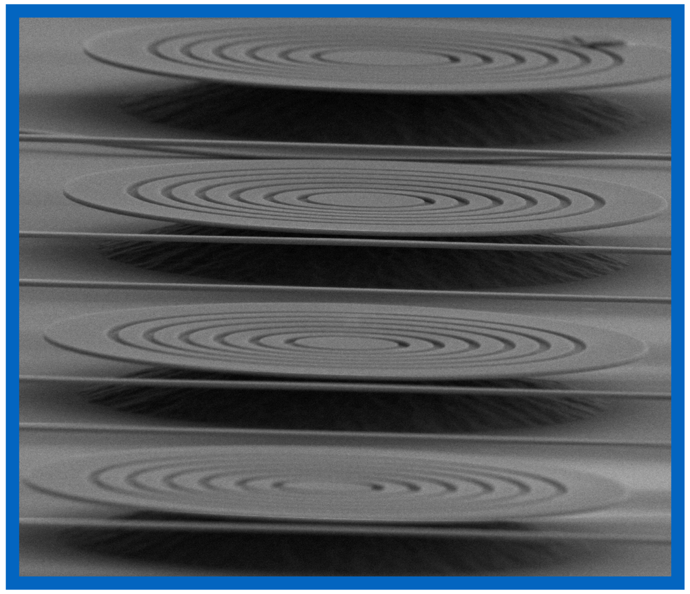
Perspective on Bull's eye arrays



Foundry based sample

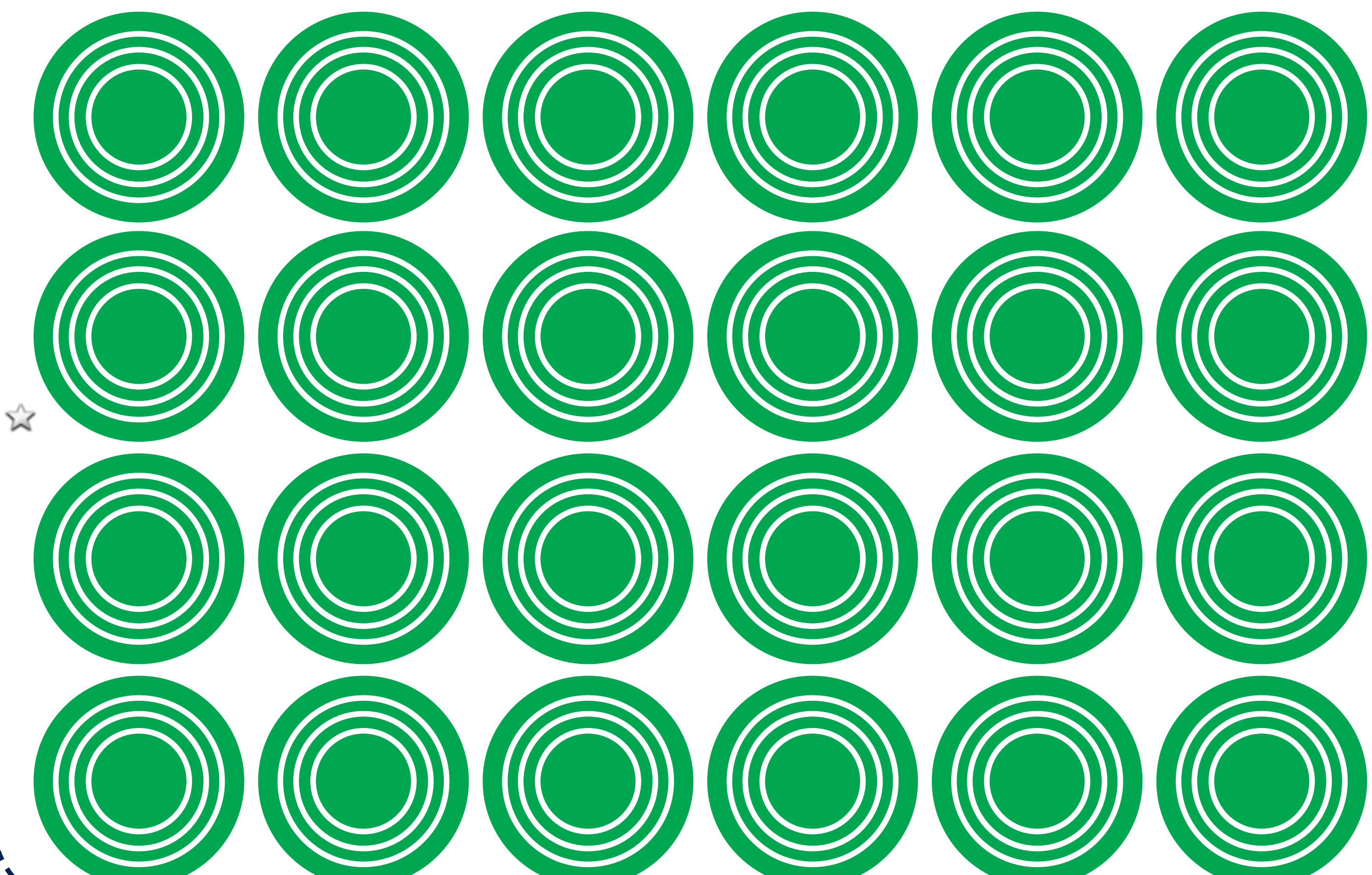
- Moderate optical damping rate (Q~100,000)
- High mechanical Q's (2300 @ 300K)
- Array-scalable
- Design flexible to different materials

Independent confinement control

Mechanical: radial gratting

Optical: total internal reflection

Array-scalable



Outlook

Florian Marquardt, 2014

Outlook

Linear Optomechanics

- Displacement detection
- Optical Spring
- Cooling & Amplification
- Two-tone drive: "Optomechanically induced transparency"
- Ground state cooling
- State transfer, pulsed operation
- Wavelength conversion
- Radiation Pressure Shot Noise
- Squeezing of Light
- Squeezing of Mechanics
- Light-Mechanics Entanglement
- Accelerometers
- Single-quadrature detection, Wigner density
- Optomechanics with an active medium
- Measure gravity or other small forces
- Mechanics-Mechanics entanglement
- Pulsed measurement
- Quantum Feedback
- Rotational Optomechanics

Multimode

- Mechanical information processing
- Bandstructure in arrays
- Synchronization/patterns in arrays
- Transport & pulses in arrays

Nonlinear Optomechanics

- Self-induced mechanical oscillations
- Attractor diagram?
- Synchronization of oscillations
- Chaos

○ White: yet unknown challenges/goals

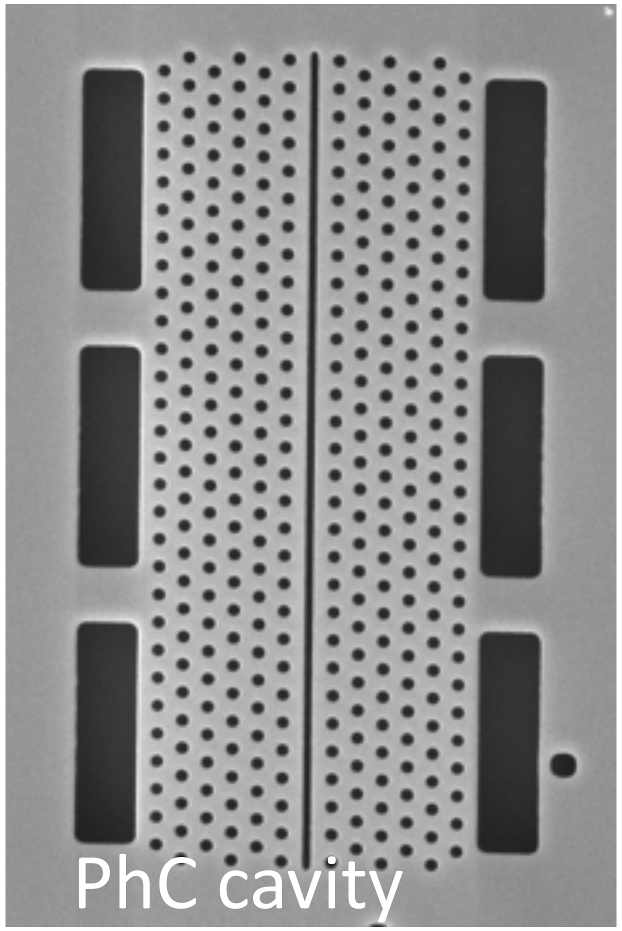
Nonlinear Quantum Optomechanics

- QND Phonon number detection
- Phonon shot noise
- Photon blockade
- Optomechanical "which-way" experiment
- Nonclassical mechanical q. states
- Nonlinear OMIT
- Noncl. via Conditional Detection
- Single-photon sources
- Coupling to other two-level systems
- Optomechanical Matter-Wave Interferometry

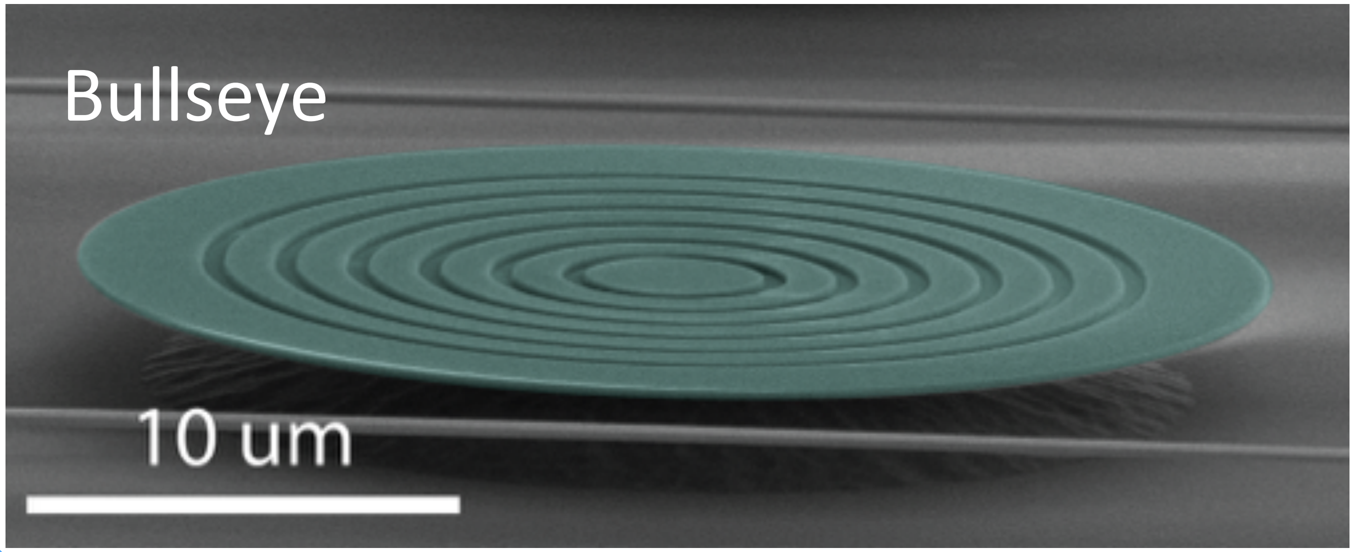
Florian Marquardt, 2014

Outlook

High Q crystal cavities

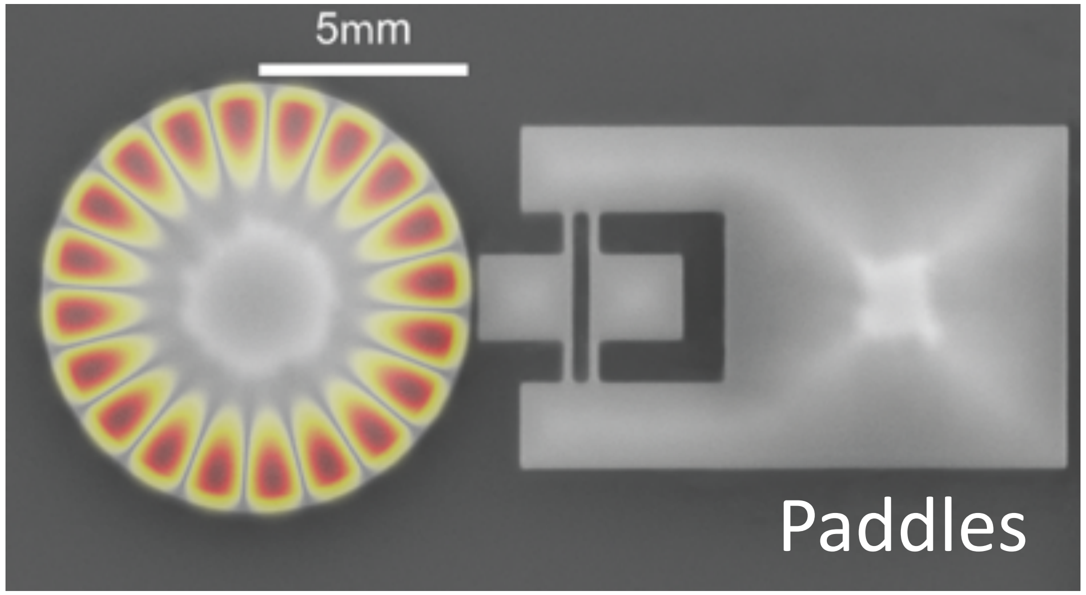


Large optomechanical coupling on disk-like structure



Third harmonic generation & Frequency combs

Suppression of mechanical radiation loss



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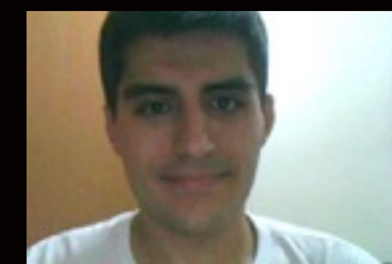
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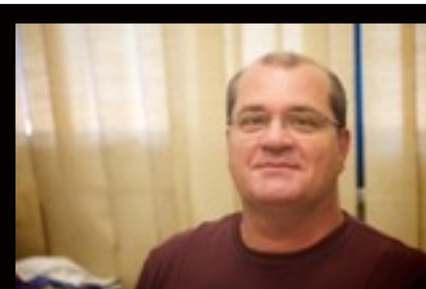
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